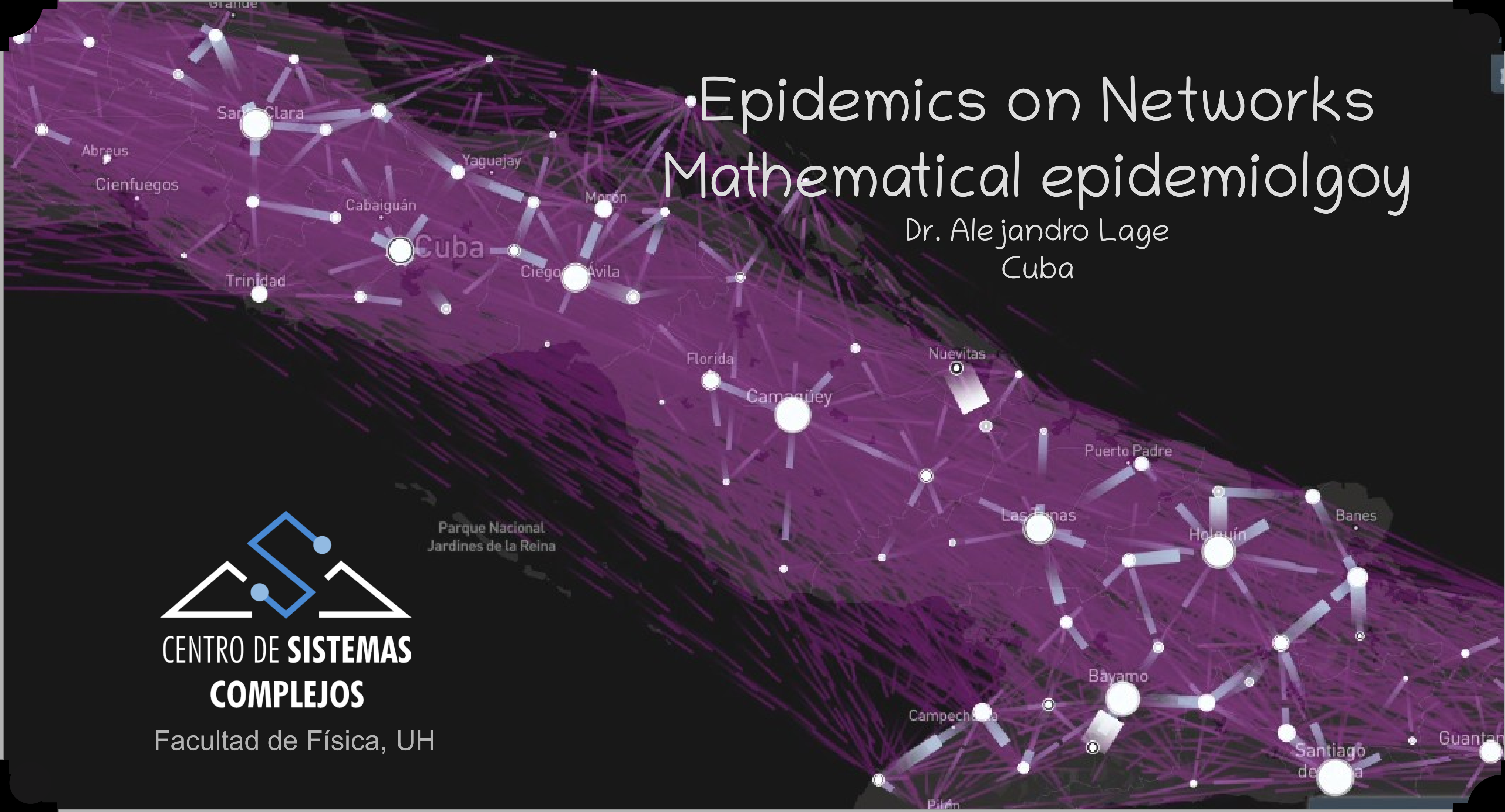
Epidemics on Networks
Mathematical epidemiolgoy

Dr. Alejandro Lage
Cuba



CENTRO DE SISTEMAS COMPLEJOS
Facultad de Física, UH

CENTER OF COMPLEX SYSTEMS

University of Havana, Cuba

Stat-mech and Systems Biolgogy
Dr. Roberto Mulet.

Granular, Active Mater and Sensors
Dr. Ernesto Altshuler.

Big Data and Population mobility
Dr. Alejandro Lage

EPIDEMICS ON NETWORKS

What you get

- Compartmental (SIR) models
- Complex networks.
- Simulations in python
- Probability theory/Statistical mechanics.
- Reconstruction of epidemics.

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- Basic calculus
- Probability theory
- Basic programming
- Statistical mechanics and/or combinatorial optimization

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Who cares?

- Working on epidemics
- Working on inference
- Working on networks
- Working on optimization

WHO ARE YOU

Field of Research

- 4 Complex Networks
- 4 Industrial engineering
- 2 Epidemics

2. Why are you taking this class?

Answered: 10 Skipped: 0

I had no other option	0%	0
I am interested in complex networks	60%	6
I am interested in epidemics	20%	2
I am interested in simulations	30%	3
I am interested in inference problems	30%	3
Somebody advice me to go	30%	3
Other (please specify)		0

WHO ARE YOU

Field of Research

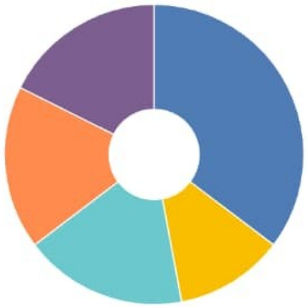
- 4 Complex Networks
- 4 Industrial engineering
- 2 Epidemics

1. What's your education level?

Answered: 10 Skipped: 0

Bachelor, undergraduate	40%	4
Graduated, master student	40%	4
Graduated, PhD student	20%	2
PhD researcher	0%	0

2. Why are you taking this class?



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Other (please specify)		0

WHO ARE YOU

Field of Research

- 4 Complex Networks
- 4 Industrial engineering
- 2 Epidemics

6. Your current level in Python is?

Answered: 10 Skipped: 0

Never used it	40%	4
I used it very little or long time ago	40%	4
I use it enough, or fairly, in recent times	20%	2

1. What's your education level?

Answered: 10 Skipped: 0

Bachelor, undergraduate	40%	4
Graduated, master student	40%	4
Graduated, PhD student	20%	2
PhD researcher	0%	0

