(1) **E - Step**

$$\gamma_{ki} = p(c_k \mid x_i) = \frac{p(x_i \mid c_k)\, p(c_{k=1})}{p(x_n)} = \frac{N(x_i \mid \mu_k, \Sigma_k) \times \pi_k}{\sum_{k=1}^{K} \pi_k \times N(x_i \mid \mu_k, \Sigma_k)}$$

$$p(x_i \mid c_k) = N(x_i \mid \mu_k, \Sigma_k) = \frac{1}{(2\pi)^{D/2} \sqrt{|\Sigma_k|}}\, e^{\left(-\frac{1}{2}(x_i - \mu_k) \times \Sigma_k^{-1} \times (x_i - \mu_k)\right)}$$

onde $D = 2$

Para c_1:

$$|\Sigma_1| = 4 \times 4 - 1 \times 1 = 15 \qquad \Sigma_1^{-1} = \frac{1}{15} \times \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \qquad \mu_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$p\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi) \times \sqrt{15}}\, e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)^T \times \frac{1}{15} \times \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)\right)} = 0{,}02944$$

$$p\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi) \times \sqrt{15}}\, e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)^T \times \frac{1}{15} \times \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)\right)} = 0{,}00487$$

$$p\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi) \times \sqrt{15}}\, e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)^T \times \frac{1}{15} \times \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)\right)} = 0{,}03596$$

Para C_2:

$$|\Sigma_2| = 4 \qquad \Sigma^{-1} = \frac{1}{4} \times \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad \mu_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$p\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \middle| C_2\right) = \frac{1}{(2\pi) \times \sqrt{4}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)^T \times \frac{1}{4} \times \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)\right)} = 0{,}06197$$

$$p\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} \middle| C_2\right) = \frac{1}{(2\pi) \times \sqrt{4}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)^T \times \frac{1}{4} \times \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)\right)} = 0{,}04827$$

$$p\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} \middle| C_2\right) = \frac{1}{(2\pi) \times \sqrt{4}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)^T \times \frac{1}{4} \times \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)\right)} = 0{,}01077$$

$$\gamma_{11} = p(C_1 \mid x_1) = \frac{0{,}02944 \times 0{,}5}{0{,}02944 \times 0{,}5 + 0{,}06197 \times 0{,}5} = 0{,}322$$

$$\gamma_{21} = p(C_2 \mid x_1) = 1 - p(C_1 \mid x_1) = 0{,}678$$

$$\gamma_{12} = p(c_1 \mid x_1) = \frac{0{,}00487 \times 0{,}5}{0{,}00487 \times 0{,}5 + 0{,}04827 \times 0{,}5} = 0{,}092$$

$$\gamma_{22} = p(c_2 \mid x_1) = 1 - p(c_1 \mid x_1) = 0{,}908$$

$$\gamma_{12} = p(c_1 \mid x_1) = \frac{0{,}03596 \times 0{,}5}{0{,}03596 \times 0{,}5 + 0{,}01077 \times 0{,}5} = 0{,}769$$

$$\gamma_{22} = p(c_2 \mid x_1) = 1 - p(c_1 \mid x_1) = 0{,}231$$

M - Step

$$N_K = \sum_{i=1} \gamma_{Ki} \qquad \mu_K = \frac{1}{N_K} \sum_{i=1}^{n} \gamma_{Ki} \times x_i \qquad \pi_K = p(c_K) = \frac{N_K}{N}$$

$$\Sigma_K = \frac{1}{N_K} \sum_{i=1}^{n} \gamma_{Ki} \times (x_i - \mu_K) \times (x_i - \mu_K)^T$$

$$N_1 = 0{,}322 + 0{,}092 + 0{,}769 = 1{,}183$$

$$N_2 = 0{,}678 + 0{,}908 + 0{,}231 = 1{,}817$$

$$\pi_1 = 0,394 \qquad \pi_2 = 0,606$$

$$\mu_1 = \begin{bmatrix} (1,183)^{-1} \times (1 \times 0,322 + 0 \times 0,092 + 3 \times 0,769 \\ (1,183)^{-1} \times (0 \times 0,322 + 2 \times 0,092 - 1 \times 0,769 \end{bmatrix} = \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}$$

$$\mu_2 = \begin{bmatrix} (1,817)^{-1} \times (1 \times 0,678 + 0 \times 0,908 + 3 \times 0,231 \\ (1,817)^{-1} \times (0 \times 0,678 + 2 \times 0,908 - 1 \times 0,231 \end{bmatrix} = \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}$$

