

ACM常用算法模板



字符串处理	3
1、KMP 算法	
2、扩展 KMP	
3、Manacher 最长回文子串	
4、AC 自动机	
5、后缀数组	
6、后缀自动机	
数学	
1、素数	
2、素数筛选和合数分解	
3、扩展欧几里得算法(求 ax+by=qcd 的解以及逆元	
4、求逆元	
5、模线性方程组	
6、随机素数测试和大数分解(POJ 1811)	
7、欧拉函数	
8、高斯消元(浮点数)	
9、FFT	
10、高斯消元法求方程组的解	
11、整数拆分	
12、求 A^B 的约数之和对 MOD 取模	
13、莫比乌斯反演	
14、Baby-Step Giant-Step	
相关公式	
数据结构	
1、划分树	
1、別が例 2、RMQ	
3、树链剖分	
4、伸展树(splay tree)	
4、甲胺树(spiay tree) 5、动态树	
6、主席树	
8论	
1、最短路	
1、取母的 2、最小生成树	
3、次小生成树 4、有向图的强连通分量	
4、有问图的强建通分重5、图的割点、桥和双连通分支的基本概念	
6、割点与桥	
8、点双连通分支	
9、最小树形图 10、二分图匹配	
11、生成树计数	
11、二分图多重匹配	
12、KM 算法(二分图最大权匹配)	
13、最大流	
14、最小费用最大流	
15、2-SAT	
16、曼哈顿最小生成树	
17、一般图匹配带花树	
18、LCA	
计算几何	
1、基本函数	
2、凸包	
3、平面最近点对(HDU 1007)	
4、旋转卡壳	

5、半平面交	131
6、三点求圆心坐标(三角形外心)	135
动态规划	136
1、最长上升子序列 O (nlogn)	136
其他	136
1、高精度	136
2、完全高精度	138
3、strtok 和 sscanf 结合输入	143
4、解决爆栈, 手动加栈	
5、STL	
6、输入输出外挂	145
7、莫队算法	146

字符串处理

1、KMP 算法

```
* next[]的含义: x[i-next[i]...i-1]=x[0...next[i]-1]
* next[i]为满足x[i-z...i-1]=x[0...z-1]的最大z值(就是x的自身匹配)
void kmp_pre(char x[],int m,int next[])
    int i,j;
    j=next[0]=-1;
    i=0;
   while(i<m)</pre>
       while(-1!=j && x[i]!=x[j])j=next[j];
       next[++i]=++j;
}
* kmpNext[]的意思: next'[i]=next[next[...[next[i]]]] (直到next'[i]<0或者
x[next'[i]]!=x[i])
* 这样的预处理可以快一些
void preKMP(char x[], int m, int kmpNext[])
    int i, j;
   j=kmpNext[0]=-1;
   i=0;
   while(i<m)</pre>
       while(-1!=j && x[i]!=x[j]) j=kmpNext[j];
       if (x[++i] ==x[++j]) kmpNext[i] = kmpNext[j];
       else kmpNext[i]=j;
    }
}
* 返回x在y中出现的次数,可以重叠
int next[10010];
```

```
int KMP_Count(char x[],int m,char y[],int n)
{//x是模式串,y是主串
   int i,j;
    int ans=0;
    //preKMP(x,m,next);
    kmp_pre(x,m,next);
    i=j=0;
    while(i<n)</pre>
        while (-1!=j \&\& y[i]!=x[j]) j=next[j];
        i++; j++;
       if(j>=m)
            ans++;
            j=next[j];
        }
    return ans;
}
经典题目: POJ 3167
* POJ 3167 Cow Patterns
* 模式串可以浮动的模式匹配问题
 * 给出模式串的相对大小,需要找出模式串匹配次数和位置
 * 比如说模式串: 1, 4, 4, 2, 3, 1 而主串: 5,6,2,10,10,7,3,2,9
 * 那么2,10,10,7,3,2就是匹配的
 * 统计比当前数小,和于当前数相等的,然后进行kmp
*/
#include <iostream>
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <vector>
using namespace std;
const int MAXN=100010;
const int MAXM=25010;
int a[MAXN];
int b[MAXN];
int n,m,s;
int as[MAXN] [30];
int bs[MAXM][30];
void init()
{
    for (int i=0; i<n; i++)</pre>
        if(i==0)
            for(int j=1; j<=25; j++) as[i][j]=0;</pre>
        }
        else
            for(int j=1; j<=25; j++) as[i][j]=as[i-1][j];</pre>
        as[i][a[i]]++;
```

```
}
    for (int i=0; i<m; i++)</pre>
         if(i==0)
         {
             for(int j=1; j<=25; j++)bs[i][j]=0;</pre>
        else
         {
             for(int j=1; j<=25; j++)bs[i][j]=bs[i-1][j];</pre>
        bs[i][b[i]]++;
int next[MAXM];
void kmp_pre()
    int i,j;
    j = next[0] = -1;
    i=0;
    while(i<m)</pre>
         int t11=0,t12=0,t21=0,t22=0;
         for (int k=1; k<b[i]; k++)</pre>
             if(i-j>0)t11+=bs[i][k]-bs[i-j-1][k];
             else t11+=bs[i][k];
         if (i-j>0) t12=bs[i][b[i]]-bs[i-j-1][b[i]];
        else t12=bs[i][b[i]];
         for (int k=1; k<b[j];k++)</pre>
             t21+=bs[j][k];
         t22=bs[j][b[j]];
         if(j==-1 || (t11==t21&&t12==t22))
            next[++i]=++j;
        else j=next[j];
    }
vector<int>ans;
void kmp()
    ans.clear();
    int i,j;
    kmp_pre();
    i=j=0;
    while(i<n)</pre>
         int t11=0,t12=0,t21=0,t22=0;
         for (int k=1; k<a[i];k++)</pre>
             if (i-j>0) t11+=as[i][k]-as[i-j-1][k];
             else t11+=as[i][k];
         }
```

```
if(i-j>0)t12=as[i][a[i]]-as[i-j-1][a[i]];
        else t12=as[i][a[i]];
        for (int k=1; k<b[j]; k++)</pre>
             t21+=bs[j][k];
        t22=bs[j][b[j]];
        if(j==-1 || (t11==t21&&t12==t22))
             i++; j++;
             if(j>=m)
                 ans.push back(i-m+1);
                 j=next[j];
             }
        else j=next[j];
    }
}
int main()
    while (scanf("%d%d%d", &n, &m, &s) ==3)
        for (int i=0; i<n; i++)</pre>
             scanf("%d",&a[i]);
        for (int i=0; i<m; i++)</pre>
             scanf("%d",&b[i]);
        init();
        kmp();
        printf("%d\n", ans.size());
        for(int i=0;i<ans.size();i++)</pre>
             printf("%d\n", ans[i]);
    }
    return 0;
}
```

2、扩展 KMP

```
/*
 * 扩展KMP算法
 */
//next[i]:x[i...m-1]与x[0...m-1]的最长公共前缀
//extend[i]:y[i...n-1]与x[0...m-1]的最长公共前缀
void pre_EKMP(char x[],int m,int next[])
{
    next[0]=m;
    int j=0;
    while(j+1<m && x[j]==x[j+1])j++;
    next[1]=j;
    int k=1;
    for(int i=2;i<m;i++)
```

```
{
        int p=next[k]+k-1;
        int L=next[i-k];
        if(i+L<p+1)next[i]=L;</pre>
        else
             j=max(0,p-i+1);
             while(i+j<m && x[i+j] ==x[j])j++;</pre>
             next[i]=j;
            k=i;
        }
    }
void EKMP(char x[],int m,char y[],int n,int next[],int extend[])
    pre EKMP(x,m,next);
    int j=0;
    while(j < n \&\& j < m \&\& x[j] == y[j]) j++;
    extend[0]=j;
    int k=0;
    for (int i=1; i<n; i++)</pre>
        int p=extend[k]+k-1;
        int L=next[i-k];
        if(i+L<p+1)extend[i]=L;</pre>
        else
             j=max(0,p-i+1);
             while(i+j<n && j<m && y[i+j]==x[j])j++;</pre>
             extend[i]=j;
             k=i;
        }
   }
}
```

3、Manacher 最长回文子串

```
* 求最长回文子串
const int MAXN=110010;
char Ma [MAXN*2];
int Mp[MAXN*2];
void Manacher(char s[],int len)
    int 1=0;
    Ma[l++]='$';
    Ma[l++]='#';
    for (int i=0; i<len; i++)</pre>
        Ma[l++]=s[i];
        Ma[1++] = ' # ';
    Ma[1]=0;
    int mx=0,id=0;
    for (int i=0; i<1; i++)</pre>
```

```
{
        Mp[i]=mx>i?min(Mp[2*id-i],mx-i):1;
        while (Ma[i+Mp[i]] ==Ma[i-Mp[i]]) Mp[i]++;
        if(i+Mp[i]>mx)
            mx=i+Mp[i];
            id=i;
    }
}
/*
* <u>abaaba</u>
* i: 0 1 2 3 4 5 6 7 8 9 10 11 12 13
* <u>Ma</u>[i]: $ # a # b # a # a $ b # a #
 * Mp[i]: 1 1 2 1 4 1 2 7 2 1 4 1 2 1
*/
char s[MAXN];
int main()
{
    while (scanf("%s", s) == 1)
        int len=strlen(s);
        Manacher(s,len);
        int ans=0;
        for (int i=0; i<2*len+2;i++)</pre>
            ans=max(ans, Mp[i]-1);
        printf("%d\n",ans);
   return 0;
}
```

4、AC 自动机

```
// HDU 2222
// 求目标串中出现了几个模式串
#include <stdio.h>
#include <algorithm>
#include <iostream>
#include <string.h>
#include <queue>
using namespace std;
struct Trie
   int next[500010][26], fail[500010], end[500010];
   int root, L;
   int newnode()
      for(int i = 0;i < 26;i++)</pre>
         next[L][i] = -1;
      end[L++] = 0;
      return L-1;
   void init()
```

```
{
    L = 0;
    root = newnode();
void insert(char buf[])
    int len = strlen(buf);
    int now = root;
    for(int i = 0;i < len;i++)</pre>
        if (next [now] [buf[i]-'a'] == -1)
           next[now][buf[i]-'a'] = newnode();
        now = next[now][buf[i]-'a'];
    end[now]++;
void build()
    queue<int>Q;
    fail[root] = root;
    for(int i = 0;i < 26;i++)</pre>
        if(next[root][i] == -1)
            next[root][i] = root;
        else
        {
            fail[next[root][i]] = root;
            Q.push(next[root][i]);
    while( !Q.empty() )
        int now = Q.front();
        Q.pop();
        for(int i = 0;i < 26;i++)</pre>
            if (next[now][i] == -1)
                next[now][i] = next[fail[now]][i];
            else
                fail[next[now][i]]=next[fail[now]][i];
                Q.push(next[now][i]);
            }
}
int query(char buf[])
    int len = strlen(buf);
    int now = root;
    int res = 0;
    for(int i = 0;i < len;i++)</pre>
        now = next[now][buf[i]-'a'];
        int temp = now;
        while( temp != root )
            res += end[temp];
            end[temp] = 0;
            temp = fail[temp];
```

```
return res;
    }
    void debug()
        for(int i = 0;i < L;i++)</pre>
             printf("id = %3d, fail = %3d, end = %3d, chi = [",i,fail[i],end[i]);
             for(int j = 0; j < 26; j++)</pre>
                 printf("%2d",next[i][j]);
             printf("]\n");
        }
    }
};
char buf[1000010];
Trie ac;
int main()
    int T;
    int n;
    scanf("%d",&T);
    while ( T-- )
        scanf("%d", &n);
        ac.init();
        for(int i = 0;i < n;i++)</pre>
             scanf("%s",buf);
             ac.insert(buf);
        ac.build();
        scanf("%s",buf);
        printf("%d\n", ac.query(buf));
    return 0;
}
```

5、后缀数组

5.1 DA 算法

```
/*
*suffix array
*倍增算法 O(n*logn)
*待排序数组长度为n,放在0~n-1中,在最后面补一个0
*da(str ,n+1,sa,rank,height, , );//注意是n+1;
*例如:
*n = 8;
*num[] = { 1, 1, 2, 1, 1, 1, 1, 2, $ };注意num最后一位为0,其他大于0
*rank[] = { 4, 6, 8, 1, 2, 3, 5, 7, 0 };rank[0~n-1]为有效值,rank[n]必定为0无效值

*sa[] = { 8, 3, 4, 5, 0, 6, 1, 7, 2 };sa[1~n]为有效值,sa[0]必定为n是无效值
*height[]= { 0, 0, 3, 2, 3, 1, 2, 0, 1 };height[2~n]为有效值
*
*/
const int MAXN=20010;
int t1[MAXN],t2[MAXN],c[MAXN];//求SA数组需要的中间变量,不需要赋值
//待排序的字符串放在s数组中,从s[0]到s[n-1],长度为n,且最大值小于m,
```

```
//除s[n-1]外的所有s[i]都大于0, r[n-1]=0
//函数结束以后结果放在sa数组中
bool cmp(int *r,int a,int b,int 1)
    return r[a] == r[b] && r[a+1] == r[b+1];
void da(int str[],int sa[],int rank[],int height[],int n,int m)
{
   n++;
   int i, j, p, *x = t1, *y = t2;
   //第一轮基数排序,如果s的最大值很大,可改为快速排序
   for (i = 0; i < m; i++)c[i] = 0;
   for(i = 0;i < n;i++)c[x[i] = str[i]]++;</pre>
   for(i = 1;i < m;i++)c[i] += c[i-1];</pre>
   for (i = n-1; i >= 0; i--) sa[--c[x[i]]] = i;
   for(j = 1; j <= n; j <<= 1)
      p = 0;
       //直接利用sa数组排序第二关键字
       for(i = n-j; i < n; i++)y[p++] = i;//后面的j个数第二关键字为空的最小
       for(i = 0; i < n; i++)if(sa[i] >= j)y[p++] = sa[i] - j;
       //这样数组y保存的就是按照第二关键字排序的结果
       //基数排序第一关键字
       for(i = 0; i < m; i++)c[i] = 0;</pre>
       for(i = 0; i < n; i++)c[x[y[i]]]++;</pre>
       for(i = 1; i < m;i++)c[i] += c[i-1];</pre>
       for(i = n-1; i >= 0;i--)sa[--c[x[y[i]]]] = y[i];
       //根据sa和x数组计算新的x数组
       swap(x, y);
       p = 1; x[sa[0]] = 0;
       for(i = 1;i < n;i++)
          x[sa[i]] = cmp(y,sa[i-1],sa[i],j)?p-1:p++;
       if(p >= n)break;
      m = p;//下次基数排序的最大值
   int k = 0;
   n--;
   for(i = 0;i <= n;i++) rank[sa[i]] = i;</pre>
   for(i = 0;i < n;i++)</pre>
       if(k)k--;
       j = sa[rank[i]-1];
       while (str[i+k] == str[j+k])k++;
      height[rank[i]] = k;
   }
}
int rank[MAXN], height[MAXN];
int RMQ[MAXN];
int mm[MAXN];
int best[20][MAXN];
void initRMQ(int n)
{
   mm[0] = -1;
   for (int i=1; i<=n; i++)</pre>
      mm[i] = ((i&(i-1)) == 0)?mm[i-1]+1:mm[i-1];
   for (int i=1; i<=n; i++) best[0][i]=i;</pre>
   for (int i=1; i<=mm[n]; i++)</pre>
       for (int j=1; j+(1<<i) -1<=n; j++)</pre>
                                     11 / 152
```

```
{
           int a=best[i-1][j];
           int b=best[i-1][j+(1<<(i-1))];</pre>
           if (RMQ[a] < RMQ[b]) best[i][j] = a;</pre>
           else best[i][j]=b;
       }
int askRMQ(int a, int b)
   int t;
   t=mm[b-a+1];
   b = (1 << t) -1;
   a=best[t][a];b=best[t][b];
   return RMQ[a]<RMQ[b]?a:b;</pre>
}
int lcp(int a,int b)
   a=rank[a];b=rank[b];
   if(a>b) swap(a,b);
   return height[askRMQ(a+1,b)];
char str[MAXN];
int r[MAXN];
int sa[MAXN];
int main()
   while(scanf("%s", str) == 1)
       int len = strlen(str);
       int n = 2*len + 1;
       for(int i = 0;i < len;i++)r[i] = str[i];</pre>
       for(int i = 0;i < len;i++)r[len + 1 + i] = str[len - 1 - i];</pre>
       r[len] = 1;
       r[n] = 0;
       da(r,sa,rank,height,n,128);
        for (int i=1; i<=n; i++) RMQ[i]=height[i];</pre>
       initRMQ(n);
       int ans=0,st;
       int tmp;
       for(int i=0; i<len; i++)</pre>
           tmp=lcp(i,n-i);//偶对称
           if(2*tmp>ans)
              ans=2*tmp;
              st=i-tmp;
           tmp=lcp(i,n-i-1);//奇数对称
           if(2*tmp-1>ans)
              ans=2*tmp-1;
              st=i-tmp+1;
           }
       str[st+ans]=0;
       printf("%s\n",str+st);
   return 0;
```

```
}
   5.2 DC3 算法
da[]和str[]数组要开大三倍,相关数组也是三倍
 * 后缀数组
* DC3算法,复杂度O(n)
 * 所有的相关数组都要开三倍
*/
const int MAXN = 2010;
#define F(x) ((x)/3+((x)%3==1?0:tb))
#define G(x) ((x) < tb? (x) *3+1: ((x) -tb) *3+2)
int wa[MAXN*3], wb[MAXN*3], wv[MAXN*3], wss[MAXN*3];
int c0(int *r,int a,int b)
    return r[a] == r[b] && r[a+1] == r[b+1] && r[a+2] == r[b+2];
int c12 (int k,int *r,int a,int b)
    if(k == 2)
       return r[a] < r[b] || ( r[a] == r[b] && c12(1, r, a+1, b+1) );
    else return r[a] < r[b] \mid | (r[a] == r[b] && wv[a+1] < wv[b+1] );
void sort(int *r, int *a, int *b, int n, int m)
{
    int i;
    for(i = 0;i < n;i++)wv[i] = r[a[i]];</pre>
    for(i = 0;i < m;i++)wss[i] = 0;</pre>
    for(i = 0;i < n;i++)wss[wv[i]]++;</pre>
    for(i = 1;i < m;i++)wss[i] += wss[i-1];</pre>
    for(i = n-1; i >= 0;i--)
        b[--wss[wv[i]]] = a[i];
}
void dc3(int *r,int *sa,int n,int m)
    int i, j, *rn = r + n;
    int *san = sa + n, ta = 0, tb = (n+1)/3, tbc = 0, p;
    r[n] = r[n+1] = 0;
    for(i = 0; i < n; i++) if(i %3 != 0) wa[tbc++] = i;
    sort(r + 2, wa, wb, tbc, m);
    sort(r + 1, wb, wa, tbc, m);
    sort(r, wa, wb, tbc, m);
    for (p = 1, rn[F(wb[0])] = 0, i = 1; i < tbc; i++)
        rn[F(wb[i])] = c0(r, wb[i-1], wb[i]) ? p - 1 : p++;
    if(p < tbc)dc3(rn,san,tbc,p);</pre>
    else for(i = 0;i < tbc;i++)san[rn[i]] = i;
    for(i = 0; i < tbc; i++) if(san[i] < tb)wb[ta++] = san[i] * 3;
   if (n % 3 == 1) wb[ta++] = n - 1;
    sort(r, wb, wa, ta, m);
    for(i = 0; i < tbc; i++)wv[wb[i] = G(san[i])] = i;
    for(i = 0, j = 0, p = 0;i < ta && j < tbc;p++)</pre>
        sa[p] = c12(wb[j] % 3, r, wa[i], wb[j]) ? wa[i++] : wb[j++];
    for(;i < ta;p++)sa[p] = wa[i++];</pre>
    for(; j < tbc; p++) sa[p] = wb[j++];</pre>
}
//str和sa也要三倍
void da(int str[],int sa[],int rank[],int height[],int n,int m)
{
```

```
for(int i = n;i < n*3;i++)</pre>
        str[i] = 0;
    dc3(str, sa, n+1, m);
    int i,j,k = 0;
    for(i = 0;i <= n;i++) rank[sa[i]] = i;</pre>
    for(i = 0;i < n; i++)</pre>
        if(k) k--;
        j = sa[rank[i]-1];
        while (str[i+k] == str[j+k]) k++;
        height[rank[i]] = k;
}
```

6、后缀自动机

```
const int CHAR = 26;
const int MAXN = 250010;
struct SAM Node
   SAM_Node *fa,*next[CHAR];
   int len;
   int id, pos;
   SAM Node(){}
   SAM_Node(int _len)
      fa = 0;
      len = len;
      memset(next, 0, sizeof(next));
};
SAM_Node SAM_node[MAXN*2], *SAM_root, *SAM_last;
int SAM size;
SAM Node *newSAM Node (int len)
   SAM node[SAM size] = SAM Node(len);
   SAM node[SAM size].id = SAM size;
   return &SAM node[SAM size++];
}
SAM_Node *newSAM_Node (SAM_Node *p)
   SAM node[SAM size] = *p;
   SAM_node[SAM_size].id = SAM_size;
   return &SAM_node[SAM_size++];
void SAM init()
   SAM size = 0;
   SAM root = SAM last = newSAM Node(0);
   SAM_node[0].pos = 0;
void SAM add(int x,int len)
   SAM_Node *p = SAM_last, *np = newSAM_Node(p->len+1);
   np->pos = len;
   SAM last = np;
                                     14 / 152
```

```
for(;p && !p->next[x];p = p->fa)
       p->next[x] = np;
   if(!p)
       np->fa = SAM root;
       return;
   SAM_Node *q = p->next[x];
   if(q->len == p->len + 1)
       np->fa = q;
       return;
   SAM Node *nq = newSAM Node(q);
   nq->len = p->len + 1;
   q \rightarrow fa = nq;
   np->fa = nq;
   for(;p && p->next[x] == q;p = p->fa)
       p->next[x] = nq;
void SAM build(char *s)
   SAM_init();
   int len = strlen(s);
   for (int i = 0;i < len;i++)</pre>
       SAM add (s[i] - 'a', i+1);
}
//加入串后进行拓扑排序。
char str[MAXN];
int topocnt[MAXN];
SAM Node *topsam[MAXN*2];
      int n = strlen(str);
       SAM_build(str);
       memset(topocnt, 0, sizeof(topocnt));
       for(int i = 0;i < SAM size;i++)</pre>
           topocnt[SAM_node[i].len]++;
       for(int i = 1;i <= n;i++)</pre>
          topocnt[i] += topocnt[i-1];
       for(int i = 0;i < SAM size;i++)</pre>
          topsam[--topocnt[SAM_node[i].len]] = &SAM_node[i];
```

数学

1、素数

1.1 素数筛选 (判断<MAXN 的数是否素数)

```
void init()
   memset(notprime, false, size of (notprime));
   notprime[0]=notprime[1]=true;
   for (int i=2; i<MAXN; i++)</pre>
      if(!notprime[i])
          if(i>MAXN/i) continue;//防止后面i*i溢出(或者i,i用long long)
          //直接从i*i开始就可以,小于i倍的已经筛选过了,注意是j+=i
          for(int j=i*i;j<MAXN;j+=i)</pre>
             notprime[j]=true;
       }
}
   1.2 素数筛选 (筛选出小于等于 MAXN 的素数)
* 素数筛选,存在小于等于MAXN的素数
 * prime[0] 存的是素数的个数
const int MAXN=10000;
int prime[MAXN+1];
void getPrime()
   memset(prime, 0, sizeof(prime));
   for (int i=2; i<=MAXN; i++)</pre>
      if(!prime[i])prime[++prime[0]]=i;
      for (int j=1; j<=prime[0] &&prime[j] <=MAXN/i; j++)</pre>
          prime[prime[j]*i]=1;
          if(i%prime[j]==0) break;
   }
}
   1.3 大区间素数筛选(POJ 2689)
* POJ 2689 Prime Distance
* 给出一个区间[L,U],找出区间内容、相邻的距离最近的两个素数和
 * 距离最远的两个素数。
 * 1<=L<U<=2,147,483,647 区间长度不超过1,000,000
 * 就是要筛选出[L,U]之间的素数
 */
#include <stdio.h>
#include <algorithm>
#include <iostream>
#include <string.h>
using namespace std;
const int MAXN=100010;
int prime[MAXN+1];
void getPrime()
{
   memset(prime, 0, sizeof(prime));
    for (int i=2; i<=MAXN; i++)</pre>
       if(!prime[i])prime[++prime[0]]=i;
       for (int j=1; j<=prime[0]&&prime[j]<=MAXN/i; j++)</pre>
```

```
{
             prime[prime[j]*i]=1;
             if(i%prime[j]==0) break;
        }
    }
}
bool notprime[1000010];
int prime2[1000010];
void getPrime2(int L, int R)
    memset(notprime, false, sizeof(notprime));
    if(L<2) L=2;
    for(int i=1;i<=prime[0]&&(long long)prime[i]*prime[i]<=R;i++)</pre>
        int s=L/prime[i]+(L%prime[i]>0);
        if(s==1)s=2;
        for(int j=s; (long long)j*prime[i]<=R;j++)</pre>
             if((long long)j*prime[i]>=L)
                 notprime[j*prime[i]-L]=true;
    }
    prime2[0]=0;
    for (int i=0; i<=R-L; i++)</pre>
        if(!notprime[i])
            prime2[++prime2[0]]=i+L;
}
int main()
    getPrime();
    int L, U;
    while (scanf ("%d%d", &L, &U) ==2)
        getPrime2(L,U);
        if(prime2[0]<2)printf("There are no adjacent primes.\n");</pre>
        else
             int x1=0,x2=100000000,y1=0,y2=0;
             for(int i=1; i<prime2[0]; i++)</pre>
             {
                 if (prime2[i+1]-prime2[i]<x2-x1)</pre>
                     x1=prime2[i];
                     x2=prime2[i+1];
                 if (prime2[i+1]-prime2[i]>y2-y1)
                     y1=prime2[i];
                     y2=prime2[i+1];
            printf("%d,%d are closest, %d,%d are most distant. \n", x1, x2, y1, y2);
        }
    }
}
```

2、素数筛选和合数分解

```
//************
//素数筛选和合数分解
const int MAXN=10000;
int prime[MAXN+1];
void getPrime()
   memset(prime, 0, sizeof(prime));
   for (int i=2; i<=MAXN; i++)</pre>
      if(!prime[i])prime[++prime[0]]=i;
      for (int j=1; j<=prime[0] &&prime[j] <=MAXN/i; j++)</pre>
          prime[prime[j]*i]=1;
          if(i%prime[j]==0) break;
   }
long long factor[100][2];
int fatCnt;
int getFactors(long long x)
   fatCnt=0;
   long long tmp=x;
   for (int i=1; prime[i] <=tmp/prime[i];i++)</pre>
      factor[fatCnt][1]=0;
      if(tmp%prime[i]==0)
          factor[fatCnt][0]=prime[i];
          while(tmp%prime[i]==0)
             factor[fatCnt][1]++;
             tmp/=prime[i];
          fatCnt++;
       }
   if (tmp! =1)
      factor[fatCnt][0]=tmp;
      factor[fatCnt++][1]=1;
   return fatCnt;
```

3、扩展欧几里得算法(求 ax+by=gcd 的解以及逆元素)

```
{
  if(a==0&&b==0) return -1;//无最大公约数
   if (b==0) {x=1;y=0; return a; }
   long long d=extend gcd(b,a%b,y,x);
   y=a/b*x;
  return d;
//ax = 1 \pmod{n}
long long mod_reverse(long long a,long long n)
   long long x, y;
   long long d=extend_gcd(a,n,x,y);
   if(d==1) return (x%n+n)%n;
   else return -1;
4、求逆元
   4.1 扩展欧几里德法(见上面)
   4.2 简洁写法
注意:这个只能求a < m的情况,而且必须保证a和m互质
//求ax = 1( mod m) 的x值, 就是逆元(0<a<m)
long long inv(long long a, long long m)
   if(a == 1) return 1;
   return inv(m%a,m) * (m-m/a)%m;
   4.3 利用欧拉函数
mod为素数,而且a和m互质
long long inv(long long a,long long mod) //mod为素数
   return pow_m (a, mod-2, mod);
}
5、模线性方程组
```

```
long long extend gcd(long long a,long long b,long long &x,long long &y)
   if(a == 0 && b == 0)return -1;
   if(b == 0 ) {x = 1; y = 0; return a;}
   long long d = extend_gcd(b,a%b,y,x);
   y = a/b*x;
   return d;
int m[10],a[10];//模数为m,余数为a, X % m = a
bool solve(int &m0,int &a0,int m,int a)
   long long y, x;
   int g = extend gcd(m0,m,x,y);
   if( abs(a - a0)%g )return false;
   x *= (a - a0)/g;
```

```
x \% = m/g;
   a0 = (x*m0 + a0);
   m0 \star = m/g;
   a0 %= m0;
   if( a0 < 0 ) a0 += m0;
   return true;
}
* 无解返回false,有解返回true;
* 解的形式最后为 a0 + m0 * t (0<=a0<m0)
*/
bool MLES(int &m0 ,int &a0,int n)//mh X = a0 + m0 * k
   bool flag = true;
   m0 = 1;
   a0 = 0;
   for(int i = 0;i < n;i++)</pre>
      if( !solve(m0,a0,m[i],a[i]) )
      flag = false;
      break;
   return flag;
}
```

