Chemical Formulas

A chemical formula indicates the elements that make up a compound. The chemical formula also indicates the relative number of atoms of each element present. For example, the chemical formula for water is H_2O, which indicates that each water molecule consists of two atoms of hydrogen and one atom of oxygen.

Chemical Equations

A chemical reaction occurs when a chemical change takes place. (During a chemical change, new substances that have new properties form.) A chemical equation is a useful way of describing a chemical reaction by means of chemical formulas. The equation indicates the substances that react and the products. For example, when carbon and oxygen combine, they can form carbon dioxide. The equation for this reaction is as follows:

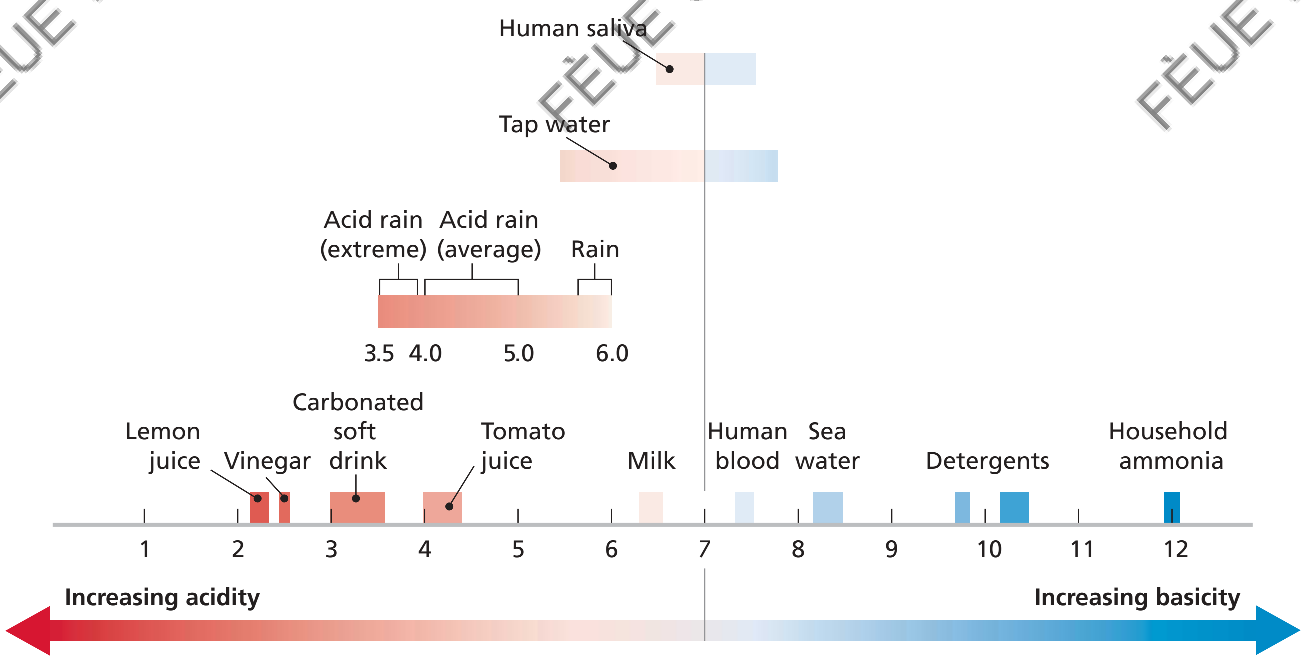
$$C + O_2 \rightarrow CO_2$$

Acids, Bases, and pH

An ion is an atom or group of atoms that has an electrical charge because it has lost or gained one or more electrons. When an acid, such as hydrochloric acid, HCl, is mixed with water, the acid separates into ions. An acid is a compound that produces hydrogen ions, H^+, in water. The hydrogen ions then combine with a water molecule to form a hydronium ion, H_3O^+. A solution that contains hydronium ions is an acidic solution. A base, on the other hand, is a substance that produces hydroxide ions, OH^-, in water.

To determine whether a solution is acidic or basic, scientists measure pH. **pH** is a measure of how many hydronium ions are in solution. The pH scale ranges from 0 to 14. The middle point, pH = 7, is neutral, neither acidic nor basic. Acids have a pH of less than 7; bases have a pH of more than 7. The lower the number is, the stronger the acid is. The higher the number is, the stronger the base is. A pH scale is shown below.

pH Measurements of Some Common Substances



SKILLS HANDBOOK Physics Skills Refresher

Mass

All matter has mass. Mass is the amount of matter that makes up an object. For example, Earth is made of a very large amount of matter and therefore has a large mass. An object's mass can be changed only by changing the amount of matter in the object.

Weight

Weight is different from mass. Weight is a measure of the gravitational force that is exerted on an object. Objects that have large mass are heavier than objects that have a small mass, even if the objects are the same size.

Density

The mass per unit of volume of a substance is density. Thus, a material's density is the amount of matter it has in a given space. To find density, both mass and volume must be measured. Density is calculated by using the following equation:

$$density = \frac{mass}{volume}$$

Density is expressed in units of mass over units of volume. Most commonly, density is expressed as grams per cubic centimeter (g/cm^3) or as kilograms per cubic meter (kg/m^3).

The density of a particular substance is always the same at a given temperature and pressure. The density of one substance is usually different from the density of other substances. Therefore, density is a useful property for identifying substances.

Concentration

A measure of the amount of one substance that is dissolved in another substance is concentration. The substance that is dissolved is the solute. The substance that dissolves another substance is the solvent. Concentration is calculated by using the following equation:

$$concentration = \frac{mass\ of\ solute}{volume\ of\ solvent}$$

Concentration is expressed as mass of solute divided by volume of solvent. Most commonly, concentration is expressed as grams per milliliter (g/mL) or as kilograms per liter (kg/L).

Forces

In science, a force is simply a push or a pull. All forces have both magnitude and direction. Force is expressed using a unit called a newton (N). All forces are exerted by one object on another object.

More than one force can be exerted on an object at the same time. The net force is the force that results from combining all the forces exerted on an object. When forces are in the same direction, net force is calculated by using the following equation:

$$net\ force = force\ A + force\ B$$

When forces are in the opposite direction, net force is calculated by using the following equation:

$$net\ force = force\ A - force\ B$$

Pressure

The force exerted over a given area is pressure. Pressure can be calculated by using the following equation:

$$pressure = \frac{force}{area}$$

The SI unit for pressure is the pascal (Pa). Other common units of pressure include bars and atmospheres.

Speed

The rate at which an object moves is its speed. Speed depends on the distance traveled and the time taken to travel that distance. Speed is calculated by using the following equation:

$$speed = \frac{distance}{time}$$

The SI unit for speed is meters per second (m/s). Other units commonly used to express speed are kilometers per hour, feet per second, and miles per hour.

Velocity

The speed of an object in a particular direction is velocity. Speed and velocity are not the same, even though they are calculated using the same equation. Velocity must include a direction, so velocity is described as speed in a certain direction. For example, the speed of a plane that is traveling south at 600 km/h is 600 km/h. The velocity of a plane that is traveling south at 600 km/h is 600 km/h south.

Velocity can also be thought of as the rate of change of an object's position. An object's velocity remains constant only if its speed and direction don't change. Therefore, constant velocity occurs only along a straight line.

Acceleration

The rate at which velocity changes is called *acceleration*. Acceleration can be calculated by using the following equation:

$$acceleration = \frac{final\ velocity - starting\ velocity}{time\ it\ takes\ to\ change\ velocity}$$

Velocity is expressed in meters per second (m/s), and time is expressed in seconds (s). Therefore, acceleration is expressed in meters per second per second (m/s/s), or meters per second squared (m/s^2).

Inertia

The tendency of an object to resist any change in motion is called *inertia*. Because of inertia, an object at rest will remain at rest until something causes it to move. A moving object continues to move at the same speed and in the same direction unless something acts on it to change its speed or direction.

Momentum

The property of a moving object that is equal to the product of the object's mass and velocity is momentum. Momentum is calculated by using the following equation:

$$momentum = mass \times velocity, \text{ or } p = mv$$

The SI unit for momentum is kilograms multiplied by meters per second (kg•m/s)

When a moving object hits another object, some or all of the momentum of the first object is transferred to the other object. If only some of the momentum is transferred, the rest of the momentum stays with the first object.

Thermodynamics

The study of the behavior of the flow of energy in natural systems is thermodynamics. The laws of thermodynamics describe some of the basic truths of how energy behaves in the universe. Many Earth processes involve the flow of energy through the Earth system.

The First Law of Thermodynamics This law is often called the Law of Conservation of Energy. Simply stated, this law states that energy can be changed from one form to another but that it cannot be created or destroyed. Energy constantly changes from one form to another, but the total amount of energy available in the universe is constant.

The Second Law of Thermodynamics This law states that in all energy exchanges, if no energy enters or leaves the system, the potential energy of the new state will always be less than that of the initial state. In other words, no form of energy converts entirely to another form of energy without losing some energy as heat. So, the entropy of an isolated system always increases as time increases. Entropy is a measure of disorder, or randomness, of energy and matter.

The Third Law of Thermodynamics This law states that if all of the thermal motion of molecules, or kinetic energy, were removed from a system, a temperature called *absolute zero* would be reached. Absolute zero is in a temperature of 0 Kelvin or –273.15 degrees Celsius.

$$\text{absolute zero} = 0\text{K} = -273.15°\text{C}$$

ANSWERS Answers to Practice Problems

Reading and Study Skills

How to Make Power Notes

1. Sample answer:

The Experimental Method

Power 1: observing
- Power 2: observation

Power 1: hypothesizing and predicting
- Power 2: hypothesis
- Power 2: prediction

Power 1: experimenting
- Power 2: experiment
  - Power 3: variable
  - Power 3: experimental group
  - Power 3: control group

Power 1: organizing and analyzing data
- Power 2: data

Power 1: drawing conclusions

Power 1: repeating experiments

Power 1: communicating results

How to Make KWL Notes

1. a. The first step is observing.
   b. A hypothesis is more than a guess. It must be based on observations and be testable by experiment.
   c. A good experiment has a single variable and a control group.

Math Skills Refresher

Geometry

1. 225,000 m^2
2. 1,230 cm^3 (rounded to three significant figures)
3. 96 cm^2

Exponents

1. a. 9
   b. 14,348,907
   c. 537,824
   d. 1

Order of Operations

1. 24
2. 7

Algebraic Rearrangements

1. a. $x = 20$
   b. $a = -1.75$
   c. $y = -6.3$
   d. $m = -4$
   e. $z = 2$
   f. $b = 5$

Scientific Notation

1. a. 1.23×10^7 m/s
   b. 4.5×10^{-12} kg
   c. 6.53×10^{-5} m
   d. 5.5432×10^{13} s
   e. 2.7315×10^2 K
   f. 6.2714×10^{-4} kg

Significant Digits

1. a. 4
   b. 5
   c. 4
   d. 3
2. a. 0.129 dm
   b. 2700 m/s
   c. 9.84 m^2
   d. 0.98 g

MAPPING EXPEDITIONS

Journey to Red River

Materials

- compass, magnetic, with degree markings (optional)
- ruler, metric

How do you get from one place to another when you don't know the route? Whether you are planning a trip on foot or by car or boat, a map can be very handy. The ability to read a map can help you reach your destination quickly and safely and can help you avoid becoming disoriented and lost.

The topographic map on the facing page shows the area around Red River, New Mexico. Imagine that you are traveling to Red River to do some camping, hiking, and sightseeing. Study the map for a few moments, and note the locations of roads, creeks, hills, and other features. Then, answer the questions below.

1. Red River lies in northeastern New Mexico near the Colorado border. The magnetic declination in Red River is about 13°E. Draw a diagram that shows how you would adjust a magnetic compass to determine true north in Red River. Why is distinguishing true north from geomagnetic north important?
2. You set up a tent at Mallette Campground. If you walk in a straight line from your campsite to the cemetery at the base of Graveyard Canyon, how far will you walk? Show your work.
3. You decide to hike from St. Edwin Chapel to location A. What is your elevation at location A? How much higher than your starting point is your destination? (Elevations on the map are given in feet.)
4. Notice that the road in the lower-right corner of the map winds back and forth to make a series of hairpin turns. Why did the road designers build the road this way?
5. Most United States Geological Survey maps, including this one, were created in the early 1960s. Like most towns, Red River has changed in the last few decades. Which features of the map might not reflect how Red River looks today? Which features are probably still accurate?

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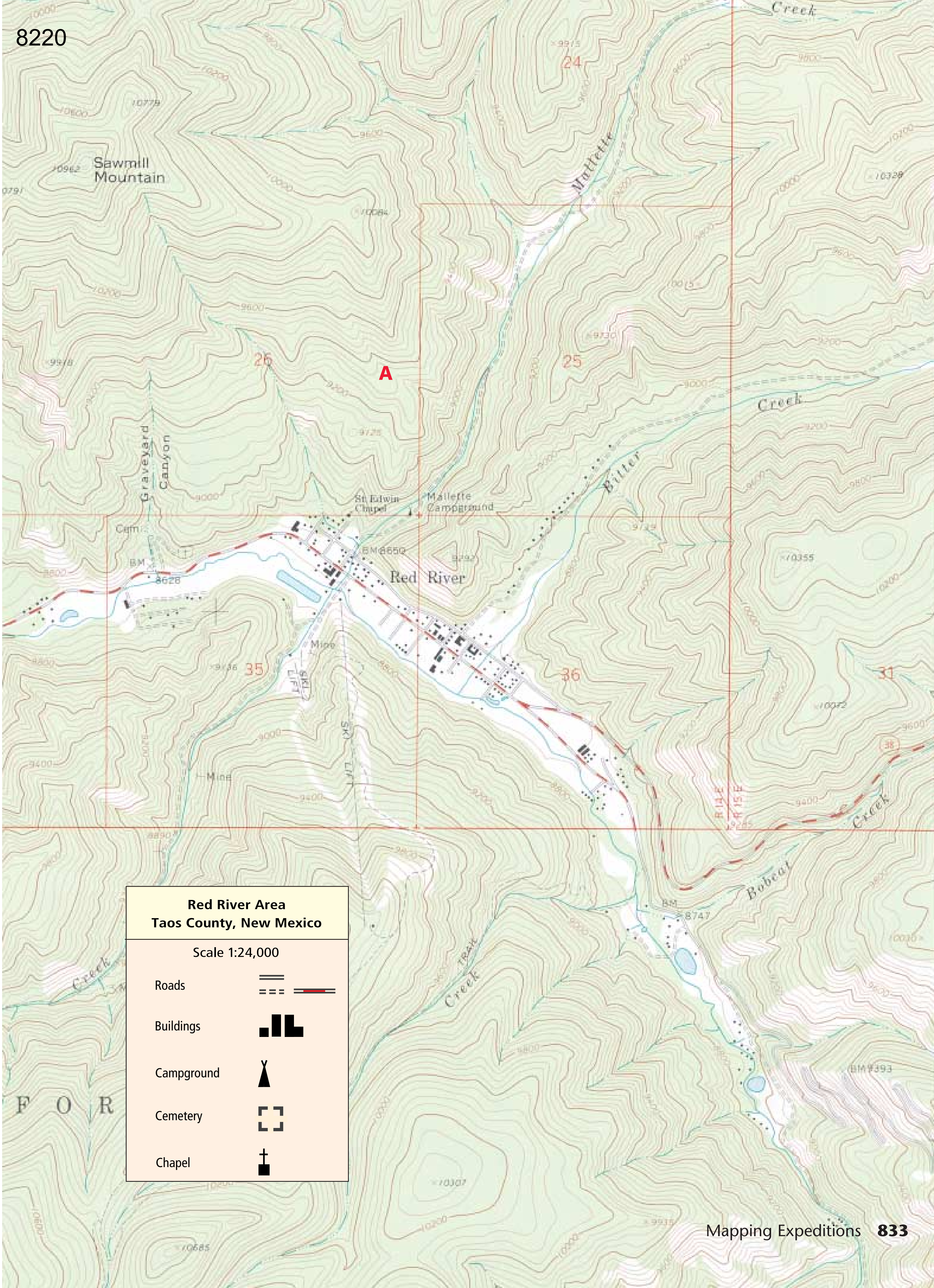

Red River Area
Taos County, New Mexico
Scale 1:24,000
Roads
Buildings
Campground
Cemetery
Chapel
Sawmill Mountain
Graveyard Canyon
Mallette
Creek
Bitter
St Edwin Chapel
Mallette Campground
Cem
Red River
BM 8650
BM 8628
Mine
SKI LIFT
SKI LIFT
Bobcat
Creek
TRAIL Creek
BM 8747
BM 9393
A
24
25
26
35
36
31
R 14 E
R 15 E
F O R

MAPPING EXPEDITIONS

A Case of the Tennessee Shakes

Materials

- road map of Tennessee

Did you know that almost 10,000 earthquakes occur every day? In fact, an earthquake likely is occurring right now somewhere in the world. Fortunately, less than 2% of the earthquakes that seismographs record are strong enough to do serious damage.

You might think that scientists are most interested in strong earthquakes. But weak earthquakes can tell a seismologist (a scientist who studies earthquakes) as much as strong ones can.

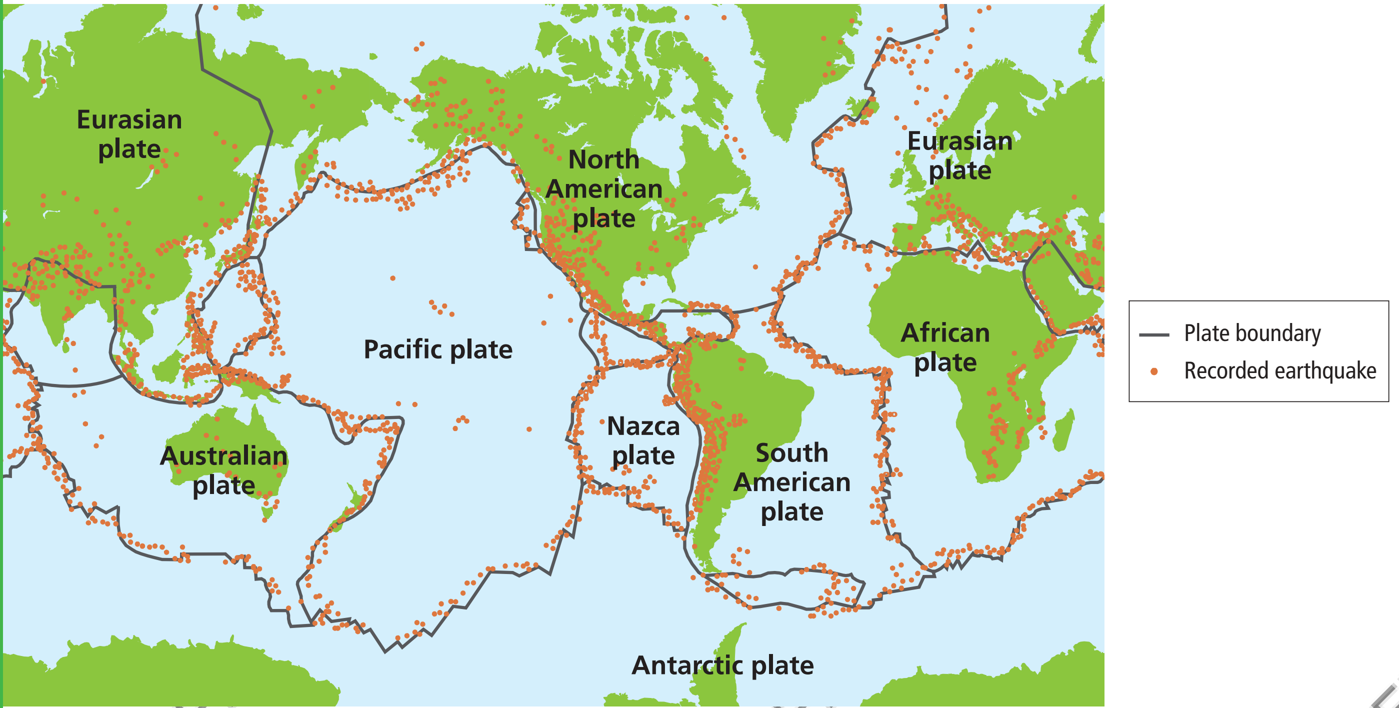
Earthquake Frequency (based on observations since 1900)

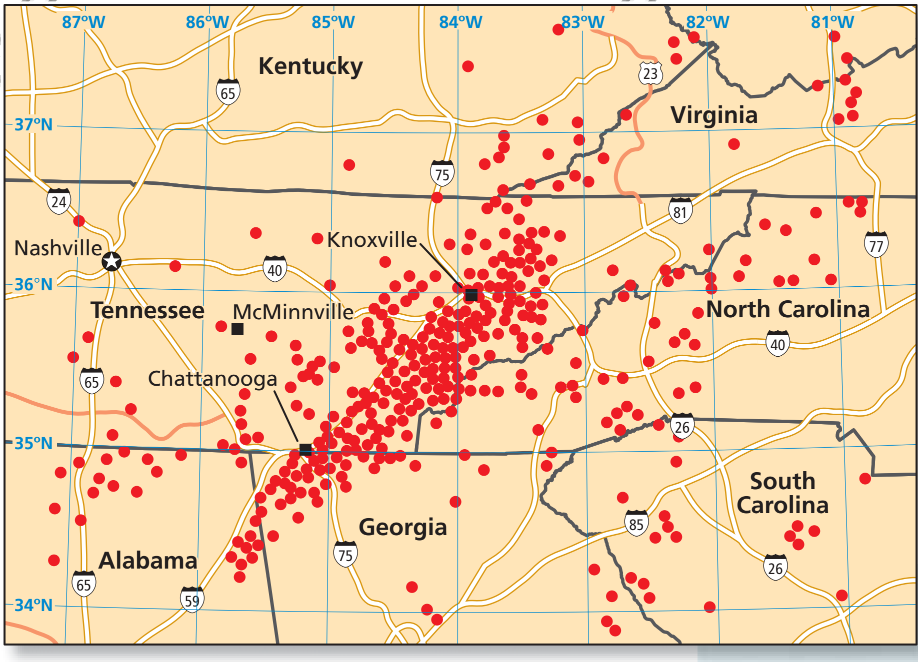
Descriptor	Magnitude	Average occurring annually
Great	8.0 and higher	1
Major	7.0 to 7.9	18
Strong	6.0 to 6.9	120
Moderate	5.0 to 5.9	800
Light	4.0 to 4.9	about 6,200
Minor	3.0 to 3.9	about 49,000
Very minor	2.0 to 2.9 1.0 to 1.9	about 365,000 about 29,200,000

Part 1

1. Examine the map below. Which tectonic plates are involved in most earthquakes that occur in North America?

2. At tectonic plate boundaries, most earthquake epicenters are densely distributed, or closely packed. Why do most earthquakes occur along tectonic plate boundaries?





▶ Each dot on this map represents the epicenter of an earthquake. Most of these earthquakes, which occurred over a 20-year period, were too weak to be felt by people.

3. Some earthquakes, however, occur in the interior of the United States, which is far from any plate boundary. Propose a hypothesis that explains these earthquakes.

Part 2

The map above shows the epicenters of earthquakes in eastern Tennessee. However, Tennessee is far from any plate boundary. Some scientists think that the earthquakes in this region are the result of an ancient fault that has been reactivated. Other scientists think that a new fault zone is forming in eastern Tennessee. If they are correct, eastern Tennessee may experience a major earthquake in this zone.

1. Use the map to describe the location of the eastern Tennessee seismic zone (ETSZ) in terms of longitude and latitude.

2. Name at least two major cities that are located in the ETSZ. How could a major earthquake affect these cities?

3. Two nuclear power plants are located in the ETSZ. Imagine that a company has plans to build a plant near McMinnville, Tennessee. The United States Geological Survey has hired you to advise this company about the risk of a major earthquake. Briefly describe what you would say in a letter to the company. Explain your reasoning as clearly as possible.

▶ By using trench excavations, seismologist Karl Mueller can study sediments across the New Madrid fault in Tennessee to estimate the dates and magnitudes of past earthquakes.

MAPPING EXPEDITIONS

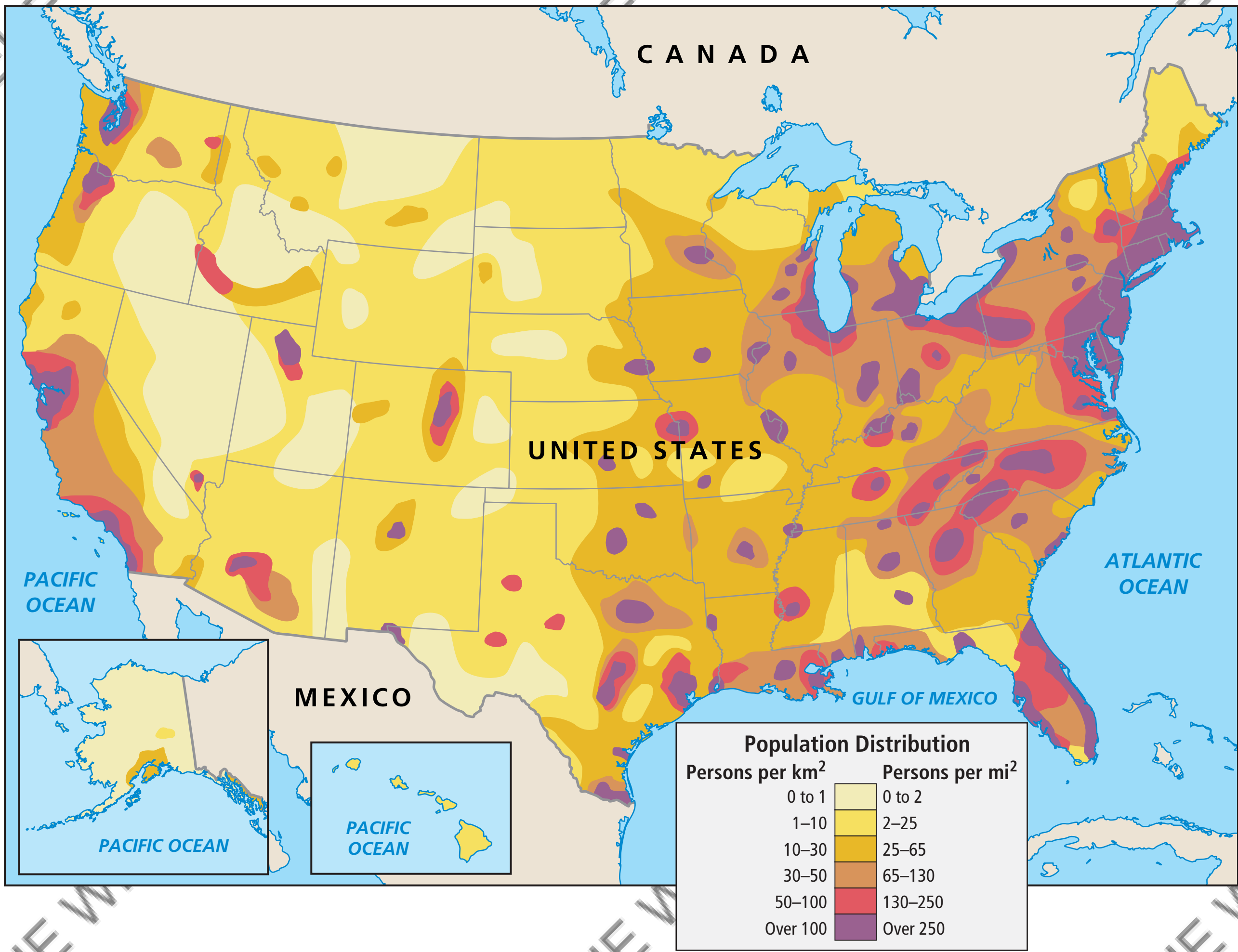
ENVIRONMENTAL CONNECTION

MAPPING EXPEDITIONS

Buried Treasure

By many standards, the United States is one of the wealthiest countries in the world. Although this wealth is largely due to the ingenuity and hard work of the people who live in the country, it is also due to good fortune. The crust that lies beneath the United States holds a huge supply of natural resources. These resources include ores that contain precious metals, such as copper, silver, and gold, as well as fossil fuels, such as petroleum, coal, and natural gas. The availability and distribution of these resources have been important in shaping U.S. history.

▶ This worker in California cuts through steel, an iron alloy, at 2,000°F!



1. Find your state on the map of natural resources on the previous page. What resources are produced in your state?

2. Look at the locations of the various resources. Which resources are commonly located near each other? Why do certain resources commonly occur together?

3. When tectonic plates collide, pockets of hot magma may come into contact with cooler, solid rock. Using what you know about plate tectonics and the ways in which minerals form, describe why Washington has more iron-ore deposits than Nebraska does.

4. Describe the conditions that existed in the United States millions of years ago and that resulted in the formation of the modern petroleum and natural-gas deposits.

5. Compare the map on this page with the map on the previous page. What areas have both high concentrations of people and a large reserve of natural resources? What areas have many people but few resources? How could resources be transported to areas in which they are needed?

6. Using these two maps, would you say that most cities have grown up in places in or near which there are natural resources? Why or why not? What other factors could have influenced the location of cities?

7. Imagine that you work for a company that builds electrical equipment made primarily of copper. Why might southern Arizona be a good place to locate a new plant? What might be a disadvantage of locating your plant there?

MAPPING EXPEDITIONS

What Comes Down Must Go...WHERE?

Materials

- cardboard, about 23 cm × 33 cm
- paper, about 23 cm × 33 cm
- pencils, red, blue, and purple
- permanent marker, fine-tipped
- plastic bag, reclosable, about 23 cm × 33 cm
- scissors
- umbrella, raincoat, or other rain gear

Imagine looking out your classroom window during a downpour. Billions of tiny raindrops splatter off of everything in sight. Streams of water fall from the roof and form dozens of puddles and streams on the ground. These miniature lakes and rivers swirl together, and tiny torrents carry away leaves, bits of trash, and other debris. A day or two later, the ground outside looks completely dry. Where did all of the water go?

In this activity, you will create a map of your school. After observing the type of ground cover and the slope of the terrain at various locations, you will predict whether rainwater will collect or run off at those locations. You will also look for possible sources of pollution and places where erosion might occur. Later, you will go outside in the rain and find out whether your predictions are correct.

Part 1: Outside on a Fair Day

1. Form a team with several of your classmates. Then, divide your school's campus into the number of equal areas that is the same as the number of teams in your class. Each team will work on one campus area.

2. Cut out a piece of paper and a piece of cardboard that fit exactly into a large reclosable plastic bag. The piece of paper will be your map.

3. On the paper, map one section of your school's campus. Include buildings, paved areas (such as sidewalks, outdoor sports courts, and parking lots), and vegetated areas (such as lawns, athletic fields, and wooded areas). The map on the next page is an example of the type of map that you will make.

4. **a.** Use a red pencil to mark the areas on your map. Draw arrows to indicate a downhill slope. Use a narrow arrow to indicate a steep slope and a wider arrow to indicate a gradual slope. Use circles to indicate flat areas.

b. Use a blue pencil to draw arrows and circles that indicate where you think surface water may collect or flow during a steady rain. These areas may include low-lying areas, the roofs of buildings, gutters, and drainage ditches.

c. Use a purple pencil to mark the locations that you think might contribute pollution to the runoff. (These areas may include parking lots that contain oil stains or places where trash is usually found on the ground, such as near a dumpster.)

5 Seal your map and the piece of cardboard in the reclosable bag. When you are outside in the rain, use a permanent marker to write your observations on the outside of the bag.

Part 2: Outside During a Steady Rain

6 Dress appropriately, and go outside. Using the marker, write on your plastic-covered map the places where water collects and runs. In places where water moves along the ground, use arrows to show the water's direction. Use the letter *P* to mark the locations of pollutants that you observe in the water. Use the letter *E* to mark the locations where erosion seems to be occurring. (Look for soil or natural debris that is being washed along by moving water.)

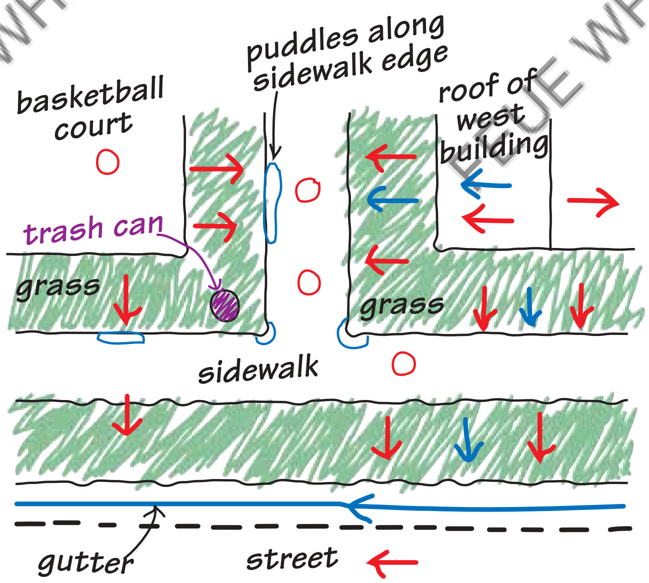
Part 3: Back in the Classroom

7 Discuss your predictions for some of the locations. Were your predictions correct? How do you explain differences between your predictions and your observations?

8 Was the pollution that you observed suspended load, bed load, or dissolved load? Explain how there may have been pollution that you could not observe.

9 Explain how erosion on your school's campus could affect the erosion and deposition that occurs downstream from the campus.

10 Assemble the maps from your class into a single map of your school's campus. In your opinion, does most of the rainfall at your school become groundwater or runoff? Where does runoff go when it leaves your school's campus? Your school is probably part of a larger, local watershed. Find out what stream or other body of water the surface runoff in your area empties into.



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MAPPING EXPEDITIONS

Where the hippos Roam

Materials

- pencils, assorted colors
- ruler, metric

Millions of years ago, ancestors of modern crocodiles lurked in the shallow waters of lakes and other bodies of water. Like their current descendents, they hunted fish and other animals. If you could travel back in time to visit one of those lakes, you might see the ancestors of today's hippopotamuses there, too. Antelopes might browse along the edges of the lake, and rodents of various sizes might scurry back and forth.

When paleontologists examine the fossil of a prehistoric organism, they may discover clues about the organism's life. They may also answer questions about the organism's environment: Was the area hot or cold? Was it humid or dry? Then, by putting all of these clues together, the paleontologists may be able to learn a little more about how organisms and environments change over time.

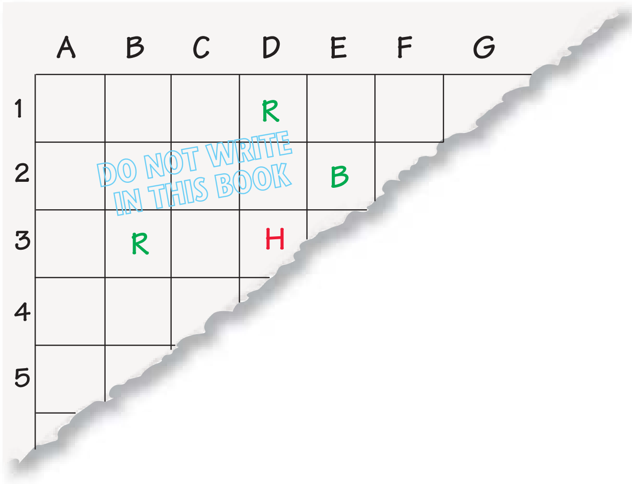
Unfortunately, studying a fossil site is no easy task! Discoveries of complete organisms are rare. More often, a paleontologist may find a few teeth scattered over a very large area. In such cases, keeping track of where the fossils were found is very important. In this activity, you will use the data from a fossil site to create a map of fossil locations at that site. Then, you will draw some conclusions about the past environment, or *paleoenvironment,* at that location.

▶ The animals that lived near lakes millions of years ago probably had lives similar to the lives of animals that live near lakes today.

Location of Fossil Teeth

Layer	Hippos	Rodents	Crocodiles	Bovids*
A	B11, C6, D3, I15, J10, L7, M6		C14, F7, G13, I3, L13, O2	
B	F2, J3, K1, K2	B10, B11, F13	H2, I7, K2, N5, N7	G14
C		B3, C10, D1, H8, M9, N4		A5, A6, E2, E4, E14, H7, H8, H12, K4, M1, N15

*Bovids are antelopes and other such animals.



The table above shows the locations of fossils that were found spread out over 22,500 m^2. A team of paleontologists decided that this site, which measured 150 m × 150 m, was too large to work on all at once. So, the paleontologists decided to create a grid of 10 m squares. Starting in the northwest corner of the site, they labeled the squares from west to east with the letters *A–O*. Then, they numbered the squares 1–15 from north to south. Thus, each fossil could be labeled with a letter and a number that would identify where the fossil was found. For example, A1 would signify the 10 m × 10 m square in the northwest corner of the site, and O15 would signify the square in the southeast corner.

1. Create a map of the fossil site by drawing a grid similar to the one described above. The scale should be 1 cm = 10 m. Use letters to label across the top edge of the grid, and use numbers to label down the left edge of the grid. For each fossil, place a letter (*H* for a hippo fossil, *R* for a rodent fossil, *C* for a crocodile fossil, and *B* for a bovid fossil) in the square that corresponds to where the fossil was found. Use pencil color to represent the different layers of sediment, and make a key that shows which layer each color represents.

2. From the distribution of fossils in the layer of sediment just below the surface layer, what part of this site might have been underwater? Explain your answer, and devise a way to show that area on your map.

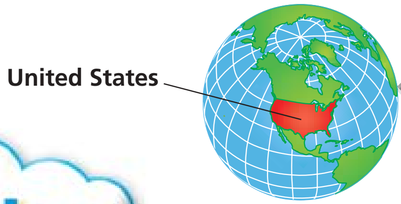
3. Describe how the environment at this site changed over time.

4. One team member wished to search this site for fossils of dry-climate plants. Which layer or layers would most likely yield fossils of such plants? Explain your answer.

5. One paleontologist suggested that tectonic uplift had raised the area's elevation over time and thus caused the climate to change. A second paleontologist thought that the area had probably lost elevation over time. With which scientist do you agree? Explain your answer.

▶ Fossils, such as these crocodile teeth, help scientists learn what an area was like millions of years ago.

MAPPING EXPEDITIONS



Snapshots of the Weather

Materials

- paper, tracing
- pencil

From looking at a weather map, you might get the impression that the clouds, fronts, and other features shown are standing still. However, weather patterns change constantly, and a weather map can show only what is happening at one particular instant. For this reason, meteorologists rely on a sequence of maps to make predictions about local weather.

In this activity, you will analyze a sequence of weather maps. The maps were taken from a daily newspaper and show weather patterns that occurred in the United States during a 4-day period. You will note what information the maps show and do not show, and you will make a few predictions based on your observations.

1. Look carefully at the maps on the next page. What weather information do they show? Now, look at the weather symbols in the Reference Tables section of the Appendix. What information is not included on these maps? Why might a newspaper exclude certain types of information on daily weather maps?

2. Why would a newspaper that serves only a specific geographic region publish the weather for the entire continental United States?

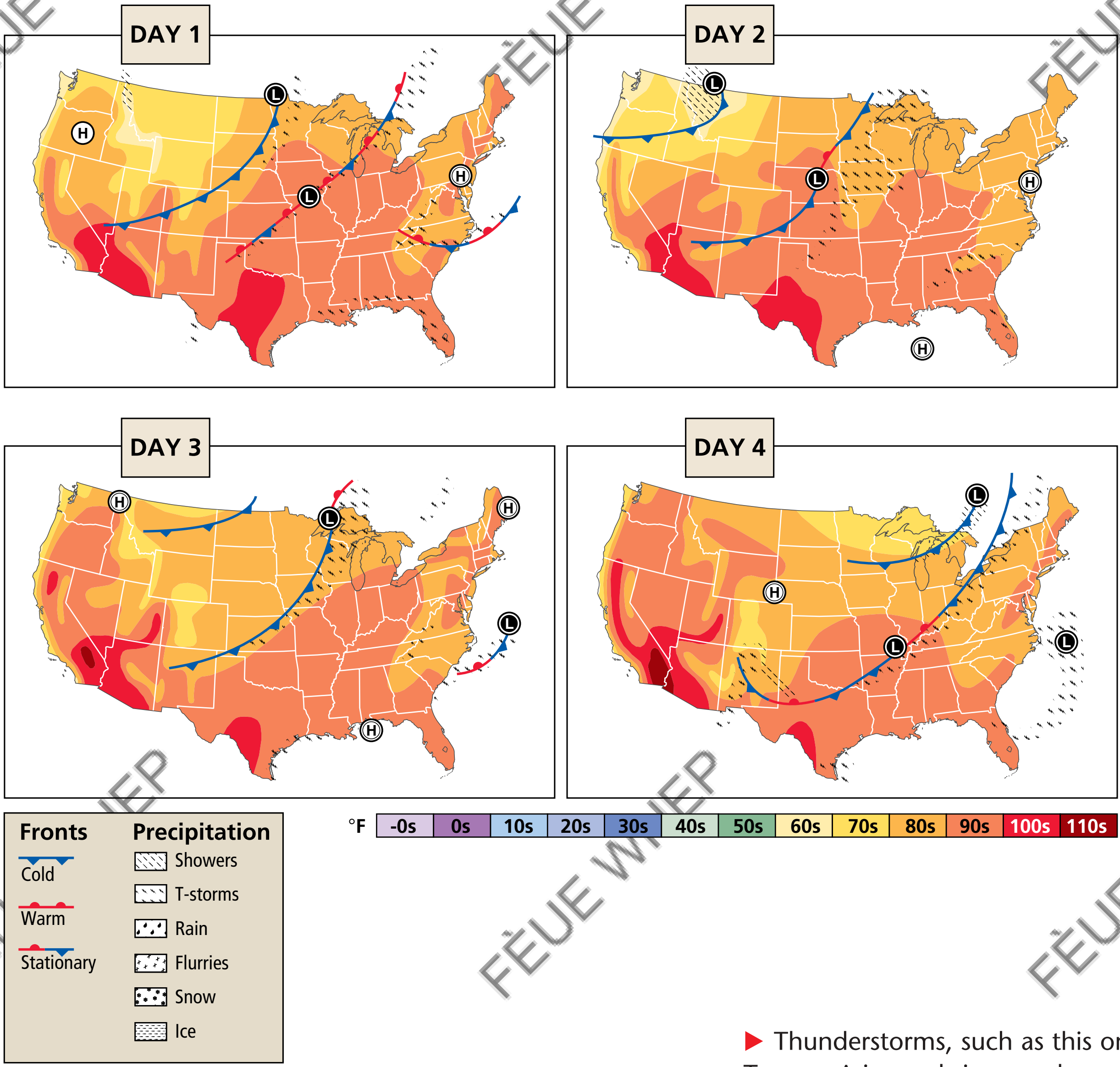
3. Describe how the weather patterns in your location changed during the 4-day period shown.

4. During what season do you think this 4-day period occurred? Explain your answer.

5. Trace the outline of one weather map on a separate piece of paper, but do not include any information on the map. Predict the locations of the fronts on the day following this 4-day period. Note the locations of the fronts on your new map.

6. Predict the temperature and precipitation patterns that occur in your location on the day following this 4-day period.

▶ This tornado twisted through Manchester, South Dakota, on June 24, 2003. On the same day, South Dakota had its largest recorded outbreak of twisters ever!



▶ Thunderstorms, such as this one in Tucson, Arizona, bring much-needed moisture to dry regions.

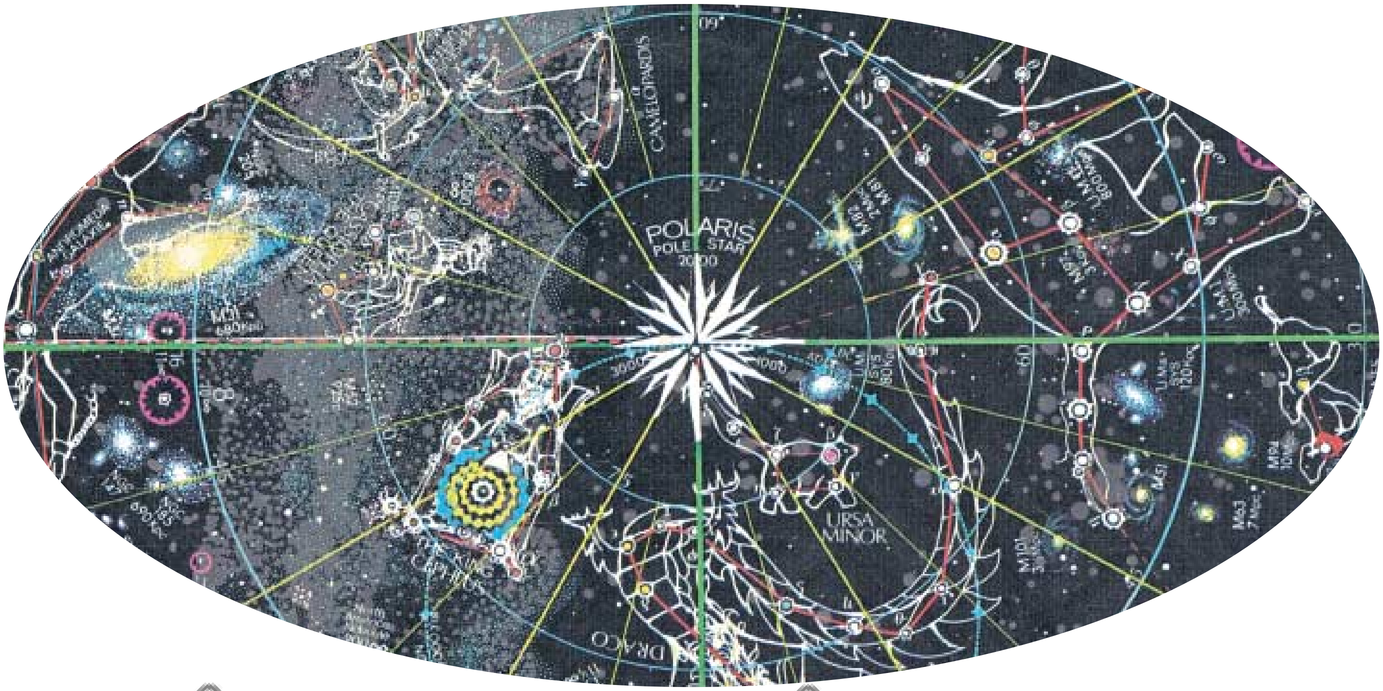
MAPPING EXPEDITIONS

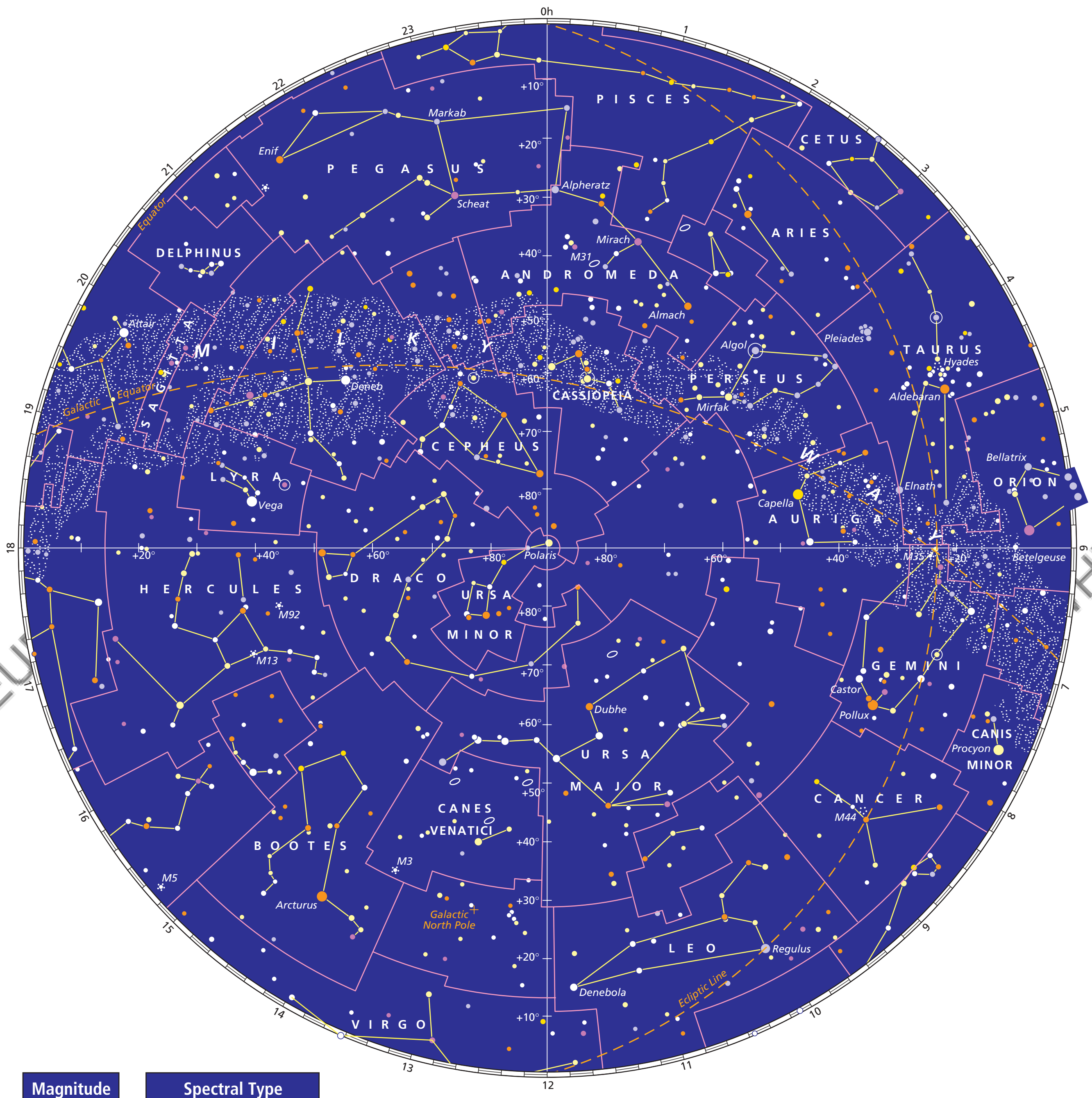
Stars in Your Eyes

A constellation is an arbitrary grouping of stars. The map to the right shows constellations that can be seen from the Northern Hemisphere of Earth. A Southern Hemisphere sky map, which is not provided, would show stars that can be seen from the Southern Hemisphere of Earth. Refer to the map to the right as you answer the questions below.

1. For many years, the best way to navigate at night was to use the stars as a guide. Many people used Polaris—the North Star—to orient themselves. This approach would not have worked for people all over the world, however. Why not?
2. What is the name of the constellation that contains Polaris?
3. What is the temperature of the star in the Bootes constellation, with magnitude 0?
4. The constellation Virgo can be observed from the Northern Hemisphere during the summer but not during the winter. Explain why.
5. Compare the constellation Draco on the map below with the constellation on the map to the right. What type of animal was this constellation named after?
6. Pick a group of stars that have not been connected into a constellation. Sketch the star group on a separate piece of paper. Connect the stars into a new constellation, and name your constellation.

▶ People, such as Sumerians, Greeks, Chinese, and Egyptians, have been grouping stars into constellations like these for thousands of years.





Magnitude Scale
0
1
2
3
4
5

Spectral Type Temperature (K)	
50,000–25,000	O-B
11,000	A
7,500	F
6,000	G
5,000	K
3,500	M

- Variable star
- Open cluster
- Globular cluster
- Galaxy
- Constellation boundary

▶ There are 88 constellations recognized by modern astronomers. The Northern Hemisphere sky map shows those that can be seen from the Northern Hemisphere of Earth.

LONG-TERM PROJECTS

Introducing Long-Term Projects

Scientific investigations that lead to important discoveries are almost never short term. Usually, these investigations last months, years, and even decades before results are considered complete and dependable. Investigations in Earth science are no exception. The long-term projects included in this section will give you practical experience in investigating Earth science the way that Earth scientists do—over extended periods of time. You will observe changes over time, keep detailed records of your observations, and draw conclusions from your data. By following these steps, you will learn firsthand what it is like to be an Earth scientist.

Safety First!

Many of the long-term projects require you to make field trips to an observation site or to conduct your activities outdoors. Advance planning is essential. You should plan carefully for these investigations and should be certain that you are aware of the safety guidelines that must be followed. The following are general guidelines for fieldwork and lab work.

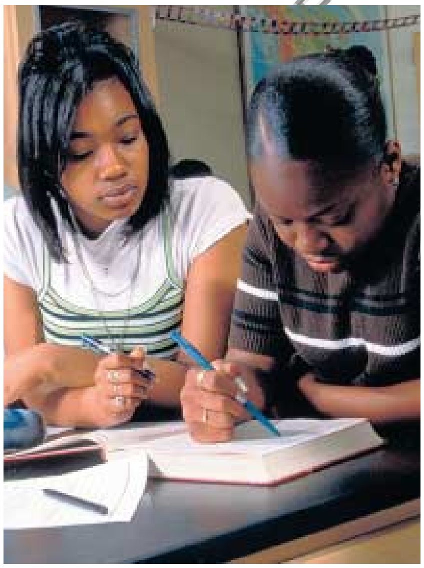
Conducting Fieldwork

Find out about on-site hazards before setting out. Determine whether there are poisonous plants or dangerous animals where you are going, and know how to identify them. Also, find out about other hazards, such as steep or slippery terrain.

Wear protective clothing. Dress in a manner that will keep you warm, comfortable, and dry. Wear sunglasses, a hat, gloves, rain gear, or other gear to suit local weather conditions. Wear waterproof shoes if you will be near water or mud.

Do not approach or touch wild animals unless you have permission from your teacher. Avoid animals that may sting, bite, scratch, or otherwise cause injury.

Do not touch wild plants or pick wildflowers without permission from your teacher. Many wild plants can cause irritation or can be toxic, and many are protected by law. Never taste a wild plant.



Do not wander away from the group. Do not go beyond where you can be seen or heard. Travel with a partner at all times.

Report any hazards or accidents to your teacher immediately. Even if an incident seems unimportant, tell your teacher about it.

Consider the safety of the ecosystem that you will be visiting as well as your own safety. Do not remove anything from a field site without your teacher's permission. Stay on trails when possible to avoid trampling delicate vegetation. Never leave garbage behind at a field site. Strive to leave natural areas just as you find them.

Conducting Lab Work

Be aware of safety hazards. Any field or lab exercises in which there are known safety hazards will include safety cautions and icons to identify specific hazards. By being aware of safety concerns, you may avoid accidents. Know where safety equipment and emergency exits are located so that you are prepared in the event of an emergency.

Do not engage in inappropriate behavior. Most laboratory accidents are caused by carelessness, lack of attention, or inappropriate behavior. Always be aware of your surroundings, and pay attention to safety cautions.

Be neat. Keep your work area free of unnecessary clutter. Tie back loose hair and loose articles of clothing. Do not wear dangling jewelry or open-toed shoes in the lab. Never eat or drink in the laboratory.

Clean your lab station when your lab time is over. Before leaving the lab, clean up your work area. Put away all equipment and supplies, and dispose of chemicals and other materials as directed by your teacher. Turn off water, gas, and burners, and unplug electrical equipment. Wash your hands with soap and water after working on any lab.

For additional information about safety in the lab and in the field, refer to the Lab and Field Safety section in the front of this book. Don't take any chances with safety!

LONG-TERM PROJECT 1

Duration
8 or 9 months

Objectives

- USING SCIENTIFIC METHODS **Observe and record** the positions of sunrise and sunset once per month.
- USING SCIENTIFIC METHODS **Graph and analyze** collected data that describe the positions of sunrise and sunset.
- **Predict** the positions of sunrise and sunset for 3 or 4 months.

Materials

- compass, magnetic
- glue
- paper
- paper, graph
- pen
- pencil
- poster board
- scissors
- twist tie (or pipe cleaner)

Safety

The sun appears to rise and set at different positions relative to landmarks, such as these skyscrapers in Los Angeles, California.

Positions of Sunrise and Sunset

You are probably aware that the sun rises in the east and sets in the west each day. What may not be obvious to you is that the positions of sunrise and sunset along the horizon differ from day to day in a specific pattern. As the positions of sunrise and sunset change, the amount of sunlight that an area receives also changes. In this investigation, you will observe the changes in the sun's position along the horizon at sunrise and at sunset. You will be making two observations on or near the 21st of each month for approximately 8 or 9 months (depending on the schedule of your school year).

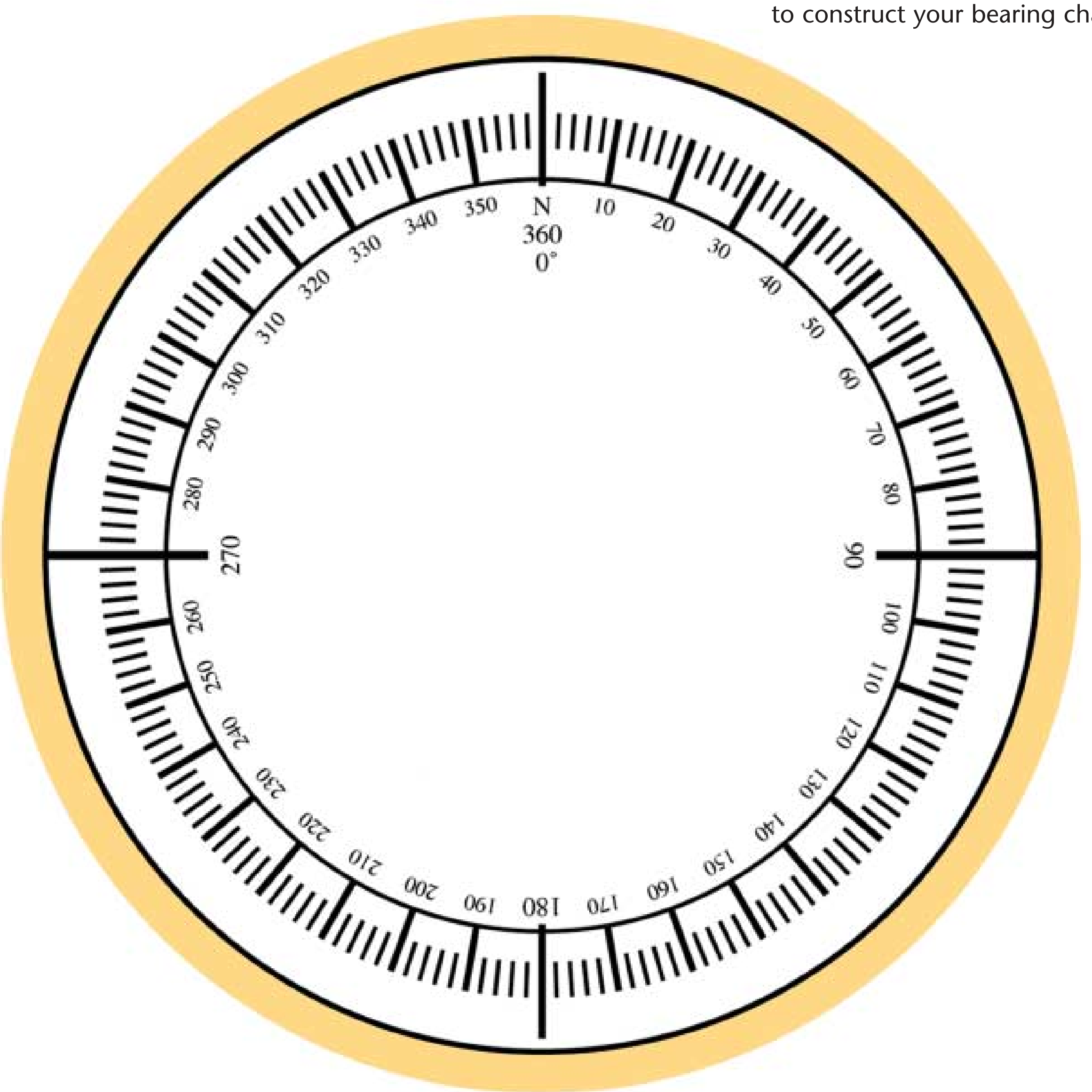
PROCEDURE

1. Construct the bearing chart before taking any measurements. Copy the bearing chart from the next page onto a piece of paper.
2. Glue your copy of the chart to a piece of poster board. When the glue is dry, trim away the excess poster board.
3. Wrap the center of a twist tie once around the center of a pencil. Poke the ends of the twist tie through the center of the bearing chart to make a pointer for your chart.

4. On a cloudless morning just before sunrise, place the bearing chart on a level spot where no buildings or trees block your view of sunrise and sunset.
CAUTION Although sunlight is less intense at sunrise and sunset, you should never stare at the sun for extended periods of time.

5. Use the magnetic compass to determine the direction of north. Set the bearing chart so that 0° is pointing north and 180° is pointing south. (Note: Have the chart face the same direction for every observation, even months from now.) Try to align an edge of the chart with some permanent object near your observation point.

Step 1 Copy the chart below on a separate piece of paper, and use it to construct your bearing chart.



Step 7

Date	Position of sunrise (in degrees)	Position of sunset (in degrees)

DO NOT WRITE IN THIS BOOK

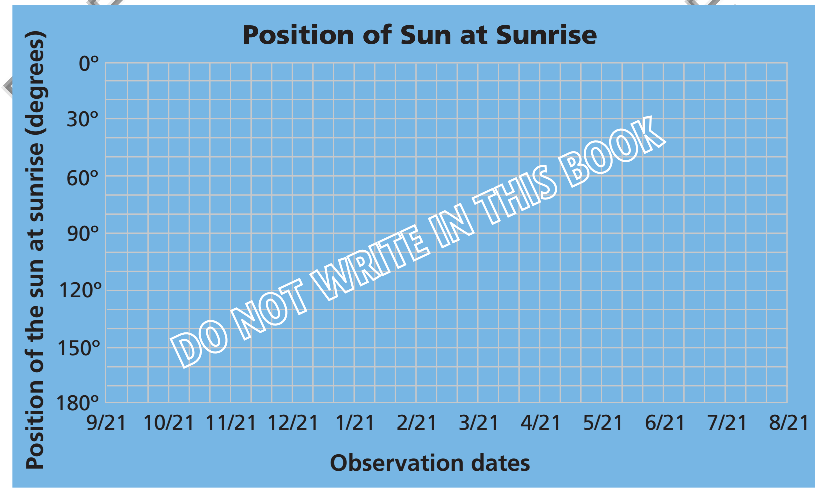
6. When the bearing chart is in place and properly aligned, measure the position of sunrise. (Note: Sunrise occurs at the instant the top of the sun appears on the horizon.) Without looking at the sun, measure its position by pointing the chart's pencil toward the sun. The shadow of the eraser end of the pencil will mark the sun's position.

7. In your lab notebook, draw a table like the one shown above. Record the position of sunrise (in degrees) in your table. That evening, measure and record the position of sunset (in degrees). (Note: Sunset occurs when the top edge of the sun drops below the horizon.)

Many scientists think that ancient people used structures, such as Stonehenge in England, to predict astronomical occurrences and to determine the timing of solstices and equinoxes.

8. Repeat steps 6 and 7 on the same date each month. (Note: On the equinoxes, the sun will rise due east and set due west. On the solstices, the sunrise and sunset will be shifted from these directions. The amount of the maximum shift will depend on the observer's latitude.)

9. On a sheet of graph paper, prepare a graph similar to the one shown at right. Label the graph's y-axis, which represents the position (in degrees) of the sun at sunrise, from 0° to 180°. The position of 90° will be in the center of the y-axis scale. The x-axis represents observation dates. Next, make a graph on which to plot the position of the sun at sunset. Label the y-axis of this graph from 180° to 360°.



Step 9

10. On each graph, connect the points by drawing a smooth line. Estimate the positions of points between the plotted points.

ANALYSIS AND CONCLUSION

1. **Analyzing Data** On which date does the sun appear to follow the highest path across the sky? What happens to the length of daylight during this season?

2. **Analyzing Data** On which date does the sun appear to follow the lowest path across the sky? What happens to the length of daylight during this season?

3. **Analyzing Results** On which dates are the positions of the sun at sunrise and again at sunset about 180° apart? Those dates mark the beginning of which seasons?

4. **Describing Events** What general statement can you make about the pattern of sunrise and sunset according to the graphs?

5. **Forming a Hypothesis** Expand your graph to include the predicted position of the sun in June and July. Describe the pattern that you predicted.

Extension

1. **Evaluating Hypotheses** Use the process described in the investigation to chart the positions of sunrise and sunset for the months of June and July. How do your observations compare with your prediction?

2. **Making Comparisons** Use the same process to chart the positions of the moon at moonrise and moonset. Refer to an almanac when you choose the times at which you will make measurements. Do the positions of sunrise and sunset correlate to those of moonrise and moonset? What can you conclude from your observations?

8239

LONG-TERM PROJECT 2

ENVIRONMENTAL CONNECTION

Duration
2 weeks

Objectives
- USING SCIENTIFIC METHODS **Count and record** the number of particulates in the air over a 2-week period.
- **Analyze** how wind direction and particulate source are related.

Materials
- compass, magnetic
- microscope
- microscope slides (8 or more)
- paper, graph
- pencil, grease
- petroleum jelly
- slide box
- tape, masking or packaging, or rubber bands

Safety

Air-Pollution Watch

When certain types of pollutants are present in high concentrations, they threaten the general health and well-being of humans. Some substances that can be air pollutants include dust and smoke particles, pollen, mold spores, and waste gases. If these tiny particles remain suspended in the air for long periods of time, they are called *particulates*.

Wind direction affects the number of particulates in the air. If there is a source of particulates in an area, there will be a large number of particulates in the air when the wind blows from the direction of that source. There will be fewer particulates in the air when the wind blows from the direction opposite the source.

In this investigation, you will collect and view a few types of particulates. You will collect particulates from an outdoor site every day for 2 weeks. Then, you will examine those particulates.

PROCEDURE

1. Select a collection site in an open area, such as a large field or pasture, where the wind can blow past the site from every direction.
2. Locate a four-sided post, such as a 4 in. × 4 in. fence post, that is firmly driven into the ground. Try to choose a post whose top is at least 1 m above ground level. If there are no fences in your area, look for another four-sided structure that you could use.

Haze and smog are common in large cities, such as Los Angeles, California.

