

Learning Useful Representations of Recurrent Neural Network Weight Matrices

Vincent Herrmann, Francesco Faccio, Jürgen Schmidhuber

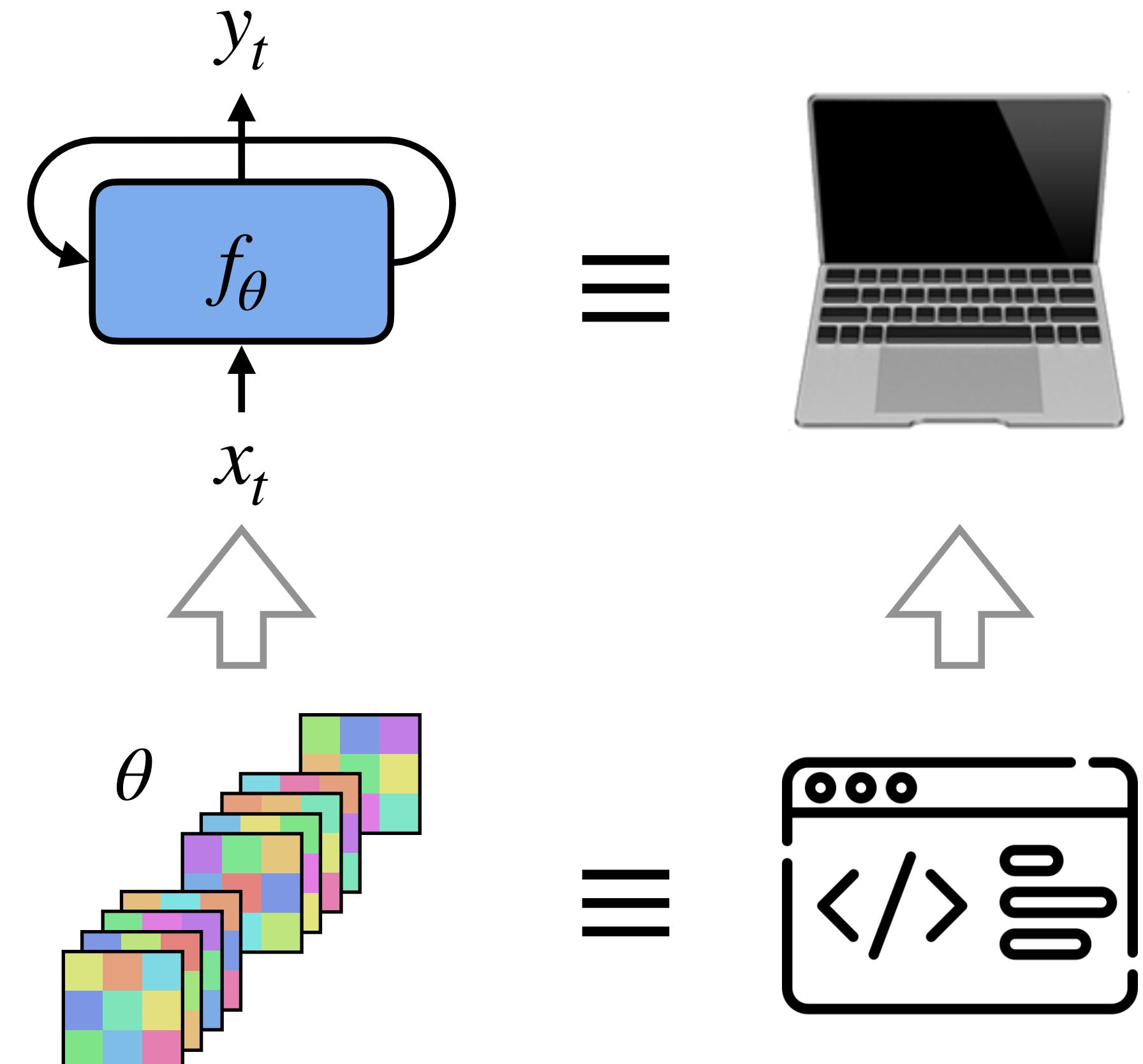


Learning RNN Weight Representations

- RNNs are universal computers
- Weight matrices are their program

Applications:

- Reinforcement Learning
- Implicit Neural Representations
- Interpretability
- ...



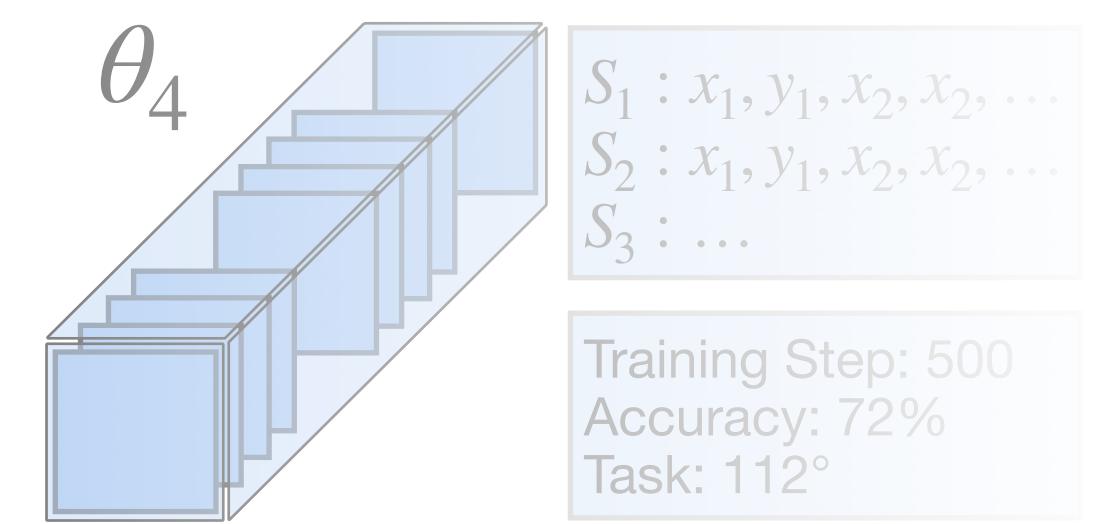
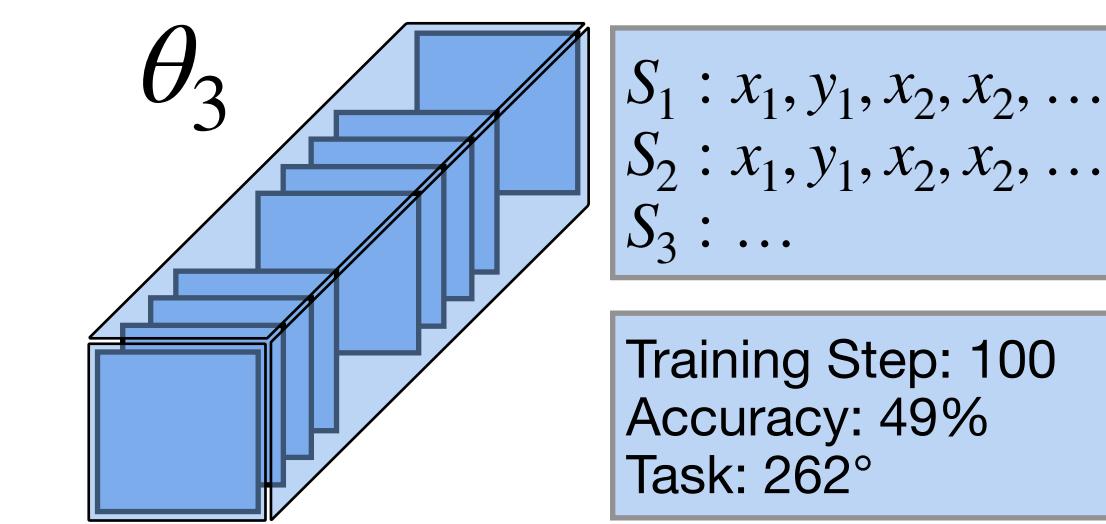
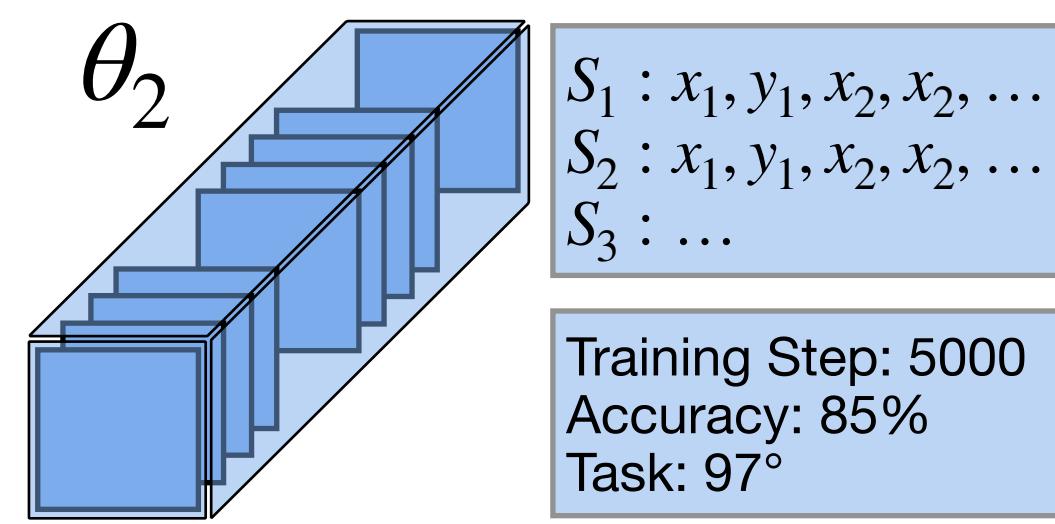
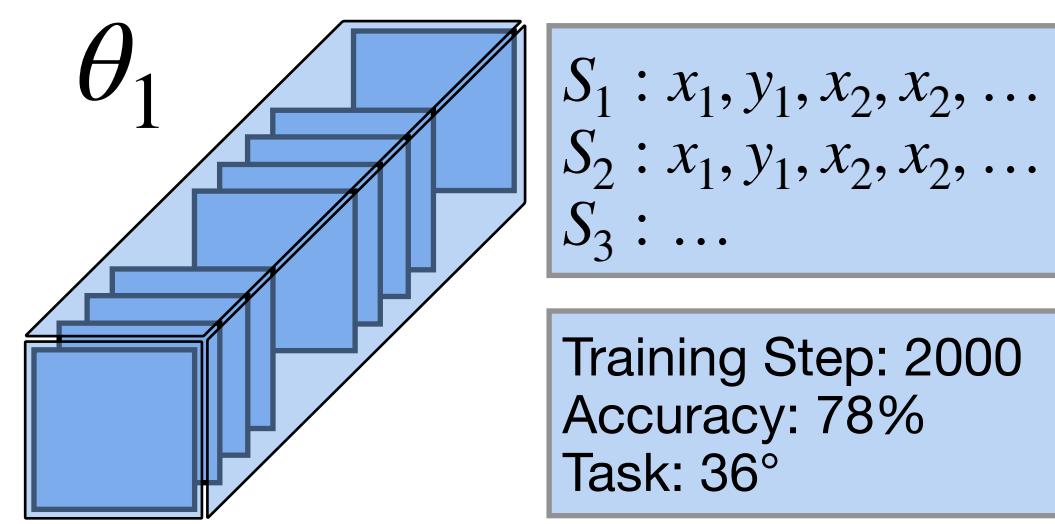
Three Ingredients

- **Datasets**
- **Learning / Pre-Training method**
- **RNN Weight encoder architecture**

Datasets

Two RNN ‘Model Zoos’

- 1000 LSTM training runs per dataset, 9 snapshots per run
- Each LSTM is trained on a different task from a task family
- Datapoint: LSTM weights θ , 100 rollouts/trajectories, metadata



Formal Languages Dataset

- Autoregressive models of 216 different formal languages
- Language family: $L_{m_a, m_b, m_c, m_d} := \{a^{n+m_a}b^{n+m_b}c^{n+m_c}d^{n+m_d} \mid n \in \mathbb{N}\}$

- Examples

$L_{1,1,1,1} :$

a	b	c	d
a	a	b	b
a	a	a	b
...			

$L_{1,2,1,2} :$

a	b	b	c	d	d
a	a	b	b	b	c
a	a	a	b	b	b
...					

$L_{2,1,3,1} :$

a	a	b	c	c	c	d
a	a	a	b	b	c	c
a	a	a	a	b	b	b
...						