6、随机素数测试和大数分解(POJ 1811)

```
/* *************
* Miller Rabin 算法进行素数测试
* 速度快,可以判断一个 < 2^63 的数是不是素数
**************************************
const int S = 8; //随机算法判定次数,一般8~10就够了
// 计算<u>ret</u> = (a*b)%c
                  a,b,c < 2^63
long long mult mod(long long a, long long b, long long c)
{
  a %= c;
  b %= c;
  long long ret = 0;
  long long tmp = a;
  while(b)
     if(b & 1)
        ret += tmp;
        if(ret > c)ret -= c;//直接取模慢很多
     tmp <<= 1;
     if(tmp > c)tmp -= c;
     b >>= 1;
  return ret;
}
```

```
// 计算 <u>ret</u> = (a^n)%<u>mod</u>
long long pow_mod(long long a,long long n,long long mod)
   long long ret = 1;
   long long temp = a%mod;
   while(n)
      if(n & 1)ret = mult mod(ret,temp,mod);
      temp = mult_mod(temp, temp, mod);
      n >>= 1;
   return ret;
}
// 通过 a^(n-1)=1 (mod n)来判断n是不是素数
// n-1 = x*2^t 中间使用二次判断
// 是合数返回true, 不一定是合数返回false
bool check(long long a,long long n,long long x,long long t)
{
   long long ret = pow_mod(a, x, n);
   long long last = ret;
   for(int i = 1;i <= t;i++)</pre>
      ret = mult mod(ret,ret,n);
      if(ret == 1 && last != 1 && last != n-1)return true;//合数
      last = ret;
   if(ret != 1) return true;
   else return false;
}
//***********
// Miller Rabin算法
// 是素数返回true, (可能是伪素数)
// 不是素数返回false
                          ******
//************
bool Miller Rabin (long long n)
{
   if( n < 2)return false;</pre>
   if( n == 2)return true;
   if((n&1) == 0)return false;//偶数
   long long x = n - 1;
   long long t = 0;
   while ( (x\&1) == 0 ) {x >>= 1; t++;}
   for(int i = 0;i < S;i++)</pre>
      long long a = rand()%(n-1) + 1;
      if(check(a,n,x,t))
        return false;
   return true;
}
//***********
// pollard rho 算法进行质因素分解
//
```

```
long long factor[100];//质因素分解结果(刚返回时时无序的)
int tol;//质因素的个数,编号0~tol-1
long long gcd(long long a, long long b)
   long long t;
   while(b)
       t = a;
       a = b;
       b = t%b;
   if(a >= 0)return a;
   else return -a;
}
//找出一个因子
long long pollard_rho(long long x,long long c)
    long long i = 1, k = 2;
    srand(time(NULL));
   long long x0 = rand() % (x-1) + 1;
    long long y = x0;
   while(1)
       i ++;
       x0 = (mult mod(x0, x0, x) + c) %x;
       long long d = gcd(y - x0,x);
       if( d != 1 && d != x) return d;
       if(y == x0)return x;
       if(i == k) {y = x0; k += k;}
    }
//对 n进行素因子分解,存入factor. k设置为107左右即可
void findfac(long long n,int k)
{
   if(n == 1)return;
   if(Miller Rabin(n))
       factor[tol++] = n;
       return;
   long long p = n;
   int c = k;
   while (p >= n)
      p = pollard_rho(p,c--);//值变化,防止死循环k
    findfac(p,k);
   findfac (n/p, k);
}
//POJ 1811
//给出一个N(2 <= N < 2^54),如果是素数,输出"Prime",否则输出最小的素因子
int main()
   int T;
   long long n;
   scanf("%d",&T);
```

```
while(T--)
{
    scanf("%I64d",&n);
    if(Miller_Rabin(n))printf("Prime\n");
    else
    {
        tol = 0;
        findfac(n,107);
        long long ans = factor[0];
        for(int i = 1;i < tol;i++)
            ans = min(ans,factor[i]);
        printf("%I64d\n",ans);
    }
}
return 0;
}</pre>
```

7、欧拉函数

6.1 分解质因素求欧拉函数

```
getFactors(n);
       int ret = n;
       for(int i = 0;i < fatCnt;i++)</pre>
          ret = ret/factor[i][0]*(factor[i][0]-1);
   6.2 筛法欧拉函数
int euler[3000001];
void getEuler()
{
   memset(euler,0,sizeof(euler));
   euler[1] = 1;
   for(int i = 2;i <= 3000000;i++)</pre>
       if(!euler[i])
         for(int j = i; j <= 3000000; j += i)</pre>
              if(!euler[j])
                 euler[j] = j;
              euler[j] = euler[j]/i*(i-1);
          }
}
   6.2 求单个数的欧拉函数
long long eular(long long n)
   long long ans = n;
   for(int i = 2;i*i <= n;i++)</pre>
      if(n % i == 0)
          ans -= ans/i;
          while (n % i == 0)
```

n /= i;

if(n > 1) ans -= ans/n;

```
return ans;
}
   6.3 线性筛 (同时得到欧拉函数和素数表)
const int MAXN = 10000000;
bool check[MAXN+10];
int phi[MAXN+10];
int prime[MAXN+10];
int tot;//素数的个数
void phi_and_prime_table(int N)
   memset(check,false,sizeof(check));
    phi[1] = 1;
    tot = 0;
    for(int i = 2; i <= N; i++)</pre>
        if( !check[i] )
            prime[tot++] = i;
            phi[i] = i-1;
        for(int j = 0; j < tot; j++)</pre>
            if(i * prime[j] > N)break;
            check[i * prime[j]] = true;
            if( i % prime[j] == 0)
               phi[i * prime[j]] = phi[i] * prime[j];
               break;
            }
            else
            {
                phi[i * prime[j]] = phi[i] * (prime[j] - 1);
        }
    }
}
```

8、高斯消元 (浮点数)

```
#define eps 1e-9
const int MAXN=220;
double a [MAXN] [MAXN], x [MAXN]; //方程的左边的矩阵和等式右边的值,求解之后 x 存的就是结果
int equ, var; //方程数和未知数个数
*返回0表示无解,1表示有解
* /
int Gauss()
   int i,j,k,col,max_r;
   for (k=0,col=0; k<equ&&col<var; k++,col++)</pre>
      max_r=k;
       for (i=k+1; i<equ; i++)</pre>
        if(fabs(a[i][col])>fabs(a[max r][col]))
          max r=i;
       if(fabs(a[max_r][col])<eps)return 0;</pre>
                                                            ACM 模板
                                     24 / 152
                                                                          kuangbin
```

```
if(k!=max r)
           for(j=col;j<var;j++)</pre>
            swap(a[k][j],a[max_r][j]);
           swap(x[k],x[max_r]);
        }
       x[k]/=a[k][col];
       for (j=col+1; j<var; j++) a[k] [j] /=a[k] [col];</pre>
       a[k][col]=1;
       for (i=0;i<equ;i++)</pre>
         if(i!=k)
         {
             x[i] = x[k] *a[i][k];
             for (j=col+1; j<var; j++)a[i][j]-=a[k][j]*a[i][col];</pre>
             a[i][col]=0;
         }
   }
   return 1;
}
```

9、FFT

```
//HDU 1402 求高精度乘法
#include <stdio.h>
#include <string.h>
#include <iostream>
#include <algorithm>
#include <math.h>
using namespace std;
const double PI = acos(-1.0);
//复数结构体
struct Complex
{
   double x,y;//实部和虚部 x+yi
   Complex (double x = 0.0, double y = 0.0)
      x = x;
      y = y;
   Complex operator - (const Complex &b) const
      return Complex(x-b.x, y-b.y);
   Complex operator +(const Complex &b) const
      return Complex(x+b.x, y+b.y);
   Complex operator *(const Complex &b) const
      return Complex(x*b.x-y*b.y,x*b.y+y*b.x);
};
* 进行FFT和IFFT前的反转变换。
* 位置i和 (i二进制反转后位置) 互换
```

```
* len必须去2的幂
void change(Complex y[],int len)
   int i,j,k;
   for(i = 1, j = len/2; i <len-1; i++)</pre>
       if(i < j)swap(y[i],y[j]);</pre>
       //交换互为小标反转的元素, i<j保证交换一次
       //i做正常的+1, j左反转类型的+1, 始终保持i和j是反转的
       k = len/2;
       while(j >= k)
          j -= k;
          k /= 2;
       if(j < k)j += k;
}
/*
* 做FFT
 * <u>len</u>必须为2<sup>k</sup>形式,
 * on==1时是DFT, on==-1时是IDFT
*/
void fft(Complex y[], int len, int on)
{
   change(y,len);
   for(int h = 2; h <= len; h <<= 1)</pre>
       Complex wn(cos(-on*2*PI/h),sin(-on*2*PI/h));
       for(int j = 0; j < len; j+=h)</pre>
          Complex w(1,0);
          for (int k = j; k < j+h/2; k++)
              Complex u = y[k];
              Complex t = w*y[k+h/2];
              y[k] = u+t;
              y[k+h/2] = u-t;
              w = w*wn;
          }
       }
   if(on == -1)
       for(int i = 0;i < len;i++)</pre>
          y[i].x /= len;
const int MAXN = 200010;
Complex x1[MAXN], x2[MAXN];
char str1[MAXN/2], str2[MAXN/2];
int sum[MAXN];
int main()
   while (scanf("%s%s", str1, str2) == 2)
       int len1 = strlen(str1);
       int len2 = strlen(str2);
       int len = 1;
```

```
while(len < len1*2 || len < len2*2)len<<=1;</pre>
       for(int i = 0;i < len1;i++)</pre>
          x1[i] = Complex(str1[len1-1-i]-'0',0);
       for(int i = len1;i < len;i++)</pre>
          x1[i] = Complex(0,0);
       for(int i = 0;i < len2;i++)</pre>
          x2[i] = Complex(str2[len2-1-i]-'0',0);
       for(int i = len2;i < len;i++)</pre>
          x2[i] = Complex(0,0);
       //求DFT
       fft(x1, len, 1);
       fft(x2,len,1);
       for(int i = 0;i < len;i++)</pre>
          x1[i] = x1[i]*x2[i];
       fft(x1, len, -1);
       for(int i = 0;i < len;i++)</pre>
          sum[i] = (int)(x1[i].x+0.5);
       for(int i = 0;i < len;i++)</pre>
          sum[i+1] += sum[i] / 10;
          sum[i] %=10;
       }
       len = len1+len2-1;
       while(sum[len] <= 0 && len > 0)len--;
       for(int i = len; i >= 0;i--)
          printf("%c", sum[i]+'0');
       printf("\n");
   return 0;
}
//HDU 4609
//给出 n 条线段长度, 问任取 3 根, 组成三角形的概率。
          用 FFT 求可以组成三角形的取法有几种
//n<=10^5
const int MAXN = 400040;
Complex x1[MAXN];
int a[MAXN/4];
long long num[MAXN];//100000*100000会超int
long long sum[MAXN];
int main()
{
   int T;
   int n;
   scanf("%d",&T);
   while (T--)
       scanf("%d", &n);
       memset(num, 0, sizeof(num));
       for(int i = 0;i < n;i++)</pre>
          scanf("%d", &a[i]);
          num[a[i]]++;
       sort(a,a+n);
       int len1 = a[n-1]+1;
```

```
int len = 1;
       while( len < 2*len1 )len <<= 1;</pre>
       for(int i = 0;i < len1;i++)</pre>
          x1[i] = Complex(num[i], 0);
       for(int i = len1;i < len;i++)</pre>
          x1[i] = Complex(0,0);
       fft(x1, len, 1);
       for(int i = 0;i < len;i++)</pre>
          x1[i] = x1[i]*x1[i];
       fft(x1, len, -1);
       for(int i = 0;i < len;i++)</pre>
          num[i] = (long long)(x1[i].x+0.5);
       len = 2*a[n-1];
       //减掉取两个相同的组合
       for(int i = 0;i < n;i++)</pre>
          num[a[i]+a[i]]--;
       for(int i = 1;i <= len;i++)num[i]/=2;</pre>
       sum[0] = 0;
       for(int i = 1;i <= len;i++)</pre>
          sum[i] = sum[i-1]+num[i];
       long long cnt = 0;
       for(int i = 0;i < n;i++)</pre>
          cnt += sum[len]-sum[a[i]];
          //减掉一个取大,一个取小的
          cnt -= (long long) (n-1-i) * i;
          //减掉一个取本身,另外一个取其它
          cnt -= (n-1);
          cnt -= (long long) (n-1-i)*(n-i-2)/2;
       long long tot = (long long)n*(n-1)*(n-2)/6;
       printf("%.71f\n", (double)cnt/tot);
   return 0;
}
```

10、高斯消元法求方程组的解

```
10.1 一类开关问题,对2取模的01方程组
```

POJ 1681 需要枚举自由变元,找解中 1 个数最少的

```
const int MAXN = 300;

//有equ个方程, var个变元。增广矩阵行数为equ,列数为var+1,分别为0到var
int equ,var;
int a[MAXN] [MAXN]; //增广矩阵
int x[MAXN]; //解集
int free_x[MAXN]; //用来存储自由变元(多解枚举自由变元可以使用)
int free_num; //自由变元的个数

//返回值为-1表示无解,为0是唯一解,否则返回自由变元个数
int Gauss()
{
   int max_r,col,k;
   free_num = 0;
   for(k = 0, col = 0; k < equ && col < var; k++, col++)
```

```
{
        max_r = k;
        for(int i = k+1;i < equ;i++)</pre>
            if(abs(a[i][col]) > abs(a[max_r][col]))
                \max r = i;
        if(a[max r][col] == 0)
            k--;
            free x[free num++] = col;//这个是自由变元
            continue;
        if(max r != k)
            for(int j = col; j < var+1; j++)</pre>
                swap(a[k][j],a[max r][j]);
        for(int i = k+1;i < equ;i++)</pre>
            if(a[i][col] != 0)
            {
                 for(int j = col; j < var+1; j++)</pre>
                     a[i][j] ^= a[k][j];
            }
        }
    for(int i = k;i < equ;i++)</pre>
        if(a[i][col] != 0)
            return -1;//无解
    if(k < var) return var-k;//自由变元个数
    //唯一解,回代
    for(int i = var-1; i >= 0;i--)
        x[i] = a[i][var];
        for(int j = i+1; j < var; j++)</pre>
            x[i] ^= (a[i][j] \&\& x[j]);
    }
    return 0;
int n;
void init()
    memset(a,0,sizeof(a));
    memset(x,0,sizeof(x));
    equ = n*n;
    var = n*n;
    for(int i = 0;i < n;i++)</pre>
        for(int j = 0; j < n; j++)</pre>
            int t = i*n+j;
            a[t][t] = 1;
            if(i > 0)a[(i-1)*n+j][t] = 1;
            if(i < n-1)a[(i+1)*n+j][t] = 1;
            if(j > 0)a[i*n+j-1][t] = 1;
            if(j < n-1)a[i*n+j+1][t] = 1;
```

}

}

```
void solve()
    int t = Gauss();
    if(t == -1)
        printf("inf\n");
        return;
    }
    else if(t == 0)
        int ans = 0;
        for(int i = 0;i < n*n;i++)</pre>
           ans += x[i];
        printf("%d\n", ans);
        return;
    }
    else
    {
        //枚举自由变元
        int ans = 0x3f3f3f3f;
        int tot = (1<<t);
        for(int i = 0;i < tot;i++)</pre>
            int cnt = 0;
            for(int j = 0; j < t; j++)
            {
                if(i&(1<<j))
                    x[free_x[j]] = 1;
                    cnt++;
                else x[free x[j]] = 0;
            for(int j = var-t-1; j >= 0; j--)
                int idx;
                for(idx = j;idx < var;idx++)</pre>
                    if(a[j][idx])
                        break;
                x[idx] = a[j][var];
                for(int 1 = idx+1;1 < var;1++)</pre>
                    if(a[j][l])
                       x[idx] ^= x[1];
                cnt += x[idx];
            }
            ans = min(ans,cnt);
        printf("%d\n",ans);
    }
char str[30][30];
int main()
   int T;
   scanf("%d",&T);
    while (T--)
        scanf("%d",&n);
```

```
init();
        for(int i = 0;i < n;i++)</pre>
            scanf("%s",str[i]);
            for(int j = 0; j < n; j++)</pre>
                if(str[i][j] == 'y')
                    a[i*n+j][n*n] = 0;
                else a[i*n+j][n*n] = 1;
            }
        solve();
   return 0;
}
   10.2 解同余方程组
POJ 2947 Widget Factory
//求解对MOD取模的方程组
const int MOD = 7;
const int MAXN = 400;
int a[MAXN][MAXN];//增广矩阵
int x[MAXN];//最后得到的解集
inline int gcd(int a, int b)
    while(b != 0)
    {
       int t = b;
       b = a%b;
        a = t;
   return a;
inline int lcm(int a, int b)
   return a/gcd(a,b) *b;
long long inv(long long a, long long m)
   if(a == 1)return 1;
   return inv(m%a,m) * (m-m/a)%m;
}
int Gauss(int equ,int var)
    int max_r,col,k;
    for(k = 0, col = 0; k < equ && col < var; k++, col++)
        \max r = k;
        for(int i = k+1; i < equ;i++)</pre>
            if(abs(a[i][col]) > abs(a[max r][col]))
               \max r = i;
        if(a[max_r][col] == 0)
        {
            k--;
            continue;
        }
```

```
if (max_r != k)
            for(int j = col; j < var+1; j++)</pre>
                swap(a[k][j],a[max_r][j]);
        for(int i = k+1; i < equ; i++)
            if(a[i][col] != 0)
                int LCM = lcm(abs(a[i][col]),abs(a[k][col]));
                int ta = LCM/abs(a[i][col]);
                int tb = LCM/abs(a[k][col]);
                if(a[i][col] *a[k][col] < 0)tb = -tb;</pre>
                for (int j = col; j < var+1; j++)</pre>
                     a[i][j] = ((a[i][j]*ta - a[k][j]*tb)%MOD + MOD)%MOD;
            }
        }
    for(int i = k;i < equ;i++)</pre>
        if(a[i][col] != 0)
            return -1;//无解
    if(k < var) return var-k;//多解
    for(int i = var-1;i >= 0;i--)
        int temp = a[i][var];
        for(int j = i+1; j < var; j++)</pre>
            if(a[i][j] != 0)
                temp -= a[i][j]*x[j];
                temp = (temp%MOD + MOD)%MOD;
            }
        }
        x[i] = (temp*inv(a[i][i],MOD))%MOD;
    return 0;
int change(char s[])
    if(strcmp(s, "MON") == 0) return 1;
    else if (strcmp(s, "TUE") == 0) return 2;
   else if (strcmp(s, "WED") == 0) return 3;
   else if(strcmp(s,"THU")==0) return 4;
   else if(strcmp(s, "FRI") == 0) return 5;
   else if (strcmp(s, "SAT") == 0) return 6;
   else return 7;
}
int main()
    int n,m;
    while(scanf("%d%d",&n,&m) == 2)
        if(n == 0 && m == 0)break;
        memset(a,0,sizeof(a));
        char str1[10],str2[10];
        int k;
        for(int i = 0;i < m;i++)</pre>
            scanf("%d%s%s",&k,str1,str2);
            a[i][n] = ((change(str2) - change(str1) + 1)%MOD + MOD)%MOD;
                                       32 / 152
                                                              ACM 模板
                                                                            kuangbin
```

}

```
int t;
            while (k--)
                scanf("%d",&t);
                t--;
                a[i][t] ++;
                a[i][t]%=MOD;
            }
        }
        int ans = Gauss(m,n);
        if(ans == 0)
            for(int i = 0;i < n;i++)</pre>
                if(x[i] <= 2)
                    x[i] += 7;
            for(int i = 0;i < n-1;i++)printf("%d ",x[i]);</pre>
            printf("%d\n",x[n-1]);
        else if (ans == -1)printf("Inconsistent data.\n");
        else printf("Multiple solutions.\n");
   return 0;
}
```

整数拆分 11、

HDU 4651

```
const int MOD = 1e9+7;
int dp[100010];
void init()
   memset(dp,0,sizeof(dp));
   dp[0] = 1;
   for(int i = 1;i <= 100000;i++)</pre>
       for (int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0; <math>j++, r *= -1)
          dp[i] += dp[i -(3 * j * j - j) / 2] * r;
          dp[i] %= MOD;
          dp[i] = (dp[i]+MOD)%MOD;
          if(i - (3 * j * j + j) / 2 >= 0)
              dp[i] += dp[i - (3 * j * j + j) / 2] * r;
              dp[i] %= MOD;
              dp[i] = (dp[i]+MOD)%MOD;
       }
   }
}
int main()
   int T;
   int n;
   init();
   scanf("%d",&T);
   while (T--)
                                      33 / 152
```

```
{
       scanf("%d", &n);
      printf("%d\n",dp[n]);
   return 0;
}
HDU 4658
数 n(<=10^5)的划分,相同的数重复不能超过 k 个。
const int MOD = 1e9+7;
int dp[100010];
void init()
   memset(dp,0,sizeof(dp));
   dp[0] = 1;
   for(int i = 1;i <= 100000;i++)</pre>
       for(int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0; <math>j++, r *= -1)
          dp[i] += dp[i -(3 * j * j - j) / 2] * r;
          dp[i] %= MOD;
          dp[i] = (dp[i]+MOD)%MOD;
          if(i - (3 * j * j + j) / 2 >= 0)
             dp[i] += dp[i - (3 * j * j + j) / 2] * r;
             dp[i] %= MOD;
             dp[i] = (dp[i]+MOD)%MOD;
          }
       }
   }
}
int solve(int n,int k)
   int ans = dp[n];
   for (int j = 1, r = -1; n - k*(3 * j * j - j) / 2 >= 0; <math>j++, r *= -1)
       ans += dp[n - k*(3 * j * j - j) / 2] * r;
      ans %= MOD;
      ans = (ans+MOD)%MOD;
       if( n - k*(3 * j * j + j) / 2 >= 0 )
          ans += dp[n - k*(3 * j * j + j) / 2] * r;
          ans %= MOD;
          ans = (ans+MOD)%MOD;
      }
   }
   return ans;
}
int main()
   init();
   int T;
   int n,k;
   scanf("%d",&T);
   while (T--)
       scanf("%d%d", &n, &k);
```

```
printf("%d\n", solve(n,k));
   return 0;
}
```

12、求 A^B 的约数之和对 MOD 取模

```
参考 POJ 1845
里面有一种求 1+p+p^2+p^3+...p^n 的方法。
需要素数筛选和合数分解的程序,需要先调用 getPrime();
long long pow_m(long long a,long long n)
   long long ret = 1;
   long long tmp = a%MOD;
   while(n)
       if(n\&1) ret = (ret*tmp) %MOD;
       tmp = tmp*tmp%MOD;
       n >>= 1;
   return ret;
//计算1+p+p^2+...+p^n
long long sum(long long p, long long n)
   if(p == 0)return 0;
   if(n == 0)return 1;
   if(n & 1)
       return ((1+pow_m(p,n/2+1))%MOD*sum(p,n/2)%MOD)%MOD;
   else return ((1+pow m(p,n/2+1)) MOD*sum(p,n/2-1)+pow m(p,n/2) MOD;
//返回A^B的约数之和 % MOD
long long solve(long long A, long long B)
   getFactors(A);
   long long ans = 1;
   for(int i = 0;i < fatCnt;i++)</pre>
       ans *= sum(factor[i][0],B*factor[i][1])%MOD;
       ans %= MOD;
   return ans;
}
```

35 / 152

13、莫比乌斯反演

莫比乌斯反演公式:

$$F(n) = \sum_{d|n} f(d)$$
 \emptyset $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$

莫比乌斯函数 μ

$$\mu(n) = \begin{cases} 1 & n = 1 \\ (-1)^k & n = p_1 p_2 \cdots p_k \\ 0 & 其余情况 \end{cases}$$

另外一种更常用的形式:

在某一范围内:
$$F(n) = \sum_{n|d} f(d)$$
 则 $f(n) = \sum_{n|d} \mu \left(\frac{d}{n}\right) F(d)$

线性筛法求解积性函数 (莫比乌斯函数)

```
const int MAXN = 1000000;
bool check[MAXN+10];
int prime[MAXN+10];
int mu[MAXN+10];
void Moblus()
   memset(check, false, sizeof(check));
   mu[1] = 1;
    int tot = 0;
    for(int i = 2; i <= MAXN; i++)</pre>
        if(!check[i])
           prime[tot++] = i;
           mu[i] = -1;
        for (int j = 0; j < tot; j++)
            if(i * prime[j] > MAXN) break;
            check[i * prime[j]] = true;
            if( i % prime[j] == 0)
                mu[i * prime[j]] = 0;
                break;
            }
            else
               mu[i * prime[j]] = -mu[i];
       }
   }
}
```

例题:

BZOJ 2301

对于给出的 n 个询问,每次求有多少个数对(x,y),满足 a \leq x \leq b, c \leq y \leq d,且 gcd(x,y) = k, gcd(x,y)函数为 x 和 y 的最大公约数。

1≤n≤50000, 1≤a≤b≤50000, 1≤c≤d≤50000, 1≤k≤50000

```
const int MAXN = 100000;
bool check[MAXN+10];
int prime[MAXN+10];
int mu[MAXN+10];
void Moblus()
   memset(check,false,sizeof(check));
   mu[1] = 1;
    int tot = 0;
    for(int i = 2; i <= MAXN; i++)</pre>
        if(!check[i])
            prime[tot++] = i;
            mu[i] = -1;
        for(int j = 0; j < tot; j ++)</pre>
            if( i * prime[j] > MAXN) break;
            check[i * prime[j]] = true;
            if( i % prime[j] == 0)
                mu[i * prime[j]] = 0;
                break;
            }
            else
                mu[i * prime[j]] = -mu[i];
        }
    }
}
int sum[MAXN+10];
//找[1,n],[1,m]内互质的数的对数
long long solve(int n,int m)
    long long ans = 0;
    if(n > m) swap(n,m);
    for(int i = 1, la = 0; i <= n; i = la+1)</pre>
        la = min(n/(n/i), m/(m/i));
        ans += (long long) (sum[la] - sum[i-1])*(n/i)*(m/i);
    return ans;
}
int main()
   Moblus();
   sum[0] = 0;
    for(int i = 1;i <= MAXN;i++)</pre>
       sum[i] = sum[i-1] + mu[i];
    int a,b,c,d,k;
    int T;
    scanf("%d",&T);
    while (T--)
    {
```

```
scanf("%d%d%d%d%d", &a, &b, &c, &d, &k);
        long long ans = solve(b/k, d/k) - solve((a-1)/k, d/k) - solve(b/k, (c-1)/k)
+ solve((a-1)/k,(c-1)/k);
       printf("%lld\n",ans);
    }
   return 0;
```

14 \ Baby-Step Giant-Step

```
(POJ 2417,3243)
```

```
//baby step giant step
// a^x = b (\underline{mod} n) n是素数和不是素数都可以
// 求解上式 0<=x < n的解
#define MOD 76543
int hs[MOD], head[MOD], next[MOD], id[MOD], top;
void insert(int x,int y)
    int k = x%MOD;
   hs[top] = x, id[top] = y, next[top] = head[k], head[k] = top++;
}
int find(int x)
   int k = x%MOD;
    for(int i = head[k]; i != -1; i = next[i])
        if(hs[i] == x)
           return id[i];
    return -1;
}
int BSGS(int a,int b,int n)
   memset(head, -1, sizeof(head));
   top = 1;
   if(b == 1)return 0;
    int m = sqrt(n*1.0), j;
    long long x = 1, p = 1;
    for (int i = 0; i < m; ++i, p = p*a%n) insert (p*b%n, i);
    for(long long i = m; ;i += m)
        if ((j = find(x = x*p%n)) != -1) return i-j;
       if(i > n)break;
   return -1;
```

相关公式

1、欧拉定理

对于互质的整数 a 和 n, 有 $a^{\varphi(n)} \equiv 1 \pmod{n}$

费马定理: a 是不能被质数 p 整除的正整数,有 $a^{p-1} \equiv 1 \pmod{p}$

2、Polya 定理

设 G 是 p 个对象的一个置换群,用 k 种颜色去染这 p 个对象,若一种染色方案在群 G 的作用下变为一种方案,则这两个方案当作是同一种方案,这样的不同染色方案数为:

$$L = \frac{1}{|G|} \times \sum (k^{C(f)}), f \in G$$

C(f)为循环节,|G|表示群的置换方法数。

对于 n 个位置的手镯, 有 n 种旋转置换和 n 种翻转置换对于旋转置换:

