Lecture 4

Binary Data

Section A: The Sample Proportion as a Summary Statistic



## Learning Objectives

- Upon completion of this lecture section you will be able to
  - Summarize a binary outcome across a group of individual observations via the sample proportion
  - Explain why, with binary data, the sample proportion is the only summary statistic (besides sample size *n*) necessary to describe characteristics of the sample
  - Compute the sample proportion based on the results of a study



## Example 1[1]

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

206 patients responded

Summary measure: sample proportion $\hat{p}$ (pronounced p-hat!), given by

$$\hat{p} = \frac{\#\text{ in sample who have outcome}}{\text{total }\#\text{ in sample}} = \frac{206}{1{,}000} = 0.206 \approx 0.21\ (21\%)$$

[1] http://inclass.kaggle.com/



## Example 1

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

Summary measure: sample proportion $\hat{p}$ (pronounced p-hat!), given by

$\hat{p}$ may be called estimated proportion, estimated probability or estimated risk of responding to treatment



## Example 1

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

Why the "hat"? To distinguish $\hat{p}$, the sample estimate from the underlying true (population) proportion *p*. (which can only be estimated)

## Example 1

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

The sample proportion $\hat{p}$ is just a sample mean of 0/1 data

Generally, binary data values are given a value of $x$=1 for observations that have the outcome, and $x$=0 for observations that do not have the outcome.

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## Example 1

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

So with 206 of the 1,000 responding, we have $x_i$=1 for 206 observations, and $x_i$=0 for 794 observations.

So

$$\frac{\sum_{i=1}^{n} x_i}{n} = \frac{206}{1{,}000} = \hat{p} = 0.206$$

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## Example 1

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

Quantifying variability: There is a formula for the standard deviation of binary data

$$s = \sqrt{n\hat{p} \times (1 - \hat{p})}$$

But this quantity is not particularly useful in understanding the distribution.

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## Example 1

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

Percentiles?

Well, if we know $\hat{p}$ , we know the sample percentiles.

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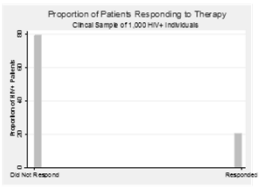
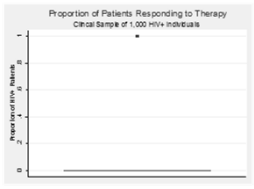
## Example 1

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

Visual Displays?

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## Example 2: Maternal/Infant HIV Transmission[2]

- Randomized Trial: HIV positive pregnant women randomized to receive AZT or placebo

**Abstract** *Background and Methods.* Maternal–infant transmission is the primary means by which young children become infected with human immunodeficiency virus type 1 (HIV). We conducted a randomized, double-blind, placebo-controlled trial of the efficacy and safety of zidovudine in reducing the risk of maternal–infant HIV transmission. HIV-infected pregnant women (14 to 34 weeks' gestation) with CD4+ T-lymphocyte counts above 200 cells per cubic millimeter who had not received antiretroviral therapy during the current pregnancy were enrolled. The zidovudine regimen included antepartum zidovudine (100 mg orally five times daily), intrapartum zidovudine (2 mg per kilogram of body weight given intravenously over a one-hour period, then 1 mg per kilogram per hour until delivery), and zidovudine for the newborn (2 mg per kilogram orally every six hours for six weeks). Infants with at least one positive HIV culture of peripheral-blood mononuclear cells were classified as HIV-infected.

[2]Connor E, et al. Reduction of Maternal-Infant Transmission of Human Immunodeficiency Virus Type 1 with Zidovudine Treatment. *New England Journal of Medicine* (1994). 331(18); 1173-1180

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## Example 2: Maternal/Infant HIV Transmission

- Results

> *Results.* From April 1991 through December 20, 1993, the cutoff date for the first interim analysis of efficacy, 477 pregnant women were enrolled; during the study period, 409 gave birth to 415 live-born infants. HIV-infection status was known for 363 births (180 in the zido-

Of the 363 births whose HIV status was assessed (up to 18 months after birth) , 53 infants were HIV infected.

$$\hat{p} = \frac{53}{363} \approx 0.15\,(15\%)$$

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## Example 3: Colorectal Cancer Screening[3]

- From Abstract

**Background:** Screening decreases colorectal cancer (CRC) incidence and mortality, yet almost half of age-eligible patients are not screened at recommended intervals.

**Objective:** To determine whether interventions using electronic health records (EHRs), automated mailings, and stepped increases in support improve CRC screening adherence over 2 years.

