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1. (a) (i) $A = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$.

~~eigenvalue~~ eigenvalue: det$(A-\lambda I)=0$.

$\therefore (0.6-\lambda)(-0.6-\lambda)-0.8^2=0$.

$\lambda^2=1 \Rightarrow \lambda=\pm 1$

when $\lambda_1=1$,

$Av_1=\lambda_1 v_1$. $\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix} v_1 = v_1 \Rightarrow v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

when $\lambda_2=-1$

$Av_2=\lambda_2 v_2$ $\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix} v_2 = -v_2 \Rightarrow v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(ii) assume v_1 and λ_1 is the eigenvector and the eigenvalue of matrix A.

~~(iii) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 2-2=0$.~~

~~$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ are orthogonal.~~

$\because Av_1=\lambda_1 v_1$

$v_1^T A^T = v_1^T \lambda_1$

$\therefore v_1^T A^T A v_1 = \lambda_1^2 v_1^T v_1$.

$\because v_1^T v_1 = \lambda_1^2 v_1^T v_1$

$\therefore \lambda_1^2 = 1$

$\lambda_1 = \|1\|$.

(iii) $Av_1=\lambda_1 v_1$ $v_2^T A^T = \lambda_2 v_2^T$

$\therefore v_2^T A^T A v_1 = \lambda_1\lambda_2 v_2^T v_1$.

$v_2^T v_1 = \lambda_1\lambda_2 v_2^T v_1$

for the formula to be consist for all time,

$v_2^T v_1 = 0$. i.e., v_2; v_1 are orthogonal to each other.

(iv) the vector may rotate accordingly to the matrix A.

counter clockwisely.

(b) (i) The left singular vectors of A is the same as the eigenvectors of AA^T, and the right singular vector of A is the same as the eigenvectors of A^TA.

(ii) the singular values of A are the square root of eigenvalues of AA^T and A^TA.

(c). (i) F.

(ii) T

(iii) T

(iv) F

(v) F

2 (a) (i). $P(H50|T)$

$$= \frac{P(H50)P(T|H50)}{P(T)}$$

$P(T) = P(H50)P(T|H50) + P(H60)P(T|H60)$.

$P(H50) = P(H60) = \frac{1}{2}$, $P(T|H50) = 0.5$, $P(T|H60) = 0.4$

$$\therefore P(H50|T) = \frac{\frac{1}{2}\times 0.5}{\frac{1}{2}\times 0.5 + \frac{1}{2}\times 0.4} = \frac{5}{9}$$

(ii) $P(H50|T,H,H,H)$

$$= \frac{P(H50)P(T,H,H,H|H50)}{P(T,H,H,H)} = \frac{0.5^4}{0.5^4 + 0.4\times 0.6^3} = 0.42$$

(iii) $$P(H50|9H1T) = \frac{P(H50)P(9H1T|H50)}{P(9H1T)}$$

$$P(9H1T) = \frac{1}{3}\times 0.5^9 \times 0.5 + \frac{1}{3}\times 0.55^9\times 0.45 + \frac{1}{3}\times 0.6^9\times 0.4$$

$$\therefore P(H50|9H1T) = \frac{0.5^{10}}{0.5^9\times 0.5 + 0.55^9\times 0.45 + 0.6^9\times 0.4} = 0.138$$

$$P(H55|9H1T) = \frac{0.55^9\times 0.45}{0.5^9\times 0.5 + 0.55^9\times 0.45 + 0.6^9\times 0.4} = 0.293.$$

$$p(H60 \mid 9H1T) = \frac{0.6^9 \times 0.4}{0.5^{10} + 0.55^9 \times 0.45 + 0.6^9 \times 0.4} = 0.569$$

(b) $$p(\text{pregnant} \mid \text{positive}) = \frac{p(\text{positive} \mid \text{pregnant})\, p(\text{pregnant})}{p(\text{positive})}$$

$$= \frac{0.99 \times 0.01}{0.99 \times 0.01 + 0.99 \times 0.1} = \frac{1}{11}$$

The result makes sense because most of the female population (99%) is not pregnant, so even if the ~~test~~ shows positive, the non-pregnancy still takes ~~the~~ ~~larger~~ a larger aspect.

(c) $$E[Ax+b] = A \cdot E[x] + b.$$

(d) $$\mathrm{cov}(Ax+b) = A\,\mathrm{cov}(X)A^T$$

3. (a) $\nabla_x x^T A y = Ay.$

(b) $\nabla_y x^T A y = A^T x$

(c) $\nabla_A x^T A y = x y^T$

(d) $\nabla_x f = \cancel{Ax+b}.\ (A+A^T)x + b$

(e) $\nabla_A f = B^T$

4. $$\min_W \frac{1}{2} \sum_{i=1}^{n} \| y^{(i)} - W x^{(i)} \|^2$$

$$= \min_W \frac{1}{2} \| Y - WX \| \quad \text{where } Y = [y^{(1)}\ y^{(2)} \cdots y^{(n)}]$$

$$X = [x^{(1)}\ x^{(2)} \cdots x^{(n)}]$$

$\therefore$ ~~W~~ according to pseudo inverse

$$\cancel{x} \quad W = (XX^T)^{-1} X^T Y$$