 $\mathit{C}(f_i) = \gcd(\mathit{n},i)$,i 表示旋转 i 颗宝石以后。i=0 时 $\gcd(\mathsf{n},\mathsf{0})$ =n

对于翻转置换:

如果 n 为偶数: 则有 n/2 个置换
$$C(f) = \frac{n}{2}$$
,有 n/2 个置换 $C(f) = \frac{n}{2} + 1$

如果 n 为奇数: 则有 n 个置换
$$C(f) = \frac{n}{2} + 1$$

3、欧拉函数 $\varphi(n)$

$$\varphi(n)$$
 积性函数,对于一个质数 p 和正整数 k ,有

$$\varphi(p^k) = p^k - p^{k-1} = (p-1)p^{k-1} = p^k(1-\frac{1}{p})$$

$$\sum_{d|n} \varphi(d) = n$$

当
$$n > 1$$
时, $1 \dots n$ 中与 n 互质的整数和为 $\frac{n\varphi(n)}{2}$

数据结构

1、划分树

```
/*
    * 划分树(查询区间第k大)
    */
const int MAXN = 100010;
int tree[20][MAXN];//表示每层每个位置的值
int sorted[MAXN];//已经排序好的数
int toleft[20][MAXN];//toleft[p][i]表示第i层从1到i有数分入左边
```

void build(int 1, int r, int dep)

```
{
   if(1 == r)return;
   int mid = (l+r)>>1;
   int same = mid - 1 + 1; //表示等于中间值而且被分入左边的个数
   for(int i = 1; i <= r; i++) //注意是1,不是one
      if(tree[dep][i] < sorted[mid])</pre>
          same--;
   int lpos = 1;
   int rpos = mid+1;
   for(int i = 1;i <= r;i++)</pre>
      if(tree[dep][i] < sorted[mid])</pre>
          tree[dep+1][lpos++] = tree[dep][i];
      else if (tree[dep][i] == sorted[mid] && same > 0)
          tree[dep+1][lpos++] = tree[dep][i];
          same--;
      }
      else
          tree[dep+1][rpos++] = tree[dep][i];
      toleft[dep][i] = toleft[dep][l-1] + lpos - 1;
   build(1,mid,dep+1);
   build(mid+1, r, dep+1);
}
//查询区间第k大的数,[L,R]是大区间,[1,r]是要查询的小区间
int query(int L,int R,int l,int r,int dep,int k)
   if(l == r)return tree[dep][l];
   int mid = (L+R)>>1;
   int cnt = toleft[dep][r] - toleft[dep][l-1];
   if(cnt >= k)
      int newl = L + toleft[dep][l-1] - toleft[dep][L-1];
      int newr = newl + cnt - 1;
      return query(L, mid, newl, newr, dep+1, k);
   }
   else
      int newr = r + toleft[dep][R] - toleft[dep][r];
      int newl = newr - (r-l-cnt);
      return query (mid+1, R, newl, newr, dep+1, k-cnt);
   }
}
int main()
   int n,m;
   while (scanf("%d%d", &n, &m) ==2)
      memset(tree, 0, sizeof(tree));
      for(int i = 1;i <= n;i++)</pre>
          scanf("%d", &tree[0][i]);
          sorted[i] = tree[0][i];
      sort(sorted+1, sorted+n+1);
      build(1,n,0);
```

```
int s,t,k;
      while (m--)
         scanf("%d%d%d", &s, &t, &k);
         printf("%d\n", query(1, n, s, t, 0, k));
   return 0;
}
2、RMQ
   2.1 一维
求最大值,数组下标从1开始。
求最小值,或者最大最小值下标,或者数组从0开始对应修改即可。
const int MAXN = 50010;
int dp[MAXN] [20];
int mm[MAXN];
//初始化RMQ, b数组下标从1开始,从0开始简单修改
void initRMQ(int n,int b[])
   mm[0] = -1;
   for(int i = 1; i <= n;i++)</pre>
      mm[i] = ((i&(i-1)) == 0)?mm[i-1]+1:mm[i-1];
      dp[i][0] = b[i];
   for (int j = 1; j <= mm[n]; j++)</pre>
      for(int i = 1;i + (1<<j) -1 <= n;i++)</pre>
         dp[i][j] = max(dp[i][j-1], dp[i+(1<<(j-1))][j-1]);
//查询最大值
int rmq(int x,int y)
   int k = mm[y-x+1];
   return max(dp[x][k],dp[y-(1<<k)+1][k]);</pre>
}
   2.2 二维
* 二维RMQ, 预处理复杂度 n*m*log*(n)*log(m)
* 数组下标从1开始
*/
int val[310][310];
int dp[310][310][9][9];//最大值
int mm[310];//二进制位数减一,使用前初始化
void initRMQ(int n,int m)
{
   for(int i = 1;i <= n;i++)</pre>
      for (int j = 1; j <= m; j++)</pre>
         dp[i][j][0][0] = val[i][j];
   for (int ii = 0; ii <= mm[n]; ii++)</pre>
```

```
for(int jj = 0; jj <= mm[m]; jj++)</pre>
          if(ii+jj)
              for(int i = 1; i + (1<<ii) - 1 <= n;i++)</pre>
                 for(int j = 1; j + (1<<jj) - 1 <= m; j++)</pre>
                     if(ii) dp[i][j][ii][jj] =
\max(dp[i][j][ii-1][jj], dp[i+(1<<(ii-1))][j][ii-1][jj]);
                     else dp[i][j][ii][jj] =
\max(dp[i][j][ii][jj-1], dp[i][j+(1<<(jj-1))][ii][jj-1]);
//查询矩形内的最大值(x1<=x2,y1<=y2)
int rmq(int x1,int y1,int x2,int y2)
   int k1 = mm[x2-x1+1];
   int k2 = mm[y2-y1+1];
   x2 = x2 - (1 << k1) + 1;
   y2 = y2 - (1 << k2) + 1;
   return
\max(\max(dp[x1][y1][k1][k2], dp[x1][y2][k1][k2]), \max(dp[x2][y1][k1][k2], dp[x2]
[y2][k1][k2]));
}
int main()
{
   //在外面对mm数组进行初始化
   mm[0] = -1;
   for(int i = 1;i <= 305;i++)</pre>
       mm[i] = ((i&(i-1))==0)?mm[i-1]+1:mm[i-1];
   int n,m;
   int Q;
   int r1, c1, r2, c2;
   while (scanf("%d%d", &n, &m) == 2)
       for(int i = 1;i <= n;i++)</pre>
          for(int j = 1; j <= m; j++)</pre>
              scanf("%d", &val[i][j]);
       initRMO(n,m);
       scanf("%d", &Q);
       while (Q--)
       {
          scanf("%d%d%d%d",&r1,&c1,&r2,&c2);
          if(r1 > r2)swap(r1,r2);
          if(c1 > c2)swap(c1,c2);
          int tmp = rmq(r1,c1,r2,c2);
          printf("%d ",tmp);
          if(tmp == val[r1][c1] || tmp == val[r1][c2] || tmp == val[r2][c1] ||
tmp == val[r2][c2]
              printf("yes\n");
          else printf("no\n");
   }
   return 0;
}
```

3、树链剖分

3.1 点权

```
基于点权,查询单点值,修改路径的上的点权(HDU 3966 树链剖分+树状数组)
```

```
const int MAXN = 50010;
struct Edge
    int to, next;
}edge[MAXN*2];
int head[MAXN],tot;
int top[MAXN];//top[v] 表示v所在的重链的顶端节点
int fa[MAXN];//父亲节点
int deep[MAXN];//深度
int num[MAXN];//num[v] 表示以v为根的子树的节点数
int p[MAXN];//p[v]表示v对应的位置
int fp[MAXN];//<u>fp</u>和p数组相反
int son[MAXN];//重儿子
int pos;
void init()
{
    tot = 0;
   memset(head, -1, sizeof(head));
   pos = 1;//使用树状数组,编号从头1开始
   memset(son,-1,sizeof(son));
void addedge (int u,int v)
    edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
void dfs1(int u,int pre,int d)
{
    deep[u] = d;
    fa[u] = pre;
   num[u] = 1;
    for(int i = head[u];i != -1; i = edge[i].next)
       int v = edge[i].to;
        if(v != pre)
        {
           dfs1(v,u,d+1);
           num[u] += num[v];
           if(son[u] == -1 \mid \mid num[v] > num[son[u]])
               son[u] = v;
        }
}
void getpos(int u,int sp)
    top[u] = sp;
    p[u] = pos++;
    fp[p[u]] = u;
    if(son[u] == -1) return;
    getpos(son[u],sp);
    for(int i = head[u];i != -1;i = edge[i].next)
    {
```

```
int v = edge[i].to;
        if(v != son[u] && v != fa[u])
           getpos(v,v);
   }
}
//树状数组
int lowbit(int x)
   return x&(-x);
}
int c[MAXN];
int n;
int sum(int i)
   int s = 0;
   while(i > 0)
       s += c[i];
       i -= lowbit(i);
   return s;
}
void add(int i,int val)
    while(i <= n)</pre>
       c[i] += val;
       i += lowbit(i);
}
void Change(int u,int v,int val)//u->v的路径上点的值改变val
    int f1 = top[u], f2 = top[v];
    int tmp = 0;
    while(f1 != f2)
        if(deep[f1] < deep[f2])</pre>
           swap(f1,f2);
           swap(u,v);
        add(p[f1],val);
        add(p[u]+1,-val);
       u = fa[f1];
        f1 = top[u];
    if(deep[u] > deep[v]) swap(u,v);
    add(p[u],val);
    add(p[v]+1,-val);
}
int a[MAXN];
int main()
{
   int M,P;
   while(scanf("%d%d%d", &n, &M, &P) == 3)
        int u, v;
```

```
int C1, C2, K;
       char op [10];
       init();
       for (int i = 1;i <= n;i++)</pre>
           scanf("%d",&a[i]);
       while (M--)
           scanf("%d%d",&u,&v);
           addedge (u, v);
           addedge (v, u);
       dfs1(1,0,0);
       getpos(1,1);
       memset(c,0,sizeof(c));
       for(int i = 1;i <= n;i++)</pre>
           add(p[i],a[i]);
           add(p[i]+1,-a[i]);
       while (P--)
           scanf("%s",op);
           if(op[0] == 'Q')
           {
               scanf("%d",&u);
               printf("%d\n", sum(p[u]));
           }
           else
           {
               scanf("%d%d%d", &C1, &C2, &K);
               if(op[0] == 'D')
                   K = -K;
               Change (C1,C2,K);
       }
    }
   return 0;
}
   3.2 边权
基于边权,修改单条边权,查询路径边权最大值(SPOJ QTREE 树链剖分+线段树 )
const int MAXN = 10010;
struct Edge
   int to, next;
}edge[MAXN*2];
int head[MAXN],tot;
int top[MAXN];//top[v]表示v所在的重链的顶端节点
int fa[MAXN]; //父亲节点
int deep[MAXN];//深度
int num[MAXN];//num[v]表示以v为根的子树的节点数
int p[MAXN];//p[v]表示v与其父亲节点的连边在线段树中的位置
int fp[MAXN];//和p数组相反
int son[MAXN];//重儿子
int pos;
void init()
```

```
{
   tot = 0;
   memset(head, -1, sizeof(head));
   pos = 0;
   memset(son,-1,sizeof(son));
void addedge (int u,int v)
    edge[tot].to = v;edge[tot].next = head[u];head[u] = tot++;
void dfs1(int u,int pre,int d) //第一遍dfs求出fa,deep,num,son
    deep[u] = d;
    fa[u] = pre;
    num[u] = 1;
    for(int i = head[u];i != -1; i = edge[i].next)
        int v = edge[i].to;
        if(v != pre)
            dfs1(v,u,d+1);
            num[u] += num[v];
            if(son[u] == -1 \mid \mid num[v] > num[son[u]])
                son[u] = v;
}
void getpos(int u,int sp) //第二遍<u>dfs</u>求出top和p
   top[u] = sp;
   p[u] = pos++;
   fp[p[u]] = u;
   if(son[u] == -1) return;
    getpos(son[u],sp);
    for(int i = head[u] ; i != -1; i = edge[i].next)
        int v = edge[i].to;
        if(v != son[u] && v != fa[u])
           getpos(v,v);
    }
}
//线段树
struct Node
{
    int 1,r;
   int Max;
}segTree[MAXN*3];
void build(int i,int l,int r)
{
   segTree[i].l = l;
   segTree[i].r = r;
   segTree[i].Max = 0;
   if(l == r)return;
   int mid = (1+r)/2;
   build(i<<1,1,mid);</pre>
   build((i<<1) |1,mid+1,r);
}
```

```
void push_up(int i)
    segTree[i].Max = max(segTree[i<<1].Max,segTree[(i<<1)|1].Max);</pre>
void update(int i,int k,int val) // 更新线段树的第k个值为val
    if(segTree[i].l == k && segTree[i].r == k)
    {
        segTree[i].Max = val;
       return;
    int mid = (segTree[i].l + segTree[i].r)/2;
    if(k <= mid) update(i<<1,k,val);</pre>
    else update((i<<1)|1, k, val);
   push up(i);
}
int query(int i,int l,int r) //查询线段树中[l,r] 的最大值
    if(segTree[i].l == l && segTree[i].r == r)
        return segTree[i].Max;
    int mid = (segTree[i].l + segTree[i].r)/2;
    if(r <= mid) return query(i<<1,1,r);</pre>
    else if (1 > mid) return query((i \le 1) | 1, 1, r);
    else return max(query(i<<1,1,mid),query((i<<1)|1,mid+1,r));
}
int find(int u,int v)//查询u->v边的最大值
    int f1 = top[u], f2 = top[v];
    int tmp = 0;
    while(f1 != f2)
        if(deep[f1] < deep[f2])</pre>
        {
            swap(f1,f2);
            swap(u,v);
        tmp = max(tmp, query(1, p[f1], p[u]));
        u = fa[f1]; f1 = top[u];
    if(u == v)return tmp;
    if(deep[u] > deep[v]) swap(u,v);
    return max(tmp,query(1,p[son[u]],p[v]));
int e[MAXN][3];
int main()
   //freopen("in.txt","r",stdin);
   //freopen("out.txt","w",stdout);
   int T;
    int n;
    scanf("%d",&T);
    while (T--)
        init();
        scanf("%d", &n);
        for(int i = 0;i < n-1;i++)</pre>
            scanf("%d%d%d", &e[i][0], &e[i][1], &e[i][2]);
                                      47 / 152
```

```
addedge(e[i][0],e[i][1]);
            addedge(e[i][1],e[i][0]);
        }
        dfs1(1,0,0);
        getpos(1,1);
        build(1,0,pos-1);
        for(int i = 0;i < n-1; i++)</pre>
            if(deep[e[i][0]] > deep[e[i][1]])
                swap(e[i][0],e[i][1]);
            update(1,p[e[i][1]],e[i][2]);
        }
        char op [10];
        int u, v;
        while(scanf("%s", op) == 1)
            if(op[0] == 'D')break;
            scanf("%d%d", &u, &v);
            if(op[0] == 'Q')
                printf("%d\n", find(u, v));//查询u->v路径上边权的最大值
            else update(1,p[e[u-1][1]],v);//修改第u条边的长度为v
        }
   }
   return 0;
}
```

4、伸展树 (splay tree)

```
题目:维修数列。
```

```
经典题,插入、删除、修改、翻转、求和、求和最大的子序列
#define Key value ch[ch[root][1]][0]
const int MAXN = 500010;
```

```
const int INF = 0x3f3f3f3f;
int pre[MAXN], ch[MAXN][2], key[MAXN], size[MAXN];
int root, tot1;
int sum[MAXN], rev[MAXN], same[MAXN];
int lx[MAXN],rx[MAXN],mx[MAXN];
int s[MAXN],tot2;//内存池和容量
int a[MAXN];
int n,q;
```

```
void Treavel(int x)
{
   if(x)
      Treavel (ch[x][0]);
      printf("结点: %2d: 左儿子 %2d 右儿子 %2d 父结点 %2d size
= 2d^n, x, ch[x][0], ch[x][1], pre[x], size[x]);
      Treavel (ch[x][1]);
}
```

48 / 152

void debug()

```
{
   printf("root:%d\n",root);
   Treavel (root);
void NewNode (int &r,int father,int k)
   if(tot2) r = s[tot2--];//取的时候是tot2--,存的时候就是++tot2
   else r = ++tot1;
   pre[r] = father;
   ch[r][0] = ch[r][1] = 0;
   key[r] = k;
   sum[r] = k;
   rev[r] = same[r] = 0;
   lx[r] = rx[r] = mx[r] = k;
   size[r] = 1;
void Update_Rev(int r)
   if(!r)return;
   swap(ch[r][0],ch[r][1]);
   swap(lx[r],rx[r]);
   rev[r] ^= 1;
}
void Update_Same(int r,int v)
   if(!r)return;
   key[r] = v;
   sum[r] = v*size[r];
   lx[r] = rx[r] = mx[r] = max(v,v*size[r]);
   same[r] = 1;
}
void push_up(int r)
   int lson = ch[r][0], rson = ch[r][1];
   size[r] = size[lson] + size[rson] + 1;
   sum[r] = sum[lson] + sum[rson] + key[r];
   lx[r] = max(lx[lson], sum[lson] + key[r] + max(0, lx[rson]));
   rx[r] = max(rx[rson], sum[rson] + key[r] + max(0, rx[lson]));
   mx[r] = max(0,rx[lson]) + key[r] + max(0,lx[rson]);
   mx[r] = max(mx[r], max(mx[lson], mx[rson]));
}
void push down(int r)
   if(same[r])
       Update_Same(ch[r][0], key[r]);
       Update Same(ch[r][1], key[r]);
       same[r] = 0;
   }
   if(rev[r])
       Update_Rev(ch[r][0]);
       Update_Rev(ch[r][1]);
       rev[r] = 0;
   }
```

```
}
void Build(int &x,int 1,int r,int father)
   if(l > r)return;
   int mid = (1+r)/2;
   NewNode (x, father, a[mid]);
   Build(ch[x][0],1, mid-1,x);
   Build(ch[x][1],mid+1,r,x);
   push_up(x);
}
void Init()
    root = tot1 = tot2 = 0;
    ch[root][0] = ch[root][1] = size[root] = pre[root] = 0;
    same[root] = rev[root] = sum[root] = key[root] = 0;
    lx[root] = rx[root] = mx[root] = -INF;
   NewNode (root, 0, -1);
   NewNode (ch[root][1], root, -1);
    for(int i = 0;i < n;i++)</pre>
        scanf("%d",&a[i]);
    Build(Key value, 0, n-1, ch[root][1]);
   push_up (ch[root][1]);
   push_up (root);
}
//旋转,0为左旋,1为右旋
void Rotate(int x,int kind)
    int y = pre[x];
   push down(y);
   push_down(x);
    ch[y][!kind] = ch[x][kind];
   pre[ch[x][kind]] = y;
    if (pre[y])
        ch[pre[y]][ch[pre[y]][1]==y] = x;
    pre[x] = pre[y];
    ch[x][kind] = y;
   pre[y] = x;
   push_up(y);
//Splay调整,将r结点调整到goal下面
void Splay(int r,int goal)
   push down(r);
    while(pre[r] != goal)
        if(pre[pre[r]] == goal)
        {
            push_down(pre[r]);
            push_down(r);
            Rotate(r,ch[pre[r]][0] == r);
        }
        else
            push_down(pre[pre[r]]);
            push_down(pre[r]);
            push_down(r);
            int y = pre[r];
            int kind = ch[pre[y]][0]==y;
                                     50 / 152
```

```
if(ch[y][kind] == r)
               Rotate(r,!kind);
               Rotate(r,kind);
            }
           else
               Rotate(y, kind);
               Rotate(r,kind);
        }
   }
   push_up(r);
   if(goal == 0) root = r;
int Get_kth(int r,int k)
   push down(r);
   int t = size[ch[r][0]] + 1;
   if(t == k)return r;
   if(t > k)return Get kth(ch[r][0],k);
   else return Get_kth(ch[r][1],k-t);
}
//在第pos个数后面插入tot个数
void Insert(int pos,int tot)
   for(int i = 0;i < tot;i++) scanf("%d",&a[i]);</pre>
   Splay(Get kth(root,pos+1),0);
   Splay(Get_kth(root,pos+2),root);
   Build(Key_value, 0, tot-1, ch [root] [1]);
   push up (ch[root][1]);
   push_up (root);
}
//删除子树
void erase(int r)
{
   if(!r)return;
   s[++tot2] = r;
   erase(ch[r][0]);
   erase(ch[r][1]);
//从第pos个数开始连续删除tot个数
void Delete(int pos,int tot)
   Splay(Get kth(root,pos),0);
   Splay(Get_kth(root,pos+tot+1),root);
   erase(Key_value);
   pre[Key_value] = 0;
   Key_value = 0;
   push_up (ch[root][1]);
   push_up (root);
//将从第pos个数开始的连续的tot个数修改为c
void Make_Same(int pos,int tot,int c)
   Splay(Get kth(root,pos),0);
```

```
Splay(Get_kth(root,pos+tot+1),root);
   Update_Same(Key_value,c);
   push_up (ch[root][1]);
   push_up (root);
}
//将第pos个数开始的连续tot个数进行反转
void Reverse (int pos, int tot)
{
   Splay(Get_kth(root,pos),0);
   Splay(Get kth(root,pos+tot+1),root);
   Update_Rev(Key_value);
   push_up (ch[root][1]);
   push up (root);
//得到第pos个数开始的tot个数的和
int Get_Sum(int pos,int tot)
   Splay(Get kth(root,pos),0);
   Splay(Get_kth(root,pos+tot+1),root);
   return sum[Key_value];
//得到第pos个数开始的tot个数中最大的子段和
int Get_MaxSum(int pos,int tot)
   Splay(Get kth(root,pos),0);
   Splay(Get kth(root,pos+tot+1),root);
   return mx[Key value];
void InOrder(int r)
   if(!r)return;
   push down(r);
   InOrder(ch[r][0]);
   printf("%d ",key[r]);
   InOrder(ch[r][1]);
int main()
   //freopen("in.txt","r", stdin);
   //freopen("out.txt","w", stdout);
   while(scanf("%d%d",&n,&q) == 2)
       Init();
       char op [20];
       int x,y,z;
       while(q--)
           scanf("%s",op);
            if(strcmp(op,"INSERT") == 0)
            {
               scanf("%d%d", &x, &y);
               Insert(x,y);
           else if (strcmp(op,"DELETE") == 0)
               scanf("%d%d", &x, &y);
               Delete(x,y);
```

```
else if (strcmp(op,"MAKE-SAME") == 0)
                scanf("%d%d%d", &x, &y, &z);
                Make_Same(x, y, z);
            else if (strcmp(op, "REVERSE") == 0)
                scanf("%d%d",&x,&y);
                Reverse(x,y);
            else if (strcmp(op, "GET-SUM") == 0)
                scanf("%d%d", &x, &y);
                printf("%d\n",Get Sum(x,y));
            else if (strcmp(op, "MAX-SUM") == 0)
                printf("%d\n",Get MaxSum(1,size[root]-2));
        }
    }
   return 0;
}
```

5、动态树

{

5.1 HDU 4010(切割、合并子树,路径上所有点的点权增加一个值,查询路径上点权的最 大值)

```
//动态维护一组森林,要求支持一下操作:
//link(a,b) : 如果a,b不在同一颗子树中,则通过在a,b之间连边的方式,连接这两颗子树
//cut(a,b) : 如果a,b在同一颗子树中,且a!=b,则将a视为这颗子树的根以后,切断b与其父亲结点
//ADD(a,b,w): 如果a,b在同一颗子树中,则将a,b之间路径上所有点的点权增加w
//query(a,b): 如果a,b在同一颗子树中,返回a,b之间路径上点权的最大值
const int MAXN = 300010;
int ch[MAXN][2],pre[MAXN], key[MAXN];
int add[MAXN], rev[MAXN], Max[MAXN];
bool rt[MAXN];
void Update_Add(int r,int d)
   if(!r)return;
   key[r] += d;
   add[r] += d;
   Max[r] += d;
}
void Update Rev(int r)
   if(!r)return;
   swap(ch[r][0],ch[r][1]);
   rev[r] ^= 1;
void push_down(int r)
   if(add[r])
```

```
Update_Add(ch[r][0],add[r]);
        Update_Add(ch[r][1],add[r]);
        add[r] = 0;
   if(rev[r])
        Update_Rev(ch[r][0]);
       Update_Rev(ch[r][1]);
       rev[r] = 0;
    }
}
void push_up (int r)
   Max[r] = max(max(Max[ch[r][0]], Max[ch[r][1]]), key[r]);
}
void Rotate(int x)
   int y = pre[x], kind = ch[y][1] ==x;
   ch[y][kind] = ch[x][!kind];
   pre[ch[y][kind]] = y;
   pre[x] = pre[y];
   pre[y] = x;
   ch[x][!kind] = y;
    if(rt[y])
       rt[y] = false, rt[x] = true;
       ch[pre[x]][ch[pre[x]][1]==y] = x;
   push up (y);
}
//P函数先将根结点到r的路径上所有的结点的标记逐级下放
void P(int r)
   if(!rt[r])P(pre[r]);
   push_down(r);
void Splay(int r)
{
    P(r);
   while( !rt[r] )
        int f = pre[r], ff = pre[f];
        if(rt[f])
            Rotate(r);
        else if ( (ch[ff][1]==f) == (ch[f][1]==r) )
           Rotate(f), Rotate(r);
        else
           Rotate(r), Rotate(r);
   push_up(r);
int Access(int x)
    int y = 0;
    for( ; x : x = pre[y=x])
        Splay(x);
        rt[ch[x][1]] = true, rt[ch[x][1]=y] = false;
        push_up(x);
                                     54 / 152
```

```
}
   return y;
//判断是否是同根(真实的树,非splay)
bool judge(int u,int v)
   while(pre[u]) u = pre[u];
   while(pre[v]) v = pre[v];
   return u == v;
}
//使r成为它所在的树的根
void mroot(int r)
   Access(r);
   Splay(r);
   Update Rev(r);
//调用后u是原来u和v的lca,v和ch[u][1]分别存着lca的2个儿子
//(原来u和v所在的2颗子树)
void lca(int &u,int &v)
   Access(v), v = 0;
   while(u)
   {
       Splay(u);
       if(!pre[u])return;
       rt[ch[u][1]] = true;
       rt[ch[u][1]=v] = false;
       push up (u);
       u = pre[v = u];
void link(int u,int v)
{
   if(judge(u,v))
       puts("-1");
       return;
   mroot(u);
   pre[u] = v;
//使u成为u所在树的根,并且v和它父亲的边断开
void cut(int u,int v)
{
   if(u == v \mid \mid !judge(u,v))
   {
       puts("-1");
       return;
   mroot(u);
   Splay(v);
   pre[ch[v][0]] = pre[v];
   pre[v] = 0;
   rt[ch[v][0]] = true;
   ch[v][0] = 0;
   push up (v);
}
                                   55 / 152
```

```
void ADD(int u,int v,int w)
    if(!judge(u, v))
        puts("-1");
        return;
    lca(u,v);
   Update_Add(ch[u][1],w);
   Update_Add(v,w);
   key[u] += w;
   push_up(u);
void query(int u, int v)
{
    if(!judge(u, v))
        puts("-1");
        return;
    lca(u,v);
   printf("%d\n", max(max(Max[v], Max[ch[u][1]]), key[u]));
}
struct Edge
    int to, next;
}edge[MAXN*2];
int head[MAXN], tot;
void addedge (int u,int v)
    edge[tot].to = v;
    edge[tot].next = head[u];
   head[u] = tot++;
}
void dfs(int u)
{
    for(int i = head[u];i != -1; i = edge[i].next)
        int v = edge[i].to;
       if(pre[v] != 0)continue;
        pre[v] = u;
        dfs(v);
    }
}
int main()
   //freopen("in.txt","r",stdin);
   //freopen("out.txt","w", stdout);
   int n,q,u,v;
   while(scanf("%d", &n) == 1)
        tot = 0;
        for(int i = 0;i <= n;i++)</pre>
            head[i] = -1;
            pre[i] = 0;
                                      56 / 152
```

```
ch[i][0] = ch[i][1] = 0;
         rev[i] = 0;
         add[i] = 0;
        rt[i] = true;
     }
    Max[0] = -20000000000;
     for(int i = 1;i < n;i++)</pre>
         scanf("%d%d",&u,&v);
         addedge(u,v);
         addedge(v,u);
     }
     for(int i = 1;i <= n;i++)</pre>
         scanf("%d", & key[i]);
        Max[i] = key[i];
     scanf("%d", &q);
    pre[1] = -1;
     dfs(1);
     pre[1] = 0;
     int op;
     while(q--)
         scanf("%d", &op);
         if(op == 1)
             int x,y;
             scanf("%d%d", &x, &y);
             link(x, y);
         }
         else if (op == 2)
             int x,y;
             scanf ("%d%d", &x, &y);
             cut(x,y);
         else if (op == 3)
             int w,x,y;
             scanf("%d%d%d", &w, &x, &y);
             ADD(x,y,w);
         }
         else
         {
             int x,y;
             scanf("%d%d", &x, &y);
             query(x,y);
         }
    printf("\n");
}
return 0;
```

