2.4 Decision-Making Under Uncertainty

(Single-Stage Decision with decision criteria that DO NOT use probabilities)

Under *complete uncertainty*, the decision maker either is unable to estimate the probabilities for the occurrence of the different states of nature or he or she lacks confidence in available estimates of probabilities. For that reason, probabilities are not included in the analysis.

When using *decision making under uncertainty* to analyze a situation, the decision criteria are based on the decision maker's towards life. The five criteria for decision-making under uncertainty are as follows:

1. Maximax.
2. Maximin.
3. Equally likely.
4. Criterion of realism.
5. Minimax regret.

The first four criteria can be analyzed directly from the decision table (or payoff table), while the minimax criterion requires the use of an *opportunity loss table*.

1. Maximax

The *maximax* approach is an *optimistic* approach. An optimistic decision maker feels that luck is always shining and whatever decision is made, the best possible state of nature corresponding to that decision will occur. Given a payoff table, the maximax criterion finds an alternative that *max*imize the *max*imum payoff for every alternative. First, locate the maximum payoff within every alternative, and then pick an alternative with the maximum number. Since this criterion locates the alternative with the *highest possible gain*, it has been called an *optimistic decision criterion*.

2. Maximin

The *maximin* strategy is a *conservative* or *pessimistic* approach. In contrast to an optimistic decision maker, a pessimistic decision maker believes that, no matter what decision is made, the worst possible result will occur. Given a payoff table, this criterion finds an alternative that *max*imizes the *min*imum payoff for every alternative. First, locate the minimum payoff within every alternative and then pick an alternative with the maximum number. Since this criterion locates the alternative that has the *least possible loss*, it has been called a *pessimistic decision criterion*.

3. Equally Likely

The *equally likely* criterion offers a method that incorporates more of the information. It treats the states of nature as if each was equally likely, and it focuses on the *average payoff* for each alternative, selecting the alternative that has the *highest average*. Given a payoff table, this decision criterion (also called *Laplace*) finds an alternative with the highest average payoff. First, calculate the average payoff for every alternative and then pick an alternative with the *maximum average*.

4. Criterion of Realism

The *criterion of realism*, also called the *Hurwicz criterion*, often referred as the *weighted average*, which offers the decision maker a compromise between an optimistic (maximax) and a pessimistic (maximin) criteria. This approach requires the decision maker to specify a *degree of optimism*, in the form of a *coefficient of optimism* or *realism*, α. The value of this coefficient is between 0 and 1. To begin with, a value of α is selected. When α is close to 1, the decision maker is optimistic about the future, and when α is close to 0, the decision maker is pessimistic about the future. This approach allows the decision maker to build in personal feelings about relative optimism and pessimism. Given a payoff table, the criterion of realism for each alternative can be determined as follows:

Criterion of realism = α(maximum payoff) + (1-α)(minimum payoff).

This criterion selects an alternative with the *highest criterion of realism* (weighted average).

5. Minimax Regret

The *minimax regret* is another approach that takes all payoffs into account, as in equally likely. Like maximin, this is also a *pessimistic* approach. This decision criterion involves calculating the regret or *loss opportunity* corresponding to each payoff. An *opportunity loss table* is required. The general format of an opportunity loss table is illustrated in Table 2.2. This table shows, for each possible state of nature, the *shortfall in profit* that would arise from making the *wrong decision*. Hence, the opportunity loss reflects the difference between each payoff and the best possible payoff for a given state of nature. Given an opportunity loss table, the minimax criterion finds the alternative that *mini*mizes the *max*imum opportunity loss within each alternative. First, find the maximum opportunity loss within each alternative, and then pick an alternative with the *minimum number* (regret).

Table 2.2 General Format of an Opportunity Loss Table (3×3)

Opportunity Losses		State of Nature		
		S_1	S_2	S_3
Alternative	A_1	l_{11}	l_{12}	l_{13}
	A_2	l_{21}	l_{22}	l_{23}
	A_3	l_{31}	l_{32}	l_{33}

Where l_{ij} is the value of opportunity loss that will be realized if alternative i is chosen and state of nature j occurs.

Example 2.1

Syed owns a bookstore. At the beginning of each week, he must decide how many copies of a weekly magazine he should order. The weekly demand has been categorized into five levels; 100, 150, 200, 250, and 300 copies. Each copy of the magazine costs $4 and sells at $6. If there are copies of the magazine left unsold at the end of the week, Syed can reclaim $3 each from the publisher. Syed also feels that it is a loss of $0.50 for each demand of the magazine that is not met as a result of loses of future sales.

Syed's possible *alternatives* are to order 100, 150, 200, 250, or 300 copies each week. The possible *states of nature* (or outcomes) are that the demand will be for 100, 150, 200, 250, or 300 copies per week. Now we can construct a decision table or payoff table, which gives the payoff, or monetary gain, for each possible combination of alternatives and states of nature. The payoffs for all combinations of alternatives and states of nature are illustrated in Table 2.3.