Day	Date	Wind direction	Slide direction	Number of particulates 1	2	3	4	5	Total
1			N						
			S						
			E						
			W						

DO NOT WRITE IN THIS BOOK

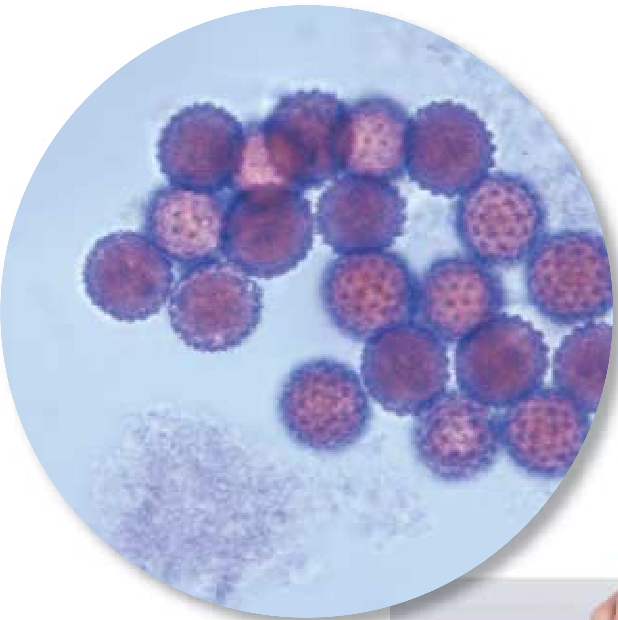
Step 3

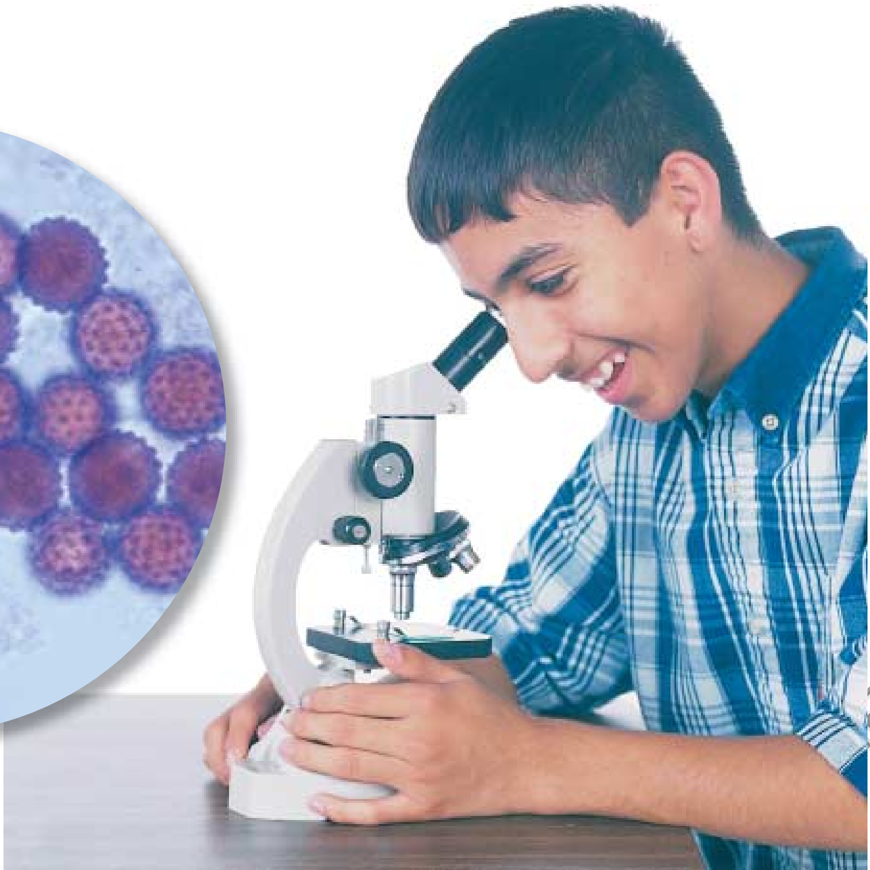
3. Use a compass to establish north, south, east, and west directions from the post's location. Determine the direction from which the wind is blowing by watching objects, such as a flag, move in the wind. Record the wind direction and the date in a table like the one shown above.
4. Look for any phenomena that may affect air quality, such as smokestacks or heavy traffic. Record these observations and the day's weather conditions.
5. Use a grease pencil to mark on the back of each microscope slide the direction that the slide will face when placed on the post. Place each slide on the appropriate side of the post.
6. Use tape or rubber bands to attach the slides to the post, as shown on the lower-right side of this page. Use your finger to spread a thin, even film of petroleum jelly on one side of each slide.
7. Return to the site the following day. Remove the slides. Place each slide carefully in the slide box. (Note: Do not touch the greased surface.)

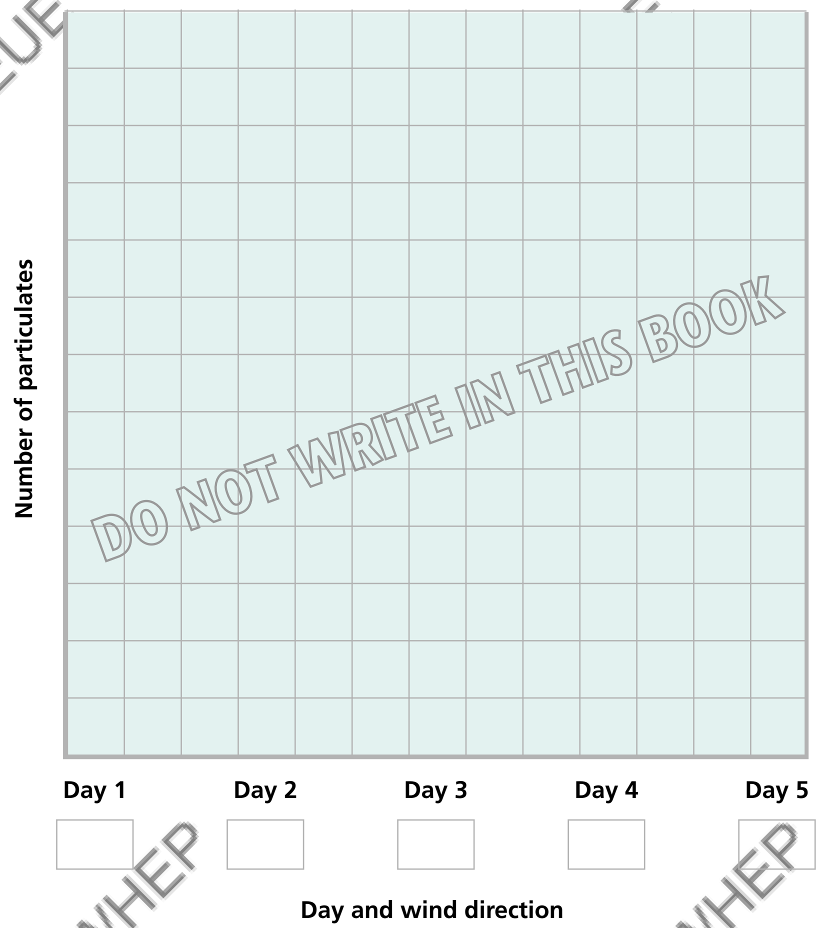
Step 6

8. Place new slides on the post, and record the wind direction and weather conditions.
9. Examine each slide under the microscope at 100×. Focus on one section that you have chosen at random. Count the number of particulates that you observe in the section. Record this number in column 1 of your table.
10. Move the slide, and examine another section that you have chosen at random. Count the number of particulates in this section. Record this number in column 2 of your table.
11. Repeat step 10 three more times so that you have a total of five observations per slide. Record the total number of particulates counted in the five sections.
12. Repeat steps 7 through 11 each weekday for 2 weeks.
13. When you have finished examining the slides and recording your results, total the number of particulates counted for each of the wind directions.
14. Using the data in your table, construct a bar graph. On the *y*-axis, plot the total number of particulates obtained in the past 5 days. On the *x*-axis, plot the day and wind direction. A sample graph is shown on the next page.

Step 9







Step 14

ANALYSIS AND CONCLUSION

1. **Analyzing Results** For your location, did any one wind direction or group of wind directions result in more particulates than any other direction or directions did?
2. **Interpreting Information** What are possible sources of these particulates?
3. **Applying Conclusions** What would your results likely be if you set up this investigation near a populated urban area?
4. **Evaluating Methods** Did weather conditions have any effect on your results? Explain your answer.
5. **Graphing Data** Construct a second graph that uses data from all 10 days on which you collected samples. How does this graph differ from your previous graph?
6. **Identifying Patterns** Did you see a different pattern in the data when you plotted data from more than 5 days? Explain your answer.

Extension

1. **Research** Use the library or the Internet to research common particulates. Use your research to identify common particulates on several of the slides from this investigation. Which particulates are most common in your area? Explain why these particulates are most common.

8243

LONG-TERM PROJECT 3

Duration

1 month

Objectives

- USING SCIENTIFIC METHODS **Measure and record** weather variables twice every day.
- **Predict** weather conditions based on data that you collected.

Materials

aneroid barometer or barograph
compass, magnetic
paper, graph
thermometer, Celsius

Safety

Correlating Weather Variables

Identifying weather patterns that exist in an area is the first step in making weather predictions. Weather records are used to identify relationships between variables. Although weather cannot be predicted with complete accuracy, the probability that certain weather conditions will happen at a given time and place can be established.

In this investigation, you will gather and organize weather information in a data table and will present the information as a graph. Then, you will analyze the relationships between the data collected and will make predictions about the weather.

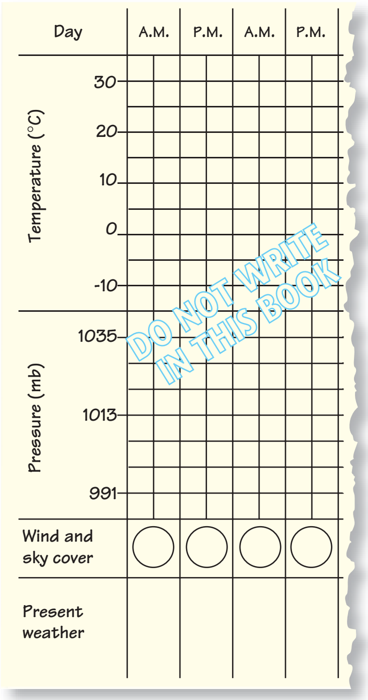
PROCEDURE

1. Use graph paper to make two charts like the one shown at lower left. One chart will hold data from the first 15 days of the project. The other will hold data from the second 15 days of the project. At the top of each column, write the date on which the observation was made.

2. Twice daily for a month, at about the same times every day (about 10 hours apart), measure and record the following weather variables: temperature (°C), barometric pressure (mb), wind direction, cloud cover, and weather conditions. The chart on the following page explains how to measure and record these variables.

3. Use the month's data to make two graphs. Do so by connecting the points for temperature with one smooth line and the points for pressure with a second smooth line.

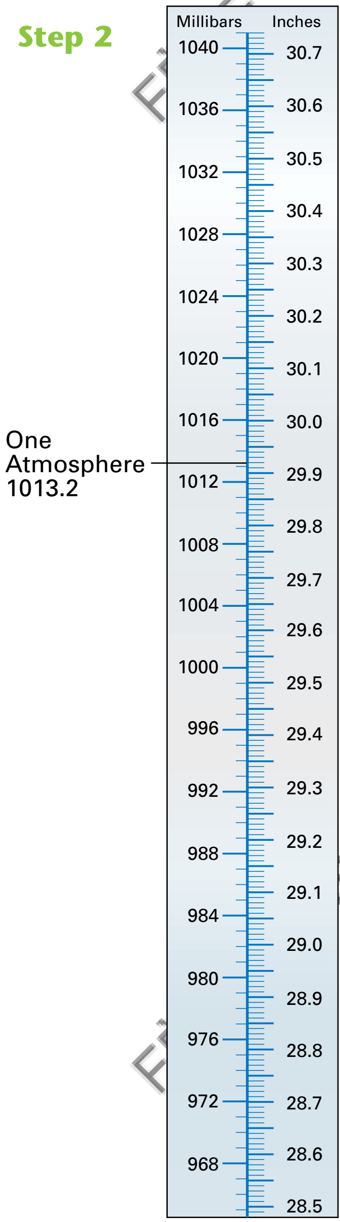
Step 1



8244

How to Measure Weather Variables	
Weather variable	**How to measure and record the variable**
Temperature	Place a thermometer where it is in the shade and is not exposed to precipitation. Wait at least 3 min, and then read the temperature. Plot a point for that temperature on your chart.
Barometric pressure	Using a barometer, record the barometric pressure to the nearest tenth of a millibar. If your barometer is calibrated in "inches of mercury," change inches to millibars by using the Barometric Conversion Scale at right. Plot a point for the barometric pressure.
Wind direction	Determine the wind direction by using a weather vane or by observing objects moved by the wind. Wind direction is named according to the direction from which the wind blows. Use a compass to help determine direction. Using the symbols shown in the Table of Weather Symbols in the Reference Tables section of the Appendix of this book, record wind direction on the circles at the bottom of your chart.
Cloud cover	Estimate the amount of sky that is covered by clouds. Using the symbols shown in the Table of Weather Symbols in the Reference Tables section of the Appendix of this book, shade the circles at the bottom of your chart.
Weather conditions	Observe the present weather conditions. Using the Table of Weather Symbols in the Reference Tables section of the Appendix of this book, draw in your chart the symbol that most accurately indicates the weather conditions.

Step 2



ANALYSIS AND CONCLUSION

1. **Evaluating Data** According to your graph, on how many days was the temperature falling? On how many days was the barometric pressure falling?
2. **Identifying Patterns** Of the days that had falling temperature, how many had rising barometric pressure?
3. **Inferring Relationships** In general, what is the relationship between temperature and pressure?
4. **Analyzing Results** What sky cover and wind direction are generally associated with falling barometric pressure?
5. **Interpreting Results** What weather conditions are generally associated with high barometric pressure? with low barometric pressure?
6. **Drawing Conclusions** How do the relationships between certain weather variables help you predict the weather?
7. **Evaluating Methods** Which weather variables are most useful in predicting precipitation?

Extension

1. **Research** Find out what the normal temperature, pressure, and precipitation are for your area during the time period in which you recorded your data. Do your observations match the normal conditions for that time period? How can you explain differences between your observations and the normal conditions?

8245

LONG-TERM PROJECT 4

Duration
1 week

Objectives

- **USING SCIENTIFIC METHODS** **Observe and record** locations of weather fronts.
- **Predict** weather conditions based on data you collected.
- **Compare** the movement of weather fronts in different seasons.

Materials

- pencils, colored
- daily weather maps for consecutive days (5)

Weather Forecasting

Every three hours, the National Weather Service collects data from about 800 weather stations located around the world. Daily newspapers summarize this weather data in the form of national weather maps. The data include temperature, precipitation, cloud cover, and barometric pressure. The patterns produced by the data allow meteorologists to identify weather fronts and to provide information about weather conditions around the globe.

In this investigation, you will use a series of daily weather maps to track the movements of weather systems in the winter months. Then, you will use these datas to predict weather conditions.

PROCEDURE

1. Find a local or national newspaper that prints a daily weather map from the National Weather Service. You may also use the Internet to find daily weather maps.
2. Cut out or print out the map, and write on it the date that it represents.
3. Make a data table similar to the sample table shown at the bottom of this page.
4. Fill in your table with the information from the weather map.
5. Make at least one copy of the blank weather map on the third page of this exercise.

	Week 1 (winter)					Week 2 (spring)		
	1	2	3	4	5	1	2	3
Temperature								
Barometric pressure								
Barometric trend (R = rising; F = falling; S = steady)								
Wind direction								
Cloud cover (C=clear; CL=cloudy; PC=partly cloudy; O=overcast)								
Present weather (rain, sleet, snow, etc.)								
Prediction								

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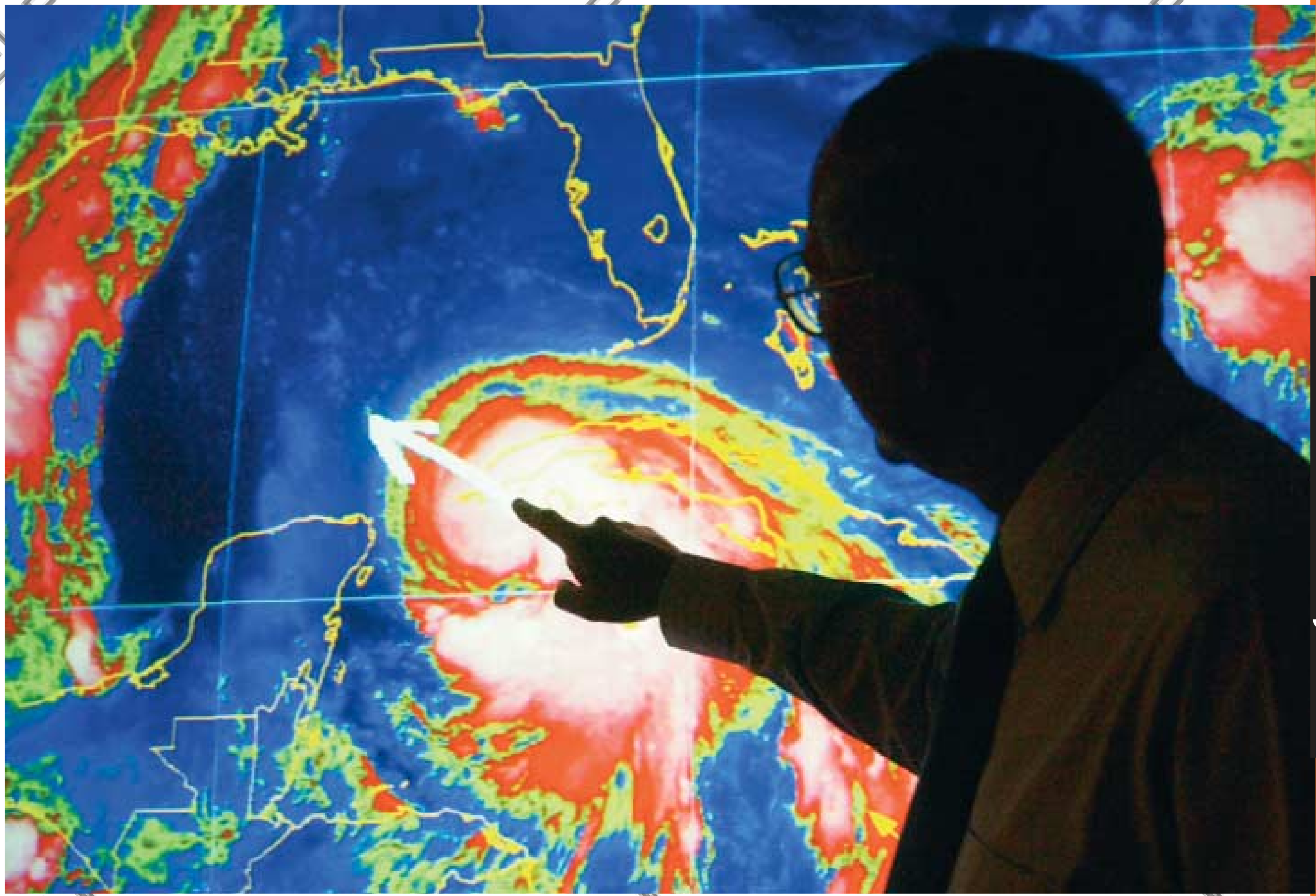
Step 3

8246



A meteorologist from the National Weather Service tracks the path of a hurricane in the Gulf of Mexico.

6 On your copy of the map, put an *L* at any locations where low-pressure centers are shown on the daily weather map that you collected in step 1. Circle the *L*s with a colored pencil, and label each circle with the date.

7 Put an *H* on your map at the locations of any high-pressure centers. Circle the *H*s with a second colored pencil, and label each circle with the date.

8 Repeat steps 1–7, but use four consecutive daily weather maps that follow the first day's map, and reuse your data table and weather map. For each symbol, use the same colors that you used for the first day's map.

9 Draw arrows to connect the daily positions of each high-pressure center and ofeach low-pressure center.

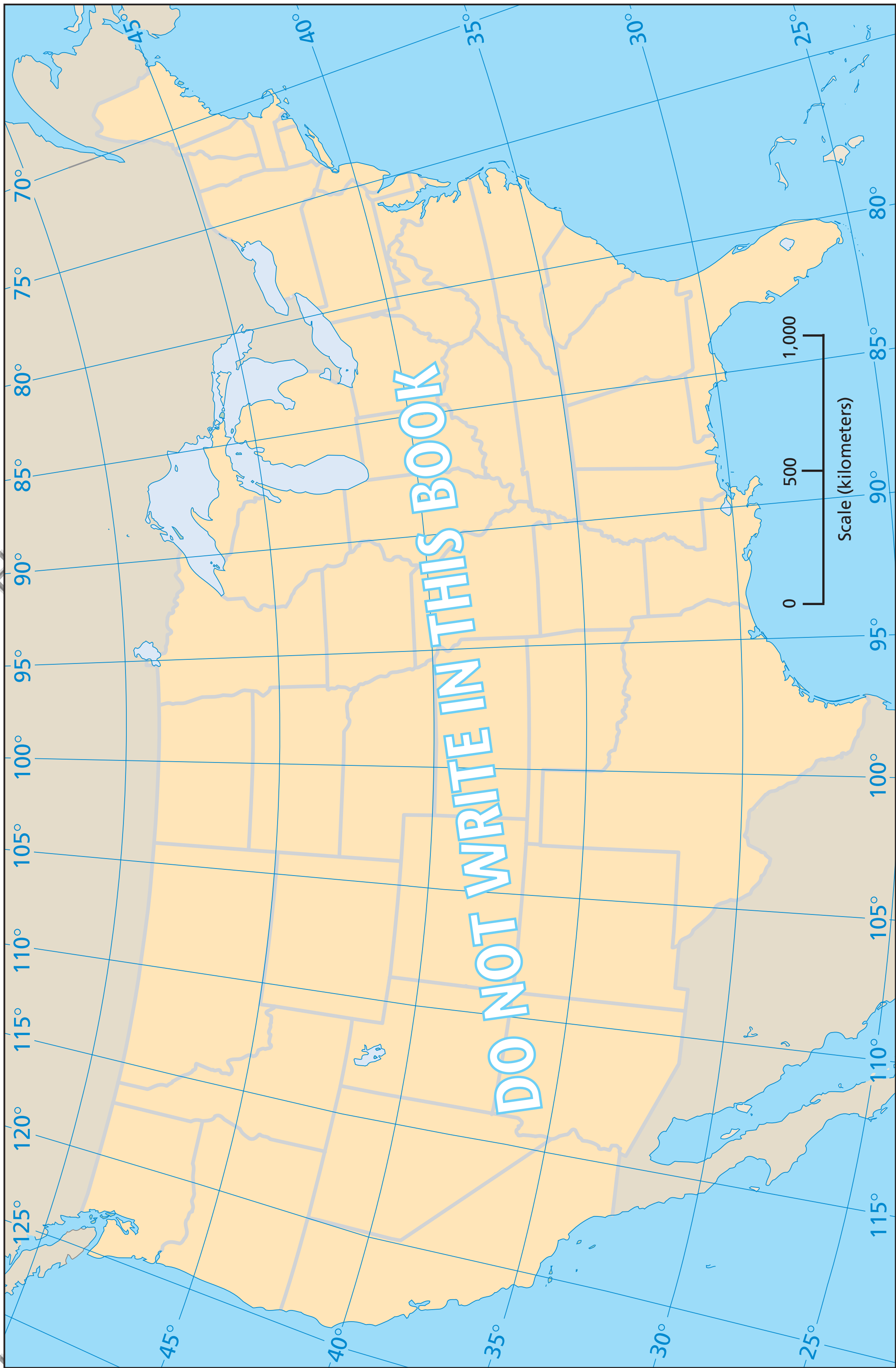
10 Use the formula below to calculate the average velocity (in kilometers per day) of each high-pressure center and each low-pressure center. The average velocity equals the total distance traveled divided by the number of days traveled, or

$$\textit{average velocity} = \frac{\textit{total distance traveled}}{\textit{number of days}}$$

8247

LONG-TERM PROJECT 4 *Weather Forecasting, continued*

Step 5



Many hurricane-prone areas have established evacuation routes to help people reach safety before a hurricane hits. This man is boarding up windows in his house to prepare for an imminent hurricane.

ANALYSIS AND CONCLUSION

1. **Analyzing Data** Generally, in which direction do the pressure centers over the United States move?
2. **Analyzing Results** From your calculations, what is the average rate of movement (in kilometers per day) of low- and high-pressure centers in winter?
3. **Making Predictions** Predict where the low- and high-pressure centers will be located on the day following the date of the last map in your series.
4. **Forming a Hypothesis** Refer to your series of daily weather maps to predict the weather for your hometown on the sixth day of the series. Write a forecast, and fill in the data table with your predictions of weather conditions.
5. **Evaluating Predictions** Was your prediction about the locations of the low- and high-pressure centers from question 3 accurate? Explain why or why not.
6. **Evaluating Hypotheses** Compare your weather prediction with the daily weather map for the appropriate day. Check the accuracy of your prediction. What factors could have caused errors in your prediction?
7. **Explaining Events** Describe the general weather conditions associated with regions of low and high atmospheric pressure.

Extension

1. **Designing Experiments** In the spring, repeat the entire investigation. What is the average rate of movement (in kilometers per day) of low- and high-pressure centers in the spring? Compare the rate of movement of pressure systems during spring and winter.

ENVIRONMENTAL CONNECTION

LONG-TERM PROJECT 5

Duration

6 months (October 1 to April 1)

Objectives

- **Record** temperature and precipitation data for eight regions.
- USING SCIENTIFIC METHODS **Graph and analyze** climate features for eight regions.
- **Classify** regions by using two climate classification systems.

Materials

- almanac
- atlas
- paper, graph
- rain gauge (optional)
- thermometer (optional)
- weather reports, daily

Safety

Comparing Climate Features

A graph of the monthly temperatures and amounts of precipitation for a region is called a *climatograph*. Climatographs can be used to compare the climates of different areas or to classify an area's climate.

In this investigation, you will use climate data to compare your local climate with the climates of other regions of the United States. You will keep a daily temperature and precipitation log. You will record data every day from the first day of October until the first day of April. You will then compare your graphed data with information about the world's climates. You will use this comparison to develop a conclusion about the type of climate that your location has.

PROCEDURE

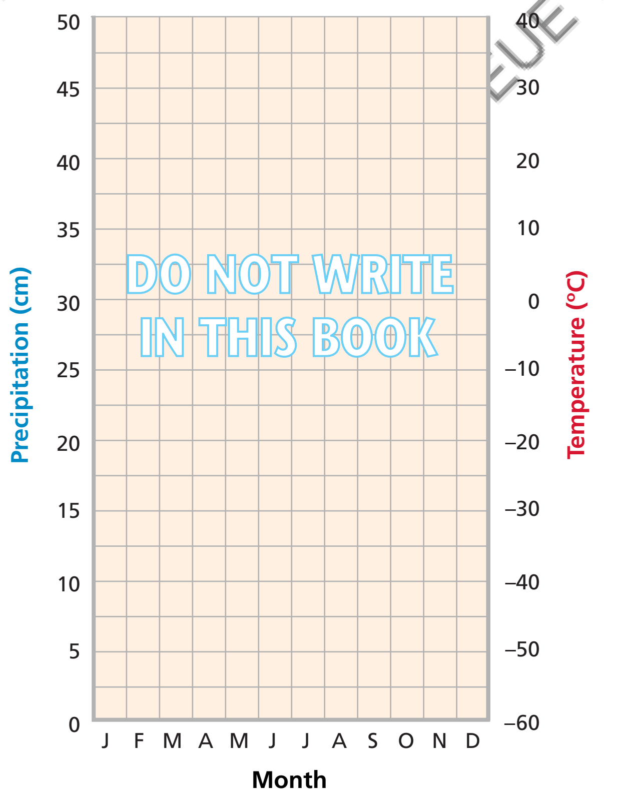
1. Listen to or watch a daily weather report for your area, or find this information in a daily newspaper or on the Internet. You may also keep your own records by using a thermometer and a rain gauge.
2. Beginning on the first day of October, keep a daily record of the high and low temperatures and of the amount of precipitation that occurs. During winter, snow should be melted before determining the amount of precipitation (in centimeters).

Rain gauges collect precipitation for scientists to measure.

3. Calculate the average temperature for each day by dividing the sum of the day's high and low temperatures by 2. Record this information.

4. At the end of each month, record the average monthly temperature given by the weather report, or calculate the average monthly temperature by dividing the sum of the daily averages by the number of days in the month. Also, record the total monthly precipitation given by the weather report.

5. Use the Internet or an almanac to look up climate data for your town or city and for seven other cities in the United States. Select one city from each of the following regions: New England, the Gulf Coast, the Midwest, the Southwest, the Pacific Northwest, the interior of Alaska, and the Hawaiian Islands.



Step 7

6. Look up the average monthly temperatures and precipitation for your town or city and for each city that you chose. Record these data in a table.

7. On your graph paper, make eight copies of the blank climatograph shown on the upper-right side of this page.

8. Label each blank climatograph with the name of one of the seven cities. Label the eighth climatograph with the name of your town or city.

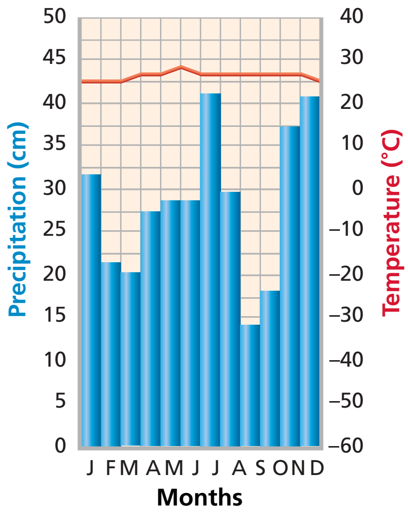
9. If you recorded temperature and precipitation in English (American) units, such as degrees Fahrenheit or inches, convert your measurements to SI units, such as degrees Celsius or centimeters. Use the SI Conversions table in the Appendix of this book to convert English units to SI units.

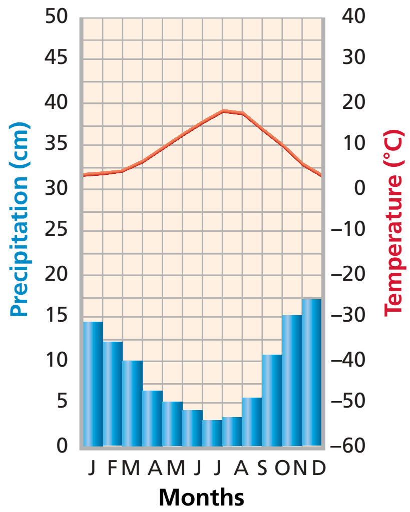
10. For each of the eight locations, plot the average temperature for January by placing a dot in the center of the square that is located in the column representing January and in the row representing that average temperature.

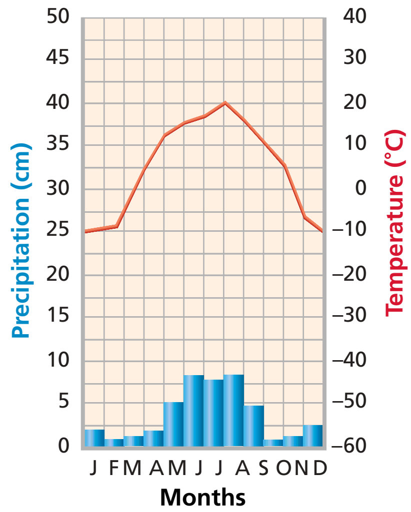
11. Using the same method, plot the average precipitation for January.

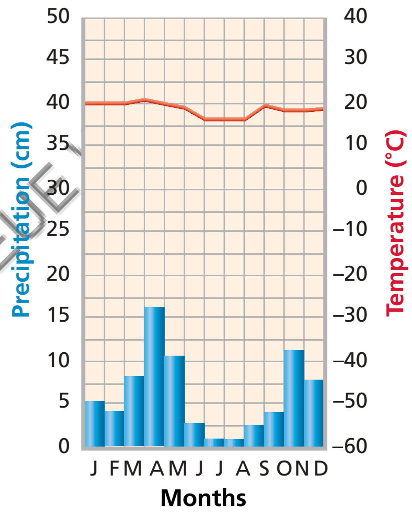
12. Repeat steps 10 and 11 for each month's data for each location. Then, connect the temperature points and connect the precipitation points in order of consecutive months.

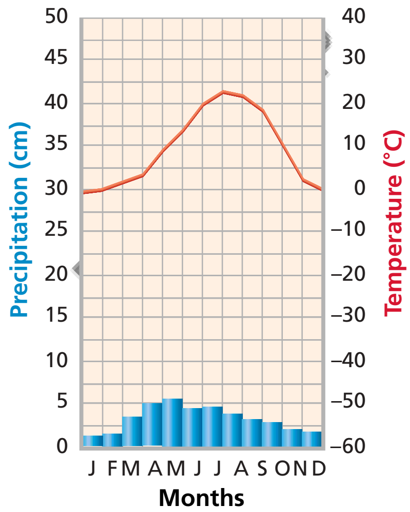
LONG-TERM PROJECT 5 *Comparing Climate Features, continued*

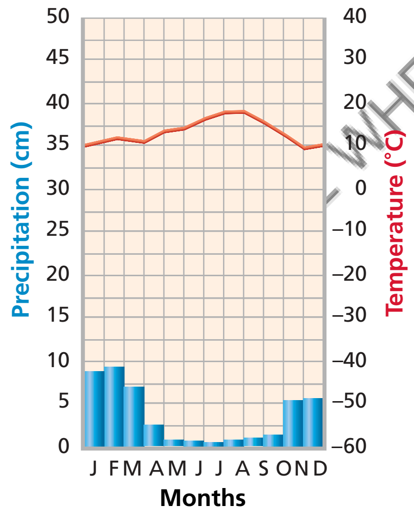
Tropical Rain Forest


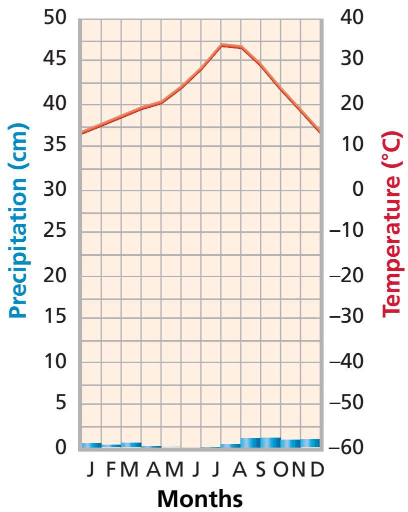
Marine West Coast


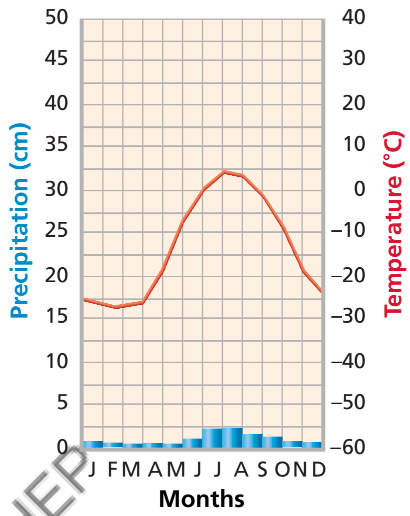
Subarctic


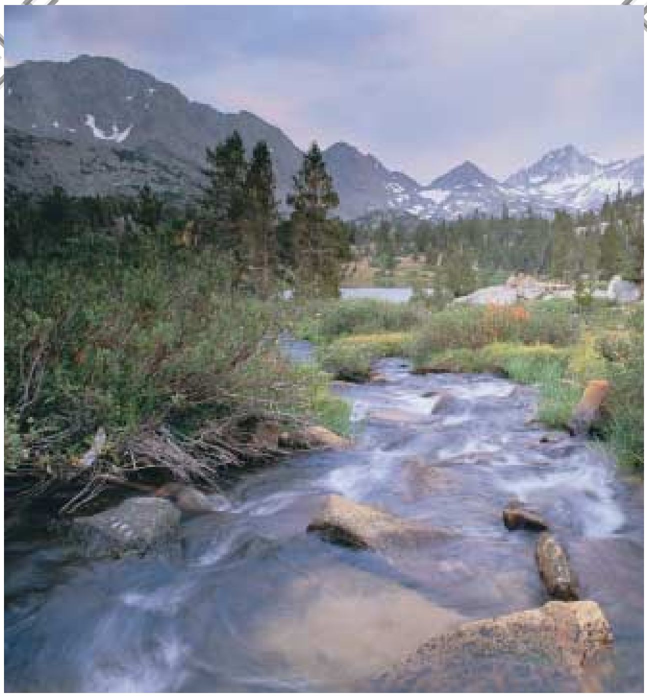
Tropical Savanna


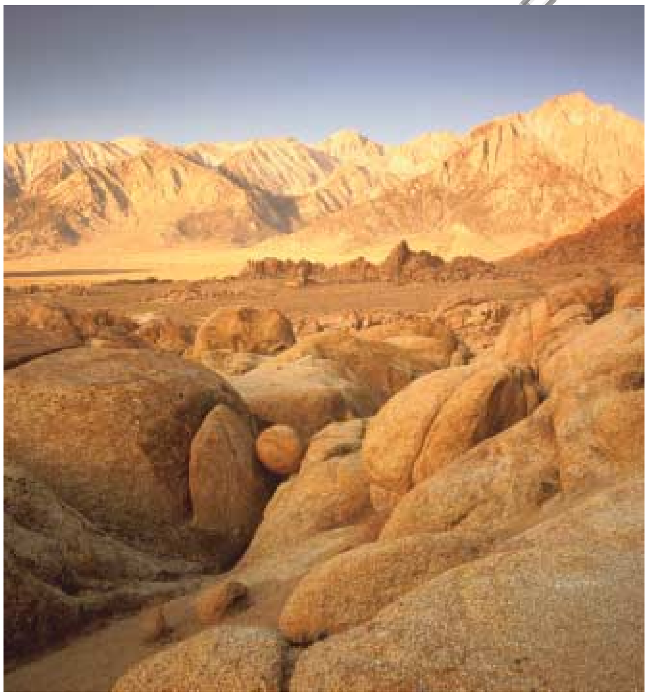
Humid Continental


Mediterranean


Tropical Desert


Tundra






Although rain forests and deserts may have similar latitudes, the amount of precipitation that rain forests receive differs greatly from the amount that deserts receive.

ANALYSIS AND CONCLUSION

1. **Making Comparisons** Compare each of the climatographs of your seven chosen U.S. cities with the sample climatographs on the previous page. Identify the climate type or types for each location that you selected. What features of each climatograph helped you classify each region?

2. **Analyzing Results** Use the climatograph for your area to classify your regional climate. What features of your climatograph helped you identify the climate type?

3. **Examining Data** Compare the average temperatures and precipitation amounts for your location that you collected to the values that you obtained from the Internet or an almanac. Do you think that this year's climate data are typical for your region? Explain your answer.

4. **Classifying Information** How would each of the climatographs, including the one for your region, fit into the climate classification system outlined in the chapter entitled "Climate"?

5. **Evaluating Methods** In this investigation, you compared climates by looking at average precipitation and temperature. What other factors might affect the climate of an area? Give examples of each factor.

Extension

1. **Making Comparisons** Bermuda is a small island in the Atlantic Ocean. Bermuda is at about the same latitude as St. Louis, Missouri, which lies in the middle of a continent. In which of the two locations does the temperature vary least from month to month? Explain the cause of the temperature pattern in the location that has the more moderate pattern.

LONG-TERM PROJECT 6

Duration

8 months

Objectives

- **Observe** the position of Mars in the night sky for 8 months.
- USING SCIENTIFIC METHODS **Graph** the apparent movement of Mars through the night sky.
- **Identify** changes in the relative positions of Earth and Mars.

Materials

- celestial sphere model (optional)
- compass, magnetic
- constellation charts
- flashlight
- metric ruler

Planetary Motions

While observing the evening sky over a period of time, you might have noticed that some objects look like stars but do not maintain a fixed position relative to the celestial sphere. These objects are the planets. As viewed from Earth at various times during the year, the patterns in the planets' motion differ from the patterns that you might expect.

In this investigation, you will observe the planet Mars in the night sky on the 1st and 15th of each month over a period of 8 months. Then, you will use your observations to draw conclusions and make predictions about planetary motion.

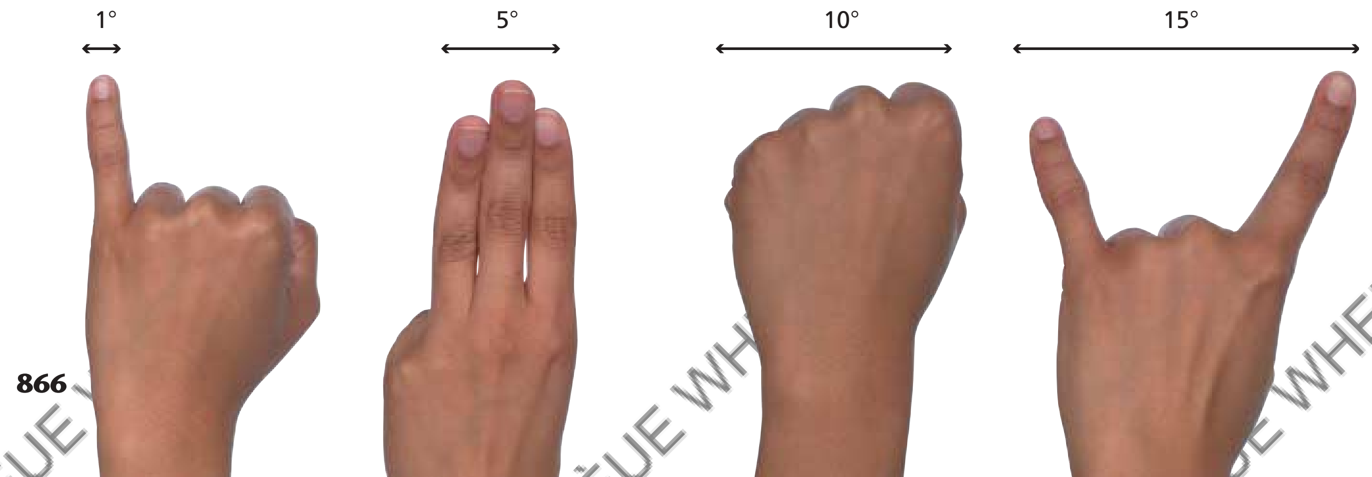
PROCEDURE

1. Obtain data on the positions of Mars in the night sky throughout the last year from an astronomical yearbook or from the Internet. Astronomical yearbooks can be found at most libraries.
2. Check the Internet, an almanac, or the weather section of a newspaper to find the time of night that Mars will be visible in your area.
3. Copy the star chart on the third and fourth pages of this lab onto a separate piece of paper.
4. Practice measuring angular distance by using the method shown at the bottom of this page. Always use the same hand when measuring angular distance. To have confidence in the accuracy of your measurements, you may need to practice measuring angular distance for a few days or weeks before beginning this lab.

Step 4

Estimating Angular Distance in Degrees

Hold your hand at arm's length.



In 2003, Mars was closer to Earth than it had been for about 60,000 years.

5. On the 1st and 15th of each month, go at night to an area that gives you a clear view of the eastern, southern, and western skies. (Note: If the skies are not clear on the 1st and 15th, make observations as close to these dates as possible.)

6. At the same time each night, locate Mars in the night sky. Mars will have a dull red appearance.

7. Use your magnetic compass to position yourself facing south, and observe the position of Mars relative to the background stars.

8. Choose a constellation. Use the method illustrated at the bottom of the previous page to estimate Mars's angular distance (in degrees) from the constellation.

9. On your star chart, locate the constellation from which you measured Mars's angular distance. Draw Mars in the appropriate position relative to the constellation, and label the planet's position with the date.

10. Compare the apparent brightness of Mars with that of the background stars. Record your observation on a separate sheet of paper.

11. Repeat steps 6–10 on the 1st and 15th of each month for the next 8 months.

12. After each observation, draw an arrow from Mars's previous position to the position that you just observed. The progression of arrows will show Mars's apparent path.

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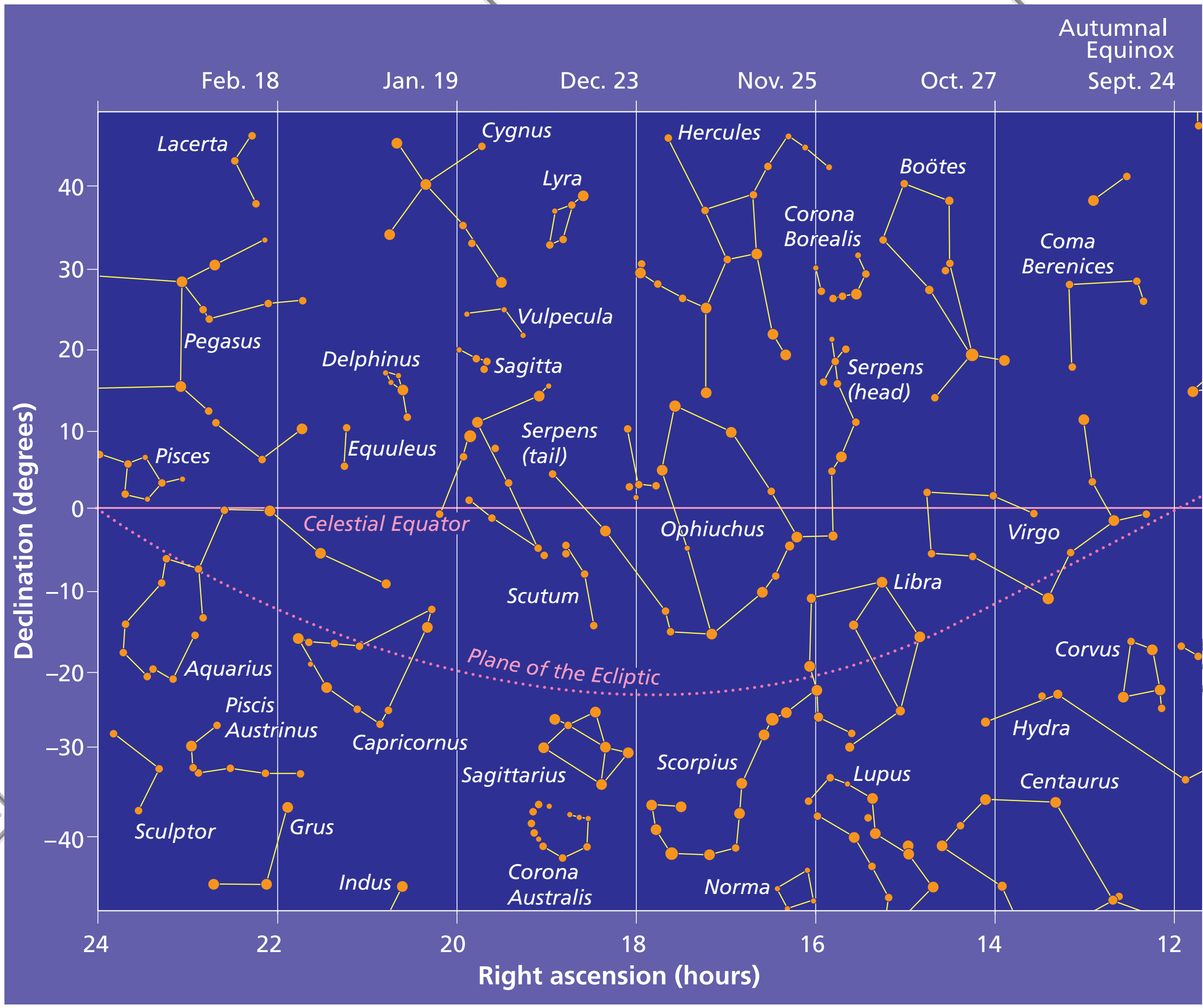


Step 3

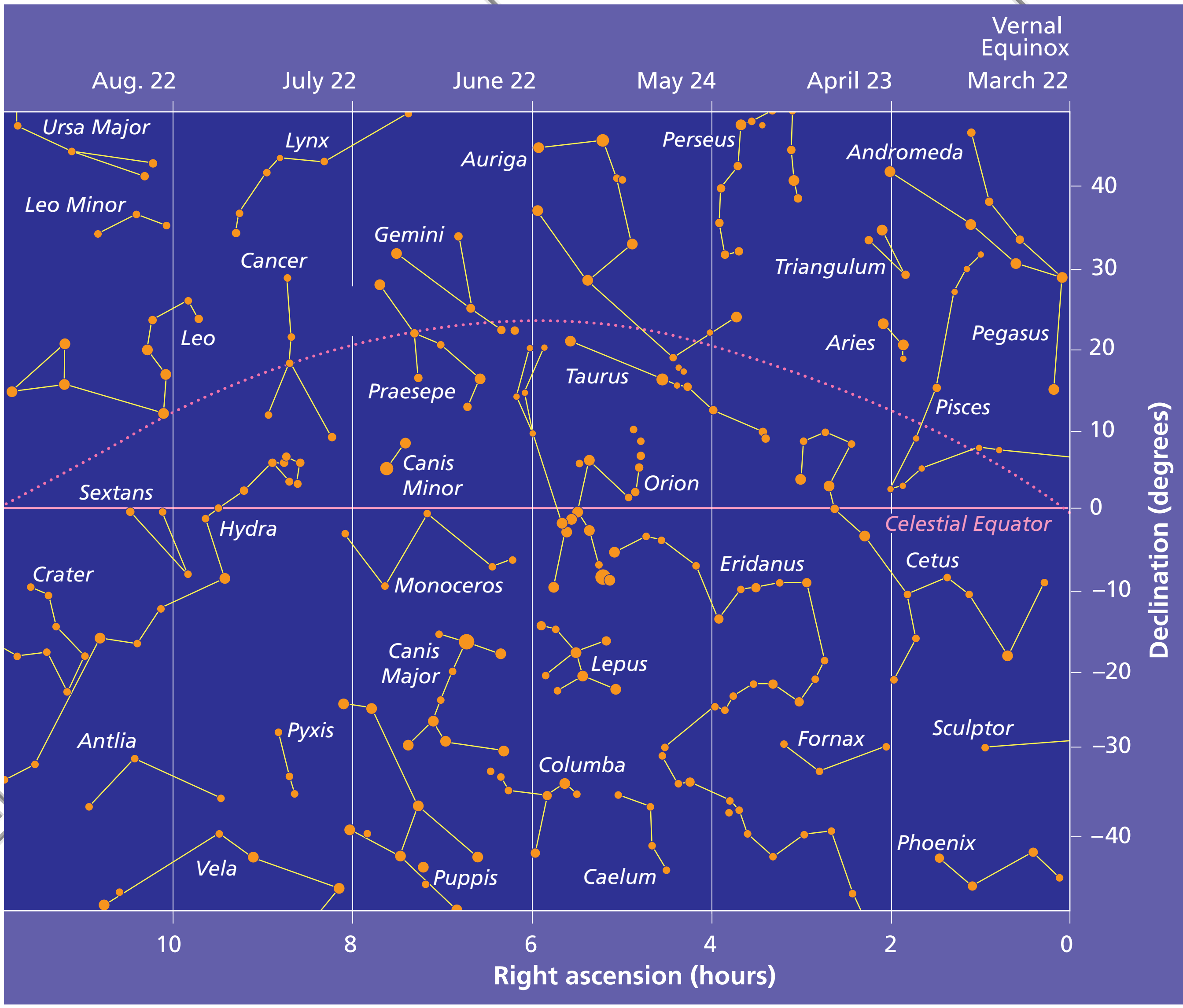
ANALYSIS AND CONCLUSION

1. **Examining Data** In which months does Mars appear highest in the night sky? lowest in the night sky?

2. **Evaluating Results** In which direction does Mars appear to move across the sky? At any point during the year, does Mars appear to deviate from this apparent path?

3. **Making Predictions** In one year from today, will Mars be in the same position that it is in today? Explain.

4. **Drawing Conclusions** From your observations of the apparent brightness of Mars at different times of the year, what can you infer about the distance between Mars and Earth? Explain your answer.



Extension

1. **Evaluating Predictions** In the Analysis and Conclusion section of this investigation, you predicted the position of Mars in one year. Use the Internet or library to research where astronomers predict Mars will be in one year. Was your prediction accurate? Explain why or why not.

2. **Evaluating Hypotheses** Repeat this investigation, but measure the positions of another planet, such as Venus, for the course of 8 months. Write a brief essay that explains how the paths of Mars and Venus differ.

REFERENCE TABLES SI Conversions

The metric system is used for making measurements in science. The official name of this system is the Système Internationale d'Unités, or International System of Measurements (SI).

SI Units	From SI to English	From English to SI
Length		
kilometer (km) = 1,000 m	1 km = 0.62 mile	1 mile = 1.609 km
meter (m) = 100 cm	1 m = 3.28 feet	1 foot = 0.305 m
centimeter (cm) = 0.01 m	1 cm = 0.394 inch	1 inch = 2.54 cm
millimeter (mm) = 0.001 m	1 mm = 0.039 inch	
micrometer (μm) = 0.000 001 m		
nanometer (nm) = 0.000 000 001 m		
Area		
square kilometer (km^2) = 100 hectares	1 km^2 = 0.386 square mile	1 square mile = 2.590 km^2
hectare (ha) = 10,000 m^2	1 ha = 2.471 acres	1 acre = 0.405 ha
square meter (m^2) = 10,000 cm^2	1 m^2 = 10.765 square feet	1 square foot = 0.093 m^2
square centimeter (cm^2) = 100 mm^2	1 cm^2 = 0.155 square inch	1 square inch = 6.452 cm^2
Volume		
liter (L) = 1,000 mL = 1 dm^3	1 L = 1.06 fluid quarts	1 fluid quart = 0.946 L
milliliter (mL) = 0.001 L = 1 cm^3	1 mL = 0.034 fluid ounce	1 fluid ounce = 29.577 mL
microliter (μL) = 0.000 001 L		
Mass		* Equivalent weight at Earth's surface
kilogram (kg) = 1,000 g	1 kg = 2.205 pounds*	1 pound* = 0.454 kg
gram (g) = 1,000 mg	1 g = 0.035 ounce*	1 ounce* = 28.35 g
milligram (mg) = 0.001 g		
microgram (μg) = 0.000 001 g		
Energy		
British Thermal Units (BTU)	1 BTU = 1,055.056 joules	1 joule = 0.00095 BTU

Temperature

Conversion of Fahrenheit to Celsius:
$°C = \frac{5}{9}(°F - 32)$

Conversion of Celsius to Fahrenheit:
$°F = \frac{9}{5}(°C) + 32$

Mineral Uses

Metallic Minerals		
Mineral and chemical formula	**Location of economically important deposits**	**Important uses**
Chalcopyrite, $CuFeS_2$	Chile, U.S., and Indonesia	electrical and electronic products, wiring, telecommunications equipment, industrial machinery and equipment
Chromite, $FeCr_2O_4$	South Africa, Kazahkstan, and India	production of stainless steel, alloys, and metal plating
Galena, PbS	Australia, China, and U.S.	batteries, ammunition, glass and ceramics, and X-ray shielding
Gold, Au	South Africa, U.S., and Australia	computers, communications equipment, spacecraft, jet engines, dentistry, jewelry, and coins
Ilmenite, $FeTiO_3$	Australia, South Africa, and Canada	jet engines; missile components; and white pigment in paints, toothpaste, and candy
Magnetite, Fe_3O_4	China, Brazil, and Australia	steelmaking
Uraninite, UO_2	Canada and Australia	fuel in nuclear reactors and manufacture of radioisotopes

Nonmetallic Minerals		
Mineral and chemical formula	**Location of economically important deposits**	**Important uses**
Barite, $BaSO_4$	China, India, and U.S.	weighting agent in oil well drilling fluids, automobile paint primer, and X-ray diagnostic work
Borax, $Na_2B_4O_7 \bullet 10H_2O$	Turkey, U.S., and Russia	glass, soaps and detergents, agriculture, fire retardants, and plastics and polymer additives
Calcite, $CaCO_3$	China, U.S., and Russia	cement, lime production, crushed stone, glassmaking, chemicals, and optics
Diamond, C	Australia, Democratic Republic of the Congo, and Russia	jewelry, cutting tools, drill bits, and manufacture of computer chips
Fluorite, CaF_2	China, Mexico, and South Africa	hydrofluoric acid, steelmaking, water fluoridation, solvents, manufacture of glass, and enamels
Gypsum, $CaSO_4 \bullet 2H_2O$	U.S., Iran, and Canada	wallboard, building plasters, and manufacture of cement
Halite, NaCl	U.S., China, and Germany	chemical production, human and animal nutrition, highway deicer, and water softener
Sulfur, S	Canada, U.S., and Russia	sulfuric acid, fertilizers, gunpowder, and tires
Kaolinite, $Al_2Si_2O_5(OH)_4$	U.S., Uzbehkistan, and Czech Republic	glossy paper and whitener and abrasive in toothpaste
Orthoclase, $KAlSi_3O_8$	Italy, Turkey, and U.S.	glass, ceramics, and soaps
Quartz, SiO_2	U.S., Germany, and France	glass, computer chips, ceramics, abrasives, and water filtration
Talc, $Mg_3Si_4O_{10}(OH)_2$	China, U.S., and Republic of Korea	ceramics, plastics, paint, paper, rubber, and cosmetics

REFERENCE TABLES

Guide to Common Minerals

This table is used in the chapter lab for the chapter entitled "Minerals of Earth's Crust."

	Luster		Hardness	Cleavage	Fracture	Color/opacity
Nonmetallic; light color	glassy to pearly	Scratches glass	6	two cleavage planes at nearly right angles		various colors but often white or pink; opaque
	glassy		6	two cleavage planes at 86° and 94°		colorless, white, pink, or various colors; translucent to opaque
	glassy and waxy		7	no cleavage	conchoidal fracture	various colors; transparent to opaque
	glassy		6.5–7	no cleavage	conchiodal to irregular fracture	olive green; transparent to translu-cent
	glassy	Does not scratch glass	2.5–3	three cleavage planes at right angles		colorless to gray; transparent to opaque
	glassy		3	three cleavage planes at 75° and 105°		colorless or white and may be tinted; transparent to opaque
	glassy, pearly, or silky		1–2.5	one perfect cleavage plane	conchoidal and fibrous fracture	white, pink, or gray to colorless; transparent to opaque
	pearly to waxy		1	one cleavage plane		white to green; opaque
	glassy or pearly		2–2.5	one cleavage plane		colorless to light gray or brown; translucent to opaque
	glassy		4	eight cleavage planes (octahedral)		green, yellow, purple, and other colors; transparent to translucent
	glassy		4.5–5	no cleavage	conchiodal to irregular fracture	green, blue, violet, brown, or color-less; translucent to opaque
	silky		3.5–4	no cleavage	irregular, splintery fracture	green; translucent to opaque
Nonmetallic; dark color	glassy and silky	Scratches glass	5–6	two cleavage planes at 56° and 124°		dark green, brown, or black; translu-cent to opaque
	resinous and glassy		6.5–7.5	no cleavage	irregular fracture	dark red or green; transparent to opaque
	pearly and glassy	Does not scratch glass	2.5–3	one cleavage plane		black to dark brown; translucent to opaque
	metallic to earthy		5.5–6.5	no cleavage	irregular fracture	reddish brown to black; opaque
Metallic	metallic or earthy	Does not scratch glass	1–2	one cleavage plane		black to gray; opaque
	metallic		2.5	three cleavage planes at right angles		lead gray; opaque
	metallic	Scratches glass	5–6	two cleavage planes at 56° and 124°		iron black; opaque
	metallic		6–6.5	no cleavage	conchoidal to irregular fracture	brass yellow; opaque

Streak	Specific gravity	Other properties	Mineral name and chemical formula
white	2.6	prismatic, columnar, or tabular crystals	orthoclase, $KAlSi_3O_8$
blue-gray to white	2.6 to 2.7	striations	plagioclase, (Na, Cl)(Al, Si)$_4$O$_8$
white	2.65	six-sided crystals	quartz, SiO_2
white to pale green	3.2 to 3.3	stubby, prismatic crystals	olivine, $(Mg, Fe)_2SiO_4$
white	2.2	cubic crystals and salty taste	halite, NaCl
white	2.7	may produce double image when you look through it	calcite, $CaCO_3$
white	2.2 to 2.4	thin layers and flexible	gypsum, $CaSO_4{\bullet}2H_2O$
white	2.7 to 2.8	soapy feel and thin scales	talc, $Mg_3Si_4O_{10}(OH)_2$
white	2.7 to 3	thin sheets	muscovite, $KAl_2Si_3O_{10}(OH)_2$
white	3.2	fluorescent under UV light; cubic and six-sided crystals	fluorite, CaF_2
white or pale red-brown	3.1	six-sided crystals	apatite, $Ca_5(OH, F, Cl)(PO_4)_3$
emerald green	4	fibrous, radiating aggregates or circular, banded structure	malachite, $CuCO_3{\bullet}Cu(OH)_2$
pale green or white	3.2	six-sided crystals	hornblende, $(Ca, Na)_{2\text{-}3}(Mg, Fe, Al)_5Si_6(Si, Al)_2O_{22}(OH)_2$
white	4.2	12- or 24-sided crystals	garnet, $Fe_3Al_2(SiO_4)_3$
white to gray	2.7 to 3.2	thin, flexible sheets	biotite, $K(Mg, Fe)_3AlSi_3O_{10}(OH)_2$
red to red-brown	5.25	granular masses	hematite, Fe_2O_3
black to dark green	2.3	greasy feel, soft, and flaky	graphite, C
lead gray to black	7.4 to 7.6	very heavy	galena, PbS
black to dark green	5.2	8- or 12-sided crystals; may be magnetic	magnetite, Fe_3O_4
greenish black	5	cubic crystals	pyrite, FeS_2

Guide to Common Rocks

This table is used in the chapter lab for the chapter entitled "Rocks."

Rock class	Grain size	Description	Rock class	Rock name
Made of crystals	Coarse grained	mostly light in color; shades of pink, gray, and white are common	igneous	granite
		dark in color; commonly black and white; heavy heft	igneous	gabbro
		foliated; layers of different minerals give a banded appearance	metamorphic	gneiss
		foliated; contains abundant amount of quartz, and may contain garnet; flaky minerals	metamorphic	schist
		nonfoliated; reacts with acid	metamorphic	marble
	Fine grained	usually light in color; many holes and spongy appearance; may float in water	igneous	pumice
		light to dark in color; glassy luster; conchoidal fracture	igneous	obsidian
		dark in color; may ring like a bell when struck with a hammer	igneous	basalt
		fine grained; foliated; cleaves into thin, flat plates	metamorphic	slate
Made of rock particles	Coarse grained	coarse-grained particles, more than 2 mm; rounded pebbles; some sorting; clay and sand are visible	sedimentary	conglomerate
		well-preserved fossils are common; can be scratched with a knife; many colors but usually white-gray; reacts with acid	sedimentary	limestone
		cube-shaped crystals; commonly colorless; does not react with acid	sedimentary	halite
	Medium grained	1/16 to 2 mm grains; mostly quartz fragments; surface feels sandy	sedimentary	sandstone
	Fine grained	soft and porous; commonly white or buff color	sedimentary	chalk
		microscopic grains; clay composition; smooth surface; hardened mud appearance	sedimentary	shale

Radiogenic Isotopes and Half-Life

Unstable isotopes, called *radiogenic isotopes* or *radioactive isotopes,* decay to form different isotopes called *daughter isotopes*. Each radiogenic isotope breaks down at a predictable rate, called its *half-life,* into a daughter isotope. Because of this predictable decay pattern, radiogenic isotopes are used to determine numeric dates for rocks. The table below describes several common radiometric dating methods.

Radiometric dating method	How it works	Parent isotope	Daughter isotope	Half-life	Effective dating range
Argon-argon dating ($^{39}Ar/^{40}Ar$)	Comparison made between ^{39}Ar and ^{40}Ar in a sample specially irradiated to form ^{39}Ar; ^{39}Ar is equivalent to ^{40}K in potassium-argon dating.	potassium-40 (^{40}K) irradiated to form argon-39 (^{39}Ar)	argon-40, ^{40}Ar	1.25 billion years	50,000 to 4.6 billion years
Fission track dating	Tracks of damage created by charged particles from radioactive decay that pass through a mineral's crystal lattice are counted under an electron microscope.	uranium, U	ultimately, lead, Pb, but also several other daughter isotopes	not applicable	500 years to 1 billion years
Potassium-argon dating ($^{40}K/^{40}Ar$)	Comparison is made between the amount of ^{40}K and amount of ^{40}Ar; over time, ^{40}K decreases and ^{40}Ar increases.	potassium-40, ^{40}K	argon-40, ^{40}Ar	1.25 billion years	50,000 to 4.6 billion years
Radiocarbon dating ($^{14}C/^{12}C$)	Comparison is made between the amount of ^{14}C in organic matter and the amount of ^{12}C; ^{12}C remains constant over time, and ^{14}C breaks down.	carbon-14, ^{14}C	nitrogen-14, ^{14}N	5,730 years	<80,000 years
Rubidium-strontium dating ($^{87}Rb/^{87}Sr$)	Comparison made between the ratio of $^{87}Sr/^{86}Sr$ and the ratio of $^{87}Rb/^{86}Sr$ to find the amount of ^{87}Sr formed by radioactive decay.	rubidium-87, ^{87}Rb	strontium-87, ^{87}Sr	48.8 billion years	10 million to 4.6 billion years
Thorium-lead dating	Comparison made between amount of ^{232}Th and the ratio of $^{208}Pb/^{204}Pb$; ^{232}Th breaks into ^{208}Pb, and ^{204}Pb remains constant.	thorium-232, ^{232}Th	lead-208, ^{208}Pb	14.0 billion years	>200 million years
Uranium-lead dating ($^{235}U/^{207}Pb$)	Comparison made between amount of ^{235}U and the ratio of $^{207}Pb/^{204}Pb$; ^{235}U breaks into ^{207}Pb, and ^{204}Pb remains constant.	uranium-235, ^{235}U	lead-207, ^{207}Pb	704 million years	10 million to 4.6 billion years
Uranium-lead dating ($^{238}U/^{206}Pb$)	Comparison made between amount of ^{238}U and the ratio of $^{206}Pb/^{204}Pb$; ^{238}U breaks into ^{206}Pb, and ^{204}Pb remains constant.	uranium-238, ^{238}U	lead-206, ^{206}Pb	4.5 billion years	10 million to 4.6 billion years

Topographic and Geologic Map Symbols

Topographic Map Symbols

Elevation markers

Contour lines	
Index contour lines	100
Depression contour lines	
Water elevation	9600
Spot elevation	x 9136

Boundaries

- National
- State
- County, parish, municipal
- Township, precinct, town
- Incorporated city, village, or town
- National or state reservation
- Small park, cemetery, airport, etc.
- Land grant

Buildings and Structures

Buildings	
School	
Church	
Cemetery	cem
Barn and warehouse	
Wells (non-water)	○ oil ○ gas
Open-pit mine, quarry, or prospect	
Tunnel	
Benchmark	⊗BM △8025
National Park	
Campsite	
Bridge	

Roads and Railroads

- Divided highway
- Road
- Trail
- Railroad

Geologic Map Symbols

Sedimentary Rocks

- Breccia
- Conglomerate
- Dolomite
- Limestone
- Mudstone
- Sandstone
- Siltstone
- Shale

Igneous and Metamorphic Rocks

- Extrusive
- Intrusive
- Metamorphic

Features

River	
Water well	water
Spring	
Lake	
Glacier	

Contour Map

This map is used in the chapter lab for the chapter entitled "Models of the Earth."