}

6、主席树

6.1 查询区间有多少个不同的数(SPOJ DQUERY)

```
* 给出一个序列,查询区间内有多少个不相同的数
const int MAXN = 30010;
const int M = MAXN * 100;
int n,q,tot;
int a[MAXN];
int T[MAXN],lson[M],rson[M],c[M];
int build(int 1,int r)
   int root = tot++;
    c[root] = 0;
    if(1 != r)
        int mid = (l+r)>>1;
        lson[root] = build(1, mid);
        rson[root] = build(mid+1,r);
    return root;
}
int update(int root,int pos,int val)
    int newroot = tot++, tmp = newroot;
    c[newroot] = c[root] + val;
    int 1 = 1, r = n;
    while(l < r)</pre>
        int mid = (1+r) >> 1;
        if(pos <= mid)</pre>
            lson[newroot] = tot++; rson[newroot] = rson[root];
            newroot = lson[newroot]; root = lson[root];
            r = mid;
        }
        else
            rson[newroot] = tot++; lson[newroot] = lson[root];
            newroot = rson[newroot]; root = rson[root];
            l = mid+1;
        c[newroot] = c[root] + val;
    }
   return tmp;
int query(int root,int pos)
    int ret = 0;
    int 1 = 1, r = n;
    while(pos < r)</pre>
        int mid = (1+r) >> 1;
        if (pos <= mid)</pre>
        {
                                      58 / 152
```

```
r = mid;
            root = lson[root];
        }
        else
        {
            ret += c[lson[root]];
            root = rson[root];
            1 = mid+1;
        }
    }
    return ret + c[root];
}
int main()
   //freopen("in.txt","r",stdin);
   //freopen("out.txt","w", stdout);
   while(scanf("%d", &n) == 1)
        tot = 0;
        for(int i = 1;i <= n;i++)</pre>
            scanf("%d",&a[i]);
        T[n+1] = build(1,n);
        map<int,int>mp;
        for(int i = n;i>= 1;i--)
            if (mp.find(a[i]) == mp.end())
                T[i] = update(T[i+1], i, 1);
            else
                int tmp = update(T[i+1], mp[a[i]], -1);
                T[i] = update(tmp,i,1);
            mp[a[i]] = i;
        scanf("%d", &q);
        while (q--)
            int 1,r;
            scanf("%d%d",&l,&r);
            printf("%d\n",query(T[1],r));
   return 0;
}
   6.2 静态区间第 k 大 (POJ 2104)
const int MAXN = 100010;
const int M = MAXN * 30;
int n,q,m,tot;
int a[MAXN], t[MAXN];
int T[MAXN], lson[M], rson[M], c[M];
void Init_hash()
    for(int i = 1; i <= n;i++)</pre>
        t[i] = a[i];
```

```
sort(t+1, t+1+n);
   m = unique(t+1, t+1+n) -t-1;
int build(int 1,int r)
   int root = tot++;
    c[root] = 0;
   if(1 != r)
        int mid = (1+r)>>1;
       lson[root] = build(1, mid);
       rson[root] = build(mid+1,r);
    return root;
}
int hash(int x)
   return lower bound(t+1,t+1+m,x) - t;
int update(int root,int pos,int val)
    int newroot = tot++, tmp = newroot;
    c[newroot] = c[root] + val;
   int 1 = 1, r = m;
   while(1 < r)</pre>
        int mid = (1+r)>>1;
        if(pos <= mid)</pre>
            lson[newroot] = tot++; rson[newroot] = rson[root];
            newroot = lson[newroot]; root = lson[root];
           r = mid;
        }
        else
            rson[newroot] = tot++; lson[newroot] = lson[root];
            newroot = rson[newroot]; root = rson[root];
            1 = mid+1;
        c[newroot] = c[root] + val;
    return tmp;
int query(int left_root,int right root,int k)
    int 1 = 1, r = m;
    while( 1 < r)
        int mid = (1+r) >> 1;
        if(c[lson[left root]]-c[lson[right root]] >= k )
            r = mid;
            left_root = lson[left_root];
           right_root = lson[right_root];
        }
        else
           1 = mid + 1;
                                      60 / 152
```

```
k -= c[lson[left_root]] - c[lson[right_root]];
            left_root = rson[left_root];
            right_root = rson[right_root];
    }
   return 1;
int main()
    //freopen("in.txt","r", stdin);
   //freopen("out.txt","w",stdout);
   while(scanf("%d%d",&n,&q) == 2)
        tot = 0;
        for(int i = 1;i <= n;i++)</pre>
            scanf("%d",&a[i]);
        Init hash();
        T[n+1] = build(1,m);
        for(int i = n;i ;i--)
            int pos = hash(a[i]);
            T[i] = update(T[i+1], pos, 1);
        }
        while(q--)
            int 1, r, k;
            scanf("%d%d%d",&1,&r,&k);
            printf("%d\n",t[query(T[1],T[r+1],k)]);
        }
   return 0;
   6.3 树上路径点权第 k 大(SPOJ COT)
LCA + 主席树
//主席树部分 ************
const int MAXN = 200010;
const int M = MAXN * 40;
int n,q,m,TOT;
int a[MAXN], t[MAXN];
int T[MAXN], lson[M], rson[M], c[M];
void Init_hash()
{
    for(int i = 1; i <= n;i++)</pre>
       t[i] = a[i];
   sort(t+1, t+1+n);
   m = unique(t+1, t+n+1) -t-1;
int build(int 1,int r)
   int root = TOT++;
   c[root] = 0;
   if(1 != r)
        int mid = (1+r) >> 1;
       lson[root] = build(1, mid);
       rson[root] = build(mid+1,r);
```

```
}
   return root;
int hash(int x)
   return lower bound(t+1,t+1+m,x) - t;
int update(int root,int pos,int val)
    int newroot = TOT++, tmp = newroot;
    c[newroot] = c[root] + val;
    int 1 = 1, r = m;
    while( 1 < r)
        int mid = (1+r) >> 1;
        if(pos <= mid)</pre>
           lson[newroot] = TOT++; rson[newroot] = rson[root];
           newroot = lson[newroot]; root = lson[root];
            r = mid;
        else
        {
            rson[newroot] = TOT++; lson[newroot] = lson[root];
            newroot = rson[newroot]; root = rson[root];
            1 = mid+1;
        c[newroot] = c[root] + val;
   return tmp;
}
int query(int left root,int right root,int LCA,int k)
    int lca_root = T[LCA];
    int pos = hash(a[LCA]);
    int 1 = 1, r = m;
    while(1 < r)</pre>
        int mid = (1+r) >> 1;
        int tmp = c[lson[left root]] + c[lson[right root]] - 2*c[lson[lca root]]
+ (pos >= 1 && pos <= mid);
        if(tmp >= k)
            left_root = lson[left_root];
           right root = lson[right root];
           lca_root = lson[lca_root];
           r = mid;
        }
        else
            k -= tmp;
           left_root = rson[left_root];
           right_root = rson[right_root];
           lca_root = rson[lca_root];
            1 = mid + 1;
        }
    return 1;
```

```
}
//LCA部分
int rmq[2*MAXN];//rmq数组,就是欧拉序列对应的深度序列
struct ST
    int mm[2*MAXN];
    int dp[2*MAXN][20];//最小值对应的下标
    void init(int n)
    {
       mm[0] = -1;
       for(int i = 1;i <= n;i++)</pre>
           mm[i] = ((i&(i-1)) == 0)?mm[i-1]+1:mm[i-1];
           dp[i][0] = i;
       for (int j = 1; j <= mm[n]; j++)</pre>
           for (int i = 1; i + (1 << j) - 1 <= n; i++)
               dp[i][j] = rmq[dp[i][j-1]] <
rmq[dp[i+(1<<(j-1))][j-1]]?dp[i][j-1]:dp[i+(1<<(j-1))][j-1];
    int query(int a,int b)//查询[a,b]之间最小值的下标
       if(a > b)swap(a,b);
       int k = mm[b-a+1];
       return rmq[dp[a][k]] <=</pre>
rmq[dp[b-(1<< k)+1][k]]?dp[a][k]:dp[b-(1<< k)+1][k];
} ;
//边的结构体定义
struct Edge
   int to, next;
} ;
Edge edge[MAXN*2];
int tot, head [MAXN];
int F[MAXN*2];//欧拉序列,就是dfs遍历的顺序,长度为2*n-1,下标从1开始
int P[MAXN];//P[i]表示点i在F中第一次出现的位置
int cnt;
ST st;
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
void addedge(int u,int v)//加边,无向边需要加两次
    edge[tot].to = v;
    edge[tot].next = head[u];
   head[u] = tot++;
}
void dfs(int u,int pre,int dep)
   F[++cnt] = u;
    rmq[cnt] = dep;
   P[u] = cnt;
```

```
for(int i = head[u];i != -1;i = edge[i].next)
         int v = edge[i].to;
         if(v == pre) continue;
         dfs(v,u,dep+1);
         F[++cnt] = u;
         rmq[cnt] = dep;
}
void LCA_init(int root,int node_num) //查询LCA前的初始化
{
    cnt = 0;
    dfs(root, root, 0);
    st.init(2*node num-1);
int query_lca(int u,int v)//查询u,v的lca编号
    return F[st.query(P[u],P[v])];
}
void dfs build(int u, int pre)
    int pos = hash(a[u]);
    T[u] = update(T[pre], pos, 1);
    for(int i = head[u]; i != -1;i = edge[i].next)
         int v = edge[i].to;
         if(v == pre) continue;
         dfs build(v,u);
}
int main()
    //freopen("in.txt","r", stdin);
    //freopen("out.txt","w",stdout);
    while (scanf("%d%d", &n, &q) == 2)
    {
         for(int i = 1;i <= n;i++)</pre>
             scanf("%d",&a[i]);
         Init hash();
         init();
         TOT = 0;
         int u, v;
         for(int i = 1;i < n;i++)</pre>
             scanf("%d%d",&u,&v);
             addedge (u, v);
             addedge (v,u);
         LCA init(1,n);
         T[n+1] = build(1, m);
         dfs_build(1, n+1);
         int k;
         while(q--)
              scanf("%d%d%d", &u, &v, &k);
             \label{eq:printf} \textbf{printf}("\$d\n", \texttt{t}[\texttt{query}(\texttt{T}[\texttt{u}], \texttt{T}[\texttt{v}], \texttt{query}\_lca(\texttt{u}, \texttt{v}), \texttt{k})]);
         }
```

```
return 0;
   }
   return 0;
   6.4 动态第 k 大(ZOJ2112)
树状数组套主席树
const int MAXN = 60010;
const int M = 2500010;
int n,q,m,tot;
int a[MAXN], t[MAXN];
int T[MAXN], lson[M], rson[M],c[M];
int S[MAXN];
struct Query
   int kind;
   int 1, r, k;
}query[10010];
void Init hash(int k)
   sort(t, t+k);
   m = unique(t,t+k) - t;
int hash(int x)
   return lower bound(t, t+m, x) -t;
}
int build(int 1,int r)
   int root = tot++;
   c[root] = 0;
    if(1 != r)
        int mid = (1+r)/2;
        lson[root] = build(1, mid);
       rson[root] = build(mid+1,r);
   return root;
int Insert(int root,int pos,int val)
    int newroot = tot++, tmp = newroot;
    int 1 = 0, r = m-1;
    c[newroot] = c[root] + val;
    while(1 < r)</pre>
        int mid = (1+r)>>1;
        if(pos <= mid)</pre>
            lson[newroot] = tot++; rson[newroot] = rson[root];
           newroot = lson[newroot]; root = lson[root];
           r = mid;
        else
        {
```

```
rson[newroot] = tot++; lson[newroot] = lson[root];
            newroot = rson[newroot]; root = rson[root];
            1 = mid+1;
        c[newroot] = c[root] + val;
   return tmp;
}
int lowbit(int x)
   return x&(-x);
int use[MAXN];
void add(int x,int pos,int val)
   while(x <= n)</pre>
       S[x] = Insert(S[x], pos, val);
        x += lowbit(x);
}
int sum(int x)
    int ret = 0;
   while (x > 0)
       ret += c[lson[use[x]]];
       x \rightarrow lowbit(x);
   return ret;
int Query(int left,int right,int k)
    int left root = T[left-1];
    int right root = T[right];
    int 1 = 0, r = m-1;
    for(int i = left-1;i;i -= lowbit(i)) use[i] = S[i];
    for(int i = right;i ;i -= lowbit(i)) use[i] = S[i];
    while(1 < r)</pre>
        int mid = (1+r)/2;
        int tmp = sum(right) - sum(left-1) + c[lson[right_root]] -
c[lson[left root]];
        if(tmp >= k)
            r = mid;
            for(int i = left-1; i ;i -= lowbit(i))
               use[i] = lson[use[i]];
            for(int i = right; i; i -= lowbit(i))
                use[i] = lson[use[i]];
            left_root = lson[left_root];
            right_root = lson[right_root];
        }
        else
            1 = mid+1;
            k -= tmp;
```

```
for(int i = left-1; i;i -= lowbit(i))
                use[i] = rson[use[i]];
            for(int i = right;i ;i -= lowbit(i))
                use[i] = rson[use[i]];
            left root = rson[left root];
            right_root = rson[right_root];
    return 1;
}
void Modify(int x,int p,int d)
    while(x <= n)</pre>
        S[x] = Insert(S[x], p, d);
        x += lowbit(x);
}
int main()
   //freopen("in.txt","r", stdin);
   //freopen("out.txt","w", stdout);
   int Tcase;
   scanf("%d", &Tcase);
    while (Tcase--)
        scanf("%d%d",&n,&q);
        tot = 0;
        m = 0;
        for(int i = 1;i <= n;i++)</pre>
            scanf("%d",&a[i]);
            t[m++] = a[i];
        char op [10];
        for(int i = 0;i < q;i++)</pre>
            scanf("%s",op);
            if(op[0] == 'Q')
                query[i].kind = 0;
                scanf("%d%d%d",&query[i].1,&query[i].r,&query[i].k);
            }
            else
                query[i].kind = 1;
                scanf("%d%d", &query[i].1, &query[i].r);
                t[m++] = query[i].r;
        }
        Init_hash(m);
        T[0] = build(0,m-1);
        for (int i = 1;i <= n;i++)</pre>
            T[i] = Insert(T[i-1], hash(a[i]), 1);
        for(int i = 1;i <= n;i++)</pre>
            S[i] = T[0];
        for(int i = 0;i < q;i++)</pre>
                                       67 / 152
                                                              ACM 模板
```

图论

1、最短路

1.1 Dijkstra 单源最短路,邻接矩阵形式

```
权值必须是非负
```

```
/*
* 单源最短路径, Dijkstra算法, 邻接矩阵形式, 复杂度为O(n^2)
* 求出源beg到所有点的最短路径,传入图的顶点数,和邻接矩阵cost[][]
*返回各点的最短路径lowcost[],路径pre[].pre[i]记录beg到i路径上的父结点,pre[beg]=-1
* 可更改路径权类型,但是权值必须为非负
*/
const int MAXN=1010;
#define typec int
const typec INF=0x3f3f3f3f;//防止后面溢出,这个不能太大
bool vis[MAXN];
int pre[MAXN];
void Dijkstra(typec cost[][MAXN],typec lowcost[],int n,int beg)
    for (int i=0; i<n; i++)</pre>
       lowcost[i]=INF; vis[i]=false; pre[i]=-1;
   lowcost[beg] =0;
   for (int j=0; j<n; j++)</pre>
       int k=-1;
       int Min=INF;
       for (int i=0; i<n; i++)</pre>
           if(!vis[i]&&lowcost[i]<Min)</pre>
               Min=lowcost[i];
               k=i;
           }
       if(k==-1)break;
       vis[k]=true;
       for (int i=0; i<n; i++)</pre>
           if(!vis[i]&&lowcost[k]+cost[k][i]<lowcost[i])</pre>
           {
```

```
lowcost[i]=lowcost[k]+cost[k][i];
                pre[i]=k;
            }
   }
}
   1.2 Dijkstar 算法+堆优化
使用优先队列优化,复杂度 O (E log E)
 * 使用优先队列优化Dijkstra算法
* 复杂度O(ElogE)
* 注意对vector<Edge>E[MAXN]进行初始化后加边
*/
const int INF=0x3f3f3f3f;
const int MAXN=1000010;
struct qnode
    int v;
    qnode(int _v=0,int _c=0):v(_v),c(_c){}
   bool operator <(const qnode &r) const</pre>
        return c>r.c;
};
struct Edge
{
    int v,cost;
   Edge(int _v=0,int _cost=0):v(_v),cost(_cost){}
vector<Edge>E[MAXN];
bool vis[MAXN];
int dist[MAXN];
void Dijkstra(int n,int start)//点的编号从1开始
{
    memset(vis,false, sizeof(vis));
    for (int i=1; i<=n; i++) dist[i]=INF;</pre>
    priority_queue<qnode>que;
   while(!que.empty())que.pop();
    dist[start]=0;
    que.push(qnode(start,0));
    qnode tmp;
    while(!que.empty())
        tmp=que.top();
        que.pop();
        int u=tmp.v;
        if(vis[u])continue;
        vis[u]=true;
        for(int i=0;i<E[u].size();i++)</pre>
            int v=E[tmp.v][i].v;
            int cost=E[u][i].cost;
            if(!vis[v]&&dist[v]>dist[u]+cost)
            {
                dist[v] =dist[u] +cost;
                que.push(qnode(v, dist[v]));
```

```
}
       }
   }
void addedge (int u,int v,int w)
   E[u].push back(Edge(v,w));
}
   1.3 单源最短路 bellman_ford 算法
 * 单源最短路bellman_ford算法,复杂度O(VE)
* 可以处理负边权图。
* 可以判断是否存在负环回路。返回true,当且仅当图中不包含从源点可达的负权回路
 * vector<Edge>E;先E.clear()初始化,然后加入所有边
 * 点的编号从1开始(从0开始简单修改就可以了)
*/
const int INF=0x3f3f3f3f3;
const int MAXN=550;
int dist[MAXN];
struct Edge
   int u, v;
   int cost;
   Edge(int u=0,int v=0,int cost=0):u(u),v(v),cost(cost){}
} ;
vector<Edge>E;
bool bellman ford(int start,int n)//点的编号从1开始
   for (int i=1; i<=n; i++) dist[i]=INF;</pre>
   dist[start]=0;
   for(int i=1;i<n;i++)//最多做n-1次
       bool flag=false;
       for(int j=0; j<E.size(); j++)</pre>
           int u=E[j].u;
           int v=E[j].v;
           int cost=E[j].cost;
           if (dist[v]>dist[u]+cost)
               dist[v] =dist[u] +cost;
               flag=true;
           }
       if(!flag)return true;//没有负环回路
   for(int j=0; j<E.size(); j++)</pre>
       if (dist[E[j].v]>dist[E[j].u]+E[j].cost)
           return false; //有负环回路
   return true; //没有负环回路
}
```

1.4 单源最短路 SPFA

```
/*
* 单源最短路SPFA
* 时间复杂度 0(kE)
* 这个是队列实现,有时候改成栈实现会更加快,很容易修改
 * 这个复杂度是不定的
*/
const int MAXN=1010;
const int INF=0x3f3f3f3f3;
struct Edge
   int v;
   int cost;
   Edge(int _v=0,int _cost=0):v(_v),cost(_cost){}
vector<Edge>E[MAXN];
void addedge (int u,int v,int w)
   E[u].push back(Edge(v,w));
}
bool vis[MAXN];//在队列标志
int cnt[MAXN];//每个点的入队列次数
int dist[MAXN];
bool SPFA(int start,int n)
{
   memset(vis, false, sizeof(vis));
   for(int i=1; i<=n; i++) dist[i]=INF;</pre>
    vis[start]=true;
    dist[start]=0;
    queue<int>que;
    while(!que.empty())que.pop();
    que.push(start);
    memset(cnt, 0, size of (cnt));
    cnt[start]=1;
    while(!que.empty())
        int u=que.front();
        que.pop();
        vis[u]=false;
        for (int i=0; i < E[u].size(); i++)</pre>
           int v=E[u][i].v;
           if (dist[v]>dist[u]+E[u][i].cost)
               dist[v] =dist[u] +E[u][i].cost;
               if(!vis[v])
                   vis[v]=true;
                   que.push(v);
                   if(++cnt[v]>n)return false;
                   //cnt[i]为入队列次数,用来判定是否存在负环回路
            }
       }
   return true;
}
```

2、最小生成树

2.1 Prim 算法

```
* Prim求MST
 * 耗费矩阵cost[][], 标号从0开始, 0~n-1
 * 返回最小生成树的权值,返回-1表示原图不连通
const int INF=0x3f3f3f3f3;
const int MAXN=110;
bool vis[MAXN];
int lowc[MAXN];
int Prim(int cost[][MAXN],int n)//点是0~n-1
    int ans=0;
   memset(vis,false, sizeof(vis));
    vis[0]=true;
    for (int i=1; i<n; i++) lowc[i]=cost[0][i];</pre>
    for (int i=1; i<n; i++)</pre>
        int minc=INF;
        int p=-1;
        for (int j=0; j<n; j++)</pre>
            if(!vis[j]&&minc>lowc[j])
                minc=lowc[j];
                p=j;
        if(minc==INF) return -1;//原图不连通
        ans+=minc;
        vis[p]=true;
        for (int j=0; j<n; j++)</pre>
            if(!vis[j]&&lowc[j]>cost[p][j])
                lowc[j] = cost[p][j];
    return ans;
}
    2.2 Kruskal 算法
 * <u>Kruskal</u>算法求MST
const int MAXN=110;//最大点数
const int MAXM=10000;//最大边数
int F[MAXN];//并查集使用
struct Edge
    int u, v, w;
}edge[MAXM];//存储边的信息,包括起点/终点/权值
int tol;//边数,加边前赋值为0
void addedge (int u,int v,int w)
```

```
edge[tol].u=u;
    edge[tol].v=v;
    edge[tol++].w=w;
bool cmp(Edge a, Edge b)
{//排序函数,讲边按照权值从小到大排序
    return a.w<b.w;</pre>
int find(int x)
    if(F[x] ==-1) return x;
   else return F[x]=find(F[x]);
int Kruskal(int n)//传入点数,返回最小生成树的权值,如果不连通返回-1
   memset(F,-1, sizeof(F));
   sort(edge,edge+tol,cmp);
    int cnt=0;//计算加入的边数
    int ans=0;
    for (int i=0; i<tol; i++)</pre>
        int u=edge[i].u;
       int v=edge[i].v;
       int w=edge[i].w;
       int t1=find(u);
        int t2=find(v);
        if(t1!=t2)
            ans+=w;
           F[t1]=t2;
           cnt++;
        if (cnt==n-1) break;
    if(cnt<n-1)return -1;//不连通
    else return ans;
}
```

3、次小生成树

```
* 次小生成树
* 求最小生成树时,用数组Max[i][j]来表示MST中i到j最大边权
* 求完后,直接枚举所有不在MST中的边,替换掉最大边权的边,更新答案
* 点的编号从0开始
const int MAXN=110;
const int INF=0x3f3f3f3f;
bool vis[MAXN];
int lowc[MAXN];
int pre[MAXN];
int Max[MAXN][MAXN];//Max[i][j]表示在最小生成树中从i到j的路径中的最大边权
bool used[MAXN][MAXN];
int Prim(int cost[][MAXN], int n)
{
```

```
int ans=0;
    memset(vis,false, sizeof(vis));
    memset(Max, 0, sizeof(Max));
   memset(used, false, sizeof(used));
    vis[0]=true;
    pre[0]=-1;
    for (int i=1; i<n; i++)</pre>
        lowc[i] = cost[0][i];
        pre[i]=0;
    lowc[0] = 0;
    for (int i=1; i<n; i++)</pre>
        int minc=INF;
        int p=-1;
        for (int j=0; j<n; j++)</pre>
             if(!vis[j]&&minc>lowc[j])
                 minc=lowc[j];
                 p=j;
        if (minc==INF) return -1;
        ans+=minc;
        vis[p]=true;
        used[p] [pre[p]]=used[pre[p]][p]=true;
        for (int j=0; j<n; j++)</pre>
             if(vis[j])Max[j][p]=Max[p][j]=max(Max[j][pre[p]],lowc[p]);
             if(!vis[j]&&lowc[j]>cost[p][j])
                 lowc[j] = cost[p][j];
                 pre[j]=p;
        }
    return ans;
}
```

4、有向图的强连通分量

4.1 Tarjan

```
/*

* Tarjan
算法

* 复杂度O(N+M)

*/

const int MAXN = 20010;//点数

const int MAXM = 50010;//边数

struct Edge
{
    int to, next;
}edge [MAXM];
int head [MAXN], tot;
int Low [MAXN], DFN [MAXN], Stack [MAXN], Belong [MAXN];//Belong数组的值是1~scc
int Index, top;
int scc;//强连通分量的个数
```

```
bool Instack[MAXN];
int num[MAXN];//各个强连通分量包含点的个数,数组编号1~scc
//num数组不一定需要,结合实际情况
void addedge (int u,int v)
    edge[tot].to = v;edge[tot].next = head[u];head[u] = tot++;
void Tarjan(int u)
{
   int v;
   Low[u] = DFN[u] = ++Index;
   Stack[top++] = u;
    Instack[u] = true;
    for (int i = head[u];i != -1;i = edge[i].next)
       v = edge[i].to;
       if( !DFN[v] )
        {
           Tarjan(v);
           if ( Low[u] > Low[v] ) Low[u] = Low[v];
       else if(Instack[v] && Low[u] > DFN[v])
           Low[u] = DFN[v];
    if(Low[u] == DFN[u])
        scc++;
        do
        {
           v = Stack[--top];
           Instack[v] = false;
           Belong[v] = scc;
           num[scc]++;
       while (v != u);
}
void solve(int N)
   memset(DFN, 0, sizeof(DFN));
   memset(Instack, false, sizeof(Instack));
   memset(num, 0, sizeof(num));
   Index = scc = top = 0;
    for(int i = 1;i <= N;i++)</pre>
       if(!DFN[i])
           Tarjan(i);
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
}
   4.2 Kosaraju
* Kosaraju算法,复杂度O(N+M)
```

```
*/
const int MAXN = 20010;
const int MAXM = 50010;
struct Edge
    int to, next;
}edge1[MAXM],edge2[MAXM];
//edge1是原图G, edge2是逆图GT
int head1[MAXN],head2[MAXN];
bool mark1[MAXN], mark2[MAXN];
int tot1,tot2;
int cnt1,cnt2;
int st[MAXN];//对原图进行dfs,点的结束时间从小到大排序
int Belong[MAXN];//每个点属于哪个连通分量(0~cnt2-1)
int num; //中间变量, 用来数某个连通分量中点的个数
int setNum[MAXN];//强连通分量中点的个数,编号0~cnt2-1
void addedge (int u,int v)
{
    edge1[tot1].to = v;edge1[tot1].next = head1[u];head1[u] = tot1++;
    edge2[tot2].to = u;edge2[tot2].next = head2[v];head2[v] = tot2++;
}
void DFS1(int u)
   mark1[u] = true;
    for(int i = head1[u];i != -1;i = edge1[i].next)
       if(!mark1[edge1[i].to])
           DFS1(edge1[i].to);
    st[cnt1++] = u;
}
void DFS2(int u)
   mark2[u] = true;
   num++;
   Belong[u] = cnt2;
    for (int i = head2[u]; i != -1; i = edge2[i].next)
       if(!mark2[edge2[i].to])
           DFS2(edge2[i].to);
void solve(int n)//点的编号从1开始
   memset(mark1, false, sizeof(mark1));
   memset(mark2,false,sizeof(mark2));
    cnt1 = cnt2 = 0;
    for(int i = 1;i <= n;i++)</pre>
       if(!mark1[i])
           DFS1(i);
    for(int i = cnt1-1;i >= 0; i--)
       if(!mark2[st[i]])
           num = 0;
           DFS2(st[i]);
           setNum[cnt2++] = num;
       }
}
```

5、图的割点、桥和双连通分支的基本概念

[点连通度与边连通度]

在一个无向连通图中,如果有一个顶点集合,删除这个顶点集合,以及这个集合中所有顶点相关联的边以后,原图变成多个连通块,就称这个点集为**割点集合**。一个图的**点连通度**的定义为,最小割点集合中的顶点数。

类似的,如果有一个边集合,删除这个边集合以后,原图变成多个连通块,就称这个点集为**割边集合**。一个图的**边连通度**的定义为,最小割边集合中的边数。

[双连通图、割点与桥]