$$\Sigma_1 = \frac{1}{1,183} \times \left(0,322 \times \begin{bmatrix} -1,22 \\ 0,494 \end{bmatrix} \times \begin{bmatrix} -1,22 & 0,494 \end{bmatrix} + 0,092 \times \begin{bmatrix} -2,22 \\ 2,494 \end{bmatrix} \times \begin{bmatrix} -2,22 & 2,494 \end{bmatrix} + 0,769 \times \begin{bmatrix} 0,78 \\ -0,506 \end{bmatrix} \times \begin{bmatrix} 0,78 & -0,506 \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 1,182 & -0,851 \\ -0,851 & 0,717 \end{bmatrix}$$

$$\Sigma_2 = \frac{1}{1,817} \times \left(0,678 \times \begin{bmatrix} 0,246 \\ -0,872 \end{bmatrix} \times \begin{bmatrix} 0,246 & -0,872 \end{bmatrix} + 0,908 \times \begin{bmatrix} -0,754 \\ 1,128 \end{bmatrix} \times \begin{bmatrix} -0,754 & 1,128 \end{bmatrix} + 0,231 \times \begin{bmatrix} 2,246 \\ -1,872 \end{bmatrix} \begin{bmatrix} 2,246 & -1,872 \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 0,948 & -1,040 \\ -1,040 & 1,365 \end{bmatrix}$$

Updated: $\mu_1 = \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}$ $\pi_1 = 0,394$ $\pi_2 = 0,606$

$$\Sigma_1 = \begin{bmatrix} 1,182 & -0,851 \\ -0,851 & 0,717 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 0,948 & -1,040 \\ -1,040 & 1,365 \end{bmatrix}$$

E-Step

Para c_1:

$$|\Sigma_1| = 1,182 \times 0,717 - 0,851^2 = 0,1233 \qquad \Sigma_1^{-1} = \frac{1}{0,1233} \times \begin{bmatrix} 0,717 & 0,851 \\ 0,851 & 1,182 \end{bmatrix}$$

$$p\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi) \times \sqrt{0,1233}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}\right)^T \times \Sigma_1^{-1} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}\right)\right)} = 0,119$$

$$p\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi) \times \sqrt{0,1233}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}\right)^T \times \Sigma_1^{-1} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}\right)\right)} = 0,0012$$

$$p\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi) \times \sqrt{0,1233}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}\right)^T \times \Sigma_1^{-1} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2,22 \\ -0,494 \end{bmatrix}\right)\right)} = 0,345$$

Para c_2:

$$|\Sigma_2| = 0,948 \times 1,365 - 1,040^2 = 0,2124 \qquad \Sigma_2^{-1} = \frac{1}{0,2124} \times \begin{bmatrix} 1,365 & 1,040 \\ 1,040 & 0,948 \end{bmatrix}$$

$$p\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \middle| c_2\right) = \frac{1}{(2\pi) \times \sqrt{0,2124}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}\right)^T \times \Sigma_2^{-1} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}\right)\right)} = 0,149$$

$$p\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} \middle| c_2\right) = \frac{1}{(2\pi) \times \sqrt{0,2124}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}\right)^T \times \Sigma_2^{-1} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}\right)\right)} = 0,209$$

$$p\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} \middle| c_2\right) = \frac{1}{(2\pi) \times \sqrt{0,2124}} e^{\left(-\frac{1}{2} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}\right)^T \times \Sigma_2^{-1} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 0,754 \\ 0,872 \end{bmatrix}\right)\right)} = 0,011$$

$$\gamma_{11} = p(c_1 \mid x_1) = \frac{0,394 \times 0,119}{0,119 \times 0,394 + 0,149 \times 0,606} = 0,342$$

$$\gamma_{21} = p(c_2 \mid x_1) = 1 - p(c_1 \mid x_1) = 0,658$$

$$\gamma_{12} = p(c_1 \mid x_1) = \frac{0{,}0012 \times 0{,}344}{0{,}0012 \times 0{,}344 + 0{,}209 \times 0{,}606} = 0{,}004$$

$$\gamma_{22} = p(c_2 \mid x_1) = 1 - p(c_1 \mid x_1) = 0{,}996$$

$$\gamma_{12} = p(c_1 \mid x_1) = \frac{0{,}345 \times 0{,}344}{0{,}345 \times 0{,}344 + 0{,}606 \times 0{,}011} = 0{,}954$$