**Design:** 4-group, parallel-design, randomized, controlled comparative effectiveness trial with concealed allocation and blinded outcome assessments. (ClinicalTrials.gov: NCT00697047)

**Setting:** 21 primary care medical centers.

**Patients:** 4675 adults aged 50 to 73 years not current for CRC screening.

**Intervention:** Usual care, EHR-linked mailings ("automated"), automated plus telephone assistance ("assisted"), or automated and assisted plus nurse navigation to testing completion or refusal ("navigated"). Interventions were repeated in year 2.

**Measurements:** The proportion of participants current for screening in both years, defined as colonoscopy or sigmoidoscopy (year 1) or fecal occult blood testing (FOBT) in year 1 and FOBT, colonoscopy, or sigmoidoscopy (year 2).

[3]Green B, et al. An Automated Intervention With Stepped Increases in Support to Increase Uptake of Colorectal Cancer Screening: A Randomized Trial. *Annals of Internal Medicine* (2013). 158(5); 301-307

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## Example 3: Colorectal Cancer Screening

- From Abstract

**Results:** Compared with those in the usual care group, participants in the intervention groups were more likely to be current for CRC screening for both years with significant increases by intensity (usual care, 26.3% [95% CI, 23.4% to 29.2%]; automated, 50.8% [CI, 47.3% to 54.4%]; assisted, 57.5% [CI, 54.5% to 60.6%]; and navigated, 64.7% [CI, 62.5% to 67.0%]; $P < 0.001$ for all pairwise comparisons). Increases in screening were primarily due to increased uptake of FOBT being completed in both years (usual care, 3.9% [CI, 2.8% to 5.1%]; automated, 27.5% [CI, 24.9% to 30.0%]; assisted, 30.5% [CI, 27.9% to 33.2%]; and navigated, 35.8% [CI, 33.1% to 38.6%]).

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## Example 3: Colorectal Cancer Screening

- From Abstract

**Results:** Compared with those in the usual care group, participants in the intervention groups were more likely to be current for CRC screening for both years with significant increases by intensity (usual care, 26.3% [95% CI, 23.4% to 29.2%]; automated, 50.8% [CI, 47.3% to 54.4%]; assisted, 57.5% [CI, 54.5% to 60.6%]; and navigated, 64.7% [CI, 62.5% to 67.0%]; $P < 0.001$ for all pairwise comparisons). Increases in screening were primarily due to increased uptake of FOBT being completed in both years (usual care, 3.9% [CI, 2.8% to 5.1%]; automated, 27.5% [CI, 24.9% to 30.0%]; assisted, 30.5% [CI, 27.9% to 33.2%]; and navigated, 35.8% [CI, 33.1% to 38.6%]).

$$\hat{p}_{usual\_care} = 0.263\,(26.3\%)$$
$$\hat{p}_{automated} = 0.508\,(50.8\%)$$
$$\hat{p}_{assisted} = 0.575\,(57.5\%)$$
$$\hat{p}_{navigated} = 0.647\,(64.7\%)$$

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## Summary

- For quantifying the distribution of binary outcomes in a sample (and hence estimating the distribution in the population from which the sample was taken), the sample proportion $\hat{p}$ is paramount
  - $\hat{p}$ not only summarizes the percentage (probability, risk) of outcomes among a sample, it gives information about the variability of individual sample observations and the sample percentiles
  - $\hat{p}$ is the sample mean of sample observations that take on the value of 1 for observations with the outcome and 0 for observations without the outcome

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Section B: Comparing Binary Outcomes Between Two (or More) Populations Using Sample Results: Risk Difference and Relative Risk

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## Learning Objectives

- Upon completion of this lecture you will be able to:
  - Compute the risk difference and relative risk for comparing binary outcomes between two samples
  - Interpret the risk difference and relative risk in a public health/personal health context
  - Understand that the risk difference and relative risk will always agree in terms of direction, but can differ greatly in magnitude
  - Understand that neither the risk difference alone, or the relative risk alone is sufficient to quantify the association of interest

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## Example 1

- Among the 1,000 subjects in the sample:
  - 503 had CD4 counts < 250 at start of therapy, and 127 responded to therapy
  - 497 had CD4 counts ≥ 250 at start of therapy, and 79 responded to therapy
- 2 X 2 Table Representation

| | CD4 < 250 | CD4 ≥ 250 | |
|---|---|---|---|
| Respond | 127 | 79 | 206 |
| Not Respond | 376 | 418 | 794 |
| | 503 | 497 | 1,000 |

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## Example 1

- How to summarize difference in response between the CD4 count groups?

| | CD4 < 250 | CD4 ≥ 250 | |
|---|---|---|---|
| Respond | 127 | 79 | 206 |
| Not Respond | 376 | 418 | 794 |
| | 503 | 497 | 1,000 |