如果一个无向连通图的点连通度大于 1,则称该图是**点双连通的(point biconnected)**,简称**双连通**或**重连通**。一个图有割点,当且仅当这个图的点连通度为 1,则割点集合的唯一元素被称为**割点(cut point)**,又叫**关节点(articulation point)**。

如果一个无向连通图的边连通度大于 1,则称该图是**边双连通的(edge biconnecte d)**,简称双连通或重连通。一个图有桥,当且仅当这个图的边连通度为 1,则割边集合的唯一元素被称为**桥(bridge)**,又叫**关节边**(articulation edge)。

可以看出,点双连通与边双连通都可以简称为双连通,它们之间是有着某种联系的,下文中提到的双连通,均既可指点双连通,又可指边双连通。

[双连通分支]

在图 G 的所有子图 G'中,如果 G'是双连通的,则称 G'为**双连通子图**。如果一个双连通子图 G'它不是任何一个双连通子图的真子集,则 G'为**极大双连通子图。双连通分支(biconnected component)**,或**重连通分支**,就是图的极大双连通子图。特殊的,点双连通分支又叫做**块**。

[求割点与桥]

该算法是 R.Tarjan 发明的。对图深度优先搜索,定义 DFS(u)为 u 在搜索树(以下简称为树)中被遍历到的次序号。定义 Low(u)为 u 或 u 的子树中能通过非父子边追溯到的最早的节点,即 DFS 序号最小的节点。根据 定义,则有:

Low(u)=Min { DFS(u) DFS(v) (u,v)为后向边(返祖边) 等价于 DFS(v)<DFS(u)且 v 不为 u 的父亲节点 Low(v) (u,v) 为树枝边(父子边) }

一个顶点 u 是割点,当且仅当满足(1)或(2)(1) u 为树根,且 u 有多于一个子树。 (2) u 不为树根,且满足存在(u,v)为树枝边(或称父子边,即 u 为 v 在搜索树中的父亲),使得 DFS(u)<=Low(v)。

一条无向边(u,v)是桥,当且仅当(u,v)为树枝边,且满足 DFS(u)<Low(v)。

[求双连通分支]

下面要分开讨论点双连通分支与边双连通分支的求法。

对于点双连通分支,实际上在求割点的过程中就能顺便把每个点双连通分支求出。建立一个栈,存储当前双连通分支,在搜索图时,每找到一条树枝边或后向边(非横叉边),就把这条边加入栈中。如果遇到某时满足 DFS(u)<=Low(v),说明 u 是一个割点,同时把边从栈顶一个个取出,直到遇到了边(u,v),取出的这些边与其关联的点,组成一个点双连通分支。割点可以属于多个点双连通分支,其余点和每条边只属于且属于一个点双连通分支。

对于边双连通分支,求法更为简单。只需在求出所有的桥以后,把桥边删除,原图变成了多个连通块,则每个连通块就是一个边双连通分支。桥不属于任何一个边双连通分支,其余的边和每个顶点都属于且只属于一个边双连通分支。

[构造双连通图]

一个有桥的连通图,如何把它通过加边变成边双连通图?方法为首先求出所有的桥,然后删除这些桥边,剩下的每个连通块都是一个双连通子图。把每个双连通子图收缩为一个顶点,再把桥边加回来,最后的这个图一定是一棵树,边连通度为1。

统计出树中度为 1 的节点的个数,即为叶节点的个数,记为 leaf。则至少在树上添加(leaf+1)/2 条边,就能使树达到边二连通,所以至少添加的边数就是(leaf+1)/2。具体方法为,首先把两个最近公共祖先最远的两个叶节点之间连接一条边,这样可以把这两个点到祖先的路径上所有点收缩到一起,因为一个形成的环一定是双连通的。然后再找两个最近公共祖先最远的两个叶节点,这样一对一对找完,恰好是(leaf+1)/2 次,把所有点收缩到了一起。

6、割点与桥

```
模板:
* 求 无向图的割点和桥
* 可以找出割点和桥,求删掉每个点后增加的连通块。
* 需要注意重边的处理,可以先用矩阵存,再转邻接表,或者进行判重
const int MAXN = 10010;
const int MAXM = 100010;
struct Edge
{
   int to, next;
   bool cut;//是否为桥的标记
}edge[MAXM];
int head[MAXN],tot;
int Low[MAXN], DFN[MAXN], Stack[MAXN];
int Index, top;
bool Instack[MAXN];
bool cut[MAXN];
int add block[MAXN];//删除一个点后增加的连通块
int bridge;
void addedge (int u,int v)
{
   edge[tot].to = v;edge[tot].next = head[u];edge[tot].cut = false;
  head[u] = tot++;
}
void Tarjan(int u,int pre)
{
   int v;
   Low[u] = DFN[u] = ++Index;
   Stack[top++] = u;
   Instack[u] = true;
   int son = 0;
   for (int i = head[u];i != -1;i = edge[i].next)
      v = edge[i].to;
      if (v == pre) continue;
      if( !DFN[v] )
      {
         son++;
         Tarjan(v,u);
         if(Low[u] > Low[v])Low[u] = Low[v];
         //一条无向边(u,v)是桥, 当且仅当(u,v)为树枝边,且满足DFS(u)<Low(v)。
         if(Low[v] > DFN[u])
            bridge++;
            edge[i].cut = true;
            edge[i^1].cut = true;
         }
         //割点
         //一个顶点u是割点, 当且仅当满足(1)或(2) (1) u为树根, 且u有多于一个子树。
```

```
//(2) u不为树根, 且满足存在(u,v)为树枝边(或称父子边,
          //即u为v在搜索树中的父亲), 使得DFS(u)<=Low(v)
          if(u != pre && Low[v] >= DFN[u])//不是树根
             cut[u] = true;
             add block[u]++;
      else if ( Low[u] > DFN[v])
          Low[u] = DFN[v];
   //树根,分支数大于1
   if(u == pre && son > 1)cut[u] = true;
   if(u == pre) add block[u] = son - 1;
   Instack[u] = false;
   top--;
}
调用:
1) UVA 796 Critical Links 给出一个无向图,按顺序输出桥
void solve(int N)
   memset(DFN, 0, sizeof(DFN));
   memset(Instack,false, sizeof(Instack));
   memset(add_block, 0, sizeof(add_block));
   memset(cut,false, sizeof(cut));
   Index = top = 0;
   bridge = 0;
   for(int i = 1;i <= N;i++)</pre>
      if(!DFN[i])
          Tarjan(i,i);
   printf("%d critical links\n",bridge);
   vector<pair<int,int> >ans;
   for (int u = 1; u <= N; u++)</pre>
      for(int i = head[u];i != -1;i = edge[i].next)
          if(edge[i].cut && edge[i].to > u)
          {
             ans.push back(make pair(u, edge[i].to));
          }
   sort(ans.begin(), ans.end());
   //按顺序输出桥
   for(int i = 0;i < ans.size();i++)</pre>
      printf("%d - %d\n", ans[i].first-1, ans[i].second-1);
   printf("\n");
}
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
//处理重边
map<int,int>mapit;
inline bool isHash(int u,int v)
   if (mapit[u*MAXN+v]) return true;
   if (mapit[v*MAXN+u]) return true;
   mapit[u*MAXN+v] = mapit[v*MAXN+u] = 1;
   return false;
}
```

```
int main()
   int n;
   while(scanf("%d", &n) == 1)
      init();
      int u;
      int k;
      int v;
       //mapit.clear();
       for(int i = 1;i <= n;i++)</pre>
          scanf("%d (%d)", &u, &k);
          u++;
          //这样加边,要保证正边和反边是相邻的,建无向图
          while (k--)
             scanf("%d", &v);
             v++;
             if (v <= u) continue;</pre>
              //if(isHash(u,v))continue;
             addedge(u,v);
             addedge(v,u);
          }
       }
       solve(n);
   return 0;
2) POJ 2117 求删除一个点后,图中最多有多少个连通块
void solve(int N)
   memset(DFN, 0, sizeof(DFN));
   memset(Instack, 0, sizeof(Instack));
   memset(add block, 0, sizeof(add block));
   memset(cut,false, sizeof(cut));
   Index = top = 0;
   int cnt = 0;//原来的连通块数
   for(int i = 1;i <= N;i++)</pre>
      if(!DFN[i])
         Tarjan(i,i);//找割点调用必须是Tarjan(i,i)
         cnt++;
      }
   int ans = 0;
   for(int i = 1;i <= N;i++)</pre>
     ans = max(ans,cnt+add block[i]);
   printf("%d\n",ans);
void init()
{
   tot = 0;
   memset(head, -1, sizeof(head));
int main()
   int n,m;
   int u, v;
```

```
while (scanf("%d%d",&n,&m) == 2)
{
    if(n==0 && m == 0)break;
    init();
    while (m--)
    {
        scanf("%d%d",&u,&v);
        u++;v++;
        addedge(u,v);
        addedge(v,u);
    }
    solve(n);
}
return 0;
}
```

7、边双连通分支

去掉桥,其余的连通分支就是边双连通分支了。一个有桥的连通图要变成边双连通图的话,把双连通子图收缩为一个点,形成一颗树。需要加的边为(leaf+1)/2 (leaf 为叶子结点个数)

```
POJ 3177 给定一个连通的无向图 G,至少要添加几条边,才能使其变为双连通图。
```

```
#include <stdio.h>
#include <string.h>
#include <iostream>
#include <algorithm>
#include <map>
using namespace std;
const int MAXN = 5010;//点数
const int MAXM = 20010;//边数,因为是无向图,所以这个值要*2
struct Edge
   int to, next;
   bool cut;//是否是桥标记
}edge[MAXM];
int head[MAXN], tot;
int Low[MAXN], DFN[MAXN], Stack[MAXN], Belong[MAXN]; //Belong数组的值是1~block
int Index, top;
int block;//边双连通块数
bool Instack[MAXN];
int bridge;//桥的数目
\textbf{void addedge} \; (\textbf{int} \;\; u \,, \textbf{int} \;\; v)
   edge[tot].to = v;edge[tot].next = head[u];edge[tot].cut=false;
   head[u] = tot++;
void Tarjan(int u,int pre)
   int v;
   Low[u] = DFN[u] = ++Index;
   Stack[top++] = u;
   Instack[u] = true;
   for (int i = head[u];i != -1;i = edge[i].next)
```

```
{
       v = edge[i].to;
       if(v == pre) continue;
       if( !DFN[v] )
          Tarjan(v,u);
          if ( Low[u] > Low[v] ) Low[u] = Low[v];
          if(Low[v] > DFN[u])
              bridge++;
              edge[i].cut = true;
              edge[i^1].cut = true;
          }
       else if ( Instack[v] && Low[u] > DFN[v] )
          Low[u] = DFN[v];
   if(Low[u] == DFN[u])
       block++;
       do
       {
          v = Stack[--top];
          Instack[v] = false;
          Belong[v] = block;
       while( v!=u );
   }
}
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
}
int du[MAXN];//缩点后形成树,每个点的度数
void solve(int n)
{
   memset(DFN, 0, sizeof(DFN));
   memset(Instack, false, sizeof(Instack));
   Index = top = block = 0;
   Tarjan(1,0);
   int ans = 0;
   memset(du,0,sizeof(du));
   for(int i = 1;i <= n;i++)</pre>
      for(int j = head[i];j != -1;j = edge[j].next)
        if(edge[j].cut)
           du[Belong[i]]++;
   for(int i = 1;i <= block;i++)</pre>
      if (du[i]==1)
         ans++;
   //找叶子结点的个数<u>ans</u>,构造边双连通图需要加边(<u>ans</u>+1)/2
   printf("%d\n", (ans+1) /2);
}
int main()
   int n,m;
   int u, v;
```

```
while (scanf("%d%d",&n,&m) == 2)
{
    init();
    while(m--)
    {
        scanf("%d%d",&u,&v);
        addedge(u,v);
        addedge(v,u);
    }
    solve(n);
}
return 0;
}
```

8、点双连通分支

对于点双连通分支,实际上在求割点的过程中就能顺便把每个点双连通分支求出。建立一个栈,存储当前双连通分支,在搜索图时,每找到一条树枝边或后向边(非横叉边),就把这条边加入栈中。如果遇到某时满足 DFS(u)<=Low(v),说明 u 是一个割点,同时把边从栈顶一个个取出,直到遇到了边(u,v),取出的这些边与其关联的点,组成一个点双连通分支。割点可以属于多个点双连通分支,其余点和每条边只属于且属于一个点双连通分支。

POJ 2942

奇圈,二分图判断的染色法,求点双连通分支

/ *

POJ 2942 Knights of the Round Table

亚瑟王要在圆桌上召开骑士会议,为了不引发骑士之间的冲突,

并且能够让会议的议题有令人满意的结果,每次开会前都必须对出席会议的骑士有如下要求:

- 1、 相互憎恨的两个骑士不能坐在直接相邻的2个位置;
- 2、 出席会议的骑士数必须是奇数,这是为了让投票表决议题时都能有结果。

注意: 1、所给出的憎恨关系一定是双向的,不存在单向憎恨关系。

2、由于是圆桌会议,则每个出席的骑士身边必定刚好有2个骑士。

即每个骑士的座位两边都必定各有一个骑士。

3、一个骑士无法开会,就是说至少有3个骑士才可能开会。

首先根据给出的互相憎恨的图中得到补图。

然后就相当于找出不能形成奇圈的点。

利用下面两个定理:

- (1) 如果一个双连通分量内的某些顶点在一个奇圈中(即双连通分量含有奇圈),
- 那么这个双连通分量的其他顶点也在某个奇圈中;
- (2) 如果一个双连通分量含有奇圈,则他必定不是一个二分图。反过来也成立,这是一个充要条件。

所以本题的做法,就是对补图求点双连通分量。

然后对于求得的点双连通分量,使用染色法判断是不是二分图,不是二分图,这个双连通分量的点是可以 存在的

```
const int MAXN = 1010;
const int MAXM = 2000010;

struct Edge
{
   int to, next;
}edge[MAXM];
int head[MAXN],tot;
int Low[MAXN],DFN[MAXN],Stack[MAXN],Belong[MAXN];
```

```
int Index, top;
int block; //点双连通分量的个数
bool Instack[MAXN];
bool can[MAXN];
bool ok[MAXN];//标记
int tmp[MAXN];//暂时存储双连通分量中的点
int cc;//tmp的计数
int color[MAXN];//染色
void addedge (int u,int v)
   edge[tot].to = v;edge[tot].next = head[u];head[u] = tot++;
bool dfs(int u,int col) //染色判断二分图
   color[u] = col;
   for(int i = head[u];i != -1;i = edge[i].next)
      int v = edge[i].to;
       if( !ok[v] ) continue;
       if(color[v] != -1)
          if(color[v]==col)return false;
          continue;
      if(!dfs(v,!col)) return false;
   return true;
}
void Tarjan(int u,int pre)
   int v;
   Low[u] = DFN[u] = ++Index;
   Stack[top++] = u;
   Instack[u] = true;
   for (int i = head[u];i != -1;i = edge[i].next)
       v = edge[i].to;
      if(v == pre) continue;
      if( !DFN[v] )
          Tarjan(v,u);
          if(Low[u] > Low[v])Low[u] = Low[v];
          if( Low[v] >= DFN[u])
             block++;
             int vn;
             cc = 0;
             memset(ok, false, sizeof(ok));
             do
                 vn = Stack[--top];
                Belong[vn] = block;
                Instack[vn] = false;
                 ok[vn] = true;
                 tmp[cc++] = vn;
              }
                                     84 / 152
```

```
while( vn!=v );
              ok[u] = 1;
              memset(color,-1, sizeof(color));
              if(!dfs(u,0))
                  can[u] = true;
                  while (cc--) can[tmp[cc]]=true;
               }
           }
       }
       else if(Instack[v] && Low[u] > DFN[v])
         Low[u] = DFN[v];
void solve(int n)
   memset(DFN, 0, sizeof(DFN));
   memset(Instack, false, sizeof(Instack));
   Index = block = top = 0;
   memset(can,false, sizeof(can));
   for (int i = 1;i <= n;i++)</pre>
      if(!DFN[i])
         Tarjan(i,-1);
   int ans = n;
   for (int i = 1;i <= n;i++)</pre>
       if(can[i])
        ans--;
   printf("%d\n",ans);
}
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
int g[MAXN][MAXN];
int main()
{
   int n,m;
   int u, v;
   while (scanf("%d%d", &n, &m) ==2)
       if (n==0 \&\& m==0) break;
       init();
       memset(g,0,sizeof(g));
       while (m--)
           scanf("%d%d", &u, &v);
           g[u][v]=g[v][u]=1;
       for(int i = 1;i <= n;i++)</pre>
         for(int j = 1; j <= n; j++)</pre>
           if(i != j && g[i][j]==0)
              addedge(i,j);
       solve(n);
   return 0;
```

9、最小树形图

```
#include <stdio.h>
#include <string.h>
#include <iostream>
#include <algorithm>
using namespace std;
/*
* 最小树形图
* <u>int</u>型
* 复杂度O(NM)
 * 点从0开始
*/
const int INF = 0x3f3f3f3f;
const int MAXN = 1010;
const int MAXM = 40010;
struct Edge
   int u, v, cost;
};
Edge edge[MAXM];
int pre[MAXN],id[MAXN],visit[MAXN],in[MAXN];
int zhuliu(int root,int n,int m,Edge edge[])
{
   int res = 0, u, v;
   while(1)
       for(int i = 0;i < n;i++)</pre>
          in[i] = INF;
       for(int i = 0;i < m;i++)</pre>
          if(edge[i].u != edge[i].v && edge[i].cost < in[edge[i].v])</pre>
              pre[edge[i].v] = edge[i].u;
             in[edge[i].v] = edge[i].cost;
       for(int i = 0;i < n;i++)</pre>
          if(i != root && in[i] == INF)
              return -1;//不存在最小树形图
       int tn = 0;
       memset(id,-1,sizeof(id));
       memset(visit,-1, sizeof(visit));
       in[root] = 0;
       for(int i = 0;i < n;i++)</pre>
          res += in[i];
          v = i;
          while( visit[v] != i && id[v] == -1 && v != root)
              visit[v] = i;
              v = pre[v];
          if( v != root && id[v] == -1 )
              for(int u = pre[v]; u != v ;u = pre[u])
                 id[u] = tn;
```

```
id[v] = tn++;
          }
       }
       if(tn == 0)break;//没有有向环
       for(int i = 0;i < n;i++)</pre>
          if(id[i] == -1)
              id[i] = tn++;
       for(int i = 0;i < m;)</pre>
          v = edge[i].v;
          edge[i].u = id[edge[i].u];
          edge[i].v = id[edge[i].v];
          if(edge[i].u != edge[i].v)
              edge[i++].cost -= in[v];
          else
              swap(edge[i],edge[--m]);
       }
       n = tn;
       root = id[root];
   return res;
}
int g[MAXN][MAXN];
int main()
   int n,m;
   int iCase = 0;
   int T;
   scanf("%d",&T);
   while ( T-- )
       iCase ++;
       scanf("%d%d",&n,&m);
       for(int i = 0;i < n;i++)</pre>
          for(int j = 0; j < n; j++)</pre>
              g[i][j] = INF;
       int u, v, cost;
       while (m--)
          scanf("%d%d%d", &u, &v, &cost);
          if(u == v)continue;
          g[u][v] = min(g[u][v], cost);
       int L = 0;
       for(int i = 0;i < n;i++)</pre>
          for (int j = 0; j < n; j++)
              if(g[i][j] < INF)
                 edge[L].u = i;
                 edge[L].v = j;
                 edge[L++].cost = g[i][j];
              }
       int ans = zhuliu(0,n,L,edge);
       printf("Case #%d: ",iCase);
       if (ans == -1)printf("Possums!\n");
       else printf("%d\n",ans);
   return 0;
```

}

10、二分图匹配

1) 一个二分图中的最大匹配数等于这个图中的最小点覆盖数

König 定理是一个二分图中很重要的定理,它的意思是,一个二分图中的最大匹配数等于这个图中的最小 点覆盖数。如果你还不知道什么是最小点覆盖,我也在这里说一下:假如选了一个点就相当于覆盖了以它 为端点的所有边, 你需要选择最少的点来覆盖所有的边。

2) 最小路径覆盖= | G | - 最大匹配数

在一个 N*N 的有向图中,路径覆盖就是在图中找一些路经,使之覆盖了图中的所有顶点,且任何一个顶点 有且只有一条路径与之关联;(如果把这些路径中的每条路径从它的起始点走到它的终点,那么恰好可以经 过图中的每个顶点一次且仅一次);如果不考虑图中存在回路,那么每每条路径就是一个弱连通子集. 由上面可以得出:

1.一个单独的顶点是一条路径;

2.如果存在一路径 p1,p2,.....pk, 其中 p1 为起点, pk 为终点, 那么在覆盖图中, 顶点 p1,p2,.....pk 不再 与其它的

顶点之间存在有向边.

最小路径覆盖就是找出最小的路径条数, 使之成为 G 的一个路径覆盖. 路径覆盖与二分图匹配的关系:最小路径覆盖= | G | 一最大匹配数;

3) 二分图最大独立集=顶点数-二分图最大匹配

独立集:图中任意两个顶点都不相连的顶点集合。

10.1 邻接矩阵(匈牙利算法)

```
/* ***********************************
//二分图匹配(匈牙利算法的DFS实现)(邻接矩阵形式)
//初始化: q[][]两边顶点的划分情况
//建立g[i][j]表示i->j的有向边就可以了,是左边向右边的匹配
//g没有边相连则初始化为0
//uN是匹配左边的顶点数, vN是匹配右边的顶点数
//调用: res=hungary();输出最大匹配数
//优点:适用于稠密图, DFS找增广路, 实现简洁易于理解
//时间复杂度:0(VE)
//顶点编号从0开始的
const int MAXN = 510;
int uN, vN; //u, v的数目, 使用前面必须赋值
int g[MAXN][MAXN];//邻接矩阵
int linker[MAXN];
bool used[MAXN];
bool dfs(int u)
  for (int v = 0; v < vN; v++)
     if(g[u][v] && !used[v])
       used[v] = true;
       if(linker[v] == -1 || dfs(linker[v]))
          linker[v] = u;
          return true;
       }
     }
  return false;
int hungary()
  int res = 0;
```

```
memset(linker,-1, sizeof(linker));
   for (int u = 0; u < uN; u++)</pre>
      memset(used, false, sizeof(used));
      if(dfs(u))res++;
   return res;
}
   10.2 邻接表 (匈牙利算法)
 * 匈牙利算法邻接表形式
*使用前用init()进行初始化,给uN赋值
* 加边使用函数<u>addedge</u>(u,v)
*/
const int MAXN = 5010;//点数的最大值
const int MAXM = 50010;//边数的最大值
struct Edge
{
   int to, next;
}edge[MAXM];
int head[MAXN],tot;
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
}
void addedge (int u,int v)
   edge[tot].to = v; edge[tot].next = head[u];
   head[u] = tot++;
}
int linker[MAXN];
bool used[MAXN];
int uN;
bool dfs(int u)
   for(int i = head[u]; i != -1; i = edge[i].next)
      int v = edge[i].to;
      if(!used[v])
          used[v] = true;
          if(linker[v] == -1 || dfs(linker[v]))
             linker[v] = u;
             return true;
          }
      }
   return false;
int hungary()
   int res = 0;
   memset(linker,-1, sizeof(linker));
   for(int u = 0; u < uN; u++) //点的编号0~uN-1
```

```
{
      memset(used, false, sizeof(used));
      if(dfs(u))res++;
   return res;
}
   10.3 Hopcroft-Carp 算法
/* **********
* 二分图匹配(<u>Hopcroft-Carp</u>算法)
 * 复杂度O(sqrt(n)*E)
 * 邻接表存图, vector实现
* vector先初始化,然后假如边
 * uN 为左端的顶点数,使用前赋值(点编号0开始)
*/
const int MAXN = 3000;
const int INF = 0x3f3f3f3f;
vector<int>G[MAXN];
int uN;
int Mx[MAXN],My[MAXN];
int dx[MAXN],dy[MAXN];
int dis;
bool used[MAXN];
bool SearchP()
   queue<int>Q;
   dis = INF;
   memset(dx,-1,sizeof(dx));
   memset(dy,-1,sizeof(dy));
   for(int i = 0 ; i < uN; i++)</pre>
      if (Mx[i] == -1)
      {
          Q.push(i);
         dx[i] = 0;
   while(!Q.empty())
      int u = Q.front();
      Q.pop();
      if(dx[u] > dis)break;
      int sz = G[u].size();
      for(int i = 0;i < sz;i++)</pre>
          int v = G[u][i];
          if(dy[v] == -1)
             dy[v] = dx[u] + 1;
             if(My[v] == -1)dis = dy[v];
             else
                dx[My[v]] = dy[v] + 1;
                Q.push(My[v]);
             }
          }
      }
   return dis != INF;
```

```
}
bool DFS(int u)
   int sz = G[u].size();
   for(int i = 0;i < sz;i++)</pre>
       int v = G[u][i];
       if(!used[v] && dy[v] == dx[u] + 1)
           used[v] = true;
           if (My[v] != -1 && dy[v] == dis) continue;
           if(My[v] == -1 \mid \mid DFS(My[v]))
              My[v] = u;
              Mx[u] = v;
              return true;
           }
       }
   return false;
int MaxMatch()
{
   int res = 0;
   memset(Mx,-1,sizeof(Mx));
   memset(My,-1,sizeof(My));
   while (SearchP())
       memset(used, false, sizeof(used));
       for(int i = 0;i < uN;i++)</pre>
           if(Mx[i] == -1 && DFS(i))
              res++;
   return res;
}
```

11、生成树计数

Matrix-Tree 定理(Kirchhoff 矩阵-树定理)

1、G 的度数矩阵 D[G]是一个 n*n 的矩阵,并且满足: 当 i≠j 时,dij=0; 当 i=j 时,dij 等于 vi 的度数。

2、G 的邻接矩阵 A[G]也是一个 n*n 的矩阵, 并且满足: 如果 vi、vj 之间有边直接相连,则 aij=1,否则 为 0。

我们定义 G 的 Kirchhoff 矩阵(也称为拉普拉斯算子)C[G]为 C[G]=D[G]-A[G],则 Matrix-Tree 定理可以描述为: G 的所有不同的生成树的个数等于其 Kirchhoff 矩阵 C[G]任何一个 n-1 阶主子式的行列式的绝对值。所谓 n-1 阶主子式,就是对于 r($1 \le r \le n$),将 C[G]的第 r 行、第 r 列同时去掉后得到的新矩阵,用 Cr[G]表示。

91 / 152

// HDU 4305

// 求生成树计数部分代码, 计数对 10007 取模

```
const int MOD = 10007;
int INV[MOD];
//求ax = 1( mod m) 的x值,就是逆元(0<a<m)</pre>
```

```
long long inv(long long a, long long m)
   if(a == 1)return 1;
   return inv(m%a,m) * (m-m/a)%m;
}
struct Matrix
   int mat[330][330];
   void init()
      memset(mat, 0, sizeof(mat));
   int det(int n)//求行列式的值模上MOD,需要使用逆元
       for(int i = 0;i < n;i++)</pre>
          for(int j = 0; j < n; j++)</pre>
              mat[i][j] = (mat[i][j]%MOD+MOD)%MOD;
       int res = 1;
       for(int i = 0;i < n;i++)</pre>
          for(int j = i; j < n; j++)
              if (mat[j][i]!=0)
              {
                  for (int k = i; k < n; k++)
                     swap(mat[i][k],mat[j][k]);
                  if(i != j)
                     res = (-res+MOD) %MOD;
                 break;
              }
          if(mat[i][i] == 0)
              res = -1; // 不存在 (也就是行列式值为0)
              break;
          for(int j = i+1; j < n; j++)</pre>
              //<u>int</u> <u>mut</u> = (mat[j][i]*INV[mat[i][i]])%MOD;//打表逆元
              int mut = (mat[j][i]*inv(mat[i][i],MOD))%MOD;
              for (int k = i; k < n; k++)
                 mat[j][k] = (mat[j][k]-(mat[i][k]*mut)%MOD+MOD)%MOD;
          res = (res * mat[i][i])%MOD;
       return res;
   }
};
       Matrix ret;
       ret.init();
       for(int i = 0;i < n;i++)</pre>
           for(int j = 0; j < n; j++)
              if(i != j && g[i][j])
                 ret.mat[i][j] = -1;
                 ret.mat[i][i]++;
              }
       printf("%d\n", ret.det(n-1));
```

```
计算生成树个数,不取模,SPOJ 104
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
const double eps = 1e-8;
const int MAXN = 110;
int sgn (double x)
   if(fabs(x) < eps) return 0;</pre>
   if(x < 0) return -1;
   else return 1;
double b[MAXN][MAXN];
double det(double a[][MAXN],int n)
   int i, j, k, sign = 0;
   double ret = 1;
   for(i = 0;i < n;i++)</pre>
       for(j = 0; j < n; j++)</pre>
          b[i][j] = a[i][j];
   for (i = 0;i < n;i++)</pre>
       if(sgn(b[i][i]) == 0)
           for(j = i + 1; j < n; j++)</pre>
              if(sgn(b[j][i]) != 0)
                 break;
           if(j == n)return 0;
           for(k = i; k < n; k++)
              swap(b[i][k],b[j][k]);
           sign++;
       ret *= b[i][i];
       for(k = i + 1; k < n; k++)
          b[i][k]/=b[i][i];
       for(j = i+1; j < n; j++)</pre>
           for(k = i+1; k < n; k++)</pre>
              b[j][k] = b[j][i]*b[i][k];
   if(sign & 1) ret = -ret;
   return ret;
double a[MAXN][MAXN];
int g[MAXN][MAXN];
int main()
   int T;
   int n,m;
   int u, v;
   scanf("%d", &T);
   while (T--)
       scanf("%d%d", &n, &m);
```

```
memset(g,0,sizeof(g));
       while (m--)
          scanf("%d%d", &u, &v);
          u--;v--;
          g[u][v] = g[v][u] = 1;
       memset(a,0,sizeof(a));
       for(int i = 0;i < n;i++)</pre>
          for(int j = 0; j < n; j++)</pre>
              if(i != j && g[i][j])
                  a[i][i]++;
                  a[i][j] = -1;
              }
       double ans = det(a, n-1);
       printf("%.01f\n",ans);
   return 0;
}
```