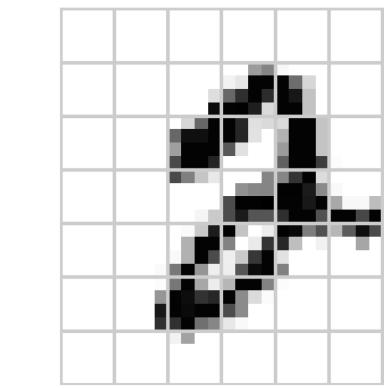
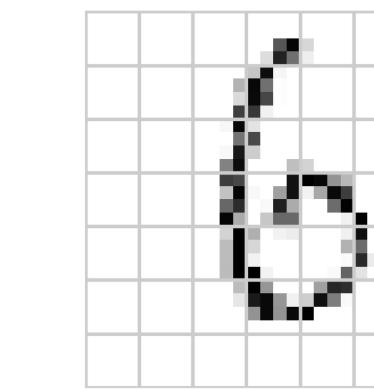
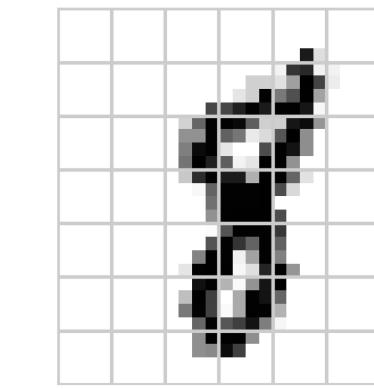
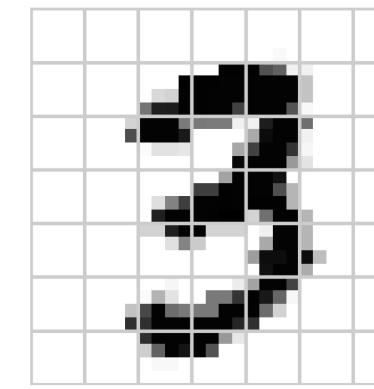
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Tiled Sequential MNIST Dataset

- Classifiers of tiled sequentialised MNIST digits (prediction at every step)
- Every model is trained on a different rotation angle of the digits

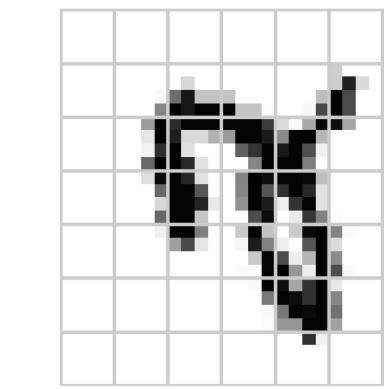
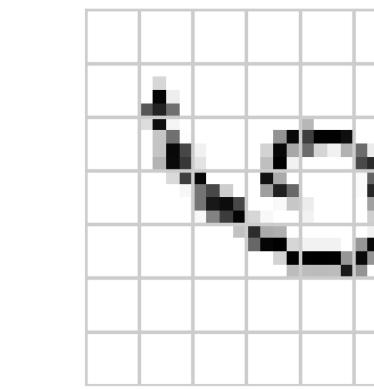
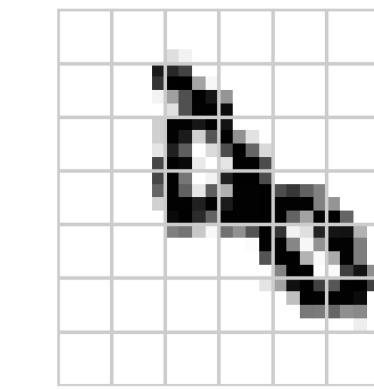
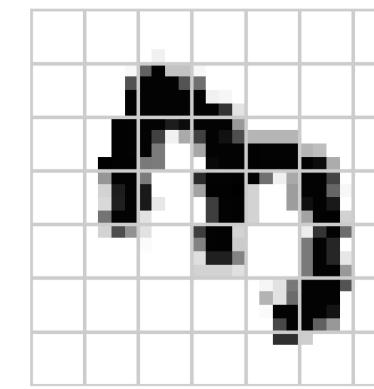
- Examples

17° :



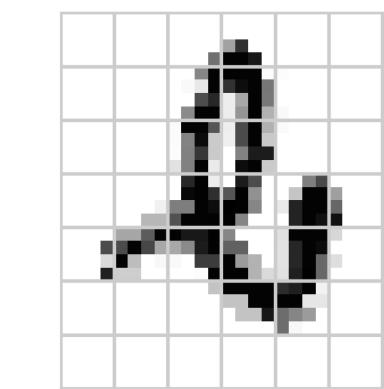
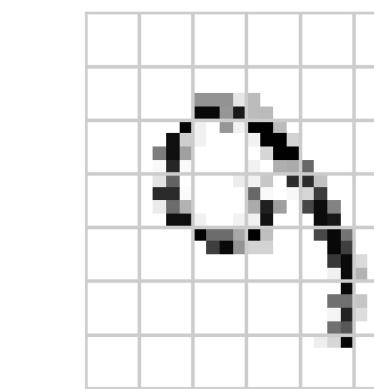
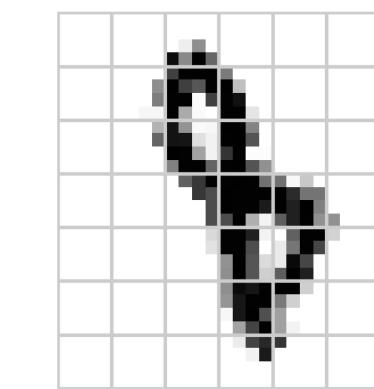
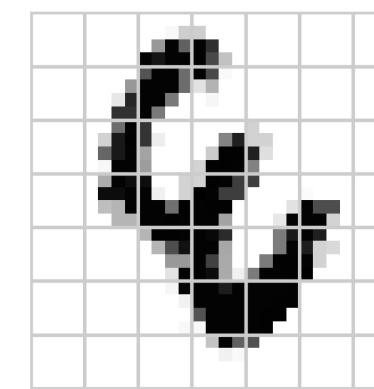
...

74° :



...

231° :



...

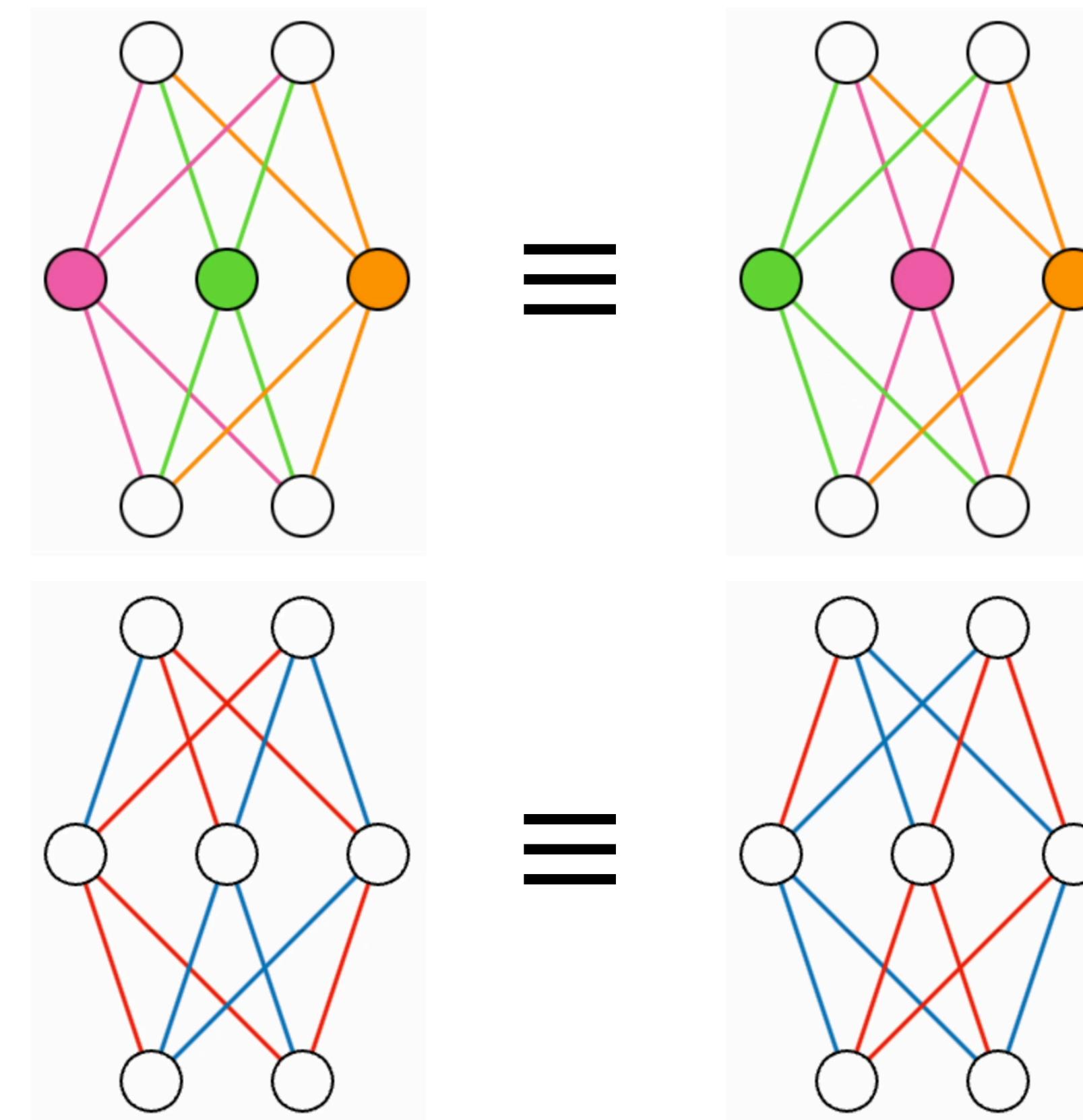
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Weight Space Symmetries

Challenge and Opportunity

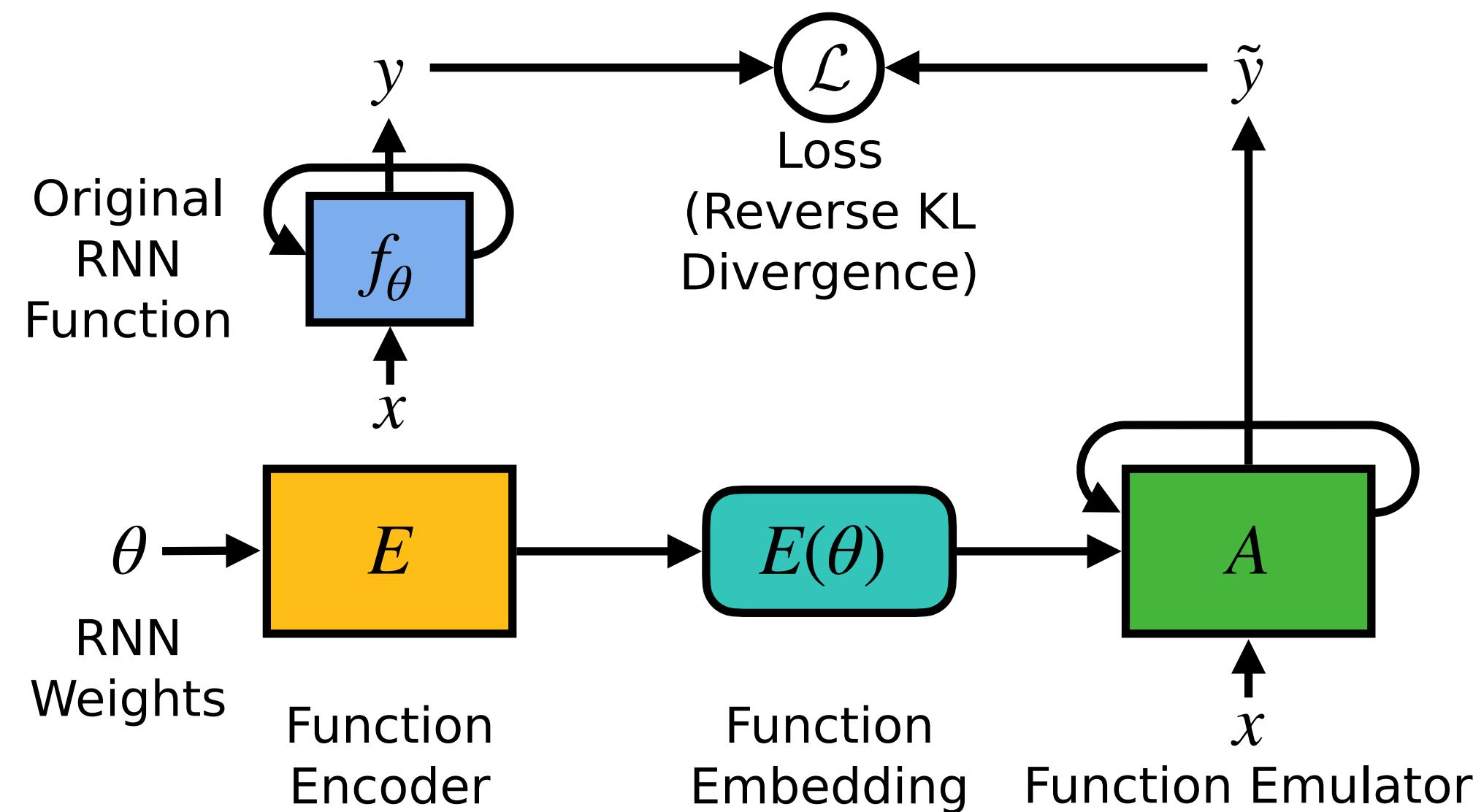
Huge number of equivalent networks

- Hidden Neuron Permutation
- Sign Flip / Scaling
(depends on non-linearity)



Self-Supervised Training Method

- Reconstruction or naive contrastive methods? 🚨 *Weight Space Symmetries* 🚨
- *Instead:* Emulate the functionality of the network!