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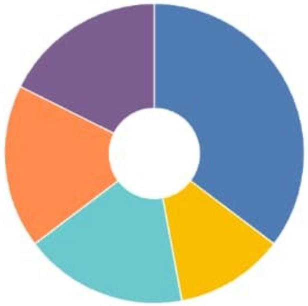

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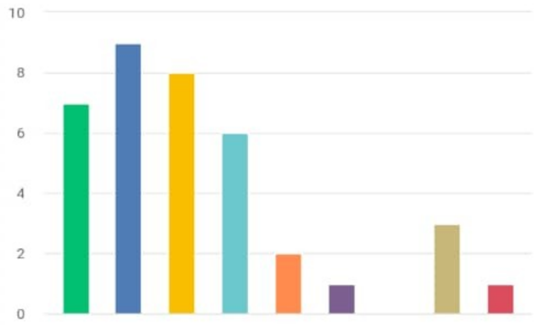
Other (please specify)		0

WHO ARE YOU

4. Which of the following courses have you passed?

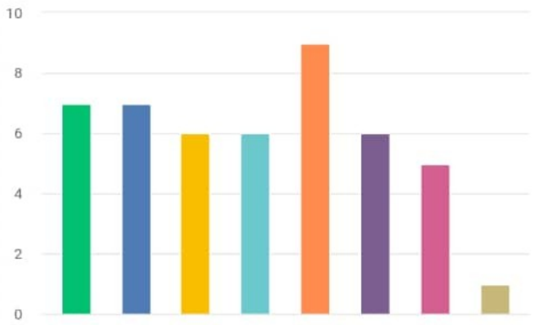


Answered: 10 Skipped: 0

Calculus or mathematical analysis	70%	7
Linear algebra	90%	9
Probability theory	80%	8
Statistics	60%	6
Differential equations	20%	2
Complex systems	10%	1
Tai chi	0%	0
Networks or graph theory	30%	3
Statistical mechanics	10%	1

WHO ARE YOU

9. Which of the following concepts in probability are known to you?



Answered: 10 Skipped: 0

Events	70%	7
Random variables	70%	7
Density distribution	60%	6
Cumulative distribution	60%	6
Mean and variance	90%	9
Conditional probability	60%	6
Likelihood	50%	5
Entropy of a distribution	10%	1

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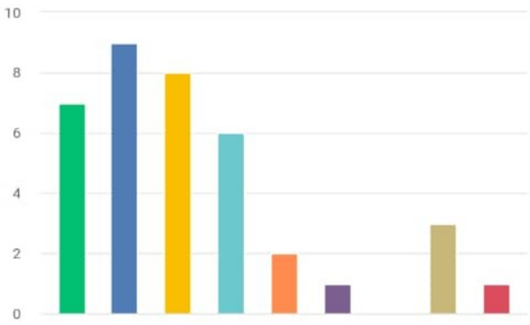

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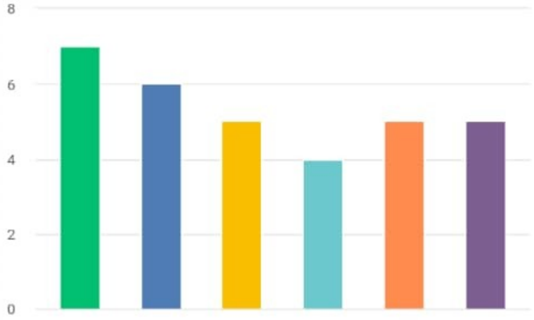
WHO ARE YOU

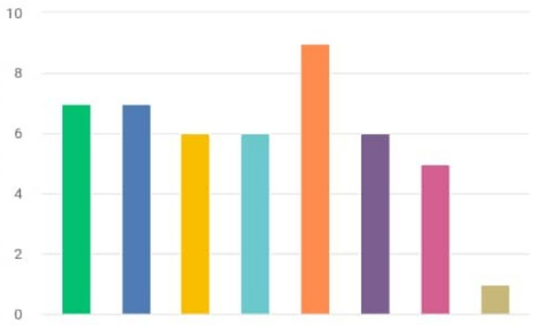
10. Which of the following concepts in graph theory are known to you?



Answered: 8 Skipped: 2

Paths and cycles?	87.5%	7
Adjacency matrix	75%	6
Diameter of a graph	62.5%	5
Scale free networks	50%	4
Small world networks	62.5%	5
Regular graph	62.5%	5

9. Which of the following concepts in probability are known to you?



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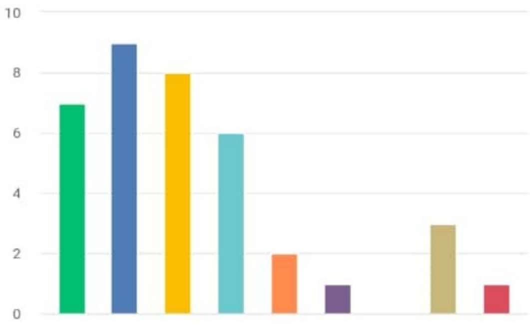

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1. Mathematical epidemiology, origins, early results. The compartmental model family.
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6. Playing with networks in Python. Networkx. Plotting. Nice plotting.
7. Master equation of epidemics on networks. The mean field solution and pair based solution.
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MATHEMATICAL EPIDEMIOLOGY