Table 2.3 Decision or Payoff Table for Example 2.1

Payoffs		*State of Nature*				
		100	150	200	250	300
Alternative	100	100(6-4) = **200**	100(6-4) – 50(0.5) = **175**	100(6-4) – 100(0.5) = **150**	100(6-4) – 150(0.5) = **125**	100(6-4) – 200(0.5) = **100**
	150	100(6-4) – 50(4-3) = **150**	150(6-4) = **300**	150(6-4) – 50(0.5) = **275**	150(6-4) – 100(0.5) = **250**	150(6-4) – 150(0.5) = **225**
	200	100(6-4) – 100(4-3) = **100**	150(6-4) – 50(4-3) = **250**	200(6-4) = **400**	200(6-4) – 50(0.5) = **375**	200(6-4) – 100(0.5) = **350**
	250	100(6-4) – 150(4-3) = **50**	150(6-4) – 100(4-3) = **200**	200(6-4) – 50(4-3) = **350**	250(6-4) = **500**	250(6-4) – 50(0.5) = **475**
	300	100(6-4) – 200(4-3) = **0**	150(6-4) – 150(4-3) = **150**	200(6-4) – 100(4-3) = **300**	250(6-4) – 50(4-3) = **450**	300(6-4) = **600**

1. **Maximax Criterion: (optimistic)**

For all possible alternatives in Table 2.3, the *maximum payoffs* are as follows:

Alternative	Maximum payoff ($)
100 copies	200
150 copies	300
200 copies	400
250 copies	500
300 copies	**600** ← maximum

Under maximax criterion, you would decide to order 300 copies of the magazine at the beginning of each week (since $600 is the highest possible gain). This is the *gambler's* approach.

2. **Maximin Criterion: (pessimistic)**

For all possible alternatives in Table 2.3, the *minimum payoffs* are as follows:

Alternative	Minimum payoff ($)
100 copies	100
150 copies	**150** ← maximum
200 copies	100
250 copies	50
300 copies	0

Under this criterion, you would decide to order 150 copies of the magazine at the beginning of each week. This is a *very cautious* decision criterion.

3. **Equally Likely Criterion (Laplace): (average)**

From Table 2.3, the *average payoffs* for all possible alternatives are as follows:

Alternative	Average payoff ($)
100 copies	(200+175+150+125+100)/5 = 150
150 copies	(150+300+275+250+225)/5 = 240
200 copies	(100+250+400+375+350)/5 = 295
250 copies	(50+200+350+500+475)/5 = **315** ← maximum
300 copies	(0+150+300+450+600)/5 = 300

Hence, under this criterion, you would decide to order 250 copies of the magazine at the beginning of each week ($315 is the *highest average* payoff).

4. **Criterion of Realism (Hurwicz Criterion): (weighted average)**

If, in Example 2.1, we assume that the coefficient of realism α to be **0.8**, the *criterion of realisms* for all alternatives are as follows:

Alternative	Criterion of realism ($)
100 copies	[(0.8)(200)+(0.2)(100)] = 180
150 copies	[(0.8)(300)+(0.2)(150)] = 270
200 copies	[(0.8)(400)+(0.2)(100)] = 340
250 copies	[(0.8)(500)+(0.2)(50)] = 410
300 copies	[(0.8)(600)+(0.2)(0)] = **480** ← maximum

Under this criterion, you would decide to order 300 copies of the magazine at the beginning of each week ($480 is the *highest weighted* average). This criterion gives the same result as in maximax since α is close to 1.

5. **Minimax Regret: (opportunity loss)**

For instance, in Example 2.1, if 100 copies were demanded, the best alternative would be to order 100 copies, giving a payoff of $200. However, if you had decided to order 150 copies, the payoff would be $150, which is $50 less than you would have received for this state of nature. This $50 is what is called the opportunity loss or *regret*. The opportunity losses for all combinations of alternatives and states of nature are illustrated in Table 2.4.

Table 2.4 Opportunity Loss Table for Example 2.1

Opportunity Losses		*State of Nature*				
		100	150	200	250	300
Alternative	100	**0**	300–175 = **125**	400–150 = **250**	500–125 = **375**	600–100 = **500**
	150	200–150 = **50**	**0**	400–275 = **125**	500–250 = **250**	600–225 = **375**
	200	200–100 = **100**	300–250 = **50**	**0**	500–375 = **125**	600–350 = **250**
	250	200–50 = **150**	300–200 = **100**	400–350 = **50**	**0**	600–475 = **125**
	300	200–0 = **200**	300–150 = **150**	400–300 = **100**	500–450 = **50**	**0**

From Table 2.4, the *maximum regrets* (opportunity losses) for all possible alternatives are as follows:

Alternative	Maximum regret	
100 copies	500	
150 copies	375	
200 copies	250	
250 copies	**150**	← minimum
300 copies	200	

Hence, you would choose 250 copies as the decision (the best alternative) based on the minimax criterion.