- Start with sample proportions:

$$\hat{p}_{CD4<250} = \frac{127}{503} = 0.253 \approx 0.25\ (25\%)$$

$$\hat{p}_{CD4\geq250} = \frac{79}{497} = 0.159 \approx 0.16\ (16\%)$$

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## Example 1

- Summary Measure 1: the difference in proportions (also called *risk difference* , or *attributable risk*)

$$\hat{p}_{CD4<250} - \hat{p}_{CD4\geq250} = 0.25 - 0.16 = 0.09\ (9\%)$$

Interpretation(s):

9% greater (absolute) response to therapy in CD4<250 group as compared to CD4 ≥ 250 group

9% greater absolute "risk" of response to therapy in CD4<250 group as compared to CD4 ≥ 250 group

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## Example 1

- Summary Measure 2: the ratio proportions (also called *relative risk*, or *risk ratio*)

$$\hat{RR} = \frac{\hat{p}_{CD4<250}}{\hat{p}_{CD4\geq250}} = \frac{0.25}{0.16} \approx 1.56$$

Interpretation(s):

Those in the CD4<250 group have 1.56 times the chances (risk) of responding to therapy as compared to CD4 ≥ 250 group

56% greater relative "risk" of response to therapy in CD4<250 group as compared to CD4 ≥ 250 group

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## Example 2: Maternal/Infant HIV Transmission[2]

- Randomized Trial: HIV positive pregnant women randomized to receive AZT or placebo

**Abstract** *Background and Methods.* Maternal–infant transmission is the primary means by which young children become infected with human immunodeficiency virus type 1 (HIV). We conducted a randomized, double-blind, placebo-controlled trial of the efficacy and safety of zidovudine in reducing the risk of maternal–infant HIV transmission. HIV-infected pregnant women (14 to 34 weeks' gestation) with CD4+ T-lymphocyte counts above 200 cells per cubic millimeter who had not received antiretroviral therapy during the current pregnancy were enrolled. The zidovudine regimen included antepartum zidovudine (100 mg orally five times daily), intrapartum zidovudine (2 mg per kilogram of body weight given intravenously over a one-hour period, then 1 mg per kilogram per hour until delivery), and zidovudine for the newborn (2 mg per kilogram orally every six hours for six weeks). Infants with at least one positive HIV culture of peripheral-blood mononuclear cells were classified as HIV-infected.

[2]Connor E, et al. Reduction of Maternal-Infant Transmission of Human Immunodeficiency Virus Type 1 with Zidovudine Treatment. *New England Journal of Medicine* (1994). 331(18); 1173-1180

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## Example 2: Maternal/Infant HIV Transmission[2]

- Results

*Results.* From April 1991 through December 20, 1993, the cutoff date for the first interim analysis of efficacy, 477 pregnant women were enrolled; during the study period, 409 gave birth to 415 live-born infants. HIV-infection status was known for 363 births (180 in the zidovudine group and 183 in the placebo group). Thirteen infants in the zidovudine group and 40 in the placebo group were HIV-infected.

| (at 18 mos) | AZT | Placebo | |
|---|---|---|---|
| HIV+ | 13 | 40 | 53 |
| HIV- | 167 | 143 | 310 |
| | 180 | 183 | 363 |

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## Example 2: Maternal/Infant HIV Transmission[2]

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*Results.* From April 1991 through December 20, 1993, the cutoff date for the first interim analysis of efficacy, 477 pregnant women were enrolled; during the study period, 409 gave birth to 415 live-born infants. HIV-infection status was known for 363 births (180 in the zidovudine group and 183 in the placebo group). Thirteen infants in the zidovudine group and 40 in the placebo group were HIV-infected.

| (at 18 mos) | AZT | Placebo | |
|---|---|---|---|
| HIV+ | 13 | 40 | 53 |
| HIV- | 167 | 143 | 310 |
| | 180 | 183 | 363 |

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## Example 2

- Summary Measure 1: the difference in proportions (also called *risk difference* , or *attributable risk*)

$$\hat{p}_{AZT} - \hat{p}_{Placebo} = 0.07 - 0.22 = -0.15 \text{ (-15\%)}$$

Interpretation(s):

15% (absolute) reduction in HIV+ transmission to children born to mothers given AZT as compared to children born to mothers given placebo

15% lower absolute "risk" of HIV+ transmission to children born to mothers given AZT…

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## Example 2

- Summary Measure 2: the ratio proportions (also called *relative risk*, or *risk ratio*)

$$\hat{RR} = \frac{\hat{p}_{AZT}}{\hat{p}_{Placebo}} = \frac{0.07}{0.22} \approx 0.32$$

Interpretation(s):

Risk of mother/child HIV transmission for mothers given AZT is 0.32 times the chances (risk) of mother/child HIV transmission for mothers given placebo