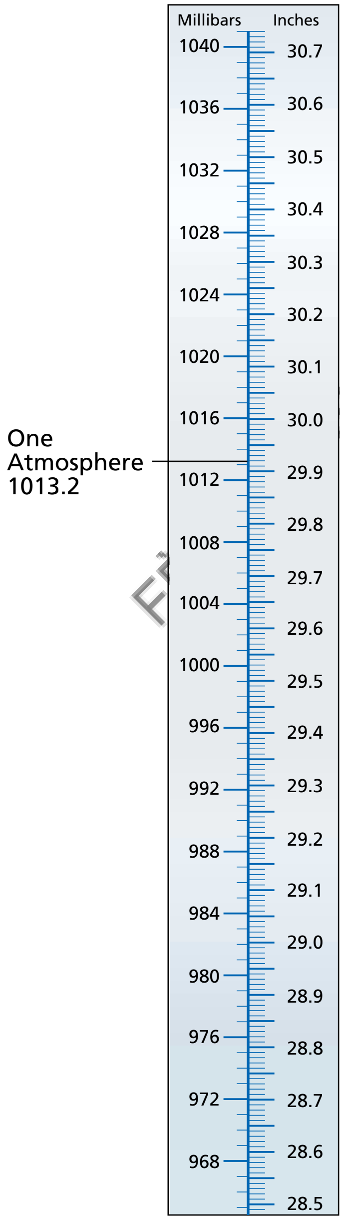
Humidity and Air Pressure

The Relative Humidity table below is used in the chapter lab for the chapter entitled "Water in the Atmosphere." The Barometric Conversion Scale is used in the Long-Term Project entitled "Correlating Weather Variables" in the Appendix.

Relative Humidity (%)

Dry-bulb temperature (°C)	Difference in temperature (°C) 1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
10	88	77	66	55	44	34	24	15	6	—
11	89	78	67	56	46	36	27	18	9	—
12	89	78	68	58	48	39	29	21	12	—
13	89	79	69	59	50	41	32	23	15	7
14	90	79	70	60	51	42	34	26	18	10
15	90	80	71	61	53	44	36	27	20	13
16	90	81	71	63	54	46	38	30	23	15
17	90	81	72	64	55	47	40	32	25	18
18	91	82	73	65	57	49	41	34	27	20
19	91	82	74	65	58	50	43	36	29	22
20	91	83	74	66	59	51	44	37	31	24
21	91	83	75	67	60	53	46	39	32	26
22	92	83	76	68	61	54	47	40	34	28
23	92	84	76	69	62	55	48	42	36	30
24	92	84	77	69	62	56	49	43	37	31
25	92	84	77	70	63	57	50	44	39	33
26	92	85	78	71	64	58	51	46	40	34
27	92	85	78	71	65	58	52	47	41	36
28	93	85	78	72	65	59	53	48	42	37
29	93	86	79	72	66	60	54	49	43	38
30	93	86	79	73	67	61	55	50	44	39
31	93	86	80	73	67	61	56	51	45	40
32	93	86	80	74	68	62	57	51	46	41
33	93	87	80	74	68	63	57	52	47	42
34	93	87	81	75	69	63	58	53	48	43
35	94	87	81	75	69	64	59	54	49	44
36	94	87	81	75	70	64	59	54	50	45
37	94	87	82	76	70	65	60	55	51	46
38	94	88	82	76	71	66	60	56	51	47
39	94	88	82	77	71	66	61	57	52	48
40	94	88	82	77	72	67	62	57	53	48

Barometric Conversion Scale



REFERENCE TABLES

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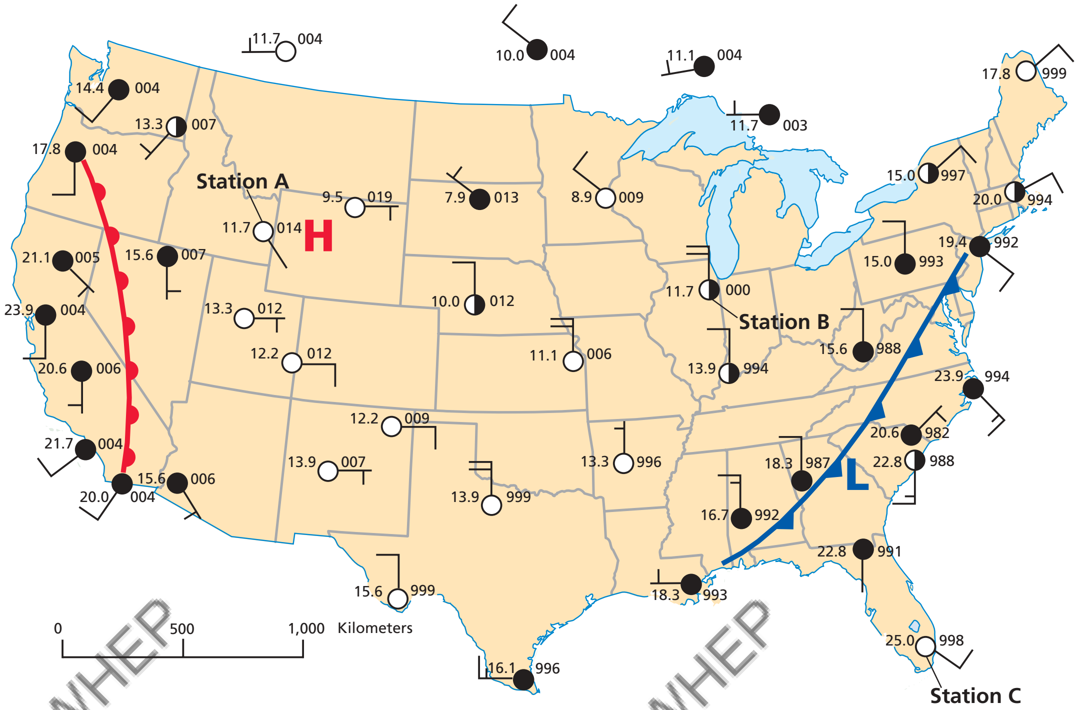
Weather Map of the United States

This map is used in the chapter lab for the chapter entitled "Weather."



Weather Map Symbols

This chart is used in the Long-Term Project entitled "Weather Forecasting."

Cloud cover (fraction of sky covered)	Clear; 1/8; Scattered; 3/8; 4/8; 5/8; Broken; 7/8; Overcast; Obscured; No data
Wind speed (knots)	Calm; 1 to 2; 3 to 7; 8 to 12; 13 to 17; 18 to 22; 23 to 27; 48 to 52; 73 to 77; 103 to 107
Wind direction	North; Northeast; East; Southeast; South; Southwest; West; Northwest
Weather conditions	Drizzle; Fog; Hail; Haze; Rain; Shower; Freezing rain; Smoke; Snow; Thunderstorm; Hurricane; Tropical storm

REFERENCE TABLES

Solar System Data

Notes

The **semimajor axis** is the average distance between an object and its primary (the body the object revolves around).

Surface gravity indicated for the gas giants is calculated for the altitude at which the atmospheric pressure equals 1 bar.

Rotation period and **orbital period** are sidereal measurements (relative to the stars, not the sun).

* This value indicates distance from the sun in AU.

† This value represents the rate of rotation at the sun's equator. The sun displays differential rotation; in other words, it rotates faster at its equator than at its poles.

R This value indicates retrograde rotation or retrograde revolution.

	Sun	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mass (10^{24} kg)	1,989,100	0.33	4.87	5.97	0.642	1,899	568	86.8	102	0.0125
Diameter (km)	1,390,000	4,879	12,104	12,756	6,794	142,984	120,536	51,118	49,528	2,390
Density (kg/m^3)	1,408	5,427	5,243	5,515	3,933	1,326	687	1,270	1,638	1,750
Surface gravity (m/s^2)	274	3.7	8.9	9.8	3.7	23.1	9	8.7	11	0.6
Escape velocity (km/s)	617.7	4.3	10.4	11.2	5	59.5	35.5	21.3	23.5	1.1
Rotation period (h)	609.12	1,407.6	5,832.5 R	23.9	24.6	9.9	10.7	17.2 R	16.1	153.3 R
Length of day (hours)	609.6†	4,222.6	2,802	24	24.7	9.9	10.7	17.2	16.1	153.3 R
Semimajor axis (10^6 km)	N/A	57.9	108.2	149.6	227.9	778.6	1,433.5	2,872.5	4,495.1	5,870
Perihelion (10^6 km)	N/A	46	107.5	147.1	206.6	740.5	1,352.6	2,741.3	4,444.5	4,435
Aphelion (10^6 km)	N/A	69.8	108.9	152.1	249.2	816.6	1,514.5	3,003.6	4,545.7	7,304.3
Orbital period (days)	N/A	88	224.7	365.2	687	4,331	10,747	30,589	59,800	90,588
Orbital velocity (km/s)	N/A	47.9	35	29.8	24.1	13.1	9.7	6.8	5.4	4.7
Orbital inclination (degrees)	N/A	7	3.4	0	1.9	1.3	2.5	0.8	1.8	17.2
Orbital eccentricity	N/A	0.205	0.007	0.017	0.094	0.049	0.057	0.046	0.011	0.244
Axial tilt (degrees)	7.25	0.01	2.6	23.5	25.2	3.1	26.7	82.2	28.3	57.5
Mean surface temperature (°C)	6,073	167	464	15	–65	–110	–140	–195	–200	–225
Global magnetic field?	yes	yes	no	yes	no	yes	yes	yes	yes	unknown

	Earth's moon	Major moons of Jupiter				Major moons of Saturn			
		Io	Europa	Ganymede	Callisto	Dione	Rhea	Titan	Iapetus
Mass (10^{20} kg)	0.073	893.2	480.0	1,481.9	1,075.9	0.375	11.0	1,345.5	15.9
Diameter (km)	3,475	3,643.2	3,121.6	5,262.4	4,820.6	1,120	1,528	5,150	1,436
Density (kg/m^3)	3,340	3,530	3,010	1,940	1,830	1,500	1,240	1,881	1,020
Rotation period (days)	655.7	1.77	3.55	7.15	16.69	2.74	4.52	15.95	79.33
Semimajor axis (10^3 km)	0.384*	421.6	670.9	1,070.4	1,882.7	377.40	527.04	1,221.83	3,561.3
Orbital period (days)	27.32	1.77	3.55	7.15	16.69	2.74	4.52	15.95	79.33

	Major moons of Uranus			Major moons of Neptune		Pluto's moon	Selected asteroids		Selected comets	
	Umbriel	Titania	Oberon	Triton	Nereid	Charon	Vesta	Ceres	Chiron	Hale-Bopp
Mass (10^{20} kg)	11.7	35.2	30.1	214	0.2	19	3	8.7	—	—
Diameter (km)	1,169	1,578	1,523	2,707	340	1,186	530	960 × 932	—	—
Density (kg/m^3)	1,400	1,710	1,630	2,050	1,000	2,000	—	—	—	—
Rotation period	4.14 days	8.71 days	13.46 days	5.87 days R	unknown	6.39 days	5.342 h	9.075 h	—	—
Semimajor axis (10^3 km)	266.30	435.91	583.52	354.76	5,513.4	19,600	2.362 *	2.767 *	13.7 *	250 *
Orbital period	4.14 days	8.71 days	13.46 days	5.87 days R	360.14 days	6.39 days	3.63 y	4.60 y	50.7 y	4,000 y

Topographic Provinces of North America

Geologic Map of North America

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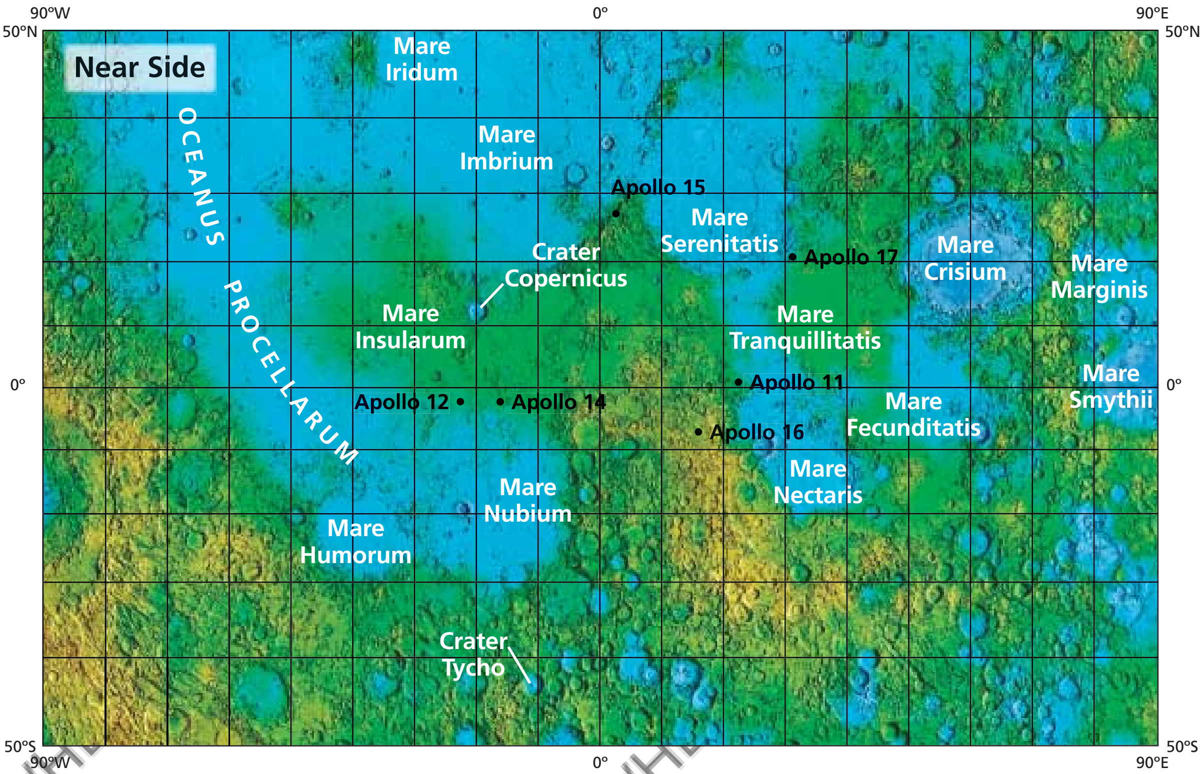
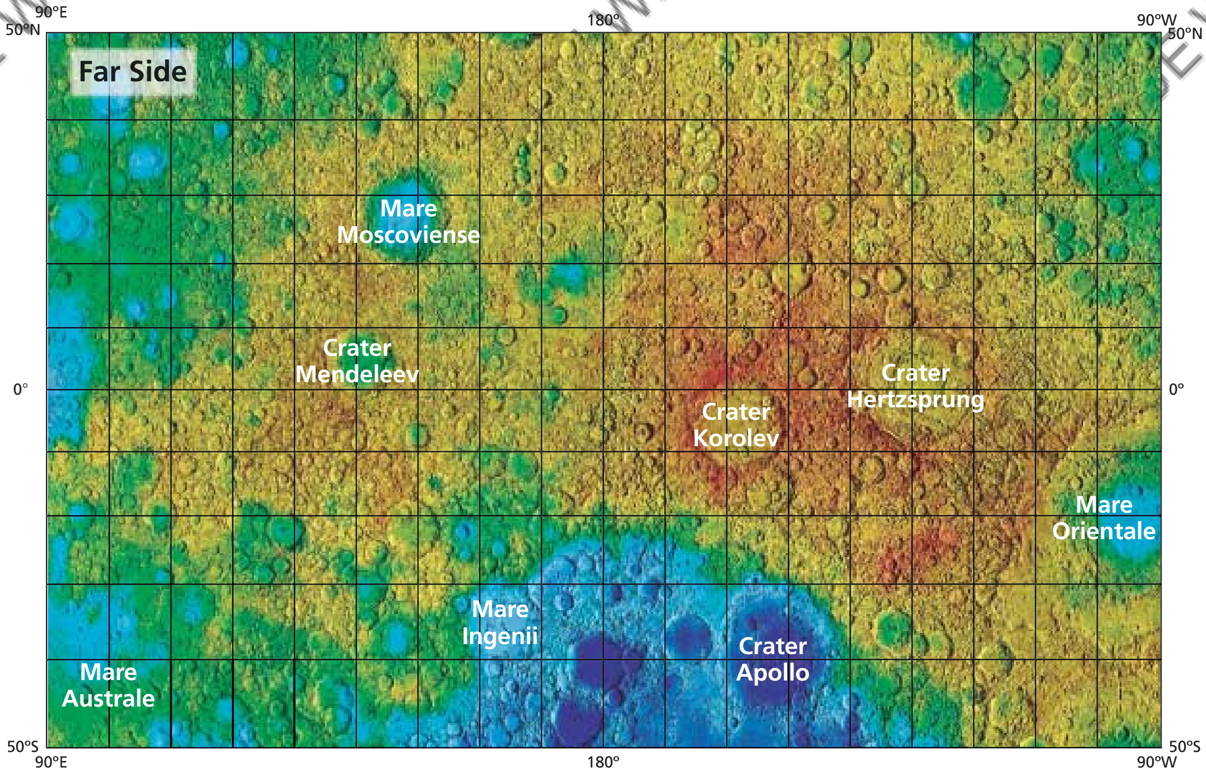
Mineral and Energy Resources of North America

REFERENCE MAPS

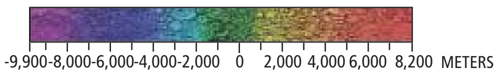
Fossil Fuel Deposits of North America

Topographic Maps of the Moon





● Apollo landing sites



Color-coded topography key

REFERENCE MAPS

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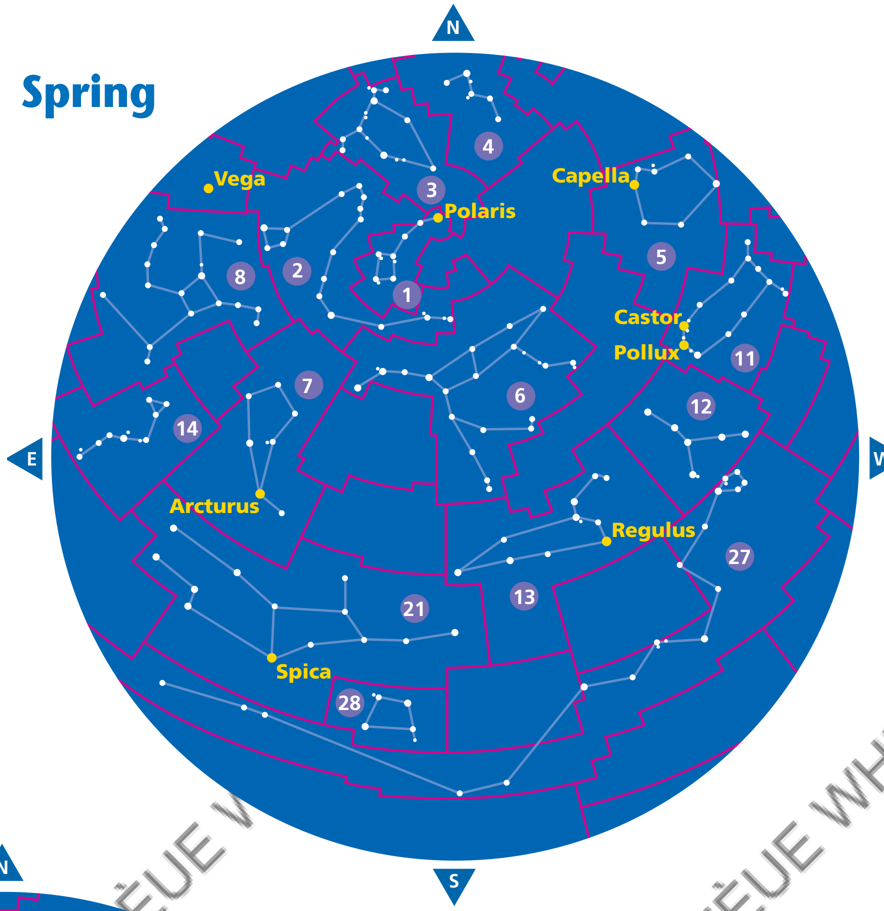
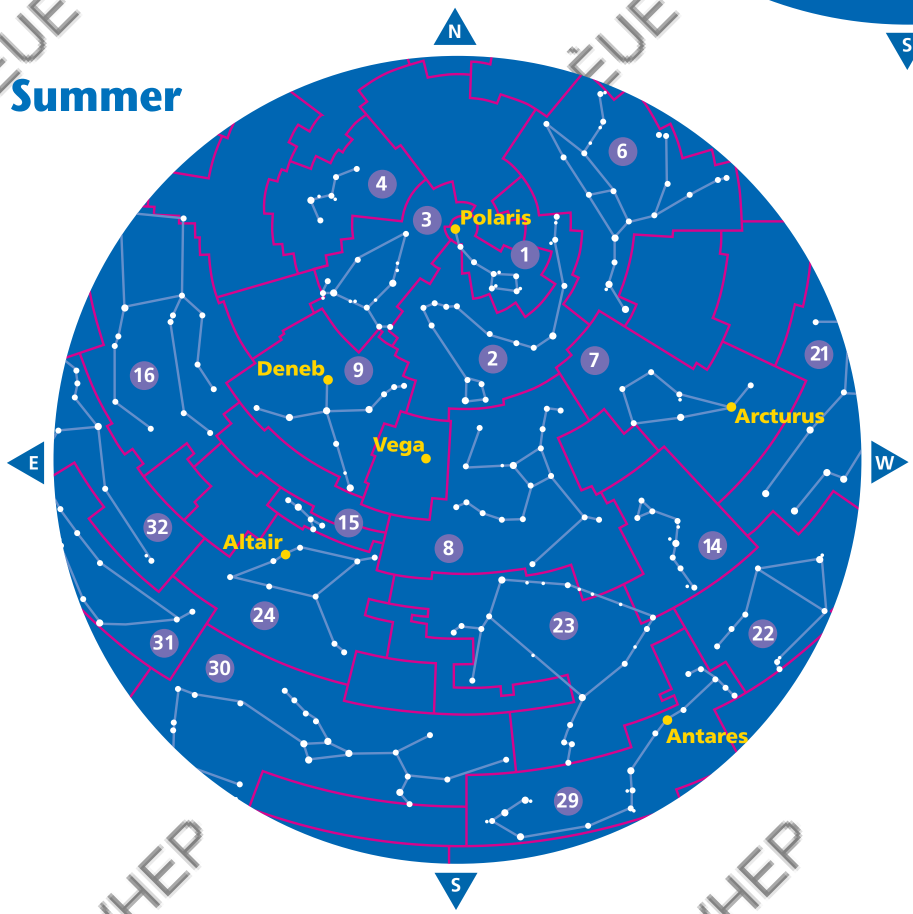
Star Charts for the Northern Hemisphere

Spring

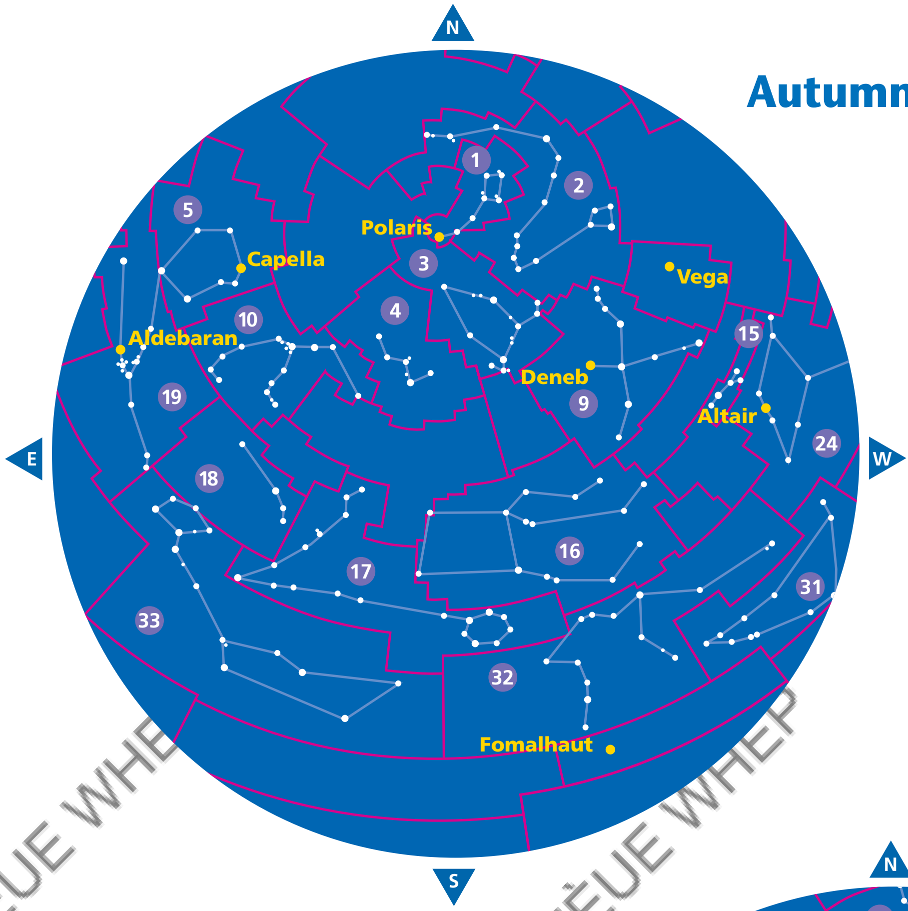


Summer

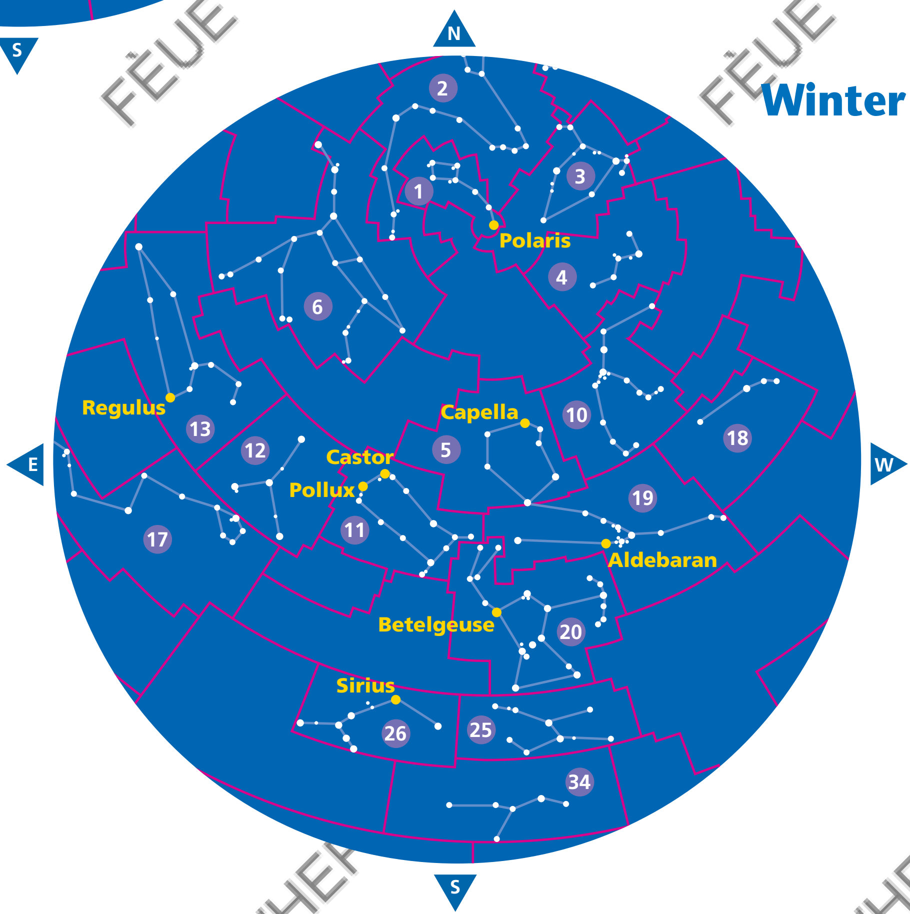


Constellations	
1	Ursa Minor
2	Draco
3	Cepheus
4	Cassiopeia
5	Auriga
6	Ursa Major
7	Boötes
8	Hercules
9	Cygnus
10	Perseus
11	Gemini
12	Cancer
13	Leo
14	Serpens
15	Sagitta
16	Pegasus
17	Pisces

Autumn



Winter

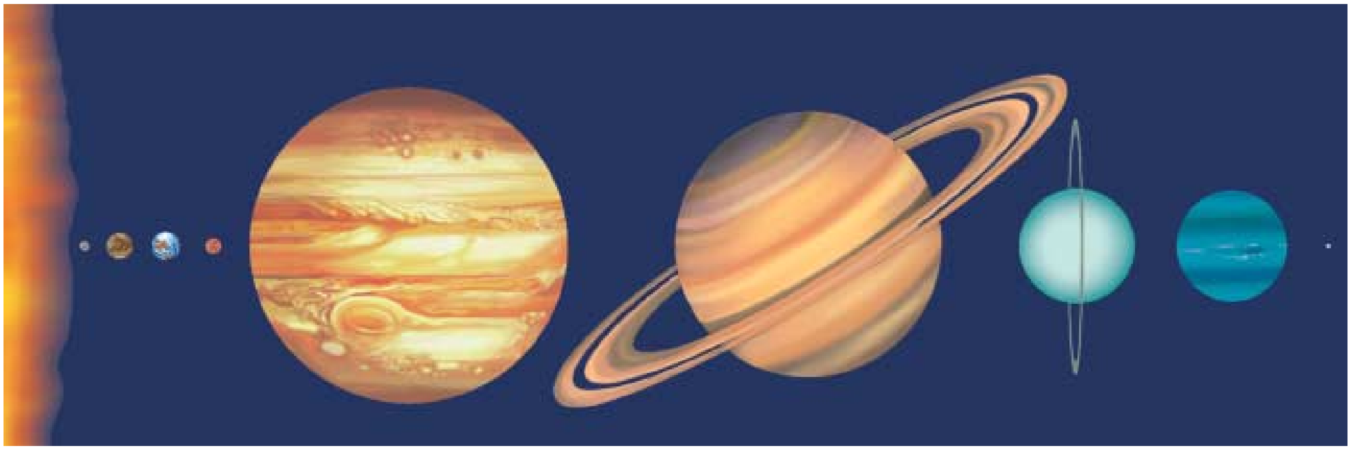


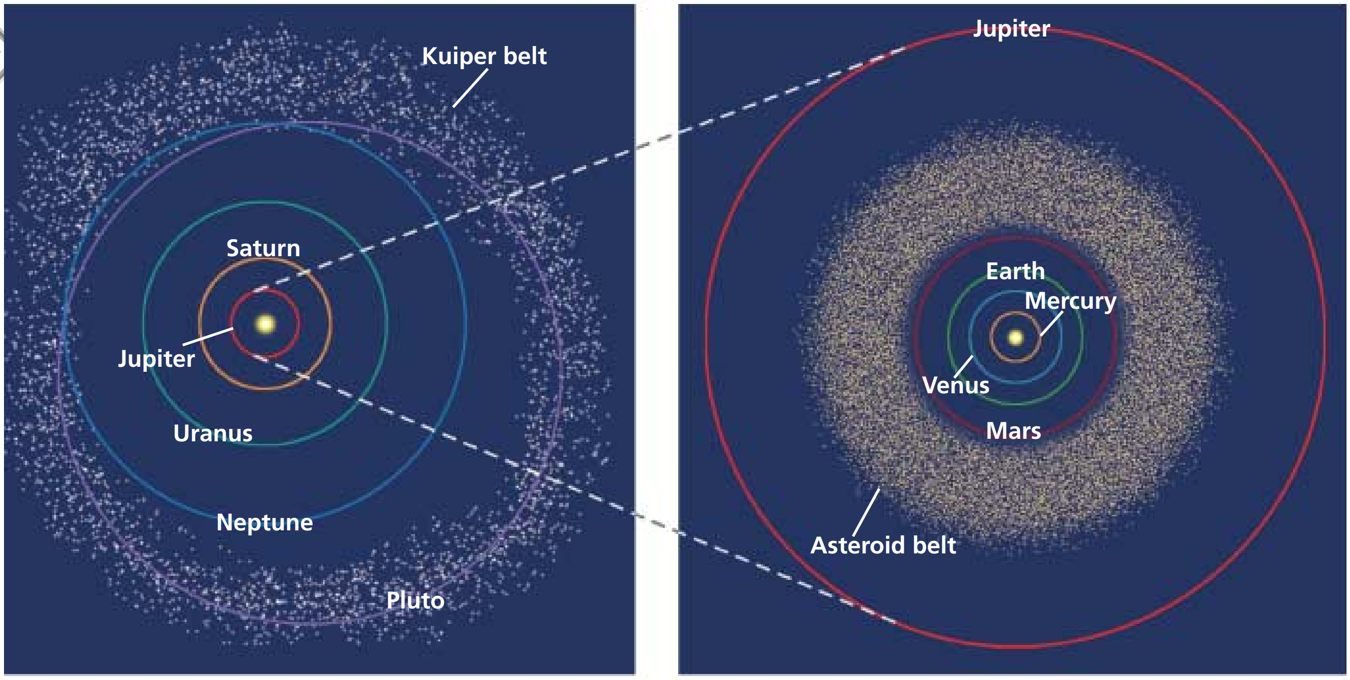
Constellations	
18	Aries
19	Taurus
20	Orion
21	Virgo
22	Libra
23	Ophiuchus
24	Aquila
25	Lepus
26	Canis Major
27	Hydra
28	Corvus
29	Scorpius
30	Sagittarius
31	Capricornus
32	Aquarius
33	Cetus
34	Columba

REFERENCE MAPS

Maps of the Solar System

The diagram at top shows the relative sizes of the nine planets. The order of the planets from the sun is the following: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto. The diagrams at bottom show the orbits of the planets around the sun.





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READING CHECK ANSWERS

Chapter 1 Introduction to Earth Science

Section 1

Page 7: The development of telescopes, satellites, and space probes has greatly expanded astronomers' understanding of the universe.

Section 2

Page 10: Observations may lead to interesting scientific questions and may help scientists formulate reasonable and testable hypotheses.

Page 13: Answers may vary but should include three of the following types of models: physical models, graphic models, conceptual models, computer models, and mathematical models.

Page 14: Scientists present the results of their work at professional meetings and in scientific journals.

Chapter 2 Earth as a System

Section 1

Page 28: Indirect observations are the only means available for exploring Earth's interior at depths too great to be reached by drilling.

Section 2

Page 32: Dust and rock come to Earth from space, while hydrogen atoms from the atmosphere enter space from Earth.

Page 34: An energy budget is the total distribution of energy to, from, and between Earth's various spheres.

Page 36: soil and plants

Section 3

Page 40: The amount of matter and energy in an ecosystem can supply a population of a given size, and no larger. This maximum population is the carrying capacity of the ecosystem.

Chapter 3 Models of the Earth

Section 1

Page 54: because the equator is the only parallel that divides Earth into halves

Section 2

Page 58: Because both the parallels and the meridians are equally spaced straight lines on a cylindrical projection, the parallels and meridians form a grid.

Page 61: by using a graphic scale, or a printed line divided into proportional parts that represent units of measure; a fractional scale, in which a ratio shows how distance on Earth relates to distance on a map; or a verbal scale, which expresses scale in sentence form

Section 3

Page 65: Water moves from areas of higher elevation to areas of lower elevation. Because the V shape points toward higher elevation, it points upstream.

Page 67: Scientists create soil maps to classify, map, and describe soils.

Chapter 4 Earth Chemistry

Section 1

Page 83: The atomic number is the number of protons in an atom's nucleus. The mass number is the sum of the number of protons and the number of neutrons in an atom. The atomic mass unit is used to express the mass of subatomic particles or atoms.

Section 2

Page 89: Atoms form chemical bonds by transferring electrons or by sharing electrons.

Page 91: The oxygen atom has a larger and more positively charged nucleus than the hydrogen atoms do. As a result, the oxygen nucleus pulls the electrons from the hydrogen atoms closer to it than the hydrogen nuclei pull the shared electrons from the oxygen. This unequal attraction forms a polar-covalent bond.

Chapter 5 Minerals of Earth's Crust

Section 1

Page 105: Nonsilicate minerals never contain compounds of silicon bonded to oxygen.

Page 106: The building block of the silicate crystalline structure is a four-sided structure known as the *silicon-oxygen tetrahedron*, which is one silicon atom surrounded by four oxygen atoms.

Section 2

Page 111: The strength and geometric arrangement of the bonds between the atoms that make up a mineral's internal structure determines the hardness of a mineral.

Page 113: Chatoyancy is the silky appearance of some minerals in reflected light. Asterism is the appearance of a six-sided star when a mineral reflects light.

Chapter 6 Rocks

Section 1

Page 127: As magma cools and solidifies, minerals crystallize out of the magma in a specific order that depends on their melting points.

Section 2

Page 131: Fine-grained igneous rock forms mainly from magma that cools rapidly; coarse-grained igneous rock forms mainly from magma that cools more slowly.

Page 133: A batholith is an intrusive structure that covers an area of at least 100 km^2. A stock covers an area of less than 100 km^2.

Section 3

Page 137: Three groups of clastic sedimentary rock are conglomerates and breccias, sandstones, and shales.

Page 139: Graded bedding is a type of stratification in which different sizes and types of sediments settle to different levels.

READING CHECK ANSWERS

Section 4

Page 142: The high pressures and temperatures that result from the movements of tectonic plates may cause chemical changes in the minerals.

Chapter 7 Resources and Energy

Section 1

Page 156: Water creates ore deposits by eroding rock and releasing minerals and by carrying the mineral fragments and depositing them in streambeds.

Section 2

Page 161: Cap rock is a layer of impermeable rock at the top of an oil- or natural gas-bearing formation through which fluids cannot flow.

Page 162: As neutrons strike neighboring nuclei, the nuclei split and release additional neutrons that strike other nuclei and cause the chain to continue.

Section 3

Page 167: Answers may vary but should include three of the following: geothermal, solar, hydroelectric, and biomass.

Section 4

Page 170: The use of fossil fuels affects the environment when coal is mined from the surface, which destroys the land. When fossil fuels are burned, they affect the environment by creating air pollution.

Chapter 8 The Rock Record

Section 1

Page 186: Hutton reasoned that the extremely slow-working forces that changed the land on his farm had also slowly changed the rocks that make up Earth's crust. He concluded that large changes must happen over a period of millions of years.

Page 188: Because ripple marks form at the top of a rock layer, scientists can use the orientation of the ripple marks to determine which direction was "up" when the rock layers formed.

Section 2

Page 192: Varves are like tree rings in that varves are laid down each year. Thus, counting varves can reveal the age of sedimentary deposits.

Page 195: An isotope that has an extremely long half-life will not show significant or measurable changes in a young rock. In a very old rock, an isotope that has a short half-life may have decayed to the point at which too little of the isotope is left to give an accurate age measurement. So, the estimated age of the rock must be correlated to the dating method used.

Section 3

Page 199: A trace fossil is fossilized evidence of past animal movement, such as tracks, footprints, borings, or burrows, that can provide information about prehistoric life.

Chapter 9 A View of Earth's Past

Section 1

Page 211: You would find fossils of extinct animals in older layers of a geologic column.

Section 2

Page 216: Earth is approximately 4.6 billion years old.

Page 218: Answers may vary but should include three of the following: trilobites, brachiopods, jellyfish, worms, snails, and sponges.

Section 3

Page 222: Answers may vary but could include *Archaeopteryx*, pterosaurs, *Apatosaurus*, and *Stegosaurus*.

Page 225: During ice ages, water from the ocean was frozen as ice, so the amount of liquid water in the seas decreased and sea level fell.

Chapter 10 Plate Tectonics

Section 1

Page 241: Many scientists rejected Wegener's hypothesis because the mechanism that Wegener suggested was easily disproved by geologic evidence.

Page 243: New sea floor forms as magma rises to fill the rift that forms when two plates pull apart at a divergent boundary.

Page 245: The symmetrical magnetic patterns in sea-floor rocks show that rock formed at one place (at a ridge) and then broke apart and moved away from the center in opposite directions.

Section 2

Page 248: Scientists use the locations of earthquakes, volcanoes, trenches, and mid-ocean ridges to outline tectonic plates.

Page 250: Collisions at convergent boundaries can happen between two oceanic plates, between two continental plates, or between one oceanic plate and one continental plate.

Page 253: When denser lithosphere sinks into the asthenosphere, the asthenosphere must move out of the way. As the asthenosphere moves, it drags or pushes on other parts of the lithosphere, which causes movement.

Section 3

Page 256: As a plate subducts beneath another plate, islands and other land features on the subducting plate are scraped off the subducting plate and become part of the overriding plate.

Page 259: The continents Africa, South America, Antarctica, and Australia formed from Gondwanaland. The subcontinent of India was also part of Gondwanaland.

Chapter 11 Deformation of the Crust

Section 1

Page 273: Tension and shear stress can both pull rock apart.

Page 275: limbs and hinges

Page 277: A thrust fault is a type of reverse fault in which the fault plane is at a low angle relative to the surface.

Section 2

Page 281: The Himalayas are growing taller because the two plates are still colliding and causing further compression of the rock, which further uplifts the mountains.

Page 283: Answers may include three of the following: folded mountains, fault-block mountains, dome mountains, and volcanic mountains.

Chapter 12 Earthquakes

Section 1

Page 297: Rayleigh waves cause the ground to move in an elliptical, rolling motion. Love waves cause rock to move side-to-side and perpendicular to the direction the waves are traveling.

Page 298: The speed of seismic waves changes as they pass through different layers of Earth.

Section 2

Page 303: Moment magnitude is more accurate for larger earthquakes than the Richter scale is. Moment magnitude is directly related to rock properties and so is more closely related to the cause of the earthquake than the Richter scale is.

Section 3

Page 307: Scientists think that stress on a fault builds up to a critical point and is then released as an earthquake. Seismic gaps are areas in which no earthquakes have happened in a long period of time and thus are likely to be under a high amount of stress.

Chapter 13 Volcanoes

Section 1

Page 321: The denser plate of oceanic lithosphere subducts beneath the less dense plate of continental lithosphere.

Page 323: As the lithosphere moves over the mantle plume, older volcanoes move away from the mantle plume. A new hot spot forms in the lithosphere above the mantle plume as a new volcano begins to form.

Section 2

Page 326: The faster the rate of flow is and the higher the gas content is, the more broken up and rough the resulting cooled lava will be.

Page 329: A caldera may form when a magma chamber empties or when large amounts of magma are discharged, causing the ground to collapse.

Chapter 14 Weathering and Erosion

Section 1

Page 344: Two types of mechanical weathering are ice wedging and abrasion. Ice wedging is caused by water that seeps into cracks in rock and freezes. When water freezes, it expands and creates pressure on the rock, which widens and deepens cracks. Abrasion is the grinding away of rock surfaces by other rocks or sand particles. Abrasive agents may be carried by gravity, water, and wind.

Page 346: Two effects of chemical weathering are changes in the chemical composition and changes in the physical appearance of a rock.

Section 2

Page 350: Fractures and joints in a rock increase surface area and allow weathering to occur more rapidly.

Section 3

Page 355: Large amounts of rainfall and high temperatures cause thick soils to form in both tropical and temperate climates. Tropical soils have thin A horizons because of the continuous leaching of topsoil. Temperate soils have three thick layers, because leaching of the A horizon in temperate climates is much less than leaching of the A horizon in tropical climates.

Section 4

Page 358: Dust storms may form during droughts when the soil is made dry and loose by lack of moisture and wind-caused sheet erosion carries it away in clouds of dust. If all of the topsoil is removed, the remaining subsoil will not contain enough nutrients to raise crops.

Page 361: Landslides are masses of loose rock combined with soil that suddenly fall down a slope. A rockfall consists of rock falling from a steep cliff.

Page 363: When a mountain is no longer being uplifted, weathering and erosion wear down its jagged peaks to low, featureless surfaces called *peneplains*.

Chapter 15 River Systems

Section 1

Page 376: Precipitation is any form of water that falls to Earth from the clouds, including rain, snow, sleet, and hail.

Section 2

Page 381: A river that has meanders probably has a low gradient.

Section 3

Page 385: Floods can be controlled indirectly through forest and soil conservation measures that reduce or prevent runoff, or directly by building artificial structures, such as dams, levees, and floodways, to redirect water flow.

Chapter 16 Groundwater

Section 1

Page 399: The two zones of groundwater are the zone of saturation and the zone of aeration.

Page 400: The depth of a water table depends on topography, aquifer permeability, the amount of rainfall, and the rate at which humans use the groundwater.

Page 403: Ordinary springs occur where the ground surface drops below the water table. An artesian spring occurs where groundwater flows to the surface through natural cracks in the overlying cap rock.

Section 2

Page 407: A natural bridge may form when two sinkholes form close to each other. The bridge is the uncollapsed rock between the sinkholes.

Chapter 17 Glaciers

Section 1

Page 420: Continental glaciers exist only in Greenland and Antarctica.

Page 424: A moving glacier forms a cirque by pulling blocks of rock from the floor and walls of a valley and leaving a bowl-shaped depression.

Section 2

Page 427: A drumlin is a long, low, tear-shaped mound of till.

Page 428: Eskers form when meltwater from receding continental glaciers flows through ice tunnels and deposits long, winding ridges of gravel and sand.

Section 3

Page 432: The sea level was up to 140 m lower than it is now.

Chapter 18 Erosion by Wind and Waves

Section 1

Page 446: Moisture makes soil heavier, so the soil sticks and is more difficult to move. Therefore, erosion happens faster in dry climates.

Page 448: Barchan dunes are crescent shaped; transverse dunes form linear ridges.

Section 2

Page 452: Answers should include three of the following: sea cliffs, sea caves, sea arches, sea stacks, wave-cut terraces, and wave-built terraces.

Section 3

Page 457: As sea levels rise over a flat coastal plain, the shoreline moves inland and isolates dunes from the old shoreline. These dunes become barrier islands.

Chapter 19 Ocean Basins

Section 1

Page 472: Oceanographers study the physical characteristics, chemical composition, and life-forms of the ocean.

Section 2

Page 476: Trenches; broad, flat plains; mountain ranges; and submerged volcanoes are part of the deep-ocean basins.

Section 3

Page 481: When chemical reactions take place in the ocean, dissolved substances can crystallize to form nodules that settle to the ocean floor.

Chapter 20 Ocean Water

Section 1

Page 495: Dissolved solids enter the oceans from the chemical weathering of rock on land, from volcanic eruptions, and from chemical reactions between sea water and newly formed sea-floor rocks.

Page 497: Ocean surface temperatures are affected by the amount of solar energy an area receives and by the movement of water in the ocean.

Page 499: Ocean water contains dissolved solids (mostly salts) that add mass to a given volume of water. The large amount of dissolved solids in ocean water makes ocean water denser than fresh water.

Section 2

Page 503: Most marine life is found in the sublittoral zone. Life in this zone is continuously submerged, but waters are still shallow enough to allow sunlight to penetrate.

Section 3

Page 507: Aquaculture provides a reliable, economical source of food. However, aquatic farms are susceptible to pollution and they may become local sources of pollution.

Chapter 21 Movements of the Ocean

Section 1

Page 521: Because no continents interrupt the flow of the Antarctic Circumpolar Current, also called the *West Wind Drift*, it completely encircles Antarctica and crosses three major oceans. All other surface currents are deflected and divided when they meet a continental barrier.

Page 523: Antarctic Bottom Water is very cold. It also has a high salinity. The extreme cold and high salinity combine to make the water extremely dense.

Section 2

Page 526: Because waves receive energy from wind that pushes against the surface of the water, the amount of energy decreases as the depth of water increases. As a result, the diameter of the water molecules' circular path also decreases.

Page 528: Contact with the ocean floor causes friction, which slows down the bottom of the wave but not the top of the wave. Because of the difference in speed between the top and bottom of the wave, the top gets farther ahead of the bottom until the wave becomes unstable and falls over.

Section 3

Page 532: When the tidal range is small, the sun and the moon are at right angles to each other relative to Earth's orbit.

Chapter 22 The Atmosphere

Section 1

Page 548: Transpiration increases the amount of water vapor in the atmosphere.

Page 551: An aneroid barometer contains a sealed metal container that has a partial vacuum.

Page 553: The lower region of the thermosphere is called the *ionosphere*.

Section 2

Page 559: Deserts are colder at night than other areas are because the air in deserts contains little water vapor that can absorb heat during the day and release heat slowly at night.

Section 3

Page 562: They flow in opposite directions from each other, and they occur at different latitudes.

Chapter 23 Water in the Atmosphere

Section 1

Page 576: When the air is very dry and the temperature is below freezing, ice and snow change directly into water vapor by sublimation.

Page 578: Dew is liquid moisture that condenses from air on cool objects when the air is nearly saturated and the temperature drops. Frost is water vapor that condenses as ice crystals onto a cool surface directly from the air when the dew point is below freezing.

READING CHECK ANSWERS

Section 2

Page 582: The source of heat that warms the air and leads to cloud formation is solar energy that is reradiated as heat by Earth's surface. As the process continues, latent heat released by the condensation may allow the clouds to expand beyond the condensation level.

Page 585: because cirrus clouds form at very high altitudes where air temperature is low

Section 3

Page 589: Doppler radar measures the location, direction of movement, and intensity of precipitation.

Chapter 24 Weather

Section 1

Page 603: a continental tropical air mass

Section 2

Page 607: The air of an anticyclone sinks and flows outward from a center of high pressure. The air of a midlatitude cyclone rotates toward the rising air of a central, low-pressure region.

Page 609: over warm tropical seas

Section 3

Page 612: A barometer is used to measure atmospheric pressure.

Section 4

Page 617: Areas of precipitation are marked by using colors or symbols.

Page 618: Meteorologists compare computer models because different models are better at predicting different weather variables. If information from two or more models matches, scientists can be more confident of their predictions.

Chapter 25 Climate

Section 1

Page 633: Waves, currents, and other water motions continually replace warm surface waters with cooler water from the ocean depths, which keeps the surface temperature of the water from increasing rapidly.

Page 634: The temperature of land increases faster than that of water does because the specific heat of land is lower than that of water, and thus the land requires less energy to heat up than the water does.

Section 2

Page 638: marine west coast, humid continental, and humid subtropical

Section 3

Page 642: Scientists use computer models to incorporate as much data as possible to sort out the complex variables that influence climate and to make predictions about climate.

Page 644: Climate change influences humans, plants, and animals. It also affects nearby climates, sea level, and precipitation rates.

Chapter 26 Studying Space

Section 1

Page 661: The only kind of electromagnetic radiation the human eye can detect is visible light.

Page 663: Images produced by refracting telescopes are subject to distortion because of the way different colors of visible light are focused at different distances from the lens and because of weight limitations on the objective lens.

Page 664: Scientists launch spacecraft into orbit to detect radiation screened out by Earth's atmosphere and to avoid light pollution and other atmospheric distortions.

Section 2

Page 669: Constellations provide two kinds of evidence of Earth's motion. As Earth rotates, the stars appear to change position during the night. As Earth revolves around the sun, Earth's night sky faces a different part of the universe. As a result, different constellations appear in the night sky as the seasons change.

Page 671: Because time zones are based on Earth's rotation, as you travel west, you eventually come to a location where, on one side of time zone border, the calendar moves ahead one day. The purpose of the International Dateline is to locate the border so that the transition would affect the least number of people. So that it will affect the least number of people, the International Dateline is in the middle of the Pacific Ocean, instead of on a continent.

Page 672: Daylight savings time is an adjustment that is made to standard time by setting clocks ahead one hour to take advantage of longer hours of daylight in the summer months and to save energy.

Chapter 27 Planets of the Solar System

Section 1

Page 687: Unlike the other outer planets, Pluto is very small and is composed of rock and frozen gas, instead of thick layers of gases.

Page 689: Green plants release free oxygen as part of photosynthesis, which caused the concentration of oxygen gas in the atmosphere to gradually increase.

Section 2

Page 692: An ellipse is a closed curve whose shape is defined by two points inside the curve. An ellipse looks like an oval.

Section 3

Page 697: Answers may vary but should address differences in distance from the sun, density, atmospheric pressure and density, and tectonics.

Page 699: Martian volcanoes are larger than volcanoes on Earth because Mars has no moving tectonic plates. Magma sources remain in the same spot for millions of years and produce volcanic material that builds the volcanic cone higher and higher.

Section 4

Page 702: When Jupiter formed, it did not have enough mass for nuclear fusion to begin.

Page 704: Saturn and Jupiter are made almost entirely of hydrogen and helium and have rocky-iron cores, ring systems, many satellites, rapid rotational periods, and bands of colored clouds.

Page 707: The Kuiper belt is located beyond the orbit of Neptune.

Chapter 28 Minor Bodies of the Solar System

Section 1

Page 720: Answers should include two of the following features: maria, highlands, craters, ridges, and rilles.

Page 722: The crust of the far side of the moon is thicker than the crust of the near side is. The crust of the far side also consists mainly of mountainous terrain and has only a few small maria.

Section 2

Page 726: The far side of the moon is never visible from Earth, because the moon's rotation and the moon's revolution around Earth take the same amount of time.

Page 728: During a total eclipse, the entire disk of the sun is blocked, and the outer layers of the sun become visible. During an annular eclipse, the disk of the sun is never completely blocked out, so the sun is too bright for observers on Earth to see the outer layers of the sun's atmosphere.

Page 731: When the lighted part of the moon is larger than a semicircle but the visible part of the moon is shrinking, the phase is called *waning gibbous*. When only a sliver of the near side is visible, the phase is a waning crescent.

Section 3

Page 735: Io's surface is covered with many active volcanoes. Europa's surface is covered by an enormous ice sheet. Ganymede is the largest moon in the solar system and has a strong magnetic field. Callisto's surface is heavily cratered.

Page 737: Charon is almost half the size of the planet it orbits. Charon's orbital period is the same length as Pluto's day, so only one side of Pluto always faces the moon.

Section 4

Page 740: The most common type is made mostly of silicate rock. Other asteroids are made mostly of metals such as iron and nickel. The third type is composed mostly of carbon-based materials.

Page 743: A meteoroid is a rocky body that travels through space. When a meteoroid enters Earth's atmosphere and begins to burn up, the meteoroid becomes a meteor.

Chapter 29 The Sun

Section 1

Page 757: Einstein's equation helped scientists understand the source of the sun's energy. The equation explained how the sun could produce huge amounts of energy without burning up.

Page 759: The sun's atmosphere consists of the photosphere, the chromosphere, and the corona.

Section 2

Page 763: Coronal mass ejections generate sudden disturbances in Earth's magnetic field. The high-energy particles that circulate during these storms can damage satellites, cause power blackouts, and interfere with radio communications.

Chapter 30 Stars, Galaxies, and the Universe

Section 1

Page 777: Polaris is almost exactly above the pole of Earth's rotational axis, so Polaris moves only slightly around the pole during one rotation of Earth.

Page 778: Starlight is shifted toward the red end of the spectrum when the star is moving away from the observer.

Section 2

Page 783: The forces balance each other and keep the star in equilibrium. As gravity increases the pressure on the matter within a star, the rate of fusion increases. This increase in fusion causes a rise in gas pressure. As a result, the energy from the increased fusion and gas pressure generates outward pressure that balances the force of gravity.

Page 784: Giants and supergiants appear in the upper-right part of the H-R diagram.

Page 787: As supergiants collapse because of gravitational forces, fusion begins and continues until the supply of fuel is used up. The core begins to collapse under its own gravity and causes energy to transfer to the outer layers of the star. The transfer of energy to the outer layers causes the explosion.

Section 3

Page 790: More than 50% of all stars are in multiple-star systems.

Section 4

Page 794: All matter and energy in the early universe were compressed into a small volume at an extremely high temperature until the temperature cooled and all of the matter and energy were forced outward in all directions.

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GLOSSARY/GLOSARIO

Terms and their definitions are listed in English in alphabetical order in the first column. The second column lists the equivalent term in Spanish.