$$\gamma_{22} = p(c_2 \mid x_1) = 1 - p(c_1 \mid x_1) = 0{,}046$$

M - Step

$$N_1 = 0{,}342 + 0{,}004 + 0{,}954 = 1{,}3$$

$$N_2 = 0{,}658 + 0{,}996 + 0{,}046 = 1{,}7$$

$$\pi_1 = 0{,}433 \qquad \pi_2 = 0{,}567$$

$$\mu_1 = \begin{bmatrix} (1{,}3)^{-1} \times (1 \times 0{,}342 + 0 \times 0{,}004 + 3 \times 0{,}954 \\ (1{,}3)^{-1} \times (0 \times 0{,}342 + 2 \times 0{,}004 - 1 \times 0{,}954 \end{bmatrix} = \begin{bmatrix} 2{,}465 \\ -0{,}728 \end{bmatrix}$$

$$\mu_2 = \begin{bmatrix} (1{,}7)^{-1} \times (1 \times 0{,}658 + 0 \times 0{,}996 + 3 \times 0{,}046) \\ (1{,}7)^{-1} \times (0 \times 0{,}658 + 2 \times 0{,}996 - 1 \times 0{,}046) \end{bmatrix} = \begin{bmatrix} 0{,}468 \\ 1{,}145 \end{bmatrix}$$

$$\Sigma_1 = \frac{1}{1{,}3} \times \left(0{,}342 \times \begin{bmatrix} -1{,}465 \\ 0{,}728 \end{bmatrix} \times \begin{bmatrix} -1{,}465 & 0{,}728 \end{bmatrix} + 0{,}004 \times \begin{bmatrix} -2{,}465 \\ 2{,}728 \end{bmatrix} \times \begin{bmatrix} -2{,}465 & 2{,}721 \end{bmatrix} + 0{,}954 \times \begin{bmatrix} 0{,}535 \\ -0{,}272 \end{bmatrix} \times \begin{bmatrix} 0{,}535 & -0{,}272 \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 0{,}793 & -0{,}408 \\ -0{,}408 & 0{,}217 \end{bmatrix}$$

$$\Sigma_2 = \frac{1}{1{,}7} \times \left(0{,}658 \times \begin{bmatrix} 0{,}532 \\ -1{,}145 \end{bmatrix} \times \begin{bmatrix} 0{,}532 & -1{,}145 \end{bmatrix} + 0{,}996 \times \begin{bmatrix} -0{,}468 \\ 0{,}855 \end{bmatrix} \times \begin{bmatrix} -0{,}468 & 0{,}855 \end{bmatrix} + 0{,}046 \times \begin{bmatrix} 2{,}532 \\ -2{,}145 \end{bmatrix} \begin{bmatrix} 2{,}532 & -2{,}145 \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 0{,}411 & -0{,}617 \\ -0{,}617 & 1{,}060 \end{bmatrix}$$

$$\tilde{\pi}_1 = 1{,}3 \qquad \mu_1 = \begin{bmatrix} 2{,}465 \\ -0{,}728 \end{bmatrix} \qquad \mu_2 = \begin{bmatrix} 0{,}468 \\ 1{,}145 \end{bmatrix}$$

$$\pi_2 = 1{,}7$$

$$\Sigma_1 = \begin{bmatrix} 0{,}793 & -0{,}408 \\ -0{,}408 & 0{,}217 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 0{,}411 & -0{,}617 \\ -0{,}617 & 1{,}060 \end{bmatrix}$$

② a) $\tilde{\pi}_1 = 1,3$ $\qquad \mu_1 = \begin{bmatrix} 2,465 \\ -0,728 \end{bmatrix}$ $\qquad \mu_2 = \begin{bmatrix} 0,468 \\ 1,145 \end{bmatrix}$

$\pi_2 = 1,7$

$$\Sigma_1 = \begin{bmatrix} 0,793 & -0,408 \\ -0,408 & 0,217 \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} 0,411 & -0,617 \\ -0,617 & 1,060 \end{bmatrix}$$