68% lower relative risk of mother/child HIV transmission for mothers given AZT

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## Example 2

- Risk Difference Versus Relative Risk: Substantive Interpretations
- Both measures use exact same information but give seemingly different results:
  - (risk difference) 15% reduction in HIV transmission
  - (relative risk) 68% reduction in HIV transmission

Notice, both agree in terms of direction of association

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## Example 2

- Risk Difference : Substantive Interpretation
  - Can be interpreted as impact (assuming causation) at the "population level"
- For example: with this risk difference of -15% :
  - In a population of 1,000 HIV pregnant positive women, we'd expect to see 150 (15%) fewer mother/child transmissions if the 1,000 women were given AZT during pregnancy
  - In a population of 50,000 HIV pregnant positive women, we'd expect to see 7,500 (15%) fewer mother/child transmissions if the 50,000 women were given AZT during pregnancy

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## Example 2

- Relative Risk : Substantive Interpretation
  - Can be interpreted as impact (assuming causation) at the "individual level"
- For example: with this relative risk of 0.32:
  - The risk that a HIV+ mother who takes AZT during pregnancy transmits HIV to her child is 0.32 times her risk if she did not take AZT
  - The risk that a HIV+ mother transmits HIV to her child is 68% lower if she takes AZT during pregnancy (as compared to if she were not taking AZT)

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## Example 3: Aspirin and CVD: Women[3]

- From Abstract

BACKGROUND

Randomized trials have shown that low-dose aspirin decreases the risk of a first myocardial infarction in men, with little effect on the risk of ischemic stroke. There are few similar data in women.

METHODS

We randomly assigned 39,876 initially healthy women 45 years of age or older to receive 100 mg of aspirin on alternate days or placebo and then monitored them for 10 years for a first major cardiovascular event (i.e., nonfatal myocardial infarction, nonfatal stroke, or death from cardiovascular causes).

RESULTS

During follow-up, 477 major cardiovascular events were confirmed in the aspirin group, as compared with 522 in the placebo group.

[3] Ridker P, et al. A Randomized Trial of Low-Dose Aspirin in the Primary Prevention of Cardiovascular Disease in Women. *New England Journal of Medicine* (2005). 352(13); 1293-1304

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## Example 3

- 2X2 table and estimates

| | Aspirin | Placebo | |
|---|---|---|---|
| CVD | 477 | 522 | 999 |
| No CVD | 19,457 | 19,420 | 38,887 |
| | 19,934 | 19,942 | 39,876 |

$$\hat{p}_{Aspirin} = \frac{477}{19{,}934} = 0.024\ (2.4\%)$$

$$\hat{p}_{Placebo} = \frac{522}{19{,}942} = 0.026\ (2.6\%)$$

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## Example 3

- Risk Difference

$$\hat{p}_{Aspirin} - \hat{p}_{Placebo} = 0.024 - 0.026 = -0.002\ (-0.2\%)$$

- 0.2 % (absolute) reduction in (10-year) risk of CVD for women on low-dose aspirin therapy compared to women not on low dose therapy
- In a population of 100,000 women, we would expect to see 0.002*100,000=200 fewer cases of CVD (developing within 10 years) if the women were given low-dose aspirin therapy

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## Example 3

- Relative Risk

$$\frac{\hat{p}_{Aspirin}}{\hat{p}_{Placebo}} = \frac{0.024}{0.026} \approx 0.92$$

- 10-year risk of CVD for 0.92 for women on low-dose aspirin regimen is 0.92 times the risk for women given placebo
- A women can reduce her personal risk of CVD (developing within 10 years) by 8% if she takes a low -dose of aspirin daily

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## Risk Difference and Relative Risk

- The risk difference and relative risk will always agree in term of the direction of estimated association

  If $\hat{p}_1 - \hat{p}_2 > 0$, then $\frac{\hat{p}_1}{\hat{p}_2} > 1$

  If $\hat{p}_1 - \hat{p}_2 < 0$, then $\frac{\hat{p}_1}{\hat{p}_2} < 1$

  If $\hat{p}_1 - \hat{p}_2 = 0$, then $\frac{\hat{p}_1}{\hat{p}_2} = 1$

- However, the two quantities can appear different in terms of magnitude

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## Risk Difference and Relative Risk

- It is possible to see a "large" effect with one measure, and a "small" effect with the other

  For example, if $\hat{p}_1 = 0.001$ and $\hat{p}_2 = 0.003$

  then $\hat{p}_1 - \hat{p}_2 = 0.001 - 0.003 = -0.002$ (-0.2%) : an absolute decrease of 0.2%

  But $\frac{\hat{p}_1}{\hat{p}_2} = \frac{0.001}{0.003} = 0.33$ : a relative decrease of 67%!