A

abrasion the grinding and wearing away of rock surfaces through the mechanical action of other rock or sand particles (344)

abrasion/abrasión proceso por el cual las super-ficies de las rocas se muelen o desgastan por medio de la acción mecánica de otras rocas y partículas de arena (344)

absolute age the numeric age of an object or event, often stated in years before the present, as established by an absolute-dating process, such as radiometric dating (191)

absolute age/edad absoluta la edad numérica de un objeto o suceso, que suele expresarse en cantidad de años antes del presente, determinada por un proceso de datación absoluta, tal como la datación radiométrica (191)

absolute humidity the mass of water vapor per unit volume of air that contains the water vapor; usually expressed as grams of water vapor per cubic meter of air (577)

absolute humidity/humedad absoluta la masa de vapor de agua por unidad de volumen de aire que contiene al vapor de agua; normalmente se expresa por metro cúbico de aire (577)

absolute magnitude the brightness that a star would have at a distance of 32.6 light-years from Earth (780)

absolute magnitude/magnitud absoluta el brillo que una estrella tendría a una distancia de 32.6 años luz de la Tierra (780)

abyssal plain a large, flat, almost level area of the deep-ocean basin (477)

abyssal plain/llanura abisal un área amplia, llana y casi plana de la cuenca oceánica profunda (477)

adiabatic cooling the process by which the temperature of an air mass decreases as the air mass rises and expands (582)

adiabatic cooling/enfriamiento adiabático el proceso por medio del cual la temperatura de una masa de aire disminuye a medida que ésta se eleva y se expande (582)

advective cooling the process by which the temperature of an air mass decreases as the air mass moves over a cold surface (583)

advective cooling/enfriamiento advectivo el proceso por medio del cual la temperatura de una masa de aire disminuye a medida que ésta se mueve sobre una superficie fría (583)

air mass a large body of air throughout which temperature and moisture content are similar (601)

air mass/masa de aire un gran volumen de aire, cuya temperatura y cuyo contenido de humedad son similares en toda su extensión (601)

albedo the fraction of solar radiation that is reflected off the surface of an object (557)

albedo/albedo porcentaje de la radiación solar que la superficie de un objeto refleja (557)

alluvial fan a fan-shaped mass of rock material deposited by a stream when the slope of the land decreases sharply; for example, alluvial fans form when streams flow from mountains to flat land (383)

alluvial fan/abanico aluvial masa de materiales rocosos en forma de abanico, depositados por un arroyo cuando la pendiente del terreno disminuye bruscamente; por ejemplo, los abanicos aluviales se forman cuando los arroyos fluyen de una montaña a un terreno llano (383)

alpine glacier a narrow, wedge-shaped mass of ice that forms in a mountainous region and that is confined to a small area by surrounding topog-raphy; examples include valley glaciers, cirque glaciers, and piedmont glaciers (420)

alpine glacier/glaciar alpino una masa de hielo angosta, parecida a una cuña, que se forma en una región montañosa y que está confinada a un área pequeña por la topografía que la rodea; los glaci-ares de valle, los circos glaciares y los glaciares de pie de monte son algunos ejemplos de esto (420)

anemometer an instrument used to measure wind speed (612)	**anemometer/anemómetro** un instrumento que se usa para medir la rapidez del viento (612)
aphelion the point in the orbit of a planet at which the planet is farthest from the sun (668)	**aphelion/afelio** el punto en la órbita de un planeta en que el planeta está más lejos del Sol (668)
apogee in the orbit of a satellite, the point at which the satellite is farthest from Earth (725)	**apogee/apogeo** en la órbita de un satélite, el punto en el que el satélite está más alejado de la Tierra (725)
apparent magnitude the brightness of a star as seen from the Earth (780)	**apparent magnitude/magnitud aparente** el brillo de una estrella como se percibe desde la Tierra (780)
aquaculture the raising of aquatic plants and animals for human use or consumption (507)	**aquaculture/acuacultura** el cultivo de plantas y animales acuáticos para uso o consumo humano (507)
aquifer a body of rock or sediment that stores groundwater and allows the flow of groundwater (397)	**aquifer/acuífero** un cuerpo rocoso o sedimento que almacena agua subterránea y permite que fluya (397)
arête a sharp, jagged ridge that forms between cirques (424)	**arête/cresta** una cumbre puntiaguda e irregular que se forma entre circos glaciares (424)
artesian formation a sloping layer of permeable rock sandwiched between two layers of impermeable rock and exposed at the surface (403)	**artesian formation/formación artesiana** capa inclinada de rocas permeables que está en medio de dos capas de rocas impermeables y expuesta en la superficie (403)
asteroid a small, rocky object that orbits the sun; most asteroids are located in a band between the orbits of Mars and Jupiter (739)	**asteroid/asteroide** un objeto pequeño y rocoso que se encuentra en órbita alrededor del Sol; la mayoría de los asteroides se ubican en una banda entre las órbitas de Marte y Júpiter (739)
asthenosphere the solid, plastic layer of the mantle beneath the lithosphere; made of mantle rock that flows very slowly, which allows tectonic plates to move on top of it (29, 247)	**asthenosphere/astenosfera** la capa sólida y plástica del manto, que se encuentra debajo de la litosfera; está formada por roca del manto que fluye muy lentamente, lo cual permite que las placas tectónicas se muevan en su superficie (29, 247)
astronomical unit the average distance between the Earth and the sun; approximately 150 million kilometers (symbol, AU) (660)	**astronomical unit/unidad astronómica** la distancia promedio entre la Tierra y el Sol; aproximadamente 150 millones de kilómetros (símbolo: UA) (660)
astronomy the scientific study of the universe (7, 659)	**astronomy/astronomía** el estudio científico del universo (7, 659)
atmosphere a mixture of gases that surrounds a planet or moon (33, 547)	**atmosphere/atmósfera** una mezcla de gases que rodea un planeta o una luna (33, 547)
atmospheric pressure the force per unit area that is exerted on a surface by the weight of the atmosphere (550)	**atmospheric pressure/presión atmosférica** la fuerza por unidad de área que el peso de la atmósfera ejerce sobre una superficie (550)
atom the smallest unit of an element that maintains the chemical properties of that element (82)	**atom/átomo** la unidad más pequeña de un elemento que conserva las propiedades químicas de ese elemento (82)

aurora colored light produced by charged particles from the solar wind and from the magnetosphere that react with and excite the oxygen and nitrogen of Earth's upper atmosphere; usually seen in the sky near Earth's magnetic poles (764)

aurora/aurora luz de colores producida por partículas con carga del viento solar y de la magnetosfera, que reaccionan con los átomos de oxígeno y nitrógeno de la parte superior de la atmósfera de la Tierra y los excitan; normalmente se ve en el cielo cerca de los polos magnéticos de la Tierra (764)

B

barometer an instrument that measures atmospheric pressure (612)

barometer/barómetro un instrumento que mide la presión atmosférica (612)

barrier island a long ridge of sand or narrow island that lies parallel to the shore (457)

barrier island/isla barrera un largo arrecife de arena o una isla angosta ubicada paralela a la costa (457)

basal slip the process that causes the ice at the base of a glacier to melt and the glacier to slide (421)

basal slip/deslizamiento basal el proceso que hace que el hielo de la base de un glaciar se derrita y que éste se deslice (421)

beach an area of the shoreline that is made up of deposited sediment (453)

beach/playa un área de la costa que está formada por sedimento depositado (453)

benthic zone the bottom region of oceans and bodies of fresh water (503)

benthic zone/zona bentónica la región del fondo de los océanos y de las masas de agua dulce (503)

benthos organisms that live at the bottom of oceans or bodies of fresh water (502)

benthos/benthos organismos que viven en el fondo de los océanos o de las masas de agua dulce (502)

big bang theory the theory that all matter and energy in the universe was compressed into an extremely small volume that 13 billion to 15 billion years ago exploded and began expanding in all directions (794)

big bang theory/teoría del Big Bang la teoría que establece que toda la materia y la energía del universo estaban comprimidas en un volumen extremadamente pequeño que explotó hace aproximadamente 13 a 15 mil millones de años y empezó a expandirse en todas direcciones (794)

biomass plant material, manure, or any other or-ganic matter that is used as an energy source (167)

biomass/biomasa materia vegetal, estiércol o cualquier otra materia orgánica que se usa como fuente de energía (167)

biosphere the part of Earth where life exists; includes all of the living organisms on Earth (33)

biosphere/biosferavla parte de la Tierra donde existe la vida; abarca a todos los organismos vivos de la Tierra (33)

black hole an object so massive and dense that even light cannot escape its gravity (788)

black hole/hoyo negro un objeto tan masivo y denso que ni siquiera la luz puede salir de su campo gravitacional (788)

body wave in geology, a seismic wave that travels through the body of a medium (296)

body wave/onda interna en geología, una onda sísmica que se desplaza a través del cuerpo de un medio (296)

Bowen's reaction series the simplified pattern that illustrates the order in which minerals crystallize from cooling magma according to their chemical composition and melting point (127)

Bowen's reaction series/serie de reacción de Bowen el patrón simplificado que ilustra el orden en que los minerales se cristalizan a partir del magma que se enfría, de acuerdo con su composición química y punto de fusión (127)

braided stream a stream or river that is composed of multiple channels that divide and rejoin around sediment bars (382)

braided stream/corriente anastomosada una corriente o río compuesto por varios canales que se dividen y se vuelven a encontrar alrededor de barreras de sedimento (382)

C

caldera a large, circular depression that forms when the magma chamber below a volcano partially empties and causes the ground above to sink (329)

caldera/caldera una depresión grande y circular que se forma cuando se vacía parcialmente la cámara de magma que hay debajo de un volcán, lo cual hace que el suelo se hunda (329)

carbonation the conversion of a compound into a carbonate (347)

carbonation/carbonación la transformación de un compuesto a un carbonato (347)

carrying capacity the largest population that an environment can support at any given time (40)

carrying capacity/capacidad de carga la población más grande que un ambiente puede sostener en cualquier momento dado (40)

cavern a natural cavity that forms in rock as a result of the dissolution of minerals; *also* a large cave that commonly contains many smaller, connecting chambers (406)

cavern/caverna una cavidad natural que se forma en la roca como resultado de la disolución de minerales; *también,* una gran cueva que generalmente contiene muchas cámaras más pequeñas comunicadas entre sí (406)

cementation the process in which minerals precipitate into pore spaces between sediment grains and bind sediments together to form rock (135)

cementation/cementación el proceso en el cual los minerales se precipitan entre los poros de granos de sedimento y unen los sedimentos para formar rocas (135)

Cenozoic Era the current geologic era, which began 65.5 million years ago; also called the *Age of Mammals* (224)

Cenozoic Era/Era Cenozoica la era geológica actual, que comenzó hace 65.5 millones de años; también llamada *Edad de los Mamíferos* (224)

chemical sedimentary rock sedimentary rock that forms when minerals precipitate from a solution or settle from a suspension (136)

chemical sedimentary rock/roca sedimentaria química roca sedimentaria que se forma cuando los minerales precipitan a partir de una solución o se depositan a partir de una suspensión (136)

chemical weathering the process by which rocks break down as a result of chemical reactions (346)

chemical weathering/desgaste químico el proceso por medio del cual las rocas se fragmentan como resultado de reacciones químicas (346)

chromosphere the thin layer of the sun that is just above the photosphere and that glows a reddish color during eclipses (760)

chromosphere/cromosfera la delgada capa del Sol que se encuentra justo encima de la fotosfera y que resplandece con un color rojizo durante los eclipses (760)

cirque a deep and steep bowl-like depression produced by glacier erosion (424)

cirque/circo una depresión profunda y empinada, con forma de tazón, producida por erosión glaciar (424)

cirrus cloud a feathery cloud that is composed of ice crystals and that has the highest altitude of any cloud in the sky (585)

cirrus cloud/nube cirro una nube liviana formada por cristales de hielo, la cual tiene la mayor altitud de todas las nubes en el cielo (585)

clastic sedimentary rock sedimentary rock that forms when fragments of preexisting rocks are compacted or cemented together (137)	**clastic sedimentary rock/roca sedimentaria clástica** roca sedimentaria que se forma cuando los fragmentos de rocas preexistentes se unen por compactación o cementación (137)
cleavage in geology, the tendency of a mineral to split along specific planes of weakness to form smooth, flat surfaces (110)	**cleavage/exfoliación** en geología, la tendencia de un mineral a agrietarse a lo largo de planos débiles específicos y formar superficies lisas y planas (110)
climate the average weather conditions in an area over a long period of time (631)	**climate/clima** las condiciones promedio del tiempo en un área durante un largo período de tiempo (631)
climatologist a scientist who gathers data to study and compare past and present climates and to predict future climate change (641)	**climatologist/climatólogo** un científico que recopila datos para estudiar y comparar los climas del pasado y del presente y para predecir cambios climáticos en el futuro (641)
cloud a collection of small water droplets or ice crystals suspended in the air, which forms when the air is cooled and condensation occurs (581)	**cloud/nube** un conjunto de pequeñas gotitas de agua o cristales de hielo suspendidos en el aire, que se forma cuando el aire se enfría y ocurre conden-sación (581)
cloud seeding the process of introducing freezing nuclei or condensation nuclei into a cloud in order to cause rain to fall (590)	**cloud seeding/sembrado de nubes** el proceso de introducir núcleos congelados o núcleos de con-densación en una nube para producir lluvia (590)
coalescence the formation of a larger droplet by the combination of smaller droplets (588)	**coalescence/coalescencia** la formación de una gota más grande al combinarse gotas más pequeñas (588)
cold front the front edge of a moving mass of cold air that pushes beneath a warmer air mass like a wedge (605)	**cold front/frente frío** el borde del frente de una masa de aire frío en movimiento que empuja por debajo de una masa de aire más caliente como una cuña (605)
comet a small body of ice, rock, and cosmic dust that follows an elliptical orbit around the sun and that gives off gas and dust in the form of a tail as it passes close to the sun (741)	**comet/cometa** un cuerpo pequeño formado por hielo, roca y polvo cósmico que sigue una órbita elíptica alrededor del Sol y que libera gas y polvo, los cuales forman una cola al pasar cerca del Sol (741)
compaction the process in which the volume and porosity of a sediment is decreased by the weight of overlying sediments as a result of burial beneath other sediments (135)	**compaction/compactación** el proceso en el que el volumen y la porosidad de un sedimento disminuyen por efecto del peso al quedar el sedimento enterrado debajo de otros sedimentos superpuestos (135)
compound a substance made up of atoms of two or more different elements joined by chemical bonds (87)	**compound/compuesto** una substancia formada por átomos de dos o más elementos diferentes unidos por enlaces químicos (87)
condensation the change of state from a gas to a liquid (376)	**condensation/condensación** el cambio de estado de gas a líquido (376)
condensation nucleus a solid particle in the atmos-phere that provides the surface on which water vapor condenses (581)	**condensation nucleus/núcleo de condensación** una partícula sólida en la atmósfera que propor-ciona la superficie en la que el vapor de agua se condensa (581)

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conduction the transfer of energy as heat through a material (560)	**conduction/conducción** transferencia de energía en forma de calor a través de un material (560)
conservation the preservation and wise use of natural resources (171)	**conservation/conservación** la preservación y el uso inteligente de los recursos naturales (171)
constellation one of 88 regions into which the sky has been divided in order to describe the locations of celestial objects; a group of stars organized in a recognizable pattern (789)	**constellation/constelación** una de las 88 regiones en las que se ha dividido el cielo con el fin de describir la ubicación de los objetos celestes; un grupo de estrellas organizadas en un patrón reconocible (789)
contact metamorphism a change in the texture, structure, or chemical composition of a rock due to contact with magma (142)	**contact metamorphism/metamorfismo de contacto** un cambio en la textura, estructura o composición química de una roca debido al contacto con el magma (142)
continental glacier a massive sheet of ice that may cover millions of square kilometers, that may be thousands of meters thick, and that is not confined by surrounding topography (420)	**continental glacier/glaciar continental** una enorme capa de hielo que puede cubrir millones de kilómetros cuadrados, tener un espesor de miles de metros y que no está confinada por la topografía que la rodea (420)
continental drift the hypothesis that states that the continents once formed a single landmass, broke up, and drifted to their present locations (239)	**continental drift/deriva continental** la hipótesis que establece que alguna vez los continentes formaron una sola masa de tierra, se dividieron y se fueron a la deriva hasta terminar en sus ubicaciones actuales (239)
continental margin the shallow sea floor that is located between the shoreline and the deep-ocean bottom (475)	**continental margin/margen continental** el suelo marino poco profundo que se ubica entre la costa y el fondo profundo del océano (475)
contour line a line that connects points of equal elevation on a map (64)	**contour line/curva de nivel** una línea en un mapa que une puntos que tienen la misma elevación (64)
convection the movement of matter due to differences in density that are caused by temperature variations; can result in the transfer of energy as heat (560)	**convection/convección** el movimiento de la materia debido a diferencias en la densidad que se producen por variaciones en la temperatura; puede resultar en la transferencia de energía en forma de calor (560)
convective zone the region of the sun's interior that is between the radiative zone and the photosphere and in which energy is carried upward by convection (759)	**convective zone/zona convectiva** la región del interior del Sol que se encuentra entre la zona radiactiva y la fotosfera y en la cual la energía se desplaza hacia arriba por convección (759)
convergent boundary the boundary between tectonic plates that are colliding (250)	**convergent boundary/límite convergente** el límite entre placas tectónicas que chocan (250)
core the central part of the Earth below the mantle; *also* the center of the sun (28)	**core/núcleo** la parte central de la Tierra, debajo del manto; *también,* el centro del Sol (28)
core sample a cylindrical piece of sediment, rock, soil, snow, or ice that is collected by drilling (479)	**core sample/muestra de sondeo** un fragmento de sedimento, roca, suelo, nieve o hielo que se obtiene taladrando (479)
Coriolis effect the curving of the path of a moving object from an otherwise straight path due to the Earth's rotation (520, 561)	**Coriolis effect/efecto de Coriolis** la desviación de la trayectoria recta que experimentan los objetos en movimiento debido a la rotación de la Tierra (520, 561)

corona the outermost layer of the sun's atmosphere (760)	**corona/corona** la capa externa de la atmósfera del Sol (760)
coronal mass ejection a part of coronal gas that is thrown into space from the sun (763)	**coronal mass ejection/eyección de masa coronal** una parte de gas coronal que el Sol expulsa al espacio (763)
cosmic background radiation radiation uniformly detected from every direction in space; considered a remnant of the big bang (795)	**cosmic background radiation/radiación cósmica de fondo** radiación que se detecta de manera uniforme desde todas las direcciones en el espacio; se considera un resto del Big Bang (795)
cosmology the study of the origin, properties, processes, and evolution of the universe (793)	**cosmology/cosmología** el estudio del origen, propiedades, procesos y evolución del universo (793)
covalent bond a bond formed when atoms share one or more pairs of electrons (91)	**covalent bond/enlace covalente** un enlace formado cuando los átomos comparten uno más pares de electrones (91)
crater a bowl-shaped depression that forms on the surface of an object when a falling body strikes the object's surface or when an explosion occurs; a similar depression around the central vent of a volcano or geyser (720)	**crater/cráter** una depresión con forma de tazón, que se forma sobre la superficie de un objeto cuando un cuerpo en caída impacta sobre ésta o cuando se produce una explosión; una depresión similar alrededor de la chimenea de un volcán o géiser (720)
creep the slow downhill movement of weathered rock material (362)	**creep/arrastre** el movimiento lento y descendente de materiales rocosos desgastados (362)
crevasse in a glacier, a large crack or fissure that results from ice movement (422)	**crevasse/grieta** en un glaciar, una fractura o fisura grande debida al movimiento del hielo (422)
crust the thin and solid outermost layer of the Earth above the mantle (28)	**crust/corteza** la capa externa, delgada y sólida de la Tierra, que se encuentra sobre el manto (28)
crystal a solid whose atoms, ions, or molecules are arranged in a regular, repeating pattern (106)	**crystal/cristal** un sólido cuyos átomos, iones o moléculas están ordenados en un patrón regular y repetitivo (106)
cumulus cloud a low-level, billowy cloud that commonly has a top that resembles cotton balls and a dark bottom (585)	**cumulus cloud/nube cúmulo** una nube esponjada ubicada en un nivel bajo, cuya parte superior normalmente parece una bola de algodón y es obscura en la parte inferior (585)
current in geology, a horizontal movement of water in a well-defined pattern, such as a river or stream; the movement of air in a certain direction (519)	**current/corriente** en geología, un movimiento horizontal de agua en un patrón bien definido, como por ejemplo, un río o arroyo; el movimiento del aire en una cierta dirección (519)

D

deep current a streamlike movement of ocean water far below the surface (523)	**deep current/corriente profunda** un movimiento del agua del océano que es similar a una corriente y ocurre debajo de la superficie (523)
deep-ocean basin the part of the ocean floor that is under deep water beyond the continental margin and that is composed of oceanic crust and a thin layer of sediment (475)	**deep-ocean basin/cuenca oceánica profunda** la parte del fondo del océano que está bajo aguas profundas más allá del margen continental y que se compone de corteza oceánica y una delgada capa de sedimento (475)

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deflation a form of wind erosion in which fine, dry soil particles are blown away (446)	**deflation/deflación** una forma de erosión del viento en la que se mueven partículas de suelo finas y secas (446)
deformation the bending, tilting, and breaking of Earth's crust; the change in the shape of rock in response to stress (271)	**deformation/deformación** el proceso de doblar, inclinar y romper la corteza de la Tierra; el cambio en la forma de una roca en respuesta a la tensión (271)
delta a fan-shaped mass of rock material deposited at the mouth of a stream; for example, deltas form where streams flow into the ocean at the edge of a continent (383)	**delta/delta** un depósito de materiales rocosos en forma de abanico ubicado en la desembocadura de un río; por ejemplo, los deltas se forman en el lugar donde las corrientes fluyen al océano en el borde de un continente (383)
density the ratio of the mass of a substance to the volume of the substance; commonly expressed as grams per cubic centimeter for solids and liquids and as grams per liter for gases (112, 499)	**density/densidad** la relación entre la masa de una substancia y su volumen; comúnmente se expresa en gramos por centímetro cúbico para los sólidos y líquidos, y como gramos por litro para los gases (112, 499)
dependent variable in an experiment, the factor that changes as a result of manipulation of one or more other factors (the independent variables) (11)	**dependent variable/variable dependiente** en un experimento, el factor que cambia como resultado de la manipulación de uno o más factores (las variables independientes) (11)
desalination a process of removing salt from ocean water (378, 505)	**desalination/desalación** (o desalinización) un proceso de remoción de sal del agua del océano (378, 505)
dew point at constant pressure and water vapor content, the temperature at which the rate of condensation equals the rate of evaporation (577)	**dew point/punto de rocío** a presión y contenido de vapor constantes, la temperatura a la cual la tasa de condensación iguala la tasa de evaporación (577)
differential weathering the process by which softer, less weather resistant rocks wear away at a faster rate than harder, more weather resistant rocks do (349)	**differential weathering/desgaste diferencial** el proceso por medio cual las rocas más blandas y menos resistentes al clima se desgastan a una tasa más rápida que las rocas más duras y resistentes al clima (349)
discharge the volume of water that flows within a given time (380)	**discharge/descarga** el volumen de agua que fluye en un tiempo determinado (380)
divergent boundary the boundary between two tectonic plates that are moving away from each other (249)	**divergent boundary/límite divergente** el límite entre dos placas tectónicas que se están separando una de la otra (249)
dome mountain a circular or elliptical, almost symmetrical elevation or structure in which the stratified rock slopes downward gently from the central point of folding (283)	**dome mountain/domo** una elevación o estructura circular o elíptica, casi simétrica, en la cual la roca estratificada se encuentra en una ligera pendiente hacia abajo a partir del punto central de plegamiento (283)
Doppler effect an observed change in the frequency of a wave when the source or observer is moving (778)	**Doppler effect/efecto Doppler** un cambio que se observa en la frecuencia de una onda cuando la fuente o el observador está en movimiento (778)
dune a mound of wind-deposited sand that moves as a result of the action of wind (447)	**dune/duna** un montículo de arena depositada por el viento que se mueve como resultado de la acción de éste (447)

E

earthquake a movement or trembling of the ground that is caused by a sudden release of energy when rocks along a fault move (295)	**earthquake/terremoto** un movimiento o temblor del suelo causado por una liberación súbita de energía que se produce cuando las rocas ubicadas a lo largo de una falla se mueven (295)
Earth science the scientific study of Earth and the universe around it (5)	**Earth science/ciencias de la Tierra** el estudio científico de la Tierra y del universo que la rodea (5)
eccentricity the degree of elongation of an elliptical orbit (symbol, *e*) (692)	**eccentricity/excentricidad** el grado de alargamiento de una orbita eliptica (símbolo: *e*) (692)
eclipse an event in which the shadow of one celestial body falls on another (727)	**eclipse/eclipse** un suceso en el que la sombra de un cuerpo celeste cubre otro cuerpo celeste (727)
ecosystem a community of organisms and their abiotic environment (39)	**ecosystem/ecosistema** una comunidad de organismos y su ambiente abiótico (39)
elastic rebound the sudden return of elastically deformed rock to its undeformed shape (295)	**elastic rebound/rebote elástico** ocurre cuando una roca deformada elásticamente vuelve súbitamente a su forma no deformada (295)
electromagnetic spectrum all of the frequencies or wavelengths of electromagnetic radiation (555, 661)	**electromagnetic spectrum/espectro electromagnético** todas las frecuencias o longitudes de onda de la radiación electromagnética (555, 661)
electron a subatomic particle that has a negative charge (82)	**electron/electrón** una partícula subatómica que tiene carga negativa (82)
element a substance that cannot be separated or broken down into simpler substances by chemical means; all atoms of an element have the same atomic number (81)	**element/elemento** una substancia que no se puede separar o descomponer en substancias más simples por medio de métodos químicos; todos los átomos de un elemento tienen el mismo número atómico (81)
elevation the height of an object above sea level (63)	**elevation/elevación** la altura de un objeto sobre el nivel del mar (63)
El Niño the warm-water phase of the El Niño–Southern Oscillation; a periodic occurrence in the eastern Pacific Ocean in which the surface-water temperature becomes unusually warm (635)	**El Niño/El Niño** la fase caliente de la Oscilación Sureña “El Niño”; un fenómeno periódico que ocurre en el océano Pacífico oriental en el que la temperatura del agua superficial se vuelve más caliente que de costumbre (635)
epicenter the point on Earth’s surface directly above an earthquake’s starting point, or focus (296)	**epicenter/epicentro** el punto de la superficie de la Tierra que queda justo arriba del punto de inicio, o foco, de un terremoto (296)
epoch a subdivision of geologic time that is longer than an age but shorter than a period (214)	**epoch/época** una subdivisión del tiempo geológico que es más larga que una edad pero más corta que un período (214)
equinox the moment when the sun appears to cross the celestial equator (673)	**equinox/equinoccio** el momento en que el Sol parece cruzar el ecuador celeste (673)
era a unit of geologic time that includes two or more periods (214)	**era/era** una unidad de tiempo geológico que incluye dos o más períodos (214)

erosion a process in which the materials of Earth's surface are loosened, dissolved, or worn away and transported from one place to another by a natural agent, such as wind, water, ice, or gravity (357)	**erosion/erosión** un proceso por medio del cual los materiales de la superficie de la Tierra se aflojan, disuelven o desgastan y son transportados de un lugar a otro por un agente natural, como el viento, el agua, el hielo o la gravedad (357)
erratic a large rock transported from a distant source by a glacier (426)	**erratic/errática** una piedra grande transportada de una fuente lejana por un glacial (426)
esker a long, winding ridge of gravel and coarse sand deposited by glacial meltwater streams (428)	**esker/esker** una cumbre larga y con curvas, compuesta por grava y arena gruesa depositada por corrientes de aguas glaciares (428)
estuary an area where fresh water from rivers mixes with salt water from the ocean; the part of a river where the tides meet the river current (456)	**estuary/estuario** un área donde el agua dulce de los ríos se mezcla con el agua salada del océano; la parte de un río donde las mareas se encuentran con la corriente del río (456)
evapotranspiration the total loss of water from an area, which equals the sum of the water lost by evaporation from the soil and other surfaces and the water lost by transpiration from organisms (376)	**evapotranspiration/evapotranspiración** la pérdida total de agua de un área, igual a la suma del agua perdida por evaporación del suelo y otras superficies, y el agua perdida debido a la transpiración de los organismos (376)
evolution a heritable change in the characteristics within a population from one generation to the next; the development of new types of organisms from preexisting types of organisms over time (215)	**evolution/evolución** un cambio hereditario en las características de una población que se produce de una generación a la siguiente; el desarrollo de nuevos tipos de organismos a partir de organismos preexistentes a lo largo del tiempo (215)
extrusive igneous rock rock that forms from the cooling and solidification of lava at Earth's surface (131)	**extrusive igneous rock/roca ígnea extrusiva** roca que se forma a partir del enfriamiento y la solidificación de la lava en la superficie de la Tierra (131)

F

fault a break in a body of rock along which one block slides relative to another; a form of brittle strain (277)	**fault/falla** una grieta en un cuerpo rocoso a lo largo de la cual un bloque se desliza respecto a otro; una forma de tensión quebradiza (277)
fault-block mountain a mountain that forms where faulting breaks Earth's crust into large blocks, which causes some blocks to drop down relative to other blocks (283)	**fault-block mountain/montaña de bloque de falla** una montaña que se forma cuando una falla rompe la corteza de la Tierra en grandes bloques, lo cual hace que algunos bloques se hundan respecto a otros bloques (283)
fault zone a region of numerous, closely spaced faults (300)	**fault zone/zona de fallas** una región donde hay muchas fallas, las cuales están cerca unas de otras (300)
felsic describes magma or igneous rock that is rich in feldspars and silica and that is generally light in color (132, 325)	**felsic/félsica** término que describe el magma o la roca ígnea que es rica en feldespato y sílice y que en general es de color claro (132, 325)
fetch the distance that wind blows across an area of the sea to generate waves (527)	**fetch/alcance** la distancia que el viento sopla en un área del mar para generar olas (527)
floodplain an area along a river that forms from sediments deposited when the river overflows its banks (384)	**floodplain/llanura de inundación** un área a lo largo de un río formada por sedimentos que se depositan cuando el río se desborda (384)

focus the location within Earth along a fault at which the first motion of an earthquake occurs (296)

focus/foco el lugar dentro de la Tierra a lo largo de una falla donde ocurre el primer movimiento de un terremoto (296)

fog water vapor that has condensed very near the surface of Earth because air close to the ground has cooled (586)

fog/niebla vapor de agua que se ha condensado muy cerca de la superficie de la Tierra debido al enfriamiento del aire próximo al suelo (586)

fold a form of ductile strain in which rock layers bend, usually as a result of compression (275)

fold/pliegue una forma de tensión dúctil en la cual las capas de roca se curvan, normalmente como resultado de la compresión (275)

folded mountain a mountain that forms when rock layers are squeezed together and uplifted (282)

folded mountain/montaña de plegamiento una montaña que se forma cuando las capas de roca se comprimen y se elevan (282)

foliation the metamorphic rock texture in which mineral grains are arranged in planes or bands (143)

foliation/foliación la textura de una roca metamórfica en la que los granos de mineral están ordenados en planos o bandas (143)

food web a diagram that shows the feeding relationships among organisms in an ecosystem (41)

food web/red alimenticia un diagrama que muestra las relaciones de alimentación entre los organismos de un ecosistema (41)

fossil the trace or remains of an organism that lived long ago, most commonly preserved in sedimentary rock (197)

fossil/fósil los indicios o los restos de un organismo que vivió hace mucho tiempo, comúnmente preservados en las rocas sedimentarias (197)

fossil fuel a nonrenewable energy resource formed from the remains of organisms that lived long ago; examples include oil, coal, and natural gas (159)

fossil fuel/combustible fósil un recurso energético no renovable formado a partir de los restos de organismos que vivieron hace mucho tiempo; algunos ejemplos incluyen el petróleo, el carbón y el gas natural (159)

fracture in geology, a break in a rock, which results from stress, with or without displacement, including cracks, joints, and faults; *also* the manner in which a mineral breaks along either curved or irregular surfaces (110)

fracture/fractura en geología, un rompimiento en una roca, que resulta de la tensión, con o sin desplazamiento, incluyendo grietas, fisuras y fallas; *también,* la forma en la que se rompe un mineral a lo largo de superficies curvas o irregulares (110)

G

galaxy a collection of stars, dust, and gas bound together by gravity (660, 790)

galaxy/galaxia un conjunto de estrellas, polvo y gas unidos por la gravedad (660, 790)

Galilean moon any one of the four largest satellites of Jupiter—Io, Europa, Ganymede, and Callisto—that were discovered by Galileo in 1610 (733)

Galilean moon/satélite galileano cualquiera de los cuatro satélites más grandes de Júpiter (Io, Europa, Ganímedes y Calisto) que fueron descubiertos por Galileo en 1610 (733)

gas giant a planet that has a deep, massive atmosphere, such as Jupiter, Saturn, Uranus, or Neptune (701)

gas giant/gigante gaseoso un planeta con una atmósfera masiva y profunda, como por ejemplo, Júpiter, Saturno, Urano o Neptuno (701)

gemstone a mineral, rock, or organic material that can be used as jewelry or an ornament when it is cut and polished (157)

gemstone/piedra preciosa un mineral, roca o material orgánico que se puede usar como joya u ornamento cuando se corta y se pule (157)

geologic column an ordered arrangement of rock layers that is based on the relative ages of the rocks and in which the oldest rocks are at the bottom (211)	**geologic column/columna geológica** un arreglo ordenado de capas de rocas que se basa en la edad relativa de las rocas y en el cual las rocas más antiguas están al fondo (211)
geology the scientific study of the origin, history, and structure of Earth and the processes that shape Earth (6)	**geology/geología** el estudio científico del origen, la historia y la estructura del planeta Tierra y los procesos que le dan forma (6)
geosphere the mostly solid, rocky part of the Earth; extends from the center of the core to the surface of the crust (33)	**geosphere/geosfera** la parte principalmente sólida y rocosa de la Tierra; se extiende del centro del núcleo a la superficie de la corteza (33)
geothermal energy the energy produced by heat within Earth (165)	**geothermal energy/energía geotérmica** la energía producida por el calor del interior de la Tierra (165)
giant a very large and bright star whose hot core has used most of its hydrogen (784)	**giant/gigante** una estrella muy grande y brillante que tiene un núcleo caliente que ha usado la mayor parte de su hidrógeno (784)
glacial drift the rock material carried and deposited by glaciers (426)	**glacial drift/deriva glacial** el material rocoso que es transportado y depositado por los glaciares (426)
glacier a large mass of moving ice (419)	**glacier/glaciar** una masa grande de hielo en movimiento (419)
global ocean the body of salt water that covers nearly three-fourths of Earth's surface (471)	**global ocean/océano global** la masa de agua salada que cubre cerca de tres cuartas partes de la superficie de la Tierra (471)
global warming a gradual increase in the average global temperature that is due to a higher concentration of gases such as carbon dioxide in the atmosphere (645)	**global warming/calentamiento global** un aumento gradual de la temperatura global promedio debido a una concentración más alta de gases (tales como dióxido de carbono) en la atmósfera (645)
gradient the change in elevation over a given distance (380)	**gradient/gradiente** el cambio en la elevación a lo largo de una distancia determinada (380)
greenhouse effect the warming of the surface and lower atmosphere of Earth that occurs when carbon dioxide, water vapor, and other gases in the air absorb and reradiate infrared radiation (558)	**greenhouse effect/efecto de invernadero** el calentamiento de la superficie terrestre y de la parte más baja de la atmósfera, el cual se produce cuando el dióxido de carbono, el vapor de agua y otros gases del aire absorben radiación infrarroja y la vuelven a irradiar (558)
groundwater the water that is beneath the Earth's surface (397)	**groundwater/agua subterránea** el agua que está debajo de la superficie de la Tierra (397)
Gulf Stream the swift, deep, and warm Atlantic current that flows along the eastern coast of the United States toward the northeast (522)	**Gulf Stream/corriente del Golfo** la corriente rápida, profunda y cálida del océano Atlántico que fluye por la costa este de los Estados Unidos hacia el noreste (522)
gyre a huge circle of moving ocean water found above and below the equator (520)	**gyre/giro** un círculo enorme de agua oceánica en movimiento que se encuentra debajo del ecuador (520)

H

half-life the time required for half of a sample of a radioactive isotope to break down by radioactive decay to form a daughter isotope (194)

half-life/vida media el tiempo que se requiere para que la mitad de una muestra de un isótopo radiactivo se descomponga por desintegración radiactiva y forme un isótopo hijo (194)

headland a high and steep formation of rock that extends out from shore into the water (452)

headland/promontorio una formación rocosa alta y empinada que se extiende de la costa hacia el agua (452)

horizon the line where the sky and the Earth appear to meet; *also* a horizontal layer of soil that can be distinguished from the layers above and below it; *also* a boundary between two rock layers that have different physical properties (354)

horizon/horizonte la línea donde parece que el cielo y la Tierra se unen; *también,* una capa horizontal de suelo que puede distinguirse de las capas que están por encima y por debajo de ella; *también,* un límite entre dos capas de roca que tienen propiedades físicas distintas (354)

horn a sharp, pyramid-like peak that forms because of the erosion of cirques (424)

horn/cuerno un pico puntiagudo en forma de pirámide que se forma debido a la erosión de los circos (424)

hot spot a volcanically active area of Earth's surface, commonly far from a tectonic plate boundary (323)

hot spot/mancha caliente un área volcánicamente activa de la superficie de la Tierra que comúnmente se encuentra lejos de un límite entre placas tectónicas (323)

humus dark, organic material formed in soil from the decayed remains of plants and animals (354)

humus/humus material orgánico obscuro que se forma en la tierra a partir de restos de plantas y animales en descomposición (354)

hurricane a severe storm that develops over tropical oceans and whose strong winds of more than 120 km/h spiral in toward the intensely low-pressure storm center (609)

hurricane/huracán tormenta severa que se desarrolla sobre océanos tropicales, con vientos fuertes que soplan a más de 120 km/h y que se mueven en espiral hacia el centro de presión extremadamente baja de la tormenta (609)

hydroelectric energy electrical energy produced by the flow of water (167)

hydroelectric energy/energía hidroeléctrica energía eléctrica producida por el flujo del agua (167)

hydrolysis a chemical reaction between water and another substance to form two or more new substances; a reaction between water and a salt to create an acid or a base (347)

hydrolysis/hidrólisis una reacción química entre el agua y otras substancias para formar dos o más substancias nuevas; una reacción entre el agua y una sal para crear un ácido o una base (347)

hydrosphere the portion of the Earth that is water (33)

hydrosphere/hidrosfera la porción de la Tierra que es agua (33)

hypothesis an idea or explanation that is based on observations and that can be tested (10)

hypothesis/hipótesis una idea o explicación que se basa en observaciones y que se puede probar (10)

I

ice age a long period of climatic cooling during which the continents are glaciated repeatedly (431)

ice age/edad de hielo un largo período de enfriamiento del clima, durante el cual los continentes se ven repetidamente sometidos a la glaciación (431)

igneous rock rock that forms when magma cools and solidifies (129)

igneous rock/roca ígnea una roca que se forma cuando el magma se enfría y se solidifica (129)

GLOSSARY/GLOSARIO

independent variable in an experiment, the factor that is deliberately manipulated (11)

independent variable/variable independiente en un experimento, el factor que se manipula deliberadamente (11)

index fossil a fossil that is used to establish the age of a rock layer because the fossil is distinct, abundant, and widespread and existed for only a short span of geologic time (200)

index fossil/fósil guía un fósil que se usa para establecer la edad de una capa de roca debido a que puede diferenciarse bien de otros y es abundante; está extendido y existió sólo por un corto período de tiempo geológico (200)

inertia the tendency of an object to resist being moved or, if the object is moving, to resist a change in speed or direction until an outside force acts on the object (694)

inertia/inercia la tendencia de un objeto a no moverse o, si el objeto se está moviendo, la tendencia a resistir un cambio en su rapidez o dirección hasta que una fuerza externa actúe en el objeto (694)

intensity in Earth science, the amount of damage caused by an earthquake (304)

intensity/intensidad en las ciencias de la Tierra, la cantidad de daño causado por un terremoto (304)

internal plastic flow the process by which glaciers flow slowly as grains of ice deform under pressure and slide over each other (421)

internal plastic flow/flujo plástico interno el proceso por medio del cual los glaciares fluyen lentamente a medida que los granos de hielo se deforman por efecto de la presión y se deslizan unos sobre otros (421)

intrusive igneous rock rock formed from the cooling and solidification of magma beneath the Earth's surface (131)

intrusive igneous rock/roca ígnea intrusiva una roca formada a partir del enfriamiento y solidificación del magma debajo de la superficie terrestre (131)

ion an atom, radical, or molecule that has gained or lost one or more electrons and has a negative or positive charge (90)

ion/ion un átomo, radical o molécula que ha ganado o perdido uno o más electrones y que tiene una carga negativa o positiva (90)

ionic bond the attractive force between oppositely charged ions, which form when electrons are transferred from one atom to another (90)

ionic bond/enlace iónico la fuerza de atracción entre iones con cargas opuestas, que se forman cuando se transfieren electrones de un átomo a otro (90)

isogram a line on a map that represents a constant or equal value of a given quantity (62)

isogram/isograma una línea en un mapa que representa un valor constante o igual de una cantidad dada (62)

isostasy a condition of gravitational and buoyant equilibrium between Earth's lithosphere and asthenosphere (271)

isostasy/isostasia una condición de equilibrio gravitacional y flotante entre la litosfera y la astenosfera de la Tierra (271)

isotope an atom that has the same number of protons (or the same atomic number) as other atoms of the same element do but that has a different number of neutrons (and thus a different atomic mass) (83)

isotope/isótopo un átomo que tiene el mismo número de protones (o el mismo número atómico) que otros átomos del mismo elemento, pero que tiene un número diferente de neutrones (y, por lo tanto, otra masa atómica) (83)

J

jet stream a narrow band of strong winds that blow in the upper troposphere (563)

jet stream/corriente en chorro un cinturón delgado de vientos fuertes que soplan en la parte superior de la troposfera (563)

K

karst topography a type of irregular topography that is characterized by caverns, sinkholes, and underground drainage and that forms on limestone or other soluble rock (408)

karst topography/topografía de karst una tipo de topografía irregular que se caracteriza por cavernas, depresiones y drenaje subterráneo y que se forma en piedra caliza o algún otro tipo de roca soluble (408)

kettle a bowl-shaped depression in a glacial drift deposit (428)

kettle/marmita una depresión con forma de tazón en un depósito de deriva glaciar (428)

Kuiper belt a region of the solar system that is just beyond the orbit of Neptune and that contains small bodies made mostly of ice (707, 742)

Kuiper belt/cinturón de Kuiper una región del Sistema Solar que se encuentra más allá de la órbita de Neptuno y que contiene cuerpos pequeños, en su mayoría formados por hielo (707, 742)

L

lagoon a small body of water separated from the sea by a low, narrow strip of land (457)

lagoon/laguna una masa pequeña de agua separada del mar por una tira de tierra baja y angosta (457)

landform a physical feature of Earth's surface (363)

landform/accidente geográfico una característica física de la superficie terrestre (363)

latent heat the heat energy that is absorbed or released by a substance during a phase change (575)

latent heat/calor latente la energía calorífica que es absorbida o liberada por una substancia durante un cambio de fase (575)

latitude the distance north or south from the equator; expressed in degrees (53)

latitude/latitud la distancia hacia el norte o hacia el sur del ecuador; se expresa en grados (53)

lava magma that flows onto Earth's surface; the rock that forms when lava cools and solidifies (320)

lava/lava magma que fluye a la superficie terrestre; la roca que se forma cuando la lava se enfría y se solidifica (320)

law of crosscutting relationships the principle that a fault or body of rock is younger than any other body of rock that it cuts through (190)

law of crosscutting relationships/ley de las relaciones entrecortadas el principio de que una falla o un cuerpo rocoso siempre es más joven que cualquier otro cuerpo rocoso que atraviese (190)

law of superposition the principle that a sedimentary rock layer is older than the layers above it and younger than the layers below it if the layers are not disturbed (187)

law of superposition/ley de la sobreposición el principio de que una capa de roca sedimentaria es más vieja que las capas que se encuentran arriba de ella y más joven que las capas que se encuentran debajo de ella si las capas no han sido alteradas (187)

legend a list of map symbols and their meanings (61)

legend/leyenda una lista de símbolos de un mapas y sus significados (61)

light-year the distance that light travels in one year; about 9.46 trillion kilometers (779)

light-year/año luz la distancia que viaja la luz en un año; aproximadamente 9.46 trillones de kilómetros (779)

lithosphere the solid, outer layer of Earth that consists of the crust and the rigid upper part of the mantle (29, 247)

lithosphere/litosfera la capa externa y sólida de la Tierra que está formada por la corteza y la parte superior y rígida del manto (29, 247)

lode a mineral deposit within a rock formation (156)	**lode/veta** un depósito mineral que se encuentra dentro de una formación rocosa (156)
loess fine-grained sediments of quartz, feldspar, hornblende, mica, and clay deposited by the wind (450)	**loess/loess** sedimentos de grano fino de cuarzo, feldespato, horneblenda, mica y arcilla depositados por el viento (450)
longitude the angular distance east or west from the prime meridian; expressed in degrees (54)	**longitude/longitud** la distancia angular hacia el este o hacia el oeste del primer meridiano; se expresa en grados (54)
longshore current a water current that travels near and parallel to the shoreline (454)	**longshore current/corriente de ribera** una corriente de agua que se desplaza cerca de la costa y paralela a ella (454)
lunar eclipse the passing of the moon through the Earth's shadow at full moon (729)	**lunar eclipse/eclipse lunar** el paso de la Luna frente a la sombra de la Tierra cuando hay luna llena (729)
luster the way in which a mineral reflects light (110)	**luster/brillo** la forma en que un mineral refleja la luz (110)

M

mafic describes magma or igneous rock that is rich in magnesium and iron and that is generally dark in color (132, 325)	**mafic/máfica** término que describe el magma o la roca ígnea que es rica en magnesio y hierro y que en general es de color obscuro (132, 325)
magma liquid rock produced under the Earth's surface; igneous rocks are made of magma (319)	**magma/magma** roca líquida producida debajo de la superficie terrestre; las rocas ígneas están hechas de magma (319)
magnitude a measure of the strength of an earthquake (303)	**magnitude/magnitud** una medida de la fuerza de un terremoto (303)
main sequence the location on the H-R diagram where most stars lie; it has a diagonal pattern from the lower right (low temperature and luminosity) to the upper left (high temperature and luminosity) (781)	**main sequence/secuencia principal** la ubicación en el diagrama H-R donde se encuentran la mayoría de las estrellas; tiene un patrón diagonal de la parte inferior derecha (baja temperatura y luminosidad) a la parte superior izquierda (alta temperatura y luminosidad) (781)
mantle in Earth science, the layer of rock between Earth's crust and core (28)	**mantle/manto** en las ciencias de la Tierra, la capa de roca que se encuentra entre la corteza terrestre y el núcleo (28)
map projection a flat map that represents a spherical surface (58)	**map projection/proyección cartográfica** un mapa plano que representa una superficie esférica (58)
mare a large, dark area of basalt on the moon (plural, *maria*) (720)	**mare/mar lunar** una gran área oscura de basalto en la Luna (720)
mass extinction an episode during which large numbers of species become extinct (221)	**mass extinction/extinción masiva** un episodio durante el cual grandes cantidades de especies se extinguen (221)
mass movement the movement of a large mass of sediment or a section of land down a slope (361)	**mass movement/movimiento masivo** el movimiento hacia abajo por una pendiente de una gran masa de sedimento o una sección de terreno (361)

matter anything that has mass and takes up space (81)	**matter/materia** cualquier cosa que tiene masa y ocupa un lugar en el espacio (81)
meander one of the bends, twists, or curves in a low-gradient stream or river (381)	**meander/meandro** una de las vueltas, giros o curvas de un arroyo o río de bajo gradiente (381)
mechanical weathering the process by which rocks break down into smaller pieces by physical means (343)	**mechanical weathering/desgaste mecánico** el proceso por medio del cual las rocas se rompen en pedazos más pequeños mediante medios físicos (343)
meridian any semicircle that runs north and south around Earth from the geographic North Pole to the geographic South Pole; a line of longitude (54)	**meridian/meridiano** cualquier semicírculo que va de norte a sur alrededor de la Tierra, del Polo Norte geográfico al Polo Sur geográfico; una línea de longitud (54)
mesosphere literally, the "middle sphere"; the strong, lower part of the mantle between the asthenosphere and the outer core; *also* the coldest layer of the atmosphere, between the stratosphere and the thermosphere, in which temperature decreases as altitude increases (29, 553)	**mesosphere/mesosfera** literalmente, la "esfera media"; la parte fuerte e inferior del manto que se encuentra entre la astenosfera y el núcleo externo; *también,* la capa más fría de la atmósfera que se encuentra entre la estratosfera y la termosfera, en la cual la temperatura disminuye al aumentar la altitud (29, 553)
Mesozoic Era the geologic era that lasted from 251 million to 65.5 million years ago; also called the *Age of Reptiles* (221)	**Mesozoic Era/Era Mesozoica** la era geológica que comenzó hace 251 millones de años y terminó hace 65.5 millones de años; también llamada *Edad de los Reptiles* (221)
metamorphism the process in which one type of rock changes into metamorphic rock because of chemical processes or changes in temperature and pressure (141)	**metamorphism/metamorfismo** el proceso en el que un tipo de roca cambia a roca metamórfica debido a procesos químicos o cambios en la temperatura y la presión (141)
meteor a bright streak of light that results when a meteoroid burns up in Earth's atmosphere (743)	**meteor/meteoro** un rayo de luz brillante que se produce cuando un meteoroide se quema en la atmósfera de la Tierra (743)
meteoroid a relatively small, rocky body that travels through space (743)	**meteoroid/meteoroide** un cuerpo rocoso relativamente pequeño que viaja en el espacio (743)
meteorology the scientific study of Earth's atmosphere, especially in relation to weather and climate (7)	**meteorology/meteorología** el estudio científico de la atmósfera de la Tierra, sobre todo en lo que se relaciona al tiempo y al clima (7)
microclimate the climate of a small area (640)	**microclimate/microclima** el clima de un área pequeña (640)
middle-latitude climate a climate that has a maximum average temperature of 8°C in the coldest month and a minimum average temperature of 10°C in the warmest month (638)	**middle-latitude climate/clima de latitud media** un clima que tiene una temperatura máxima promedio de 8ºC en el mes más frío y una temperatura mínima promedio de 10ºC en el mes más caliente (638)
mid-latitude cyclone an area of low pressure that is characterized by rotating wind that moves toward the rising air of the central low-pressure region; the motion is counterclockwise in the Northern Hemisphere (606)	**mid-latitude cyclone/ciclón de latitud media** un área de baja presión caracterizada por la presencia de viento en rotación que se desplaza hacia el aire ascendente de la región central de baja presión; en el hemisferio norte, el movimiento se produce en sentido contrario al de las manecillas del reloj (606)

mid-ocean ridge a long, undersea mountain chain that has a steep, narrow valley at its center, that forms as magma rises from the asthenosphere, and that creates new oceanic lithosphere (sea floor) as tectonic plates move apart (242)	**mid-ocean ridge/dorsal oceánica** una larga cadena submarina de montañas que tiene un valle empinado y angosto en el centro, se forma a medida que el magma se eleva a partir de la astenosfera y produce una nueva litosfera oceánica (suelo marino) a medida que las placas tectónicas se separan (242)
Milankovitch theory the theory that cyclical changes in Earth's orbit and in the tilt of Earth's axis occur over thousands of years and cause climatic changes (433)	**Milankovitch theory/teoría de Milankovitch** la teoría que establece que los cambios cíclicos en la órbita de la Tierra y en la inclinación de su eje se producen a lo largo de miles de años y provocan cambios climáticos (433)
mineral a natural, usually inorganic solid that has a characteristic chemical composition, an orderly internal structure, and a characteristic set of physical properties (103)	**mineral/mineral** un sólido natural, normalmente inorgánico, que tiene una composición química característica, una estructura interna ordenada y propiedades físicas y químicas características (103)
mineralogist a person who examines, analyzes, and classifies minerals (109)	**mineralogist/minerólogo** una persona que examina, analiza y clasifica los minerales (109)
mixture a combination of two or more substances that are not chemically combined (92)	**mixture/mezcla** una combinación de dos o más substancias que no están combinadas químicamente (92)
Mohs hardness scale the standard scale against which the hardness of minerals is rated (111)	**Mohs hardness scale/escala de dureza de Mohs** la escala estándar que se usa para clasificar la dureza de un mineral (111)
molecule a group of atoms that are held together by chemical forces; a molecule is the smallest unit of matter that can exist by itself and retain all of a substance's chemical properties (87)	**molecule/molécula** un conjunto de átomos que se mantienen unidos por acción de las fuerzas químicas; una molécula es la unidad más pequeña de la materia capaz de existir en forma independiente y conservar todas las propiedades químicas de una substancia (87)
monsoon a seasonal wind that blows toward the land in the summer, bringing heavy rains, and that blows away from the land in the winter, bringing dry weather (635)	**monsoon/monzón** viento estacional que sopla hacia la tierra en el verano, ocasionando fuertes lluvias, y que se aleja de la tierra en el invierno, ocasionando tiempo seco (635)
moon a body that revolves around a planet and that has less mass than the planet does (719)	**moon/luna** un cuerpo que gira alrededor de un planeta y que tiene menos que el planeta (719)
moraine a landform that is made from unsorted sediments deposited by a glacier (427)	**moraine/morrena** un accidente geográfico que se forma a partir de varios tipos de sedimentos depositados por un glaciar (427)
mountain range a series of mountains that are closely related in orientation, age, and mode of formation (279)	**mountain range/cinturón de montañas** una serie de montañas que están íntimamente relacionadas en orientación, edad y modo de formación (279)

N

nebula a large cloud of gas and dust in interstellar space; a region in space where stars are born (782)

nebula/nebulosa una nube grande de gas y polvo en el espacio interestelar; una región en el espacio donde las estrellas nacen (782)

nekton all organisms that swim actively in open water, independent of currents (502)

nekton/necton todos los organismos que nadan activamente en las aguas abiertas, de manera independiente de las corrientes (502)

neutron a subatomic particle that has no charge and that is located in the nucleus of an atom (82)

neutron/neutrón una partícula subatómica que no tiene carga y que está ubicada en el núcleo de un átomo (82)

neutron star a star that has collapsed under gravity to the point that the electrons and protons have smashed together to form neutrons (787)

neutron star/estrella de neutrones una estrella que se ha colapsado debido a la gravedad hasta el punto en que los electrones y protones han chocado unos contra otros para formar neutrones (787)

nodule a lump of minerals whose composition differs from the composition of the surrounding sediment or rock; *also* a lump of minerals that is made of oxides of manganese, iron, copper, or nickel and that is found in scattered groups on the ocean floor (481)

nodule/nódulo un bulto de minerales que tienen una composición diferente a la de los sedimentos o rocas de los alrededores; *también,* un bulto de minerales compuesto por óxidos de manganeso, hierro, cobre o níquel y que se encuentra en grupos esparcidos en el fondo del océano (481)

nonfoliated the metamorphic rock texture in which mineral grains are not arranged in planes or bands (144)

nonfoliated/no foliada la textura de una roca metamórfica en la que los granos de mineral no están ordenados en planos ni bandas (144)

nonrenewable resource a resource that forms at a rate that is much slower than the rate at which it is consumed (159)

nonrenewable resource/recurso no renovable un recurso que se forma a una tasa que es mucho más lenta que la tasa a la que se consume (159)

nonsilicate mineral a mineral that does not contain compounds of silicon and oxygen (105)

nonsilicate mineral/mineral no-silicato un mineral que no contiene compuestos de sílice y oxígeno (105)

nova a star that suddenly becomes brighter (786)

nova/nova una estrella que súbitamente se vuelve más brillante (786)

nuclear fission the process by which the nucleus of a heavy atom splits into two or more fragments; the process releases neutrons and energy (162)

nuclear fission/fisión nuclear el proceso por medio del cual el núcleo de un átomo pesado se divide en dos o más fragmentos; el proceso libera neutrones y energía (162)

nuclear fusion the process by which nuclei of small atoms combine to form new, more massive nuclei; the process releases energy (164, 756)

nuclear fusion/fusión nuclear el proceso por medio del cual los núcleos de átomos pequeños se combinan y forman núcleos nuevos con mayor masa; el proceso libera energía (164, 756)

O

observation the process of obtaining information by using the senses; the information obtained by using the senses (10)

observation/observación el proceso de obtener información por medio de los sentidos; la información que se obtiene al usar los sentidos (10)

8301

occluded front a front that forms when a cold air mass overtakes a warm air mass and lifts the warm air mass off the ground and over another air mass (606)

occluded front/frente ocluido un frente que se forma cuando una masa de aire frío supera a una masa de aire caliente y la levanta del suelo por encima de otra masa de aire (606)

oceanography the scientific study of the ocean, including the properties and movements of ocean water, the characteristics of the ocean floor, and the organisms that live in the ocean (6, 472)

oceanography/oceanografía el estudio científico del océano, incluyendo las propiedades y los movimientos del agua, las características del fondo y los organismos que viven en él (6, 472)

Oort cloud a spherical region that surrounds the solar system, that extends from just beyond Pluto's orbit to almost halfway to the nearest star, and that contains trillions of comets (742)

Oort cloud/nube de Oort una región esférica que rodea al Sistema Solar; comienza justo después del inicio de la órbita de Plutón y termina a medio camino entre Plutón y la estrella más cercana; contiene billones de cometas (742)

orbital period the time required for a body to complete a single orbit (693)

orbital period/período de órbita el tiempo que se requiere para que un cuerpo complete una órbita (693)

ore a natural material whose concentration of economically valuable minerals is high enough for the material to be mined profitably (155)

ore/mena un material natural cuya concentración de minerales con valor económico es suficientemente alta como para que el material pueda ser explotado de manera rentable (155)

organic sedimentary rock sedimentary rock that forms from the remains of plants or animals (136)

organic sedimentary rock/roca sedimentaria orgánica roca sedimentaria que se forma a partir de los restos de plantas o animales (136)

oxidation a reaction that removes one or more electrons from a substance such that the substance's valence or oxidation state increases; in geology, the process by which a metallic element combines with oxygen (346)

oxidation/oxidación una reacción en la que uno o más electrones son removidos de una substancia, aumentado su valencia o estado de oxidación; en geología, el proceso por medio del cual un elemento metálico se combina con oxígeno (346)

ozone a gas molecule that is made up of three oxygen atoms (549)

ozone/ozono una molécula de gas que está formada por tres átomos de oxígeno (549)

P

pack ice a floating layer of sea ice that completely covers an area of the ocean surface (497)

pack ice/manto de hielo marino una capa flotante de hielo marino que cubre completamente un área de la superficie del océano (497)

paleomagnetism the study of the alignment of magnetic minerals in rock, specifically as it relates to the reversal of Earth's magnetic poles; *also* the magnetic properties that rock acquires during formation (243)

paleomagnetism/paleomagnetismo el estudio de la alineación de los minerales magnéticos en la roca, específicamente en lo que se relaciona con la inversión de los polos magnéticos de la Tierra; *también,* las propiedades magnéticas que la roca adquiere durante su formación (243)

paleontology the scientific study of fossils (197)

paleontology/paleontología el estudio científico de los fósiles (197)

Paleozoic Era the geologic era that followed Precambrian time and that lasted from 542 million to 251 million years ago (218)

Paleozoic Era/Era Paleozoica la era geológica que vino después del período Precámbrico; comenzó hace 542 millones de años y terminó hace 251 millones de años (218)

English	Español
Pangaea the supercontinent that formed 300 million years ago and that began to break up 250 million years ago (258)	**Pangaea/Pangea** el supercontinente que se formó hace 300 millones de años y que comenzó a separarse hace 250 millones de años (258)
Panthalassa the single, large ocean that covered Earth's surface during the time the supercontinent Pangaea existed (258)	**Panthalassa/Panthalassa** el único gran océano que cubría la superficie de la Tierra cuando existía el supercontinente Pangea (258)
parallax an apparent shift in the position of an object when viewed from different locations (779)	**parallax/paralaje** un cambio aparente en la posición de un objeto cuando se ve desde lugares distintos (779)
parallel any circle that runs east and west around Earth and that is parallel to the equator; a line of latitude (53)	**parallel/paralelo** cualquier círculo que va hacia el Este o hacia el Oeste alrededor de la Tierra y que es paralelo al ecuador; una línea de latitud (53)
peer review the process in which experts in a given field examine the results and conclusions of a scientist's study before that study is accepted for publication (14)	**peer review/evaluación de pares** el proceso en el cual los expertos en un campo dado examinan los resultados y las conclusiones de un estudio científico antes de aceptar su publicación (14)
pelagic zone the region of an ocean or body of fresh water above the benthic zone (503)	**pelagic zone/zona pelágica** la región de un océano o una masa de agua dulce sobre la zona bentónica (503)
perigee in the orbit of a satellite, the point at which the satellite is closest to Earth (725)	**perigee/perigeo** en la órbita de un satélite, el punto en el que el satélite está más cerca de la Tierra (725)
perihelion the point in the orbit of a planet at which the planet is closest to the sun (668)	**perihelion/perihelio** el punto en la órbita de un planeta en el que el planeta está más cerca del Sol (668)
period a unit of geologic time that is longer than an epoch but shorter than an era (214)	**period/período** una unidad de tiempo geológico que es más larga que una época pero más corta que una era (214)
permeability the ability of a rock or sediment to let fluids pass through its open spaces, or pores (398)	**permeability/permeabilidad** la capacidad de una roca o sedimento de permitir que los fluidos pasen a través de sus espacios abiertos o poros (398)
phase in astronomy, the change in the illuminated area of one celestial body as seen from another celestial body; phases of the moon are caused by the changing positions of the Earth, the sun, and the moon (730)	**phase/fase** en astronomía, el cambio en el área iluminada de un cuerpo celeste según se ve desde otro cuerpo celeste; las fases de la Luna se producen como resultado de los cambios en la posición de la Tierra, el Sol y la Luna (730)
photosphere the visible surface of the sun (759)	**photosphere/fotosfera** la superficie visible del Sol (759)
placer deposit a deposit that contains a valuable mineral that has been concentrated by mechanical action (156)	**placer deposit/yacimiento de aluvión** un yacimiento que contiene un mineral valioso que se ha concentrado debido a la acción mecánica (156)
planet any of the primary bodies that orbit the sun; a similar body that orbits another star (685)	**planet/planeta** cualquiera de los cuerpos principales que giran en órbita alrededor del Sol; un cuerpo similar que gira en órbita alrededor de otra estrella (685)
planetesimal a small body from which a planet originated in the early stages of development of the solar system (686)	**planetesimal/planetesimal** un cuerpo pequeño a partir del cual se originó un planeta en las primeras etapas de desarrollo del Sistema Solar (686)

plankton the mass of mostly microscopic organisms that float or drift freely in the waters of aquatic (freshwater and marine) environments (502)

plankton/plancton la masa de organismos casi microscópicos que flotan o se encuentran a la deriva en aguas (dulces y marinas) de ambientes acuáticos (502)

plate tectonics the theory that explains how large pieces of the lithosphere, called plates, move and change shape (247)

plate tectonics/tectónica de placas la teoría que explica cómo las grandes partes de litosfera, denominadas placas, se mueven y cambian de forma (247)

polar climate a climate that is characterized by average temperatures that are near or below freezing; typical of polar regions (639)

polar climate/clima polar un clima caracterizado por temperaturas cercanas o inferiores al punto de congelación; típico de las regiones polares (639)

polar easterlies prevailing winds that blow from east to west between 60° and 90° latitude in both hemispheres (562)

polar easterlies/vientos polares del este vientos preponderantes que soplan de este a oeste entre los 60° y los 90° de latitud en ambos hemisferios (562)

porosity the percentage of the total volume of a rock or sediment that consists of open spaces (397)

porosity/porosidad el porcentaje del volumen total de una roca o sedimento que está formado por espacios abiertos (397)

Precambrian time the interval of time in the geologic time scale from Earth's formation to the beginning of the Paleozoic Era, from 4.6 billion to 542 million years ago (216)

Precambrian time/período Precámbrico el intervalo en la escala de tiempo geológico que abarca desde la formación de la Tierra hasta el comienzo de la Era Paleozoica; comenzó hace 4,600 millones de años y terminó hace 542 millones de años (216)

precipitation any form of water that falls to Earth's surface from the clouds; includes rain, snow, sleet, and hail (376, 587)

precipitation/precipitación cualquier forma de agua que cae de las nubes a la superficie de la Tierra; incluye a la lluvia, nieve, aguanieve y granizo (376, 587)

prominence a loop of relatively cool, incandescent gas that extends above the photosphere and above the sun's edge as seen from Earth (763)

prominence/protuberancia una espiral de gas incandescente y relativamente frío que, vista desde la Tierra, se extiende por encima de la fotosfera y la superficie del Sol (763)

proton a subatomic particle that has a positive charge and that is located in the nucleus of an atom; the number of protons of the nucleus is the atomic number, which determines the identity of an element (82)

proton/protón una partícula subatómica que tiene una carga positiva y que está ubicada en el núcleo de un átomo; el número de protones que hay en el núcleo es el número atómico, y éste determina la identidad del elemento (82)

pulsar a rapidly spinning neutron star that emits pulses of radio and optical energy (788)

pulsar/pulsar una estrella de neutrones que gira rápidamente y emite pulsaciones de energía radioeléctrica y óptica (788)

P wave a primary wave, or compression wave; a seismic wave that causes particles of rock to move in a back-and-forth direction parallel to the direction in which the wave is traveling; P waves are the fastest seismic waves and can travel through solids, liquids, and gases (297)

P wave/onda P una onda primaria u onda de compresión; una onda sísmica que hace que las partículas de roca se muevan en una dirección de atrás hacia delante en forma paralela a la dirección en que viaja la onda; las ondas P son las ondas sísmicas más rápidas y pueden viajar a través de sólidos, líquidos y gases (297)

pyroclastic material fragments of rock that form during a volcanic eruption (326)

pyroclastic material/material piroclástico fragmentos de roca que se forman durante una erupción volcánica (326)

Q

quasar quasi-stellar radio source; a very luminous object that produces energy at a high rate; quasars are thought to be the most distant objects in the universe (792)

quasar/cuasar fuente de radio cuasi-estelar; un objeto muy luminoso que produce energía a una gran velocidad; se piensa que los cuasares son los objetos más distantes del universo (792)

R

radar radio detection and ranging, a system that uses reflected radio waves to determine the velocity and location of objects (613)

radar/radar detección y exploración a gran distancia por medio de ondas de radio; un sistema que usa ondas de radio reflejadas para determinar la velocidad y ubicación de los objetos (613)

radiative zone the zone of the sun's interior that is between the core and the convection zone and in which energy moves by radiation (759)

radiative zone/zona radiactiva la zona del interior del Sol que se encuentra entre el núcleo y la zona de convección y en la cual la energía se mueve por radiación (759)

radiometric dating a method of determining the absolute age of an object by comparing the relative percentages of a radioactive (parent) isotope and a stable (daughter) isotope (193)

radiometric dating/datación radiométrica un método para determinar la edad absoluta de un objeto comparando los porcentajes relativos de un isótopo radiactivo (precursor) y un isótopo estable (hijo) (193)

radiosonde a package of instruments that is carried aloft by balloons to measure upper atmosphere conditions, including temperature, dew point, and wind velocity (613)

radiosonde/radiosonda un conjunto de instrumentos que llevan los globos para medir condiciones de la atmósfera superior, como la temperatura, el punto de rocío y la velocidad del viento (613)

recycling the process of recovering valuable or useful materials from waste or scrap; the process of reusing some items (171)

recycling/reciclar el proceso de recuperar materiales valiosos o útiles de los desechos o de la basura; el proceso de reutilizar algunas cosas (171)

reflecting telescope a telescope that uses a curved mirror to gather and focus light from distant objects (663)

reflecting telescope/telescopio reflector un telescopio que utiliza un espejo curvo para captar y enfocar la luz de objetos lejanos (663)

refracting telescope a telescope that uses a set of lenses to gather and focus light from distant objects (663)

refracting telescope/telescopio refractante un telescopio que utiliza un conjunto de lentes para captar y enfocar la luz de objetos lejanos (663)

refraction the bending of a wavefront as the wavefront passes between two substances in which the speed of the wave differs; *also* the process by which ocean waves bend directly toward the coastline as they approach shallow water (529)

refraction/refracción el curvamiento de un frente de ondas a medida que el frente pasa entre dos substancias en las que la velocidad de las ondas difiere; *también,* el proceso por medio del cual las olas oceánicas se curvan directamente hacia la costa a medida que se acercan a agua poco profunda (529)

regional metamorphism a change in the texture, structure, or chemical composition of a rock due to changes in temperature and pressure over a large area, generally as a result of tectonic forces (142)

regional metamorphism/metamorfismo regional un cambio en la textura, estructura o composición química de una roca debido a cambios en la temperatura y presión en un área extensa, generalmente como resultado de la acción de fuerzas tectónicas (142)

relative age the age of an object in relation to the ages of other objects (186)

relative age/edad relativa la edad de un objeto en relación con la edad de otros objetos (186)

relative humidity the ratio of the amount of water vapor in the air to the amount of water vapor needed to reach saturation at a given temperature (578)

relative humidity/humedad relativa la proporción de la cantidad de vapor de agua que hay en el aire respecto a la cantidad de vapor de agua necesaria para alcanzar la saturación a una temperatura dada (578)

relief the difference between the highest and lowest elevations in a given area; the variations in elevation of a land surface (64)

relief/relieve la diferencia entre las elevaciones más altas y las más bajas en un área dada; las variaciones en elevación de una superficie de terreno (64)

remote sensing the process of gathering and analyzing information about an object without physically being in touch with the object (57)

remote sensing/teledetección el proceso de recopilar y analizar información acerca de un objeto sin estar en contacto físico con el objeto (57)

renewable resource a natural resource that can be replaced at the same rate at which the resource is consumed (165)

renewable resource/recurso renovable un recurso natural que puede reemplazarse a la misma tasa a la que se consume (165)

revolution the motion of a body that travels around another body in space; one complete trip along an orbit (668)

revolution/revolución el movimiento de un cuerpo que viaja alrededor de otro cuerpo en el espacio; un viaje completo a lo largo de una órbita (668)

rifting the process by which Earth's crust breaks apart; can occur within continental crust or oceanic crust (255)

rifting/fracturación el proceso por medio del cual la corteza de la Tierra se fractura; puede producirse dentro de la corteza continental u oceánica (255)

rock cycle the series of processes in which rock forms, changes from one type to another, is destroyed, and forms again by geologic processes (126)

rock cycle/ciclo de las rocas la serie de procesos por medio de los cuales una roca se forma, cambia de un tipo a otro, se destruye y se forma nuevamente por procesos geológicos (126)

rotation the spin of a body on its axis (667)

rotation/rotación el giro de un cuerpo alrededor de su eje (667)

S

salinity a measure of the amount of dissolved salts in a given amount of liquid (496)

salinity/salinidad una medida de la cantidad de sales disueltas en una cantidad determinada de líquido (496)

saltation the movement of sand or other sediments by short jumps and bounces that is caused by wind or water (445)

saltation/saltación el movimiento de la arena u otros sedimentos por medio de saltos pequeños y rebotes debido al viento o al agua (445)

satellite a natural or artificial body that revolves around a planet (719)

satellite/satélite un cuerpo natural o artificial que gira alrededor de un planeta (719)

scale the relationship between the distance shown on a map and the actual distance (61)

scale/escala la relación entre la distancia que se muestra en un mapa y la distancia real (61)

sea a large, commonly saline body of water that is smaller than an ocean and that may be partially or completely surrounded by land; *also* a subdivision of an ocean (471)

sea/mar una gran masa de agua, generalmente salada, que es más pequeña que un océano y que puede estar parcial o totalmente rodeada de tierra; *también,* una subdivisión de un océano (471)

sea-floor spreading the process by which new oceanic lithosphere (sea floor) forms as magma rises to Earth's surface and solidifies at a mid-ocean ridge (243)	**sea-floor spreading/expansión del suelo marino** el proceso por medio del cual se forma nueva litosfera oceánica (suelo marino) a medida que el magma se eleva a la superficie de la Tierra y se solidifica en una dorsal oceánica (243)
seismic gap an area along a fault where relatively few earthquakes have occurred recently but where strong earthquakes are known to have occurred in the past (307)	**seismic gap/brecha sísmica** un área a lo largo de una falla donde han ocurrido relativamente pocos terremotos recientemente, pero donde se sabe que han ocurrido terremotos fuertes en el pasado (307)
seismogram a tracing of earthquake motion that is recorded by a seismograph (301)	**seismogram/sismograma** una traza del movimiento de un terremoto registrada por un sismógrafo (301)
seismograph an instrument that records vibrations in the ground (301)	**seismograph/sismógrafo** un instrumento que registra las vibraciones en el suelo (301)
shadow zone an area on Earth's surface where no direct seismic waves from a particular earthquake can be detected (298)	**shadow zone/zona de sombra** un área de la superficie de la Tierra donde no se detectan ondas sísmicas directas de un determinado terremoto (298)
sheet erosion the process by which water flows over a layer of soil and removes the topsoil (358)	**sheet erosion/erosión laminar** el proceso por medio del cual el agua fluye sobre el suelo y remueve la capa superior de éste (358)
silicate mineral a mineral that contains a combination of silicon and oxygen and that may also contain one or more metals (104)	**silicate mineral/mineral silicato** un mineral que contiene una combinación de silicio y oxígeno y que también puede contener uno o más metales (104)
silicon-oxygen tetrahedron the basic unit of the structure of silicate minerals; a silicon ion chemically bonded to and surrounded by four oxygen ions (106)	**silicon-oxygen tetrahedron/tetraedro de sílice-oxígeno** la unidad fundamental de la estructura de los minerales silicatos: un ion de silicio unido químicamente a cuatro iones de oxígeno, los cuales lo rodean (106)
sinkhole a circular depression that forms when rock dissolves, when overlying sediment fills an existing cavity, or when the roof of an underground cavern or mine collapses (407)	**sinkhole/depresión** una depresión circular que se forma cuando la roca se funde, cuando el sedimento suprayacente llena una cavidad existente, o al colapsarse el techo de una caverna o mina subterránea (407)
soil a loose mixture of rock fragments and organic material that can support the growth of vegetation (353)	**soil/suelo** una mezcla suelta de fragmentos de roca y material orgánico en la que puede crecer vegetación (353)
soil profile a vertical section of soil that shows the layers of horizons (354)	**soil profile/perfil del suelo** una sección vertical de suelo que muestra las capas u horizontes (354)
solar eclipse the passing of the moon between Earth and the sun; during a solar eclipse, the shadow of the moon falls on Earth (727)	**solar eclipse/eclipse solar** el paso de la Luna entre la Tierra y el Sol; durante un eclipse solar, la sombra de la Luna cae sobre la Tierra (727)
solar energy the energy received by Earth from the sun in the form of radiation (166)	**solar energy/energía solar** la energía que la Tierra recibe del Sol en forma de radiación (166)
solar flare an explosive release of energy that comes from the sun and that is associated with magnetic disturbances on the sun's surface (763)	**solar flare/erupción solar** una liberación explosiva de energía que proviene del Sol y que se asocia con disturbios magnéticos en la superficie solar (763)