11、二分图多重匹配

```
const int MAXN = 1010;
const int MAXM = 510;
int uN, vN;
int g[MAXN][MAXM];
int linker[MAXM][MAXN];
bool used[MAXM];
int num[MAXM];//右边最大的匹配数
bool dfs(int u)
   for (int v = 0; v < vN; v++)
       if(g[u][v] && !used[v])
          used[v] = true;
          if(linker[v][0] < num[v])</pre>
              linker[v][++linker[v][0]] = u;
              return true;
          for(int i = 1;i <= num[0];i++)</pre>
              if(dfs(linker[v][i]))
                 linker[v][i] = u;
                 return true;
              }
   return false;
}
int hungary()
   int res = 0;
   for(int i = 0;i < vN;i++)</pre>
       linker[i][0] = 0;
   for (int u = 0; u < uN; u++)
```

```
{
    memset(used, false, sizeof(used));
    if(dfs(u))res++;
}
return res;
}
```

12、KM 算法(二分图最大权匹配)

```
/* KM算法
   复杂度0 (<u>nx</u>*<u>nx</u>*<u>ny</u>)
 * 求最大权匹配
   若求最小权匹配,可将权值取相反数,结果取相反数
 * 点的编号从0开始
 */
const int N = 310;
const int INF = 0x3f3f3f3f;
int nx,ny;//两边的点数
int g[N][N];//二分图描述
int linker[N], lx[N], ly[N]; //y中各点匹配状态, x, y中的点标号
int slack[N];
bool visx[N], visy[N];
bool DFS(int x)
{
   visx[x] = true;
   for (int y = 0; y < ny; y++)
       if(visy[y])continue;
       int tmp = lx[x] + ly[y] - g[x][y];
       if(tmp == 0)
          visy[y] = true;
          if(linker[y] == -1 || DFS(linker[y]))
             linker[y] = x;
             return true;
       else if(slack[y] > tmp)
          slack[y] = tmp;
   return false;
}
int KM()
   memset(linker,-1, sizeof(linker));
   memset(ly,0,sizeof(ly));
   for(int i = 0;i < nx;i++)</pre>
      lx[i] = -INF;
       for(int j = 0; j < ny; j++)</pre>
          if(g[i][j] > lx[i])
             lx[i] = g[i][j];
   for (int x = 0; x < nx; x++)
```

```
{
       for(int i = 0;i < ny;i++)</pre>
          slack[i] = INF;
       while(true)
           memset(visx, false, sizeof(visx));
           memset(visy, false, sizeof(visy));
           if(DFS(x))break;
           int d = INF;
           for(int i = 0;i < ny;i++)</pre>
               if(!visy[i] && d > slack[i])
                  d = slack[i];
           for(int i = 0;i < nx;i++)</pre>
               if(visx[i])
                  lx[i] -= d;
           for(int i = 0;i < ny;i++)</pre>
               if(visy[i])ly[i] += d;
               else slack[i] -= d;
           }
       }
   int res = 0;
   for(int i = 0;i < ny;i++)</pre>
       if(linker[i] != -1)
          res += g[linker[i]][i];
   return res;
}
//HDU 2255
int main()
   int n;
   while(scanf("%d", &n) == 1)
       for(int i = 0;i < n;i++)</pre>
           for(int j = 0; j < n; j++)</pre>
               scanf("%d", &g[i][j]);
       nx = ny = n;
       printf("%d\n",KM());
   return 0;
```

13、最大流

13.1 SAP 邻接矩阵形式

```
/*

* SAP算法 (矩阵形式)

* 结点编号从0开始

*/

const int MAXN=1100;
int maze[MAXN] [MAXN];
int gap[MAXN],dis[MAXN],pre[MAXN],cur[MAXN];
```

```
int sap(int start,int end,int nodenum)
   memset(cur, 0, size of (cur));
   memset(dis,0,sizeof(dis));
   memset(gap, 0, sizeof(gap));
   int u=pre[start]=start,maxflow=0,aug=-1;
   gap[0]=nodenum;
   while (dis[start] < nodenum)</pre>
      loop:
        for (int v=cur[u]; v<nodenum; v++)</pre>
          if(maze[u][v] && dis[u]==dis[v]+1)
             if(aug==-1 || aug>maze[u][v])aug=maze[u][v];
             pre[v]=u;
             u=cur[u]=v;
             if (v==end)
                 maxflow+=aug;
                 for (u=pre[u];v!=start;v=u, u=pre[u])
                    maze[u][v]-=aug;
                    maze[v][u]+=aug;
                 }
                 aug=-1;
              }
             goto loop;
          int mindis=nodenum-1;
          for (int v=0; v<nodenum; v++)</pre>
             if (maze [u] [v] &&mindis>dis[v])
                cur[u]=v;
                mindis=dis[v];
          if((--gap[dis[u]])==0)break;
          gap[dis[u]=mindis+1]++;
          u=pre[u];
   return maxflow;
}
   13.2 SAP 邻接矩阵形式 2
保留原矩阵, 可用于多次使用最大流
* SAP邻接矩阵形式
 * 点的编号从0开始
 * 增加个flow数组,保留原矩阵maze,可用于多次使用最大流
*/
const int MAXN=1100;
int maze[MAXN][MAXN];
int gap[MAXN],dis[MAXN],pre[MAXN],cur[MAXN];
int flow[MAXN][MAXN];//存最大流的容量
int sap(int start,int end,int nodenum)
   memset(cur, 0, sizeof(cur));
   memset(dis, 0, sizeof(dis));
   memset(gap, 0, sizeof(gap));
```

```
memset(flow, 0, sizeof(flow));
   int u=pre[start]=start,maxflow=0,aug=-1;
   gap[0]=nodenum;
   while (dis[start] < nodenum)</pre>
       loop:
         for(int v=cur[u];v<nodenum;v++)</pre>
          if (maze [u] [v] -flow[u] [v] && dis [u] == dis[v]+1)
              if(aug==-1 ||
aug>maze[u][v]-flow[u][v]) aug=maze[u][v]-flow[u][v];
              pre[v]=u;
              u=cur[u]=v;
              if(v==end)
                 maxflow+=aug;
                 for (u=pre[u]; v!=start; v=u, u=pre[u])
                     flow[u][v]+=aug;
                     flow[v][u]-=aug;
                 aug=-1;
              }
              goto loop;
          int mindis=nodenum-1;
          for (int v=0; v<nodenum; v++)</pre>
             if (maze[u][v]-flow[u][v]&&mindis>dis[v])
                cur[u]=v;
                mindis=dis[v];
          if((--gap[dis[u]])==0)break;
          gap[dis[u]=mindis+1]++;
          u=pre[u];
   return maxflow;
}
   13.3 ISAP 邻接表形式
const int MAXN = 100010;//点数的最大值
const int MAXM = 400010;//边数的最大值
const int INF = 0x3f3f3f3f;
struct Edge
    int to, next, cap, flow;
}edge[MAXM];//注意是MAXM
int tol;
int head[MAXN];
int gap[MAXN], dep[MAXN], pre[MAXN], cur[MAXN];
void init()
    tol = 0;
    memset(head, -1, sizeof(head));
}
//加边,单向图三个参数,双向图四个参数
void addedge (int u,int v,int w,int rw=0)
{
```

```
edge[tol].to = v;edge[tol].cap = w;edge[tol].next = head[u];
    edge[tol].flow = 0;head[u] = tol++;
    edge[tol].to = u;edge[tol].cap = rw;edge[tol].next = head[v];
    edge[tol].flow = 0;head[v]=tol++;
//输入参数:起点、终点、点的总数
//点的编号没有影响,只要输入点的总数
int sap(int start,int end,int N)
{
   memset(gap, 0, size of (gap));
   memset(dep, 0, sizeof(dep));
   memcpy(cur,head,sizeof(head));
    int u = start;
    pre[u] = -1;
    gap[0] = N;
    int ans = 0;
    while(dep[start] < N)</pre>
       if(u == end)
          int Min = INF;
          for(int i = pre[u];i != -1; i = pre[edge[i^1].to])
             if (Min > edge[i].cap - edge[i].flow)
                   Min = edge[i].cap - edge[i].flow;
          for(int i = pre[u];i != -1; i = pre[edge[i^1].to])
             edge[i].flow += Min;
             edge[i^1].flow -= Min;
          u = start;
          ans += Min;
          continue;
      }
      bool flag = false;
      int v;
      for(int i = cur[u]; i != -1;i = edge[i].next)
          v = edge[i].to;
          if(edge[i].cap - edge[i].flow && dep[v]+1 == dep[u])
             flag = true;
             cur[u] = pre[v] = i;
             break;
          }
       }
      if(flag)
       {
          u = v:
          continue;
      int Min = N;
      for(int i = head[u]; i != -1;i = edge[i].next)
          if(edge[i].cap - edge[i].flow && dep[edge[i].to] < Min)</pre>
          {
             Min = dep[edge[i].to];
             cur[u] = i;
      gap[dep[u]]--;
```

```
if(!gap[dep[u]]) return ans;
      dep[u] = Min+1;
      gap[dep[u]]++;
      if(u != start) u = edge[pre[u]^1].to;
   return ans;
   13.4 ISAP+bfs 初始化+栈优化
const int MAXN = 100010;//点数的最大值
const int MAXM = 400010;//边数的最大值
const int INF = 0x3f3f3f3f;
struct Edge
   int to, next, cap, flow;
}edge[MAXM];//注意是MAXM
int tol;
int head[MAXN];
int gap[MAXN],dep[MAXN],cur[MAXN];
void init()
   tol = 0;
   memset(head, -1, sizeof(head));
void addedge (int u,int v,int w,int rw = 0)
   edge[tol].to = v; edge[tol].cap = w; edge[tol].flow = 0;
   edge[tol].next = head[u]; head[u] = tol++;
   edge[tol].to = u; edge[tol].cap = rw; edge[tol].flow = 0;
   edge[tol].next = head[v]; head[v] = tol++;
int Q[MAXN];
void BFS(int start,int end)
{
   memset(dep,-1,sizeof(dep));
   memset(gap, 0, sizeof(gap));
   gap[0] = 1;
   int front = 0, rear = 0;
   dep[end] = 0;
   Q[rear++] = end;
   while(front != rear)
      int u = Q[front++];
      for(int i = head[u]; i != -1; i = edge[i].next)
          int v = edge[i].to;
          if (dep[v] != -1) continue;
          Q[rear++] = v;
          dep[v] = dep[u] + 1;
          gap[dep[v]]++;
       }
   }
int S[MAXN];
int sap(int start,int end,int N)
   BFS (start, end);
   memcpy(cur,head,sizeof(head));
```

```
int top = 0;
int u = start;
int ans = 0;
while (dep[start] < N)</pre>
   if(u == end)
       int Min = INF;
       int inser;
       for(int i = 0;i < top;i++)</pre>
          if (Min > edge[S[i]].cap - edge[S[i]].flow)
              Min = edge[S[i]].cap - edge[S[i]].flow;
              inser = i;
          }
       for(int i = 0;i < top;i++)</pre>
          edge[S[i]].flow += Min;
          edge[S[i]^1].flow -= Min;
       ans += Min;
       top = inser;
       u = edge[S[top]^1].to;
       continue;
   bool flag = false;
   int v;
   for(int i = cur[u]; i != -1; i = edge[i].next)
       v = edge[i].to;
       if(edge[i].cap - edge[i].flow && dep[v]+1 == dep[u])
          flag = true;
          cur[u] = i;
          break;
   }
   if(flag)
       S[top++] = cur[u];
       u = v;
       continue;
   int Min = N;
   for(int i = head[u]; i != -1; i = edge[i].next)
       if(edge[i].cap - edge[i].flow && dep[edge[i].to] < Min)</pre>
          Min = dep[edge[i].to];
          cur[u] = i;
   gap[dep[u]]--;
   if(!gap[dep[u]]) return ans;
   dep[u] = Min + 1;
   gap[dep[u]]++;
   if(u != start)u = edge[S[--top]^1].to;
return ans;
```

}

14、最小费用最大流

```
最小费用最大流,求最大费用只需要取相反数,结果取相反数即可。
点的总数为 N, 点的编号 0~N-1
const int MAXN = 10000;
const int MAXM = 100000;
const int INF = 0x3f3f3f3f;
struct Edge
   int to, next, cap, flow, cost;
}edge[MAXM];
int head[MAXN],tol;
int pre[MAXN], dis[MAXN];
bool vis[MAXN];
int N; //节点总个数, 节点编号从0~N-1
void init(int n)
   N = n;
   tol = 0;
   memset(head, -1, sizeof(head));
void addedge (int u,int v,int cap,int cost)
   edge[tol].to = v;
   edge[tol].cap = cap;
   edge[tol].cost = cost;
   edge[tol].flow = 0;
   edge[tol].next = head[u];
   head[u] = tol++;
   edge[tol].to = u;
   edge[tol].cap = 0;
   edge[tol].cost = -cost;
   edge[tol].flow = 0;
   edge[tol].next = head[v];
   head[v] = tol++;
bool spfa(int s,int t)
   queue<int>q;
   for(int i = 0;i < N;i++)</pre>
      dis[i] = INF;
      vis[i] = false;
      pre[i] = -1;
   dis[s] = 0;
   vis[s] = true;
   q.push(s);
   while(!q.empty())
      int u = q.front();
      q.pop();
      vis[u] = false;
      for(int i = head[u]; i != -1;i = edge[i].next)
          int v = edge[i].to;
```

```
if(edge[i].cap > edge[i].flow &&
            dis[v] > dis[u] + edge[i].cost )
             dis[v] = dis[u] + edge[i].cost;
             pre[v] = i;
             if(!vis[v])
                vis[v] = true;
                q.push(v);
             }
          }
      }
   if(pre[t] == -1)return false;
   else return true;
}
//返回的是最大流, cost存的是最小费用
int minCostMaxflow(int s,int t,int &cost)
{
   int flow = 0;
   cost = 0;
   while(spfa(s,t))
      int Min = INF;
      for (int i = pre[t];i != -1;i = pre[edge[i^1].to])
          if (Min > edge[i].cap - edge[i].flow)
             Min = edge[i].cap - edge[i].flow;
      for(int i = pre[t];i != -1;i = pre[edge[i^1].to])
          edge[i].flow += Min;
          edge[i^1].flow -= Min;
         cost += edge[i].cost * Min;
      flow += Min;
   return flow;
}
```

15、2-SAT

15.1 染色法(可以得到字典序最小的解)

```
HDU 1814
const int MAXN = 20020;
const int MAXM = 100010;
struct Edge
{
   int to, next;
}edge[MAXM];
int head[MAXN],tot;
void init()
{
   tot = 0;
   memset(head, -1, sizeof(head));
}
```

```
void addedge (int u,int v)
   edge[tot].to = v;edge[tot].next = head[u];head[u] = tot++;
bool vis[MAXN];//染色标记,为true表示选择
int S[MAXN],top;//栈
bool dfs(int u)
   if(vis[u^1]) return false;
   if(vis[u])return true;
   vis[u] = true;
   S[top++] = u;
   for(int i = head[u];i != -1;i = edge[i].next)
       if(!dfs(edge[i].to))
          return false;
   return true;
}
bool Twosat(int n)
   memset(vis,false, sizeof(vis));
   for(int i = 0;i < n;i += 2)</pre>
       if(vis[i] || vis[i^1])continue;
       top = 0;
       if(!dfs(i))
          while(top)vis[S[--top]] = false;
          if(!dfs(i^1)) return false;
       }
   return true;
}
int main()
   int n,m;
   int u, v;
   while (scanf("%d%d",&n,&m) == 2)
       init();
       while (m--)
          scanf("%d%d",&u,&v);
          u--;v--;
          addedge (u, v^1);
          addedge (v,u^1);
       }
       if(Twosat(2*n))
          for(int i = 0;i < 2*n;i++)</pre>
              if(vis[i])
                printf("%d\n",i+1);
       }
       else printf("NIE\n");
   return 0;
}
```

```
POJ 3648 Wedding
//***********
//2-SAT 强连通缩点
const int MAXN = 1010;
const int MAXM = 100010;
struct Edge
   int to, next;
}edge[MAXM];
int head[MAXN],tot;
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
void addedge (int u,int v)
   edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
int Low[MAXN],DFN[MAXN],Stack[MAXN],Belong[MAXN];//Belong数组的值1~scc
int Index, top;
int scc;
bool Instack[MAXN];
int num[MAXN];
void Tarjan(int u)
{
   int v;
   Low[u] = DFN[u] = ++Index;
   Stack[top++] = u;
   Instack[u] = true;
   for(int i = head[u];i != -1;i = edge[i].next)
      v = edge[i].to;
      if( !DFN[v] )
         Tarjan(v);
         if(Low[u] > Low[v])Low[u] = Low[v];
      else if(Instack[v] && Low[u] > DFN[v])
         Low[u] = DFN[v];
   if(Low[u] == DFN[u])
      scc++;
      do
         v = Stack[--top];
         Instack[v] = false;
         Belong[v] = scc;
         num[scc]++;
      while(v != u);
   }
bool solvable(int n)//n是总个数,需要选择一半
   memset(DFN, 0, sizeof(DFN));
   memset(Instack, false, sizeof(Instack));
                                   105 / 152
```

```
memset(num, 0, sizeof(num));
   Index = scc = top = 0;
   for(int i = 0;i < n;i++)</pre>
      if(!DFN[i])
          Tarjan(i);
   for(int i = 0;i < n;i += 2)</pre>
      if(Belong[i] == Belong[i^1])
          return false;
   }
   return true;
//*************
//拓扑排序求任意一组解部分
queue<int>q1,q2;
vector<vector<int> > dag;//缩点后的逆向DAG图
char color[MAXN];//染色,为'R'是选择的
int indeg[MAXN];//入度
int cf[MAXN];
void solve(int n)
   dag.assign(scc+1, vector<int>());
   memset(indeg, 0, sizeof(indeg));
   memset(color, 0, sizeof(color));
   for (int u = 0; u < n; u++)
      for(int i = head[u];i != -1;i = edge[i].next)
          int v = edge[i].to;
          if(Belong[u] != Belong[v])
             dag[Belong[v]].push back(Belong[u]);
             indeg[Belong[u]]++;
       }
   for (int i = 0; i < n; i += 2)
      cf[Belong[i]] = Belong[i^1];
      cf[Belong[i^1]] = Belong[i];
   while(!q1.empty())q1.pop();
   while(!q2.empty())q2.pop();
   for(int i = 1;i <= scc;i++)</pre>
      if(indeg[i] == 0)
          q1.push(i);
   while(!q1.empty())
   {
      int u = q1.front();
      q1.pop();
      if(color[u] == 0)
          color[u] = 'R';
         color[cf[u]] = 'B';
      int sz = dag[u].size();
      for(int i = 0;i < sz;i++)</pre>
          indeg[dag[u][i]]--;
```

```
if(indeg[dag[u][i]] == 0)
              q1.push (dag[u][i]);
   }
}
int change(char s[])
   int ret = 0;
   int i = 0;
   while(s[i]>='0' && s[i]<='9')</pre>
       ret *= 10;
       ret += s[i]-'0';
       i++;
   if(s[i] == 'w')return 2*ret;
   else return 2*ret+1;
}
int main()
   int n,m;
   char s1[10], s2[10];
   while(scanf("%d%d",&n,&m) == 2)
       if(n == 0 && m == 0)break;
       init();
       while (m--)
          scanf("%s%s",s1,s2);
          int u = change(s1);
          int v = change(s2);
          addedge (u^1, v);
          addedge (v^1, u);
       addedge (1,0);
       if(solvable(2*n))
       {
          solve(2*n);
          for(int i = 1;i < n;i++)</pre>
              //注意这一定是判断color[Belong[
              if(color[Belong[2*i]] == 'R')printf("%dw",i);
              else printf("%dh",i);
              if(i < n-1)printf(" ");</pre>
              else printf("\n");
          }
       }
       else printf("bad luck\n");
   return 0;
}
```

16、曼哈顿最小生成树

POJ 3241 求曼哈顿最小生成树上第 k 大的边

```
const int MAXN = 100010;
const int INF = 0x3f3f3f3f;
struct Point
    int x,y,id;
}p[MAXN];
bool cmp(Point a, Point b)
{
    if(a.x != b.x) return a.x < b.x;</pre>
    else return a.y < b.y;</pre>
//树状数组,找y-x大于当前的,但是y+x最小的
struct BIT
    int min val, pos;
    void init()
       min val = INF;
       pos = -1;
    }
}bit[MAXN];
//所有有效边
struct Edge
    int u, v, d;
}edge[MAXN<<2];</pre>
bool cmpedge (Edge a, Edge b)
   return a.d < b.d;</pre>
}
int tot;
int n;
int F[MAXN];
int find(int x)
    if(F[x] == -1) return x;
    else return F[x] = find(F[x]);
}
void addedge (int u,int v,int d)
    edge[tot].u = u;
    edge[tot].v = v;
    edge[tot++].d = d;
}
int lowbit(int x)
   return x&(-x);
void update(int i,int val,int pos)
    while (i > 0)
        if(val < bit[i].min_val)</pre>
            bit[i].min_val = val;
           bit[i].pos = pos;
        i -= lowbit(i);
```

```
}
}
int ask(int i,int m)//查询[i,m]的最小值位置
    int min_val = INF,pos = -1;
    while(i <= m)</pre>
        if(bit[i].min_val < min_val)</pre>
            min_val = bit[i].min_val;
            pos = bit[i].pos;
        i += lowbit(i);
    return pos;
}
int dist(Point a, Point b)
    return abs(a.x - b.x) + abs(a.y - b.y);
void Manhattan minimum spanning tree(int n,Point p[])
    int a[MAXN], b[MAXN];
    tot = 0;
    for(int dir = 0; dir < 4;dir++)</pre>
        //4种坐标变换
        if(dir == 1 || dir == 3)
             for(int i = 0;i < n;i++)</pre>
                 swap(p[i].x,p[i].y);
        else if (dir == 2)
             for(int i = 0;i < n;i++)</pre>
                 p[i].x = -p[i].x;
        sort(p,p+n,cmp);
        for(int i = 0;i < n;i++)</pre>
             a[i] = b[i] = p[i].y - p[i].x;
        sort(b, b+n);
        int m = unique(b, b+n) - b;
        for(int i = 1;i <= m;i++)</pre>
            bit[i].init();
        for(int i = n-1; i >= 0; i--)
             int pos = lower bound(b,b+m,a[i]) - b + 1;
             int ans = ask(pos,m);
             if(ans != -1)
                 addedge (p[i].id, p[ans].id, dist(p[i], p[ans]));
             update(pos,p[i].x+p[i].y,i);
        }
    }
}
\quad \textbf{int solve} \, (\textbf{int} \ k)
    Manhattan minimum spanning tree (n,p);
    memset(F,-1, sizeof(F));
```

```
sort(edge,edge+tot,cmpedge);
    for(int i = 0;i < tot;i++)</pre>
        int u = edge[i].u;
        int v = edge[i].v;
        int t1 = find(u), t2 = find(v);
        if(t1 != t2)
            F[t1] = t2;
            k--;
            if(k == 0)return edge[i].d;
        }
int main()
   //freopen("in.txt", "r", stdin);
   //freopen("out.txt","w",stdout);
   int k;
   while (scanf("%d%d", &n, &k) == 2 && n)
        for(int i = 0;i < n;i++)</pre>
            scanf("%d%d",&p[i].x,&p[i].y);
            p[i].id = i;
        printf("%d\n", solve(n-k));
   return 0;
}
```

17、一般图匹配带花树

```
URAL 1099
const int MAXN = 250;
int N; //点的个数,点的编号从1到N
bool Graph[MAXN][MAXN];
int Match[MAXN];
bool InQueue [MAXN], InPath[MAXN], InBlossom[MAXN];
int Head, Tail;
int Queue[MAXN];
int Start,Finish;
int NewBase;
int Father[MAXN], Base[MAXN];
int Count;//匹配数, 匹配对数是Count/2
void CreateGraph()
   int u, v;
   memset(Graph, false, sizeof(Graph));
   scanf("%d", &N);
    while(scanf("%d%d",&u,&v) == 2)
        Graph[u][v] = Graph[v][u] = true;
```

```
}
}
void Push(int u)
{
   Queue[Tail] = u;
   Tail++;
    InQueue[u] = true;
}
int Pop()
    int res = Queue[Head];
   Head++;
   return res;
int FindCommonAncestor(int u,int v)
   memset(InPath, false, sizeof(InPath));
    while(true)
    {
       u = Base[u];
        InPath[u] = true;
        if(u == Start) break;
       u = Father[Match[u]];
    }
    while(true)
    {
       v = Base[v];
        if(InPath[v])break;
        v = Father[Match[v]];
    return v;
void ResetTrace(int u)
{
    int v;
    while (Base[u] != NewBase)
        v = Match[u];
        InBlossom[Base[u]] = InBlossom[Base[v]] = true;
        u = Father[v];
        if(Base[u] != NewBase) Father[u] = v;
void BloosomContract(int u,int v)
{
   NewBase = FindCommonAncestor(u, v);
   memset(InBlossom, false, sizeof(InBlossom));
   ResetTrace(u);
   ResetTrace(v);
    if(Base[u] != NewBase) Father[u] = v;
    if(Base[v] != NewBase) Father[v] = u;
    for(int tu = 1; tu <= N; tu++)</pre>
        if(InBlossom[Base[tu]])
            Base[tu] = NewBase;
            if(!InQueue[tu]) Push(tu);
}
```

```
void FindAugmentingPath()
    memset(InQueue, false, sizeof(InQueue));
    memset(Father, 0, sizeof(Father));
    for(int i = 1;i <= N;i++)</pre>
        Base[i] = i;
    Head = Tail = 1;
    Push (Start);
    Finish = 0;
    while (Head < Tail)</pre>
        int u = Pop();
        for(int v = 1; v <= N; v++)</pre>
            if(Graph[u][v] && (Base[u] != Base[v]) && (Match[u] != v))
                if((v == Start) \mid | ((Match[v] > 0) && Father[Match[v]] > 0))
                     BloosomContract(u,v);
                 else if (Father[v] == 0)
                     Father[v] = u;
                     if(Match[v] > 0)
                         Push (Match[v]);
                     else
                         Finish = v;
                         return;
                }
            }
}
void AugmentPath()
    int u, v, w;
    u = Finish;
    while (u > 0)
        v = Father[u];
        w = Match[v];
        Match[v] = u;
        Match[u] = v;
        u = w;
}
void Edmonds()
    memset(Match, 0, sizeof(Match));
    for(int u = 1; u <= N; u++)</pre>
        if(Match[u] == 0)
            Start = u;
            FindAugmentingPath();
            if(Finish > 0)AugmentPath();
void PrintMatch()
   Count = 0;
```

```
for(int u = 1; u <= N;u++)</pre>
       if(Match[u] > 0)
           Count++;
   printf("%d\n",Count);
   for(int u = 1; u <= N; u++)</pre>
        if(u < Match[u])</pre>
           printf("%d %d\n", u, Match[u]);
}
int main()
   CreateGraph();//建图
   Edmonds();//进行匹配
   PrintMatch();//输出匹配数和匹配
   return 0;
}
18 LCA
   18.1 dfs+ST 在线算法
* LCA (POJ 1330)
* 在线算法 DFS + ST
*/
const int MAXN = 10010;
int rmq[2*MAXN];//rmq数组,就是欧拉序列对应的深度序列
struct ST
   int mm[2*MAXN];
    int dp[2*MAXN][20];//最小值对应的下标
   void init(int n)
       mm[0] = -1;
        for(int i = 1;i <= n;i++)</pre>
           mm[i] = ((i&(i-1)) == 0)?mm[i-1]+1:mm[i-1];
           dp[i][0] = i;
        for(int j = 1; j <= mm[n]; j++)</pre>
            for(int i = 1; i + (1<<j) - 1 <= n; i++)</pre>
               dp[i][j] = rmq[dp[i][j-1]] <
rmq[dp[i+(1<<(j-1))][j-1]]?dp[i][j-1]:dp[i+(1<<(j-1))][j-1];
   }
    int query(int a, int b) //查询[a,b]之间最小值的下标
       if(a > b)swap(a,b);
       int k = mm[b-a+1];
        return rmq[dp[a][k]] <=</pre>
rmq[dp[b-(1<< k)+1][k]]?dp[a][k]:dp[b-(1<< k)+1][k];
   }
//边的结构体定义
struct Edge
   int to, next;
};
```