Para c_1:

$$|\Sigma_1| = 0,0056 \qquad \Sigma_1^{-1} = \frac{1}{0,0056}\begin{bmatrix} 0,217 & 0,408 \\ 0,408 & 0,793 \end{bmatrix}$$

$$p\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi)\times\sqrt{0,0056}}\, e^{\left(-\frac{1}{2}\times\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2,467 \\ -0,728 \end{bmatrix}\right)\right)^T \times \Sigma_1^{-1} \times\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2,467 \\ -0,728 \end{bmatrix}\right)} = 0,519$$

$$p\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi)\times\sqrt{0,0056}}\, e^{\left(-\frac{1}{2}\times\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 2,461 \\ -0,728 \end{bmatrix}\right)\right)^T \times \Sigma_1^{-1} \times\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 2,461 \\ -0,728 \end{bmatrix}\right)} = 1,68\times 10^{-67}$$

$$p\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} \middle| c_1\right) = \frac{1}{(2\pi)\times\sqrt{0,0056}}\, e^{\left(-\frac{1}{2}\times\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2,461 \\ -0,728 \end{bmatrix}\right)\right)^T \times \Sigma_1^{-1} \times\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 2,461 \\ -0,728 \end{bmatrix}\right)} = 1,779$$

Para C_2:

$$|\Sigma_2| = 0{,}411 \times 1{,}06 - 0{,}617^2 = 0{,}055 \qquad \Sigma_2^{-1} = \frac{1}{0{,}055}\begin{bmatrix} 1{,}060 & 0{,}617 \\ 0{,}617 & 0{,}411 \end{bmatrix}$$

$$p\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \middle| C_2\right) = \frac{1}{(2\pi)\times\sqrt{0{,}055}}\, e^{\left(-\frac{1}{2}\times\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0{,}968 \\ 1{,}145 \end{bmatrix}\right)^T \times \Sigma_2^{-1} \times \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0{,}968 \\ 1{,}145 \end{bmatrix}\right)\right)} = 0{,}307$$

$$p\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} \middle| C_2\right) = \frac{1}{(2\pi)\times\sqrt{0{,}055}}\, e^{\left(-\frac{1}{2}\times\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0{,}968 \\ 1{,}145 \end{bmatrix}\right)^T \times \Sigma_2^{-1} \times \left(\begin{bmatrix} 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0{,}968 \\ 1{,}145 \end{bmatrix}\right)\right)} = 0{,}477$$

$$p\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} \middle| C_2\right) = \frac{1}{(2\pi)\times\sqrt{0{,}055}}\, e^{\left(-\frac{1}{2}\times\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 0{,}968 \\ 1{,}145 \end{bmatrix}\right)^T \times \Sigma_2^{-1} \times \left(\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 0{,}968 \\ 1{,}145 \end{bmatrix}\right)\right)} = 9{,}88 \times 10^{-9}$$

$$p(C_1 \mid x_1) = \frac{0{,}519 \times 1{,}3}{0{,}519 \times 1{,}3 + 0{,}307 \times 1{,}7} = 0{,}564 \quad \text{Logo} \quad x_1 \in C_1$$

$$p(C_2 \mid x_2) = 1 - p(C_1 \mid x_1) = 0{,}436$$

$$p(c_1 \mid x_3) \quad \frac{1{,}68 \times 10^{-67} \times 1{,}3}{1{,}68 \times 10^{-67} \times 1{,}3 + 0{,}477 \times 1{,}7} \simeq 0$$

$$p(c_2 \mid x_2) \simeq 1 \qquad x_2 \in c_2$$

$$p(c_1 \mid x_3) = \frac{1{,}779 \times 1{,}3}{1{,}779 \times 1{,}3 + 9{,}88 \times 10^{-9} \times 1{,}7} \simeq 1 \qquad x_3 \in c_1$$

$$p(c_2 \mid x_3) \simeq 0$$

$$\text{clusters} = \{ c_1 = \{ x_1, x_3 \}, c_2 = \{ x_2 \} \}$$

b)

$$s(x_i) = \frac{b(x_i) - a(x_i)}{\max\{b(x_i), a(x_i)\}}$$

Para x_1 :

$$b(x_1) = \| x_1 - x_2 \| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$a(x_1) = \| x_1 - x_3 \| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Luego $s(x_1) = 0$

Para x_3:

$$a(x_3) = \|x_3 - x_1\|_2 = \sqrt{5}$$

$$b(x_3) = \|x_3 - x_2\|_2 = \sqrt{3^2 + 3^2} = \sqrt{18}$$

Luego $s(x_3) = 1 - \dfrac{\sqrt{5}}{\sqrt{18}} = 0{,}473$

Luego $s(C_1) = \dfrac{0{,}473}{2} = 0{,}2365$