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solar nebula a rotating cloud of gas and dust from which the sun and planets formed; *also* any nebula from which stars and planets may form (685)	**solar nebula/nebulosa solar** una nube de gas y polvo en rotación a partir de la cual se formaron el Sol y los planetas; *también,* cualquier nebulosa a partir de la cual se pueden formar estrellas y planetas (685)
solar system the sun and all of the planets and other bodies that travel around it (685)	**solar system/Sistema Solar** el Sol y todos los planetas y otros cuerpos que se desplazan alrededor de él (685)
solifluction the slow, downslope flow of soil saturated with water in areas surrounding glaciers at high elevations (362)	**solifluction/soliflucción** el flujo lento y descendente de suelo saturado con agua en áreas que rodean glaciares a altas elevaciones (362)
solstice the point at which the sun is as far north or as far south of the equator as possible (674)	**solstice/solsticio** el punto en el que el Sol está tan lejos del ecuador como es posible, ya sea hacia el norte o hacia el sur (674)
solution a homogeneous mixture throughout which two or more substances are uniformly dispersed (92)	**solution/solución** una mezcla homogénea en la cual dos o más sustancias se dispersan de manera uniforme (92)
sonar sound navigation and ranging, a system that uses acoustic signals and returned echoes to determine the location of objects or to communicate (473)	**sonar/sonar** navegación y exploración por medio del sonido; un sistema que usa señales acústicas y ondas de eco que regresan para determinar la ubicación de los objetos o para comunicarse (473)
specific heat the quantity of heat required to raise a unit mass of homogeneous material 1 K or 1°C in a specified way given constant pressure and volume (634)	**specific heat/calor específico** la cantidad de calor que se requiere para aumentar una unidad de masa de un material homogéneo 1 K ó 1°C de una manera especificada, dados un volumen y una presión constantes (634)
star a large celestial body that is composed of gas and that emits light; the sun is a typical star (775)	**star/estrella** un cuerpo celeste grande que está compuesto de gas y emite luz; el Sol es una estrella típica (775)
stationary front a front of air masses that moves either very slowly or not at all (606)	**stationary front/frente estacionario** un frente de masas de aire que se mueve muy lentamente o que no se mueve (606)
station model a pattern of meteorological symbols that represents the weather at a particular observing station and that is recorded on a weather map (616)	**station model/estación modelo** el modelo de símbolos meteorológicos que representan el tiempo en una estación de observación determinada y que se registra en un mapa meteorológico (616)
strain any change in a rock's shape or volume caused by stress; deformation (274)	**strain/tensión** cualquier cambio en la forma o volumen de una roca causado por el estrés; deformación (274)
stratosphere the layer of the atmosphere that lies between the troposphere and the mesosphere and in which temperature increases as altitude increases; contains the ozone layer (553)	**stratosphere/estratosfera** la capa de la atmósfera que se encuentra entre la troposfera y la mesosfera y en la cual la temperatura aumenta al aumentar la altitud; contiene la capa de ozono (553)
stratus cloud a gray cloud that has a flat, uniform base and that commonly forms at very low altitudes (584)	**stratus cloud/nube estrato** una nube gris que tiene una base plana y uniforme y que comúnmente se forma a altitudes muy bajas (584)
streak the color of a mineral in powdered form (110)	**streak/veta** el color de un mineral en forma de polvo (110)

stream load the materials other than the water that are carried by a stream (380)	**stream load/carga de un arroyo** los materiales que lleva un arroyo, además del agua (380)
stress the amount of force per unit area that acts on a rock (273)	**stress/estrés** la cantidad de fuerza por unidad de área que se ejerce sobre una roca (273)
sublimation the process in which a solid changes directly into a gas (the term is sometimes also used for the reverse process) (576)	**sublimation/sublimación** el proceso por medio del cual un sólido se transforma directamente en un gas (en ocasiones, este término también se usa para describir el proceso inverso) (576)
sunspot a dark area of the photosphere of the sun that is cooler than the surrounding areas and that has a strong magnetic field (761)	**sunspot/mancha solar** un área oscura en la fotosfera del Sol que es más fría que las áreas que la rodean y que tiene un campo magnético fuerte (761)
supercontinent cycle the process by which supercontinents form and break apart over millions of years (258)	**supercontinent cycle/ciclo de los supercontinentes** el proceso por medio del cual los supercontinentes se forman y se separan a lo largo de millones de años (258)
supercooling a condition in which a substance is cooled below its freezing point, condensation point, or sublimation point without going through a change of state (588)	**supercooling/superfrío** una condición en la que una sustancia se enfría por debajo de su punto de congelación, punto de condensación o punto de sublimación sin pasar por un cambio de estado (588)
surface current a horizontal movement of ocean water that is caused by wind and that occurs at or near the ocean's surface (519)	**surface current/corriente superficial** un movimiento horizontal del agua del océano que es producido por el viento y que ocurre en la superficie del océano o cerca de ella (519)
surface wave in geology, a seismic wave that travels along the surface of a medium and that has a stronger effect near the surface of the medium than it has in the interior (296)	**surface wave/onda superficial** en geología, una onda sísmica que se desplaza a lo largo de la superficie de un medio, cuyo efecto es más fuerte cerca de la superficie del medio que en el interior de éste (296)
S wave a secondary wave, or shear wave; a seismic wave that causes particles of rock to move in a side-to-side direction perpendicular to the direction in which the wave is traveling; S waves are the second-fastest seismic waves and can travel only through solids (297)	**S wave/onda S** una onda secundaria u onda rotacional; una onda sísmica que hace que las partículas de roca se muevan en una dirección de lado a lado, en forma perpendicular a la dirección en la que viaja la onda; las ondas S son las segundas ondas sísmicas en cuanto a velocidad y únicamente pueden viajar a través de sólidos (297)
system a set of particles or interacting components considered to be a distinct physical entity for the purpose of study (31)	**system/sistema** un conjunto de partículas o componentes que interactúan unos con otros, el cual se considera una entidad física independiente para fines de estudio (31)

T

telescope an instrument that collects electromagnetic radiation from the sky and concentrates it for better observation (662)	**telescope/telescopio** un instrumento que capta la radiación electromagnética del cielo y la concentra para mejorar la observación (662)
terrane a piece of lithosphere that has a unique geologic history and that may be part of a larger piece of lithosphere, such as a continent (256)	**terrane/macizo autóctono** un fragmento de litosfera que tiene una historia geológica única y que puede formar parte de un fragmento de litosfera mayor, como por ejemplo, un continente (256)

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English	Español
terrestrial planet one of the highly dense planets nearest to the sun; Mercury, Venus, Mars, and Earth (695)	**terrestrial planet/planeta terrestre** uno de los planetas muy densos que se encuentran más cerca del Sol; Mercurio, Venus, Marte y la Tierra (695)
theory an explanation for some phenomenon that is based on observation, experimentation, and reasoning; that is supported by a large quantity of evidence; and that does not conflict with any existing experimental results or observations (15)	**theory/teoría** una explicación sobre algún fenómeno que está basada en la observación, experimentación y razonamiento; que está respaldada por una gran cantidad de pruebas; y que no contradice ningún resultado experimental ni observación existente (15)
thermocline a layer in a body of water in which water temperature drops with increased depth faster than it does in other layers (498)	**thermocline/termoclinal** una capa en una masa de agua en la que, al aumentar la profundidad, la temperatura del agua disminuye más rápido de lo que lo hace en otras capas (498)
thermometer an instrument that measures and indicates temperature (611)	**thermometer/termómetro** un instrumento que mide e indica la temperatura (611)
thermosphere the uppermost layer of the atmosphere, in which temperature increases as altitude increases; includes the ionosphere (553)	**thermosphere/termosfera** la capa más alta de la atmósfera, en la cual la temperatura aumenta a medida que la altitud aumenta; incluye la ionosfera (553)
thunderstorm a usually brief, heavy storm that consists of rain, strong winds, lightning, and thunder (608)	**thunderstorm/tormenta eléctrica** una tormenta fuerte y normalmente breve que consiste en lluvia, vientos fuertes, relámpagos y truenos (608)
tidal current the movement of water toward and away from the coast as a result of the rise and fall of the tides (534)	**tidal current/corriente de marea** el movimiento del agua hacia la costa y de la costa hacia el mar, como resultado del ascenso y descenso de las mareas (534)
tidal oscillation the slow, rocking motion of ocean water that occurs as the tidal bulges move around the ocean basins (533)	**tidal oscillation/oscilación de las mareas** el movimiento lento y mecedor del agua del océano que se produce cuando los abultamientos de marea se mueven alrededor de las cuencas oceánicas (533)
tidal range the difference in levels of ocean water at high tide and low tide (532)	**tidal range/rango de marea** la diferencia en los niveles del agua del océano entre la marea alta y la marea baja (532)
tide the periodic rise and fall of the water level in the oceans and other large bodies of water (531)	**tide/marea** el ascenso y descenso periódico del nivel del agua en los océanos y otras masas grandes de agua (531)
till unsorted rock material that is deposited directly by a melting glacier (426)	**till/arcilla** glaciárica material rocoso desordenado que deposita directamente un glaciar que se está derritiendo (426)
topography the size and shape of the land surface features of a region, including its relief (63)	**topography/topografía** el tamaño y la forma de las características de una superficie de terreno, incluyendo su relieve (63)
tornado a destructive, rotating column of air that has very high wind speeds and that may be visible as a funnel-shaped cloud (610)	**tornado/tornado** una columna destructiva de aire en rotación cuyos vientos se mueven a velocidades muy altas y que puede verse como una nube con forma de embudo (610)

trace fossil a fossilized mark that formed in sedimentary rock by the movement of an animal on or within soft sediment (199)

trace fossil/fósil traza una marca fosilizada que se formó en una roca sedimentaria por el movimiento de un animal sobre sedimento blando o dentro de éste (199)

trade winds prevailing winds that blow from east to west from 30° latitude to the equator in both hemispheres (562)

trade winds/vientos alisios vientos prevalecientes que soplan de este a oeste desde los 30° de latitud hacia el ecuador en ambos hemisferios (562)

transform boundary the boundary between tectonic plates that are sliding past each other horizontally (251)

transform boundary/límite de transformación el límite entre placas tectónicas que se están deslizando horizontalmente una sobre otra (251)

trench a long, narrow, and steep depression that forms on the ocean floor as a result of subduction of a tectonic plate, that runs parallel to the trend of a chain of volcanic islands or the coastline of a continent, and that may be as deep as 11 km below sea level; also called an *ocean trench* or a *deep-ocean trench* (477)

trench/fosa submarina una depresión larga, angosta y empinada que se forma en el fondo del océano debido a la subducción de una placa tectónica; corre paralela al curso de una cadena de islas montañosas o a la costa de un continente; y puede tener una profundidad de hasta 11 km bajo el nivel del mar; también denominada *fosa oceánica* o *fosa oceánica profunda* (477)

tributary a stream that flows into a lake or into a larger stream (379)

tributary/afluente un arroyo que fluye a un lago o a otro arroyo más grande (379)

tropical climate a climate characterized by high temperatures and heavy precipitation during at least part of the year; typical of equatorial regions (637)

tropical climate/clima tropical un clima caracterizado por temperaturas altas y precipitación fuerte durante al menos una parte del año; típico de las regiones ecuatoriales (637)

troposphere the lowest layer of the atmosphere, in which temperature drops at a constant rate as altitude increases; the part of the atmosphere where weather conditions exist (552)

troposphere/troposfera la capa inferior de la atmósfera, en la que la temperatura disminuye a una tasa constante a medida que la altitud aumenta; la parte de la atmósfera donde se dan las condiciones del tiempo (552)

tsunami a giant ocean wave that forms after a volcanic eruption, submarine earthquake, or landslide (305)

tsunami/tsunami una ola gigante del océano que se forma después de una erupción volcánica, terremoto submarino o desprendimiento de tierras (305)

U

unconformity a break in the geologic record created when rock layers are eroded or when sediment is not deposited for a long period of time (189)

unconformity/disconformidad una ruptura en el registro geológico, creada cuando las capas de roca se erosionan o cuando el sedimento no se deposita durante un largo período de tiempo (189)

uniformitarianism a principle that geologic processes that occurred in the past can be explained by current geologic processes (185)

uniformitarianism/uniformitarianismo un principio que establece que es posible explicar los procesos geológicos que ocurrieron en el pasado en función de los procesos geológicos actuales (185)

upwelling the movement of deep, cold, and nutrient-rich water to the surface (502)

upwelling/surgencia el movimiento de las aguas profundas, frías y ricas en nutrientes hacia la superficie (502)

V

varve a banded layer of sand and silt that is de-posited annually in a lake, especially near ice sheets or glaciers, and that can be used to determine absolute age (192)

varve/sedimentos cíclicos estacionales una capa de arena y limo dispuestos en bandas, que se deposita en un lago durante un año, especialmente cerca de las capas de hielo o los glaciares, y que puede usarse para determinar la edad absoluta (192)

ventifact any rock that is pitted, grooved, or polished by wind abrasion (447)

ventifact/ventifacto cualquier roca que es marcada, estriada o pulida por la abrasión del viento (447)

volcanism any activity that includes the movement of magma toward or onto the Earth's surface (320)

volcanism/volcanismo cualquier actividad que incluye el movimiento de magma hacia la superficie de la Tierra o sobre ella (320)

volcano a vent or fissure in the Earth's surface through which magma and gases are expelled (320)

volcano/volcán una chimenea o fisura en la superficie de la Tierra a través de la cual se expulsan magma y gases (320)

W

warm front the front edge of an advancing warm air mass that replaces colder air with warmer air (606)

warm front/frente cálido el borde del frente de una masa de aire caliente en movimiento que reemplaza al aire más frío (606)

water cycle the continuous movement of water between the atmosphere, the land, and the oceans (375)

water cycle/ciclo del agua el movimiento continuo del agua entre la atmósfera, la tierra y los océanos (375)

watershed the area of land that is drained by a river system (379)

watershed/cuenca hidrográfica el área del terreno que es drenada por un sistema de ríos (379)

water table the upper surface of underground water; the upper boundary of the zone of saturation (399)

water table/capa freática el nivel más alto del agua subterránea; el límite superior de la zona de saturación (399)

wave a periodic disturbance in a solid, liquid, or gas as energy is transmitted through a medium (525)

wave/onda una perturbación periódica en un sólido, líquido o gas que se transmite a través de un medio en forma de energía (525)

wave period the time required for identical points on consecutive waves to pass a given point (525)

wave period/período de onda el tiempo que se requiere para que puntos idénticos de ondas consecutivas pasen por un punto dado (525)

weathering the natural process by which atmospheric and environmental agents, such as wind, rain, and temperature changes, disintegrate and decompose rocks (343)

weathering/meteorización el proceso natural por medio del cual los agentes atmosféricos o ambientales, como el viento, la lluvia y los cambios de temperatura, desintegran y descomponen las rocas (343)

westerlies prevailing winds that blow from west to east between 30° and 60° latitude in both hemispheres (562)

westerlies/vientos del oeste vientos preponderantes que soplan de oeste a este entre 30° y 60° de latitud en ambos hemisferios (562)

white dwarf a small, hot, dim star that is the left-over center of an old star (785)

white dwarf/enana blanca una estrella pequeña, caliente y tenue que es el centro sobrante de una estrella vieja (785)

wind vane an instrument used to determine the direction of the wind (612)

wind vane/veleta un instrumento que se usa para determinar la dirección del viento (612)

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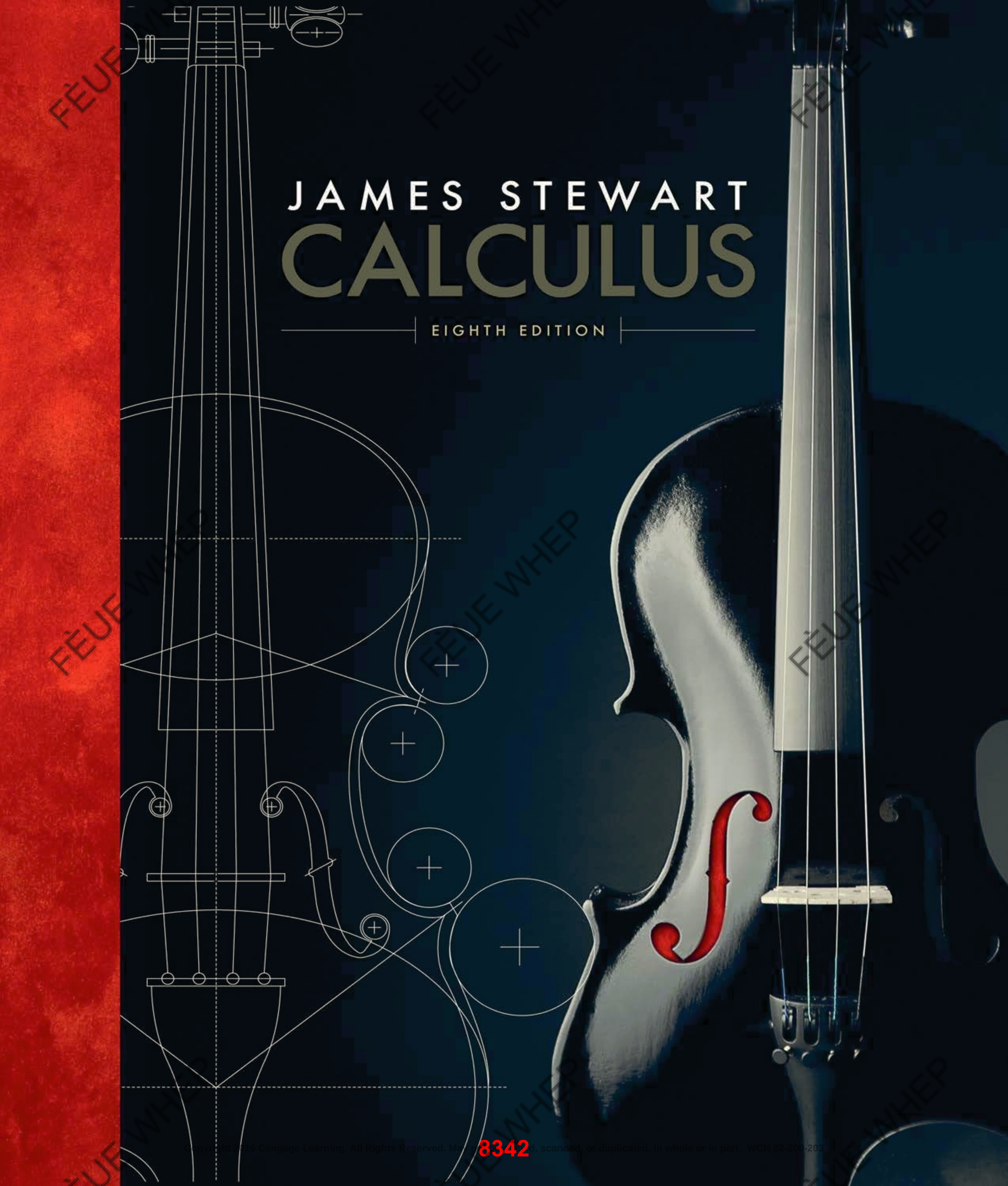
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Library of Congress Control Number: 2015937035

Student Edition:
ISBN: 978-1-285-74062-1

Loose-leaf Edition:
ISBN: 978-1-305-27176-0

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K04T16

Printed in the United States of America
Print Number: 04 Print Year: 2016

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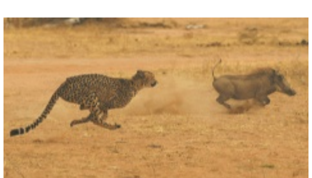

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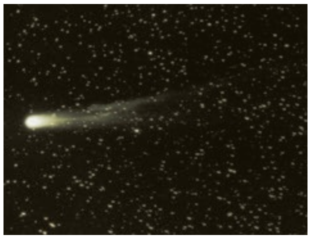

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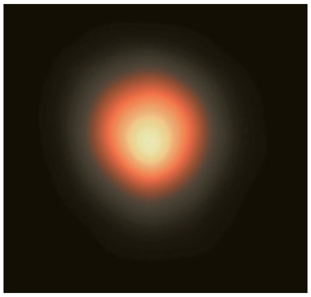

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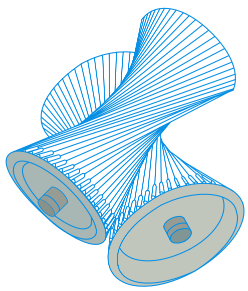

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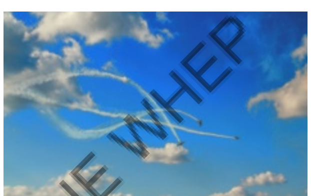

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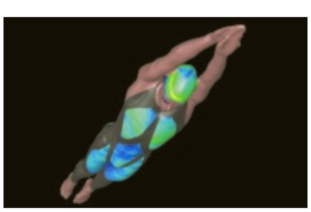

Preface

> A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.
>
> GEORGE POLYA

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first seven editions, I aim to convey to the student a sense of the utility of calculus and develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding concepts. I think that nearly everybody agrees that this should be the primary goal of calculus instruction. In fact, the impetus for the current calculus reform movement came from the Tulane Conference in 1986, which formulated as their first recommendation:

Focus on conceptual understanding.

I have tried to implement this goal through the *Rule of Three:* "Topics should be presented geometrically, numerically, and algebraically." Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well.

In writing the eighth edition my premise has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. The book contains elements of reform, but within the context of a traditional curriculum.

Alternate Versions

I have written several other calculus textbooks that might be preferable for some instructors. Most of them also come in single variable and multivariable versions.

- *Calculus: Early Transcendentals,* Eighth Edition, is similar to the present textbook except that the exponential, logarithmic, and inverse trigonometric functions are covered in the first semester.
- *Essential Calculus*, Second Edition, is a much briefer book (840 pages), though it contains almost all of the topics in *Calculus,* Eighth Edition. The relative brevity is achieved through briefer exposition of some topics and putting some features on the website.

- *Essential Calculus: Early Transcendentals,* Second Edition, resembles *Essential Calculus,* but the exponential, logarithmic, and inverse trigonometric functions are covered in Chapter 3.
- *Calculus: Concepts and Contexts*, Fourth Edition, emphasizes conceptual understanding even more strongly than this book. The coverage of topics is not encyclopedic and the material on transcendental functions and on parametric equations is woven throughout the book instead of being treated in separate chapters.
- *Calculus: Early Vectors* introduces vectors and vector functions in the first semester and integrates them throughout the book. It is suitable for students taking engineering and physics courses concurrently with calculus.
- *Brief Applied Calculus* is intended for students in business, the social sciences, and the life sciences.
- *Biocalculus: Calculus for the Life Sciences* is intended to show students in the life sciences how calculus relates to biology.
- *Biocalculus: Calculus, Probability, and Statistics for the Life Sciences* contains all the content of *Biocalculus: Calculus for the Life Sciences* as well as three additional chapters covering probability and statistics.

What's New in the Eighth Edition?

The changes have resulted from talking with my colleagues and students at the University of Toronto and from reading journals, as well as suggestions from users and reviewers. Here are some of the many improvements that I've incorporated into this edition:

- The data in examples and exercises have been updated to be more timely.
- New examples have been added (see Examples 5.1.5, 11.2.5, and 14.3.3, for instance). And the solutions to some of the existing examples have been amplified.
- Three new projects have been added: The project *Planes and Birds: Minimizing Energy* (page 271) asks how birds can minimize power and energy by flapping their wings versus gliding. The project *Controlling Red Blood Cell Loss During Surgery* (page 473) describes the ANH procedure, in which blood is extracted from the patient before an operation and is replaced by saline solution. This dilutes the patient's blood so that fewer red blood cells are lost during bleeding and the extracted blood is returned to the patient after surgery. In the project *The Speedo LZR Racer* (page 976) it is explained that this suit reduces drag in the water and, as a result, many swimming records were broken. Students are asked why a small decrease in drag can have a big effect on performance.
- I have streamlined Chapter 15 (Multiple Integrals) by combining the first two sections so that iterated integrals are treated earlier.
- More than 20% of the exercises in each chapter are new. Here are some of my favorites: 2.1.61, 2.2.34–36, 3.3.30, 3.3.54, 3.7.39, 3.7.67, 4.1.19–20, 4.2.67–68, 4.4.63, 5.1.51, 6.2.79, 6.7.54, 6.8.90, 8.1.39, 12.5.81, 12.6.29–30, 14.6.65–66. In addition, there are some good new Problems Plus. (See Problems 10–12 on page 201, Problem 10 on page 290, Problems 14–15 on pages 353–54, and Problem 8 on page 1026.)

Features

Conceptual Exercises

The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first few exercises in Sections 1.5, 1.8, 11.2, 14.2, and 14.3.) Similarly, all the review sections begin with a Concept Check and a True-False Quiz. Other exercises test conceptual understanding through graphs or tables (see Exercises 2.1.17, 2.2.33–36, 2.2.45–50, 9.1.11–13, 10.1.24–27, 11.10.2, 13.2.1–2, 13.3.33–39, 14.1.1–2, 14.1.32–38, 14.1.41–44, 14.3.3–10, 14.6.1–2, 14.7.3–4, 15.1.6–8, 16.1.11–18, 16.2.17–18, and 16.3.1–2).

Another type of exercise uses verbal description to test conceptual understanding (see Exercises 1.8.10, 2.2.64, 3.3.57–58, and 7.8.67). I particularly value problems that combine and compare graphical, numerical, and algebraic approaches (see Exercises 2.7.25, 3.4.33–34, and 9.4.4).

Graded Exercise Sets

Each exercise set is carefully graded, progressing from basic conceptual exercises and skill-development problems to more challenging problems involving applications and proofs.

Real-World Data

My assistants and I spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate, and illustrate the concepts of calculus. As a result, many of the examples and exercises deal with functions defined by such numerical data or graphs. See, for instance, Figure 1 in Section 1.1 (seismograms from the Northridge earthquake), Exercise 2.2.33 (unemployment rates), Exercise 4.1.16 (velocity of the space shuttle *Endeavour*), and Figure 4 in Section 4.4 (San Francisco power consumption). Functions of two variables are illustrated by a table of values of the wind-chill index as a function of air temperature and wind speed (Example 14.1.2). Partial derivatives are introduced in Section 14.3 by examining a column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. This example is pursued further in connection with linear approximations (Example 14.4.3). Directional derivatives are introduced in Section 14.6 by using a temperature contour map to estimate the rate of change of temperature at Reno in the direction of Las Vegas. Double integrals are used to estimate the average snowfall in Colorado on December 20–21, 2006 (Example 15.1.9). Vector fields are introduced in Section 16.1 by depictions of actual velocity vector fields showing San Francisco Bay wind patterns.

Projects

One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed. I have included four kinds of projects: *Applied Projects* involve applications that are designed to appeal to the imagination of students. The project after Section 9.3 asks whether a ball thrown upward takes longer to reach its maximum height or to fall back to its original height. (The answer might surprise you.) The project after Section 14.8 uses Lagrange multipliers to determine the masses of the three stages of a rocket so as to minimize the total mass while enabling the rocket to reach a desired velocity. *Laboratory Projects* involve technology; the one following Section 10.2 shows how to use Bézier curves to design shapes that represent letters for a laser printer. *Writing Projects* ask students to compare present-day methods with those of the founders of calculus—Fermat's method for finding tangents, for instance. Suggested references are supplied. *Discovery Projects* anticipate results to be discussed later or encourage discovery through pattern recognition (see the one following Section 7.6). Others explore aspects of geometry: tetrahedra (after Section 12.4), hyperspheres (after Section 15.6), and intersections of three cylinders (after Section 15.7). Additional projects can be found in the *Instructor's Guide* (see, for instance, Group Exercise 4.1: Position from Samples).

Problem Solving

Students usually have difficulties with problems for which there is no single well-defined procedure for obtaining the answer. I think nobody has improved very much on George Polya's four-stage problem-solving strategy and, accordingly, I have included a version of his problem-solving principles following Chapter 1. They are applied, both explicitly and implicitly, throughout the book. After the other chapters I have placed sections called *Problems Plus*, which feature examples of how to tackle challenging calculus problems. In selecting the varied problems for these sections I kept in mind the following advice from David Hilbert: "A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts." When I put these challenging problems on assignments and tests I grade them in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant.

Dual Treatment of Exponential and Logarithmic Functions

There are two possible ways of treating the exponential and logarithmic functions and each method has its passionate advocates. Because one often finds advocates of both approaches teaching the same course, I include full treatments of both methods. In Sections 6.2, 6.3, and 6.4 the exponential function is defined first, followed by the logarithmic function as its inverse. (Students have seen these functions introduced this way since high school.) In the alternative approach, presented in Sections 6.2*, 6.3*, and 6.4*, the logarithm is defined as an integral and the exponential function is its inverse. This latter method is, of course, less intuitive but more elegant. You can use whichever treatment you prefer.

If the first approach is taken, then much of Chapter 6 can be covered before Chapters 4 and 5, if desired. To accommodate this choice of presentation there are specially identified problems involving integrals of exponential and logarithmic functions at the end of the appropriate sections of Chapters 4 and 5. This order of presentation allows a faster-paced course to teach the transcendental functions and the definite integral in the first semester of the course.

For instructors who would like to go even further in this direction I have prepared an alternate edition of this book, called *Calculus: Early Transcendentals,* Eighth Edition, in which the exponential and logarithmic functions are introduced in the first chapter. Their limits and derivatives are found in the second and third chapters at the same time as polynomials and the other elementary functions.

Tools for Enriching Calculus

TEC is a companion to the text and is intended to enrich and complement its contents. (It is now accessible in the eBook via CourseMate and Enhanced WebAssign. Selected Visuals and Modules are available at www.stewartcalculus.com.) Developed by Harvey Keynes, Dan Clegg, Hubert Hohn, and myself, TEC uses a discovery and exploratory approach. In sections of the book where technology is particularly appropriate, marginal icons direct students to TEC Modules that provide a laboratory environment in which they can explore the topic in different ways and at different levels. **Visuals are animations of figures in text; Modules are more elaborate activities and include exercises**. Instructors can choose to become involved at several different levels, ranging from simply encouraging students to use the Visuals and Modules for independent exploration, to assigning specific exercises from those included with each Module, or to creating additional exercises, labs, and projects that make use of the Visuals and Modules.

TEC also includes Homework Hints for representative exercises (usually odd-numbered) in every section of the text, indicated by printing the exercise number in red. These hints are usually presented in the form of questions and try to imitate an effective teaching assistant by functioning as a silent tutor. They are constructed so as not to reveal any more of the actual solution than is minimally necessary to make further progress.

Enhanced WebAssign

Technology is having an impact on the way homework is assigned to students, particularly in large classes. The use of online homework is growing and its appeal depends on ease of use, grading precision, and reliability. With the Eighth Edition we have been working with the calculus community and WebAssign to develop an online homework system. Up to 70% of the exercises in each section are assignable as online homework, including free response, multiple choice, and multi-part formats.

The system also includes Active Examples, in which students are guided in step-by-step tutorials through text examples, with links to the textbook and to video solutions.

Website

Visit CengageBrain.com or stewartcalculus.com for these additional materials:

- Homework Hints
- Algebra Review
- Lies My Calculator and Computer Told Me
- History of Mathematics, with links to the better historical websites
- Additional Topics (complete with exercise sets): Fourier Series, Formulas for the Remainder Term in Taylor Series, Rotation of Axes
- Archived Problems (drill exercises that appeared in previous editions, together with their solutions)
- Challenge Problems (some from the Problems Plus sections from prior editions)
- Links, for particular topics, to outside Web resources
- Selected Visuals and Modules from Tools for Enriching Calculus (TEC)

Content

Diagnostic Tests The book begins with four diagnostic tests, in Basic Algebra, Analytic Geometry, Functions, and Trigonometry.

A Preview of Calculus This is an overview of the subject and includes a list of questions to motivate the study of calculus.

1 Functions and Limits

From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. A discussion of mathematical models leads to a review of the standard functions from these four points of view. The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view. Section 1.7, on the precise epsilon-delta defintion of a limit, is an optional section.

2 Derivatives

The material on derivatives is covered in two sections in order to give students more time to get used to the idea of a derivative as a function. The examples and exercises explore the meanings of derivatives in various contexts. Higher derivatives are introduced in Section 2.2.

3 Applications of Differentiation

The basic facts concerning extreme values and shapes of curves are deduced from the Mean Value Theorem. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves. Some substantial optimization problems are provided, including an explanation of why you need to raise your head 42° to see the top of a rainbow.

4 Integrals

The area problem and the distance problem serve to motivate the definite integral, with sigma notation introduced as needed. (Full coverage of sigma notation is provided in Appendix E.) Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables.

5 Applications of Integration

Here I present the applications of integration—area, volume, work, average value—that can reasonably be done without specialized techniques of integration. General methods are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral.

6 Inverse Functions:
Exponential, Logarithmic, and Inverse Trigonometric Functions

As discussed more fully on page xiv, only one of the two treatments of these functions need be covered. Exponential growth and decay are covered in this chapter.

7 Techniques of Integration

All the standard methods are covered but, of course, the real challenge is to be able to recognize which technique is best used in a given situation. Accordingly, in Section 7.5, I present a strategy for integration. The use of computer algebra systems is discussed in Section 7.6.

8 Further Applications of Integration

Here are the applications of integration—arc length and surface area—for which it is useful to have available all the techniques of integration, as well as applications to biology, economics, and physics (hydrostatic force and centers of mass). I have also included a section on probability. There are more applications here than can realistically be covered in a given course. Instructors should select applications suitable for their students and for which they themselves have enthusiasm.

9 Differential Equations

Modeling is the theme that unifies this introductory treatment of differential equations. Direction fields and Euler's method are studied before separable and linear equations are solved explicitly, so that qualitative, numerical, and analytic approaches are given equal consideration. These methods are applied to the exponential, logistic, and other models for population growth. The first four or five sections of this chapter serve as a good introduction to first-order differential equations. An optional final section uses predator-prey models to illustrate systems of differential equations.

10 Parametric Equations and Polar Coordinates

This chapter introduces parametric and polar curves and applies the methods of calculus to them. Parametric curves are well suited to laboratory projects; the two presented here involve families of curves and Bézier curves. A brief treatment of conic sections in polar coordinates prepares the way for Kepler's Laws in Chapter 13.

11 Infinite Sequences and Series

The convergence tests have intuitive justifications (see page 759) as well as formal proofs. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.

12 Vectors and the Geometry of Space

The material on three-dimensional analytic geometry and vectors is divided into two chapters. Chapter 12 deals with vectors, the dot and cross products, lines, planes, and surfaces.

13 Vector Functions

This chapter covers vector-valued functions, their derivatives and integrals, the length and curvature of space curves, and velocity and acceleration along space curves, culminating in Kepler's laws.

14 Partial Derivatives

Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. In particular, I introduce partial derivatives by looking at a specific column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity.

15 Multiple Integrals

Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, surface areas, and (in projects) volumes of hyperspheres and volumes of intersections of three cylinders. Cylindrical and spherical coordinates are introduced in the context of evaluating triple integrals.

16 Vector Calculus

Vector fields are introduced through pictures of velocity fields showing San Francisco Bay wind patterns. The similarities among the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem are emphasized.

17 Second-Order Differential Equations

Since first-order differential equations are covered in Chapter 9, this final chapter deals with second-order linear differential equations, their application to vibrating springs and electric circuits, and series solutions.

Ancillaries

Calculus, Eighth Edition, is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction. The tables on pages xxi–xxii describe each of these ancillaries.

Acknowledgments

The preparation of this and previous editions has involved much time spent reading the reasoned (but sometimes contradictory) advice from a large number of astute reviewers. I greatly appreciate the time they spent to understand my motivation for the approach taken. I have learned something from each of them.

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Mary Wright, *Southern Illinois University–Carbondale*
Paul M. Wright, *Austin Community College*
Xian Wu, *University of South Carolina*

In addition, I would like to thank R. B. Burckel, Bruce Colletti, David Behrman, John Dersch, Gove Effinger, Bill Emerson, Dan Kalman, Quyan Khan, Alfonso Gracia-Saz, Allan MacIsaac, Tami Martin, Monica Nitsche, Lamia Raffo, Norton Starr, and Jim Trefzger for their suggestions; Al Shenk and Dennis Zill for permission to use exercises from their calculus texts; COMAP for permission to use project material; George Bergman, David Bleecker, Dan Clegg, Victor Kaftal, Anthony Lam, Jamie Lawson, Ira Rosenholtz, Paul Sally, Lowell Smylie, and Larry Wallen for ideas for exercises; Dan Drucker for the roller derby project; Thomas Banchoff, Tom Farmer, Fred Gass, John Ramsay, Larry Riddle, Philip Straffin, and Klaus Volpert for ideas for projects; Dan Anderson, Dan Clegg, Jeff Cole, Dan Drucker, and Barbara Frank for solving the new exercises and suggesting ways to improve them; Marv Riedesel and Mary Johnson for accuracy in proofreading; Andy Bulman-Fleming, Lothar Redlin, Gina Sanders, and Saleem Watson for additional proofreading; and Jeff Cole and Dan Clegg for their careful preparation and proofreading of the answer manuscript.

In addition, I thank those who have contributed to past editions: Ed Barbeau, Jordan Bell, George Bergman, Fred Brauer, Andy Bulman-Fleming, Bob Burton, David Cusick, Tom DiCiccio, Garret Etgen, Chris Fisher, Leon Gerber, Stuart Goldenberg, Arnold Good, Gene Hecht, Harvey Keynes, E. L. Koh, Zdislav Kovarik, Kevin Kreider, Emile LeBlanc, David Leep, Gerald Leibowitz, Larry Peterson, Mary Pugh, Lothar Redlin, Carl Riehm, John Ringland, Peter Rosenthal, Dusty Sabo, Doug Shaw, Dan Silver, Simon Smith, Norton Starr, Saleem Watson, Alan Weinstein, and Gail Wolkowicz.

I also thank Kathi Townes, Stephanie Kuhns, Kristina Elliott, and Kira Abdallah of TECHarts for their production services and the following Cengage Learning staff: Cheryll Linthicum, content project manager; Stacy Green, senior content developer; Samantha Lugtu, associate content developer; Stephanie Kreuz, product assistant; Lynh Pham, media developer; Ryan Ahern, marketing manager; and Vernon Boes, art director. They have all done an outstanding job.

I have been very fortunate to have worked with some of the best mathematics editors in the business over the past three decades: Ron Munro, Harry Campbell, Craig Barth, Jeremy Hayhurst, Gary Ostedt, Bob Pirtle, Richard Stratton, Liz Covello, and now Neha Taleja. All of them have contributed greatly to the success of this book.

JAMES STEWART

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Tools for Enriching Calculus (TEC) functions as both a powerful tool for instructors and as a tutorial environment in which students can explore and review selected topics. The Flash simulation modules in TEC include instructions, written and audio explanations of the concepts, and exercises. TEC is accessible in the eBook via CourseMate and Enhanced WebAssign. Selected Visuals and Modules are available at www.stewartcalculus.com.

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by Konrad J. Heuvers, William P. Francis, John H. Kuisti, Deborah F. Lockhart, Daniel S. Moak, and Gene M. Ortner

ISBN 978-0-534-25248-9

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To the Student

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms. And before you read each example, I suggest that you cover up the solution and try solving the problem yourself. You'll get a lot more from looking at the solution if you do so.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix H. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon indicates an exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Maple, Mathematica, or the TI-89) are required.

You will also encounter the symbol ⊘, which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Tools for Enriching Calculus, which is a companion to this text, is referred to by means of the symbol **TEC** and can be accessed in the eBook via Enhanced WebAssign and CourseMate (selected Visuals and Modules are available at www.stewartcalculus.com). It directs you to modules in which you can explore aspects of calculus for which the computer is particularly useful.

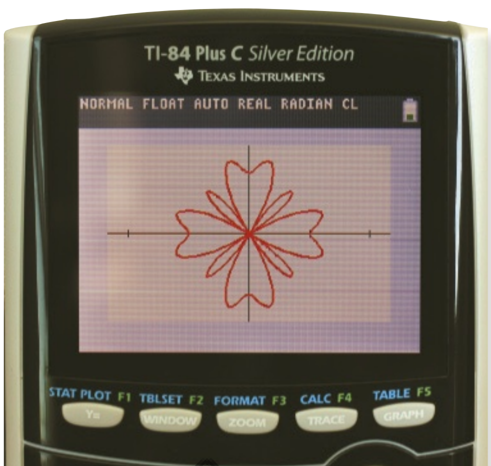
You will notice that some exercise numbers are printed in red: 5. This indicates that *Homework Hints* are available for the exercise. These hints can be found on stewartcalculus.com as well as Enhanced WebAssign and CourseMate. The homework hints ask you questions that allow you to make progress toward a solution without actually giving you the answer. You need to pursue each hint in an active manner with pencil and paper to work out the details. If a particular hint doesn't enable you to solve the problem, you can click to reveal the next hint.

I recommend that you keep this book for reference purposes after you finish the course. Because you will likely forget some of the specific details of calculus, the book will serve as a useful reminder when you need to use calculus in subsequent courses. And, because this book contains more material than can be covered in any one course, it can also serve as a valuable resource for a working scientist or engineer.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

JAMES STEWART

Calculators, Computers, and Other Graphing Devices



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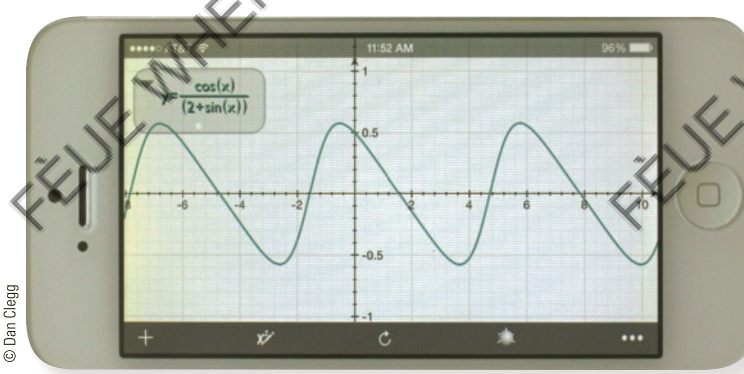
Advances in technology continue to bring a wider variety of tools for doing mathematics. Handheld calculators are becoming more powerful, as are software programs and Internet resources. In addition, many mathematical applications have been released for smartphones and tablets such as the iPad.

Some exercises in this text are marked with a graphing icon, which indicates that the use of some technology is required. Often this means that we intend for a graphing device to be used in drawing the graph of a function or equation. You might also need technology to find the zeros of a graph or the points of intersection of two graphs. In some cases we will use a calculating device to solve an equation or evaluate a definite integral numerically. Many scientific and graphing calculators have these features built in, such as the Texas Instruments TI-84 or TI-Nspire CX. Similar calculators are made by Hewlett Packard, Casio, and Sharp.



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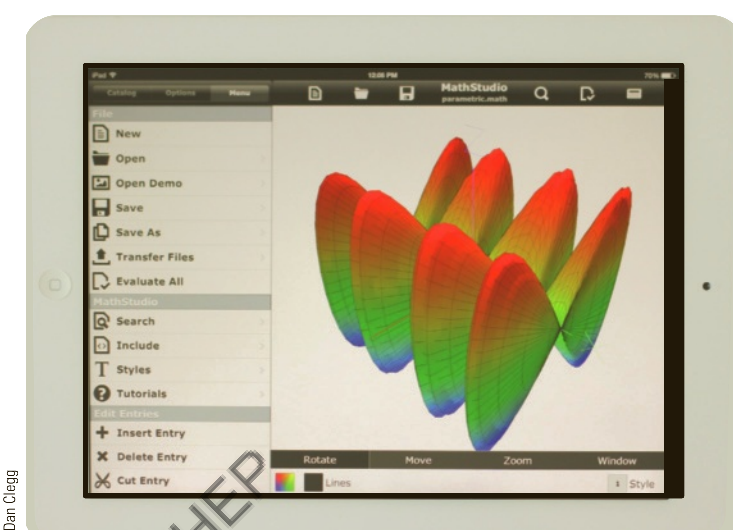
You can also use computer software such as *Graphing Calculator* by Pacific Tech (www.pacifict.com) to perform many of these functions, as well as apps for phones and tablets, like Quick Graph (Colombiamug) or Math-Studio (Pomegranate Apps). Similar functionality is available using a web interface at WolframAlpha.com.



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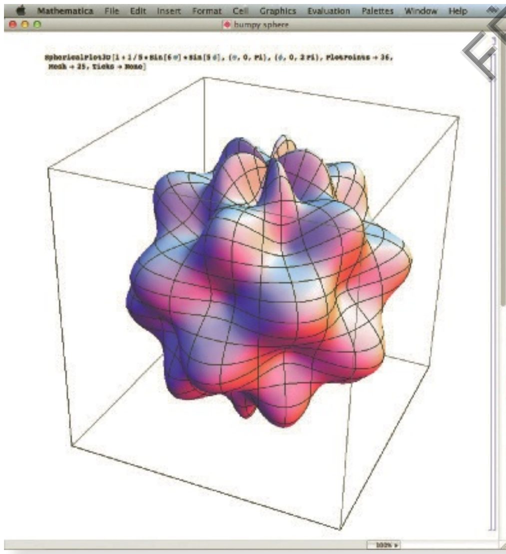

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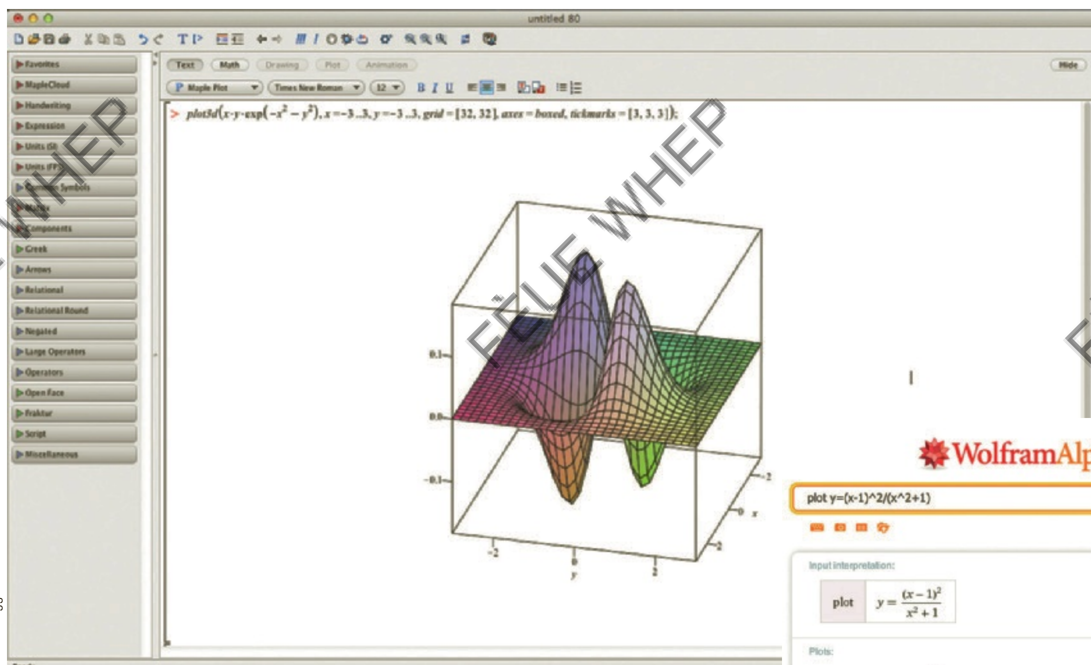
In general, when we use the term "calculator" in this book, we mean the use of any of the resources we have mentioned.

The CAS icon is reserved for problems in which the full resources of a *computer algebra system* (CAS) are required. A CAS is capable of doing mathematics (like solving equations, computing derivatives or integrals) *symbolically* rather than just numerically.

Examples of well-established computer algebra systems are the computer software packages Maple and Mathematica. The WolframAlpha website uses the Mathematica engine to provide CAS functionality via the Web.

Many handheld graphing calculators have CAS capabilities, such as the TI-89 and TI-Nspire CX CAS from Texas Instruments. Some tablet and smartphone apps also provide these capabilities, such as the previously mentioned MathStudio.



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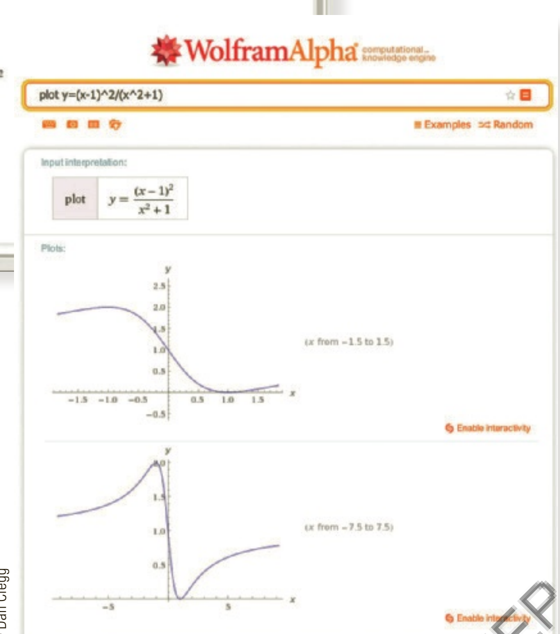

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Diagnostic Tests

Success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. The following tests are intended to diagnose weaknesses that you might have in these areas. After taking each test you can check your answers against the given answers and, if necessary, refresh your skills by referring to the review materials that are provided.

A Diagnostic Test: Algebra

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}

(d) $\dfrac{5^{23}}{5^{21}}$ (e) $\left(\dfrac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$

(b) $(3a^3b^3)(4ab^2)^2$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplify.

(a) $3(x + 6) + 4(2x - 5)$ (b) $(x + 3)(4x - 5)$

(c) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right)$ (d) $(2x + 3)^2$

(e) $(x + 2)^3$

4. Factor each expression.

(a) $4x^2 - 25$ (b) $2x^2 + 5x - 12$

(c) $x^3 - 3x^2 - 4x + 12$ (d) $x^4 + 27x$

(e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ (f) $x^3y - 4xy$

5. Simplify the rational expression.

(a) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$ (b) $\dfrac{2x^2 - x - 1}{x^2 - 9} \cdot \dfrac{x + 3}{2x + 1}$

(c) $\dfrac{x^2}{x^2 - 4} - \dfrac{x + 1}{x + 2}$ (d) $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

6. Rationalize the expression and simplify.

(a) $\dfrac{\sqrt{10}}{\sqrt{5}-2}$ (b) $\dfrac{\sqrt{4+h}-2}{h}$

7. Rewrite by completing the square.

(a) x^2+x+1 (b) $2x^2-12x+11$

8. Solve the equation. (Find only the real solutions.)

(a) $x+5=14-\frac{1}{2}x$ (b) $\dfrac{2x}{x+1}=\dfrac{2x-1}{x}$

(c) $x^2-x-12=0$ (d) $2x^2+4x+1=0$

(e) $x^4-3x^2+2=0$ (f) $3|x-4|=10$

(g) $2x(4-x)^{-1/2}-3\sqrt{4-x}=0$

9. Solve each inequality. Write your answer using interval notation.

(a) $-4<5-3x\leqslant 17$ (b) $x^2<2x+8$

(c) $x(x-1)(x+2)>0$ (d) $|x-4|<3$

(e) $\dfrac{2x-3}{x+1}\leqslant 1$

10. State whether each equation is true or false.

(a) $(p+q)^2=p^2+q^2$ (b) $\sqrt{ab}=\sqrt{a}\sqrt{b}$

(c) $\sqrt{a^2+b^2}=a+b$ (d) $\dfrac{1+TC}{C}=1+T$

(e) $\dfrac{1}{x-y}=\dfrac{1}{x}-\dfrac{1}{y}$ (f) $\dfrac{1/x}{a/x-b/x}=\dfrac{1}{a-b}$

ANSWERS TO DIAGNOSTIC TEST A: ALGEBRA

1. (a) 81 (b) -81 (c) $\frac{1}{81}$
(d) 25 (e) $\frac{9}{4}$ (f) $\frac{1}{8}$

2. (a) $6\sqrt{2}$ (b) $48a^5b^7$ (c) $\dfrac{x}{9y^7}$

3. (a) $11x-2$ (b) $4x^2+7x-15$
(c) $a-b$ (d) $4x^2+12x+9$
(e) $x^3+6x^2+12x+8$

4. (a) $(2x-5)(2x+5)$ (b) $(2x-3)(x+4)$
(c) $(x-3)(x-2)(x+2)$ (d) $x(x+3)(x^2-3x+9)$
(e) $3x^{-1/2}(x-1)(x-2)$ (f) $xy(x-2)(x+2)$

5. (a) $\dfrac{x+2}{x-2}$ (b) $\dfrac{x-1}{x-3}$
(c) $\dfrac{1}{x-2}$ (d) $-(x+y)$

6. (a) $5\sqrt{2}+2\sqrt{10}$ (b) $\dfrac{1}{\sqrt{4+h}+2}$

7. (a) $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$ (b) $2(x-3)^2-7$

8. (a) 6 (b) 1 (c) $-3, 4$
(d) $-1\pm\frac{1}{2}\sqrt{2}$ (e) $\pm1, \pm\sqrt{2}$ (f) $\frac{2}{3}, \frac{22}{3}$
(g) $\frac{12}{5}$

9. (a) $[-4, 3)$ (b) $(-2, 4)$
(c) $(-2, 0)\cup(1, \infty)$ (d) $(1, 7)$
(e) $(-1, 4]$

10. (a) False (b) True (c) False
(d) False (e) False (f) True

If you had difficulty with these problems, you may wish to consult the Review of Algebra on the website www.stewartcalculus.com.

B Diagnostic Test: Analytic Geometry

1. Find an equation for the line that passes through the point $(2, -5)$ and
   (a) has slope -3
   (b) is parallel to the x-axis
   (c) is parallel to the y-axis
   (d) is parallel to the line $2x - 4y = 3$

2. Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.

3. Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

4. Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
   (a) Find the slope of the line that contains A and B.
   (b) Find an equation of the line that passes through A and B. What are the intercepts?
   (c) Find the midpoint of the segment AB.
   (d) Find the length of the segment AB.
   (e) Find an equation of the perpendicular bisector of AB.
   (f) Find an equation of the circle for which AB is a diameter.

5. Sketch the region in the xy-plane defined by the equation or inequalities.
   (a) $-1 \leqslant y \leqslant 3$
   (b) $|x| < 4$ and $|y| < 2$
   (c) $y < 1 - \frac{1}{2}x$
   (d) $y \geqslant x^2 - 1$
   (e) $x^2 + y^2 < 4$
   (f) $9x^2 + 16y^2 = 144$

ANSWERS TO DIAGNOSTIC TEST B: ANALYTIC GEOMETRY

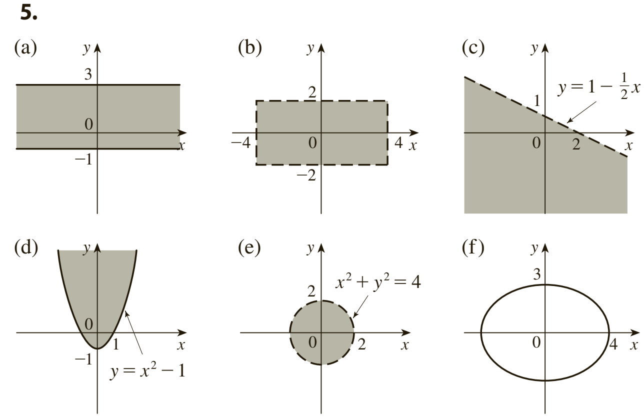
1. (a) $y = -3x + 1$ (b) $y = -5$
   (c) $x = 2$ (d) $y = \frac{1}{2}x - 6$

2. $(x + 1)^2 + (y - 4)^2 = 52$

3. Center $(3, -5)$, radius 5

4. (a) $-\frac{4}{3}$
   (b) $4x + 3y + 16 = 0$; x-intercept -4, y-intercept $-\frac{16}{3}$
   (c) $(-1, -4)$
   (d) 20
   (e) $3x - 4y = 13$
   (f) $(x + 1)^2 + (y + 4)^2 = 100$

5.



If you had difficulty with these problems, you may wish to consult the review of analytic geometry in Appendixes B and C.

C Diagnostic Test: Functions

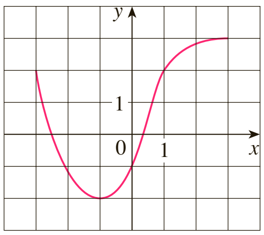


FIGURE FOR PROBLEM 1

1. The graph of a function f is given at the left.
(a) State the value of $f(-1)$.
(b) Estimate the value of $f(2)$.
(c) For what values of x is $f(x) = 2$?
(d) Estimate the values of x such that $f(x) = 0$.
(e) State the domain and range of f.

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

(a) $f(x) = \dfrac{2x+1}{x^2+x-2}$ (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2+1}$ (c) $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$

4. How are graphs of the functions obtained from the graph of f?
(a) $y = -f(x)$ (b) $y = 2f(x) - 1$ (c) $y = f(x-3) + 2$

5. Without using a calculator, make a rough sketch of the graph.
(a) $y = x^3$ (b) $y = (x+1)^3$ (c) $y = (x-2)^3 + 3$
(d) $y = 4 - x^2$ (e) $y = \sqrt{x}$ (f) $y = 2\sqrt{x}$
(g) $y = -2^x$ (h) $y = 1 + x^{-1}$

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leqslant 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$
(a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.
(a) $f \circ g$ (b) $g \circ f$ (c) $g \circ g \circ g$

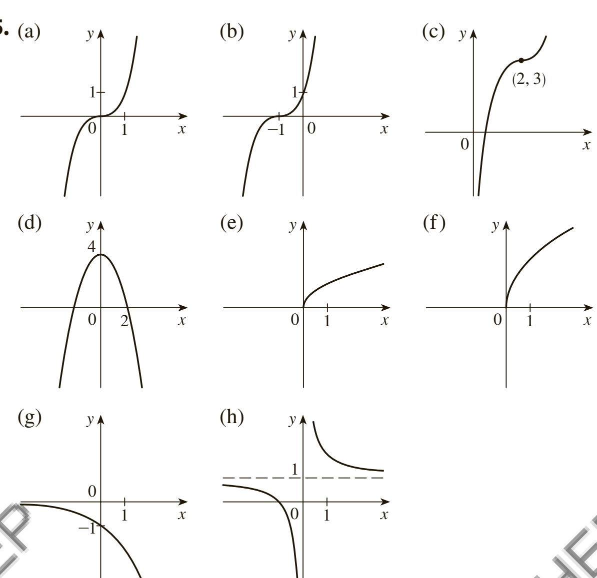
ANSWERS TO DIAGNOSTIC TEST C: FUNCTIONS

1. (a) -2 (b) 2.8
(c) $-3, 1$ (d) $-2.5, 0.3$
(e) $[-3, 3], [-2, 3]$

2. $12 + 6h + h^2$

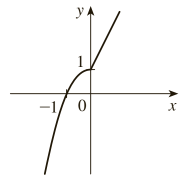
3. (a) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
(b) $(-\infty, \infty)$
(c) $(-\infty, -1] \cup [1, 4]$

4. (a) Reflect about the x-axis
(b) Stretch vertically by a factor of 2, then shift 1 unit downward
(c) Shift 3 units to the right and 2 units upward

5.


6. (a) $-3, 3$

(b)



7. (a) $(f \circ g)(x) = 4x^2 - 8x + 2$

(b) $(g \circ f)(x) = 2x^2 + 4x - 5$

(c) $(g \circ g \circ g)(x) = 8x - 21$

If you had difficulty with these problems, you should look at sections 1.1–1.3 of this book.

D Diagnostic Test: Trigonometry

1. Convert from degrees to radians.
(a) $300°$ (b) $-18°$

2. Convert from radians to degrees.
(a) $5\pi/6$ (b) 2

3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of $30°$.

4. Find the exact values.
(a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$

5. Express the lengths a and b in the figure in terms of θ.

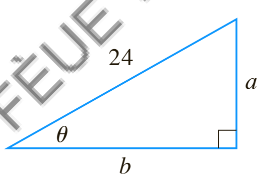


FIGURE FOR PROBLEM 5

6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

7. Prove the identities.

(a) $\tan\theta \sin\theta + \cos\theta = \sec\theta$ (b) $\dfrac{2\tan x}{1 + \tan^2 x} = \sin 2x$

8. Find all values of x such that $\sin 2x = \sin x$ and $0 \leqslant x \leqslant 2\pi$.

9. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

ANSWERS TO DIAGNOSTIC TEST D: TRIGONOMETRY

1. (a) $5\pi/3$ (b) $-\pi/10$

2. (a) $150°$ (b) $360°/\pi \approx 114.6°$

3. 2π cm

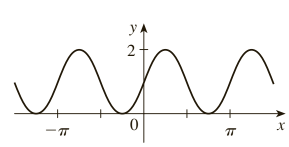
4. (a) $\sqrt{3}$ (b) $-\frac{1}{2}$ (c) 2

5. (a) $24\sin\theta$ (b) $24\cos\theta$

6. $\frac{1}{15}\left(4 + 6\sqrt{2}\right)$

8. $0, \pi/3, \pi, 5\pi/3, 2\pi$

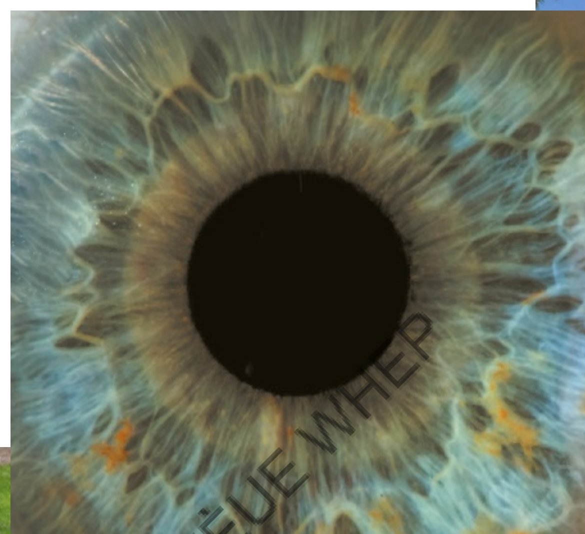
9.



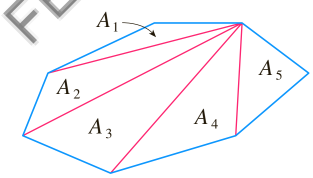
If you had difficulty with these problems, you should look at Appendix D of this book.

A Preview of Calculus

By the time you finish this course, you will be able to calculate the length of the curve used to design the Gateway Arch in St. Louis, determine where a pilot should start descent for a smooth landing, compute the force on a baseball bat when it strikes the ball, and measure the amount of light sensed by the human eye as the pupil changes size.



CALCULUS IS FUNDAMENTALLY DIFFERENT FROM the mathematics that you have studied previously: calculus is less static and more dynamic. It is concerned with change and motion; it deals with quantities that approach other quantities. For that reason it may be useful to have an overview of the subject before beginning its intensive study. Here we give a glimpse of some of the main ideas of calculus by showing how the concept of a limit arises when we attempt to solve a variety of problems.



$A = A_1 + A_2 + A_3 + A_4 + A_5$

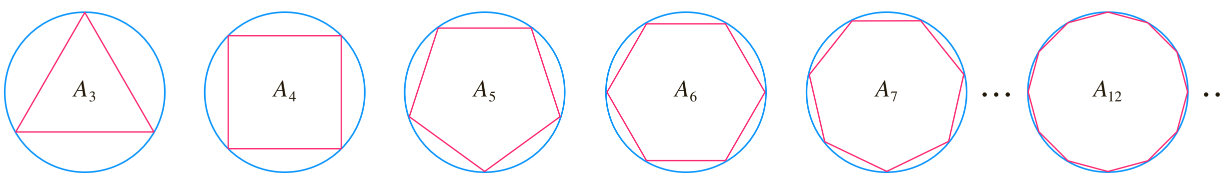
FIGURE 1

The Area Problem

The origins of calculus go back at least 2500 years to the ancient Greeks, who found areas using the "method of exhaustion." They knew how to find the area A of any polygon by dividing it into triangles as in Figure 1 and adding the areas of these triangles.

It is a much more difficult problem to find the area of a curved figure. The Greek method of exhaustion was to inscribe polygons in the figure and circumscribe polygons about the figure and then let the number of sides of the polygons increase. Figure 2 illustrates this process for the special case of a circle with inscribed regular polygons.

FIGURE 2



Let A_n be the area of the inscribed polygon with n sides. As n increases, it appears that A_n becomes closer and closer to the area of the circle. We say that the area of the circle is the limit of the areas of the inscribed polygons, and we write

TEC In the Preview Visual, you can see how areas of inscribed and circumscribed polygons approximate the area of a circle.

$$A = \lim_{n \to \infty} A_n$$

The Greeks themselves did not use limits explicitly. However, by indirect reasoning, Eudoxus (fifth century BC) used exhaustion to prove the familiar formula for the area of a circle: $A = \pi r^2$.

We will use a similar idea in Chapter 4 to find areas of regions of the type shown in Figure 3. We will approximate the desired area A by areas of rectangles (as in Figure 4), let the width of the rectangles decrease, and then calculate A as the limit of these sums of areas of rectangles.

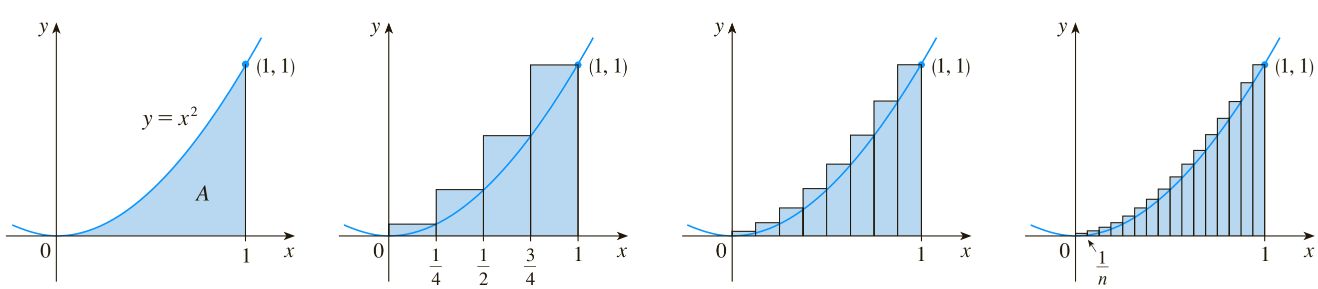


FIGURE 3

The area problem is the central problem in the branch of calculus called *integral calculus*. The techniques that we will develop in Chapter 4 for finding areas will also enable us to compute the volume of a solid, the length of a curve, the force of water against a dam, the mass and center of gravity of a rod, and the work done in pumping water out of a tank.