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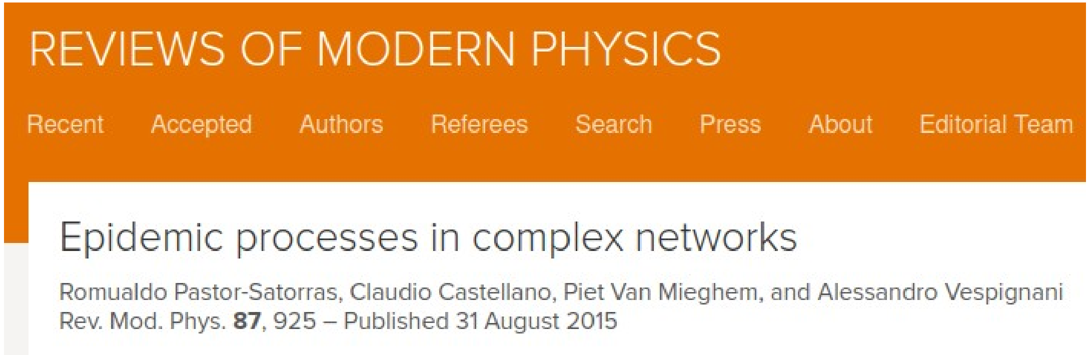
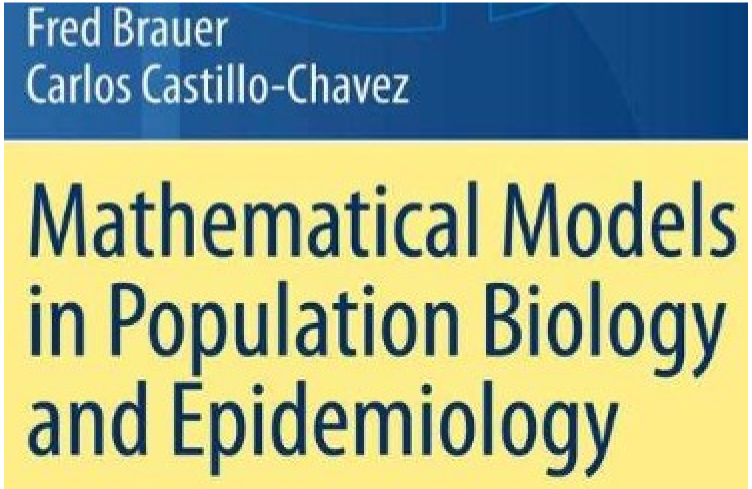




MASTER EQUATION

Stochastic procceses:

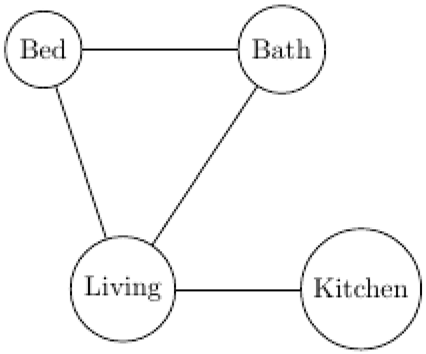
- Set of states
- Transition rates
- Distribution over states

$$\frac{d}{dt}P_i(t) = \sum_j W_{ji}P_j(t) - \sum_j W_{ij}P_i(t),$$

Conservation of mass → probability

$$\frac{d}{dt}\int_V \rho\, \mathrm{d}V + \oint_S \rho\mathbf{v}\cdot \mathrm{d}\mathbf{S} = 0$$

Classical example: **Random walk in a graph**

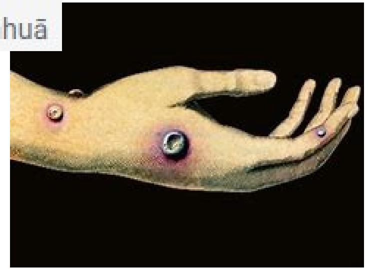


SMALLPOX IN XIII CENTURY

1760-1766 Daniel Bernoulli at Royal Academy of Science
Effects of smallpox on life expentancy.

天花

Tiānhuā



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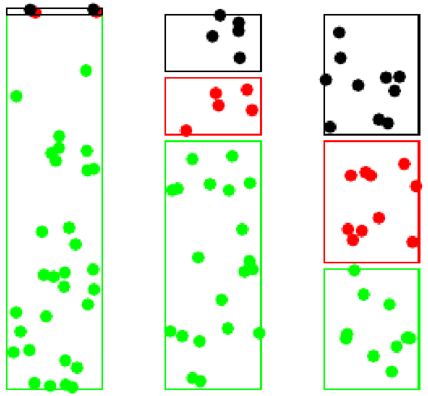
Effects of smallpox on life expentancy.

- $u(a)$ the probability for an individual of being alive and susceptible (un-infected) at age a
- $w(a)$ the probability for an individual of being alive and immune at age a
- $d(a)$ is the probability of being dead at age a, but since probabilities add up to 1, and there are only three posible states, $d(a) = 1 - u(a) - w(a)$.

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}a} &= -(\lambda + \mu(a))u(a) \\ \frac{\mathrm{d}w}{\mathrm{d}a} &= \lambda(1-c)\mu(a)u(a) - \mu(a)w(a)\end{aligned}$$

SMALLPOX IN XIII CENTURY



Compartments contains people of a certain category

Compartments are:

- Susceptible
- Immune
- Dead

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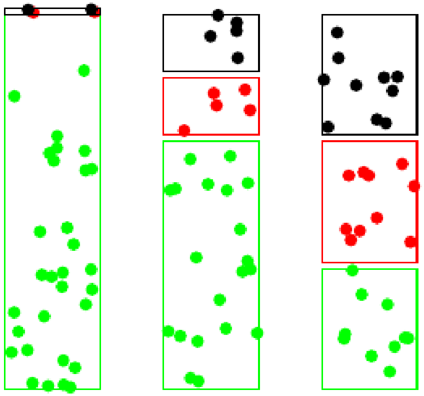

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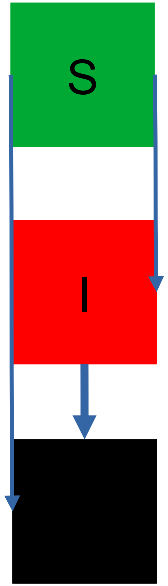
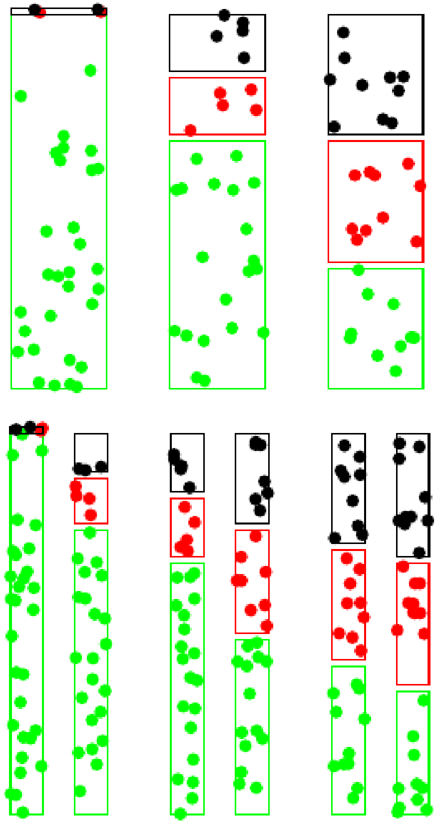


- $\mu(a)$ the rate of death of all causes except smallpox, for an individual of age a
- β is the **force of infection**: the rate at which susceptible individual get infected
- $c = 1 - s$ the fraction of individuals infected that dies of smallpox.