Edge edge[MAXN*2];

```
int tot,head[MAXN];
int F[MAXN*2];//欧拉序列,就是dfs遍历的顺序,长度为2*n-1,下标从1开始
int P[MAXN];//P[i]表示点i在F中第一次出现的位置
int cnt;
ST st;
void init()
   tot = 0;
   memset(head, -1, sizeof(head));
}
void addedge(int u,int v)//加边,无向边需要加两次
   edge[tot].to = v;
   edge[tot].next = head[u];
   head[u] = tot++;
void dfs(int u,int pre,int dep)
    F[++cnt] = u;
    rmq[cnt] = dep;
    P[u] = cnt;
    for(int i = head[u];i != -1;i = edge[i].next)
       int v = edge[i].to;
       if(v == pre) continue;
       dfs(v,u,dep+1);
       F[++cnt] = u;
       rmq[cnt] = dep;
    }
void LCA_init(int root,int node num) //查询LCA前的初始化
{
    cnt = 0;
   dfs(root,root,0);
   st.init(2*node num-1);
}
int query lca(int u,int v)//查询u,v的lca编号
   return F[st.query(P[u],P[v])];
bool flag[MAXN];
int main()
{
    int T;
   int N;
   int u, v;
    scanf("%d",&T);
    while (T--)
    {
       scanf("%d", &N);
       init();
       memset(flag, false, sizeof(flag));
        for(int i = 1; i < N;i++)</pre>
           scanf("%d%d",&u,&v);
           addedge (u, v);
```

```
addedge (v,u);
           flag[v] = true;
        }
       int root;
       for(int i = 1; i <= N;i++)</pre>
           if(!flag[i])
               root = i;
               break;
           }
       LCA init(root, N);
       scanf("%d%d",&u,&v);
       printf("%d\n",query_lca(u,v));
   return 0;
}
   18.2 离线 Tarjan 算法
* POJ 1470
* 给出一颗有向树, Q个查询
* 输出查询结果中每个点出现次数
*/
/*
* LCA离线算法, <u>Tarjan</u>
* 复杂度O(n+Q);
*/
const int MAXN = 1010;
const int MAXQ = 500010;//查询数的最大值
//并查集部分
int F[MAXN];//需要初始化为-1
int find(int x)
{
    if(F[x] == -1)return x;
   return F[x] = find(F[x]);
}
void bing(int u,int v)
   int t1 = find(u);
   int t2 = find(v);
   if(t1 != t2)
       F[t1] = t2;
//**********
bool vis[MAXN];//访问标记
int ancestor[MAXN];//祖先
struct Edge
    int to, next;
}edge[MAXN*2];
int head[MAXN],tot;
void addedge (int u,int v)
    edge[tot].to = v;
    edge[tot].next = head[u];
   head[u] = tot++;
}
```

```
struct Query
   int q,next;
   int index;//查询编号
}query[MAXQ*2];
int answer[MAXQ];//存储最后的查询结果,下标0~Q-1
int h[MAXQ];
int tt;
int Q;
void add_query(int u, int v, int index)
    query[tt].q = v;
    query[tt].next = h[u];
    query[tt].index = index;
   h[u] = tt++;
    query[tt].q = u;
    query[tt].next = h[v];
    query[tt].index = index;
   h[v] = tt++;
}
void init()
{
    tot = 0;
   memset(head, -1, sizeof(head));
    tt = 0;
   memset(h,-1, sizeof(h));
    memset(vis,false, sizeof(vis));
   memset(F,-1, sizeof(F));
   memset(ancestor, 0, sizeof(ancestor));
}
void LCA(int u)
    ancestor[u] = u;
   vis[u] = true;
    for(int i = head[u];i != -1;i = edge[i].next)
        int v = edge[i].to;
        if(vis[v])continue;
        LCA(v);
        bing(u, v);
        ancestor[find(u)] = u;
    for(int i = h[u]; i != -1; i = query[i].next)
        int v = query[i].q;
        if(vis[v])
            answer[query[i].index] = ancestor[find(v)];
    }
}
bool flag[MAXN];
int Count num[MAXN];
```

```
int main()
   int n;
    int u, v, k;
    while(scanf("%d", &n) == 1)
        init();
        memset(flag, false, sizeof(flag));
        for (int i = 1;i <= n;i++)</pre>
            scanf("%d:(%d)",&u,&k);
            while(k--)
                 scanf("%d",&v);
                 flag[v] = true;
                 addedge(u,v);
                 addedge (v,u);
            }
        scanf("%d", &Q);
        for(int i = 0;i < Q;i++)</pre>
            char ch;
            cin>>ch;
            scanf("%d %d)", &u, &v);
            add_query(u,v,i);
        int root;
        for(int i = 1;i <= n;i++)</pre>
            if(!flag[i])
                root = i;
                break;
             }
        LCA(root);
        memset(Count num, 0, sizeof(Count num));
        for(int i = 0;i < Q;i++)</pre>
            Count_num[answer[i]]++;
        for(int i = 1;i <= n;i++)</pre>
            if(Count num[i] > 0)
                printf("%d:%d\n",i,Count_num[i]);
   return 0;
}
   18.3 LCA 倍增法
* POJ 1330
* LCA 在线算法
*/
const int MAXN = 10010;
const int DEG = 20;
struct Edge
    int to, next;
}edge[MAXN*2];
int head[MAXN],tot;
```

```
void addedge (int u,int v)
    edge[tot].to = v;
    edge[tot].next = head[u];
   head[u] = tot++;
void init()
{
   tot = 0;
   memset(head, -1, sizeof(head));
int fa[MAXN][DEG];//<u>fa</u>[i][j]表示结点i的第2^j个祖先
int deg[MAXN];//深度数组
void BFS(int root)
    queue<int>que;
    deg[root] = 0;
    fa[root][0] = root;
    que.push(root);
    while(!que.empty())
        int tmp = que.front();
        que.pop();
        for(int i = 1;i < DEG;i++)</pre>
            fa[tmp][i] = fa[fa[tmp][i-1]][i-1];
        for(int i = head[tmp]; i != -1; i = edge[i].next)
            int v = edge[i].to;
            if(v == fa[tmp][0])continue;
            deg[v] = deg[tmp] + 1;
            fa[v][0] = tmp;
            que.push(v);
        }
}
int LCA(int u,int v)
    if(deg[u] > deg[v])swap(u,v);
    int hu = deg[u], hv = deg[v];
    int tu = u, tv = v;
    for(int det = hv-hu, i = 0; det ;det>>=1, i++)
        if(det&1)
           tv = fa[tv][i];
    if(tu == tv) return tu;
    for(int i = DEG-1; i >= 0; i--)
        if(fa[tu][i] == fa[tv][i])
            continue;
        tu = fa[tu][i];
        tv = fa[tv][i];
    return fa[tu][0];
bool flag[MAXN];
int main()
{
```

```
int T;
int n;
int u, v;
scanf("%d",&T);
while (T--)
     scanf("%d", &n);
     init();
     memset(flag, false, sizeof(flag));
     for(int i = 1;i < n;i++)</pre>
         scanf("%d%d",&u,&v);
         addedge (u, v);
         addedge (v,u);
         flag[v] = true;
     }
     int root;
     for(int i = 1;i <= n;i++)</pre>
         if(!flag[i])
             root = i;
             break;
         }
     BFS(root);
     scanf("%d%d", &u, &v);
     printf("%d\n", LCA(u, v));
return 0;
```

计算几何

1、基本函数

1.1 Point 定义

```
const double eps = 1e-8;
const double PI = acos(-1.0);
int sgn(double x)
{
   if(fabs(x) < eps) return 0;
   if(x < 0) return -1;
   else return 1;
}
struct Point
{
   double x,y;
   Point() {}
   Point(double _x,double _y)
   {
       x = _x;y = _y;
   }
   Point operator -(const Point &b)const</pre>
```

return Point (x - b.x, y - b.y);

```
}
   //叉积
   double operator ^(const Point &b)const
       return x*b.y - y*b.x;
   }
   //点积
   double operator * (const Point &b) const
       return x*b.x + y*b.y;
   //绕原点旋转角度B(弧度值),后x,y的变化
   void transXY(double B)
       double tx = x, ty = y;
       x = tx*cos(B) - ty*sin(B);
       y = tx*sin(B) + ty*cos(B);
};
   1.2 Line 定义
struct Line
   Point s,e;
   Line() { }
   Line (Point s, Point e)
       s = _s;e = _e;
   //两直线相交求交点
   //第一个值为0表示直线重合,为1表示平行,为0表示相交,为2是相交
   //只有第一个值为2时,交点才有意义
   pair<int, Point> operator & (const Line &b) const
   {
      Point res = s;
      if(sgn((s-e)^(b.s-b.e)) == 0)
         if(sgn((s-b.e)^(b.s-b.e)) == 0)
            return make_pair(0,res);//重合
         else return make_pair(1,res);//平行
      double t = ((s-b.s)^(b.s-b.e))/((s-e)^(b.s-b.e));
      res.x += (e.x-s.x)*t;
      res.y += (e.y-s.y)*t;
      return make pair(2,res);
   }
};
   1.3 两点间距离
//*两点间距离
double dist(Point a, Point b)
{
   return sqrt((a-b) *(a-b));
}
   1.4 判断: 线段相交
```

```
//*判断线段相交
bool inter(Line 11, Line 12)
   \max(11.s.x,11.e.x) >= \min(12.s.x,12.e.x) &&
   \max(12.s.x, 12.e.x) >= \min(11.s.x, 11.e.x) &&
   \max(11.s.y, 11.e.y) >= \min(12.s.y, 12.e.y) &&
   \max(12.s.y, 12.e.y) >= \min(11.s.y, 11.e.y) &&
   sgn((12.s-11.e)^(11.s-11.e))*sgn((12.e-11.e)^(11.s-11.e)) <= 0 &&
   sgn((11.s-12.e)^(12.s-12.e))*sgn((11.e-12.e)^(12.s-12.e)) <= 0;
}
   1.5 判断: 直线和线段相交
//判断直线和线段相交
bool Seg_inter_line(Line 11, Line 12) //判断直线11和线段12是否相交
{
   return sgn((12.s-11.e)^(11.s-11.e))*sgn((12.e-11.e)^(11.s-11.e)) <= 0;
}
   1.6 点到直线距离
//点到直线距离
//返回为result,是点到直线最近的点
Point PointToLine (Point P, Line L)
{
   Point result;
   double t = ((P-L.s)*(L.e-L.s))/((L.e-L.s)*(L.e-L.s));
   result.x = L.s.x + (L.e.x-L.s.x)*t;
   result.y = L.s.y + (L.e.y-L.s.y)*t;
   return result;
}
   1.7 点到线段距离
//点到线段的距离
//返回点到线段最近的点
Point NearestPointToLineSeg (Point P, Line L)
{
   Point result;
   double t = ((P-L.s)*(L.e-L.s))/((L.e-L.s)*(L.e-L.s));
   if(t >= 0 && t <= 1)
      result.x = L.s.x + (L.e.x - L.s.x)*t;
      result.y = L.s.y + (L.e.y - L.s.y)*t;
   }
   else
      if(dist(P,L.s) < dist(P,L.e))</pre>
         result = L.s;
      else result = L.e;
   return result;
   1.8 计算多边形面积
//计算多边形面积
//点的编号从0~n-1
double CalcArea(Point p[], int n)
{
   double res = 0;
```

```
for (int i = 0;i < n;i++)</pre>
      res += (p[i]^p[(i+1)%n])/2;
   return fabs(res);
   1.9 判断点在线段上
//*判断点在线段上
bool OnSeg(Point P, Line L)
   return
   sgn((L.s-P)^(L.e-P)) == 0 &&
   sgn((P.x - L.s.x) * (P.x - L.e.x)) \le 0 &&
   sgn((P.y - L.s.y) * (P.y - L.e.y)) <= 0;
   1.10 判断点在凸多边形内
//*判断点在凸多边形内
//点形成一个凸包,而且按逆时针排序(如果是顺时针把里面的<0改为>0)
//点的编号:0~n-1
//返回值:
//-1:点在凸多边形外
//0:点在凸多边形边界上
//1:点在凸多边形内
int inConvexPoly(Point a, Point p[], int n)
   for(int i = 0;i < n;i++)</pre>
       if(sgn((p[i]-a)^(p[(i+1)%n]-a)) < 0) return -1;</pre>
       else if (OnSeg(a,Line(p[i],p[(i+1)%n])))return 0;
   return 1;
}
   1.11 判断点在任意多边形内
//*判断点在任意多边形内
//射线法, poly[]的顶点数要大于等于3,点的编号0~n-1
//返回值
//-1:点在凸多边形外
//0:点在凸多边形边界上
//1:点在凸多边形内
int inPoly(Point p,Point poly[],int n)
   int cnt;
   Line ray, side;
   cnt = 0;
   ray.s = p;
   ray.e.y = p.y;
   ray.e.x = -100000000000.0;//-INF,注意取值防止越界
   for(int i = 0;i < n;i++)</pre>
       side.s = poly[i];
       side.e = poly[(i+1)%n];
```

```
if (OnSeg(p, side)) return 0;
       //如果平行轴则不考虑
       if(sgn(side.s.y - side.e.y) == 0)
           continue;
       if(OnSeg(side.s,ray))
           if(sgn(side.s.y - side.e.y) > 0)cnt++;
       else if (OnSeg(side.e, ray))
           if(sgn(side.e.y - side.s.y) > 0)cnt++;
       else if (inter(ray, side))
           cnt++;
   if(cnt % 2 == 1)return 1;
   else return -1;
   1.12 判断凸多边形
//判断凸多边形
//允许共线边
//点可以是顺时针给出也可以是逆时针给出
//点的编号1~n-1
bool isconvex(Point poly[],int n)
   bool s[3];
   memset(s,false,sizeof(s));
   for(int i = 0;i < n;i++)</pre>
       s[sgn( (poly[(i+1)%n]-poly[i])^(poly[(i+2)%n]-poly[i]) )+1] = true;
       if(s[0] && s[2])return false;
   return true;
}
2、凸包
* 求凸包, Graham算法
* 点的编号0~n-1
* 返回凸包结果Stack[0~top-1]为凸包的编号
const int MAXN = 1010;
Point list[MAXN];
int Stack[MAXN],top;
//相对于list[0]的极角排序
bool _cmp(Point p1, Point p2)
{
```

```
double tmp = (p1-list[0])^(p2-list[0]);
    if(sgn(tmp) > 0)return true;
    else if (sgn(tmp) == 0 \&\& sgn(dist(p1,list[0]) - dist(p2,list[0])) <= 0)
       return true;
    else return false;
void Graham(int n)
   Point p0;
   int k = 0;
   p0 = list[0];
    //找最下边的一个点
    for(int i = 1;i < n;i++)</pre>
        if( (p0.y > list[i].y) || (p0.y == list[i].y && p0.x > list[i].x) )
           p0 = list[i];
           k = i;
    }
    swap(list[k],list[0]);
    sort(list+1, list+n, cmp);
    if(n == 1)
        top = 1;
       Stack[0] = 0;
       return;
    if(n == 2)
       top = 2;
       Stack[0] = 0;
       Stack[1] = 1;
       return ;
    Stack[0] = 0;
    Stack[1] = 1;
   top = 2;
    for(int i = 2;i < n;i++)</pre>
        while(top > 1 &&
sgn((list[Stack[top-1]]-list[Stack[top-2]])^(list[i]-list[Stack[top-2]])) <= 0
           top--;
       Stack[top++] = i;
   }
}
```

3、平面最近点对(HDU 1007)

```
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <iostream>
#include <math.h>
using namespace std;
```

```
const double eps = 1e-6;
const int MAXN = 100010;
const double INF = 1e20;
struct Point
   double x, y;
double dist(Point a, Point b)
{
   return sqrt((a.x-b.x) *(a.x-b.x) + (a.y-b.y) *(a.y-b.y));
}
Point p[MAXN];
Point tmpt[MAXN];
bool cmpxy(Point a, Point b)
   if(a.x != b.x)return a.x < b.x;</pre>
   else return a.y < b.y;</pre>
bool cmpy(Point a, Point b)
   return a.y < b.y;</pre>
}
double Closest_Pair(int left,int right)
   double d = INF;
   if(left == right) return d;
   if(left + 1 == right)
       return dist(p[left],p[right]);
   int mid = (left+right)/2;
   double d1 = Closest_Pair(left,mid);
   double d2 = Closest_Pair(mid+1, right);
   d = min(d1,d2);
   int k = 0;
   for(int i = left;i <= right;i++)</pre>
       if(fabs(p[mid].x - p[i].x) \le d)
          tmpt[k++] = p[i];
   }
   sort(tmpt, tmpt+k, cmpy);
   for(int i = 0;i <k;i++)</pre>
       for(int j = i+1; j < k && tmpt[j].y - tmpt[i].y < d; j++)</pre>
          d = min(d,dist(tmpt[i],tmpt[j]));
       }
   }
   return d;
}
int main()
   int n;
   while(scanf("%d", &n) ==1 && n)
       for(int i = 0;i < n;i++)</pre>
          scanf("%1f%1f",&p[i].x,&p[i].y);
       sort(p,p+n,cmpxy);
       printf("%.21f\n",Closest Pair(0,n-1)/2);
   }
```

```
return 0;
}
```

4、旋转卡壳

```
4.1 求解平面最远点对(POJ 2187 Beauty Contest)
struct Point
   int x,y;
   Point(int _x = 0,int _y = 0)
      x = _x; y = _y;
   Point operator - (const Point &b) const
      return Point(x - b.x, y - b.y);
   int operator ^(const Point &b)const
      return x*b.y - y*b.x;
   int operator *(const Point &b)const
      return x*b.x + y*b.y;
   void input()
      scanf("%d%d", &x, &y);
};
//距离的平方
int dist2(Point a, Point b)
   return (a-b) * (a-b);
//*****二维凸包, <u>int</u>*********
const int MAXN = 50010;
Point list[MAXN];
int Stack[MAXN],top;
bool _cmp(Point p1, Point p2)
   int tmp = (p1-list[0])^(p2-list[0]);
   if(tmp > 0)return true;
   else if (tmp == 0 && dist2(p1,list[0]) <= dist2(p2,list[0]))
      return true;
   else return false;
void Graham(int n)
{
   Point p0;
   int k = 0;
   p0 = list[0];
   for (int i = 1;i < n;i++)</pre>
      if(p0.y > list[i].y || (p0.y == list[i].y && p0.x > list[i].x))
```

p0 = list[i];

```
k = i;
      }
   swap(list[k],list[0]);
   sort(list+1, list+n, cmp);
   if(n == 1)
      top = 1;
      Stack[0] = 0;
      return;
   }
   if(n == 2)
      top = 2;
      Stack[0] = 0; Stack[1] = 1;
      return;
   Stack[0] = 0; Stack[1] = 1;
   top = 2;
   for(int i = 2;i < n;i++)</pre>
      while(top > 1 &&
((list[Stack[top-1]]-list[Stack[top-2]])^(list[i]-list[Stack[top-2]])) \le 0)
          top--;
      Stack[top++] = i;
   }
//旋转卡壳,求两点间距离平方的最大值
int rotating calipers (Point p[],int n)
   int ans = 0;
   Point v;
   int cur = 1;
   for(int i = 0;i < n;i++)</pre>
      v = p[i]-p[(i+1)%n];
      while ((v^{p}(cur+1)%n)-p(cur)) < 0)
         cur = (cur+1)%n;
      ans = \max(ans, \max(dist2(p[i], p[cur]), dist2(p[(i+1)%n], p[(cur+1)%n])));
   }
   return ans;
}
Point p[MAXN];
int main()
   int n;
   while(scanf("%d", &n) == 1)
      for(int i = 0;i < n;i++)list[i].input();</pre>
      Graham(n);
      for(int i = 0;i < top;i++)p[i] = list[Stack[i]];</pre>
      printf("%d\n", rotating_calipers(p, top));
   }
   return 0;
}
   4.2 求解平面点集最大三角形
//旋转卡壳计算平面点集最大三角形面积
int rotating_calipers (Point p[],int n)
```

```
{
   int ans = 0;
   Point v;
   for (int i = 0;i < n;i++)</pre>
      int j = (i+1)%n;
      int k = (j+1) %n;
       while(j != i && k != i)
          ans = \max(ans, abs((p[j]-p[i])^(p[k]-p[i])));
          while( ((p[i]-p[j])^(p[(k+1)%n]-p[k])) < 0)
            k = (k+1) %n;
          j = (j+1) %n;
   return ans;
}
Point p[MAXN];
int main()
   int n;
   while(scanf("%d", &n) == 1)
      if(n == -1)break;
      for(int i = 0;i < n;i++)list[i].input();</pre>
      Graham(n);
      for(int i = 0;i < top;i++)p[i] = list[Stack[i]];</pre>
      printf("%.2f\n", (double) rotating calipers (p, top) /2);
   return 0;
}
   4.3 求解两凸包最小距离(POJ 3608)
const double eps = 1e-8;
int sgn (double x)
   if(fabs(x) < eps) return 0;</pre>
   if(x < 0) return -1;
   else return 1;
}
struct Point
   double x,y;
   Point(double x = 0, double y = 0)
      x = _x; y = _y;
   Point operator - (const Point &b)const
      return Point(x - b.x, y - b.y);
   double operator ^(const Point &b)const
      return x*b.y - y*b.x;
   double operator *(const Point &b)const
      return x*b.x + y*b.y;
```

```
}
   void input()
      scanf("%lf%lf",&x,&y);
};
struct Line
   Point s,e;
  Line(){}
   Line(Point _s,Point _e)
      s = _s; e = _e;
};
//两点间距离
double dist(Point a, Point b)
   return sqrt((a-b) *(a-b));
//点到线段的距离,返回点到线段最近的点
Point NearestPointToLineSeg(Point P, Line L)
{
   Point result;
   double t = ((P-L.s)*(L.e-L.s))/((L.e-L.s)*(L.e-L.s));
   if(t >= 0 && t <= 1)
      result.x = L.s.x + (L.e.x - L.s.x)*t;
      result.y = L.s.y + (L.e.y - L.s.y)*t;
   }
   else
      if(dist(P,L.s) < dist(P,L.e))</pre>
         result = L.s;
      else result = L.e;
   return result;
}
/*
* 求凸包, Graham算法
* 点的编号0~n-1
* 返回凸包结果Stack[0~top-1]为凸包的编号
const int MAXN = 10010;
Point list[MAXN];
int Stack[MAXN],top;
//相对于list[0]的极角排序
bool _cmp(Point p1, Point p2)
   double tmp = (p1-list[0])^(p2-list[0]);
   if(sgn(tmp) > 0)return true;
   else if (sgn(tmp) == 0 \& sgn(dist(p1,list[0]) - dist(p2,list[0])) <= 0)
       return true;
   else return false;
void Graham(int n)
{
```

```
Point p0;
    int k = 0;
   p0 = list[0];
    //找最下边的一个点
    for(int i = 1;i < n;i++)</pre>
        if((p0.y > list[i].y) || (p0.y == list[i].y && p0.x > list[i].x))
           p0 = list[i];
           k = i;
    }
    swap(list[k],list[0]);
    sort(list+1, list+n, cmp);
    if(n == 1)
       top = 1;
       Stack[0] = 0;
       return;
    if(n == 2)
       top = 2;
       Stack[0] = 0;
       Stack[1] = 1;
       return ;
    Stack[0] = 0;
    Stack[1] = 1;
    top = 2;
    for(int i = 2;i < n;i++)</pre>
       while(top > 1 &&
sgn((list[Stack[top-1]]-list[Stack[top-2]])^(list[i]-list[Stack[top-2]])) <= 0
0)
            top--;
        Stack[top++] = i;
    }
//点p0到线段p1p2的距离
double pointtoseg(Point p0,Point p1,Point p2)
   return dist(p0,NearestPointToLineSeg(p0,Line(p1,p2)));
}
//平行线段p0p1和p2p3的距离
double dispallseg(Point p0, Point p1, Point p2, Point p3)
{
   double ans1 = min(pointtoseg(p0,p2,p3),pointtoseg(p1,p2,p3));
   double ans2 = min(pointtoseg(p2,p0,p1),pointtoseg(p3,p0,p1));
   return min(ans1,ans2);
//得到向量a1a2和b1b2的位置关系
double Get_angle(Point a1, Point a2, Point b1, Point b2)
{
   return (a2-a1) ^ (b1-b2);
double rotating calipers(Point p[],int np,Point q[],int nq)
{
```

```
int sp = 0, sq = 0;
   for(int i = 0;i < np;i++)</pre>
      if(sgn(p[i].y - p[sp].y) < 0)
          sp = i;
   for (int i = 0;i < nq;i++)</pre>
      if(sgn(q[i].y - q[sq].y) > 0)
          sq = i;
   double tmp;
   double ans = dist(p[sp],q[sq]);
   for(int i = 0;i < np;i++)</pre>
      sq = (sq+1) %nq;
      if(sgn(tmp) == 0)
          ans = min(ans, dispallseg(p[sp], p[(sp+1)%np], q[sq], q[(sq+1)%nq]));
      else ans = min(ans,pointtoseg(q[sq],p[sp],p[(sp+1)%np]));
      sp = (sp+1) %np;
   return ans;
}
double solve (Point p[], int n, Point q[], int m)
{
   return min(rotating_calipers(p, n, q, m), rotating_calipers(q, m, p, n));
}
Point p[MAXN],q[MAXN];
int main()
   int n,m;
   while(scanf("%d%d",&n,&m) == 2)
      if(n == 0 && m == 0)break;
      for(int i = 0;i < n;i++)</pre>
          list[i].input();
      Graham(n);
      for(int i = 0;i < top;i++)</pre>
          p[i] = list[i];
      n = top;
      for(int i = 0;i < m;i++)</pre>
          list[i].input();
      Graham(m);
      for(int i = 0;i < top;i++)</pre>
         q[i] = list[i];
      m = top;
      printf("%.4f\n", solve(p,n,q,m));
   return 0;
}
```