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## Example 4: HRT and Risk of CHD

- Marilyn Vos Savant takes on a serious question

PARADE articles

I'm a middle-aged woman on hormone replacement therapy (HRT), and the news about HRT is very confusing. For example, I read that heart disease increased by almost a third as a result of the medication. Yet I also read that the increase was slight. Which is it? Now I read that the use of HRT is being questioned for this and other serious findings. What do the numbers really mean?
—Jeannie R., St. Louis, Mo.

A study that followed 16,608 women for 5.2 years—testing 8506 with HRT and 8102 with a placebo—reported a total of 286 cases of heart disease among them all. The incidence was 29% higher in the HRT group (164 cases) than in the placebo group (122 cases). (The percentage of increase is calculated with additional factors, so the numbers don't quite match.)

BY MARILYN VOS SAVANT

Ask Marilyn

The incidence of heart disease in two groups of similar women will vary from group to group, so we asked Jacques Rossouw, director of the study, how many cases it took for the difference to be significant. He replied that it took few, so most of the difference in cases would be counted, "leaving little doubt that the effect on cardiovascular disease is real." Note that this effect is still statistical. As yet, we know no cause-and-effect relationship.

And is the effect large? Not really. The lower the incidence of problems, the larger the percentage of change. For example: If only one woman in the placebo group had a problem, and two women in the HRT group had problems, the increase would be a shocking 100%, even though the actual risk would be minute.

The data on hormone replacement therapy show that the results are far from black and white.

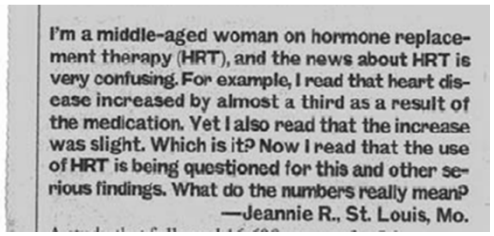
So, although we should not minimize the recent findings, we should not maximize them either. Interested readers should look at the actual study, available online at www.jama.com, site of *The Journal of the American Medical Association*. (Click on "Past Issues," then on "July 17, 2002.") You'll see that the results are not nearly as black-and-white as published reports indicate.

Perhaps as important, the quality-of-life data (energy level, sexual interest, skin tone, hair quality and so on) from the study have not yet been analyzed. When this information is released, we surely will see more front-page news.

Where's The Chicken?!

## Example 4: HRT and Risk of CHD

- Marilyn Vos Savant takes on a serious question



I'm a middle-aged woman on hormone replacement therapy (HRT), and the news about HRT is very confusing. For example, I read that heart disease increased by almost a third as a result of the medication. Yet I also read that the increase was slight. Which is it? Now I read that the use of HRT is being questioned for this and other serious findings. What do the numbers really mean?
—Jeannie R., St. Louis, Mo.

## HRT and Risk of CHD

- Results

Proportion of Women Developing CHD (Incidence)

| | HRT | Placebo | |
|---|---|---|---|
| CHD | 163 | 122 | 285 |
| No CHD | 8,345 | 7,980 | 16,325 |
| | 8,508 | 8,102 | 16,610 |

$\hat{p}_{HRT} = \frac{163}{8,508} = 0.019$ (1.9%)

$\hat{p}_{Placebo} = \frac{122}{8,102} = 0.015$ (1.5%)

## HRT and Risk of CHD

- Results

Risk Difference: $\hat{p}_{HRT} - \hat{p}_{Placebo} = 0.019 - 0.015 = 0.004$ (0.4%)

Relative Risk: $\frac{\hat{p}_{HRT}}{\hat{p}_{Placebo}} = \frac{0.019}{0.015} \approx 1.27$

Which value do you think was most quoted in the press?

## Example 5: Public Insurance Among Dialysis Patients[4]

- From Abstract

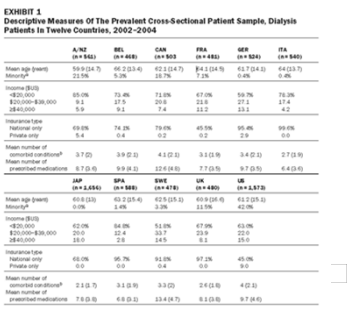


**EXHIBIT 1**
**Descriptive Measures Of The Prevalent Cross-Sectional Patient Sample, Dialysis Patients In Twelve Countries, 2002–2004**