SMALLPOX IN XIII CENTURY



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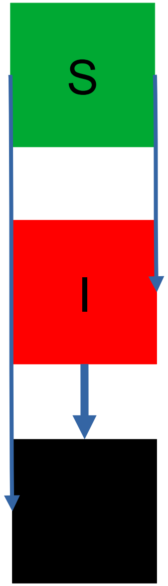
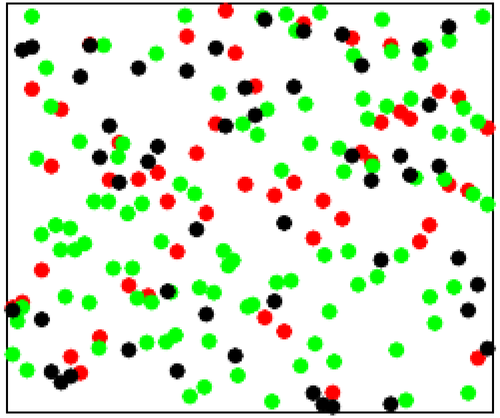


$$\frac{du}{da} = -(\lambda + \mu(a))u(a)$$
$$\frac{dw}{da} = \lambda(1-c)\mu(a)u(a) - \mu(a)w(a)$$

COMPARTMENTAL MODELS

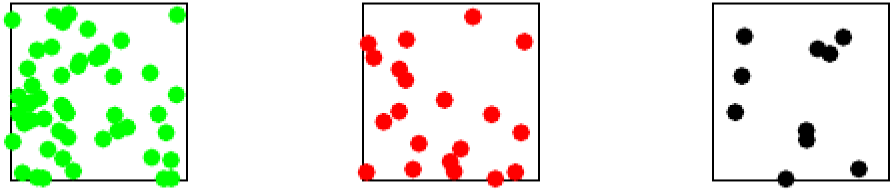
A **gas** of persons



Catalytic reactions

$$\begin{aligned} S + I &\xrightarrow{\beta} 2I \\ I &\xrightarrow{\mu} R \end{aligned}$$

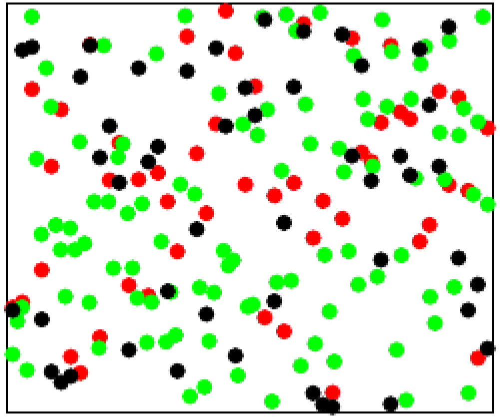
- Stochastic process
- Monte Carlo simulation
- Gillespie Alrogirthm



Compartment S   Compartment I   Compartment R

COMPARTMENTAL MODELS

A **gas** of persons



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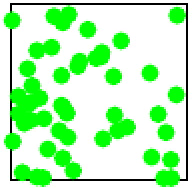
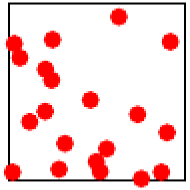
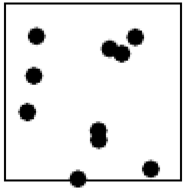


Compartment S



Compartment I



Compartment R

Compartmental model:

- Mean field approximation
- Well mixed
- No spatial correlations

$$S \xrightarrow{-\beta I} I \xrightarrow{\mu} R$$

COMPARTMENTAL MODELS

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \mu I$$
$$\frac{dR}{dt} = \mu I$$

Catalytic reactions

$$S + I \xrightarrow{\beta} 2I$$
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- Stochastic process
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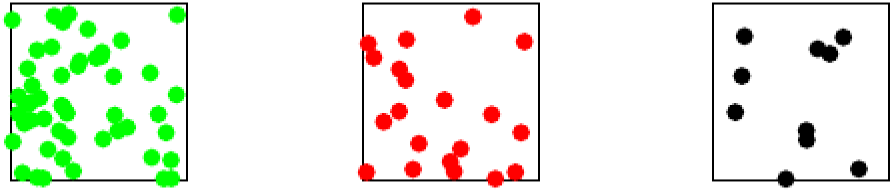

Compartment S Compartment I Compartment R

Compartmental model:

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Kermack, W.O. and A.G. McKendrick (1932) Contributions to the mathematical theory of epidemics, part. II, *Proc. Roy. Soc. London*, **138**:55-83.

COMPARTMENTAL MODELS

Probabilities

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \mu I$$
$$\frac{dR}{dt} = \mu I$$

Catalytic reactions

$$S + I \xrightarrow{\beta} 2I$$
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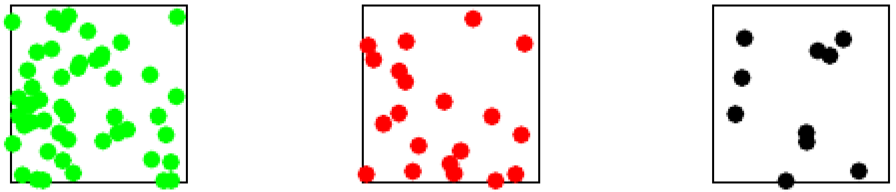