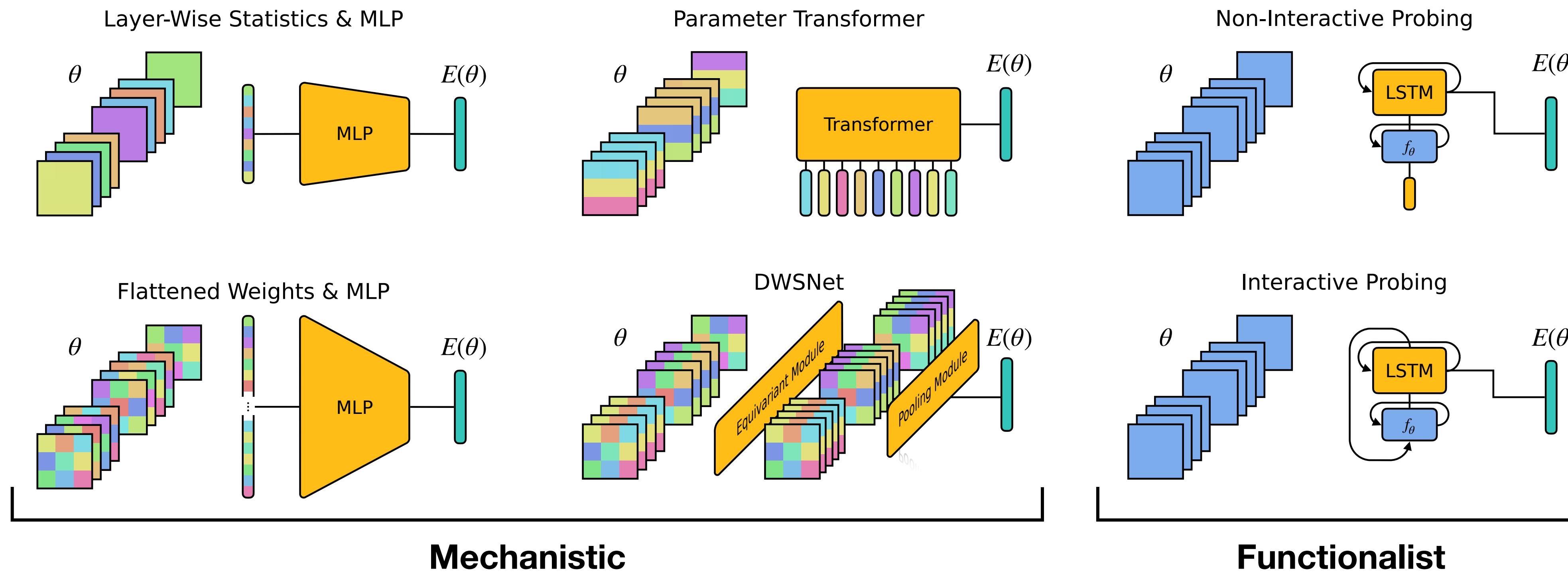
- Encoder E generates representation $E(\theta)$ of RNN f_θ
- Emulator A is conditioned on $E(\theta)$ and imitates f_θ

compare:

Raileanu et al., 2020, “Fast Adaptation via Policy-Dynamics Value Functions”

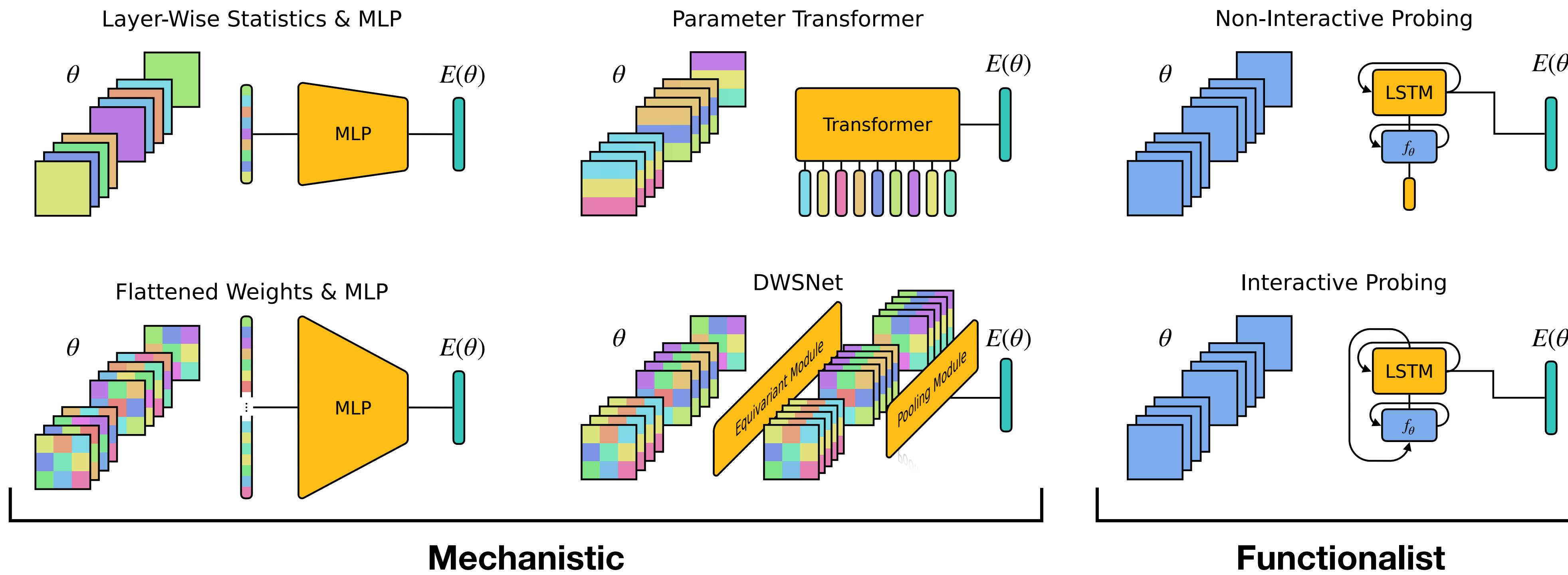
RNN Weight Encoder Architectures

- Distinction: Mechanistic Encoders & Functionalist Encoders
- **Mechanistic** encoders look at the weights θ directly
- **Functionalist** encoders look at the input-output mapping of f_θ



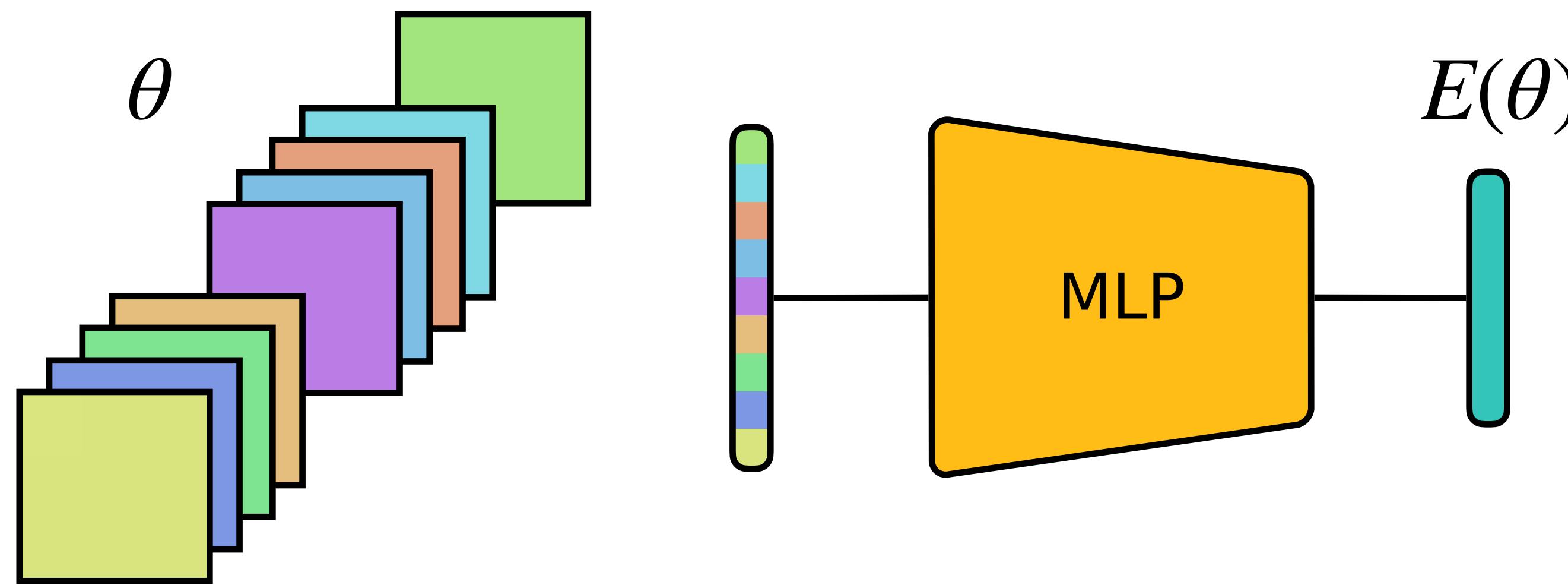
Layer-Wise Statistics & MLP (Mechanistic)

- Concatenate various global statistics of each weight matrix and feed into MLP
- Invariant to hidden neuron permutation
- Not universal



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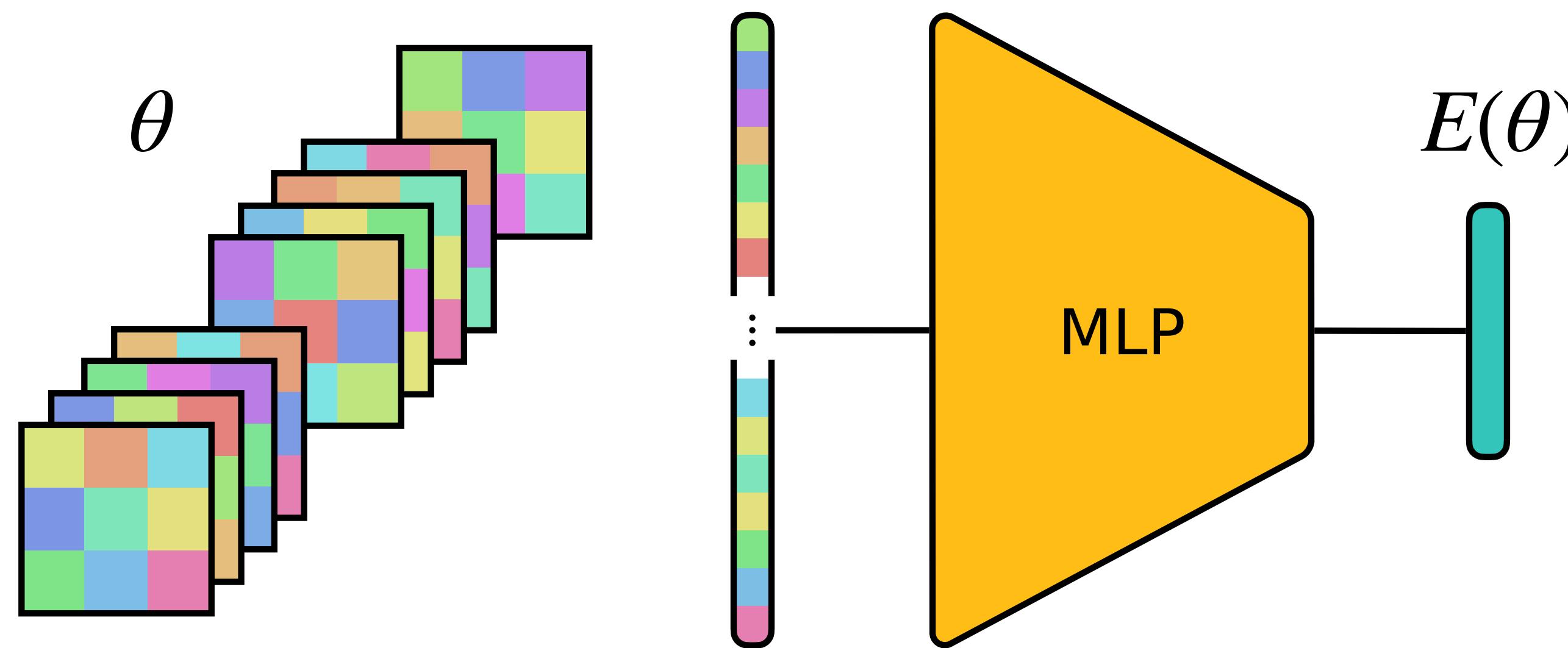


compare:

Unterthiner et al., 2020, "Predicting Neural Network Accuracy from Weights"

Flattened Weights & MLP (Mechanistic)

- Flatten all weights into one big vector and feed into MLP
- Not invariant to hidden neuron permutation 🤢
- Universal 🤘

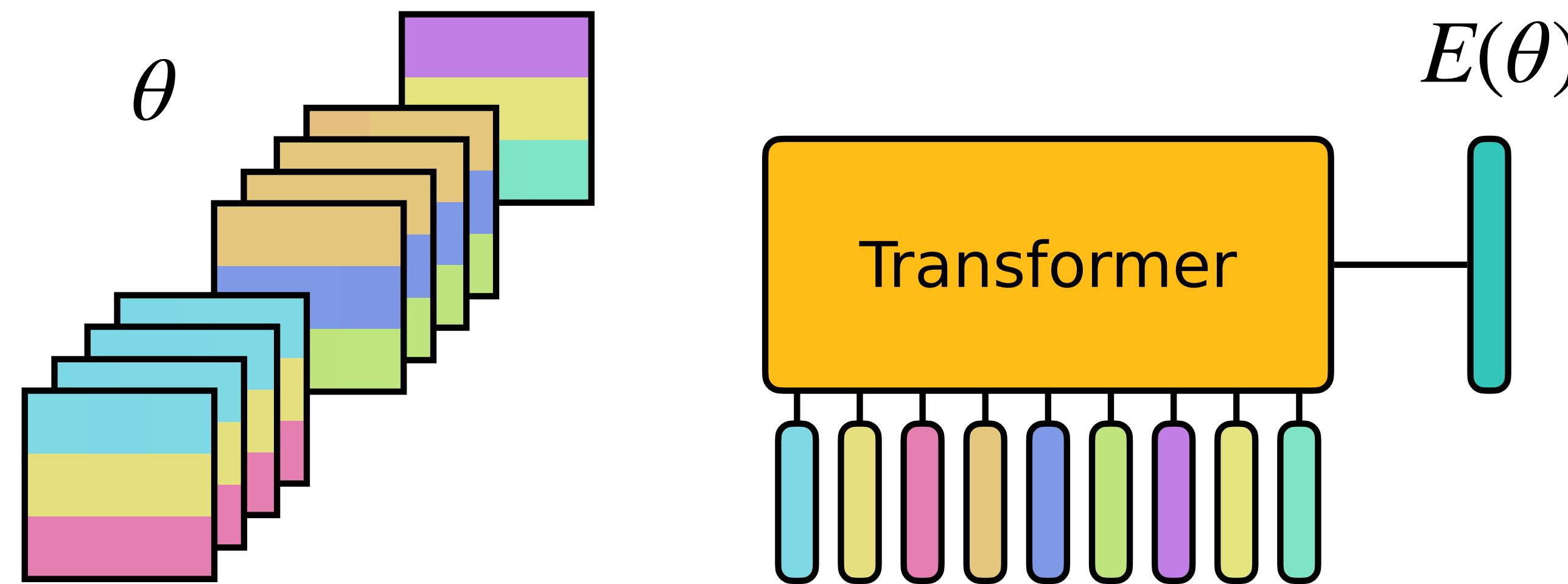


compare:
Faccio, Kirsch, Schmidhuber, 2020,
“Parameter-Based Value Functions”

Herrmann, Kirsch, Schmidhuber, 2022,
“Learning One Abstract Bit at a Time
through Self-Invented Experiments
Encoded as Neural Networks”

Parameter Transformer (Mechanistic)

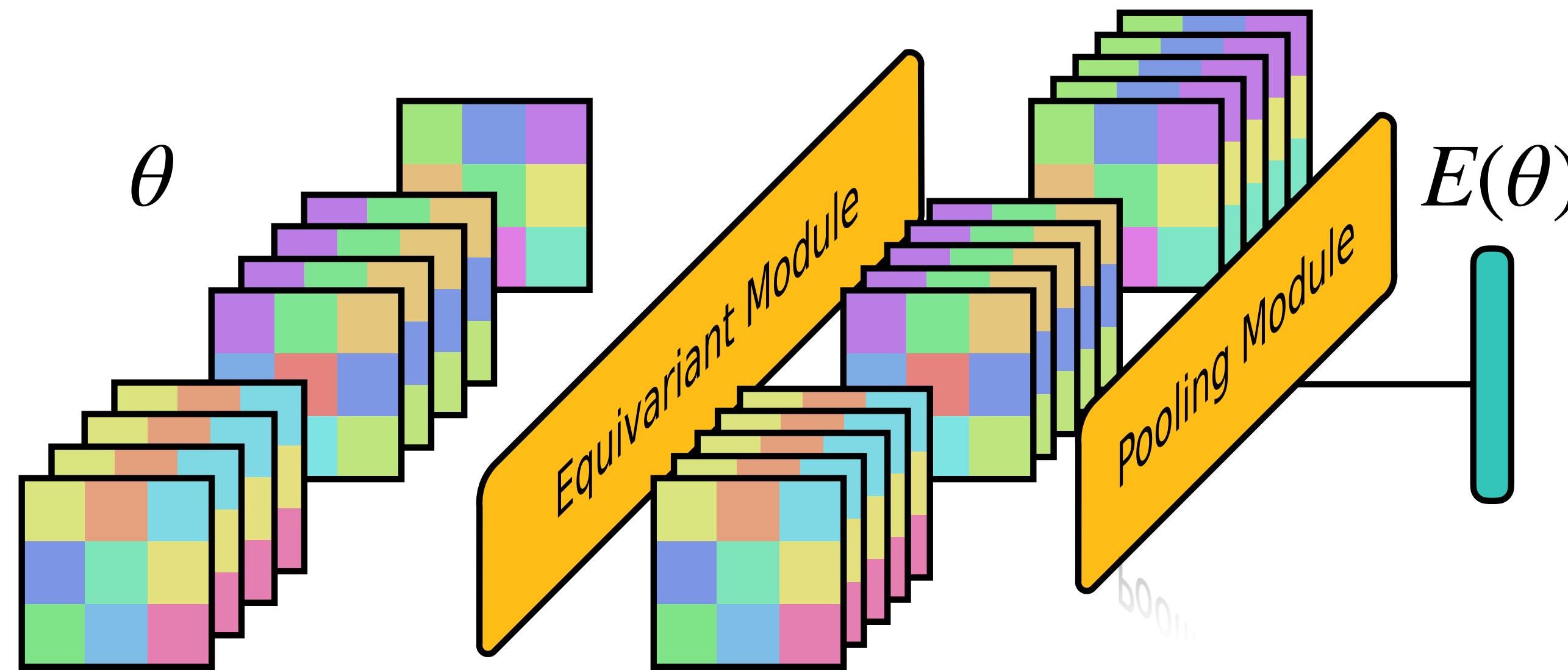
- Feed neuron weight vectors as sequence to a transformer model
- Not invariant to hidden neuron permutation 🤢
- Universal 👍



compare:
Schürholt, Kostadinov, Borth, 2021,
“Self-Supervised Representation
Learning on Neural Network Weights
for Model Characteristic Prediction”

Deep Weight Space Net (Mechanistic)

- Equivariant weight processing modules followed by invariant pooling layer
- Invariant precisely to hidden neuron permutation 
- Universal 



compare:

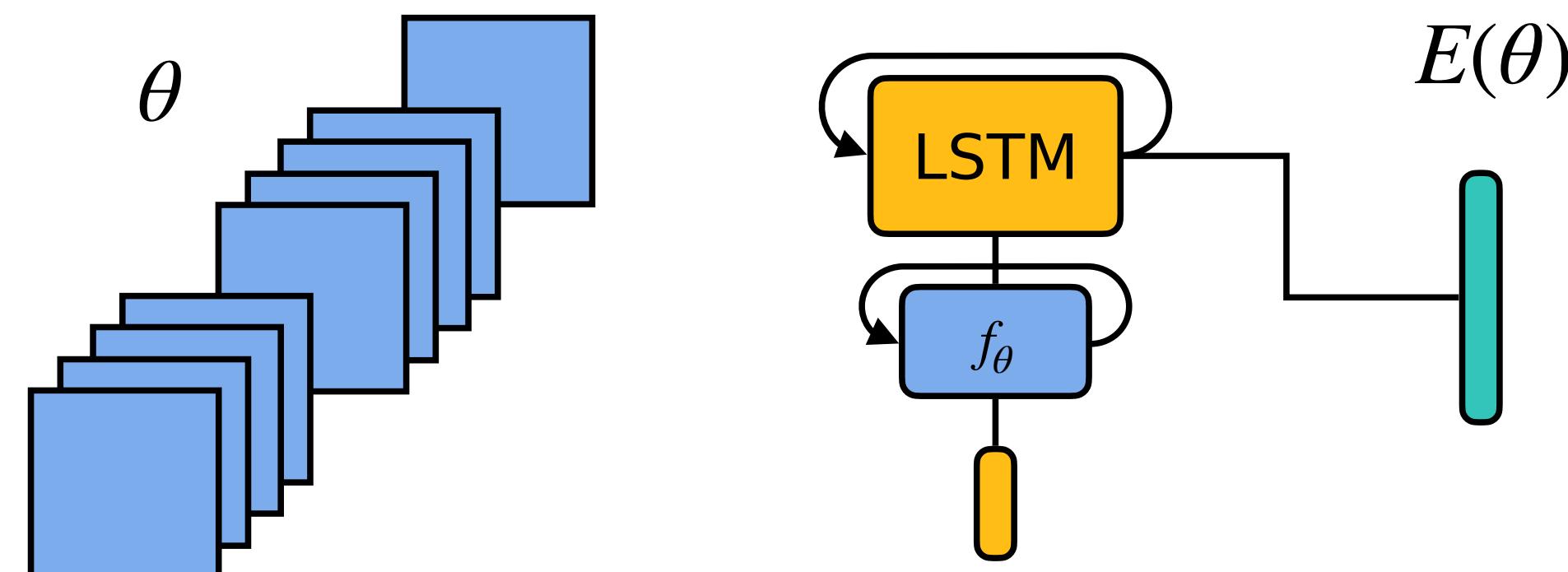
Navon et al., 2023, “Equivariant Architectures for Learning in Deep Weight Spaces”

Zhou et al., 2023, “Permutation Equivariant Neural Functionals”

Kirsch, Schmidhuber, 2020, “Meta Learning Backpropagation and Improving it”

Non-Interactive Probing (Functionalist)

- Fixed but learnable probing sequences are given to f_θ
- Based on the corresponding probing outputs, $E(\theta)$ is computed
- Invariant to everything that leaves f_θ 's functionality intact 
- Not fully universal 

