机器人导论第二次作业

- 米家龙
- 18342075

1

1. 图 4.20 所示为二自由度机械手，杆长为 $l_1 = l_2 = 0.5\text{m}$，试求表 4.1 中 3 种情况下的关节瞬时速度 $\dot{\theta}_1$ 和 $\dot{\theta}_2$。

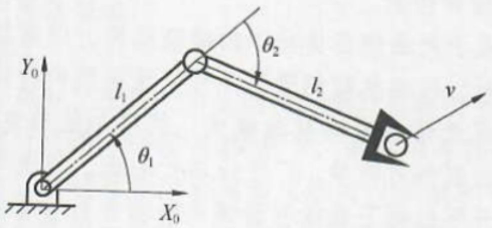

图 4.20 二自由度机械手

表 4.1 末端执行器速度和关节位置

v_x / (m · s^{-1})	-1.0	0	1.0
v_y / (m · s^{-1})	0	1.0	1.0
θ_1	30°	30°	30°
θ_2	-60°	120°	-30°

已知二自由度雅克比行列式为：

$$
\begin{aligned}
J &= \begin{bmatrix} -l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) & -l_2\sin(\theta_1+\theta_2) \\ l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) & l_2\cos(\theta_1+\theta_2) \end{bmatrix} \\
\dot{q} &= J^{-1} \cdot q \cdot V
\end{aligned}
$$

可以得到

$$
\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) & -l_2\sin(\theta_1+\theta_2) \\ l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) & l_2\cos(\theta_1+\theta_2) \end{bmatrix}^{-1} \cdot \begin{bmatrix} v_x \\ v_y \end{bmatrix}
$$

分别代入一下三种情况，可以得到

$$
\begin{aligned}
1.\ \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} &= \begin{bmatrix} 2 \\ -4 \end{bmatrix} \\
2.\ \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} &= \begin{bmatrix} \frac{2}{\sqrt{3}} \\ -\frac{4}{\sqrt{3}} \end{bmatrix} \\
3.\ \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 2\sqrt{3}-2 \end{bmatrix}
\end{aligned}
$$

2

3. 图 4.22 所示为一个三自由度机械手，其手部夹持一质量 $m=10\mathrm{kg}$ 的重物，$l_1=l_2=0.8\mathrm{m}$，$l_3=0.4\mathrm{m}$，$\theta_1=60°$，$\theta_2=-60°$，$\theta_3=-90°$。若不计机械手的重量，求机械手处于平衡状态时各关节力矩。

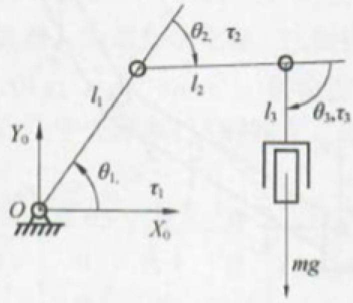

图 4.22 三自由度机械手

当机械手平衡时，各个关节的角速度和加速度都为0，即

$$\dot{\theta}_1=\dot{\theta}_2=\dot{\theta}_3=\ddot{\theta}_1=\ddot{\theta}_2=\ddot{\theta}_3=0$$

因此对于连杆1，可以得到

$$\begin{aligned}E_{k1}&=\frac{1}{2}m_1v_1^2\\E_{p1}&=m_1gl_1\sin\theta_1\end{aligned}$$

对于连杆2，可以得到

$$\begin{aligned}x_2&=l_1\cos\theta_1+l_2\cos(\theta_1+\theta_2)\\y_2&=l_1\sin\theta_1+l_2\sin(\theta_1+\theta_2)\\v_2^2&=\dot{x_2}^2+\dot{y_2}^2\\&\\E_{k2}&=\frac{1}{2}m_2v_2^2\\E_{p2}&=m_2gy_2\end{aligned}$$

由于忽略机器手自身重力，因此 $m_1=m_2=0$,可以得到上述两连杆的动能和势能都为0

对于连杆3，可以得到

$$\begin{aligned}x_3&=l_1\cos\theta_1+l_2\cos(\theta_1+\theta_2)+l_3\cos(\theta_1+\theta_2+\theta_3)\\y_3&=l_1\sin\theta_1+l_2\sin(\theta_1+\theta_2)+l_3\sin(\theta_1+\theta_2+\theta_3)\\v_3^2&=\dot{x_3}^2+\dot{y_3}^2\\&\\E_{k3}&=\frac{1}{2}m_3v_3^2\\E_{p3}&=m_3gy_3\end{aligned}$$