The Tangent Problem

Consider the problem of trying to find an equation of the tangent line t to a curve with equation $y = f(x)$ at a given point P. (We will give a precise definition of a tangent line in

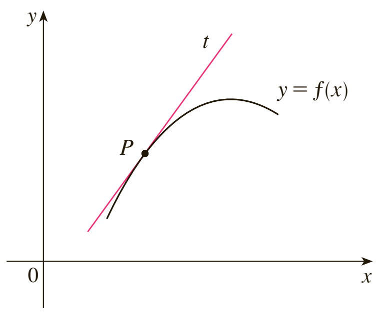


FIGURE 5
The tangent line at P

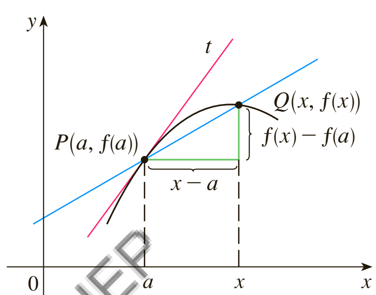


FIGURE 6
The secant line at PQ

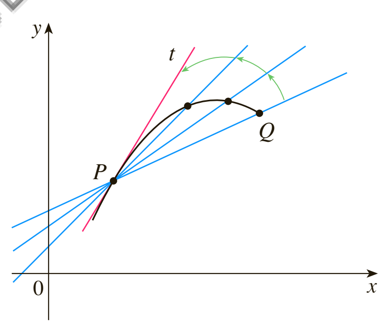


FIGURE 7
Secant lines approaching the tangent line

Chapter 1. For now you can think of it as a line that touches the curve at P as in Figure 5.) Since we know that the point P lies on the tangent line, we can find the equation of t if we know its slope m. The problem is that we need two points to compute the slope and we know only one point, P, on t. To get around the problem we first find an approximation to m by taking a nearby point Q on the curve and computing the slope m_{PQ} of the secant line PQ. From Figure 6 we see that

1
$$m_{PQ} = \frac{f(x) - f(a)}{x - a}$$

Now imagine that Q moves along the curve toward P as in Figure 7. You can see that the secant line rotates and approaches the tangent line as its limiting position. This means that the slope m_{PQ} of the secant line becomes closer and closer to the slope m of the tangent line. We write

$$m = \lim_{Q \to P} m_{PQ}$$

and we say that m is the limit of m_{PQ} as Q approaches P along the curve. Because x approaches a as Q approaches P, we could also use Equation 1 to write

2
$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Specific examples of this procedure will be given in Chapter 1.

The tangent problem has given rise to the branch of calculus called *differential calculus,* which was not invented until more than 2000 years after integral calculus. The main ideas behind differential calculus are due to the French mathematician Pierre Fermat (1601–1665) and were developed by the English mathematicians John Wallis (1616–1703), Isaac Barrow (1630–1677), and Isaac Newton (1642–1727) and the German mathematician Gottfried Leibniz (1646–1716).

The two branches of calculus and their chief problems, the area problem and the tangent problem, appear to be very different, but it turns out that there is a very close connection between them. The tangent problem and the area problem are inverse problems in a sense that will be described in Chapter 4.

Velocity

When we look at the speedometer of a car and read that the car is traveling at 48 mi/h, what does that information indicate to us? We know that if the velocity remains constant, then after an hour we will have traveled 48 mi. But if the velocity of the car varies, what does it mean to say that the velocity at a given instant is 48 mi/h?

In order to analyze this question, let's examine the motion of a car that travels along a straight road and assume that we can measure the distance traveled by the car (in feet) at l-second intervals as in the following chart:

t = Time elapsed (s)	0	1	2	3	4	5
d = Distance (ft)	0	2	9	24	42	71

As a first step toward finding the velocity after 2 seconds have elapsed, we find the average velocity during the time interval $2 \leqslant t \leqslant 4$:

$$\begin{aligned} \text{average velocity} &= \frac{\text{change in position}}{\text{time elapsed}} \\ &= \frac{42 - 9}{4 - 2} \\ &= 16.5 \text{ ft/s} \end{aligned}$$

Similarly, the average velocity in the time interval $2 \leqslant t \leqslant 3$ is

$$\text{average velocity} = \frac{24 - 9}{3 - 2} = 15 \text{ ft/s}$$

We have the feeling that the velocity at the instant $t = 2$ can't be much different from the average velocity during a short time interval starting at $t = 2$. So let's imagine that the distance traveled has been measured at 0.1-second time intervals as in the following chart:

t	2.0	2.1	2.2	2.3	2.4	2.5
d	9.00	10.02	11.16	12.45	13.96	15.80

Then we can compute, for instance, the average velocity over the time interval [2, 2.5]:

$$\text{average velocity} = \frac{15.80 - 9.00}{2.5 - 2} = 13.6 \text{ ft/s}$$

The results of such calculations are shown in the following chart:

Time interval	[2, 3]	[2, 2.5]	[2, 2.4]	[2, 2.3]	[2, 2.2]	[2, 2.1]
Average velocity (ft/s)	15.0	13.6	12.4	11.5	10.8	10.2

The average velocities over successively smaller intervals appear to be getting closer to a number near 10, and so we expect that the velocity at exactly $t = 2$ is about 10 ft/s. In Chapter 2 we will define the instantaneous velocity of a moving object as the limiting value of the average velocities over smaller and smaller time intervals.

In Figure 8 we show a graphical representation of the motion of the car by plotting the distance traveled as a function of time. If we write $d = f(t)$, then $f(t)$ is the number of feet traveled after t seconds. The average velocity in the time interval $[2, t]$ is

$$\text{average velocity} = \frac{\text{change in position}}{\text{time elapsed}} = \frac{f(t) - f(2)}{t - 2}$$

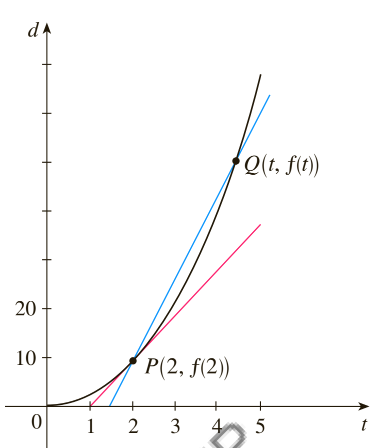


FIGURE 8

which is the same as the slope of the secant line PQ in Figure 8. The velocity v when $t = 2$ is the limiting value of this average velocity as t approaches 2; that is,

$$v = \lim_{t \to 2} \frac{f(t) - f(2)}{t - 2}$$

and we recognize from Equation 2 that this is the same as the slope of the tangent line to the curve at P.

Thus, when we solve the tangent problem in differential calculus, we are also solving problems concerning velocities. The same techniques also enable us to solve problems involving rates of change in all of the natural and social sciences.

The Limit of a Sequence

In the fifth century BC the Greek philosopher Zeno of Elea posed four problems, now known as *Zeno's paradoxes*, that were intended to challenge some of the ideas concerning space and time that were held in his day. Zeno's second paradox concerns a race between the Greek hero Achilles and a tortoise that has been given a head start. Zeno argued, as follows, that Achilles could never pass the tortoise: Suppose that Achilles starts at position a_1 and the tortoise starts at position t_1. (See Figure 9.) When Achilles reaches the point $a_2 = t_1$, the tortoise is farther ahead at position t_2. When Achilles reaches $a_3 = t_2$, the tortoise is at t_3. This process continues indefinitely and so it appears that the tortoise will always be ahead! But this defies common sense.

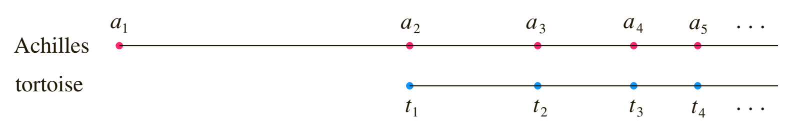


FIGURE 9

One way of explaining this paradox is with the idea of a *sequence*. The successive positions of Achilles $(a_1, a_2, a_3, \ldots)$ or the successive positions of the tortoise $(t_1, t_2, t_3, \ldots)$ form what is known as a sequence.

In general, a sequence $\{a_n\}$ is a set of numbers written in a definite order. For instance, the sequence

$$\left\{1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \tfrac{1}{5}, \ldots\right\}$$

can be described by giving the following formula for the nth term:

$$a_n = \frac{1}{n}$$

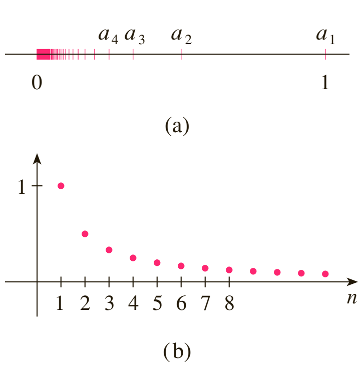


FIGURE 10

We can visualize this sequence by plotting its terms on a number line as in Figure 10(a) or by drawing its graph as in Figure 10(b). Observe from either picture that the terms of the sequence $a_n = 1/n$ are becoming closer and closer to 0 as n increases. In fact, we can find terms as small as we please by making n large enough. We say that the limit of the sequence is 0, and we indicate this by writing

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

In general, the notation

$$\lim_{n \to \infty} a_n = L$$

is used if the terms a_n approach the number L as n becomes large. This means that the numbers a_n can be made as close as we like to the number L by taking n sufficiently large.

The concept of the limit of a sequence occurs whenever we use the decimal representation of a real number. For instance, if

$$\begin{aligned} a_1 &= 3.1 \\ a_2 &= 3.14 \\ a_3 &= 3.141 \\ a_4 &= 3.1415 \\ a_5 &= 3.14159 \\ a_6 &= 3.141592 \\ a_7 &= 3.1415926 \\ &\vdots \end{aligned}$$

then $$\lim_{n \to \infty} a_n = \pi$$

The terms in this sequence are rational approximations to π.

Let's return to Zeno's paradox. The successive positions of Achilles and the tortoise form sequences $\{a_n\}$ and $\{t_n\}$, where $a_n < t_n$ for all n. It can be shown that both sequences have the same limit:

$$\lim_{n \to \infty} a_n = p = \lim_{n \to \infty} t_n$$

It is precisely at this point p that Achilles overtakes the tortoise.

The Sum of a Series

Another of Zeno's paradoxes, as passed on to us by Aristotle, is the following: "A man standing in a room cannot walk to the wall. In order to do so, he would first have to go half the distance, then half the remaining distance, and then again half of what still remains. This process can always be continued and can never be ended." (See Figure 11.)

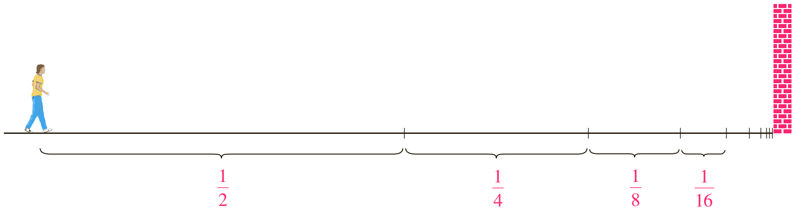


FIGURE 11

Of course, we know that the man can actually reach the wall, so this suggests that perhaps the total distance can be expressed as the sum of infinitely many smaller distances as follows:

$$\boxed{3} \qquad 1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n} + \cdots$$

Zeno was arguing that it doesn't make sense to add infinitely many numbers together. But there are other situations in which we implicitly use infinite sums. For instance, in decimal notation, the symbol $0.\overline{3} = 0.3333\ldots$ means

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10,000} + \cdots$$

and so, in some sense, it must be true that

$$\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10,000} + \cdots = \frac{1}{3}$$

More generally, if d_n denotes the nth digit in the decimal representation of a number, then

$$0.d_1d_2d_3d_4\ldots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \cdots + \frac{d_n}{10^n} + \cdots$$

Therefore some infinite sums, or infinite series as they are called, have a meaning. But we must define carefully what the sum of an infinite series is.

Returning to the series in Equation 3, we denote by s_n the sum of the first n terms of the series. Thus

$$\begin{aligned}
s_1 &= \tfrac{1}{2} = 0.5\\
s_2 &= \tfrac{1}{2} + \tfrac{1}{4} = 0.75\\
s_3 &= \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} = 0.875\\
s_4 &= \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} = 0.9375\\
s_5 &= \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32} = 0.96875\\
s_6 &= \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32} + \tfrac{1}{64} = 0.984375\\
s_7 &= \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{32} + \tfrac{1}{64} + \tfrac{1}{128} = 0.9921875\\
&\;\vdots\\
s_{10} &= \tfrac{1}{2} + \tfrac{1}{4} + \cdots + \tfrac{1}{1024} \approx 0.99902344\\
&\;\vdots\\
s_{16} &= \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{16}} \approx 0.99998474
\end{aligned}$$

Observe that as we add more and more terms, the partial sums become closer and closer to 1. In fact, it can be shown that by taking n large enough (that is, by adding sufficiently many terms of the series), we can make the partial sum s_n as close as we please to the number 1. It therefore seems reasonable to say that the sum of the infinite series is 1 and to write

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots = 1$$

In other words, the reason the sum of the series is 1 is that

$$\lim_{n \to \infty} s_n = 1$$

In Chapter 11 we will discuss these ideas further. We will then use Newton's idea of combining infinite series with differential and integral calculus.

Summary

We have seen that the concept of a limit arises in trying to find the area of a region, the slope of a tangent to a curve, the velocity of a car, or the sum of an infinite series. In each case the common theme is the calculation of a quantity as the limit of other, easily calculated quantities. It is this basic idea of a limit that sets calculus apart from other areas of mathematics. In fact, we could define calculus as the part of mathematics that deals with limits.

After Sir Isaac Newton invented his version of calculus, he used it to explain the motion of the planets around the sun. Today calculus is used in calculating the orbits of satellites and spacecraft, in predicting population sizes, in estimating how fast oil prices rise or fall, in forecasting weather, in measuring the cardiac output of the heart, in calculating life insurance premiums, and in a great variety of other areas. We will explore some of these uses of calculus in this book.

In order to convey a sense of the power of the subject, we end this preview with a list of some of the questions that you will be able to answer using calculus:

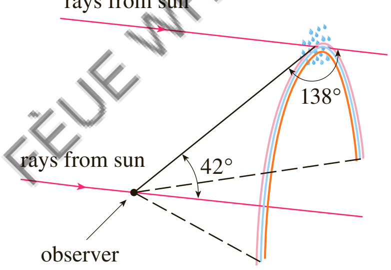


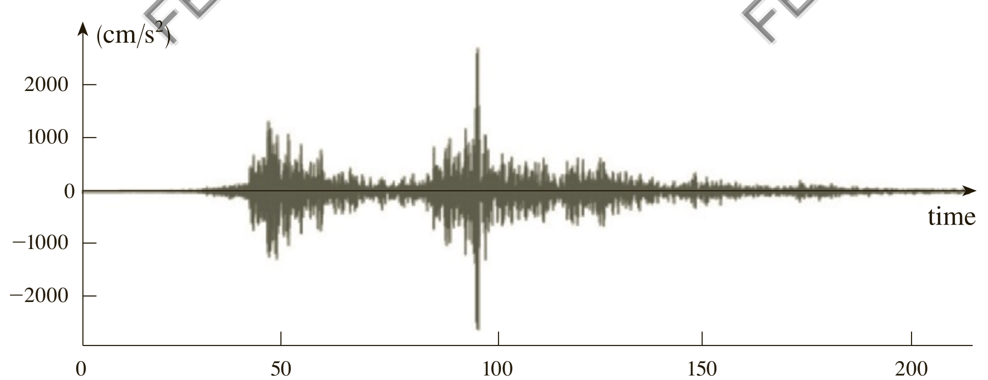
FIGURE 12

1. How can we explain the fact, illustrated in Figure 12, that the angle of elevation from an observer up to the highest point in a rainbow is 42°? (See page 213.)
2. How can we explain the shapes of cans on supermarket shelves? (See page 270.)
3. Where is the best place to sit in a movie theater? (See page 483.)
4. How can we design a roller coaster for a smooth ride? (See page 144.)
5. How far away from an airport should a pilot start descent? (See page 161.)
6. How can we fit curves together to design shapes to represent letters on a laser printer? (See page 697.)
7. How can we estimate the number of workers that were needed to build the Great Pyramid of Khufu in ancient Egypt? (See page 388.)
8. Where should an infielder position himself to catch a baseball thrown by an outfielder and relay it to home plate? (See page 392.)
9. Does a ball thrown upward take longer to reach its maximum height or to fall back to its original height? (See page 649.)
10. How can we explain the fact that planets and satellites move in elliptical orbits? (See page 916.)
11. How can we distribute water flow among turbines at a hydroelectric station so as to maximize the total energy production? (See page 1020.)
12. If a marble, a squash ball, a steel bar, and a lead pipe roll down a slope, which of them reaches the bottom first? (See page 1092.)

1 Functions and Limits

Pictura Collectus/Alamy

Often a graph is the best way to represent a function because it conveys so much information at a glance. Shown is a graph of the vertical ground acceleration created by the 2011 earthquake near Tohoku, Japan. The earthquake had a magnitude of 9.0 on the Richter scale and was so powerful that it moved northern Japan 8 feet closer to North America.



Seismological Society of America

THE FUNDAMENTAL OBJECTS THAT WE deal with in calculus are functions. We stress that a function can be represented in different ways: by an equation, in a table, by a graph, or in words. We look at the main types of functions that occur in calculus and describe the process of using these functions as mathematical models of real-world phenomena.

In *A Preview of Calculus* (page 1) we saw how the idea of a limit underlies the various branches of calculus. It is therefore appropriate to begin our study of calculus by investigating limits of functions and their properties.

1.1 Four Ways to Represent a Function

Functions arise whenever one quantity depends on another. Consider the following four situations.

A. The area A of a circle depends on the radius r of the circle. The rule that connects r and A is given by the equation $A = \pi r^2$. With each positive number r there is associated one value of A, and we say that A is a *function* of r.

B. The human population of the world P depends on the time t. The table gives estimates of the world population $P(t)$ at time t, for certain years. For instance,

$$P(1950) \approx 2{,}560{,}000{,}000$$

But for each value of the time t there is a corresponding value of P, and we say that P is a function of t.

Year	Population (millions)
1900	1650
1910	1750
1920	1860
1930	2070
1940	2300
1950	2560
1960	3040
1970	3710
1980	4450
1990	5280
2000	6080
2010	6870

C. The cost C of mailing an envelope depends on its weight w. Although there is no simple formula that connects w and C, the post office has a rule for determining C when w is known.

D. The vertical acceleration a of the ground as measured by a seismograph during an earthquake is a function of the elapsed time t. Figure 1 shows a graph generated by seismic activity during the Northridge earthquake that shook Los Angeles in 1994. For a given value of t, the graph provides a corresponding value of a.

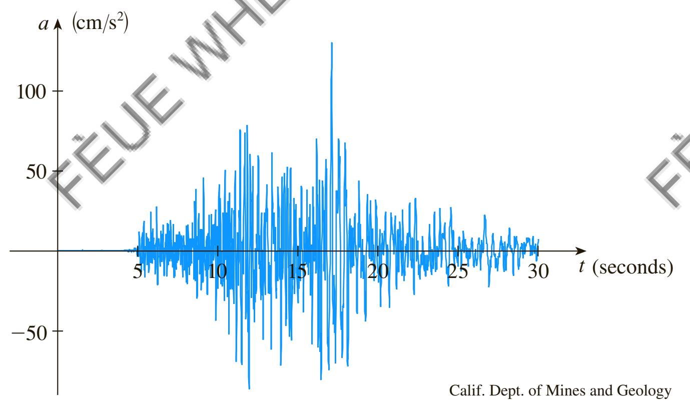


FIGURE 1
Vertical ground acceleration during the Northridge earthquake

Each of these examples describes a rule whereby, given a number (r, t, w, or t), another number (A, P, C, or a) is assigned. In each case we say that the second number is a function of the first number.

> A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

We usually consider functions for which the sets D and E are sets of real numbers. The set D is called the **domain** of the function. The number $f(x)$ is the **value of f at x** and is read "f of x." The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain. A symbol that represents an arbitrary number in the *domain* of a function f is called an **independent variable**. A symbol that represents a number in the *range* of f is called a **dependent variable**. In Example A, for instance, r is the independent variable and A is the dependent variable.

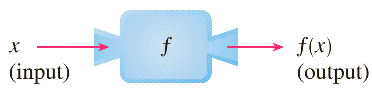


FIGURE 2
Machine diagram for a function f

It's helpful to think of a function as a **machine** (see Figure 2). If x is in the domain of the function f, then when x enters the machine, it's accepted as an input and the machine produces an output $f(x)$ according to the rule of the function. Thus we can think of the domain as the set of all possible inputs and the range as the set of all possible outputs.

The preprogrammed functions in a calculator are good examples of a function as a machine. For example, the square root key on your calculator computes such a function. You press the key labeled $\sqrt{\ }$ (or $\sqrt{x}$) and enter the input x. If $x < 0$, then x is not in the domain of this function; that is, x is not an acceptable input, and the calculator will indicate an error. If $x \geqslant 0$, then an *approximation* to $\sqrt{x}$ will appear in the display. Thus the $\sqrt{x}$ key on your calculator is not quite the same as the exact mathematical function f defined by $f(x) = \sqrt{x}$.

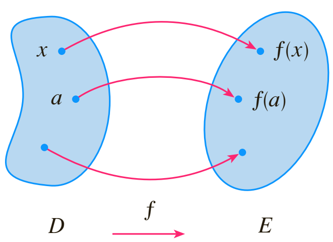


FIGURE 3
Arrow diagram for f

Another way to picture a function is by an **arrow diagram** as in Figure 3. Each arrow connects an element of D to an element of E. The arrow indicates that $f(x)$ is associated with x, $f(a)$ is associated with a, and so on.

The most common method for visualizing a function is its graph. If f is a function with domain D, then its **graph** is the set of ordered pairs

$$\{(x, f(x)) \mid x \in D\}$$

(Notice that these are input-output pairs.) In other words, the graph of f consists of all points (x, y) in the coordinate plane such that $y = f(x)$ and x is in the domain of f.

The graph of a function f gives us a useful picture of the behavior or "life history" of a function. Since the y-coordinate of any point (x, y) on the graph is $y = f(x)$, we can read the value of $f(x)$ from the graph as being the height of the graph above the point x (see Figure 4). The graph of f also allows us to picture the domain of f on the x-axis and its range on the y-axis as in Figure 5.

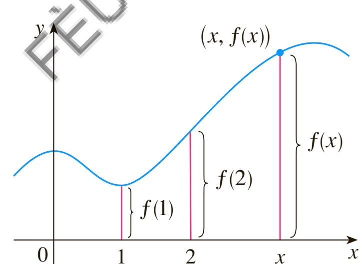


FIGURE 4

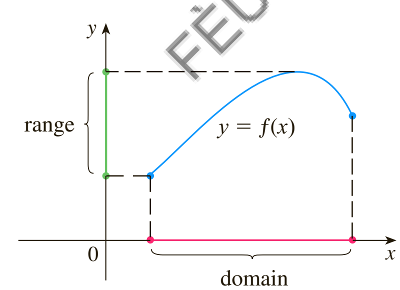


FIGURE 5

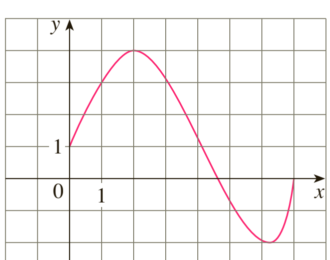


FIGURE 6

The notation for intervals is given in Appendix A.

EXAMPLE 1 The graph of a function f is shown in Figure 6.
(a) Find the values of $f(1)$ and $f(5)$.
(b) What are the domain and range of f?

SOLUTION
(a) We see from Figure 6 that the point $(1, 3)$ lies on the graph of f, so the value of f at 1 is $f(1) = 3$. (In other words, the point on the graph that lies above $x = 1$ is 3 units above the x-axis.)

When $x = 5$, the graph lies about 0.7 units below the x-axis, so we estimate that $f(5) \approx -0.7$.

(b) We see that $f(x)$ is defined when $0 \leqslant x \leqslant 7$, so the domain of f is the closed interval $[0, 7]$. Notice that f takes on all values from -2 to 4, so the range of f is

$$\{y \mid -2 \leqslant y \leqslant 4\} = [-2, 4]$$ ■

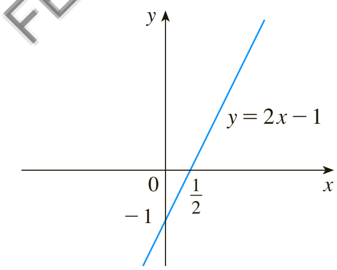


FIGURE 7

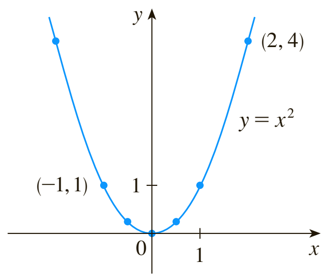


FIGURE 8

EXAMPLE 2 Sketch the graph and find the domain and range of each function.
(a) $f(x) = 2x - 1$ (b) $g(x) = x^2$

SOLUTION
(a) The equation of the graph is $y = 2x - 1$, and we recognize this as being the equation of a line with slope 2 and y-intercept -1. (Recall the slope-intercept form of the equation of a line: $y = mx + b$. See Appendix B.) This enables us to sketch a portion of the graph of f in Figure 7. The expression $2x - 1$ is defined for all real numbers, so the domain of f is the set of all real numbers, which we denote by $\mathbb{R}$. The graph shows that the range is also $\mathbb{R}$.

(b) Since $g(2) = 2^2 = 4$ and $g(-1) = (-1)^2 = 1$, we could plot the points $(2, 4)$ and $(-1, 1)$, together with a few other points on the graph, and join them to produce the graph (Figure 8). The equation of the graph is $y = x^2$, which represents a parabola (see Appendix C). The domain of g is $\mathbb{R}$. The range of g consists of all values of $g(x)$, that is, all numbers of the form x^2. But $x^2 \geqslant 0$ for all numbers x and any positive number y is a square. So the range of g is $\{y \mid y \geqslant 0\} = [0, \infty)$. This can also be seen from Figure 8. ■

EXAMPLE 3 If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\dfrac{f(a+h) - f(a)}{h}$.

SOLUTION We first evaluate $f(a + h)$ by replacing x by $a + h$ in the expression for $f(x)$:

$$\begin{aligned} f(a+h) &= 2(a+h)^2 - 5(a+h) + 1 \\ &= 2(a^2 + 2ah + h^2) - 5(a+h) + 1 \\ &= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1 \end{aligned}$$

Then we substitute into the given expression and simplify:

$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 - 5a - 5h + 1) - (2a^2 - 5a + 1)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h + 1 - 2a^2 + 5a - 1}{h} \\ &= \frac{4ah + 2h^2 - 5h}{h} = 4a + 2h - 5 \end{aligned}$$
■

The expression

$$\frac{f(a+h) - f(a)}{h}$$

in Example 3 is called a **difference quotient** and occurs frequently in calculus. As we will see in Chapter 2, it represents the average rate of change of $f(x)$ between $x = a$ and $x = a + h$.

Representations of Functions

There are four possible ways to represent a function:

- verbally (by a description in words)
- numerically (by a table of values)
- visually (by a graph)
- algebraically (by an explicit formula)

If a single function can be represented in all four ways, it's often useful to go from one representation to another to gain additional insight into the function. (In Example 2, for instance, we started with algebraic formulas and then obtained the graphs.) But certain functions are described more naturally by one method than by another. With this in mind, let's reexamine the four situations that we considered at the beginning of this section.

A. The most useful representation of the area of a circle as a function of its radius is probably the algebraic formula $A(r) = \pi r^2$, though it is possible to compile a table of values or to sketch a graph (half a parabola). Because a circle has to have a positive radius, the domain is $\{r \mid r > 0\} = (0, \infty)$, and the range is also $(0, \infty)$.

t (years since 1900)	Population (millions)
0	1650
10	1750
20	1860
30	2070
40	2300
50	2560
60	3040
70	3710
80	4450
90	5280
100	6080
110	6870

B. We are given a description of the function in words: $P(t)$ is the human population of the world at time t. Let's measure t so that $t = 0$ corresponds to the year 1900. The table of values of world population provides a convenient representation of this function. If we plot these values, we get the graph (called a *scatter plot*) in Figure 9. It too is a useful representation; the graph allows us to absorb all the data at once. What about a formula? Of course, it's impossible to devise an explicit formula that gives the exact human population $P(t)$ at any time t. But it is possible to find an expression for a function that *approximates* $P(t)$. In fact, using methods explained in Section 1.2, we obtain the approximation

$$P(t) \approx f(t) = (1.43653 \times 10^9) \cdot (1.01395)^t$$

Figure 10 shows that it is a reasonably good "fit." The function f is called a *mathematical model* for population growth. In other words, it is a function with an explicit formula that approximates the behavior of our given function. We will see, however, that the ideas of calculus can be applied to a table of values; an explicit formula is not necessary.

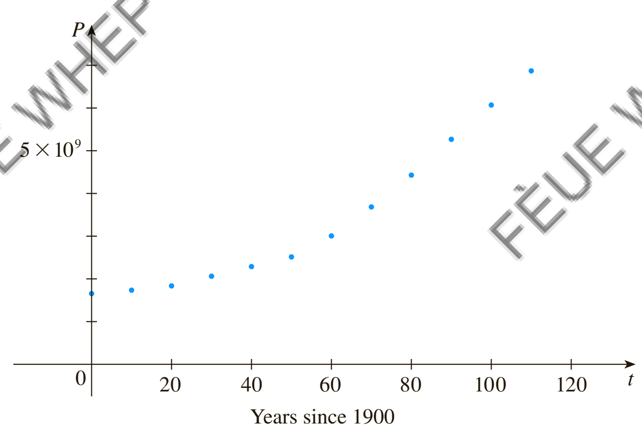


FIGURE 9

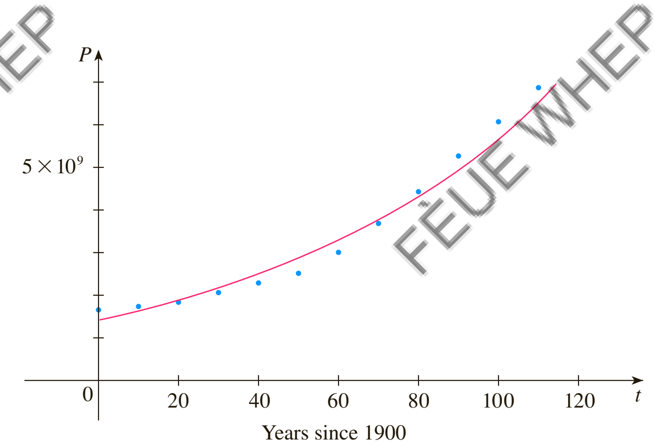


FIGURE 10

The function P is typical of the functions that arise whenever we attempt to apply calculus to the real world. We start with a verbal description of a function. Then we may be able to construct a table of values of the function, perhaps from instrument readings in a scientific experiment. Even though we don't have complete knowledge of the values of the function, we will see throughout the book that it is still possible to perform the operations of calculus on such a function.

A function defined by a table of values is called a *tabular* function.

w (ounces)	$C(w)$ (dollars)
$0 < w \leqslant 1$	0.98
$1 < w \leqslant 2$	1.19
$2 < w \leqslant 3$	1.40
$3 < w \leqslant 4$	1.61
$4 < w \leqslant 5$	1.82
⋮	⋮

C. Again the function is described in words: Let $C(w)$ be the cost of mailing a large envelope with weight w. The rule that the US Postal Service used as of 2015 is as follows: The cost is 98 cents for up to 1 oz, plus 21 cents for each additional ounce (or less) up to 13 oz. The table of values shown in the margin is the most convenient representation for this function, though it is possible to sketch a graph (see Example 10).

D. The graph shown in Figure 1 is the most natural representation of the vertical acceleration function $a(t)$. It's true that a table of values could be compiled, and it is even possible to devise an approximate formula. But everything a geologist needs to know—amplitudes and patterns—can be seen easily from the graph. (The same is true for the patterns seen in electrocardiograms of heart patients and polygraphs for lie-detection.)

In the next example we sketch the graph of a function that is defined verbally.

EXAMPLE 4 When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running. Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.

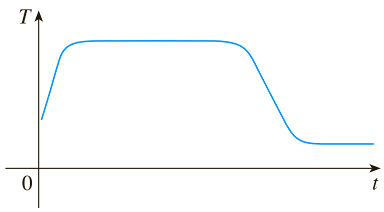


FIGURE 11

SOLUTION The initial temperature of the running water is close to room temperature because the water has been sitting in the pipes. When the water from the hot-water tank starts flowing from the faucet, T increases quickly. In the next phase, T is constant at the temperature of the heated water in the tank. When the tank is drained, T decreases to the temperature of the water supply. This enables us to make the rough sketch of T as a function of t in Figure 11. ■

In the following example we start with a verbal description of a function in a physical situation and obtain an explicit algebraic formula. The ability to do this is a useful skill in solving calculus problems that ask for the maximum or minimum values of quantities.

EXAMPLE 5 A rectangular storage container with an open top has a volume of 10 m^3. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.

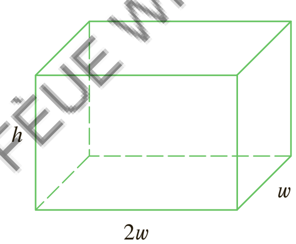


FIGURE 12

SOLUTION We draw a diagram as in Figure 12 and introduce notation by letting w and $2w$ be the width and length of the base, respectively, and h be the height.

The area of the base is $(2w)w = 2w^2$, so the cost, in dollars, of the material for the base is $10(2w^2)$. Two of the sides have area wh and the other two have area $2wh$, so the cost of the material for the sides is $6[2(wh) + 2(2wh)]$. The total cost is therefore

$$C = 10(2w^2) + 6[2(wh) + 2(2wh)] = 20w^2 + 36wh$$

To express C as a function of w alone, we need to eliminate h and we do so by using the fact that the volume is 10 m^3. Thus

$$w(2w)h = 10$$

which gives

$$h = \frac{10}{2w^2} = \frac{5}{w^2}$$

Substituting this into the expression for C, we have

$$C = 20w^2 + 36w\left(\frac{5}{w^2}\right) = 20w^2 + \frac{180}{w}$$

Therefore the equation

$$C(w) = 20w^2 + \frac{180}{w} \qquad w > 0$$

expresses C as a function of w. ■

PS In setting up applied functions as in Example 5, it may be useful to review the principles of problem solving as discussed on page 98, particularly *Step 1: Understand the Problem.*

EXAMPLE 6 Find the domain of each function.

(a) $f(x) = \sqrt{x + 2}$ (b) $g(x) = \dfrac{1}{x^2 - x}$

Domain Convention
If a function is given by a formula and the domain is not stated explicitly, the convention is that the domain is the set of all numbers for which the formula makes sense and defines a real number.

SOLUTION
(a) Because the square root of a negative number is not defined (as a real number), the domain of f consists of all values of x such that $x + 2 \geqslant 0$. This is equivalent to $x \geqslant -2$, so the domain is the interval $[-2, \infty)$.

(b) Since

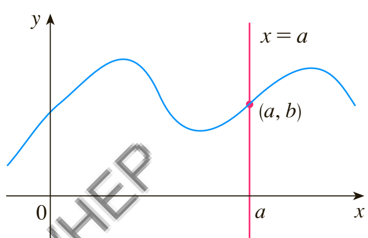
$$g(x) = \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$$

and division by 0 is not allowed, we see that $g(x)$ is not defined when $x = 0$ or $x = 1$. Thus the domain of g is

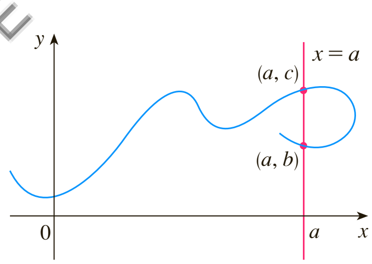
$$\{x \mid x \neq 0, x \neq 1\}$$

which could also be written in interval notation as

$$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$$



(a) This curve represents a function.



(b) This curve doesn't represent a function.

FIGURE 13

The graph of a function is a curve in the xy-plane. But the question arises: which curves in the xy-plane are graphs of functions? This is answered by the following test.

The Vertical Line Test A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

The reason for the truth of the Vertical Line Test can be seen in Figure 13. If each vertical line $x = a$ intersects a curve only once, at (a, b), then exactly one function value is defined by $f(a) = b$. But if a line $x = a$ intersects the curve twice, at (a, b) and (a, c), then the curve can't represent a function because a function can't assign two different values to a.

For example, the parabola $x = y^2 - 2$ shown in Figure 14(a) is not the graph of a function of x because, as you can see, there are vertical lines that intersect the parabola twice. The parabola, however, does contain the graphs of *two* functions of x. Notice that the equation $x = y^2 - 2$ implies $y^2 = x + 2$, so $y = \pm\sqrt{x+2}$. Thus the upper and lower halves of the parabola are the graphs of the functions $f(x) = \sqrt{x+2}$ [from Example 6(a)] and $g(x) = -\sqrt{x+2}$. [See Figures 14(b) and (c).]

We observe that if we reverse the roles of x and y, then the equation $x = h(y) = y^2 - 2$ *does* define x as a function of y (with y as the independent variable and x as the dependent variable) and the parabola now appears as the graph of the function h.

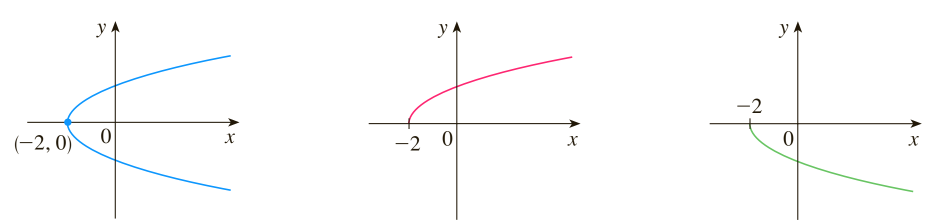


FIGURE 14 (a) $x = y^2 - 2$ (b) $y = \sqrt{x+2}$ (c) $y = -\sqrt{x+2}$

Piecewise Defined Functions

The functions in the following four examples are defined by different formulas in different parts of their domains. Such functions are called **piecewise defined functions**.

EXAMPLE 7 A function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Evaluate $f(-2)$, $f(-1)$, and $f(0)$ and sketch the graph.

SOLUTION Remember that a function is a rule. For this particular function the rule is the following: First look at the value of the input x. If it happens that $x \leqslant -1$, then the value of $f(x)$ is $1 - x$. On the other hand, if $x > -1$, then the value of $f(x)$ is x^2.

$$\text{Since } -2 \leqslant -1, \text{ we have } f(-2) = 1 - (-2) = 3.$$

$$\text{Since } -1 \leqslant -1, \text{ we have } f(-1) = 1 - (-1) = 2.$$

$$\text{Since } 0 > -1, \text{ we have } f(0) = 0^2 = 0.$$

How do we draw the graph of f? We observe that if $x \leqslant -1$, then $f(x) = 1 - x$, so the part of the graph of f that lies to the left of the vertical line $x = -1$ must coincide with the line $y = 1 - x$, which has slope -1 and y-intercept 1. If $x > -1$, then $f(x) = x^2$, so the part of the graph of f that lies to the right of the line $x = -1$ must coincide with the graph of $y = x^2$, which is a parabola. This enables us to sketch the graph in Figure 15. The solid dot indicates that the point $(-1, 2)$ is included on the graph; the open dot indicates that the point $(-1, 1)$ is excluded from the graph. ■

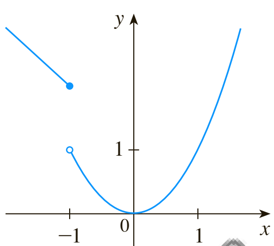


FIGURE 15

The next example of a piecewise defined function is the absolute value function. Recall that the **absolute value** of a number a, denoted by $|a|$, is the distance from a to 0 on the real number line. Distances are always positive or 0, so we have

For a more extensive review of absolute values, see Appendix A.

$$|a| \geqslant 0 \qquad \text{for every number } a$$

For example,

$$|3| = 3 \qquad |-3| = 3 \qquad |0| = 0 \qquad |\sqrt{2} - 1| = \sqrt{2} - 1 \qquad |3 - \pi| = \pi - 3$$

In general, we have

$$\boxed{\begin{aligned} |a| &= a \quad && \text{if } a \geqslant 0 \\ |a| &= -a \quad && \text{if } a < 0 \end{aligned}}$$

(Remember that if a is negative, then $-a$ is positive.)

EXAMPLE 8 Sketch the graph of the absolute value function $f(x) = |x|$.

SOLUTION From the preceding discussion we know that

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$$

Using the same method as in Example 7, we see that the graph of f coincides with the line $y = x$ to the right of the y-axis and coincides with the line $y = -x$ to the left of the y-axis (see Figure 16). ■

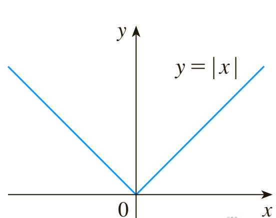


FIGURE 16

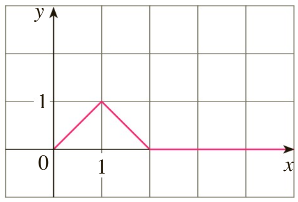


FIGURE 17

Point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

See Appendix B.

EXAMPLE 9 Find a formula for the function f graphed in Figure 17.

SOLUTION The line through $(0, 0)$ and $(1, 1)$ has slope $m = 1$ and y-intercept $b = 0$, so its equation is $y = x$. Thus, for the part of the graph of f that joins $(0, 0)$ to $(1, 1)$, we have

$$f(x) = x \qquad \text{if } 0 \leqslant x \leqslant 1$$

The line through $(1, 1)$ and $(2, 0)$ has slope $m = -1$, so its point-slope form is

$$y - 0 = (-1)(x - 2) \qquad \text{or} \qquad y = 2 - x$$

So we have $$f(x) = 2 - x \qquad \text{if } 1 < x \leqslant 2$$

We also see that the graph of f coincides with the x-axis for $x > 2$. Putting this information together, we have the following three-piece formula for f:

$$f(x) = \begin{cases} x & \text{if } 0 \leqslant x \leqslant 1 \\ 2 - x & \text{if } 1 < x \leqslant 2 \\ 0 & \text{if } x > 2 \end{cases}$$

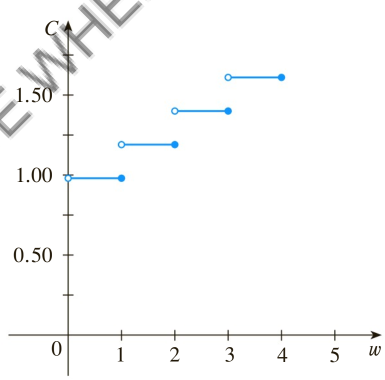


FIGURE 18

EXAMPLE 10 In Example C at the beginning of this section we considered the cost $C(w)$ of mailing a large envelope with weight w. In effect, this is a piecewise defined function because, from the table of values on page 13, we have

$$C(w) = \begin{cases} 0.98 & \text{if } 0 < w \leqslant 1 \\ 1.19 & \text{if } 1 < w \leqslant 2 \\ 1.40 & \text{if } 2 < w \leqslant 3 \\ 1.61 & \text{if } 3 < w \leqslant 4 \\ \vdots & \end{cases}$$

The graph is shown in Figure 18. You can see why functions similar to this one are called **step functions**—they jump from one value to the next. Such functions will be studied in Chapter 2.

Symmetry

If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called an **even function**. For instance, the function $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x)$$

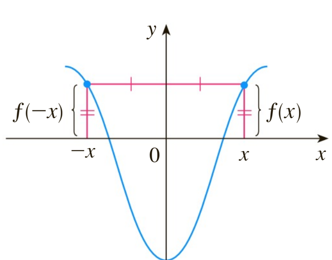


FIGURE 19
An even function

The geometric significance of an even function is that its graph is symmetric with respect to the y-axis (see Figure 19). This means that if we have plotted the graph of f for $x \geqslant 0$, we obtain the entire graph simply by reflecting this portion about the y-axis.

If f satisfies $f(-x) = -f(x)$ for every number x in its domain, then f is called an **odd function**. For example, the function $f(x) = x^3$ is odd because

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

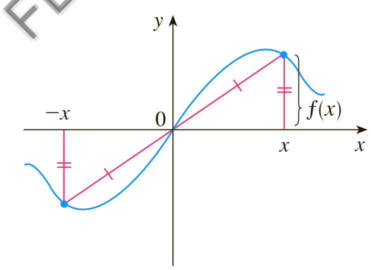


FIGURE 20
An odd function

The graph of an odd function is symmetric about the origin (see Figure 20). If we already have the graph of f for $x \geqslant 0$, we can obtain the entire graph by rotating this portion through 180° about the origin.

EXAMPLE 11 Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $f(x) = x^5 + x$ (b) $g(x) = 1 - x^4$ (c) $h(x) = 2x - x^2$

SOLUTION

(a)
$$\begin{aligned} f(-x) &= (-x)^5 + (-x) = (-1)^5 x^5 + (-x) \\ &= -x^5 - x = -(x^5 + x) \\ &= -f(x) \end{aligned}$$

Therefore f is an odd function.

(b)
$$g(-x) = 1 - (-x)^4 = 1 - x^4 = g(x)$$

So g is even.

(c)
$$h(-x) = 2(-x) - (-x)^2 = -2x - x^2$$

Since $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$, we conclude that h is neither even nor odd. ■

The graphs of the functions in Example 11 are shown in Figure 21. Notice that the graph of h is symmetric neither about the y-axis nor about the origin.

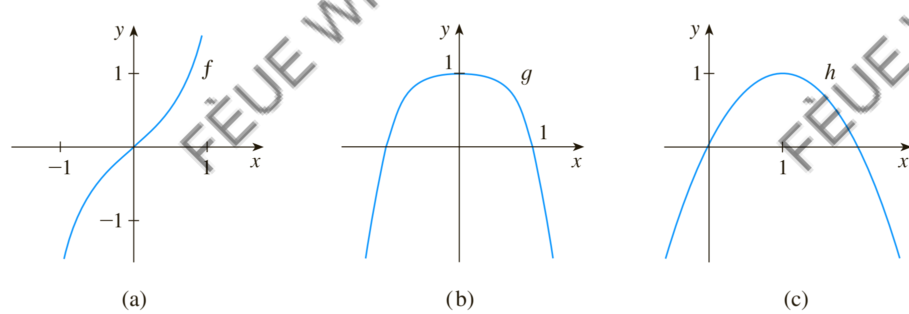


FIGURE 21

Increasing and Decreasing Functions

The graph shown in Figure 22 rises from A to B, falls from B to C, and rises again from C to D. The function f is said to be increasing on the interval $[a, b]$, decreasing on $[b, c]$, and increasing again on $[c, d]$. Notice that if x_1 and x_2 are any two numbers between a and b with $x_1 < x_2$, then $f(x_1) < f(x_2)$. We use this as the defining property of an increasing function.

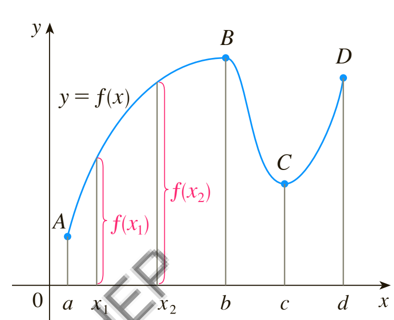


FIGURE 22

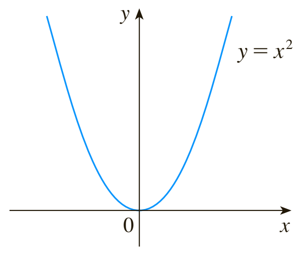


FIGURE 23

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

In the definition of an increasing function it is important to realize that the inequality $f(x_1) < f(x_2)$ must be satisfied for *every* pair of numbers x_1 and x_2 in I with $x_1 < x_2$.

You can see from Figure 23 that the function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

1.1 EXERCISES

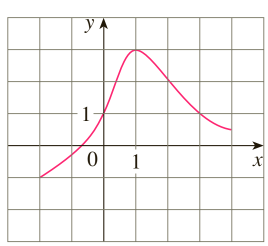
1. If $f(x) = x + \sqrt{2 - x}$ and $g(u) = u + \sqrt{2 - u}$, is it true that $f = g$?

2. If

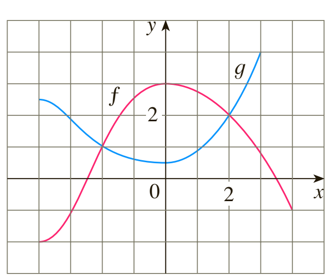
$$f(x) = \frac{x^2 - x}{x - 1} \qquad \text{and} \qquad g(x) = x$$

is it true that $f = g$?

3. The graph of a function f is given.
(a) State the value of $f(1)$.
(b) Estimate the value of $f(-1)$.
(c) For what values of x is $f(x) = 1$?
(d) Estimate the value of x such that $f(x) = 0$.
(e) State the domain and range of f.
(f) On what interval is f increasing?



4. The graphs of f and g are given.



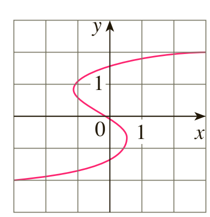
(a) State the values of $f(-4)$ and $g(3)$.
(b) For what values of x is $f(x) = g(x)$?
(c) Estimate the solution of the equation $f(x) = -1$.
(d) On what interval is f decreasing?
(e) State the domain and range of f.
(f) State the domain and range of g.

5. Figure 1 was recorded by an instrument operated by the California Department of Mines and Geology at the University Hospital of the University of Southern California in Los Angeles. Use it to estimate the range of the vertical ground acceleration function at USC during the Northridge earthquake.

6. In this section we discussed examples of ordinary, everyday functions: Population is a function of time, postage cost is a function of weight, water temperature is a function of time. Give three other examples of functions from everyday life that are described verbally. What can you say about the domain and range of each of your functions? If possible, sketch a rough graph of each function.

7–10 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

7.



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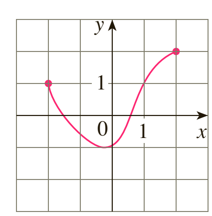
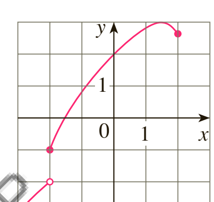


9.



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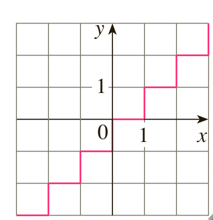


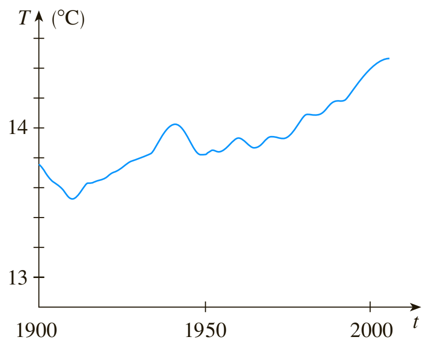
11. Shown is a graph of the global average temperature T during the 20th century. Estimate the following.

(a) The global average temperature in 1950

(b) The year when the average temperature was 14.2°C

(c) The year when the temperature was smallest; the year it was largest

(d) The range of T

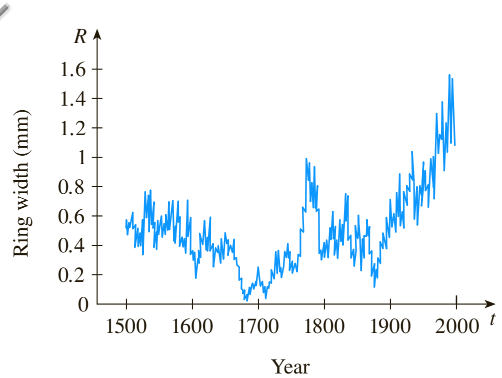


Source: Adapted from *Globe and Mail* [Toronto], 5 Dec. 2009. Print.

12. Trees grow faster and form wider rings in warm years and grow more slowly and form narrower rings in cooler years. The figure shows ring widths of a Siberian pine from 1500 to 2000.

(a) What is the range of the ring width function?

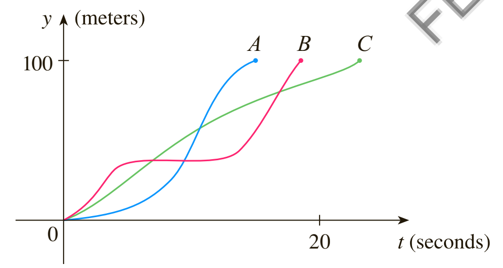
(b) What does the graph tend to say about the temperature of the earth? Does the graph reflect the volcanic eruptions of the mid-19th century?



Source: Adapted from G. Jacoby et al., "Mongolian Tree Rings and 20th-Century Warming," *Science* 273 (1996): 771–73.

13. You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.

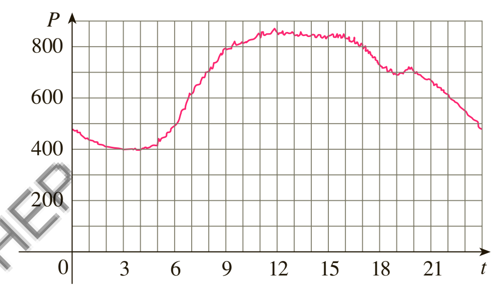
14. Three runners compete in a 100-meter race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race?



15. The graph shows the power consumption for a day in September in San Francisco. (P is measured in megawatts; t is measured in hours starting at midnight.)

(a) What was the power consumption at 6 AM? At 6 PM?

(b) When was the power consumption the lowest? When was it the highest? Do these times seem reasonable?



Pacific Gas & Electric

16. Sketch a rough graph of the number of hours of daylight as a function of the time of year.

17. Sketch a rough graph of the outdoor temperature as a function of time during a typical spring day.

18. Sketch a rough graph of the market value of a new car as a function of time for a period of 20 years. Assume the car is well maintained.

19. Sketch the graph of the amount of a particular brand of coffee sold by a store as a function of the price of the coffee.

20. You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time.

21. A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.

22. An airplane takes off from an airport and lands an hour later at another airport, 400 miles away. If t represents the time in minutes since the plane has left the terminal building, let $x(t)$ be the horizontal distance traveled and $y(t)$ be the altitude of the plane.

(a) Sketch a possible graph of $x(t)$.

(b) Sketch a possible graph of $y(t)$.

(c) Sketch a possible graph of the ground speed.
(d) Sketch a possible graph of the vertical velocity.

23. Temperature readings T (in °F) were recorded every two hours from midnight to 2:00 PM in Atlanta on June 4, 2013. The time t was measured in hours from midnight.

t	0	2	4	6	8	10	12	14
T	74	69	68	66	70	78	82	86

(a) Use the readings to sketch a rough graph of T as a function of t.
(b) Use your graph to estimate the temperature at 9:00 AM.

24. Researchers measured the blood alcohol concentration (BAC) of eight adult male subjects after rapid consumption of 30 mL of ethanol (corresponding to two standard alcoholic drinks). The table shows the data they obtained by averaging the BAC (in g/dL) of the eight men.
(a) Use the readings to sketch the graph of the BAC as a function of t.
(b) Use your graph to describe how the effect of alcohol varies with time.

t (hours)	BAC	t (hours)	BAC
0	0	1.75	0.022
0.2	0.025	2.0	0.018
0.5	0.041	2.25	0.015
0.75	0.040	2.5	0.012
1.0	0.033	3.0	0.007
1.25	0.029	3.5	0.003
1.5	0.024	4.0	0.001

Source: Adapted from P. Wilkinson et al., "Pharmacokinetics of Ethanol after Oral Administration in the Fasting State," *Journal of Pharmacokinetics and Biopharmaceutics* 5 (1977): 207–24.

25. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a+h)$.

26. A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 1$ inches.

27–30 Evaluate the difference quotient for the given function. Simplify your answer.

27. $f(x) = 4 + 3x - x^2$, $\quad \dfrac{f(3+h) - f(3)}{h}$

28. $f(x) = x^3$, $\quad \dfrac{f(a+h) - f(a)}{h}$

29. $f(x) = \dfrac{1}{x}$, $\quad \dfrac{f(x) - f(a)}{x - a}$

30. $f(x) = \dfrac{x+3}{x+1}$, $\quad \dfrac{f(x) - f(1)}{x - 1}$

31–37 Find the domain of the function.

31. $f(x) = \dfrac{x+4}{x^2 - 9}$

32. $f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$

33. $f(t) = \sqrt[3]{2t - 1}$

34. $g(t) = \sqrt{3 - t} - \sqrt{2 + t}$

35. $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$

36. $f(u) = \dfrac{u+1}{1 + \dfrac{1}{u+1}}$

37. $F(p) = \sqrt{2 - \sqrt{p}}$

38. Find the domain and range and sketch the graph of the function $h(x) = \sqrt{4 - x^2}$.

39–40 Find the domain and sketch the graph of the function.

39. $f(x) = 1.6x - 2.4$

40. $g(t) = \dfrac{t^2 - 1}{t + 1}$

41–44 Evaluate $f(-3)$, $f(0)$, and $f(2)$ for the piecewise defined function. Then sketch the graph of the function.

41. $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 1 - x & \text{if } x \geqslant 0 \end{cases}$

42. $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x < 2 \\ 2x - 5 & \text{if } x \geqslant 2 \end{cases}$

43. $f(x) = \begin{cases} x + 1 & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$

44. $f(x) = \begin{cases} -1 & \text{if } x \leqslant 1 \\ 7 - 2x & \text{if } x > 1 \end{cases}$

45–50 Sketch the graph of the function.

45. $f(x) = x + |x|$

46. $f(x) = |x + 2|$

47. $g(t) = |1 - 3t|$

48. $h(t) = |t| + |t + 1|$

49. $f(x) = \begin{cases} |x| & \text{if } |x| \leqslant 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

50. $g(x) = \big||x| - 1\big|$

51–56 Find an expression for the function whose graph is the given curve.

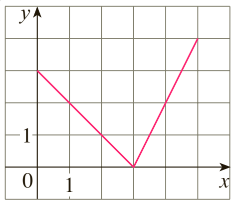
51. The line segment joining the points $(1, -3)$ and $(5, 7)$

52. The line segment joining the points $(-5, 10)$ and $(7, -10)$

53. The bottom half of the parabola $x + (y - 1)^2 = 0$

54. The top half of the circle $x^2 + (y - 2)^2 = 4$

55.



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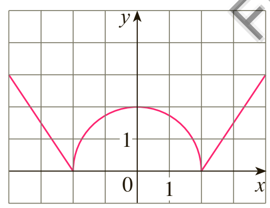


57–61 Find a formula for the described function and state its domain.

57. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

58. A rectangle has area 16 m^2. Express the perimeter of the rectangle as a function of the length of one of its sides.

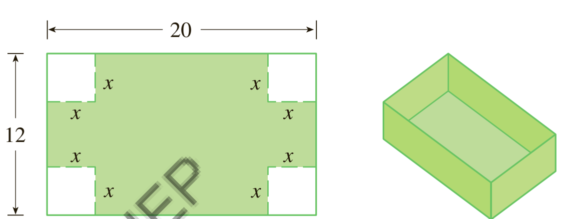
59. Express the area of an equilateral triangle as a function of the length of a side.

60. A closed rectangular box with volume 8 ft^3 has length twice the width. Express the height of the box as a function of the width.

61. An open rectangular box with volume 2 m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

62. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window.

63. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



64. A cell phone plan has a basic charge of \$35 a month. The plan includes 400 free minutes and charges 10 cents for each additional minute of usage. Write the monthly cost C as a function of the number x of minutes used and graph C as a function of x for $0 \leqslant x \leqslant 600$.

65. In a certain state the maximum speed permitted on freeways is 65 mi/h and the minimum speed is 40 mi/h. The fine for violating these limits is \$15 for every mile per hour above the maximum speed or below the minimum speed. Express the amount of the fine F as a function of the driving speed x and graph $F(x)$ for $0 \leqslant x \leqslant 100$.

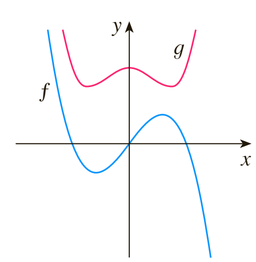
66. An electricity company charges its customers a base rate of \$10 a month, plus 6 cents per kilowatt-hour (kWh) for the first 1200 kWh and 7 cents per kWh for all usage over 1200 kWh. Express the monthly cost E as a function of the amount x of electricity used. Then graph the function E for $0 \leqslant x \leqslant 2000$.

67. In a certain country, income tax is assessed as follows. There is no tax on income up to \$10,000. Any income over \$10,000 is taxed at a rate of 10%, up to an income of \$20,000. Any income over \$20,000 is taxed at 15%.
(a) Sketch the graph of the tax rate R as a function of the income I.
(b) How much tax is assessed on an income of \$14,000? On \$26,000?
(c) Sketch the graph of the total assessed tax T as a function of the income I.

68. The functions in Example 10 and Exercise 67 are called *step functions* because their graphs look like stairs. Give two other examples of step functions that arise in everyday life.

69–70 Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

69.



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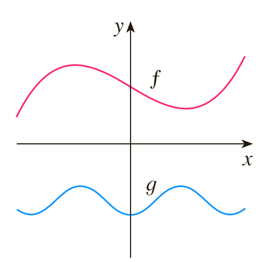
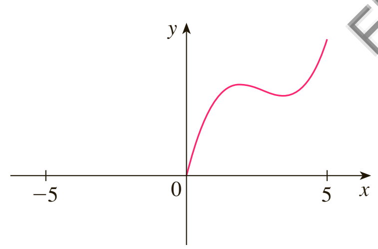


71. (a) If the point (5, 3) is on the graph of an even function, what other point must also be on the graph?
(b) If the point (5, 3) is on the graph of an odd function, what other point must also be on the graph?

72. A function f has domain $[-5, 5]$ and a portion of its graph is shown.
(a) Complete the graph of f if it is known that f is even.
(b) Complete the graph of f if it is known that f is odd.



73–78 Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

73. $f(x) = \dfrac{x}{x^2 + 1}$

74. $f(x) = \dfrac{x^2}{x^4 + 1}$

75. $f(x) = \dfrac{x}{x + 1}$

76. $f(x) = x|x|$

77. $f(x) = 1 + 3x^2 - x^4$

78. $f(x) = 1 + 3x^3 - x^5$

79. If f and g are both even functions, is $f + g$ even? If f and g are both odd functions, is $f + g$ odd? What if f is even and g is odd? Justify your answers.

80. If f and g are both even functions, is the product fg even? If f and g are both odd functions, is fg odd? What if f is even and g is odd? Justify your answers.

1.2 Mathematical Models: A Catalog of Essential Functions

A **mathematical model** is a mathematical description (often by means of a function or an equation) of a real-world phenomenon such as the size of a population, the demand for a product, the speed of a falling object, the concentration of a product in a chemical reaction, the life expectancy of a person at birth, or the cost of emission reductions. The purpose of the model is to understand the phenomenon and perhaps to make predictions about future behavior.

Figure 1 illustrates the process of mathematical modeling. Given a real-world problem, our first task is to formulate a mathematical model by identifying and naming the independent and dependent variables and making assumptions that simplify the phenomenon enough to make it mathematically tractable. We use our knowledge of the physical situation and our mathematical skills to obtain equations that relate the variables. In situations where there is no physical law to guide us, we may need to collect data (either from a library or the Internet or by conducting our own experiments) and examine the data in the form of a table in order to discern patterns. From this numerical representation of a function we may wish to obtain a graphical representation by plotting the data. The graph might even suggest a suitable algebraic formula in some cases.

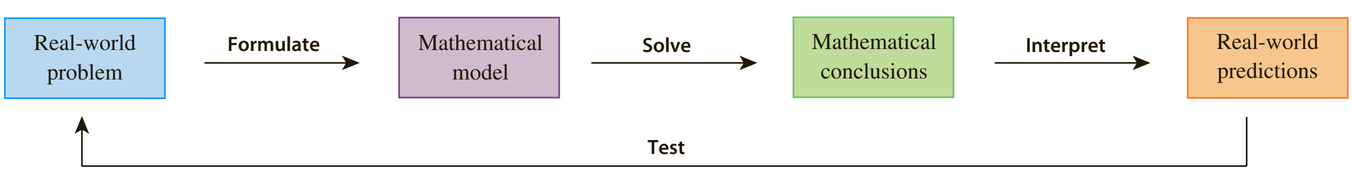


FIGURE 1
The modeling process

The second stage is to apply the mathematics that we know (such as the calculus that will be developed throughout this book) to the mathematical model that we have formulated in order to derive mathematical conclusions. Then, in the third stage, we take those mathematical conclusions and interpret them as information about the original real-world phenomenon by way of offering explanations or making predictions. The final step is to test our predictions by checking against new real data. If the predictions don't compare well with reality, we need to refine our model or to formulate a new model and start the cycle again.

A mathematical model is never a completely accurate representation of a physical situation—it is an *idealization*. A good model simplifies reality enough to permit mathematical calculations but is accurate enough to provide valuable conclusions. It is important to realize the limitations of the model. In the end, Mother Nature has the final say.

There are many different types of functions that can be used to model relationships observed in the real world. In what follows, we discuss the behavior and graphs of these functions and give examples of situations appropriately modeled by such functions.

Linear Models

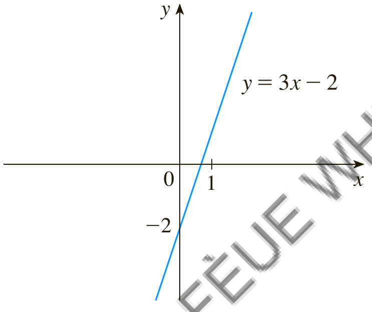
The coordinate geometry of lines is reviewed in Appendix B.

When we say that y is a **linear function** of x, we mean that the graph of the function is a line, so we can use the slope-intercept form of the equation of a line to write a formula for the function as

$$y = f(x) = mx + b$$

where m is the slope of the line and b is the y-intercept.

A characteristic feature of linear functions is that they grow at a constant rate. For instance, Figure 2 shows a graph of the linear function $f(x) = 3x - 2$ and a table of sample values. Notice that whenever x increases by 0.1, the value of $f(x)$ increases by 0.3. So $f(x)$ increases three times as fast as x. Thus the slope of the graph of $y = 3x - 2$, namely 3, can be interpreted as the rate of change of y with respect to x.



x	$f(x) = 3x - 2$
1.0	1.0
1.1	1.3
1.2	1.6
1.3	1.9
1.4	2.2
1.5	2.5

FIGURE 2

EXAMPLE 1

(a) As dry air moves upward, it expands and cools. If the ground temperature is 20°C and the temperature at a height of 1 km is 10°C, express the temperature T (in °C) as a function of the height h (in kilometers), assuming that a linear model is appropriate.
(b) Draw the graph of the function in part (a). What does the slope represent?
(c) What is the temperature at a height of 2.5 km?