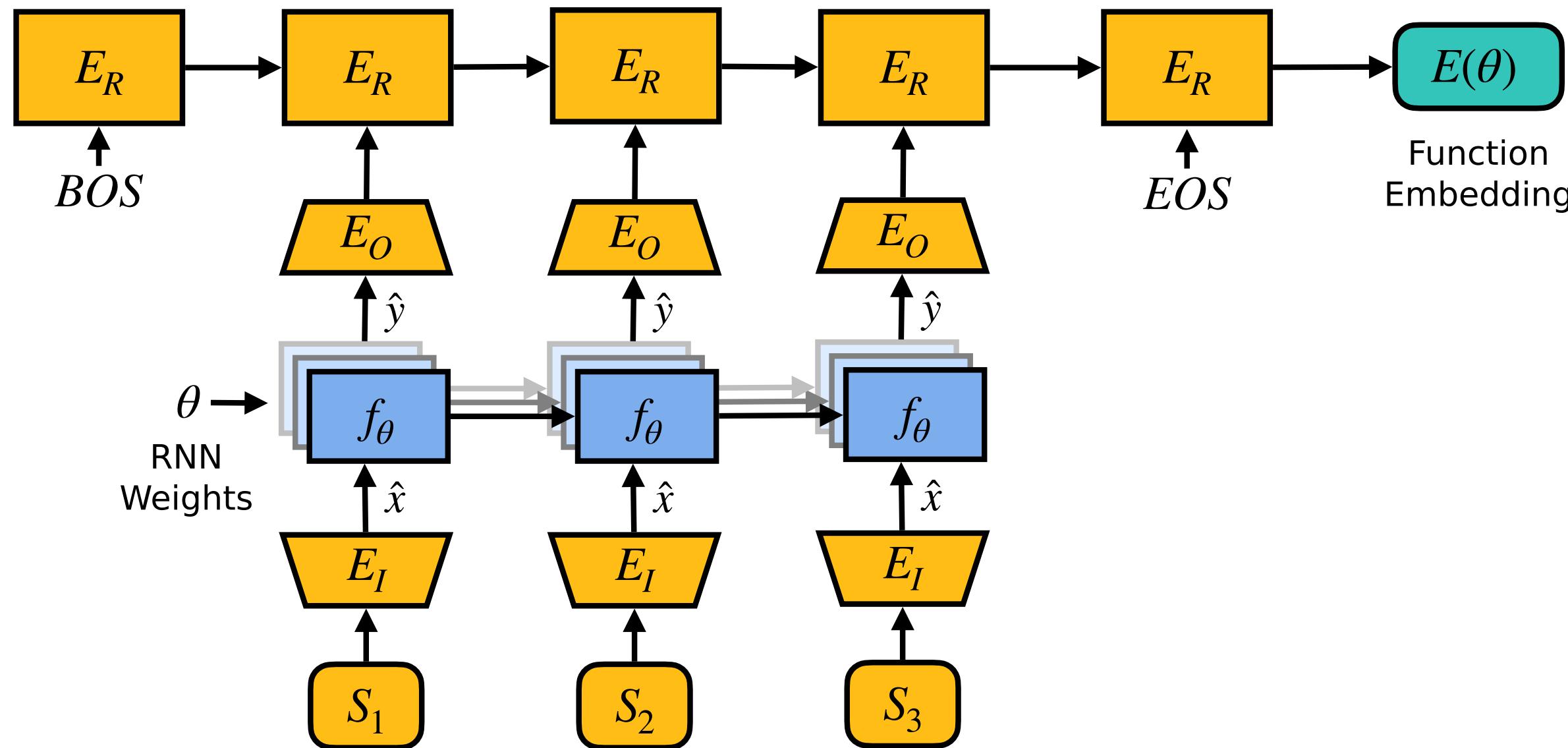


compare:
Harb et al., 2020, “Policy Evaluation Networks”

Faccio et al., 2022, “Goal-Conditioned Generator of Deep Policies”

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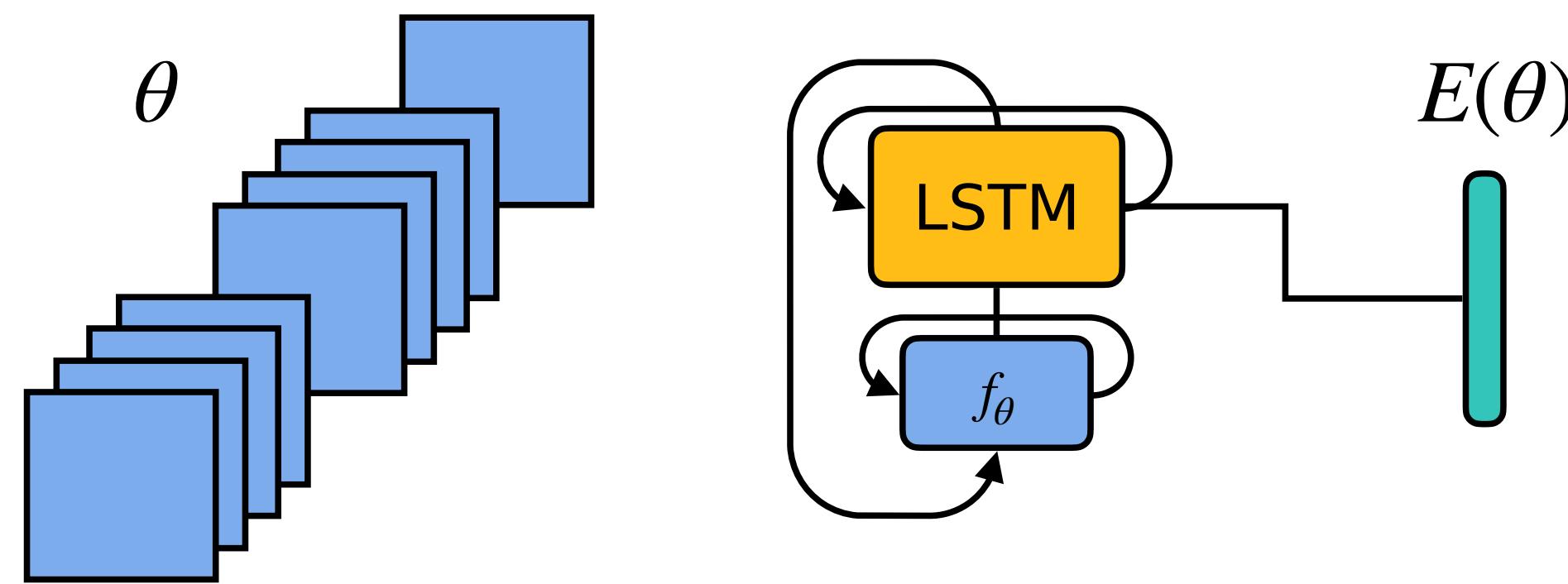


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Interactive Probing (Functionalist)

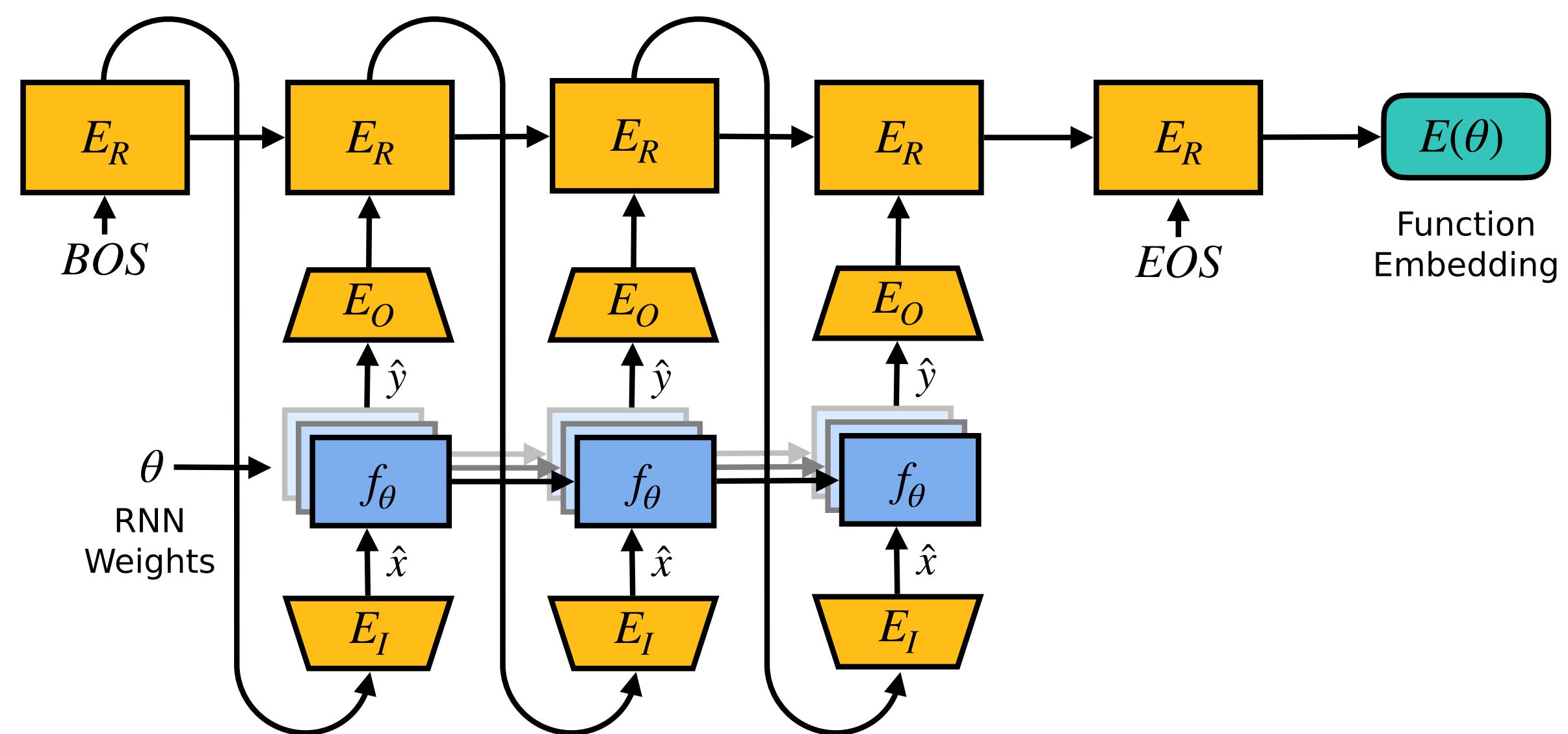
- Probing sequences are dynamically generated by core LSTM
- Next probing inputs depend on f_θ 's response to all previous probing inputs
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compare:
Schmidhuber, 2015, "On Learning to Think"

Interactive Probing (Functionalist)

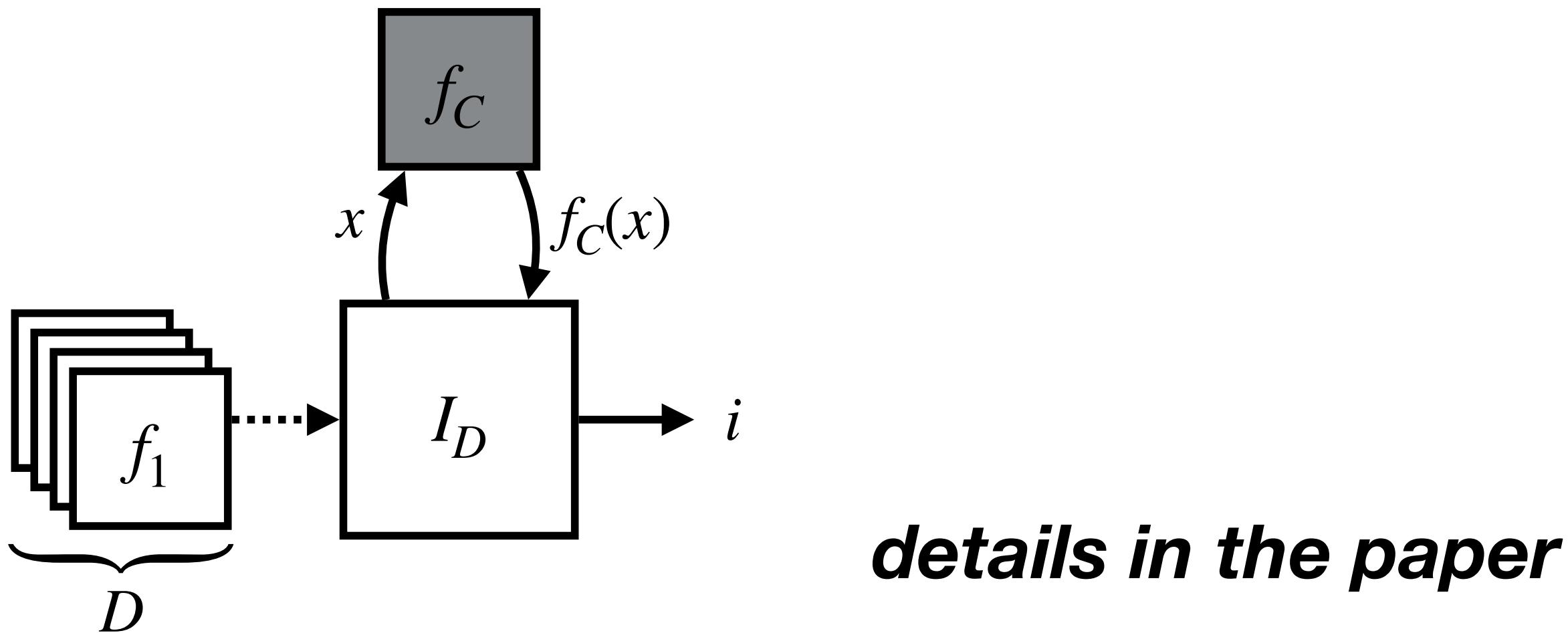
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Functionalist Approach: Theoretical Results

What's the difference between interactive and non-interactive probing?