| | A/NZ (n = 561) | BEL (n = 468) | CAN (n = 503 | FRA (n = 481) | GER (n = 524) | ITA (n = 540) |
|---|---|---|---|---|---|---|
| Mean age (years)<br>Minority[a] | 59.9 (14.7)<br>23.5% | 66.2 (13.4)<br>5.3% | 62.1 (14.7)<br>18.7% | 64.1 (14.5)<br>7.3% | 63.7 (14.2)<br>0.4% | 64 (13.7)<br>0.4% |
| Income ($US)<br><$20,000<br>$20,000–$39,000<br>≥$40,000 | 85.0%<br>9.1<br>5.9 | 73.4%<br>17.5<br>9.1 | 71.6%<br>20.8<br>7.4 | 67.0%<br>21.8<br>11.2 | 59.7%<br>27.1<br>13.1 | 78.3%<br>17.4<br>4.2 |
| Insurance type<br>National only<br>Private only | 69.8%<br>5.4 | 74.1%<br>0.4 | 79.6%<br>0.2 | 45.5%<br>0.2 | 95.4%<br>2.9 | 99.6%<br>0.0 |
| Mean number of comorbid conditions[b]<br>Mean number of prescribed medications | 3.7 (2)<br>8.7 (3.6) | 3.9 (2.1)<br>9.9 (4.2) | 4.1 (2.1)<br>12.6 (4.8) | 3.1 (1.9)<br>7.7 (3.5) | 3.4 (2.1)<br>9.7 (3.5) | 2.7 (1.9)<br>6.4 (3.4) |
| | **JAP (n = 2,454)** | **SPA (n = 588)** | **SWE (n = 472)** | **UK (n = 440)** | **US (n = 2,573)** | |
| Mean age (years)<br>Minority[a] | 60.6 (13)<br>0.0% | 63.2 (15.4)<br>1.4% | 62.5 (15.1)<br>3.3% | 60.9 (16.4)<br>11.5% | 61.2 (15.1)<br>42.0% | |
| Income ($US)<br><$20,000<br>$20,000–$39,000<br>≥$40,000 | 62.0%<br>20.0<br>18.0 | 64.8%<br>12.4<br>2.8 | 51.8%<br>33.7<br>14.5 | 47.9%<br>23.9<br>8.1 | 63.0%<br>22.0<br>15.0 | |
| Insurance type<br>National only<br>Private only | 66.0%<br>0.0 | 95.7%<br>0.0 | 91.8%<br>0.4 | 97.2%<br>0.0 | 45.0%<br>9.0 | |
| Mean number of comorbid conditions[b]<br>Mean number of prescribed medications | 2.1 (1.7)<br>7.6 (3.9) | 3.1 (1.9)<br>6.8 (3.1) | 3.3 (2)<br>13.4 (4.7) | 2.4 (1.6)<br>8.1 (3.6) | 4 (2.1)<br>9.7 (4.4) | |

[4] Hirth R, et al. Out-Of-Pocket Spending And Medication Adherence Among Dialysis Patients In Twelve Countries. *Health Affairs* (2008). 27(1); 89-101

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## Example 5: Public Insurance Among Dialysis Patients[4]

- With more than 2 categories, common practice is to designate one of the categories as he "reference group", and present comparisons of all other categories to this reference

  While the choice of reference group is arbitrary, in many cases it is purposely chosen to highlight the substantive emphasis

  For example, for this article written in a US published journal, the primary question of interest may be how the other 11 countries compared to the United States (with secondary interest in how these countries compared to each other)

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## Summary

- Risk difference ( $\hat{p}_1 - \hat{p}_2$ ) and relative risk $R\hat{R} = \frac{\hat{p}_1}{\hat{p}_2}$ are two different estimates of the magnitude and direction of association for binary outcomes (between groups
  - These two estimates are based on the exact same inputs and will always agree in terms of the direction of association, but not necessarily magnitude
- The risk difference helps to quantify the potential impact of a treatment or exposure for a group of individuals
- The relative risk helps quantify the potential impact of a treatment or exposure for an individual
- Neither estimate alone is sufficient to tell the whole "story"

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Section C: Comparing Binary Outcomes Between Two (or More) Populations Using Sample Results: The Odds Ratio

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## Example 1:

- Response to therapy in random sample of 1,000 HIV+ positive patients from a citywide clinical population

| | CD4 <250 | CD4 ≥ 250 | |
|---|---|---|---|
| Respond | 127 | 79 | 206 |
| Not Respond | 376 | 418 | 794 |
| | 503 | 497 | 1,000 |

- Start with sample proportions:

$$\hat{p}_{CD4<250} = \frac{127}{503} = 0.253 \approx 0.25\ (25\%)$$

$$\hat{p}_{CD4\geq250} = \frac{79}{497} = 0.159 \approx 0.16\ (16\%)$$

[1] http://inclass.kaggle.com/

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## Example 1

- Summary Measure 1: the difference in proportions (also called *risk difference* , or *attributable risk*)

$$\hat{p}_{CD4<250} - \hat{p}_{CD4\geq250} = 0.25 - 0.16 = 0.09\ (9\%)$$

- Summary Measure 2: the ratio of proportions (also called *relative risk*, or *risk ratio*)

$$R\hat{R} = \frac{\hat{p}_{CD4<250}}{\hat{p}_{CD4\geq250}} = \frac{0.25}{0.16} \approx 1.56$$

- Summary Measure 3: the odds ratio (also called relative odds)

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## Odds

- What is odds?