SOLUTION

(a) Because we are assuming that T is a linear function of h, we can write

$$T = mh + b$$

We are given that $T = 20$ when $h = 0$, so

$$20 = m \cdot 0 + b = b$$

In other words, the y-intercept is $b = 20$.

We are also given that $T = 10$ when $h = 1$, so

$$10 = m \cdot 1 + 20$$

The slope of the line is therefore $m = 10 - 20 = -10$ and the required linear function is

$$T = -10h + 20$$

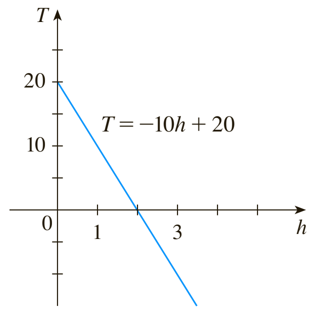


FIGURE 3

(b) The graph is sketched in Figure 3. The slope is $m = -10°C/km$, and this represents the rate of change of temperature with respect to height.

(c) At a height of $h = 2.5$ km, the temperature is

$$T = -10(2.5) + 20 = -5°C$$

If there is no physical law or principle to help us formulate a model, we construct an **empirical model**, which is based entirely on collected data. We seek a curve that "fits" the data in the sense that it captures the basic trend of the data points.

EXAMPLE 2 Table 1 lists the average carbon dioxide level in the atmosphere, measured in parts per million at Mauna Loa Observatory from 1980 to 2012. Use the data in Table 1 to find a model for the carbon dioxide level.

SOLUTION We use the data in Table 1 to make the scatter plot in Figure 4, where t represents time (in years) and C represents the CO_2 level (in parts per million, ppm).

Table 1

Year	CO_2 level (in ppm)	Year	CO_2 level (in ppm)
1980	338.7	1998	366.5
1982	341.2	2000	369.4
1984	344.4	2002	373.2
1986	347.2	2004	377.5
1988	351.5	2006	381.9
1990	354.2	2008	385.6
1992	356.3	2010	389.9
1994	358.6	2012	393.8
1996	362.4		

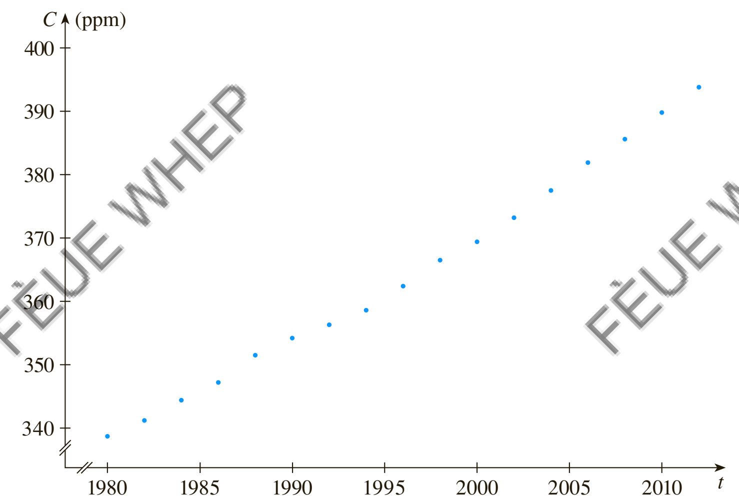


FIGURE 4 Scatter plot for the average CO_2 level

Notice that the data points appear to lie close to a straight line, so it's natural to choose a linear model in this case. But there are many possible lines that approximate these data points, so which one should we use? One possibility is the line that passes through the first and last data points. The slope of this line is

$$\frac{393.8 - 338.7}{2012 - 1980} = \frac{55.1}{32} = 1.721875 \approx 1.722$$

We write its equation as

$$C - 338.7 = 1.722(t - 1980)$$

or

$$\boxed{1} \qquad C = 1.722t - 3070.86$$

Equation 1 gives one possible linear model for the carbon dioxide level; it is graphed in Figure 5.

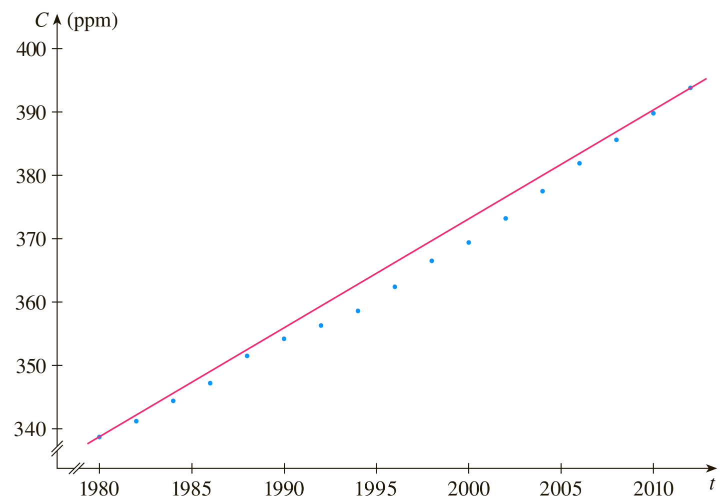


FIGURE 5
Linear model through first and last data points

A computer or graphing calculator finds the regression line by the method of **least squares**, which is to minimize the sum of the squares of the vertical distances between the data points and the line. The details are explained in Section 14.7.

Notice that our model gives values higher than most of the actual CO_2 levels. A better linear model is obtained by a procedure from statistics called *linear regression*. If we use a graphing calculator, we enter the data from Table 1 into the data editor and choose the linear regression command. (With Maple we use the fit[leastsquare] command in the stats package; with Mathematica we use the Fit command.) The machine gives the slope and y-intercept of the regression line as

$$m = 1.71262 \qquad b = -3054.14$$

So our least squares model for the CO_2 level is

$$\boxed{2} \qquad C = 1.71262t - 3054.14$$

In Figure 6 we graph the regression line as well as the data points. Comparing with Figure 5, we see that it gives a better fit than our previous linear model.

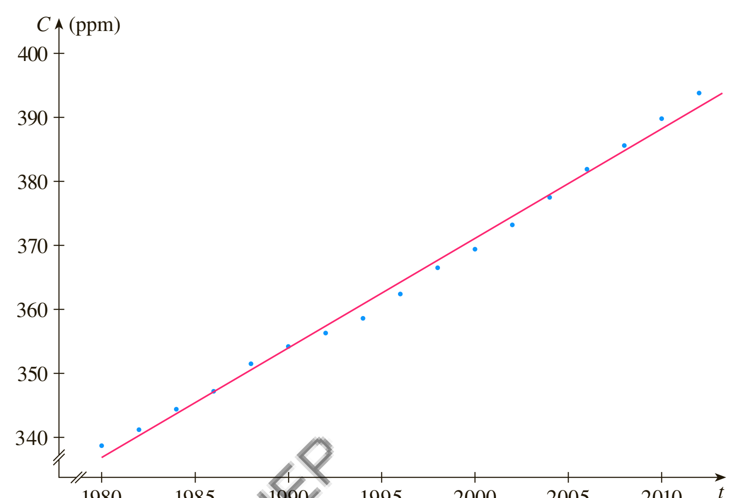


FIGURE 6
The regression line

EXAMPLE 3 Use the linear model given by Equation 2 to estimate the average CO_2 level for 1987 and to predict the level for the year 2020. According to this model, when will the CO_2 level exceed 420 parts per million?

SOLUTION Using Equation 2 with $t = 1987$, we estimate that the average CO_2 level in 1987 was

$$C(1987) = (1.71262)(1987) - 3054.14 \approx 348.84$$

This is an example of *interpolation* because we have estimated a value *between* observed values. (In fact, the Mauna Loa Observatory reported that the average CO_2 level in 1987 was 348.93 ppm, so our estimate is quite accurate.)

With $t = 2020$, we get

$$C(2020) = (1.71262)(2020) - 3054.14 \approx 405.35$$

So we predict that the average CO_2 level in the year 2020 will be 405.4 ppm. This is an example of *extrapolation* because we have predicted a value *outside* the time frame of observations. Consequently, we are far less certain about the accuracy of our prediction.

Using Equation 2, we see that the CO_2 level exceeds 420 ppm when

$$1.71262t - 3054.14 > 420$$

Solving this inequality, we get

$$t > \frac{3474.14}{1.71262} \approx 2028.55$$

We therefore predict that the CO_2 level will exceed 420 ppm by the year 2029. This prediction is risky because it involves a time quite remote from our observations. In fact, we see from Figure 6 that the trend has been for CO_2 levels to increase rather more rapidly in recent years, so the level might exceed 420 ppm well before 2029. ■

Polynomials

A function P is called a **polynomial** if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are constants called the **coefficients** of the polynomial. The domain of any polynomial is $\mathbb{R} = (-\infty, \infty)$. If the leading coefficient $a_n \neq 0$, then the **degree** of the polynomial is n. For example, the function

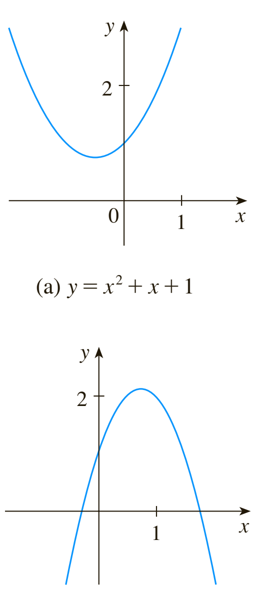
$$P(x) = 2x^6 - x^4 + \tfrac{2}{5}x^3 + \sqrt{2}$$

is a polynomial of degree 6.

A polynomial of degree 1 is of the form $P(x) = mx + b$ and so it is a linear function. A polynomial of degree 2 is of the form $P(x) = ax^2 + bx + c$ and is called a **quadratic function**. Its graph is always a parabola obtained by shifting the parabola $y = ax^2$, as we will see in the next section. The parabola opens upward if $a > 0$ and downward if $a < 0$. (See Figure 7.)

A polynomial of degree 3 is of the form

$$P(x) = ax^3 + bx^2 + cx + d \qquad a \neq 0$$



(a) $y = x^2 + x + 1$

(b) $y = -2x^2 + 3x + 1$

FIGURE 7
The graphs of quadratic functions are parabolas.

and is called a **cubic function**. Figure 8 shows the graph of a cubic function in part (a) and graphs of polynomials of degrees 4 and 5 in parts (b) and (c). We will see later why the graphs have these shapes.

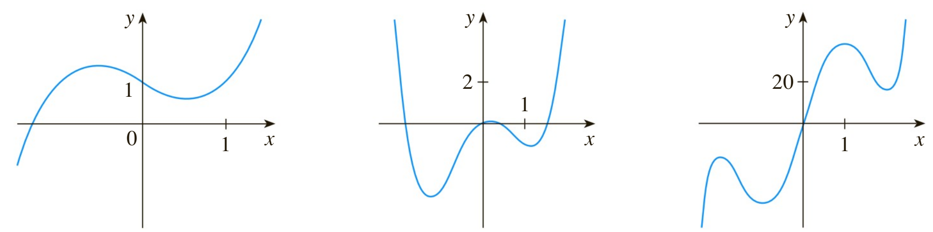


FIGURE 8

(a) $y = x^3 - x + 1$ (b) $y = x^4 - 3x^2 + x$ (c) $y = 3x^5 - 25x^3 + 60x$

Polynomials are commonly used to model various quantities that occur in the natural and social sciences. For instance, in Section 2.7 we will explain why economists often use a polynomial $P(x)$ to represent the cost of producing x units of a commodity. In the following example we use a quadratic function to model the fall of a ball.

Table 2

Time (seconds)	Height (meters)
0	450
1	445
2	431
3	408
4	375
5	332
6	279
7	216
8	143
9	61

EXAMPLE 4 A ball is dropped from the upper observation deck of the CN Tower, 450 m above the ground, and its height h above the ground is recorded at 1-second intervals in Table 2. Find a model to fit the data and use the model to predict the time at which the ball hits the ground.

SOLUTION We draw a scatter plot of the data in Figure 9 and observe that a linear model is inappropriate. But it looks as if the data points might lie on a parabola, so we try a quadratic model instead. Using a graphing calculator or computer algebra system (which uses the least squares method), we obtain the following quadratic model:

$$h = 449.36 + 0.96t - 4.90t^2 \tag{3}$$

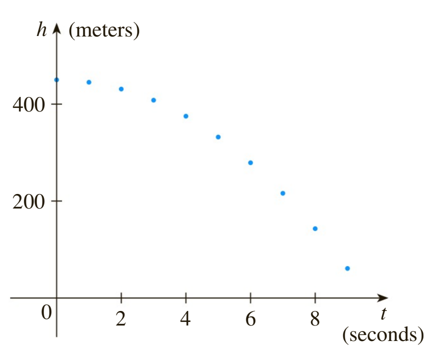


FIGURE 9
Scatter plot for a falling ball

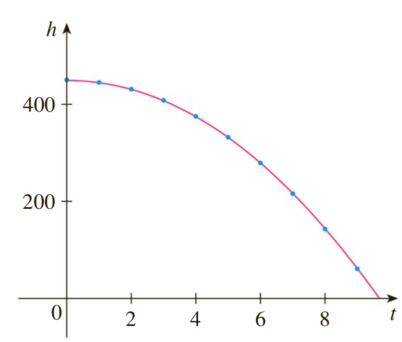


FIGURE 10
Quadratic model for a falling ball

In Figure 10 we plot the graph of Equation 3 together with the data points and see that the quadratic model gives a very good fit.

The ball hits the ground when $h = 0$, so we solve the quadratic equation

$$-4.90t^2 + 0.96t + 449.36 = 0$$

The quadratic formula gives

$$t = \frac{-0.96 \pm \sqrt{(0.96)^2 - 4(-4.90)(449.36)}}{2(-4.90)}$$

The positive root is $t \approx 9.67$, so we predict that the ball will hit the ground after about 9.7 seconds. ■

Power Functions

A function of the form $f(x) = x^a$, where a is a constant, is called a **power function**. We consider several cases.

(i) $a = n$, where n is a positive integer

The graphs of $f(x) = x^n$ for $n = 1, 2, 3, 4$, and 5 are shown in Figure 11. (These are polynomials with only one term.) We already know the shape of the graphs of $y = x$ (a line through the origin with slope 1) and $y = x^2$ [a parabola, see Example 1.1.2(b)].

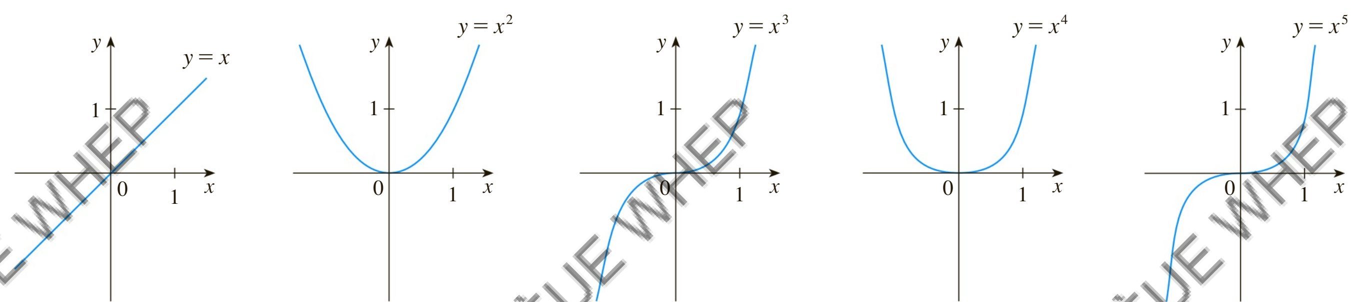


FIGURE 11 Graphs of $f(x) = x^n$ for $n = 1, 2, 3, 4, 5$

The general shape of the graph of $f(x) = x^n$ depends on whether n is even or odd. If n is even, then $f(x) = x^n$ is an even function and its graph is similar to the parabola $y = x^2$. If n is odd, then $f(x) = x^n$ is an odd function and its graph is similar to that of $y = x^3$. Notice from Figure 12, however, that as n increases, the graph of $y = x^n$ becomes flatter near 0 and steeper when $|x| \geqslant 1$. (If x is small, then x^2 is smaller, x^3 is even smaller, x^4 is smaller still, and so on.)

A **family of functions** is a collection of functions whose equations are related. Figure 12 shows two families of power functions, one with even powers and one with odd powers.

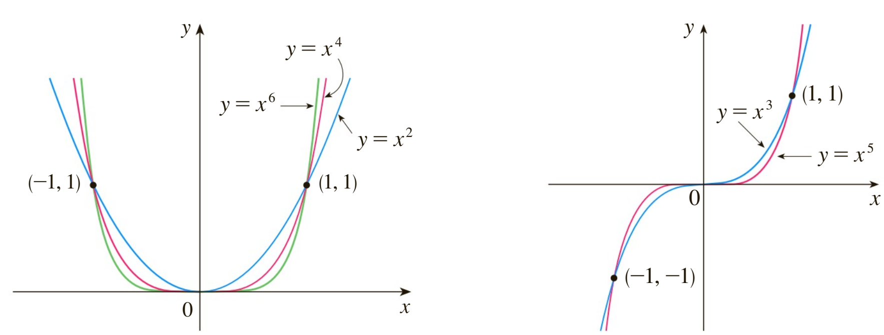
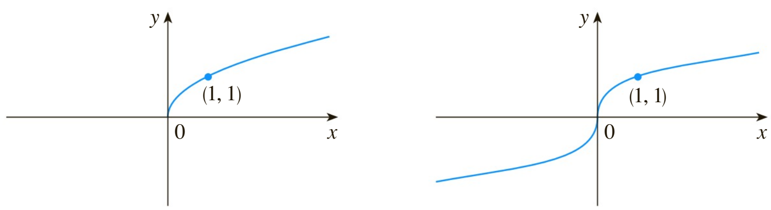


FIGURE 12

(ii) $a = 1/n$, where n is a positive integer

The function $f(x) = x^{1/n} = \sqrt[n]{x}$ is a **root function**. For $n = 2$ it is the square root function $f(x) = \sqrt{x}$, whose domain is $[0, \infty)$ and whose graph is the upper half of the parabola $x = y^2$. [See Figure 13(a).] For other even values of n, the graph of $y = \sqrt[n]{x}$ is similar to that of $y = \sqrt{x}$. For $n = 3$ we have the cube root function $f(x) = \sqrt[3]{x}$ whose domain is $\mathbb{R}$ (recall that every real number has a cube root) and whose graph is shown in Figure 13(b). The graph of $y = \sqrt[n]{x}$ for n odd ($n > 3$) is similar to that of $y = \sqrt[3]{x}$.



(a) $f(x) = \sqrt{x}$ (b) $f(x) = \sqrt[3]{x}$

FIGURE 13
Graphs of root functions

(iii) $\boldsymbol{a = -1}$

The graph of the **reciprocal function** $f(x) = x^{-1} = 1/x$ is shown in Figure 14. Its graph has the equation $y = 1/x$, or $xy = 1$, and is a hyperbola with the coordinate axes as its asymptotes. This function arises in physics and chemistry in connection with Boyle's Law, which says that, when the temperature is constant, the volume V of a gas is inversely proportional to the pressure P:

$$V = \frac{C}{P}$$

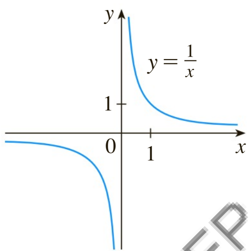


FIGURE 14
The reciprocal function

where C is a constant. Thus the graph of V as a function of P (see Figure 15) has the same general shape as the right half of Figure 14.

Power functions are also used to model species-area relationships (Exercises 30–31), illumination as a function of distance from a light source (Exercise 29), and the period of revolution of a planet as a function of its distance from the sun (Exercise 32).

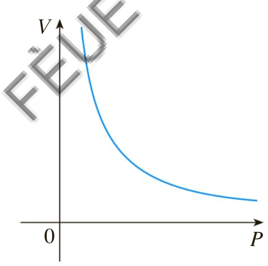


FIGURE 15
Volume as a function of pressure at constant temperature

Rational Functions

A **rational function** f is a ratio of two polynomials:

$$f(x) = \frac{P(x)}{Q(x)}$$

where P and Q are polynomials. The domain consists of all values of x such that $Q(x) \neq 0$. A simple example of a rational function is the function $f(x) = 1/x$, whose domain is $\{x \mid x \neq 0\}$; this is the reciprocal function graphed in Figure 14. The function

$$f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}$$

is a rational function with domain $\{x \mid x \neq \pm 2\}$. Its graph is shown in Figure 16.

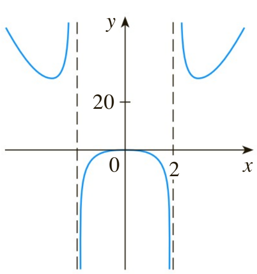


FIGURE 16
$f(x) = \dfrac{2x^4 - x^2 + 1}{x^2 - 4}$

Algebraic Functions

A function f is called an **algebraic function** if it can be constructed using algebraic operations (such as addition, subtraction, multiplication, division, and taking roots) starting with polynomials. Any rational function is automatically an algebraic function. Here are two more examples:

$$f(x) = \sqrt{x^2 + 1} \qquad g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$$

When we sketch algebraic functions in Chapter 3, we will see that their graphs can assume a variety of shapes. Figure 17 illustrates some of the possibilities.

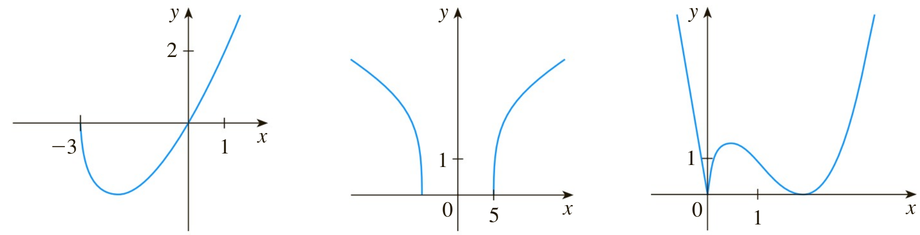


FIGURE 17 (a) $f(x) = x\sqrt{x+3}$ (b) $g(x) = \sqrt[4]{x^2 - 25}$ (c) $h(x) = x^{2/3}(x-2)^2$

An example of an algebraic function occurs in the theory of relativity. The mass of a particle with velocity v is

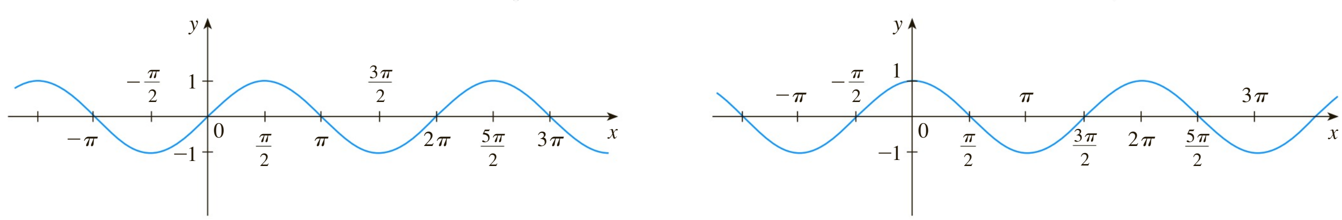
$$m = f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle and $c = 3.0 \times 10^5$ km/s is the speed of light in a vacuum.

Trigonometric Functions

The Reference Pages are located at the back of the book.

Trigonometry and the trigonometric functions are reviewed on Reference Page 2 and also in Appendix D. In calculus the convention is that radian measure is always used (except when otherwise indicated). For example, when we use the function $f(x) = \sin x$, it is understood that $\sin x$ means the sine of the angle whose radian measure is x. Thus the graphs of the sine and cosine functions are as shown in Figure 18.



(a) $f(x) = \sin x$ (b) $g(x) = \cos x$

FIGURE 18

Notice that for both the sine and cosine functions the domain is $(-\infty, \infty)$ and the range is the closed interval $[-1, 1]$. Thus, for all values of x, we have

$$\boxed{-1 \leqslant \sin x \leqslant 1 \qquad -1 \leqslant \cos x \leqslant 1}$$

or, in terms of absolute values,

$$|\sin x| \leqslant 1 \qquad |\cos x| \leqslant 1$$

Also, the zeros of the sine function occur at the integer multiples of π; that is,

$$\sin x = 0 \quad \text{when} \quad x = n\pi \quad n \text{ an integer}$$

An important property of the sine and cosine functions is that they are periodic functions and have period 2π. This means that, for all values of x,

$$\sin(x + 2\pi) = \sin x \qquad \cos(x + 2\pi) = \cos x$$

The periodic nature of these functions makes them suitable for modeling repetitive phenomena such as tides, vibrating springs, and sound waves. For instance, in Example 1.3.4 we will see that a reasonable model for the number of hours of daylight in Philadelphia t days after January 1 is given by the function

$$L(t) = 12 + 2.8 \sin\left[\frac{2\pi}{365}(t - 80)\right]$$

EXAMPLE 5 What is the domain of the function $f(x) = \dfrac{1}{1 - 2\cos x}$?

SOLUTION This function is defined for all values of x except for those that make the denominator 0. But

$$1 - 2\cos x = 0 \iff \cos x = \frac{1}{2} \iff x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad x = \frac{5\pi}{3} + 2n\pi$$

where n is any integer (because the cosine function has period 2π). So the domain of f is the set of all real numbers except for the ones noted above. ■

The tangent function is related to the sine and cosine functions by the equation

$$\tan x = \frac{\sin x}{\cos x}$$

and its graph is shown in Figure 19. It is undefined whenever $\cos x = 0$, that is, when $x = \pm\pi/2, \pm 3\pi/2, \ldots$. Its range is $(-\infty, \infty)$. Notice that the tangent function has period π:

$$\tan(x + \pi) = \tan x \qquad \text{for all } x$$

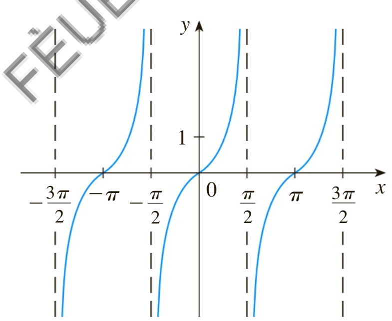


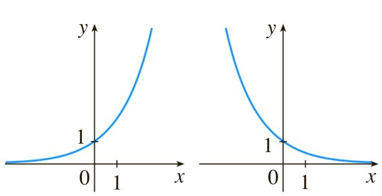
FIGURE 19
$y = \tan x$

The remaining three trigonometric functions (cosecant, secant, and cotangent) are the reciprocals of the sine, cosine, and tangent functions. Their graphs are shown in Appendix D.

Exponential Functions

The **exponential functions** are the functions of the form $f(x) = b^x$, where the base b is a positive constant. The graphs of $y = 2^x$ and $y = (0.5)^x$ are shown in Figure 20. In both cases the domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.

Exponential functions will be studied in detail in Chapter 6, and we will see that they are useful for modeling many natural phenomena, such as population growth (if $b > 1$) and radioactive decay (if $b < 1$).



(a) $y = 2^x$ (b) $y = (0.5)^x$

FIGURE 20

Logarithmic Functions

The **logarithmic functions** $f(x) = \log_b x$, where the base b is a positive constant, are the inverse functions of the exponential functions. They will be studied in Chapter 6. Figure

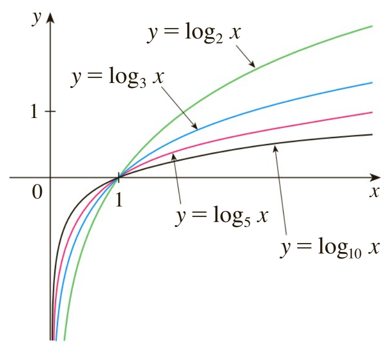


FIGURE 21

21 shows the graphs of four logarithmic functions with various bases. In each case the domain is $(0, \infty)$, the range is $(-\infty, \infty)$, and the function increases slowly when $x > 1$.

EXAMPLE 6 Classify the following functions as one of the types of functions that we have discussed.

(a) $f(x) = 5^x$ (b) $g(x) = x^5$

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ (d) $u(t) = 1 - t + 5t^4$

SOLUTION

(a) $f(x) = 5^x$ is an exponential function. (The x is the exponent.)

(b) $g(x) = x^5$ is a power function. (The x is the base.) We could also consider it to be a polynomial of degree 5.

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ is an algebraic function.

(d) $u(t) = 1 - t + 5t^4$ is a polynomial of degree 4. ■

1.2 EXERCISES

1–2 Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

1. (a) $f(x) = \log_2 x$ (b) $g(x) = \sqrt[4]{x}$

(c) $h(x) = \dfrac{2x^3}{1 - x^2}$ (d) $u(t) = 1 - 1.1t + 2.54t^2$

(e) $v(t) = 5^t$ (f) $w(\theta) = \sin\theta \cos^2\theta$

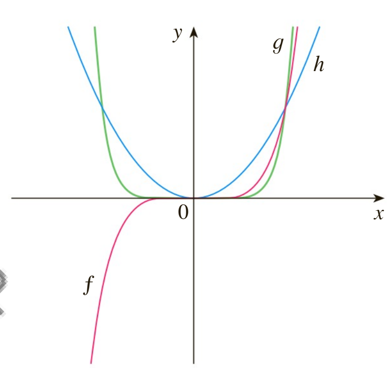
2. (a) $y = \pi^x$ (b) $y = x^\pi$

(c) $y = x^2(2 - x^3)$ (d) $y = \tan t - \cos t$

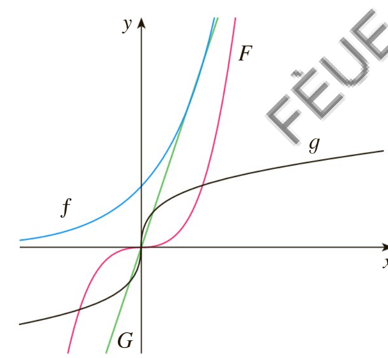
(e) $y = \dfrac{s}{1 + s}$ (f) $y = \dfrac{\sqrt{x^3 - 1}}{1 + \sqrt[3]{x}}$

3–4 Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator.)

3. (a) $y = x^2$ (b) $y = x^5$ (c) $y = x^8$



4. (a) $y = 3x$ (b) $y = 3^x$ (c) $y = x^3$ (d) $y = \sqrt[3]{x}$



5–6 Find the domain of the function.

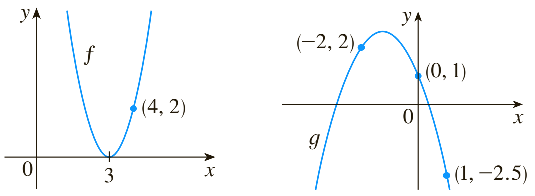
5. $f(x) = \dfrac{\cos x}{1 - \sin x}$ **6.** $g(x) = \dfrac{1}{1 - \tan x}$

7. (a) Find an equation for the family of linear functions with slope 2 and sketch several members of the family.
(b) Find an equation for the family of linear functions such that $f(2) = 1$ and sketch several members of the family.
(c) Which function belongs to both families?

8. What do all members of the family of linear functions $f(x) = 1 + m(x + 3)$ have in common? Sketch several members of the family.

9. What do all members of the family of linear functions $f(x) = c - x$ have in common? Sketch several members of the family.

10. Find expressions for the quadratic functions whose graphs are shown.



11. Find an expression for a cubic function f if $f(1) = 6$ and $f(-1) = f(0) = f(2) = 0$.

12. Recent studies indicate that the average surface temperature of the earth has been rising steadily. Some scientists have modeled the temperature by the linear function $T = 0.02t + 8.50$, where T is temperature in °C and t represents years since 1900.

(a) What do the slope and T-intercept represent?

(b) Use the equation to predict the average global surface temperature in 2100.

13. If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation $c = 0.0417D(a + 1)$. Suppose the dosage for an adult is 200 mg.

(a) Find the slope of the graph of c. What does it represent?

(b) What is the dosage for a newborn?

14. The manager of a weekend flea market knows from past experience that if he charges x dollars for a rental space at the market, then the number y of spaces he can rent is given by the equation $y = 200 - 4x$.

(a) Sketch a graph of this linear function. (Remember that the rental charge per space and the number of spaces rented can't be negative quantities.)

(b) What do the slope, the y-intercept, and the x-intercept of the graph represent?

15. The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by the linear function $F = \frac{9}{5}C + 32$.

(a) Sketch a graph of this function.

(b) What is the slope of the graph and what does it represent? What is the F-intercept and what does it represent?

16. Jason leaves Detroit at 2:00 PM and drives at a constant speed west along I-94. He passes Ann Arbor, 40 mi from Detroit, at 2:50 PM.

(a) Express the distance traveled in terms of the time elapsed.

(b) Draw the graph of the equation in part (a).

(c) What is the slope of this line? What does it represent?

17. Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 113 chirps per minute at 70°F and 173 chirps per minute at 80°F.

(a) Find a linear equation that models the temperature T as a function of the number of chirps per minute N.

(b) What is the slope of the graph? What does it represent?

(c) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

18. The manager of a furniture factory finds that it costs \$2200 to manufacture 100 chairs in one day and \$4800 to produce 300 chairs in one day.

(a) Express the cost as a function of the number of chairs produced, assuming that it is linear. Then sketch the graph.

(b) What is the slope of the graph and what does it represent?

(c) What is the y-intercept of the graph and what does it represent?

19. At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.

(a) Express the water pressure as a function of the depth below the ocean surface.

(b) At what depth is the pressure 100 lb/in^2?

20. The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her \$380 to drive 480 mi and in June it cost her \$460 to drive 800 mi.

(a) Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.

(b) Use part (a) to predict the cost of driving 1500 miles per month.

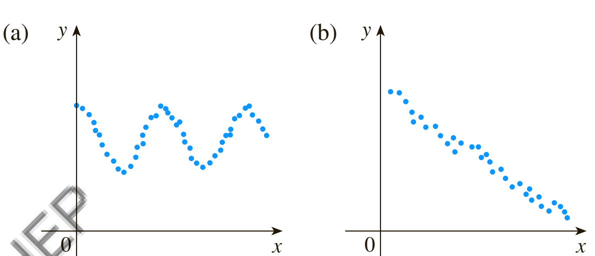
(c) Draw the graph of the linear function. What does the slope represent?

(d) What does the C-intercept represent?

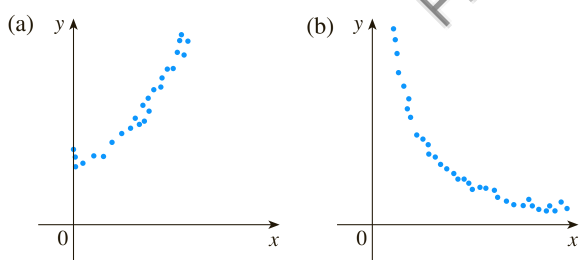
(e) Why does a linear function give a suitable model in this situation?

21–22 For each scatter plot, decide what type of function you might choose as a model for the data. Explain your choices.

21.



22.



23. The table shows (lifetime) peptic ulcer rates (per 100 population) for various family incomes as reported by the National Health Interview Survey.

Income	Ulcer rate (per 100 population)
\$4,000	14.1
\$6,000	13.0
\$8,000	13.4
\$12,000	12.5
\$16,000	12.0
\$20,000	12.4
\$30,000	10.5
\$45,000	9.4
\$60,000	8.2

(a) Make a scatter plot of these data and decide whether a linear model is appropriate.
(b) Find and graph a linear model using the first and last data points.
(c) Find and graph the least squares regression line.
(d) Use the linear model in part (c) to estimate the ulcer rate for an income of \$25,000.
(e) According to the model, how likely is someone with an income of \$80,000 to suffer from peptic ulcers?
(f) Do you think it would be reasonable to apply the model to someone with an income of \$200,000?

24. Biologists have observed that the chirping rate of crickets of a certain species appears to be related to temperature. The table shows the chirping rates for various temperatures.
(a) Make a scatter plot of the data.
(b) Find and graph the regression line.
(c) Use the linear model in part (b) to estimate the chirping rate at 100°F.

Temperature (°F)	Chirping rate (chirps/min)	Temperature (°F)	Chirping rate (chirps/min)
50	20	75	140
55	46	80	173
60	79	85	198
65	91	90	211
70	113		

25. Anthropologists use a linear model that relates human femur (thighbone) length to height. The model allows an anthropologist to determine the height of an individual when only a partial skeleton (including the femur) is found. Here we find the model by analyzing the data on femur length and height for the eight males given in the following table.
(a) Make a scatter plot of the data.
(b) Find and graph the regression line that models the data.
(c) An anthropologist finds a human femur of length 53 cm. How tall was the person?

Femur length (cm)	Height (cm)	Femur length (cm)	Height (cm)
50.1	178.5	44.5	168.3
48.3	173.6	42.7	165.0
45.2	164.8	39.5	155.4
44.7	163.7	38.0	155.8

26. When laboratory rats are exposed to asbestos fibers, some of them develop lung tumors. The table lists the results of several experiments by different scientists.
(a) Find the regression line for the data.
(b) Make a scatter plot and graph the regression line. Does the regression line appear to be a suitable model for the data?
(c) What does the y-intercept of the regression line represent?

Asbestos exposure (fibers/mL)	Percent of mice that develop lung tumors	Asbestos exposure (fibers/mL)	Percent of mice that develop lung tumors
50	2	1600	42
400	6	1800	37
500	5	2000	38
900	10	3000	50
1100	26		

27. The table shows world average daily oil consumption from 1985 to 2010 measured in thousands of barrels per day.
(a) Make a scatter plot and decide whether a linear model is appropriate.
(b) Find and graph the regression line.
(c) Use the linear model to estimate the oil consumption in 2002 and 2012.

Years since 1985	Thousands of barrels of oil per day
0	60,083
5	66,533
10	70,099
15	76,784
20	84,077
25	87,302

Source: US Energy Information Administration

28. The table shows average US retail residential prices of electricity from 2000 to 2012, measured in cents per kilowatt hour.
(a) Make a scatter plot. Is a linear model appropriate?
(b) Find and graph the regression line.
(c) Use your linear model from part (b) to estimate the average retail price of electricity in 2005 and 2013.

Years since 2000	Cents/kWh
0	8.24
2	8.44
4	8.95
6	10.40
8	11.26
10	11.54
12	11.58

Source: US Energy Information Administration

29. Many physical quantities are connected by *inverse square laws*, that is, by power functions of the form $f(x) = kx^{-2}$. In particular, the illumination of an object by a light source is inversely proportional to the square of the distance from the source. Suppose that after dark you are in a room with just one lamp and you are trying to read a book. The light is too dim and so you move halfway to the lamp. How much brighter is the light?

30. It makes sense that the larger the area of a region, the larger the number of species that inhabit the region. Many ecologists have modeled the species-area relation with a power function and, in particular, the number of species S of bats living in caves in central Mexico has been related to the surface area A of the caves by the equation $S = 0.7A^{0.3}$.
(a) The cave called *Misión Imposible* near Puebla, Mexico, has a surface area of $A = 60$ m^2. How many species of bats would you expect to find in that cave?
(b) If you discover that four species of bats live in a cave, estimate the area of the cave.

31. The table shows the number N of species of reptiles and amphibians inhabiting Caribbean islands and the area A of the island in square miles.
(a) Use a power function to model N as a function of A.
(b) The Caribbean island of Dominica has area 291 mi^2. How many species of reptiles and amphibians would you expect to find on Dominica?

Island	A	N
Saba	4	5
Monserrat	40	9
Puerto Rico	3,459	40
Jamaica	4,411	39
Hispaniola	29,418	84
Cuba	44,218	76

32. The table shows the mean (average) distances d of the planets from the sun (taking the unit of measurement to be the distance from planet Earth to the sun) and their periods T (time of revolution in years).
(a) Fit a power model to the data.
(b) Kepler's Third Law of Planetary Motion states that "The square of the period of revolution of a planet is proportional to the cube of its mean distance from the sun."
Does your model corroborate Kepler's Third Law?

Planet	d	T
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.000	1.000
Mars	1.523	1.881
Jupiter	5.203	11.861
Saturn	9.541	29.457
Uranus	19.190	84.008
Neptune	30.086	164.784

1.3 New Functions from Old Functions

In this section we start with the basic functions we discussed in Section 1.2 and obtain new functions by shifting, stretching, and reflecting their graphs. We also show how to combine pairs of functions by the standard arithmetic operations and by composition.

Transformations of Functions

By applying certain transformations to the graph of a given function we can obtain the graphs of related functions. This will give us the ability to sketch the graphs of many functions quickly by hand. It will also enable us to write equations for given graphs.

Let's first consider **translations**. If c is a positive number, then the graph of $y = f(x) + c$ is just the graph of $y = f(x)$ shifted upward a distance of c units (because each y-coordinate is increased by the same number c). Likewise, if $g(x) = f(x - c)$, where $c > 0$, then the value of g at x is the same as the value of f at $x - c$ (c units to the left of x). Therefore the graph of $y = f(x - c)$ is just the graph of $y = f(x)$ shifted c units to the right (see Figure 1).

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward
$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward
$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right
$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

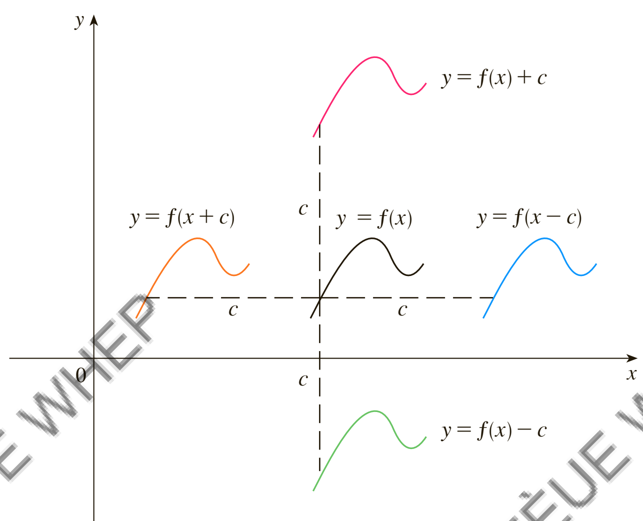


FIGURE 1 Translating the graph of f

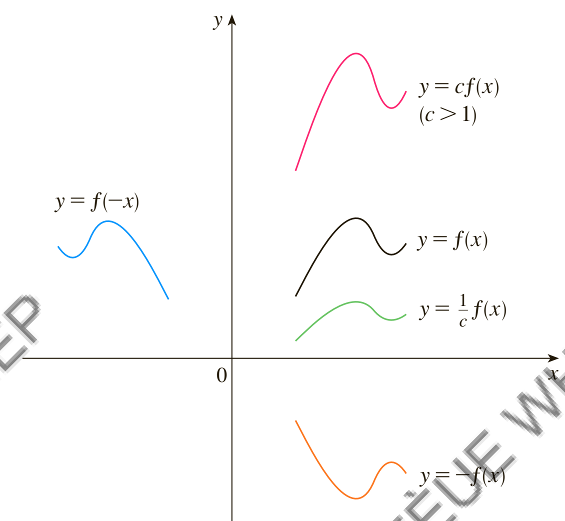


FIGURE 2 Stretching and reflecting the graph of f

Now let's consider the **stretching** and **reflecting** transformations. If $c > 1$, then the graph of $y = cf(x)$ is the graph of $y = f(x)$ stretched by a factor of c in the vertical direction (because each y-coordinate is multiplied by the same number c). The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis because the point (x, y) is replaced by the point $(x, -y)$. (See Figure 2 and the following chart, where the results of other stretching, shrinking, and reflecting transformations are also given.)

Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c
$y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c
$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c
$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c
$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

Figure 3 illustrates these stretching transformations when applied to the cosine function with $c = 2$. For instance, in order to get the graph of $y = 2 \cos x$ we multiply the y-coordinate of each point on the graph of $y = \cos x$ by 2. This means that the graph of $y = \cos x$ gets stretched vertically by a factor of 2.

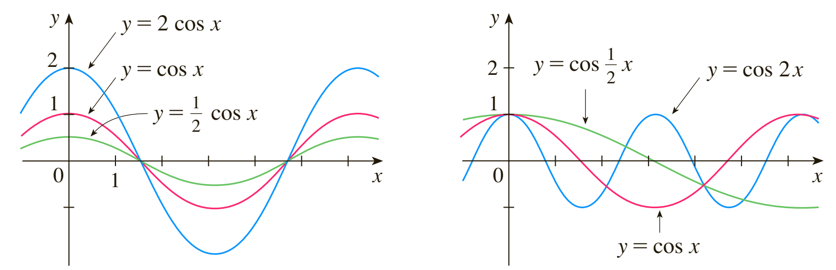
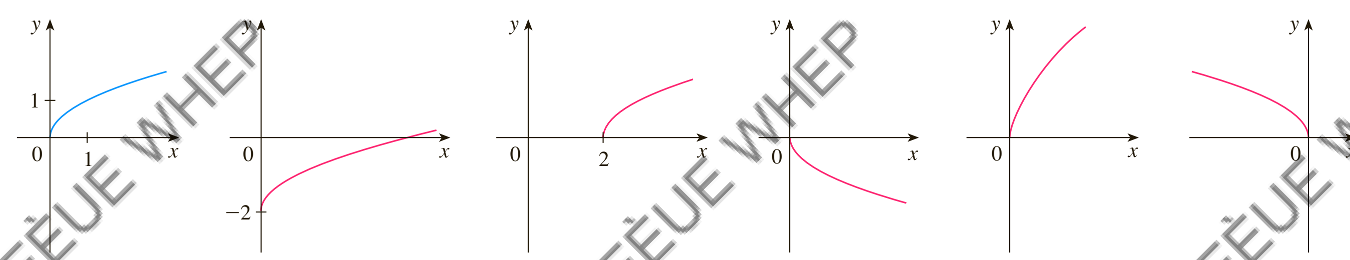


FIGURE 3

EXAMPLE 1 Given the graph of $y = \sqrt{x}$, use transformations to graph $y = \sqrt{x} - 2$, $y = \sqrt{x - 2}$, $y = -\sqrt{x}$, $y = 2\sqrt{x}$, and $y = \sqrt{-x}$.

SOLUTION The graph of the square root function $y = \sqrt{x}$, obtained from Figure 1.2.13(a), is shown in Figure 4(a). In the other parts of the figure we sketch $y = \sqrt{x} - 2$ by shifting 2 units downward, $y = \sqrt{x - 2}$ by shifting 2 units to the right, $y = -\sqrt{x}$ by reflecting about the x-axis, $y = 2\sqrt{x}$ by stretching vertically by a factor of 2, and $y = \sqrt{-x}$ by reflecting about the y-axis.



(a) $y = \sqrt{x}$ (b) $y = \sqrt{x} - 2$ (c) $y = \sqrt{x - 2}$ (d) $y = -\sqrt{x}$ (e) $y = 2\sqrt{x}$ (f) $y = \sqrt{-x}$

FIGURE 4

■

EXAMPLE 2 Sketch the graph of the function $f(x) = x^2 + 6x + 10$.

SOLUTION Completing the square, we write the equation of the graph as

$$y = x^2 + 6x + 10 = (x + 3)^2 + 1$$

This means we obtain the desired graph by starting with the parabola $y = x^2$ and shifting 3 units to the left and then 1 unit upward (see Figure 5).

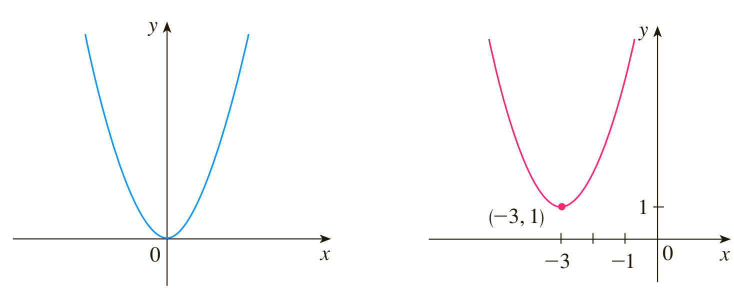


FIGURE 5 (a) $y = x^2$ (b) $y = (x + 3)^2 + 1$

■

EXAMPLE 3 Sketch the graphs of the following functions.

(a) $y = \sin 2x$ (b) $y = 1 - \sin x$

SOLUTION
(a) We obtain the graph of $y = \sin 2x$ from that of $y = \sin x$ by compressing horizontally by a factor of 2. (See Figures 6 and 7.) Thus, whereas the period of $y = \sin x$ is 2π, the period of $y = \sin 2x$ is $2\pi/2 = \pi$.

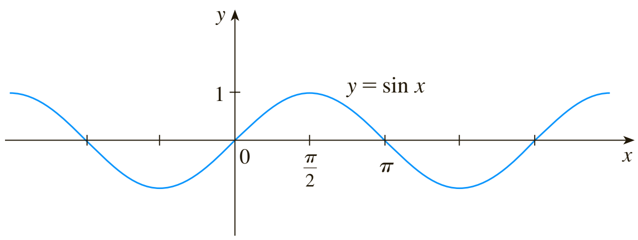


FIGURE 6

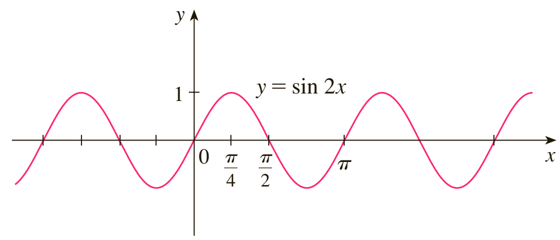


FIGURE 7

(b) To obtain the graph of $y = 1 - \sin x$, we again start with $y = \sin x$. We reflect about the x-axis to get the graph of $y = -\sin x$ and then we shift 1 unit upward to get $y = 1 - \sin x$. (See Figure 8.)

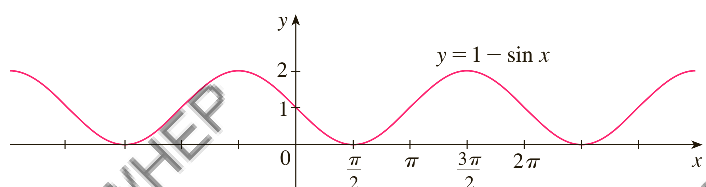


FIGURE 8

EXAMPLE 4 Figure 9 shows graphs of the number of hours of daylight as functions of the time of the year at several latitudes. Given that Philadelphia is located at approximately 40°N latitude, find a function that models the length of daylight at Philadelphia.

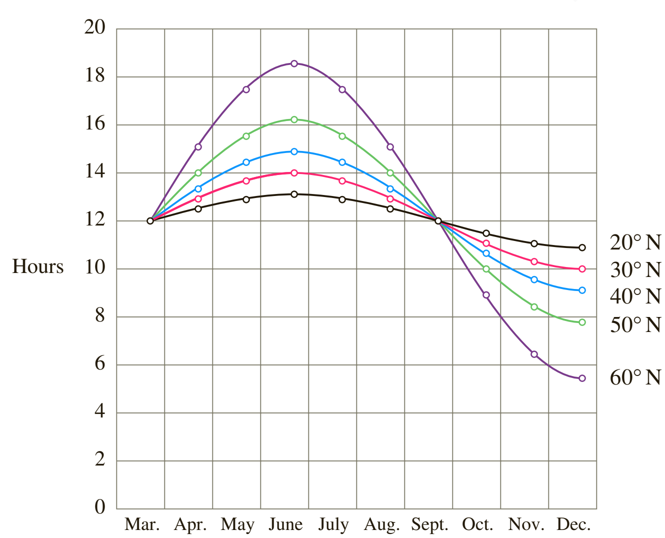


FIGURE 9
Graph of the length of daylight from March 21 through December 21 at various latitudes

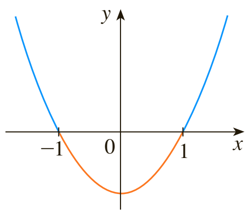
Source: Adapted from L. Harrison, *Daylight, Twilight, Darkness and Time* (New York: Silver, Burdett, 1935), 40.

SOLUTION Notice that each curve resembles a shifted and stretched sine function. By looking at the blue curve we see that, at the latitude of Philadelphia, daylight lasts about 14.8 hours on June 21 and 9.2 hours on December 21, so the amplitude of the curve (the factor by which we have to stretch the sine curve vertically) is $\frac{1}{2}(14.8 - 9.2) = 2.8$.

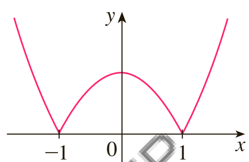
By what factor do we need to stretch the sine curve horizontally if we measure the time t in days? Because there are about 365 days in a year, the period of our model should be 365. But the period of $y = \sin t$ is 2π, so the horizontal stretching factor is $2\pi/365$.

We also notice that the curve begins its cycle on March 21, the 80th day of the year, so we have to shift the curve 80 units to the right. In addition, we shift it 12 units upward. Therefore we model the length of daylight in Philadelphia on the tth day of the year by the function

$$L(t) = 12 + 2.8 \sin\left[\frac{2\pi}{365}(t - 80)\right]$$



(a) $y = x^2 - 1$



(b) $y = |x^2 - 1|$

FIGURE 10

Another transformation of some interest is taking the *absolute value* of a function. If $y = |f(x)|$, then according to the definition of absolute value, $y = f(x)$ when $f(x) \geqslant 0$ and $y = -f(x)$ when $f(x) < 0$. This tells us how to get the graph of $y = |f(x)|$ from the graph of $y = f(x)$: the part of the graph that lies above the x-axis remains the same; the part that lies below the x-axis is reflected about the x-axis.

EXAMPLE 5 Sketch the graph of the function $y = |x^2 - 1|$.

SOLUTION We first graph the parabola $y = x^2 - 1$ in Figure 10(a) by shifting the parabola $y = x^2$ downward 1 unit. We see that the graph lies below the x-axis when $-1 < x < 1$, so we reflect that part of the graph about the x-axis to obtain the graph of $y = |x^2 - 1|$ in Figure 10(b).

Combinations of Functions

Two functions f and g can be combined to form new functions $f + g$, $f - g$, fg, and f/g in a manner similar to the way we add, subtract, multiply, and divide real numbers. The sum and difference functions are defined by

$$(f + g)(x) = f(x) + g(x) \qquad (f - g)(x) = f(x) - g(x)$$

If the domain of f is A and the domain of g is B, then the domain of $f + g$ is the intersection $A \cap B$ because both $f(x)$ and $g(x)$ have to be defined. For example, the domain of $f(x) = \sqrt{x}$ is $A = [0, \infty)$ and the domain of $g(x) = \sqrt{2 - x}$ is $B = (-\infty, 2]$, so the domain of $(f + g)(x) = \sqrt{x} + \sqrt{2 - x}$ is $A \cap B = [0, 2]$.

Similarly, the product and quotient functions are defined by

$$(fg)(x) = f(x)g(x) \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

The domain of fg is $A \cap B$. Because we can't divide by 0, the domain of f/g is therefore $\{x \in A \cap B \mid g(x) \neq 0\}$. For instance, if $f(x) = x^2$ and $g(x) = x - 1$, then the domain of the rational function $(f/g)(x) = x^2/(x - 1)$ is $\{x \mid x \neq 1\}$, or $(-\infty, 1) \cup (1, \infty)$.

There is another way of combining two functions to obtain a new function. For example, suppose that $y = f(u) = \sqrt{u}$ and $u = g(x) = x^2 + 1$. Since y is a function of u and u is, in turn, a function of x, it follows that y is ultimately a function of x. We compute this by substitution:

$$y = f(u) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$$

The procedure is called *composition* because the new function is *composed* of the two given functions f and g.

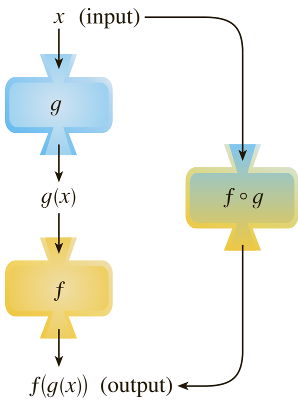


FIGURE 11
The $f \circ g$ machine is composed of the g machine (first) and then the f machine.

In general, given any two functions f and g, we start with a number x in the domain of g and calculate $g(x)$. If this number $g(x)$ is in the domain of f, then we can calculate the value of $f(g(x))$. Notice that the output of one function is used as the input to the next function. The result is a new function $h(x) = f(g(x))$ obtained by substituting g into f. It is called the *composition* (or *composite*) of f and g and is denoted by $f \circ g$ ("f circle g").

Definition Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. In other words, $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined. Figure 11 shows how to picture $f \circ g$ in terms of machines.

EXAMPLE 6 If $f(x) = x^2$ and $g(x) = x - 3$, find the composite functions $f \circ g$ and $g \circ f$.

SOLUTION We have

$$(f \circ g)(x) = f(g(x)) = f(x - 3) = (x - 3)^2$$

$$(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 3$$ ■

NOTE You can see from Example 6 that, in general, $f \circ g \neq g \circ f$. Remember, the notation $f \circ g$ means that the function g is applied first and then f is applied second. In Example 6, $f \circ g$ is the function that *first* subtracts 3 and *then* squares; $g \circ f$ is the function that *first* squares and *then* subtracts 3.

EXAMPLE 7 If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2 - x}$, find each of the following functions and their domains.

(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

SOLUTION

(a) $$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{2 - x}\right) = \sqrt{\sqrt{2 - x}} = \sqrt[4]{2 - x}$$

The domain of $f \circ g$ is $\{x \mid 2 - x \geqslant 0\} = \{x \mid x \leqslant 2\} = (-\infty, 2]$.

(b) $$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt{2 - \sqrt{x}}$$

If $0 \leqslant a \leqslant b$, then $a^2 \leqslant b^2$.

For $\sqrt{x}$ to be defined we must have $x \geqslant 0$. For $\sqrt{2 - \sqrt{x}}$ to be defined we must have $2 - \sqrt{x} \geqslant 0$, that is, $\sqrt{x} \leqslant 2$, or $x \leqslant 4$. Thus we have $0 \leqslant x \leqslant 4$, so the domain of $g \circ f$ is the closed interval $[0, 4]$.

(c) $$(f \circ f)(x) = f(f(x)) = f\left(\sqrt{x}\right) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$$

The domain of $f \circ f$ is $[0, \infty)$.

(d) $$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{2 - x}\right) = \sqrt{2 - \sqrt{2 - x}}$$

This expression is defined when both $2 - x \geqslant 0$ and $2 - \sqrt{2 - x} \geqslant 0$. The first inequality means $x \leqslant 2$, and the second is equivalent to $\sqrt{2 - x} \leqslant 2$, or $2 - x \leqslant 4$, or $x \geqslant -2$. Thus $-2 \leqslant x \leqslant 2$, so the domain of $g \circ g$ is the closed interval $[-2, 2]$. ■

It is possible to take the composition of three or more functions. For instance, the composite function $f \circ g \circ h$ is found by first applying h, then g, and then f as follows:

$$(f \circ g \circ h)(x) = f(g(h(x)))$$

EXAMPLE 8 Find $f \circ g \circ h$ if $f(x) = x/(x + 1)$, $g(x) = x^{10}$, and $h(x) = x + 3$.

SOLUTION

$$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x + 3))$$
$$= f((x + 3)^{10}) = \frac{(x + 3)^{10}}{(x + 3)^{10} + 1}$$

So far we have used composition to build complicated functions from simpler ones. But in calculus it is often useful to be able to *decompose* a complicated function into simpler ones, as in the following example.

EXAMPLE 9 Given $F(x) = \cos^2(x + 9)$, find functions f, g, and h such that $F = f \circ g \circ h$.

SOLUTION Since $F(x) = [\cos(x + 9)]^2$, the formula for F says: First add 9, then take the cosine of the result, and finally square. So we let

$$h(x) = x + 9 \qquad g(x) = \cos x \qquad f(x) = x^2$$

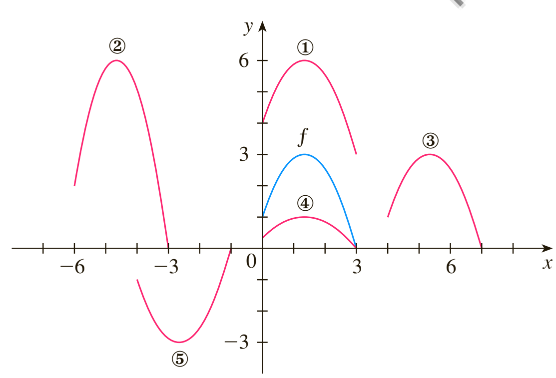
Then $$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x + 9)) = f(\cos(x + 9))$$
$$= [\cos(x + 9)]^2 = F(x)$$

1.3 EXERCISES

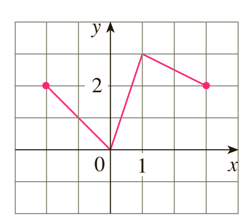
1. Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.
   (a) Shift 3 units upward.
   (b) Shift 3 units downward.
   (c) Shift 3 units to the right.
   (d) Shift 3 units to the left.
   (e) Reflect about the x-axis.
   (f) Reflect about the y-axis.
   (g) Stretch vertically by a factor of 3.
   (h) Shrink vertically by a factor of 3.

2. Explain how each graph is obtained from the graph of $y = f(x)$.
   (a) $y = f(x) + 8$ (b) $y = f(x + 8)$
   (c) $y = 8f(x)$ (d) $y = f(8x)$
   (e) $y = -f(x) - 1$ (f) $y = 8f\left(\frac{1}{8}x\right)$

3. The graph of $y = f(x)$ is given. Match each equation with its graph and give reasons for your choices.
   (a) $y = f(x - 4)$ (b) $y = f(x) + 3$
   (c) $y = \frac{1}{3}f(x)$ (d) $y = -f(x + 4)$
   (e) $y = 2f(x + 6)$



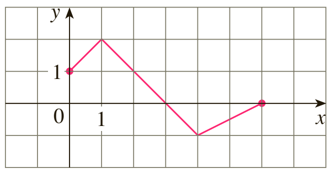
4. The graph of f is given. Draw the graphs of the following functions.
   (a) $y = f(x) - 3$ (b) $y = f(x + 1)$
   (c) $y = \frac{1}{2}f(x)$ (d) $y = -f(x)$



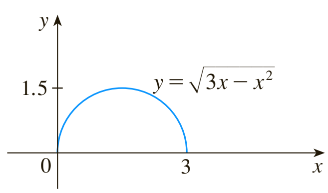
5. The graph of f is given. Use it to graph the following functions.

(a) $y = f(2x)$ (b) $y = f(\frac{1}{2}x)$

(c) $y = f(-x)$ (d) $y = -f(-x)$



6–7 The graph of $y = \sqrt{3x - x^2}$ is given. Use transformations to create a function whose graph is as shown.



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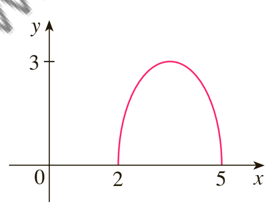

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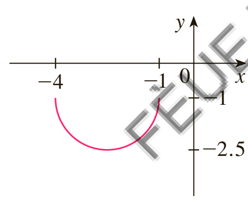


8. (a) How is the graph of $y = 2 \sin x$ related to the graph of $y = \sin x$? Use your answer and Figure 6 to sketch the graph of $y = 2 \sin x$.

(b) How is the graph of $y = 1 + \sqrt{x}$ related to the graph of $y = \sqrt{x}$? Use your answer and Figure 4(a) to sketch the graph of $y = 1 + \sqrt{x}$.

9–24 Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Section 1.2, and then applying the appropriate transformations.

9. $y = -x^2$ **10.** $y = (x - 3)^2$

11. $y = x^3 + 1$ **12.** $y = 1 - \dfrac{1}{x}$

13. $y = 2 \cos 3x$ **14.** $y = 2\sqrt{x + 1}$

15. $y = x^2 - 4x + 5$ **16.** $y = 1 + \sin \pi x$

17. $y = 2 - \sqrt{x}$ **18.** $y = 3 - 2 \cos x$

19. $y = \sin(\frac{1}{2}x)$ **20.** $y = |x| - 2$

21. $y = |x - 2|$ **22.** $y = \dfrac{1}{4} \tan\left(x - \dfrac{\pi}{4}\right)$

23. $y = |\sqrt{x} - 1|$ **24.** $y = |\cos \pi x|$

25. The city of New Orleans is located at latitude 30°N. Use Figure 9 to find a function that models the number of hours of daylight at New Orleans as a function of the time of year. To check the accuracy of your model, use the fact that on March 31 the sun rises at 5:51 AM and sets at 6:18 PM in New Orleans.

26. A variable star is one whose brightness alternately increases and decreases. For the most visible variable star, Delta Cephei, the time between periods of maximum brightness is 5.4 days, the average brightness (or magnitude) of the star is 4.0, and its brightness varies by ± 0.35 magnitude. Find a function that models the brightness of Delta Cephei as a function of time.

27. Some of the highest tides in the world occur in the Bay of Fundy on the Atlantic Coast of Canada. At Hopewell Cape the water depth at low tide is about 2.0 m and at high tide it is about 12.0 m. The natural period of oscillation is about 12 hours and on June 30, 2009, high tide occurred at 6:45 AM. Find a function involving the cosine function that models the water depth $D(t)$ (in meters) as a function of time t (in hours after midnight) on that day.

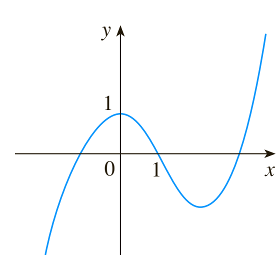
28. In a normal respiratory cycle the volume of air that moves into and out of the lungs is about 500 mL. The reserve and residue volumes of air that remain in the lungs occupy about 2000 mL and a single respiratory cycle for an average human takes about 4 seconds. Find a model for the total volume of air $V(t)$ in the lungs as a function of time.

29. (a) How is the graph of $y = f(|x|)$ related to the graph of f?

(b) Sketch the graph of $y = \sin |x|$.

(c) Sketch the graph of $y = \sqrt{|x|}$.

30. Use the given graph of f to sketch the graph of $y = 1/f(x)$. Which features of f are the most important in sketching $y = 1/f(x)$? Explain how they are used.



31–32 Find (a) $f + g$, (b) $f - g$, (c) fg, and (d) f/g and state their domains.

31. $f(x) = x^3 + 2x^2, \quad g(x) = 3x^2 - 1$

32. $f(x) = \sqrt{3 - x}, \quad g(x) = \sqrt{x^2 - 1}$

33–38 Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

33. $f(x) = 3x + 5, \quad g(x) = x^2 + x$

34. $f(x) = x^3 - 2, \quad g(x) = 1 - 4x$

35. $f(x) = \sqrt{x + 1}, \quad g(x) = 4x - 3$

36. $f(x) = \sin x, \quad g(x) = x^2 + 1$

37. $f(x) = x + \dfrac{1}{x}, \quad g(x) = \dfrac{x + 1}{x + 2}$

38. $f(x) = \dfrac{x}{1 + x}, \quad g(x) = \sin 2x$

39–42 Find $f \circ g \circ h$.