Compartment S | Compartment I | Compartment R

Extensive

$$\frac{dS}{dt} = -\frac{\beta IS}{N},$$
$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I,$$
$$\frac{dR}{dt} = \gamma I,$$

$$S \xrightarrow{-\beta I} I \xrightarrow{\mu} R$$

TECHNICAL TERMS

Technical terms

EARLY EPIDEMICS

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \mu I$$
$$\frac{dR}{dt} = \mu I$$

$$\frac{dI}{dt} = \beta S(t)I(t) - \mu I(t),$$

$$\frac{dI}{dt} = \lambda I(t), \qquad \text{where } \lambda = \beta - \mu.$$

Attention, exponentials

$$I(t) = I(0)\exp(\lambda t).$$

Basic reproduction number

$$R_0 = \beta\tau = \beta/\mu$$

$$\tau = 1/\mu$$

LARGE TIME LIMIT

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \mu I$$
$$\frac{dR}{dt} = \mu I$$

$$S'(t) + I'(t) = -\mu I(t) \qquad 1 - S_\infty = \mu \int_0^\infty I(t)\mathrm{d}t$$

$$\frac{\mathrm{d}\log S(t)}{\mathrm{d}t} = -\beta I(t)$$

$$\log(S_\infty/S_0) = -\beta \int_0^\infty I(t)\mathrm{d}t$$

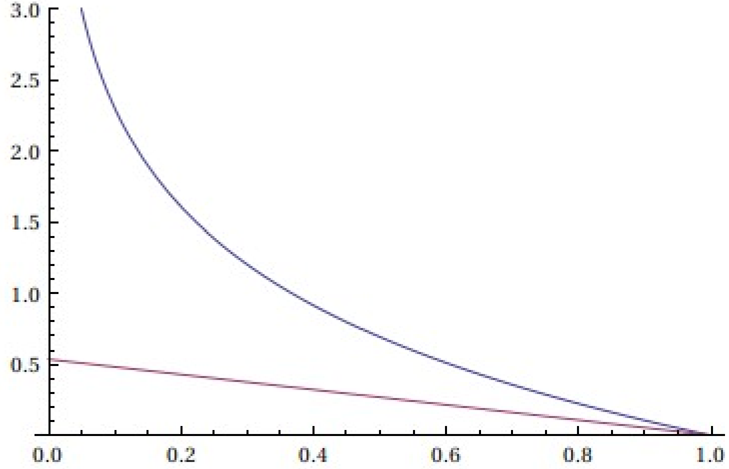
$$\log(S_0/S_\infty) = \frac{\beta}{\mu}(1 - S_\infty)$$

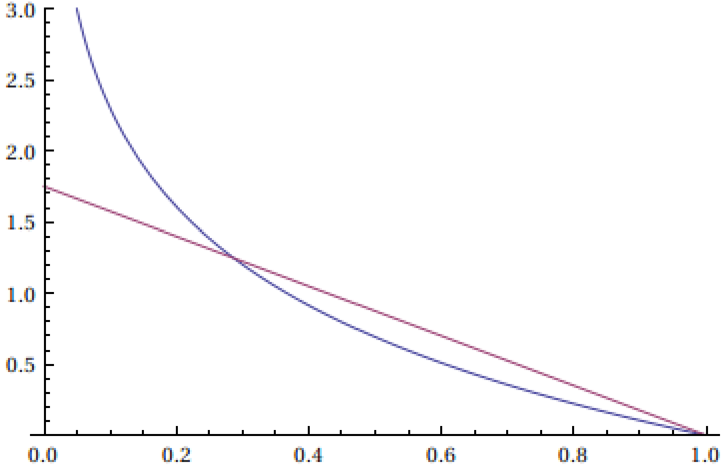
$1 - I_\infty$ or $1 - \frac{I_\infty}{N}$ is called the *attack rate*

$$) = R_0(1 - S_\infty)$$

LARGE TIME LIMIT

$$\begin{aligned}\frac{dS}{dt} &= -\beta S I \\ \frac{dI}{dt} &= \beta S I - \mu I \\ \frac{dR}{dt} &= \mu I\end{aligned}$$





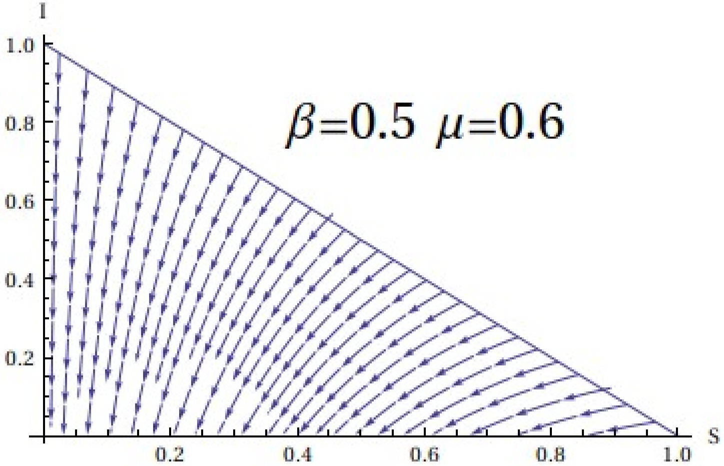
$$\log(S_0/S_\infty) = \frac{\beta}{\mu}(1 - S_\infty)$$

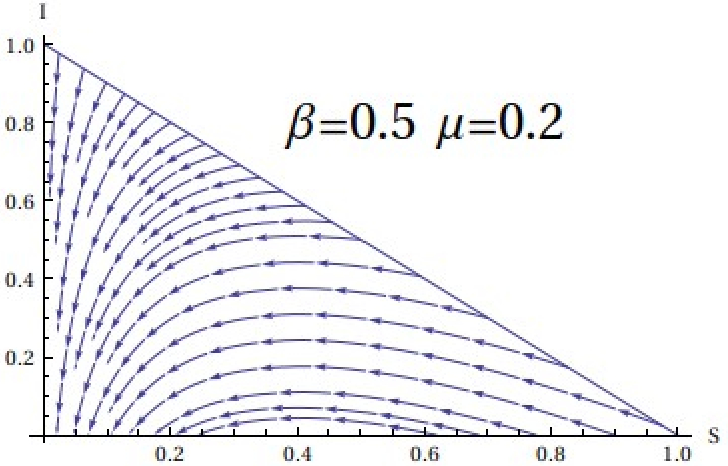
$$= R_0(1 - S_\infty)$$

LARGE TIME LIMIT

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \mu I$$
$$\frac{dR}{dt} = \mu I$$