5、半平面交

5.1 半平面交模板(from UESTC)

```
const double eps = 1e-8;
const double PI = acos(-1.0);
int sgn(double x)
{
```

```
if(fabs(x) < eps) return 0;</pre>
    if(x < 0) return -1;
    else return 1;
}
struct Point
    double x,y;
    Point() { }
    Point(double _x,double _y)
       x = _x; y = _y;
    }
    Point operator - (const Point &b) const
       return Point(x - b.x, y - b.y);
    }
    double operator ^(const Point &b)const
       return x*b.y - y*b.x;
    double operator *(const Point &b) const
       return x*b.x + y*b.y;
    }
};
struct Line
   Point s,e;
   double k;
    Line() { }
    Line(Point _s, Point _e)
       s = _s; e = _e;
       k = atan2(e.y - s.y,e.x - s.x);
    Point operator & (const Line &b) const
       Point res = s;
       double t = ((s - b.s)^(b.s - b.e))/((s - e)^(b.s - b.e));
       res.x += (e.x - s.x)*t;
       res.y += (e.y - s.y)*t;
       return res;
} ;
//半平面交,直线的左边代表有效区域
bool HPIcmp(Line a, Line b)
{
   if(fabs(a.k - b.k) > eps)return a.k < b.k;</pre>
   return ((a.s - b.s)^(b.e - b.s)) < 0;
Line Q[110];
void HPI(Line line[], int n, Point res[], int &resn)
   int tot = n;
   sort(line,line+n,HPIcmp);
   tot = 1;
    for(int i = 1;i < n;i++)</pre>
        if(fabs(line[i].k - line[i-1].k) > eps)
                                     132 / 152
```

```
line[tot++] = line[i];
    int head = 0, tail = 1;
    Q[0] = line[0];
    Q[1] = line[1];
    resn = 0;
    for(int i = 2; i < tot; i++)</pre>
        if(fabs((Q[tail].e-Q[tail].s)^(Q[tail-1].e-Q[tail-1].s)) < eps ||</pre>
\textbf{fabs} ( (Q[head].e-Q[head].s) ^ (Q[head+1].e-Q[head+1].s) ) < eps)
            return;
        while(head < tail && (((Q[tail] &Q[tail-1]) -</pre>
line[i].s)^(line[i].e-line[i].s)) > eps)
            tail--;
        while(head < tail && (((Q[head]&Q[head+1]) -</pre>
line[i].s)^(line[i].e-line[i].s)) > eps)
            head++;
        Q[++tail] = line[i];
    while(head < tail && (((Q[tail] &Q[tail-1]) -</pre>
Q[head].s)^(Q[head].e-Q[head].s)) > eps)
        tail--;
    while(head < tail && (((Q[head] &Q[head-1]) -</pre>
Q[tail].s)^(Q[tail].e-Q[tail].e)) > eps)
        head++;
    if(tail <= head + 1) return;</pre>
    for(int i = head; i < tail; i++)</pre>
        res[resn++] = Q[i]&Q[i+1];
    if(head < tail - 1)</pre>
        res[resn++] = Q[head] &Q[tail];
}
   5.2 普通半平面交写法
POJ 1750
const double eps = 1e-18;
int sgn (double x)
    if(fabs(x) < eps) return 0;</pre>
    if(x < 0) return -1;
    else return 1;
struct Point
{
    double x, y;
    Point(){}
    Point(double x,double y)
        x = _x; y = _y;
    Point operator - (const Point &b) const
        return Point(x - b.x, y - b.y);
    double operator ^(const Point &b)const
       return x*b.y - y*b.x;
    double operator *(const Point &b)const
    {
```

```
return x*b.x + y*b.y;
   }
};
//计算多边形面积
double CalcArea(Point p[], int n)
   double res = 0;
    for(int i = 0;i < n;i++)</pre>
       res += (p[i]^p[(i+1)%n]);
   return fabs(res/2);
//通过两点,确定直线方程
void Get_equation(Point p1,Point p2,double &a,double &b,double &c)
   a = p2.y - p1.y;
   b = p1.x - p2.x;
   c = p2.x*p1.y - p1.x*p2.y;
//求交点
Point Intersection (Point p1, Point p2, double a, double b, double c)
    double u = fabs(a*p1.x + b*p1.y + c);
   double v = fabs(a*p2.x + b*p2.y + c);
   Point t;
   t.x = (p1.x*v + p2.x*u)/(u+v);
   t.y = (p1.y*v + p2.y*u)/(u+v);
   return t;
Point tp[110];
void Cut(double a, double b, double c, Point p[], int &cnt)
    int tmp = 0;
    for(int i = 1;i <= cnt;i++)</pre>
        //当前点在左侧, 逆时针的点
        if(a*p[i].x + b*p[i].y + c < eps)tp[++tmp] = p[i];
        else
        {
            if(a*p[i-1].x + b*p[i-1].y + c < -eps)
               tp[++tmp] = Intersection(p[i-1],p[i],a,b,c);
            if(a*p[i+1].x + b*p[i+1].y + c < -eps)
                tp[++tmp] = Intersection(p[i],p[i+1],a,b,c);
    }
    for(int i = 1;i <= tmp;i++)</pre>
       p[i] = tp[i];
   p[0] = p[tmp];
   p[tmp+1] = p[1];
    cnt = tmp;
double V[110],U[110],W[110];
int n;
const double INF = 100000000000.0;
Point p[110];
bool solve(int id)
   p[1] = Point(0,0);
    p[2] = Point(INF, 0);
```

```
p[3] = Point(INF, INF);
    p[4] = Point(0, INF);
    p[0] = p[4];
   p[5] = p[1];
    int cnt = 4;
    for(int i = 0;i < n;i++)</pre>
        if(i != id)
            double a = (V[i] - V[id])/(V[i]*V[id]);
            double b = (U[i] - U[id])/(U[i]*U[id]);
            double c = (W[i] - W[id]) / (W[i] *W[id]);
            if(sgn(a) == 0 \&\& sgn(b) == 0)
                if(sgn(c) >= 0)return false;
                else continue;
            Cut(a,b,c,p,cnt);
    if(sgn(CalcArea(p,cnt)) == 0)return false;
    else return true;
int main()
{
   while(scanf("%d", &n) == 1)
        for(int i = 0;i < n;i++)</pre>
            scanf("%lf%lf%lf",&V[i],&U[i],&W[i]);
        for(int i = 0;i < n;i++)</pre>
            if(solve(i))printf("Yes\n");
            else printf("No\n");
    }
   return 0;
}
```

6、三点求圆心坐标(三角形外心)

```
//过三点求圆心坐标
```

```
Point waixin (Point a, Point b, Point c)
{
    double a1 = b.x - a.x, b1 = b.y - a.y, c1 = (a1*a1 + b1*b1)/2;
    double a2 = c.x - a.x, b2 = c.y - a.y, c2 = (a2*a2 + b2*b2)/2;
    double d = a1*b2 - a2*b1;
    return Point (a.x + (c1*b2 - c2*b1)/d, a.y + (a1*c2 -a2*c1)/d);
}
```

动态规划

1、最长上升子序列 O(nlogn)

```
const int MAXN=500010;
int a[MAXN], b[MAXN];
//用二分查找的方法找到一个位置,使得\underline{num}>b[i-1] 并且\underline{num}<b[i],并用\underline{num}代替b[i]
int Search(int num, int low, int high)
{
   int mid;
   while (low<=high)</pre>
      mid=(low+high)/2;
      if(num>=b[mid]) low=mid+1;
      else high=mid-1;
   return low;
int DP(int n)
   int i,len,pos;
   b[1]=a[1];
   len=1;
   for (i=2;i<=n;i++)</pre>
      if(a[i]>=b[len])//如果a[i]比b[]数组中最大还大直接插入到后面即可
          len=len+1;
          b[len]=a[i];
      else//用二分的方法在b[]数组中找出第一个比a[i]大的位置并且让a[i]替代这个位置
          pos=Search(a[i],1,len);
          b[pos]=a[i];
      }
   }
   return len;
}
```

其他

1、高精度

```
/*
 * 高精度,支持乘法和加法
 */
struct BigInt
{
    const static int mod = 10000;
```

```
const static int DLEN = 4;
int a[600],len;
BigInt()
    memset(a, 0, sizeof(a));
    len = 1;
BigInt(int v)
    memset(a,0,sizeof(a));
    len = 0;
    do
        a[len++] = v%mod;
        v /= mod;
    }while(v);
BigInt(const char s[])
    memset(a, 0, sizeof(a));
    int L = strlen(s);
    len = L/DLEN;
    if (L%DLEN) len++;
    int index = 0;
    for(int i = L-1;i >= 0;i -= DLEN)
        int t = 0;
        int k = i - DLEN + 1;
        if(k < 0)k = 0;
        for(int j = k; j <= i; j++)</pre>
            t = t*10 + s[j] - '0';
        a[index++] = t;
    }
BigInt operator + (const BigInt &b) const
    BigInt res;
    res.len = max(len,b.len);
    for(int i = 0;i <= res.len;i++)</pre>
        res.a[i] = 0;
    for(int i = 0;i < res.len;i++)</pre>
        res.a[i] += ((i < len)?a[i]:0)+((i < b.len)?b.a[i]:0);
        res.a[i+1] += res.a[i]/mod;
        res.a[i] %= mod;
    if(res.a[res.len] > 0)res.len++;
    return res;
BigInt operator *(const BigInt &b)const
    BigInt res;
    for(int i = 0; i < len;i++)</pre>
        int up = 0;
        for(int j = 0; j < b.len; j++)</pre>
            int temp = a[i]*b.a[j] + res.a[i+j] + up;
                                  137 / 152
```

2、完全高精度

```
HDU 1134 求卡特兰数
#include <iostream>
#include <stdio.h>
#include <algorithm>
#include <string.h>
using namespace std;
* 完全大数模板
* 输出cin>>a
* 输出a.print();
*注意这个输入不能自动去掉前导0的,可以先读入到char数组,去掉前导0,再用构造函数。
#define MAXN 9999
#define MAXSIZE 1010
#define DLEN 4
class BigNum
private:
  int a[500]; //可以控制大数的位数
  int len;
public:
  BigNum() {len=1; memset(a,0, sizeof(a));} //构造函数
                     //将一个int类型的变量转化成大数
  BigNum(const int);
  BigNum(const char*); //将一个字符串类型的变量转化为大数
  BigNum(const BigNum &); //拷贝构造函数
  BigNum & operator=(const BigNum &); //重载赋值运算符,大数之间进行赋值运算
  friend istream& operator>>(istream&,BigNum&); //重载输入运算符
  friend ostream& operator<<(ostream&,BigNum&); //重载输出运算符
  BigNum operator+(const BigNum &)const; //重载加法运算符,两个大数之间的相加运算
  BigNum operator-(const BigNum &) const; //重载减法运算符,两个大数之间的相减运算
  BigNum operator*(const BigNum &)const; //重载乘法运算符,两个大数之间的相乘运算
  BigNum operator/(const int &)const;
                                    //重载除法运算符,大数对一个整数进行相除
运算
```

```
BigNum operator^(const int &)const;
                                         //大数的n次方运算
   int operator% (const int &) const;
                                         //大数对一个<u>int</u>类型的变量进行取模运算
   bool operator>(const BigNum &T)const; //大数和另一个大数的大小比较
                                         //大数和一个int类型的变量的大小比较
   bool operator>(const int &t)const;
                   //输出大数
   void print();
};
BigNum::BigNum(const int b) //将一个<u>int</u>类型的变量转化为大数
   int c, d=b;
   len=0;
   memset(a,0,sizeof(a));
   while (d>MAXN)
      c=d-(d/(MAXN+1))*(MAXN+1);
      d=d/(MAXN+1);
      a[len++]=c;
   a[len++]=d;
BigNum::BigNum(const char *s) //将一个字符串类型的变量转化为大数
   int t,k,index,L,i;
   memset(a,0,sizeof(a));
   L=strlen(s);
   len=L/DLEN;
   if(L%DLEN)len++;
   index=0;
   for (i=L-1; i>=0; i-=DLEN)
      t=0;
      k=i-DLEN+1;
      if(k<0) k=0;
      for (int j=k; j<=i; j++)</pre>
         t=t*10+s[j]-'0';
      a[index++]=t;
   }
}
BigNum::BigNum(const BigNum &T):len(T.len) //拷贝构造函数
{
   int i;
   memset(a,0,sizeof(a));
   for (i=0;i<len;i++)</pre>
      a[i]=T.a[i];
}
BiqNum & BiqNum::operator=(const BiqNum &n) //重载赋值运算符,大数之间赋值运算
   int i;
   len=n.len;
   memset(a, 0, sizeof(a));
   for (i=0;i<len;i++)</pre>
      a[i]=n.a[i];
   return *this;
}
istream& operator>>(istream &in,BigNum &b)
   char ch [MAXSIZE*4];
```

```
int i=-1;
   in>>ch;
   int L=strlen(ch);
   int count=0, sum=0;
   for (i=L-1; i>=0;)
       sum=0;
       int t=1;
      for(int j=0; j<4&&i>=0; j++, i--, t*=10)
          sum+=(ch[i]-'0')*t;
       }
      b.a[count]=sum;
       count++;
   b.len=count++;
   return in;
}
ostream& operator<<(ostream& out,BigNum& b) //重载输出运算符
   int i;
   cout<<b.a[b.len-1];</pre>
   for (i=b.len-2;i>=0;i--)
      printf("%04d",b.a[i]);
   return out;
BigNum BigNum::operator+(const BigNum &T)const //两个大数之间的相加运算
   BigNum t(*this);
   int i,big;
   big=T.len>len?T.len:len;
   for (i=0;i<big;i++)</pre>
      t.a[i]+=T.a[i];
      if(t.a[i]>MAXN)
          t.a[i+1]++;
          t.a[i]-=MAXN+1;
       }
   }
   if(t.a[big]!=0)
      t.len=big+1;
   else t.len=big;
   return t;
BigNum BigNum::operator-(const BigNum &T)const //两个大数之间的相减运算
   int i,j,big;
   bool flag;
   BigNum t1,t2;
   if(*this>T)
      t1=*this;
      t2=T;
      flag=0;
   }
```

```
else
       t1=T;
       t2=*this;
       flag=1;
   big=t1.len;
   for (i=0;i<big;i++)</pre>
       if(t1.a[i]<t2.a[i])</pre>
          j=i+1;
          while(t1.a[j]==0)
              j++;
          t1.a[j--]--;
          while(j>i)
              t1.a[j--] += MAXN;
          t1.a[i] +=MAXN+1-t2.a[i];
       else t1.a[i] -=t2.a[i];
   t1.len=big;
   while (t1.a[len-1] == 0 \&\& t1.len>1)
       t1.len--;
       big--;
   if(flag)
       t1.a[big-1]=0-t1.a[big-1];
   return t1;
}
BigNum BigNum::operator*(const BigNum &T)const //两个大数之间的相乘
   BigNum ret;
   int i,j,up;
   int temp, temp1;
   for (i=0;i<len;i++)</pre>
       up=0;
       for (j=0; j<T.len; j++)</pre>
           temp=a[i]*T.a[j]+ret.a[i+j]+up;
          if (temp>MAXN)
              temp1=temp-temp/(MAXN+1) * (MAXN+1);
              up=temp/(MAXN+1);
              ret.a[i+j]=temp1;
           }
          else
              up=0;
              ret.a[i+j]=temp;
           }
       if(up!=0)
          ret.a[i+j]=up;
   ret.len=i+j;
```

```
while (ret.a[ret.len-1] == 0 && ret.len > 1) ret.len--;
   return ret;
}
BigNum BigNum::operator/(const int &b)const //大数对一个整数进行相除运算
   BigNum ret;
   int i,down=0;
   for (i=len-1; i>=0; i--)
      ret.a[i] = (a[i] + down* (MAXN+1))/b;
       down=a[i]+down*(MAXN+1)-ret.a[i]*b;
   ret.len=len;
   while (ret.a[ret.len-1]==0 && ret.len>1)
      ret.len--;
   return ret;
}
int BigNum::operator%(const int &b)const //大数对一个 int类型的变量进行取模
   int i,d=0;
   for (i=len-1; i>=0; i--)
       d=((d*(MAXN+1))%b+a[i])%b;
   return d;
}
BigNum BigNum::operator^(const int &n)const //大数的n次方运算
{
   BigNum t, ret(1);
   int i;
   if(n<0) exit(-1);
   if (n==0) return 1;
   if(n==1)return *this;
   int m=n;
   while (m>1)
      t=*this;
      for (i=1; (i<<1) <=m; i<<=1)</pre>
         t=t*t;
      m-=i;
      ret=ret*t;
      if (m==1) ret=ret* (*this);
   }
   return ret;
bool BigNum::operator>(const BigNum &T) const //大数和另一个大数的大小比较
{
   int ln;
   if(len>T.len)return true;
   else if (len==T.len)
      ln=len-1;
      while (a[ln]==T.a[ln] & &ln>=0)
       if(ln>=0 && a[ln]>T.a[ln])
         return true;
       else
         return false;
   else
```

```
return false;
}
bool BigNum::operator>(const int &t)const //大数和一个<u>int</u>类型的变量的大小比较
   BigNum b(t);
   return *this>b;
                     //输出大数
void BigNum::print()
   int i;
   printf("%d", a[len-1]);
   for (i=len-2; i>=0; i--)
    printf("%04d",a[i]);
   printf("\n");
BigNum f[110];//卡特兰数
int main()
{
    f[0]=1;
    for (int i=1; i<=100; i++)</pre>
        f[i]=f[i-1]*(4*i-2)/(i+1);//卡特兰数递推式
    int n;
    while(scanf("%d", &n)==1)
        if (n==-1) break;
       f[n].print();
   return 0;
}
```

3、strtok 和 sscanf 结合输入

空格作为分隔输入,读取一行的整数:

```
gets(buf);
int v;
char *p = strtok(buf, " ");
while(p)
{
    sscanf(p, "%d", &v);
    p = strtok(NULL, " ");
}
```

4、解决爆栈, 手动加栈

#pragma comment(linker, "/STACK:1024000000,1024000000")

5、STL

```
5.1 优先队列 priority_queue
empty() 如果队列为空返回真
pop() 删除对顶元素
push() 加入一个元素
size()返回优先队列中拥有的元素个数
top() 返回优先队列队顶元素
在默认的优先队列中,优先级高的先出队。在默认的 int 型中先出队的为较大的
数。
priority_queue<int>q1;//大的先出对
priority queue<int, vector<int>, greater<int> >q2; //小的先出队
自定义比较函数:
struct cmp
{
  bool operator () (int x, int y)
      return x > y; // x小的优先级高
   //也可以写成其他方式,如: return p[x] > p[y];表示p[i]小的优先级高
}
};
priority queue<int, vector<int>, cmp>q;//定义方法
//其中,第二个参数为容器类型。第三个参数为比较函数。
结构体排序:
struct node
  int x, y;
  friend bool operator < (node a, node b)</pre>
     return a.x > b.x; //结构体中, x小的优先级高
priority_queue<node>q;//定义方法
//在该结构中, y为值, x为优先级。
//通过自定义operator<操作符来比较元素中的优先级。
//在重载"<"时,最好不要重载">",可能会发生编译错误
  5.2 set 和 multiset
set 和 multiset 用法一样,就是 multiset 允许重复元素。
元素放入容器时,会按照一定的排序法则自动排序,默认是按照 less<>排序规则来排序。不
能修改容器里面的元素值,只能插入和删除。
自定义 int 排序函数: (默认的是从小到大的,下面这个从大到小)
struct classcomp {
 bool operator() (const int& lhs, const int& rhs) const
 {return lhs>rhs;}
};//这里有个逗号的,注意
multiset<int,classcomp> fifth;
                                  // class as Compare
```

上面这样就定义成了从大到小排列了。

```
结构体自定义排序函数:
```

```
(定义 set 或者 multiset 的时候定义了排序函数, 定义迭代器时一样带上排序函数)
struct Node
{
    int x,y;
};
struct classcomp//先按照 x 从小到大排序, x相同则按照y从大到小排序
{
    bool operator()(const Node &a,const Node &b)const
    {
        if(a.x!=b.x)return a.x<b.x;
        else return a.y>b.y;
    }
}; //注意这里有个逗号
multiset<Node,classcomp>mt;
multiset<Node,classcomp>::iterator it;
```

主要函数:

```
begin() 返回指向第一个元素的迭代器 clear() 清除所有元素 count() 返回某个值元素的个数 empty() 如果集合为空,返回 true end() 返回指向最后一个元素的迭代器 erase() 删除集合中的元素 (参数是一个元素值,或者迭代器) find() 返回一个指向被查找到元素的迭代器 insert() 在集合中插入元素 size() 集合中元素的数目 lower_bound() 返回指向大于(或等于)某值的第一个元素的迭代器 upper_bound() 返回大于某个值元素的迭代器 equal_range() 返回集合中与给定值相等的上下限的两个迭代器 (注意对于 multiset 删除操作之间删除值会把所以这个值的都删掉,删除一个要用迭代器)
```

6、输入输出外挂

```
//适用于正负整数
```

```
template <class T>
inline bool scan_d(T &ret) {
   char c; int sgn;
   if(c=getchar(),c==EOF) return 0; //EOF
   while(c!='-'&&(c<'0'||c>'9')) c=getchar();
   sgn=(c=='-')?-1:1;
   ret=(c=='-')?0:(c-'0');
   while(c=getchar(),c>='0'&&c<='9') ret=ret*10+(c-'0');
   ret*=sgn;
   return 1;
}
inline void out(int x) {</pre>
```

```
if(x>9) out(x/10);
putchar(x%10+'0');
```

7、莫队算法

莫队算法,可以解决一类静态,离线区间查询问题。

BZOJ 2038: [2009 国家集训队]小 Z 的袜子(hose)

Description

作为一个生活散漫的人,小 Z 每天早上都要耗费很久从一堆五颜六色的袜子中找出一双来穿。终于有一天,小 Z 再也无法忍受这恼人的找袜子过程,于是他决定听天由命…… 具体来说,小 Z 把这 N 只袜子从 1 到 N 编号,然后从编号 L 到 R(L)

Input

输入文件第一行包含两个正整数 N 和 M。N 为袜子的数量, M 为小 Z 所提的询问的数量。接下来一行包含 N 个正整数 Ci, 其中 Ci 表示第 i 只袜子的颜色,相同的颜色用相同的数字表示。再接下来 M 行,每行两个正整数 L,R 表示一个询问。

Output

包含 M 行,对于每个询问在一行中输出分数 A/B 表示从该询问的区间[L,R]中随机抽出两只 袜子颜色相同的概率。若该概率为 0 则输出 0/1, 否则输出的 A/B 必须为最简分数。(详见样例)

Sample Input

64

 $1\,2\,3\,3\,3\,2$

26

13

35

16

Sample Output

2/5

0/1

1/1

4/15

题解:
$$P = \frac{\sum_{\tau_i} C_{\tau_i}^2}{C_{R-L+1}^2} = \frac{\sum_{\tau_i} * (\tau_i - 1) / 2}{(R-L+1) * (R-L) / 2} = \frac{\sum_{\tau_i} \tau_i^2 - \sum_{\tau_i} \tau_i}{(R-L+1) * (R-L)}$$

只需要统计区间内各个数出现次数的平方和

莫队算法,两种方法,一种是直接分成 sqrt(n)块,分块排序。

另外一种是求得曼哈顿距离最小生成树,根据 manhattan MST 的 dfs 序求解。

7.1 分块

```
const int MAXN = 50010;
const int MAXM = 50010;
struct Query
```

```
int L,R,id;
}node[MAXM];
long long gcd(long long a, long long b)
    if(b == 0)return a;
    return gcd(b,a%b);
struct Ans
    long long a,b;//分数a/b
    void reduce()//分数化简
        long long d = gcd(a,b);
        a /= d; b /= d;
}ans[MAXM];
int a[MAXN];
int num[MAXN];
int n,m,unit;
bool cmp(Query a, Query b)
    if(a.L/unit != b.L/unit)return a.L/unit < b.L/unit;</pre>
    else return a.R < b.R;</pre>
}
void work()
{
    long long temp = 0;
    memset(num, 0, size of (num));
    int L = 1;
    int R = 0;
    for(int i = 0;i < m;i++)</pre>
        while(R < node[i].R)</pre>
        {
            R++;
            temp -= (long long)num[a[R]]*num[a[R]];
            num[a[R]]++;
            temp += (long long)num[a[R]]*num[a[R]];
        }
        while(R > node[i].R)
            temp -= (long long)num[a[R]]*num[a[R]];
            num[a[R]]--;
            temp += (long long)num[a[R]]*num[a[R]];
            R--;
        }
        while(L < node[i].L)</pre>
            temp -= (long long)num[a[L]]*num[a[L]];
            num[a[L]]--;
            temp += (long long)num[a[L]]*num[a[L]];
            L++;
        }
        while(L > node[i].L)
            temp -= (long long)num[a[L]]*num[a[L]];
            num[a[L]]++;
```

```
temp += (long long)num[a[L]]*num[a[L]];
        }
        ans[node[i].id].a = temp - (R-L+1);
        ans[node[i].id].b = (long long)(R-L+1)*(R-L);
        ans[node[i].id].reduce();
int main()
   while(scanf("%d%d",&n,&m) == 2)
        for(int i = 1;i <= n;i++)</pre>
            scanf("%d",&a[i]);
        for(int i = 0;i < m;i++)</pre>
            node[i].id = i;
            scanf("%d%d", &node[i].L, &node[i].R);
        unit = (int) sqrt(n);
        sort(node, node+m, cmp);
        work();
        for(int i = 0;i < m;i++)</pre>
            printf("%11d/%11d\n", ans[i].a, ans[i].b);
    }
   return 0;
}
   7.2 Manhattan MST 的 dfs 顺序求解
const int MAXN = 50010;
const int MAXM = 50010;
const int INF = 0x3f3f3f3f;
struct Point
    int x,y,id;
}p[MAXN],pp[MAXN];
bool cmp(Point a, Point b)
    if(a.x != b.x) return a.x < b.x;</pre>
    else return a.y < b.y;</pre>
//树状数组,找y-x大于当前的,但是y+x最小的
struct BIT
    int min val, pos;
    void init()
        min val = INF;
        pos = -1;
    }
}bit[MAXN];
struct Edge
    int u, v, d;
}edge[MAXN<<2];</pre>
bool cmpedge (Edge a, Edge b)
   return a.d < b.d;</pre>
}
```

```
int tot;
int n;
int F[MAXN];
int find(int x)
    if(F[x] == -1) return x;
    else return F[x] = find(F[x]);
}
void addedge (int u,int v,int d)
    edge[tot].u = u;
    edge[tot].v = v;
    edge[tot++].d = d;
struct Graph
    int to, next;
}e[MAXN<<1];
int total,head[MAXN];
void _addedge(int u,int v)
    e[total].to = v;
    e[total].next = head[u];
    head[u] = total++;
}
int lowbit(int x)
   return x&(-x);
void update(int i,int val,int pos)
    while (i > 0)
    {
        if(val < bit[i].min_val)</pre>
           bit[i].min val = val;
            bit[i].pos = pos;
        }
        i -= lowbit(i);
}
int ask(int i,int m)
    int min_val = INF,pos = -1;
    while(i <= m)</pre>
        if(bit[i].min_val < min_val)</pre>
            min_val = bit[i].min_val;
            pos = bit[i].pos;
        }
        i += lowbit(i);
    return pos;
int dist(Point a, Point b)
    return abs(a.x - b.x) + abs(a.y - b.y);
                                      149 / 152
```

```
}
void Manhattan_minimum_spanning_tree (int n, Point p[])
    int a[MAXN], b[MAXN];
    tot = 0;
    for(int dir = 0;dir < 4;dir++)</pre>
        if(dir == 1 || dir == 3)
            for(int i = 0;i < n;i++)</pre>
                swap(p[i].x,p[i].y);
        }
        else if(dir == 2)
            for(int i = 0;i < n;i++)</pre>
                p[i].x = -p[i].x;
        sort(p, p+n, cmp);
        for(int i = 0;i < n;i++)</pre>
            a[i] = b[i] = p[i].y - p[i].x;
        sort(b, b+n);
        int m = unique(b, b+n) - b;
        for(int i = 1;i <= m;i++)</pre>
            bit[i].init();
        for(int i = n-1;i >= 0;i--)
            int pos = lower bound(b,b+m,a[i]) - b + 1;
            int ans = ask(pos,m);
            if (ans !=-1)
                addedge(p[i].id,p[ans].id,dist(p[i],p[ans]));
            update(pos,p[i].x+p[i].y,i);
        }
    }
    memset(F,-1, sizeof(F));
    sort(edge,edge+tot,cmpedge);
    total = 0;
    memset(head, -1, sizeof(head));
    for(int i = 0;i < tot;i++)</pre>
        int u = edge[i].u, v = edge[i].v;
        int t1 = find(u), t2 = find(v);
        if(t1 != t2)
            F[t1] = t2;
            addedge(u,v);
            addedge(v,u);
        }
    }
}
int m;
int a[MAXN];
struct Ans
    long long a, b;
}ans[MAXM];
long long temp ;
int num[MAXN];
void add(int l,int r)
```

```
{
    for(int i = 1;i <= r;i++)</pre>
        temp -= (long long)num[a[i]]*num[a[i]];
        num[a[i]]++;
        temp += (long long)num[a[i]]*num[a[i]];
}
void del(int l,int r)
    for(int i = 1;i <= r;i++)</pre>
        temp -= (long long)num[a[i]]*num[a[i]];
        num[a[i]]--;
        temp += (long long)num[a[i]]*num[a[i]];
void dfs(int l1,int r1,int l2,int r2,int idx,int pre)
    if(12 < 11) add(12,11-1);</pre>
    if(r2 > r1) add(r1+1, r2);
    if(12 > 11) del(11,12-1);
   if(r2 < r1) del(r2+1, r1);</pre>
    ans[pp[idx].id].a = temp - (r2-12+1);
    ans[pp[idx].id].b = (long long) (r2-12+1) * (r2-12);
    for(int i = head[idx];i != -1;i = e[i].next)
        int v = e[i].to;
        if(v == pre) continue;
        dfs(12, r2, pp[v].x, pp[v].y, v, idx);
    if(12 < 11)del(12,11-1);</pre>
    if(r2 > r1)del(r1+1,r2);
    if(12 > 11)add(11,12-1);
    if(r2 < r1) add(r2+1,r1);
long long gcd(long long a, long long b)
    if(b == 0) return a;
    else return gcd(b,a%b);
int main()
   while (scanf("%d%d", &n, &m) == 2)
        for(int i = 1;i <= n;i++)</pre>
            scanf("%d", &a[i]);
        for(int i = 0;i < m;i++)</pre>
            scanf("%d%d", &p[i].x, &p[i].y);
            p[i].id = i;
            pp[i] = p[i];
        Manhattan minimum spanning tree (m,p);
        memset(num, 0, sizeof(num));
        temp = 0;
        dfs(1,0,pp[0].x,pp[0].y,0,-1);
        for(int i = 0;i < m;i++)</pre>
```

151 / 152

```
{
    long long d = gcd(ans[i].a,ans[i].b);
    printf("%lld/%lld\n",ans[i].a/d,ans[i].b/d);
}
return 0;
}
```