39. $f(x) = 3x - 2, \quad g(x) = \sin x, \quad h(x) = x^2$

40. $f(x) = |x - 4|, \quad g(x) = 2^x, \quad h(x) = \sqrt{x}$

41. $f(x) = \sqrt{x - 3}, \quad g(x) = x^2, \quad h(x) = x^3 + 2$

42. $f(x) = \tan x, \quad g(x) = \dfrac{x}{x - 1}, \quad h(x) = \sqrt[3]{x}$

43–48 Express the function in the form $f \circ g$.

43. $F(x) = (2x + x^2)^4$

44. $F(x) = \cos^2 x$

45. $F(x) = \dfrac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$

46. $G(x) = \sqrt[3]{\dfrac{x}{1 + x}}$

47. $v(t) = \sec(t^2)\tan(t^2)$

48. $u(t) = \dfrac{\tan t}{1 + \tan t}$

49–51 Express the function in the form $f \circ g \circ h$.

49. $R(x) = \sqrt{\sqrt{x} - 1}$

50. $H(x) = \sqrt[8]{2 + |x|}$

51. $S(t) = \sin^2(\cos t)$

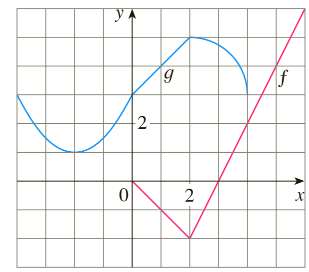
52. Use the table to evaluate each expression.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $f(f(1))$
(d) $g(g(1))$ (e) $(g \circ f)(3)$ (f) $(f \circ g)(6)$

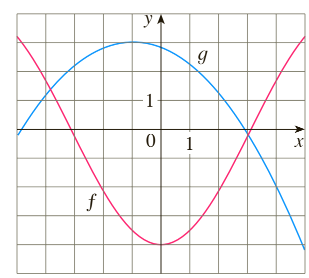
x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

53. Use the given graphs of f and g to evaluate each expression, or explain why it is undefined.

(a) $f(g(2))$ (b) $g(f(0))$ (c) $(f \circ g)(0)$
(d) $(g \circ f)(6)$ (e) $(g \circ g)(-2)$ (f) $(f \circ f)(4)$



54. Use the given graphs of f and g to estimate the value of $f(g(x))$ for $x = -5, -4, -3, \ldots, 5$. Use these estimates to sketch a rough graph of $f \circ g$.



55. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s.
(a) Express the radius r of this circle as a function of the time t (in seconds).
(b) If A is the area of this circle as a function of the radius, find $A \circ r$ and interpret it.

56. A spherical balloon is being inflated and the radius of the balloon is increasing at a rate of 2 cm/s.
(a) Express the radius r of the balloon as a function of the time t (in seconds).
(b) If V is the volume of the balloon as a function of the radius, find $V \circ r$ and interpret it.

57. A ship is moving at a speed of 30 km/h parallel to a straight shoreline. The ship is 6 km from shore and it passes a lighthouse at noon.
(a) Express the distance s between the lighthouse and the ship as a function of d, the distance the ship has traveled since noon; that is, find f so that $s = f(d)$.
(b) Express d as a function of t, the time elapsed since noon; that is, find g so that $d = g(t)$.
(c) Find $f \circ g$. What does this function represent?

58. An airplane is flying at a speed of 350 mi/h at an altitude of one mile and passes directly over a radar station at time $t = 0$.
(a) Express the horizontal distance d (in miles) that the plane has flown as a function of t.
(b) Express the distance s between the plane and the radar station as a function of d.
(c) Use composition to express s as a function of t.

59. The **Heaviside function** H is defined by

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geqslant 0 \end{cases}$$

It is used in the study of electric circuits to represent the sudden surge of electric current, or voltage, when a switch is instantaneously turned on.
(a) Sketch the graph of the Heaviside function.
(b) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 0$ and 120 volts are applied instantaneously to the circuit. Write a formula for $V(t)$ in terms of $H(t)$.
(c) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 5$ seconds and 240 volts are applied instantaneously to the circuit. Write a formula for $V(t)$ in terms of $H(t)$. (Note that starting at $t = 5$ corresponds to a translation.)

60. The Heaviside function defined in Exercise 59 can also be used to define the **ramp function** $y = ctH(t)$, which represents a gradual increase in voltage or current in a circuit.
(a) Sketch the graph of the ramp function $y = tH(t)$.
(b) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 0$ and the voltage is gradually increased to 120 volts over a 60-second time interval. Write a formula for $V(t)$ in terms of $H(t)$ for $t \leqslant 60$.
(c) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 7$ seconds and the voltage is gradually increased to 100 volts over a period of 25 seconds. Write a formula for $V(t)$ in terms of $H(t)$ for $t \leqslant 32$.

61. Let f and g be linear functions with equations $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$. Is $f \circ g$ also a linear function? If so, what is the slope of its graph?

62. If you invest x dollars at 4% interest compounded annually, then the amount $A(x)$ of the investment after one year is $A(x) = 1.04x$. Find $A \circ A$, $A \circ A \circ A$, and $A \circ A \circ A \circ A$. What do these compositions represent? Find a formula for the composition of n copies of A.

63. (a) If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$, find a function f such that $f \circ g = h$. (Think about what operations you would have to perform on the formula for g to end up with the formula for h.)
(b) If $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$, find a function g such that $f \circ g = h$.

64. If $f(x) = x + 4$ and $h(x) = 4x - 1$, find a function g such that $g \circ f = h$.

65. Suppose g is an even function and let $h = f \circ g$. Is h always an even function?

66. Suppose g is an odd function and let $h = f \circ g$. Is h always an odd function? What if f is odd? What if f is even?

1.4 The Tangent and Velocity Problems

In this section we see how limits arise when we attempt to find the tangent to a curve or the velocity of an object.

The Tangent Problem

The word *tangent* is derived from the Latin word *tangens*, which means "touching." Thus a tangent to a curve is a line that touches the curve. In other words, a tangent line should have the same direction as the curve at the point of contact. How can this idea be made precise?

For a circle we could simply follow Euclid and say that a tangent is a line that intersects the circle once and only once, as in Figure 1(a). For more complicated curves this definition is inadequate. Figure 1(b) shows two lines l and t passing through a point P on a curve C. The line l intersects C only once, but it certainly does not look like what we think of as a tangent. The line t, on the other hand, looks like a tangent but it intersects C twice.

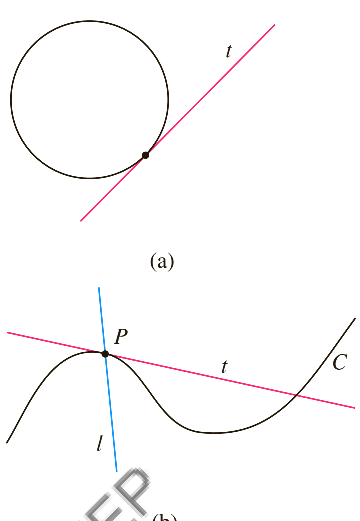


FIGURE 1

To be specific, let's look at the problem of trying to find a tangent line t to the parabola $y = x^2$ in the following example.

EXAMPLE 1 Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

SOLUTION We will be able to find an equation of the tangent line t as soon as we know its slope m. The difficulty is that we know only one point, P, on t, whereas we need two points to compute the slope. But observe that we can compute an approximation to m

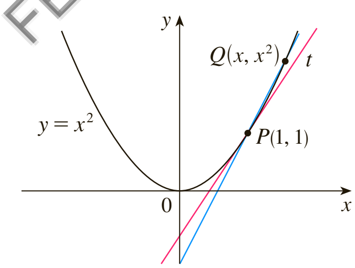


FIGURE 2

by choosing a nearby point $Q(x, x^2)$ on the parabola (as in Figure 2) and computing the slope m_{PQ} of the secant line PQ. [A **secant line**, from the Latin word *secans*, meaning cutting, is a line that cuts (intersects) a curve more than once.]

We choose $x \neq 1$ so that $Q \neq P$. Then

$$m_{PQ} = \frac{x^2 - 1}{x - 1}$$

For instance, for the point $Q(1.5, 2.25)$ we have

$$m_{PQ} = \frac{2.25 - 1}{1.5 - 1} = \frac{1.25}{0.5} = 2.5$$

x	m_{PQ}
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001

x	m_{PQ}
0	1
0.5	1.5
0.9	1.9
0.99	1.99
0.999	1.999

The tables in the margin show the values of m_{PQ} for several values of x close to 1. The closer Q is to P, the closer x is to 1 and, it appears from the tables, the closer m_{PQ} is to 2. This suggests that the slope of the tangent line t should be $m = 2$.

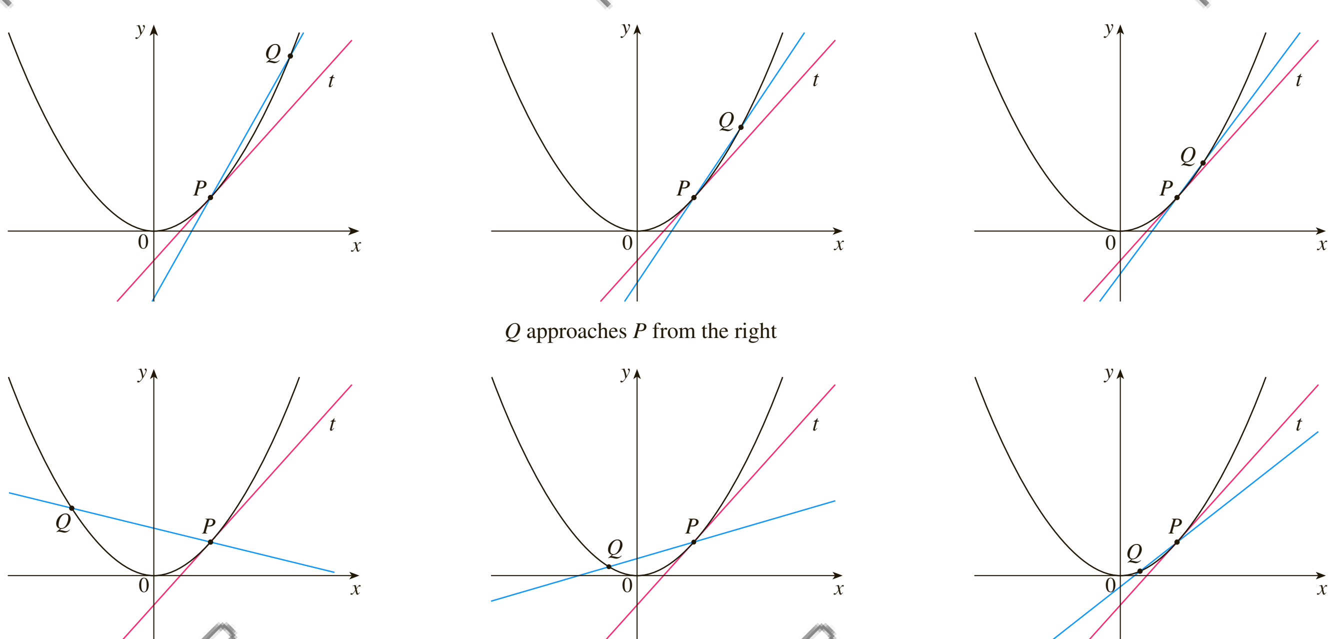
We say that the slope of the tangent line is the *limit* of the slopes of the secant lines, and we express this symbolically by writing

$$\lim_{Q \to P} m_{PQ} = m \qquad \text{and} \qquad \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$

Assuming that the slope of the tangent line is indeed 2, we use the point-slope form of the equation of a line [$y - y_1 = m(x - x_1)$, see Appendix B] to write the equation of the tangent line through $(1, 1)$ as

$$y - 1 = 2(x - 1) \qquad \text{or} \qquad y = 2x - 1$$

Figure 3 illustrates the limiting process that occurs in this example. As Q approaches P along the parabola, the corresponding secant lines rotate about P and approach the tangent line t.



Q approaches P from the right

Q approaches P from the left

FIGURE 3

TEC In Visual 1.4 you can see how the process in Figure 3 works for additional functions.

Many functions that occur in science are not described by explicit equations; they are defined by experimental data. The next example shows how to estimate the slope of the tangent line to the graph of such a function.

t	Q
0.00	100.00
0.02	81.87
0.04	67.03
0.06	54.88
0.08	44.93
0.10	36.76

EXAMPLE 2 The flash unit on a camera operates by storing charge on a capacitor and releasing it suddenly when the flash is set off. The data in the table describe the charge Q remaining on the capacitor (measured in microcoulombs) at time t (measured in seconds after the flash goes off). Use the data to draw the graph of this function and estimate the slope of the tangent line at the point where $t = 0.04$. [*Note:* The slope of the tangent line represents the electric current flowing from the capacitor to the flash bulb (measured in microamperes).]

SOLUTION In Figure 4 we plot the given data and use them to sketch a curve that approximates the graph of the function.

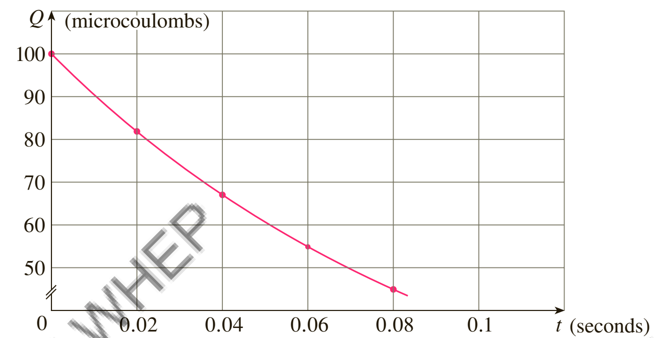


FIGURE 4

Given the points $P(0.04, 67.03)$ and $R(0.00, 100.00)$ on the graph, we find that the slope of the secant line PR is

$$m_{PR} = \frac{100.00 - 67.03}{0.00 - 0.04} = -824.25$$

R	m_{PR}
(0.00, 100.00)	-824.25
(0.02, 81.87)	-742.00
(0.06, 54.88)	-607.50
(0.08, 44.93)	-552.50
(0.10, 36.76)	-504.50

The table at the left shows the results of similar calculations for the slopes of other secant lines. From this table we would expect the slope of the tangent line at $t = 0.04$ to lie somewhere between -742 and -607.5. In fact, the average of the slopes of the two closest secant lines is

$$\tfrac{1}{2}(-742 - 607.5) = -674.75$$

So, by this method, we estimate the slope of the tangent line to be about -675.

Another method is to draw an approximation to the tangent line at P and measure the sides of the triangle ABC, as in Figure 5.

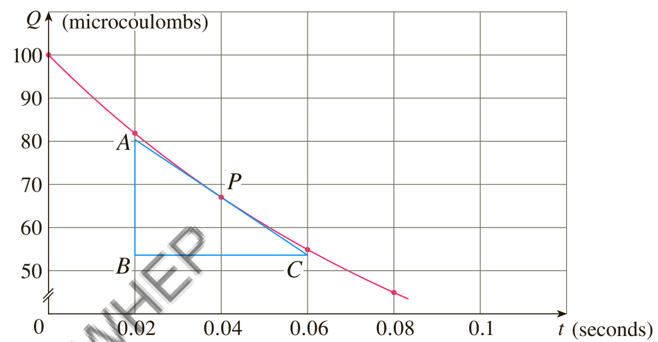


FIGURE 5

The physical meaning of the answer in Example 2 is that the electric current flowing from the capacitor to the flash bulb after 0.04 seconds is about −670 microamperes.

This gives an estimate of the slope of the tangent line as

$$-\frac{|AB|}{|BC|} \approx -\frac{80.4 - 53.6}{0.06 - 0.02} = -670$$ ■

■ The Velocity Problem

If you watch the speedometer of a car as you travel in city traffic, you see that the speed doesn't stay the same for very long; that is, the velocity of the car is not constant. We assume from watching the speedometer that the car has a definite velocity at each moment, but how is the "instantaneous" velocity defined? Let's investigate the example of a falling ball.

Steve Allen / Stockbyte / Getty Images

The CN Tower in Toronto was the tallest freestanding building in the world for 32 years.

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

SOLUTION Through experiments carried out four centuries ago, Galileo discovered that the distance fallen by any freely falling body is proportional to the square of the time it has been falling. (This model for free fall neglects air resistance.) If the distance fallen after t seconds is denoted by $s(t)$ and measured in meters, then Galileo's law is expressed by the equation

$$s(t) = 4.9t^2$$

The difficulty in finding the velocity after 5 seconds is that we are dealing with a single instant of time ($t = 5$), so no time interval is involved. However, we can approximate the desired quantity by computing the average velocity over the brief time interval of a tenth of a second from $t = 5$ to $t = 5.1$:

$$\begin{aligned}
\text{average velocity} &= \frac{\text{change in position}}{\text{time elapsed}} \\
&= \frac{s(5.1) - s(5)}{0.1} \\
&= \frac{4.9(5.1)^2 - 4.9(5)^2}{0.1} = 49.49 \text{ m/s}
\end{aligned}$$

The following table shows the results of similar calculations of the average velocity over successively smaller time periods.

Time interval	Average velocity (m/s)
$5 \leqslant t \leqslant 6$	53.9
$5 \leqslant t \leqslant 5.1$	49.49
$5 \leqslant t \leqslant 5.05$	49.245
$5 \leqslant t \leqslant 5.01$	49.049
$5 \leqslant t \leqslant 5.001$	49.0049

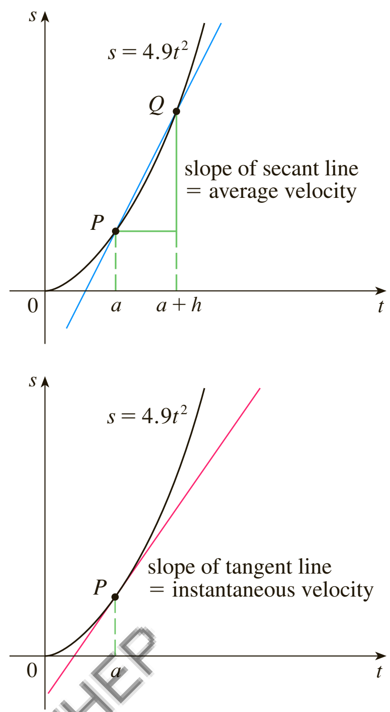


FIGURE 6

It appears that as we shorten the time period, the average velocity is becoming closer to 49 m/s. The **instantaneous velocity** when $t = 5$ is defined to be the limiting value of these average velocities over shorter and shorter time periods that start at $t = 5$. Thus it appears that the (instantaneous) velocity after 5 seconds is

$$v = 49 \text{ m/s}$$

■

You may have the feeling that the calculations used in solving this problem are very similar to those used earlier in this section to find tangents. In fact, there is a close connection between the tangent problem and the problem of finding velocities. If we draw the graph of the distance function of the ball (as in Figure 6) and we consider the points $P(a, 4.9a^2)$ and $Q(a + h, 4.9(a + h)^2)$ on the graph, then the slope of the secant line PQ is

$$m_{PQ} = \frac{4.9(a + h)^2 - 4.9a^2}{(a + h) - a}$$

which is the same as the average velocity over the time interval $[a, a + h]$. Therefore the velocity at time $t = a$ (the limit of these average velocities as h approaches 0) must be equal to the slope of the tangent line at P (the limit of the slopes of the secant lines).

Examples 1 and 3 show that in order to solve tangent and velocity problems we must be able to find limits. After studying methods for computing limits in the next four sections, we will return to the problems of finding tangents and velocities in Chapter 2.

1.4 EXERCISES

1. A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

t (min)	5	10	15	20	25	30
V (gal)	694	444	250	111	28	0

(a) If P is the point (15, 250) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5$, 10, 20, 25, and 30.
(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.
(c) Use a graph of the function to estimate the slope of the tangent line at P. (This slope represents the rate at which the water is flowing from the tank after 15 minutes.)

2. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute.

t (min)	36	38	40	42	44
Heartbeats	2530	2661	2806	2948	3080

The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of t.
(a) $t = 36$ and $t = 42$ (b) $t = 38$ and $t = 42$
(c) $t = 40$ and $t = 42$ (d) $t = 42$ and $t = 44$
What are your conclusions?

3. The point $P(2, -1)$ lies on the curve $y = 1/(1 - x)$.
(a) If Q is the point $(x, 1/(1 - x))$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:
(i) 1.5 (ii) 1.9 (iii) 1.99 (iv) 1.999
(v) 2.5 (vi) 2.1 (vii) 2.01 (viii) 2.001

(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(2, -1)$.

(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(2, -1)$.

4. The point $P(0.5, 0)$ lies on the curve $y = \cos \pi x$.

(a) If Q is the point $(x, \cos \pi x)$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:

(i) 0 (ii) 0.4 (iii) 0.49
(iv) 0.499 (v) 1 (vi) 0.6
(vii) 0.51 (viii) 0.501

(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.

(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 0)$.

(d) Sketch the curve, two of the secant lines, and the tangent line.

5. If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $y = 40t - 16t^2$.

(a) Find the average velocity for the time period beginning when $t = 2$ and lasting

(i) 0.5 seconds (ii) 0.1 seconds
(iii) 0.05 seconds (iv) 0.01 seconds

(b) Estimate the instantaneous velocity when $t = 2$.

6. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$.

(a) Find the average velocity over the given time intervals:

(i) [1, 2] (ii) [1, 1.5] (iii) [1, 1.1]
(iv) [1, 1.01] (v) [1, 1.001]

(b) Estimate the instantaneous velocity when $t = 1$.

7. The table shows the position of a motorcyclist after accelerating from rest.

t (seconds)	0	1	2	3	4	5	6
s (feet)	0	4.9	20.6	46.5	79.2	124.8	176.7

(a) Find the average velocity for each time period:

(i) [2, 4] (ii) [3, 4]
(iii) [4, 5] (iv) [4, 6]

(b) Use the graph of s as a function of t to estimate the instantaneous velocity when $t = 3$.

8. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion $s = 2 \sin \pi t + 3 \cos \pi t$, where t is measured in seconds.

(a) Find the average velocity during each time period:

(i) [1, 2] (ii) [1, 1.1]
(iii) [1, 1.01] (iv) [1, 1.001]

(b) Estimate the instantaneous velocity of the particle when $t = 1$.

9. The point $P(1, 0)$ lies on the curve $y = \sin(10\pi/x)$.

(a) If Q is the point $(x, \sin(10\pi/x))$, find the slope of the secant line PQ (correct to four decimal places) for $x = 2, 1.5, 1.4, 1.3, 1.2, 1.1, 0.5, 0.6, 0.7, 0.8$, and 0.9. Do the slopes appear to be approaching a limit?

(b) Use a graph of the curve to explain why the slopes of the secant lines in part (a) are not close to the slope of the tangent line at P.

(c) By choosing appropriate secant lines, estimate the slope of the tangent line at P.

1.5 The Limit of a Function

Having seen in the preceding section how limits arise when we want to find the tangent to a curve or the velocity of an object, we now turn our attention to limits in general and numerical and graphical methods for computing them.

Let's investigate the behavior of the function f defined by $f(x) = x^2 - x + 2$ for values of x near 2. The following table gives values of $f(x)$ for values of x close to 2 but not equal to 2.

x	$f(x)$	x	$f(x)$
1.0	2.000000	3.0	8.000000
1.5	2.750000	2.5	5.750000
1.8	3.440000	2.2	4.640000
1.9	3.710000	2.1	4.310000
1.95	3.852500	2.05	4.152500
1.99	3.970100	2.01	4.030100
1.995	3.985025	2.005	4.015025
1.999	3.997001	2.001	4.003001

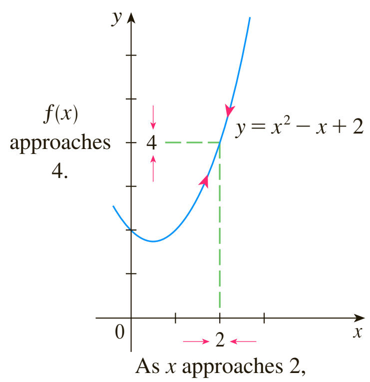


FIGURE 1

From the table and the graph of f (a parabola) shown in Figure 1 we see that the closer x is to 2 (on either side of 2), the closer $f(x)$ is to 4. In fact, it appears that we can make the values of $f(x)$ as close as we like to 4 by taking x sufficiently close to 2. We express this by saying "the limit of the function $f(x) = x^2 - x + 2$ as x approaches 2 is equal to 4." The notation for this is

$$\lim_{x \to 2} (x^2 - x + 2) = 4$$

In general, we use the following notation.

1 Intuitive Definition of a Limit Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.

Roughly speaking, this says that the values of $f(x)$ approach L as x approaches a. In other words, the values of $f(x)$ tend to get closer and closer to the number L as x gets closer and closer to the number a (from either side of a) but $x \neq a$. (A more precise definition will be given in Section 1.7.)

An alternative notation for

$$\lim_{x \to a} f(x) = L$$

is

$$f(x) \to L \qquad \text{as} \qquad x \to a$$

which is usually read "$f(x)$ approaches L as x approaches a."

Notice the phrase "but $x \neq a$" in the definition of limit. This means that in finding the limit of $f(x)$ as x approaches a, we never consider $x = a$. In fact, $f(x)$ need not even be defined when $x = a$. The only thing that matters is how f is defined *near* a.

Figure 2 shows the graphs of three functions. Note that in part (c), $f(a)$ is not defined and in part (b), $f(a) \neq L$. But in each case, regardless of what happens at a, it is true that $\lim_{x \to a} f(x) = L$.

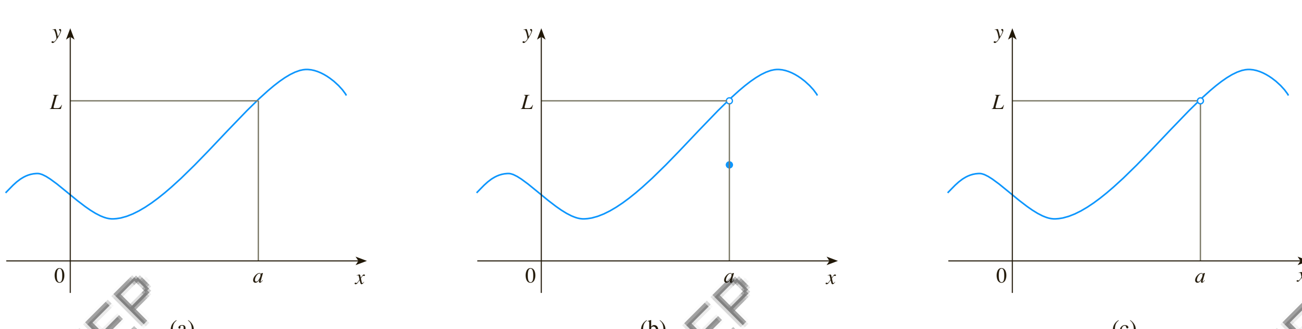


FIGURE 2 $\lim_{x \to a} f(x) = L$ in all three cases

$x < 1$	$f(x)$
0.5	0.666667
0.9	0.526316
0.99	0.502513
0.999	0.500250
0.9999	0.500025

$x > 1$	$f(x)$
1.5	0.400000
1.1	0.476190
1.01	0.497512
1.001	0.499750
1.0001	0.499975

EXAMPLE 1 Guess the value of $\displaystyle\lim_{x\to 1} \frac{x-1}{x^2-1}$.

SOLUTION Notice that the function $f(x) = (x-1)/(x^2-1)$ is not defined when $x = 1$, but that doesn't matter because the definition of $\lim_{x\to a} f(x)$ says that we consider values of x that are close to a but not equal to a.

The tables at the left give values of $f(x)$ (correct to six decimal places) for values of x that approach 1 (but are not equal to 1). On the basis of the values in the tables, we make the guess that

$$\lim_{x\to 1} \frac{x-1}{x^2-1} = 0.5$$ ■

Example 1 is illustrated by the graph of f in Figure 3. Now let's change f slightly by giving it the value 2 when $x = 1$ and calling the resulting function g:

$$g(x) = \begin{cases} \dfrac{x-1}{x^2-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

This new function g still has the same limit as x approaches 1. (See Figure 4.)

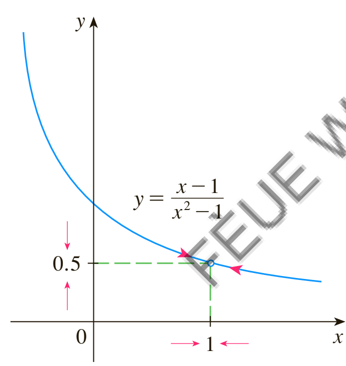


FIGURE 3

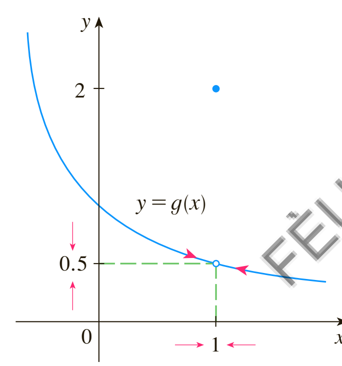


FIGURE 4

EXAMPLE 2 Estimate the value of $\displaystyle\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.

SOLUTION The table lists values of the function for several values of t near 0.

t	$\dfrac{\sqrt{t^2+9}-3}{t^2}$
± 1.0	0.162277 . . .
± 0.5	0.165525 . . .
± 0.1	0.166620 . . .
± 0.05	0.166655 . . .
± 0.01	0.166666 . . .

As t approaches 0, the values of the function seem to approach 0.1666666 . . . and so we guess that

$$\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} = \frac{1}{6}$$ ■

t	$\dfrac{\sqrt{t^2+9}-3}{t^2}$
± 0.001	0.166667
± 0.0001	0.166670
± 0.00001	0.167000
± 0.000001	0.000000

www.stewartcalculus.com
For a further explanation of why calculators sometimes give false values, click on *Lies My Calculator and Computer Told Me.* In particular, see the section called *The Perils of Subtraction.*

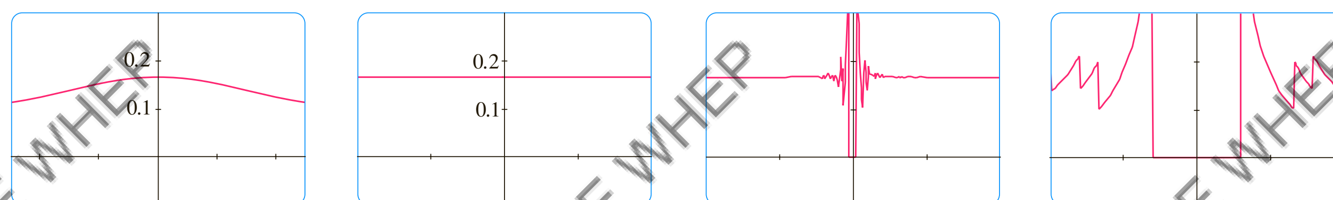
In Example 2 what would have happened if we had taken even smaller values of t? The table in the margin shows the results from one calculator; you can see that something strange seems to be happening.

If you try these calculations on your own calculator you might get different values, but eventually you will get the value 0 if you make t sufficiently small. Does this mean that the answer is really 0 instead of $\frac{1}{6}$? No, the value of the limit is $\frac{1}{6}$, as we will show in the next section. The problem is that the calculator gave false values because $\sqrt{t^2+9}$ is very close to 3 when t is small. (In fact, when t is sufficiently small, a calculator's value for $\sqrt{t^2+9}$ is 3.000. . . to as many digits as the calculator is capable of carrying.)

Something similar happens when we try to graph the function

$$f(t) = \frac{\sqrt{t^2+9}-3}{t^2}$$

of Example 2 on a graphing calculator or computer. Parts (a) and (b) of Figure 5 show quite accurate graphs of f, and when we use the trace mode (if available) we can estimate easily that the limit is about $\frac{1}{6}$. But if we zoom in too much, as in parts (c) and (d), then we get inaccurate graphs, again because of rounding errors from the subtraction.



(a) $-5 \leqslant t \leqslant 5$ (b) $-0.1 \leqslant t \leqslant 0.1$ (c) $-10^{-6} \leqslant t \leqslant 10^{-6}$ (d) $-10^{-7} \leqslant t \leqslant 10^{-7}$

FIGURE 5

EXAMPLE 3 Guess the value of $\displaystyle\lim_{x \to 0} \frac{\sin x}{x}$.

SOLUTION The function $f(x) = (\sin x)/x$ is not defined when $x = 0$. Using a calculator (and remembering that, if $x \in \mathbb{R}$, $\sin x$ means the sine of the angle whose *radian* measure is x), we construct a table of values correct to eight decimal places. From the table at the left and the graph in Figure 6 we guess that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

x	$\dfrac{\sin x}{x}$
± 1.0	0.84147098
± 0.5	0.95885108
± 0.4	0.97354586
± 0.3	0.98506736
± 0.2	0.99334665
± 0.1	0.99833417
± 0.05	0.99958339
± 0.01	0.99998333
± 0.005	0.99999583
± 0.001	0.99999983

This guess is in fact correct, as will be proved in Chapter 2 using a geometric argument.

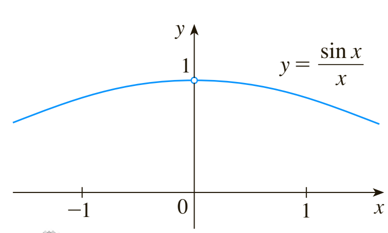


FIGURE 6

EXAMPLE 4 Investigate $\displaystyle\lim_{x \to 0} \sin\frac{\pi}{x}$.

Computer Algebra Systems
Computer algebra systems (CAS) have commands that compute limits. In order to avoid the types of pitfalls demonstrated in Examples 2, 4, and 5, they don't find limits by numerical experimentation. Instead, they use more sophisticated techniques such as computing infinite series. If you have access to a CAS, use the limit command to compute the limits in the examples of this section and to check your answers in the exercises of this chapter.

SOLUTION Again the function $f(x) = \sin(\pi/x)$ is undefined at 0. Evaluating the function for some small values of x, we get

$$f(1) = \sin \pi = 0 \qquad f\left(\tfrac{1}{2}\right) = \sin 2\pi = 0$$

$$f\left(\tfrac{1}{3}\right) = \sin 3\pi = 0 \qquad f\left(\tfrac{1}{4}\right) = \sin 4\pi = 0$$

$$f(0.1) = \sin 10\pi = 0 \qquad f(0.01) = \sin 100\pi = 0$$

Similarly, $f(0.001) = f(0.0001) = 0$. On the basis of this information we might be tempted to guess that

$$\lim_{x \to 0} \sin \frac{\pi}{x} = 0$$

but this time our guess is wrong. Note that although $f(1/n) = \sin n\pi = 0$ for any integer n, it is also true that $f(x) = 1$ for infinitely many values of x (such as $2/5$ or $2/101$) that approach 0. You can see this from the graph of f shown in Figure 7.

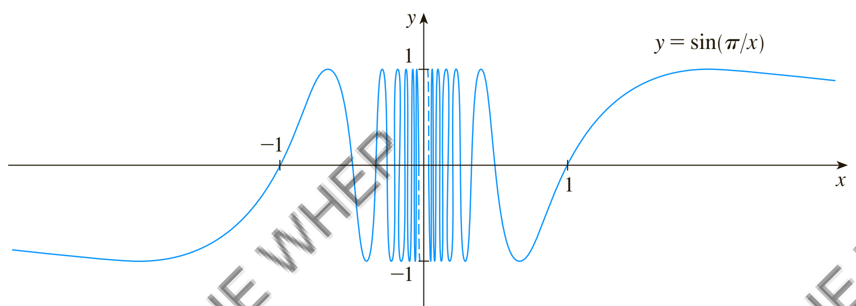


FIGURE 7

The dashed lines near the y-axis indicate that the values of $\sin(\pi/x)$ oscillate between 1 and -1 infinitely often as x approaches 0. (See Exercise 45.)

Since the values of $f(x)$ do not approach a fixed number as x approaches 0,

$$\lim_{x \to 0} \sin \frac{\pi}{x} \quad \text{does not exist}$$

■

x	$x^3 + \dfrac{\cos 5x}{10{,}000}$
1	1.000028
0.5	0.124920
0.1	0.001088
0.05	0.000222
0.01	0.000101

x	$x^3 + \dfrac{\cos 5x}{10{,}000}$
0.005	0.00010009
0.001	0.00010000

EXAMPLE 5 Find $\displaystyle\lim_{x \to 0} \left(x^3 + \frac{\cos 5x}{10{,}000} \right)$.

SOLUTION As before, we construct a table of values. From the first table in the margin it appears that

$$\lim_{x \to 0} \left(x^3 + \frac{\cos 5x}{10{,}000} \right) = 0$$

But if we persevere with smaller values of x, the second table suggests that

$$\lim_{x \to 0} \left(x^3 + \frac{\cos 5x}{10{,}000} \right) = 0.000100 = \frac{1}{10{,}000}$$

In Section 1.8 we will see that $\lim_{x \to 0} \cos 5x = 1$; then it follows that the limit is 0.0001. ■

Examples 4 and 5 illustrate some of the pitfalls in guessing the value of a limit. It is easy to guess the wrong value if we use inappropriate values of x, but it is difficult to know when to stop calculating values. And, as the discussion after Example 2 shows, sometimes calculators and computers give the wrong values. In the next section, however, we will develop foolproof methods for calculating limits.

One-Sided Limits

EXAMPLE 6 The Heaviside function H is defined by

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geqslant 0 \end{cases}$$

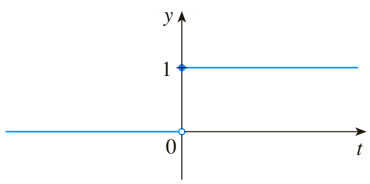


FIGURE 8
The Heaviside function

[This function is named after the electrical engineer Oliver Heaviside (1850–1925) and can be used to describe an electric current that is switched on at time $t = 0$.] Its graph is shown in Figure 8.

As t approaches 0 from the left, $H(t)$ approaches 0. As t approaches 0 from the right, $H(t)$ approaches 1. There is no single number that $H(t)$ approaches as t approaches 0. Therefore $\lim_{t \to 0} H(t)$ does not exist. ■

We noticed in Example 6 that $H(t)$ approaches 0 as t approaches 0 from the left and $H(t)$ approaches 1 as t approaches 0 from the right. We indicate this situation symbolically by writing

$$\lim_{t \to 0^-} H(t) = 0 \qquad \text{and} \qquad \lim_{t \to 0^+} H(t) = 1$$

The notation $t \to 0^-$ indicates that we consider only values of t that are less than 0. Likewise, $t \to 0^+$ indicates that we consider only values of t that are greater than 0.

2 Definition of One-Sided Limits We write

$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x *less than* a.

Notice that Definition 2 differs from Definition 1 only in that we require x to be less than a. Similarly, if we require that x be greater than a, we get "the **right-hand limit of $f(x)$ as x approaches a** is equal to L" and we write

$$\lim_{x \to a^+} f(x) = L$$

Thus the notation $x \to a^+$ means that we consider only x *greater than* a. These definitions are illustrated in Figure 9.

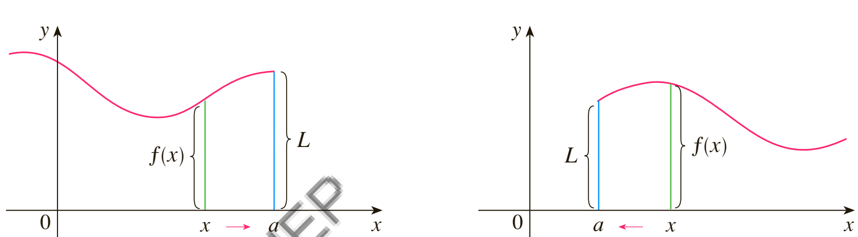


FIGURE 9
(a) $\lim_{x \to a^-} f(x) = L$
(b) $\lim_{x \to a^+} f(x) = L$

By comparing Definition 1 with the definitions of one-sided limits, we see that the following is true.

> **3** $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

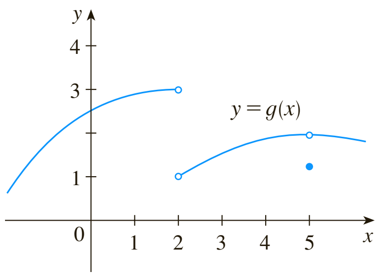


FIGURE 10

EXAMPLE 7 The graph of a function g is shown in Figure 10. Use it to state the values (if they exist) of the following:

(a) $\lim_{x \to 2^-} g(x)$ (b) $\lim_{x \to 2^+} g(x)$ (c) $\lim_{x \to 2} g(x)$

(d) $\lim_{x \to 5^-} g(x)$ (e) $\lim_{x \to 5^+} g(x)$ (f) $\lim_{x \to 5} g(x)$

SOLUTION From the graph we see that the values of $g(x)$ approach 3 as x approaches 2 from the left, but they approach 1 as x approaches 2 from the right. Therefore

$$\text{(a)} \lim_{x \to 2^-} g(x) = 3 \quad \text{and} \quad \text{(b)} \lim_{x \to 2^+} g(x) = 1$$

(c) Since the left and right limits are different, we conclude from (3) that $\lim_{x \to 2} g(x)$ does not exist.

The graph also shows that

$$\text{(d)} \lim_{x \to 5^-} g(x) = 2 \quad \text{and} \quad \text{(e)} \lim_{x \to 5^+} g(x) = 2$$

(f) This time the left and right limits are the same and so, by (3), we have

$$\lim_{x \to 5} g(x) = 2$$

Despite this fact, notice that $g(5) \neq 2$. ■

x	$\dfrac{1}{x^2}$
± 1	1
± 0.5	4
± 0.2	25
± 0.1	100
± 0.05	400
± 0.01	10,000
± 0.001	1,000,000

Infinite Limits

EXAMPLE 8 Find $\lim_{x \to 0} \dfrac{1}{x^2}$ if it exists.

SOLUTION As x becomes close to 0, x^2 also becomes close to 0, and $1/x^2$ becomes very large. (See the table in the margin.) In fact, it appears from the graph of the function $f(x) = 1/x^2$ shown in Figure 11 that the values of $f(x)$ can be made arbitrarily large by taking x close enough to 0. Thus the values of $f(x)$ do not approach a number, so $\lim_{x \to 0} (1/x^2)$ does not exist. ■

To indicate the kind of behavior exhibited in Example 8, we use the notation

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$

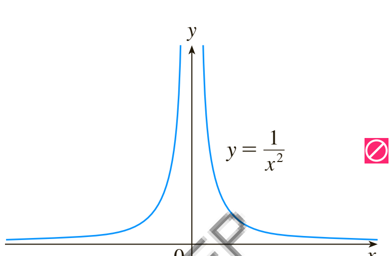


FIGURE 11

This does not mean that we are regarding ∞ as a number. Nor does it mean that the limit exists. It simply expresses the particular way in which the limit does not exist: $1/x^2$ can be made as large as we like by taking x close enough to 0.

In general, we write symbolically

$$\lim_{x \to a} f(x) = \infty$$

to indicate that the values of $f(x)$ tend to become larger and larger (or "increase without bound") as x becomes closer and closer to a.

4 Intuitive Definition of an Infinite Limit Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.

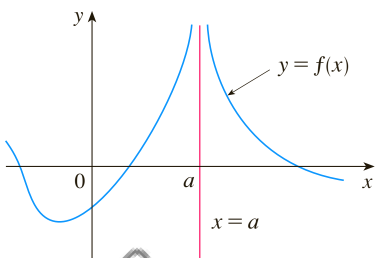


FIGURE 12
$\lim_{x \to a} f(x) = \infty$

Another notation for $\lim_{x \to a} f(x) = \infty$ is

$$f(x) \to \infty \qquad \text{as} \qquad x \to a$$

Again, the symbol ∞ is not a number, but the expression $\lim_{x \to a} f(x) = \infty$ is often read as

"the limit of $f(x)$, as x approaches a, is infinity"

or "$f(x)$ becomes infinite as x approaches a"

or "$f(x)$ increases without bound as x approaches a"

This definition is illustrated graphically in Figure 12.

When we say a number is "large negative," we mean that it is negative but its magnitude (absolute value) is large.

A similar sort of limit, for functions that become large negative as x gets close to a, is defined in Definition 5 and is illustrated in Figure 13.

5 Definition Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to a, but not equal to a.

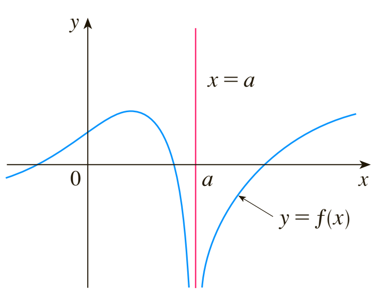


FIGURE 13
$\lim_{x \to a} f(x) = -\infty$

The symbol $\lim_{x \to a} f(x) = -\infty$ can be read as "the limit of $f(x)$, as x approaches a, is negative infinity" or "$f(x)$ decreases without bound as x approaches a." As an example we have

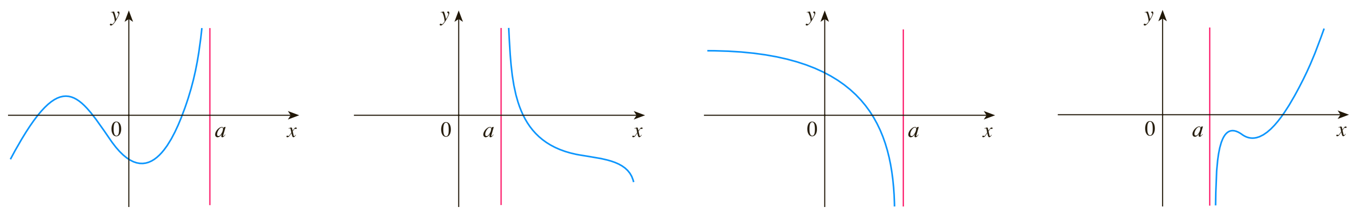
$$\lim_{x \to 0} \left(-\frac{1}{x^2} \right) = -\infty$$

Similar definitions can be given for the one-sided infinite limits

$$\lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

remembering that $x \to a^-$ means that we consider only values of x that are less than a, and similarly $x \to a^+$ means that we consider only $x > a$. Illustrations of these four cases are given in Figure 14.



(a) $\lim_{x \to a^-} f(x) = \infty$ (b) $\lim_{x \to a^+} f(x) = \infty$ (c) $\lim_{x \to a^-} f(x) = -\infty$ (d) $\lim_{x \to a^+} f(x) = -\infty$

FIGURE 14

6 Definition The vertical line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

For instance, the y-axis is a vertical asymptote of the curve $y = 1/x^2$ because $\lim_{x \to 0} (1/x^2) = \infty$. In Figure 14 the line $x = a$ is a vertical asymptote in each of the four cases shown. In general, knowledge of vertical asymptotes is very useful in sketching graphs.

EXAMPLE 9 Find $\displaystyle\lim_{x \to 3^+} \frac{2x}{x - 3}$ and $\displaystyle\lim_{x \to 3^-} \frac{2x}{x - 3}$.

SOLUTION If x is close to 3 but larger than 3, then the denominator $x - 3$ is a small positive number and $2x$ is close to 6. So the quotient $2x/(x - 3)$ is a large *positive* number. [For instance, if $x = 3.01$ then $2x/(x - 3) = 6.02/0.01 = 602$.] Thus, intuitively, we see that

$$\lim_{x \to 3^+} \frac{2x}{x - 3} = \infty$$

Likewise, if x is close to 3 but smaller than 3, then $x - 3$ is a small negative number but $2x$ is still a positive number (close to 6). So $2x/(x - 3)$ is a numerically large *negative* number. Thus

$$\lim_{x \to 3^-} \frac{2x}{x - 3} = -\infty$$

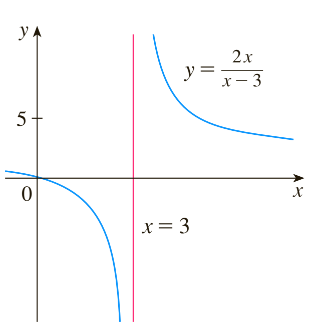


FIGURE 15

The graph of the curve $y = 2x/(x - 3)$ is given in Figure 15. The line $x = 3$ is a vertical asymptote. ■

EXAMPLE 10 Find the vertical asymptotes of $f(x) = \tan x$.

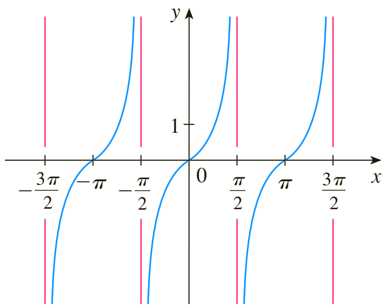


FIGURE 16
$y = \tan x$

SOLUTION Because

$$\tan x = \frac{\sin x}{\cos x}$$

there are potential vertical asymptotes where $\cos x = 0$. In fact, since $\cos x \to 0^+$ as $x \to (\pi/2)^-$ and $\cos x \to 0^-$ as $x \to (\pi/2)^+$, whereas $\sin x$ is positive (near 1) when x is near $\pi/2$, we have

$$\lim_{x \to (\pi/2)^-} \tan x = \infty \qquad \text{and} \qquad \lim_{x \to (\pi/2)^+} \tan x = -\infty$$

This shows that the line $x = \pi/2$ is a vertical asymptote. Similar reasoning shows that the lines $x = \pi/2 + n\pi$, where n is an integer, are all vertical asymptotes of $f(x) = \tan x$. The graph in Figure 16 confirms this. ■

1.5 EXERCISES

1. Explain in your own words what is meant by the equation

$$\lim_{x \to 2} f(x) = 5$$

Is it possible for this statement to be true and yet $f(2) = 3$? Explain.

2. Explain what it means to say that

$$\lim_{x \to 1^-} f(x) = 3 \qquad \text{and} \qquad \lim_{x \to 1^+} f(x) = 7$$

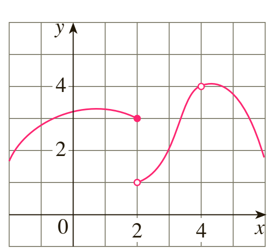
In this situation is it possible that $\lim_{x \to 1} f(x)$ exists? Explain.

3. Explain the meaning of each of the following.

(a) $\lim_{x \to -3} f(x) = \infty$ (b) $\lim_{x \to 4^+} f(x) = -\infty$

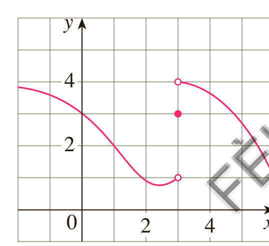
4. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 2^-} f(x)$ (b) $\lim_{x \to 2^+} f(x)$ (c) $\lim_{x \to 2} f(x)$
(d) $f(2)$ (e) $\lim_{x \to 4} f(x)$ (f) $f(4)$



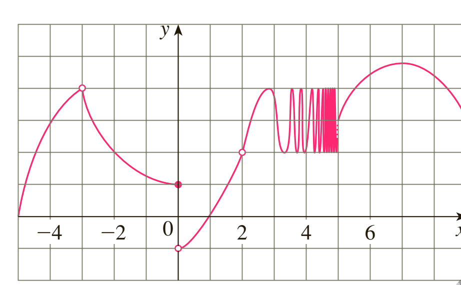
5. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 1} f(x)$ (b) $\lim_{x \to 3^-} f(x)$ (c) $\lim_{x \to 3^+} f(x)$
(d) $\lim_{x \to 3} f(x)$ (e) $f(3)$



6. For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to -3^-} h(x)$ (b) $\lim_{x \to -3^+} h(x)$ (c) $\lim_{x \to -3} h(x)$
(d) $h(-3)$ (e) $\lim_{x \to 0^-} h(x)$ (f) $\lim_{x \to 0^+} h(x)$
(g) $\lim_{x \to 0} h(x)$ (h) $h(0)$ (i) $\lim_{x \to 2} h(x)$
(j) $h(2)$ (k) $\lim_{x \to 5^+} h(x)$ (l) $\lim_{x \to 5^-} h(x)$

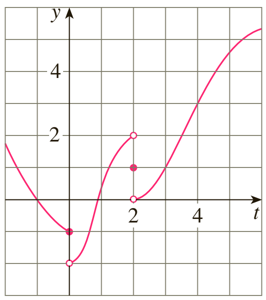


7. For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{t\to 0^-} g(t)$ (b) $\lim_{t\to 0^+} g(t)$ (c) $\lim_{t\to 0} g(t)$

(d) $\lim_{t\to 2^-} g(t)$ (e) $\lim_{t\to 2^+} g(t)$ (f) $\lim_{t\to 2} g(t)$

(g) $g(2)$ (h) $\lim_{t\to 4} g(t)$

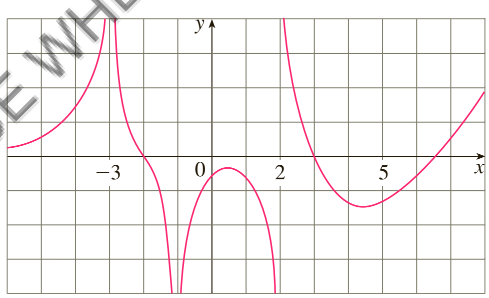


8. For the function A whose graph is shown, state the following.

(a) $\lim_{x\to -3} A(x)$ (b) $\lim_{x\to 2^-} A(x)$

(c) $\lim_{x\to 2^+} A(x)$ (d) $\lim_{x\to -1} A(x)$

(e) The equations of the vertical asymptotes

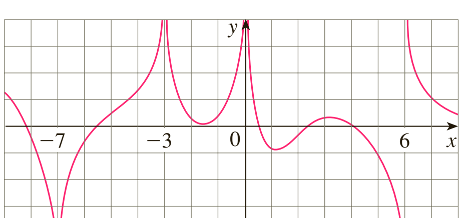


9. For the function f whose graph is shown, state the following.

(a) $\lim_{x\to -7} f(x)$ (b) $\lim_{x\to -3} f(x)$ (c) $\lim_{x\to 0} f(x)$

(d) $\lim_{x\to 6^-} f(x)$ (e) $\lim_{x\to 6^+} f(x)$

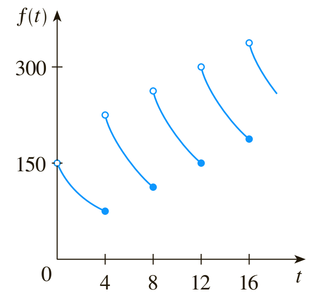
(f) The equations of the vertical asymptotes.



10. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t\to 12^-} f(t) \qquad \text{and} \qquad \lim_{t\to 12^+} f(t)$$

and explain the significance of these one-sided limits.



11–12 Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

11. $f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leqslant x < 1 \\ 2 - x & \text{if } x \geqslant 1 \end{cases}$

12. $f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leqslant x \leqslant \pi \\ \sin x & \text{if } x > \pi \end{cases}$

13–14 Use the graph of the function f to state the value of each limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$ (c) $\lim_{x\to 0} f(x)$

13. $f(x) = \dfrac{1}{1 + 2^{1/x}}$ **14.** $f(x) = \dfrac{x^2 + x}{\sqrt{x^3 + x^2}}$

15–18 Sketch the graph of an example of a function f that satisfies all of the given conditions.

15. $\lim_{x\to 0^-} f(x) = -1$, $\lim_{x\to 0^+} f(x) = 2$, $f(0) = 1$

16. $\lim_{x\to 0} f(x) = 1$, $\lim_{x\to 3^-} f(x) = -2$, $\lim_{x\to 3^+} f(x) = 2$, $f(0) = -1$, $f(3) = 1$

17. $\lim_{x\to 3^+} f(x) = 4$, $\lim_{x\to 3^-} f(x) = 2$, $\lim_{x\to -2} f(x) = 2$, $f(3) = 3$, $f(-2) = 1$

18. $\lim_{x\to 0^-} f(x) = 2$, $\lim_{x\to 0^+} f(x) = 0$, $\lim_{x\to 4^-} f(x) = 3$, $\lim_{x\to 4^+} f(x) = 0$, $f(0) = 2$, $f(4) = 1$

19–22 Guess the value of the limit (if it exists) by evaluating the function at the given numbers (correct to six decimal places).

19. $\lim_{x\to 3} \dfrac{x^2 - 3x}{x^2 - 9}$,

$x = 3.1, 3.05, 3.01, 3.001, 3.0001,$

$2.9, 2.95, 2.99, 2.999, 2.9999$

20. $\lim_{x \to -3} \dfrac{x^2 - 3x}{x^2 - 9}$,
$x = -2.5, -2.9, -2.95, -2.99, -2.999, -2.9999,$
$-3.5, -3.1, -3.05, -3.01, -3.001, -3.0001$

21. $\lim_{x \to 0} \dfrac{\sin x}{x + \tan x}$, $x = \pm 1, \pm 0.5, \pm 0.2, \pm 0.1, \pm 0.05, \pm 0.01$

22. $\lim_{h \to 0} \dfrac{(2 + h)^5 - 32}{h}$,
$h = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$

23–26 Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically.

23. $\lim_{\theta \to 0} \dfrac{\sin 3\theta}{\tan 2\theta}$

24. $\lim_{p \to -1} \dfrac{1 + p^9}{1 + p^{15}}$

25. $\lim_{x \to 0^+} x^x$

26. $\lim_{t \to 0} \dfrac{5^t - 1}{t}$

27. (a) By graphing the function $f(x) = (\cos 2x - \cos x)/x^2$ and zooming in toward the point where the graph crosses the y-axis, estimate the value of $\lim_{x \to 0} f(x)$.
(b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approach 0.

28. (a) Estimate the value of

$$\lim_{x \to 0} \frac{\sin x}{\sin \pi x}$$

by graphing the function $f(x) = (\sin x)/(\sin \pi x)$. State your answer correct to two decimal places.
(b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approach 0.

29–39 Determine the infinite limit.

29. $\lim_{x \to 5^+} \dfrac{x + 1}{x - 5}$

30. $\lim_{x \to 5^-} \dfrac{x + 1}{x - 5}$

31. $\lim_{x \to 1} \dfrac{2 - x}{(x - 1)^2}$

32. $\lim_{x \to 3^-} \dfrac{\sqrt{x}}{(x - 3)^5}$

33. $\lim_{x \to -2^+} \dfrac{x - 1}{x^2(x + 2)}$

34. $\lim_{x \to 0} \dfrac{x - 1}{x^2(x + 2)}$

35. $\lim_{x \to (\pi/2)^+} \dfrac{1}{x} \sec x$

36. $\lim_{x \to \pi^-} \cot x$

37. $\lim_{x \to 2\pi^-} x \csc x$

38. $\lim_{x \to 2^-} \dfrac{x^2 - 2x}{x^2 - 4x + 4}$

39. $\lim_{x \to 2^+} \dfrac{x^2 - 2x - 8}{x^2 - 5x + 6}$

40. (a) Find the vertical asymptotes of the function

$$y = \frac{x^2 + 1}{3x - 2x^2}$$

(b) Confirm your answer to part (a) by graphing the function.

41. Determine $\lim_{x \to 1^-} \dfrac{1}{x^3 - 1}$ and $\lim_{x \to 1^+} \dfrac{1}{x^3 - 1}$
(a) by evaluating $f(x) = 1/(x^3 - 1)$ for values of x that approach 1 from the left and from the right,
(b) by reasoning as in Example 9, and
(c) from a graph of f.

42. (a) By graphing the function $f(x) = (\tan 4x)/x$ and zooming in toward the point where the graph crosses the y-axis, estimate the value of $\lim_{x \to 0} f(x)$.
(b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approach 0.

43. (a) Evaluate the function $f(x) = x^2 - (2^x/1000)$ for $x = 1$, 0.8, 0.6, 0.4, 0.2, 0.1, and 0.05, and guess the value of

$$\lim_{x \to 0} \left(x^2 - \frac{2^x}{1000} \right)$$

(b) Evaluate $f(x)$ for $x = 0.04, 0.02, 0.01, 0.005, 0.003$, and 0.001. Guess again.

44. (a) Evaluate $h(x) = (\tan x - x)/x^3$ for $x = 1, 0.5, 0.1$, 0.05, 0.01, and 0.005.
(b) Guess the value of $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$.
(c) Evaluate $h(x)$ for successively smaller values of x until you finally reach a value of 0 for $h(x)$. Are you still confident that your guess in part (b) is correct? Explain why you eventually obtained values of 0 for $h(x)$. (In Section 6.8 a method for evaluating this limit will be explained.)
(d) Graph the function h in the viewing rectangle $[-1, 1]$ by $[0, 1]$. Then zoom in toward the point where the graph crosses the y-axis to estimate the limit of $h(x)$ as x approaches 0. Continue to zoom in until you observe distortions in the graph of h. Compare with the results of part (c).

45. Graph the function $f(x) = \sin(\pi/x)$ of Example 4 in the viewing rectangle $[-1, 1]$ by $[-1, 1]$. Then zoom in toward the origin several times. Comment on the behavior of this function.

46. Consider the function $f(x) = \tan \dfrac{1}{x}$.
(a) Show that $f(x) = 0$ for $x = \dfrac{1}{\pi}, \dfrac{1}{2\pi}, \dfrac{1}{3\pi}, \ldots$
(b) Show that $f(x) = 1$ for $x = \dfrac{4}{\pi}, \dfrac{4}{5\pi}, \dfrac{4}{9\pi}, \ldots$
(c) What can you conclude about $\lim_{x \to 0^+} \tan \dfrac{1}{x}$?

47. Use a graph to estimate the equations of all the vertical asymptotes of the curve

$$y = \tan(2 \sin x) \qquad -\pi \leqslant x \leqslant \pi$$

Then find the exact equations of these asymptotes.

48. In the theory of relativity, the mass of a particle with velocity v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the mass of the particle at rest and c is the speed of light. What happens as $v \to c^-$?

49. (a) Use numerical and graphical evidence to guess the value of the limit

$$\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$$

(b) How close to 1 does x have to be to ensure that the function in part (a) is within a distance 0.5 of its limit?

1.6 Calculating Limits Using the Limit Laws

In Section 1.5 we used calculators and graphs to guess the values of limits, but we saw that such methods don't always lead to the correct answer. In this section we use the following properties of limits, called the *Limit Laws*, to calculate limits.

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \qquad \text{and} \qquad \lim_{x \to a} g(x)$$

exist. Then

1. $\displaystyle \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\displaystyle \lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\displaystyle \lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\displaystyle \lim_{x \to a} [f(x)\, g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\displaystyle \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{if } \lim_{x \to a} g(x) \neq 0$

These five laws can be stated verbally as follows:

Sum Law 1. The limit of a sum is the sum of the limits.

Difference Law 2. The limit of a difference is the difference of the limits.

Constant Multiple Law 3. The limit of a constant times a function is the constant times the limit of the function.

Product Law 4. The limit of a product is the product of the limits.

Quotient Law 5. The limit of a quotient is the quotient of the limits (provided that the limit of the denominator is not 0).

It is easy to believe that these properties are true. For instance, if $f(x)$ is close to L and $g(x)$ is close to M, it is reasonable to conclude that $f(x) + g(x)$ is close to $L + M$. This gives us an intuitive basis for believing that Law 1 is true. In Section 1.7 we give a precise definition of a limit and use it to prove this law. The proofs of the remaining laws are given in Appendix F.

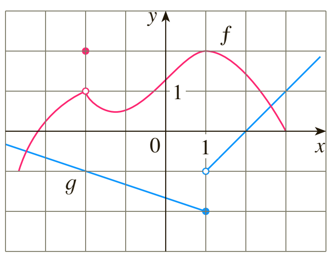


FIGURE 1

EXAMPLE 1 Use the Limit Laws and the graphs of f and g in Figure 1 to evaluate the following limits, if they exist.

(a) $\lim\limits_{x \to -2} [f(x) + 5g(x)]$ (b) $\lim\limits_{x \to 1} [f(x)g(x)]$ (c) $\lim\limits_{x \to 2} \dfrac{f(x)}{g(x)}$

SOLUTION

(a) From the graphs of f and g we see that

$$\lim_{x \to -2} f(x) = 1 \qquad \text{and} \qquad \lim_{x \to -2} g(x) = -1$$

Therefore we have

$$\begin{aligned} \lim_{x \to -2} [f(x) + 5g(x)] &= \lim_{x \to -2} f(x) + \lim_{x \to -2} [5g(x)] && \text{(by Limit Law 1)} \\ &= \lim_{x \to -2} f(x) + 5 \lim_{x \to -2} g(x) && \text{(by Limit Law 3)} \\ &= 1 + 5(-1) = -4 \end{aligned}$$

(b) We see that $\lim_{x \to 1} f(x) = 2$. But $\lim_{x \to 1} g(x)$ does not exist because the left and right limits are different:

$$\lim_{x \to 1^-} g(x) = -2 \qquad \lim_{x \to 1^+} g(x) = -1$$

So we can't use Law 4 for the desired limit. But we *can* use Law 4 for the one-sided limits:

$$\lim_{x \to 1^-} [f(x)g(x)] = \lim_{x \to 1^-} f(x) \cdot \lim_{x \to 1^-} g(x) = 2 \cdot (-2) = -4$$

$$\lim_{x \to 1^+} [f(x)g(x)] = \lim_{x \to 1^+} f(x) \cdot \lim_{x \to 1^+} g(x) = 2 \cdot (-1) = -2$$

The left and right limits aren't equal, so $\lim_{x \to 1} [f(x)g(x)]$ does not exist.

(c) The graphs show that

$$\lim_{x \to 2} f(x) \approx 1.4 \qquad \text{and} \qquad \lim_{x \to 2} g(x) = 0$$

Because the limit of the denominator is 0, we can't use Law 5. The given limit does not exist because the denominator approaches 0 while the numerator approaches a nonzero number. ■

If we use the Product Law repeatedly with $g(x) = f(x)$, we obtain the following law.

Power Law

6. $\lim\limits_{x \to a} [f(x)]^n = \left[\lim\limits_{x \to a} f(x)\right]^n$ where n is a positive integer

In applying these six limit laws, we need to use two special limits:

7. $\lim\limits_{x \to a} c = c$ **8.** $\lim\limits_{x \to a} x = a$

These limits are obvious from an intuitive point of view (state them in words or draw graphs of $y = c$ and $y = x$), but proofs based on the precise definition are requested in the exercises for Section 1.7.