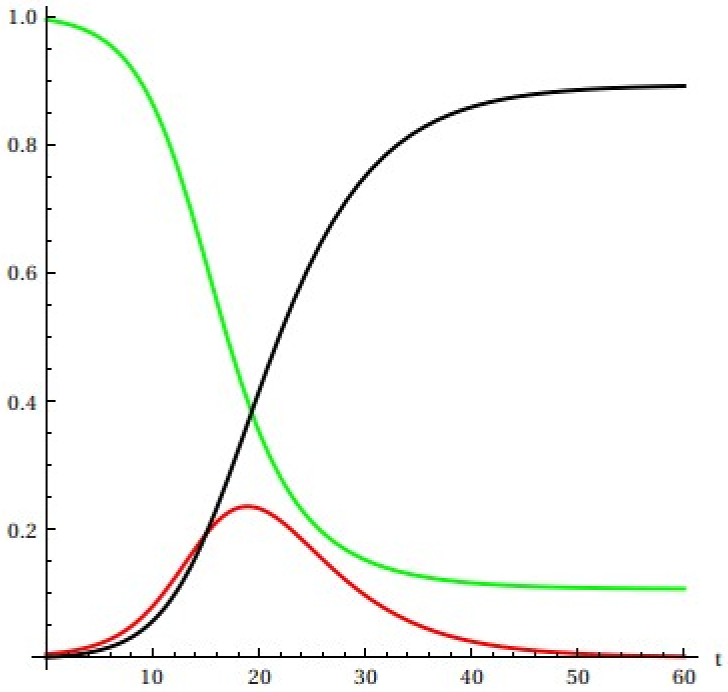
$$I(t) + S(t) - \frac{\mu}{\beta}\log S(t) = 1 - \frac{\mu}{\beta}\log S_0$$



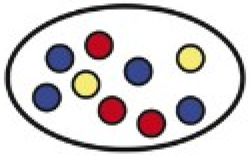


FINITE TIME INTEGRATION

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta S I \\
\frac{dI}{dt} &= \beta S I - \mu I \\
\frac{dR}{dt} &= \mu I
\end{aligned}
$$



Beyond SIR



Homogeneous mixing

$$\frac{dS}{dt} = -\frac{\beta IS}{N},$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

"Contributions to the mathematical theory of epidemics – I and II". Bulletin of Mathematical Biology. 53 (1–2) 33–55

SIR

Space

Compartments

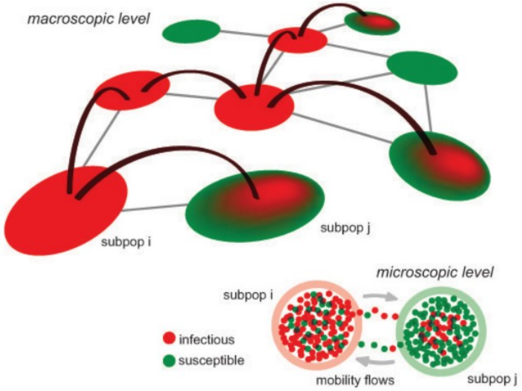
Time

Geographic patches:

- Countries, Cities, provinces, municipalities
- Mobility patterns
- Household, wokrplace, schools, commuters

Age patches:

- Seasonality
- Virus mutation
- Non pharmocolical measu



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Age patches:

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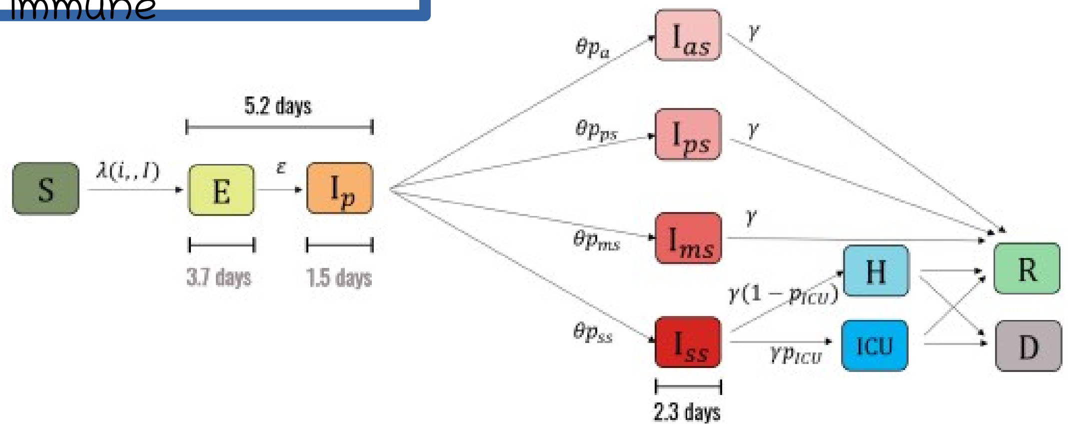
Compartments

- Exposed
- Symptomatic vs Asymptomatic
- Quarantined
- Immune

Time



SIR

Space

Geographic patches:

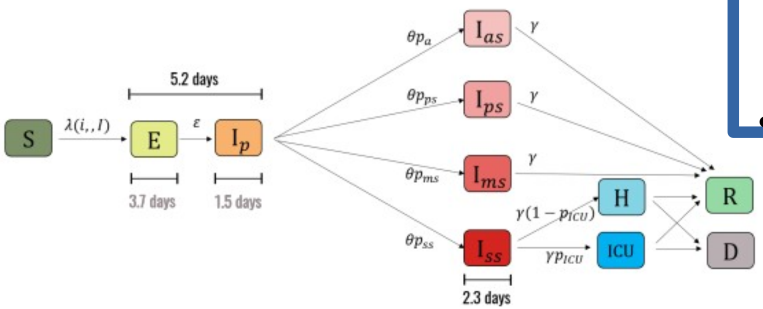
- Countries, Cities, provinces, municipalities
- Mobility patterns
- Household, wokrplace, schools, commuters

Age patches:

- Seasonality
- Virus mutation
- Non pharmocolical measu

Compartments

- Exposed
- Symptomatic vs Asymptomatic
- Quarantined
- Immune

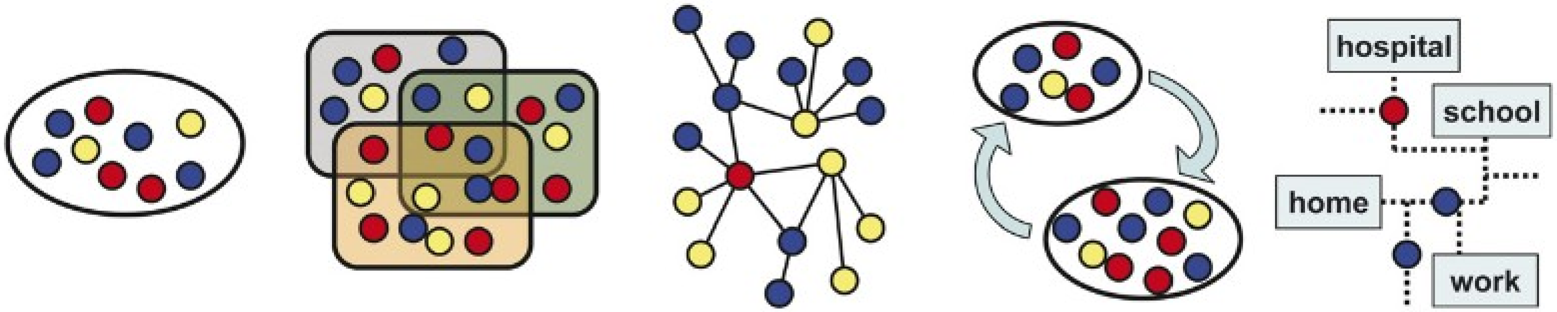


Time

Parameters in time:

- Seasonality
- Virus mutation
- Non pharmocolical measures
- Vaccination

MODELING EPIDEMICS



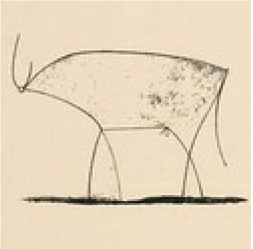
Homogeneous mixing	Social structure	Contact network models	Multi-scale models	Agent Based models

V. Colizza et al. / C. R. Biologies 330 (2007) 364–374

MODELING EPIDEMICS

Simple

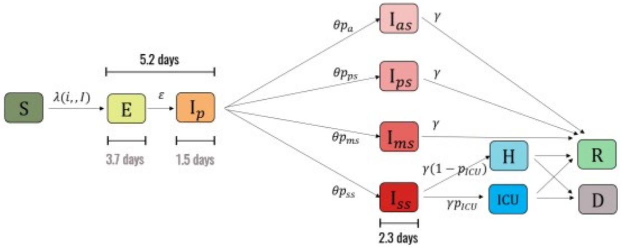
Un-specific

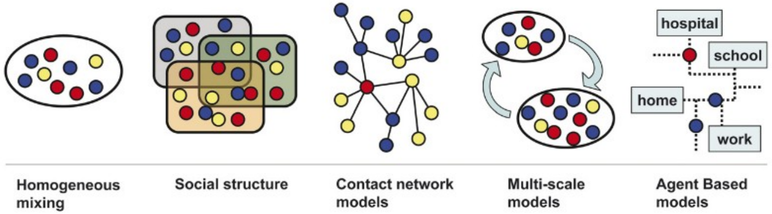


$$\frac{dS}{dt} = -\frac{\beta IS}{N},$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

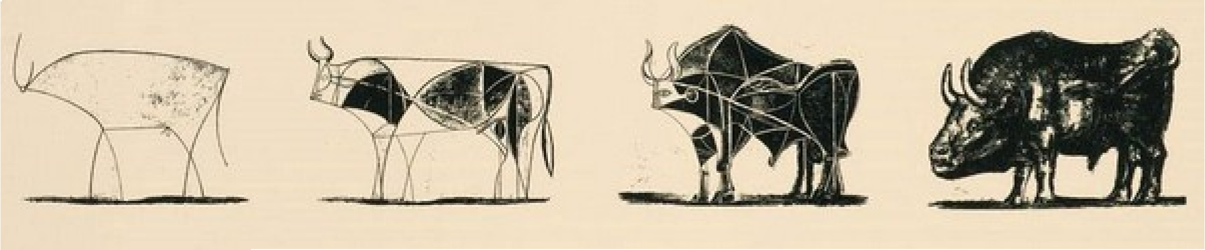




MODELING EPIDEMICS

Simple

Un-specific



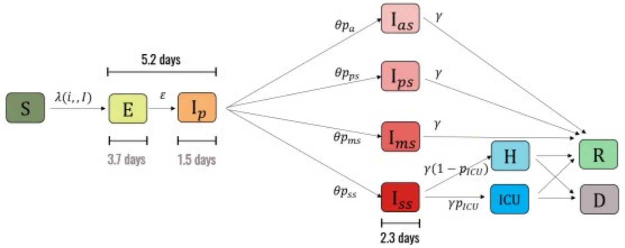
Complex

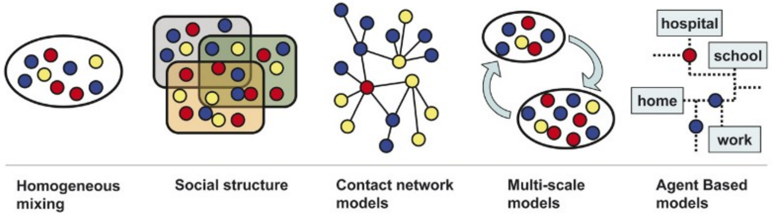
Specific

$$\frac{dS}{dt} = -\frac{\beta IS}{N},$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