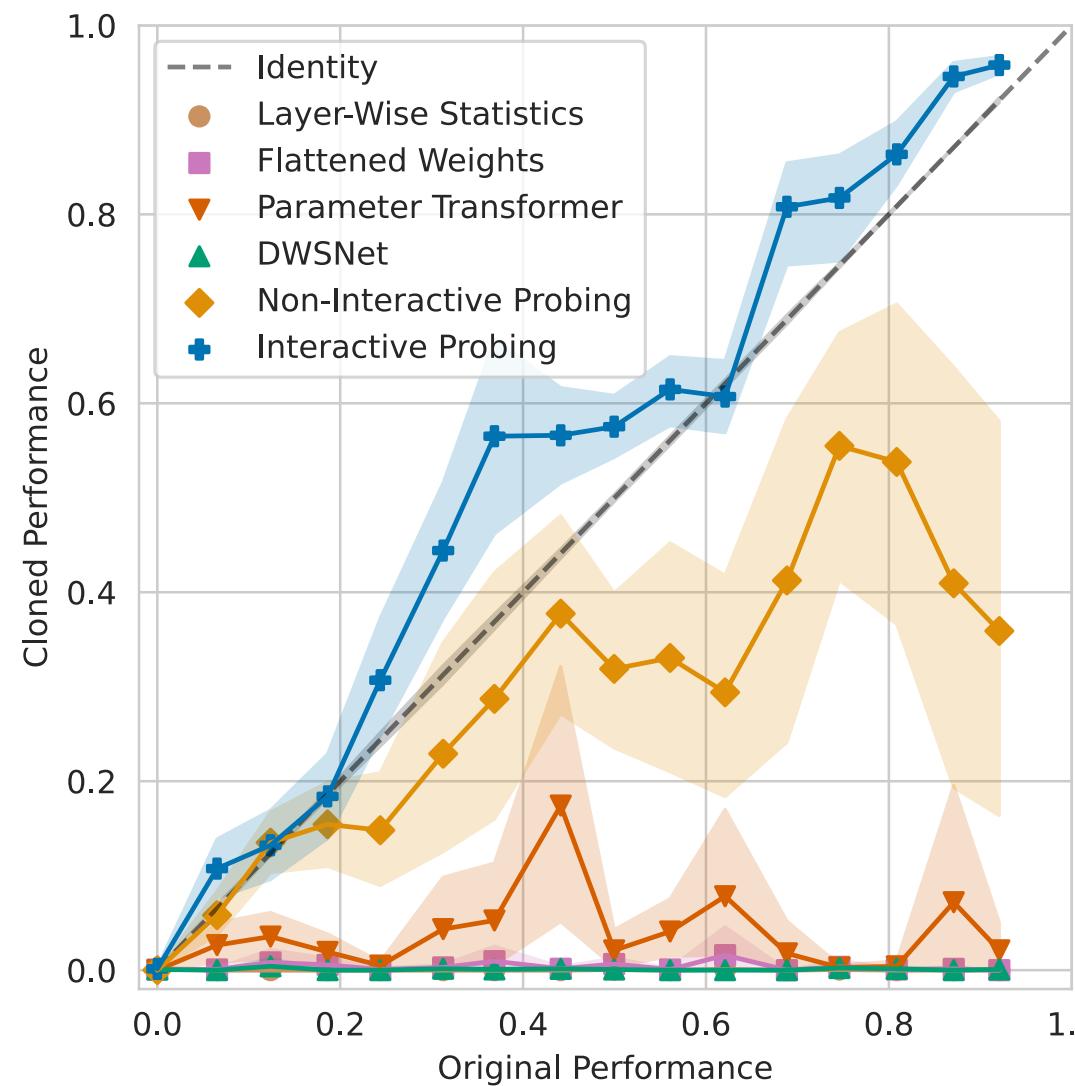
Results

- General upper bound for required interactions is the same
- In certain settings, interactive probing is exponentially more efficient