根据拉格朗日函数，可以得到

$$
\begin{aligned}
L &= E_{k3} - E_{p3} \\
&= (E_{k1} + E_{k2} + E_{k3}) - (E_{p1} + E_{p2} + E_{p3}) \\
&= -m_3 g\left[l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3)\right]
\end{aligned}
$$

$$
\begin{aligned}
\tau_1 &= \frac{\mathrm{d}}{\mathrm{dt}} \frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} \\
&= m_3 g\left[l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)\right] \\
&= 120 \\
\tau_2 &= \frac{\mathrm{d}}{\mathrm{dt}} \frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2} \\
&= m_3 g\left[l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)\right] \\
\tau_3 &= \frac{\mathrm{d}}{\mathrm{dt}} \frac{\partial L}{\partial \dot{\theta}_3} - \frac{\partial L}{\partial \theta_3} \\
&= m_3 g\, l_3 \cos(\theta_1+\theta_2+\theta_3) \\
&= 0
\end{aligned}
$$

3

8. 二连杆机器人如图 4.27 所示。连杆长度为 d_i，质量为 m_i，重心位置为 $(0.5d_i,\ 0,\ 0)$，连杆惯量为 $I_{zzi}=\frac{1}{3}m_i d_i^2$，$I_{yyi}=\frac{1}{3}m_i d_i^2$，$I_{xxi}=0$，传动机构的惯量为 $I_{ai}=0(i=1,\ 2)$。用拉格朗日矩阵法确定动力学方程的参数 D_{ij}，D_{ijk}，D_i。

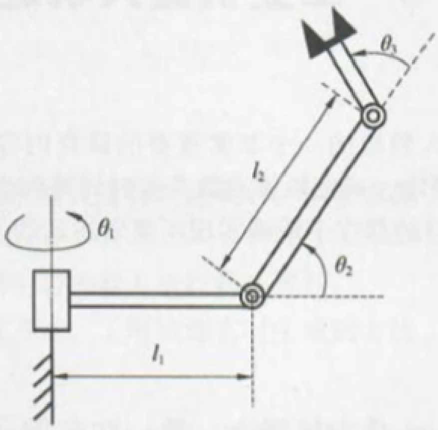

图 4.27　二连杆机器人

对于上述两个连杆，可以得到

$$
\begin{aligned}
E_{k1} &= \frac{1}{2} I_{zz1}\, \dot{\theta}_1^{\,2} + \frac{1}{8} m_1 d_1^2 \dot{\theta}_1^{\,2} \\
E_{p1} &= 0 \\
E_{k2} &= \frac{1}{2} I_{zz2}\, \dot{\theta}_2^{\,2} + \frac{1}{8} m_2 d_2^2 \dot{\theta}_2^{\,2} + \frac{1}{8} m_2 (l_2 \cos\theta_2 + d_1)^2 \theta_2^2 + (\frac{1}{2} I_{yy2} \cos^2\theta_2)\dot{\theta}_1 \\
E_{p2} &= \frac{1}{2}\, m g d_2\, \sin\theta_2
\end{aligned}
$$

根据拉格朗日方程可以得到

$$
\begin{aligned}
L &= E_k - E_p \\
&= (E_{k1} + E_{k2}) - (E_{p1} + E_{p2})
\end{aligned}
$$

$$
\begin{aligned}
\tau_1 &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} \\
&= \ddot{\theta}_2\left(\frac{7}{12}\, m_1 d_1^2 + \frac{7}{12} m_2 l_2^2 \cos^2\theta_2 + m_2 d_1^2 + m_2 d_1 d_2 \cos\theta_2\right) \\
\tau_2 &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2} \\
&= \frac{7}{12} m_2 d_2^2 \ddot{\theta}_2 + \left(\frac{7}{12} m_2 d_2^2 \sin\theta_2 \cos\theta_2\right)\dot{\theta}_1^{\ 2} + \left(\frac{1}{2} m_2 d_1 d_2 \sin\theta_2\right)\theta_1^2 + \frac{1}{2} m_2 g d_2 \cos\theta_2
\end{aligned}
$$