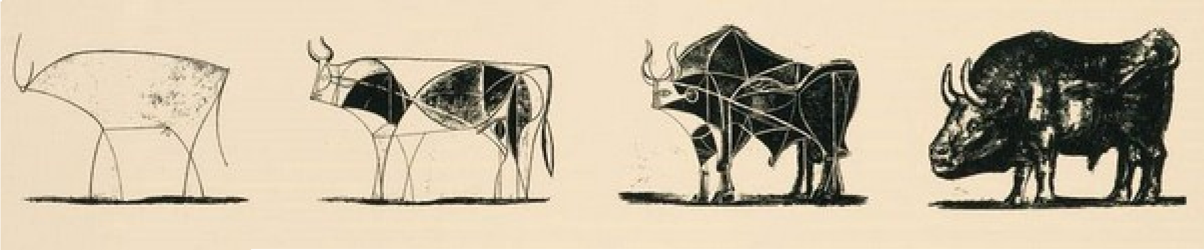




MODELING EPIDEMICS

Simple

Un-especific



Complex

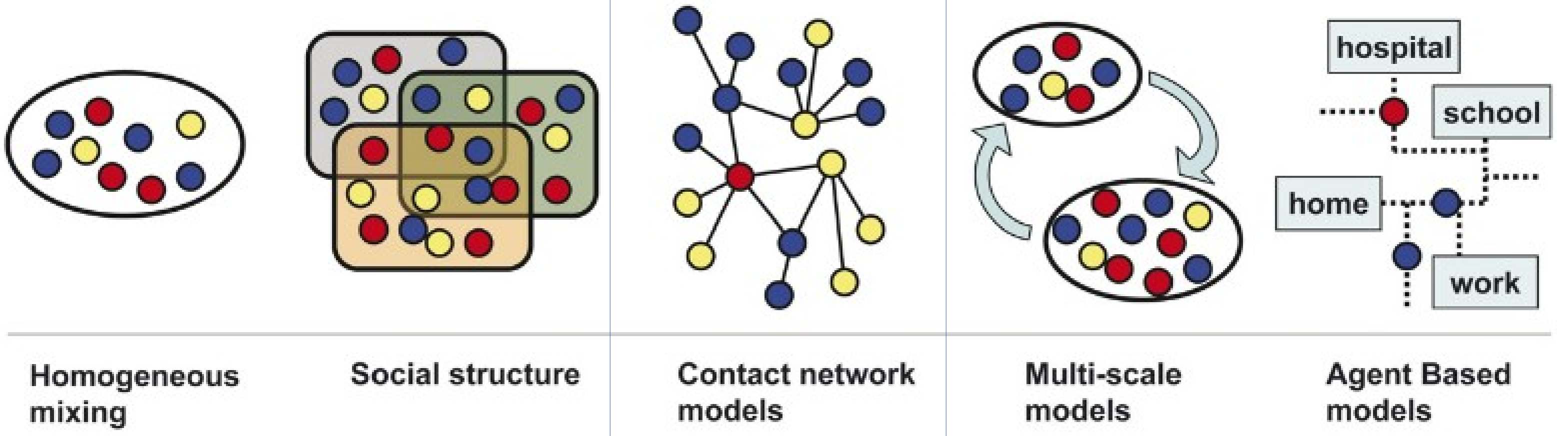
Specific

Hard to fit

$$\frac{dS}{dt} = -\frac{\beta I S}{N},$$
$$\frac{dI}{dt} = \frac{\beta I S}{N} - \gamma I,$$
$$\frac{dR}{dt} = \gamma I,$$

EPIDEMICS ON NETWORKS

Nc



NEXT CLASS

Introduction to graphs and networks

Graph metrics

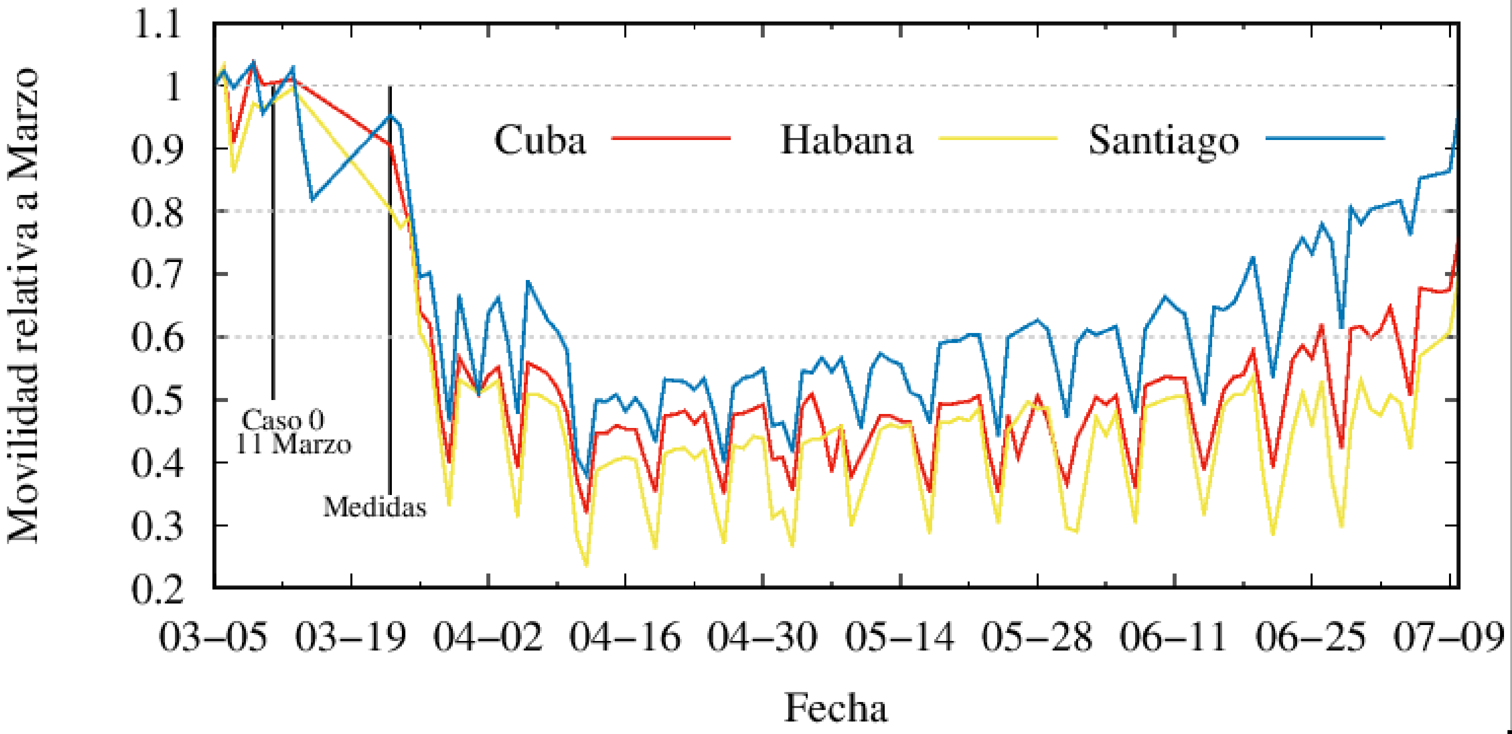
Graphs for benchmarking

Representing graphs on Python

MOVILIDAD Y COVID

COVID 2020

- **Primer caso** 11 de marzo
- **Cierre de escuelas,** 23 de marzo
- **Cierre transporte,** 3 de abril



Orlando Martínez Durive, Tesis de maestría

你好

CENTRO DE **SISTEMAS COMPLEJOS**

Facultad de Física
Universidad de La Habana

WHY SHOULD YOU CARE?

Recent epidemics:

Covid, SARS, HIV, Ebola

Each epidemic is different. And you have one oportunity.

You can not do experiments with epidemics.

Epidemics is part of a more general set of problems, like the spread of information, the expansion of memes, computer viruses. Stochastics processes running on networks.

A field that is changing: big data, cellphones, social networks, allow for new descriptions of the phenomena.