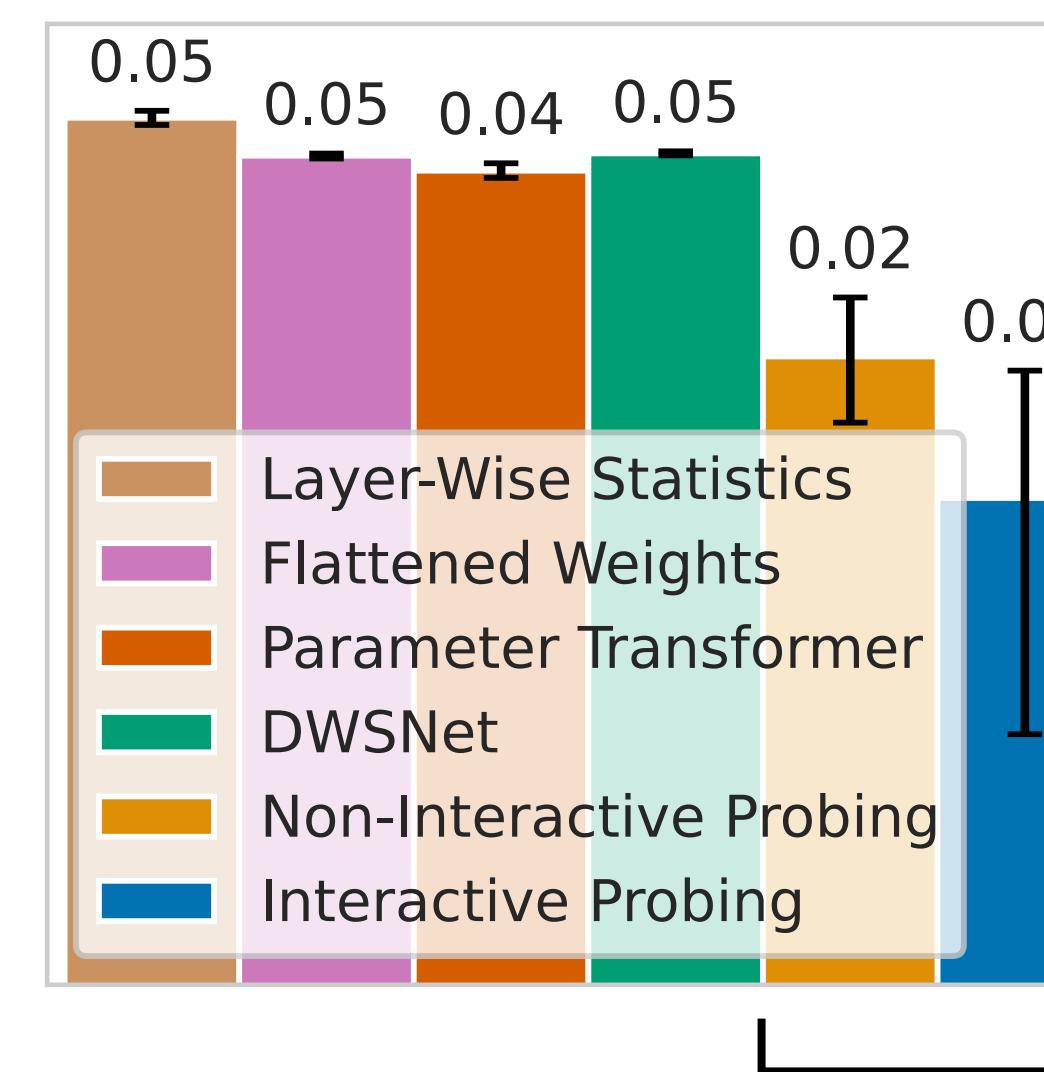
Pre-Training Results

Formal Languages

Original vs. Emulated



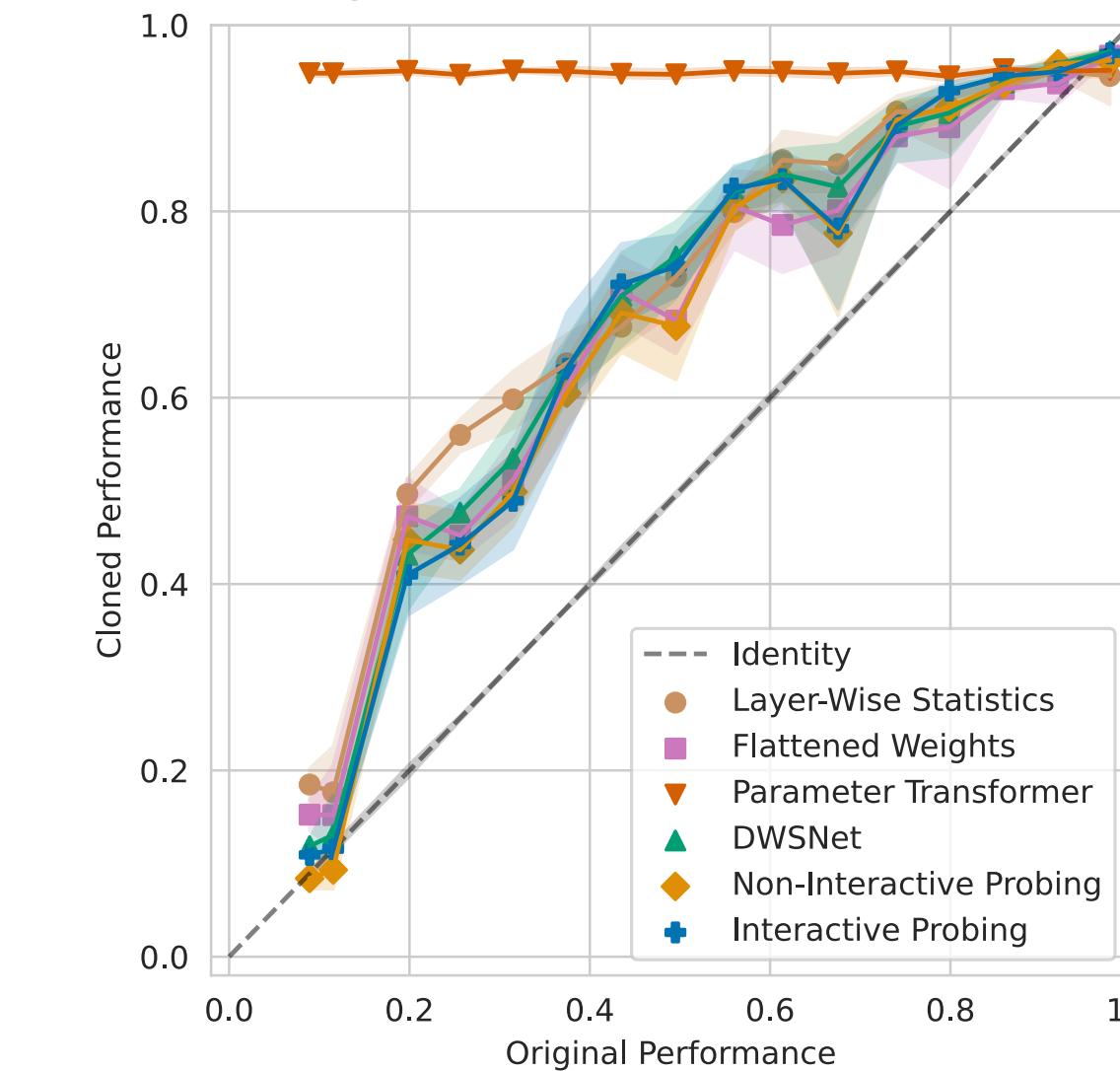
Validation Loss



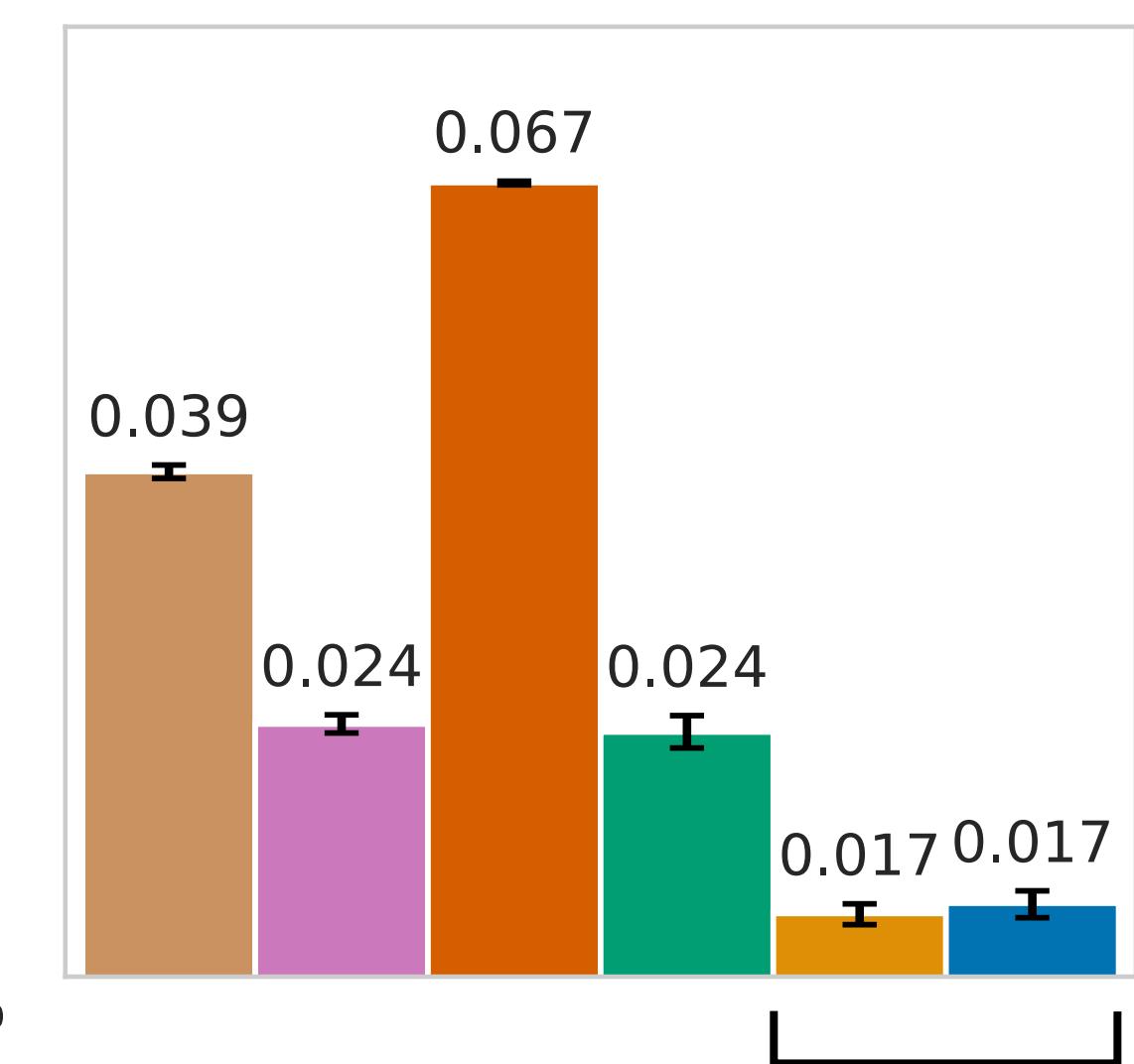
Functionalist

Sequential MNIST

Original vs. Emulated



Validation Loss

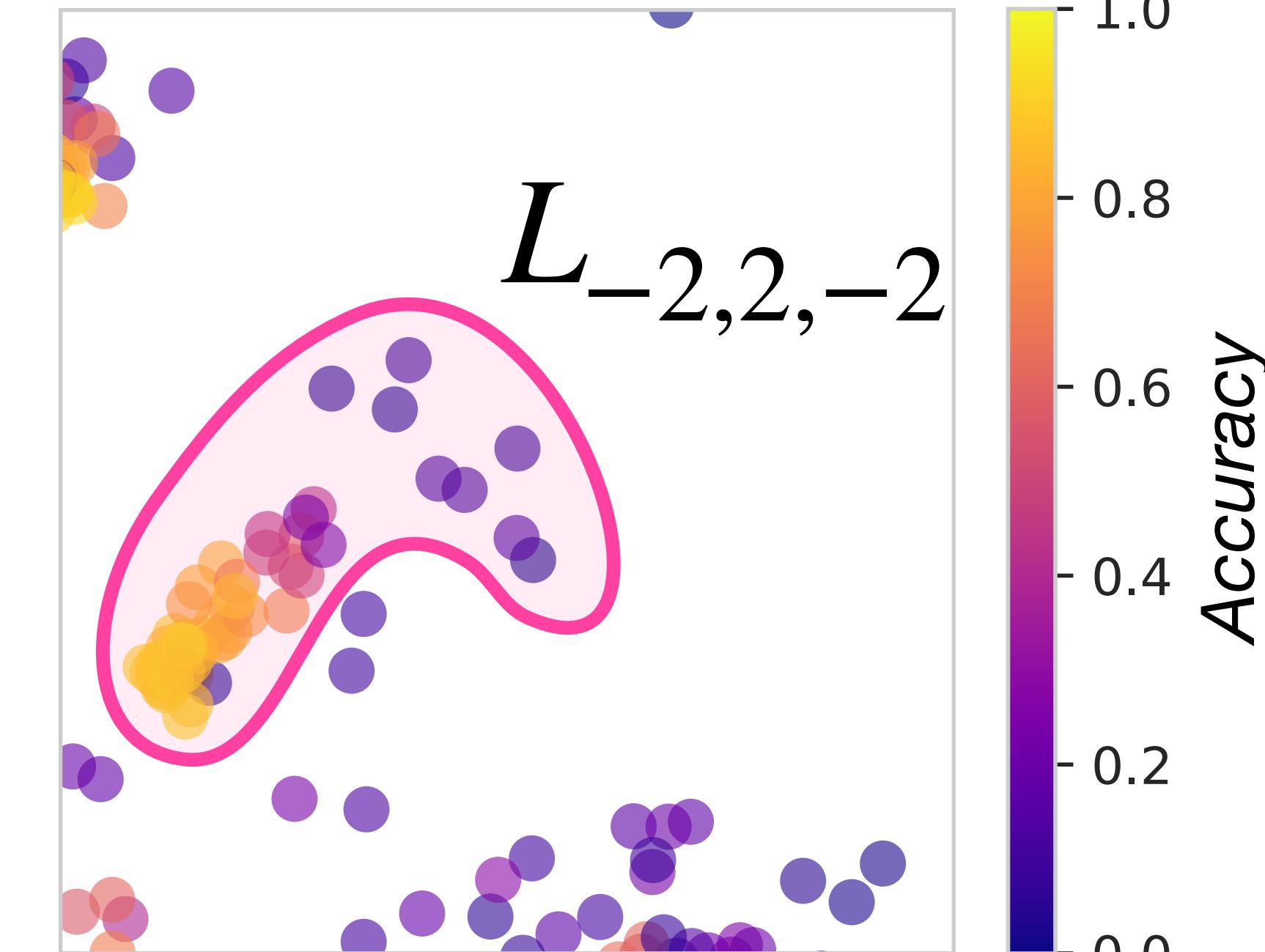
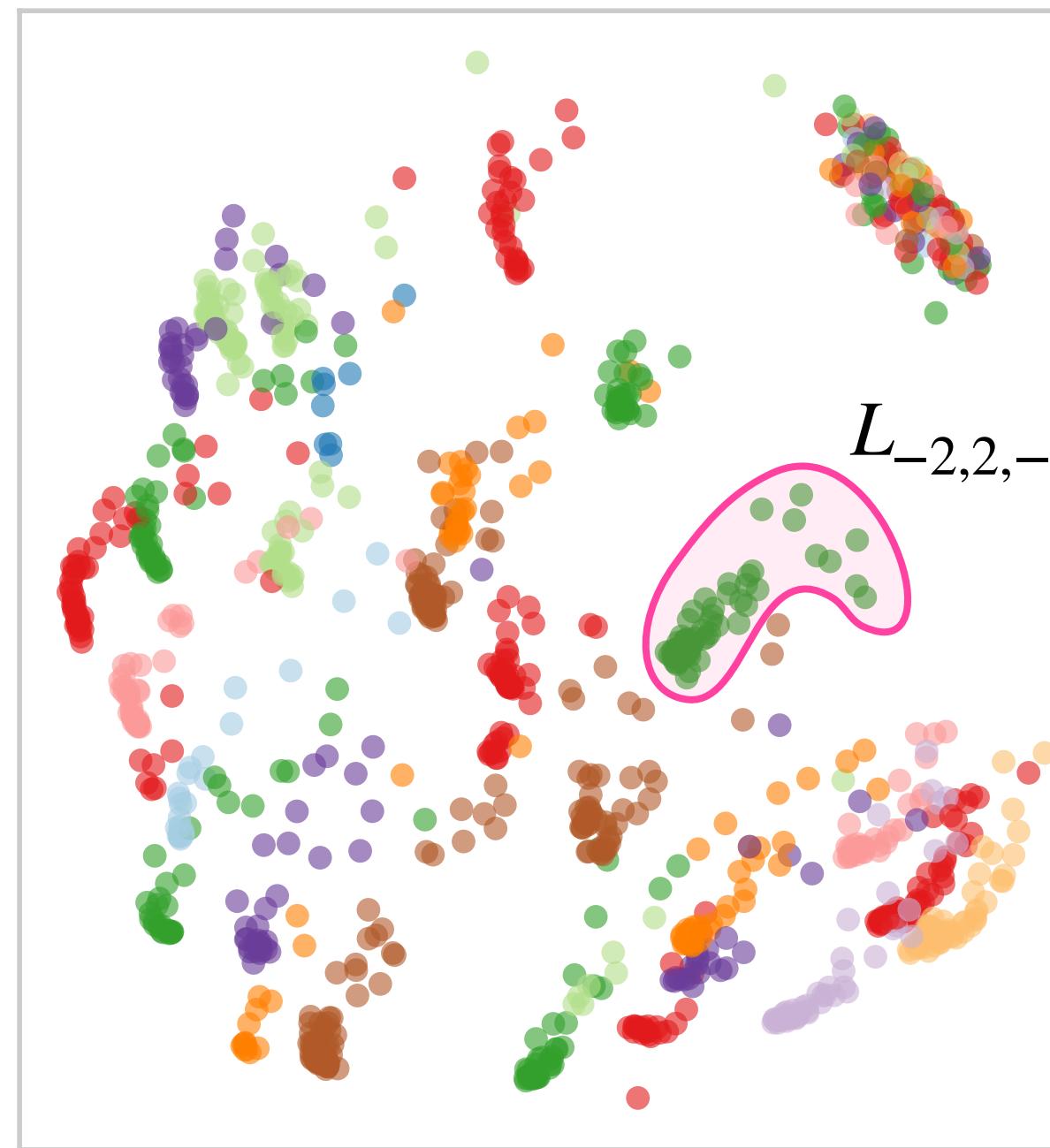


Functionalist

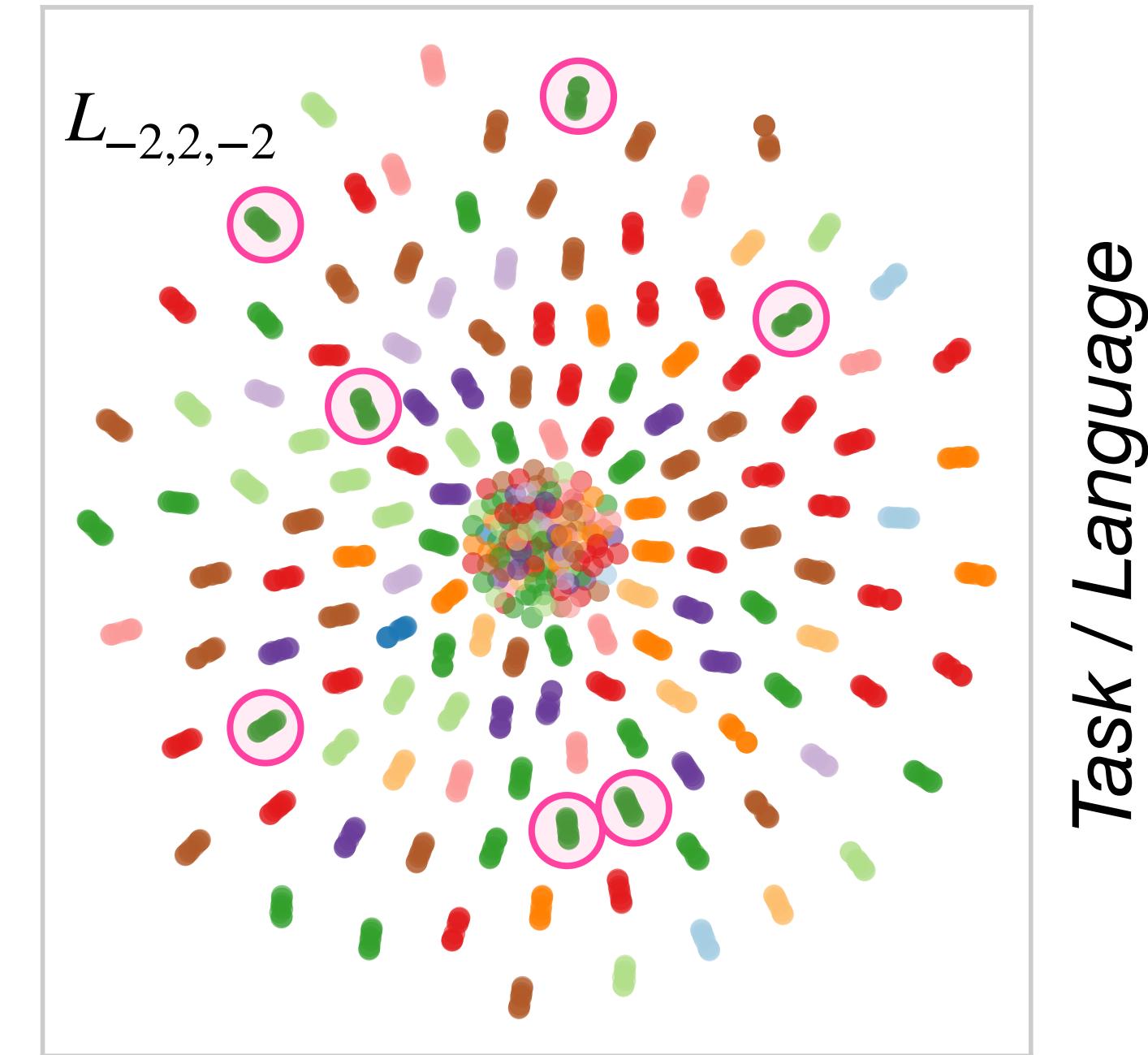
Learned Embedding Spaces

Formal Languages

Interactive Fingerprinting Embeddings (PCA)