The (estimated) odds of an event is the (estimated) probability of the event occurring, divided by the (estimated) probability of it not occurring:

$$\hat{odds} = \frac{\hat{p}}{1-\hat{p}}$$

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## Risk Versus Odds

- As risk ( $\hat{p}$ ) increases, so does odds

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## Example 1:

- What is odds?

The (estimated) odds of an event is the (estimated) probability of the event occurring, divided by the (estimated) probability of it not occurring:

$$\hat{odds} = \frac{\hat{p}}{1-\hat{p}}$$

- So for our data:

$$\hat{p}_{CD4<250} \approx 0.25 \; ; \hat{\text{odds}} = \frac{0.25}{0.75}$$

$$\hat{p}_{CD4\geq250} \approx 0.16 \; ; \text{odds} = \frac{0.16}{0.84}$$

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## Example 1:

- Odds Ratio: ratio of the odds of an event for two groups
- So for our data:

$$\hat{p}_{CD4<250} \approx 0.25 \; ; \hat{\text{odds}} = \frac{0.25}{0.75}$$

$$\hat{p}_{CD4\geq250} \approx 0.16 \; ; \text{odds} = \frac{0.16}{0.84}$$

$$\hat{OR} = \frac{\left(\frac{\hat{p}_{CD4<250}}{1-\hat{p}_{CD4<250}}\right)}{\left(\frac{\hat{p}_{CD4\geq250}}{1-\hat{p}_{CD4\geq250}}\right)} = 1.75$$

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## Example 1:

- Odds Ratio: Interpretation
  - The <250 CD4 count group has 1.75 times the odds of responding to therapy as the ≥ 250 CD4 count group
  - The <250 CD4 count group has 75% greater odds of responding to therapy than the ≥ 250 CD4 count group
- Odds Ratio: not a direct comparison of risks, but a comparison of a function of risks
  - Relative Risk and odds ratio will always agree in terms of direction, but not always be the same value
  - In this example, $\hat{RR} = 1.56$ and $\hat{OR} = 1.75$

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## Example 2: Maternal/Infant HIV Transmission[2]

- Randomized Trial: HIV positive pregnant women randomized to receive AZT or placebo

**Abstract** *Background and Methods.* Maternal–infant transmission is the primary means by which young children become infected with human immunodeficiency virus type 1 (HIV). We conducted a randomized, double-blind, placebo-controlled trial of the efficacy and safety of zidovudine in reducing the risk of maternal–infant HIV transmission. HIV-infected pregnant women (14 to 34 weeks' gestation) with CD4+ T-lymphocyte counts above 200 cells per cubic millimeter who had not received antiretroviral therapy during the current pregnancy were enrolled. The zidovudine regimen included antepartum zidovudine (100 mg orally five times daily), intrapartum zidovudine (2 mg per kilogram of body weight given intravenously over a one-hour period, then 1 mg per kilogram per hour until delivery), and zidovudine for the newborn (2 mg per kilogram orally every six hours for six weeks). Infants with at least one positive HIV culture of peripheral-blood mononuclear cells were classified as HIV-infected.

[2]Connor E, et al. Reduction of Maternal-Infant Transmission of Human Immunodeficiency Virus Type 1 with Zidovudine Treatment. *New England Journal of Medicine* (1994). 331(18); 1173-1180

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## Example 2: Maternal/Infant HIV Transmission

- Results

*Results.* From April 1991 through December 20, 1993, the cutoff date for the first interim analysis of efficacy, 477 pregnant women were enrolled; during the study period, 409 gave birth to 415 live-born infants. HIV-infection status was known for 363 births (180 in the zidovudine group and 183 in the placebo group). Thirteen infants in the zidovudine group and 40 in the placebo group were HIV-infected.

| (at 18 mos) | AZT | Placebo | |
|---|---|---|---|
| HIV+ | 13 | 40 | 53 |
| HIV- | 167 | 143 | 310 |
| | 180 | 183 | 363 |

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## Example 2: Maternal/Infant HIV Transmission

- Results

*Results.* From April 1991 through December 20, 1993, the cutoff date for the first interim analysis of efficacy, 477 pregnant women were enrolled; during the study period, 409 gave birth to 415 live-born infants. HIV-infection status was known for 363 births (180 in the zidovudine group and 183 in the placebo group). Thirteen infants in the zidovudine group and 40 in the placebo group were HIV-infected.