最终可以得到

$$
\begin{aligned}
D_{11} &= \frac{7}{12}\, m_1 d_1^2 + \frac{7}{12} m_2 l_2^2 \cos^2\theta_2 + m_2 d_1^2 + m_2 d_1 d_2 \cos\theta_2 \\
D_{22} &= \frac{7}{12} m_2 d_2^2 \\
D_{211} &= \frac{7}{12} m_2 d_2^2 + \frac{1}{2} m_2 d_1 d_2 \sin\theta_2 \\
D_2 &= \frac{1}{2} m_2 g d_2 \cos\theta_2
\end{aligned}
$$

4

9. 试求图 4.28 所示三连杆机器人的动力学方程。已知机器人参数如下：$l_1 = l_2 = 0.5\mathrm{m}$，$m_1 = 4.6\mathrm{kg}$，$m_2 = 2.3\mathrm{kg}$，$m_3 = 1.0\mathrm{kg}$，$g = 9.8\mathrm{m/s^2}$，假设连杆 1 和 2 的质量都集中在其连杆的末端（远端），而连杆 3 的质心位于坐标系{3}的原点，即在连杆 3 的近端上，连杆 3 的惯量矩阵为

$$
{}^{C_3}I = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \mathrm{kg \cdot m^2}
$$

决定两个质心位置与每个连杆坐标系的关系为 ${}^1p_{C_1} = l_1 X_1$，${}^2p_{C_2} = l_2 X_2$，${}^3p_{C_3} = 0$。

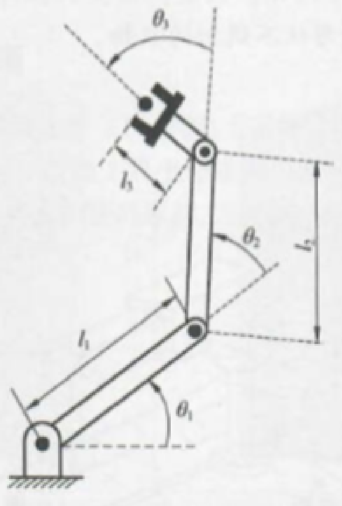

图 4.28　三连杆机器人

通过对连杆进行分析，得到：

$$
\begin{aligned}
E_{k1} &= \frac{1}{2} m_1 l_1^2 \dot{\theta_1}^2 \\
E_{p1} &= m_1 g l_1 \ \sin\theta_1
\end{aligned}
$$

$$
\begin{aligned}
x_2 &= l_1 \ \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\
y_2 &= l_1 \ \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \\
v_2^2 &= \dot{x_2}^2 + \dot{y_2}^2 \\
E_{k2} &= \frac{1}{2} m_2 v_2^2 \\
E_{p2} &= m_2 g l y_2
\end{aligned}
$$

由于连杆3的质心在坐标｛3｝的原点，因此

$$
\begin{aligned}
x_3 &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\
y_3 &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \\
v_3^2 &= \dot{x_3}^2 + \dot{y_3}^2 \\
E_{k3} &= \frac{1}{2} m_3 v_3^2 = \frac{1}{2} I_{zz} (\dot{\theta_1} + \dot{\theta_2} + \dot{\theta_3}) \\
E_{p3} &= m_3 g y_3
\end{aligned}
$$

通过拉格朗日方程可以得到

$$
\begin{aligned}
L &= E_{k3} - E_{p3} \\
&= (E_{k1} + E_{k2} + E_{k3}) - (E_{p1} + E_{p2} + E_{p3})
\end{aligned}
$$

$$
\begin{aligned}
\tau_1 =& \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\theta_1}} - \frac{\partial L}{\partial \theta_1} \\
=& (2.4 + 1.15 \ \cos\theta_2) \ \ddot{\theta_1} + (0.675 + 0.575 \ \cos\theta_2) \ \ddot{\theta_2} \\
& + 0.1 \ \ddot{\theta_3} + 38.71 \ \cos\theta_1 + 16.17 \ \cos(\theta_1 + \theta_2) \\
\tau_2 =& \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\theta_2}} - \frac{\partial L}{\partial \theta_2} \\
=& (0.675 + 0.575 \ \cos\theta_2) \ \ddot{\theta_1} + 0.675 \ \ddot{\theta_2} + 0.1 \ \ddot{\theta_3} \\
& + 0.575 \ \sin\theta_2 (\dot{\theta_1}^2 + \dot{\theta_1} \dot{\theta_2}) + 16.17 \ \cos(\theta_1 + \theta_2) \\
\tau_3 =& \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\theta_3}} - \frac{\partial L}{\partial \theta_3} \\
=& 0.1 \ (\ddot{\theta_1} + \ddot{\theta_2} + \ddot{\theta_3})
\end{aligned}
$$