Comparison:
t-SNE of weights

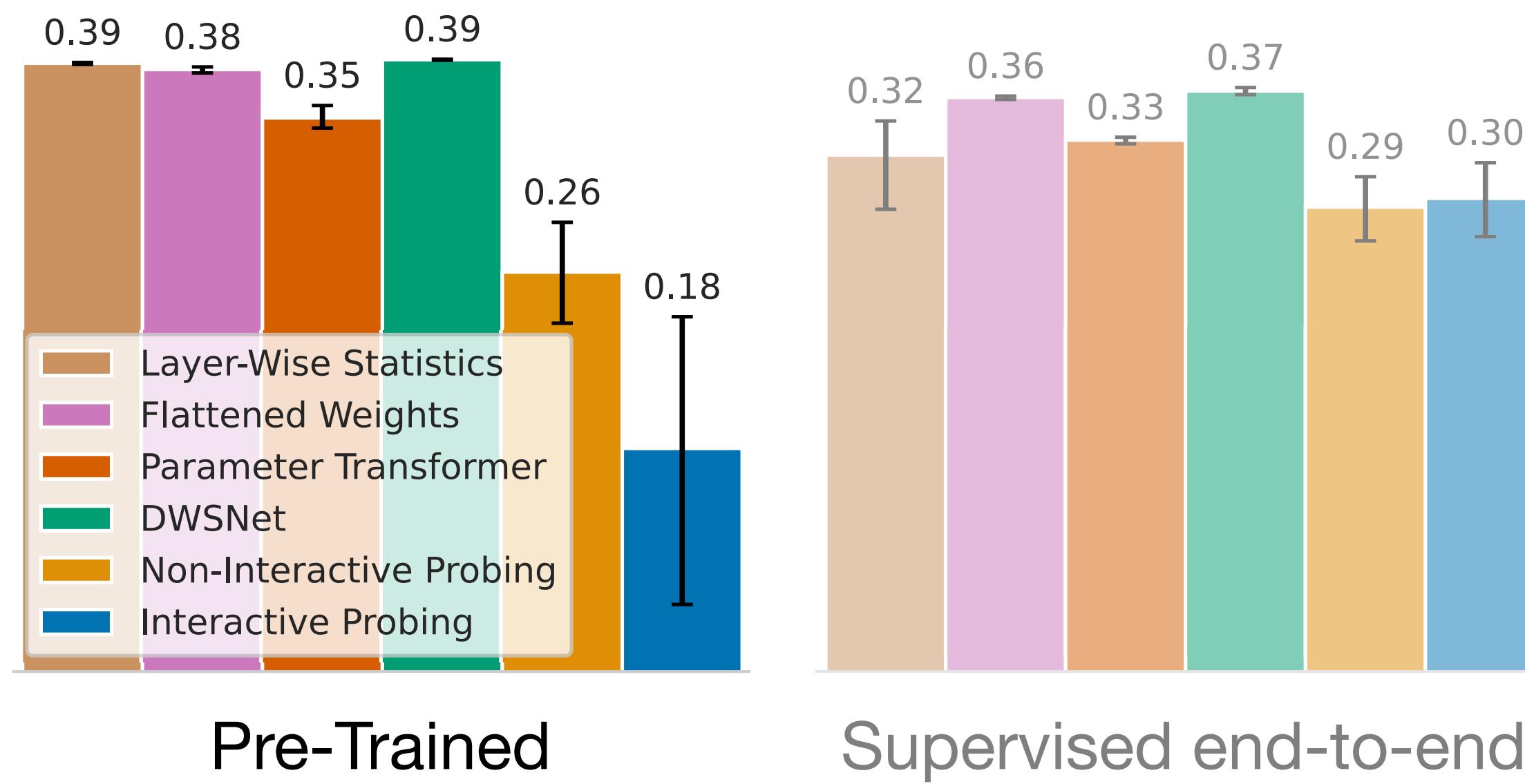


Downstream Results

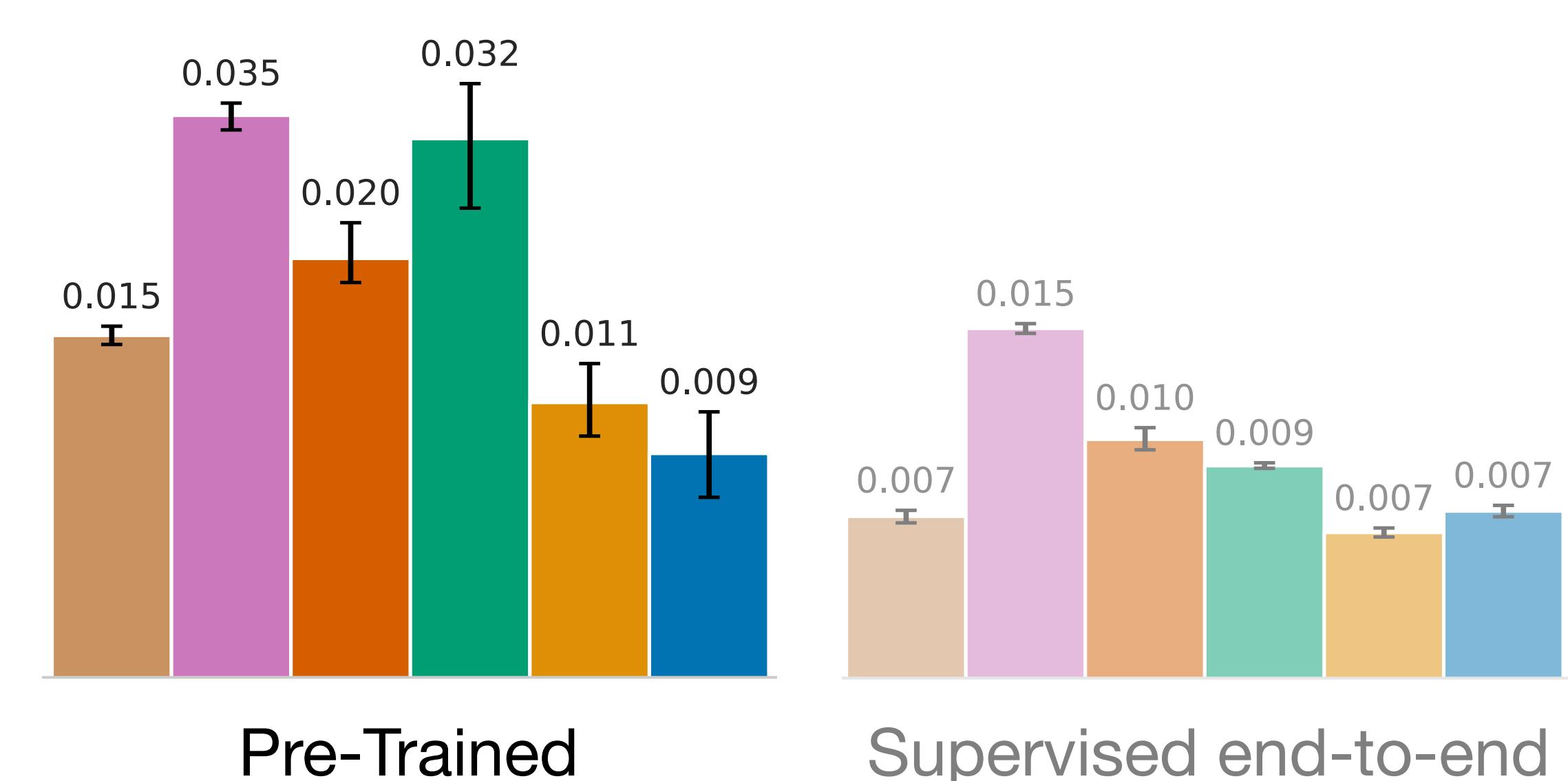
Predictor MLP on top of learned representations

Formal Languages

Task prediction loss



Accuracy prediction loss



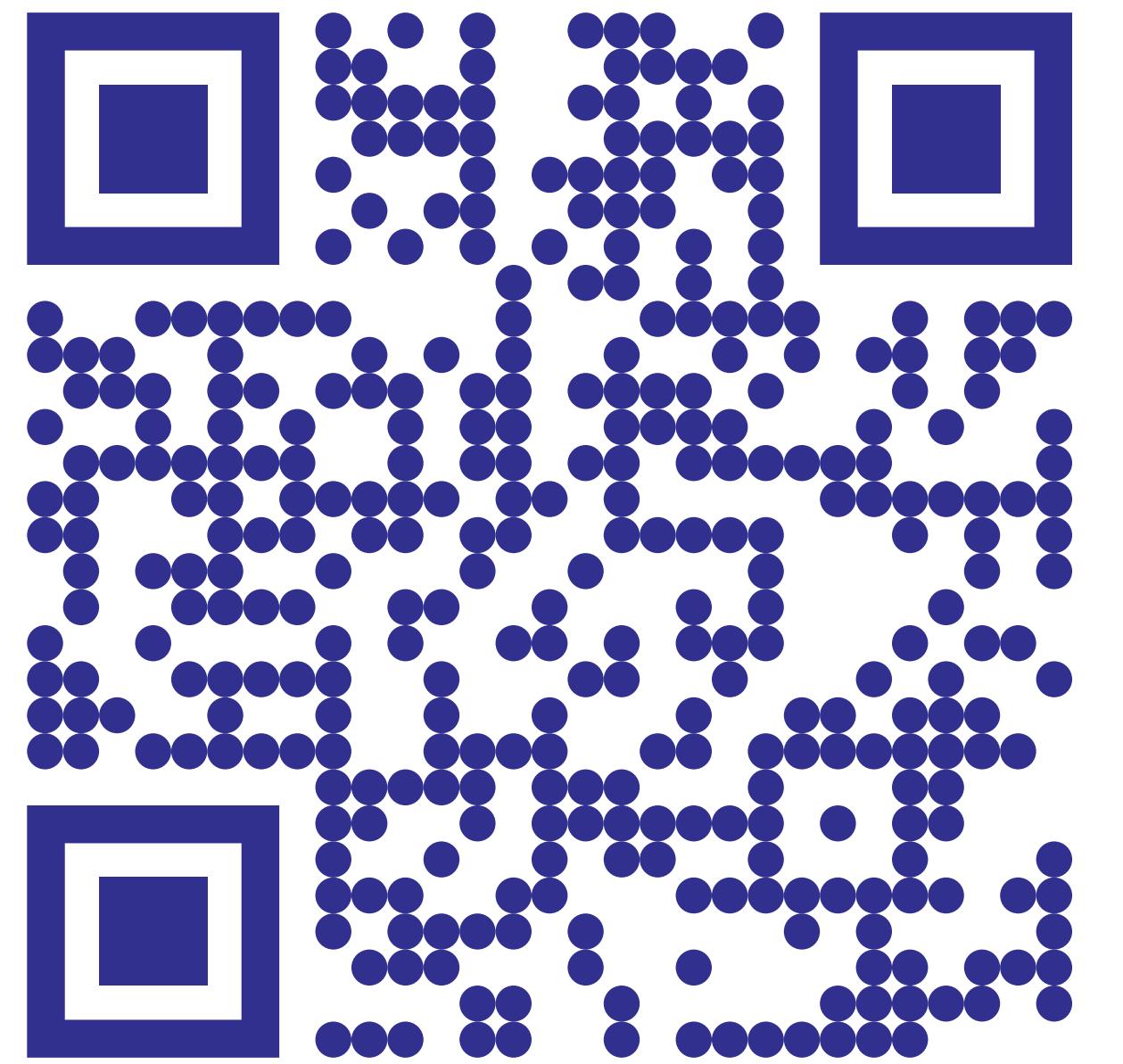
more results in the paper

Conclusion

Learning RNN Weight Representations

- Two RNN ‘Model Zoo’ datasets
- Emulation-based pre-training method
- Distinction between **mechanistic** and **functionalist** weight encoders
- Two novel functionalist encoder types
- Comparison of six different RNN weight encoder architectures

Functionalist encoders are superior at complex tasks



Paper

Thank you!



Code & Datasets