| (at 18 mos) | AZT | Placebo | |
|---|---|---|---|
| HIV+ | 13 | 40 | 53 |
| HIV- | 167 | 143 | 310 |
| | 180 | 183 | 363 |

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## Example 2

- Summary Measure 1: the difference in proportions (also called *risk difference* , or *attributable risk*)

$$\hat{p}_{AZT} - \hat{p}_{Placebo} = 0.07 - 0.22 = -0.15 \text{ (-15\%)}$$

- Summary Measure 2: the ratio proportions (also called *relative risk*, or *risk ratio*)

$$R\hat{R} = \frac{\hat{p}_{AZT}}{\hat{p}_{Placebo}} = \frac{0.07}{0.22} \approx 0.32$$

- Summary Measure 3: the odds ratio

$$O\hat{R} = \frac{\left(\frac{\hat{p}_{AZT}}{1-\hat{p}_{AZT}}\right)}{\left(\frac{\hat{p}_{Placebo}}{1-\hat{p}_{Placebo}}\right)} = \frac{\left(\frac{0.07}{0.93}\right)}{\left(\frac{0.22}{0.78}\right)} \approx 0.27$$

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## Example 2:

- Odds Ratio: Interpretation
  - The AZT group has 0.27 times the odds (of HIV to child transmission) of the placebo group
  - The AZT group has 73% lower odds of HIV to child transmission than the placebo group
- Relative Risk versus Odds Ratio: in this example the relative risk and odds ratio are

$$R\hat{R} = 0.32 \quad \text{and} \quad O\hat{R} = 0.27$$

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## Odds Ratio: Substantive Interpretation

- Odds Ratio: Substantive Interpretation
  - As with the relative risk, the odds ratio can be interpreted as impact (assuming causation) at the "individual level"
  - The odds ratio does not directly compare the probabilities (risks, proportions) of an outcome, but instead compares a function of risk: the odds

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## Relative Risk Versus Odds Ratio

- Both measures use exact same information but can give numerically different results : both will always agree in terms of direction of association

$$\text{If } \frac{\hat{p}_1}{\hat{p}_2} > 1, \text{ then } \frac{\left(\frac{\hat{p}_1}{1-\hat{p}_1}\right)}{\left(\frac{\hat{p}_2}{1-\hat{p}_2}\right)} > 1 \quad \text{ie:}$$

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## Relative Risk Versus Odds Ratio

- The smaller $\hat{p}_1$ and $\hat{p}_2$ are, the closer in value are $\hat{RR}$ and $\hat{OR}$

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## Example 3: Aspirin and CVD: Women[3]

- From Abstract

BACKGROUND
Randomized trials have shown that low-dose aspirin decreases the risk of a first myocardial infarction in men, with little effect on the risk of ischemic stroke. There are few similar data in women.

METHODS
We randomly assigned 39,876 initially healthy women 45 years of age or older to receive 100 mg of aspirin on alternate days or placebo and then monitored them for 10 years for a first major cardiovascular event (i.e., nonfatal myocardial infarction, nonfatal stroke, or death from cardiovascular causes).

RESULTS
During follow-up, 477 major cardiovascular events were confirmed in the aspirin group, as compared with 522 in the placebo group.

[3] Ridker P, et al. A Randomized Trial of Low-Dose Aspirin in the Primary Prevention of Cardiovascular Disease in Women. *New England Journal of Medicine* (2005). 352(13); 1293-1304

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## Example 3

- 2X2 table and estimates

| | Aspirin | Placebo | |
|---|---|---|---|
| CVD | 477 | 522 | 999 |
| No CVD | 19,457 | 19,420 | 38,887 |
| | 19,934 | 19,942 | 39,876 |

$$\hat{p}_{Aspirin} = \frac{477}{19{,}934} = 0.024\ (2.4\%)$$

$$\hat{p}_{Placebo} = \frac{522}{19{,}942} = 0.026\ (2.6\%)$$

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## Example 3

- Risk Difference

$$\hat{p}_{Aspirin} - \hat{p}_{Placebo} = 0.024 - 0.026 = -0.002\ (-0.2\%)$$

- Relative Risk

$$\hat{RR} = \frac{\hat{p}_{Aspirin}}{\hat{p}_{Placebo}} = \frac{0.024}{0.026} \approx 0.92$$

- Odds Ratio

$$\hat{OR} = \frac{\left(\frac{\hat{p}_{Aspirin}}{1-\hat{p}_{Aspirin}}\right)}{\left(\frac{\hat{p}_{Placebo}}{1-\hat{p}_{Placebo}}\right)} = \frac{\left(\frac{0.024}{0.976}\right)}{\left(\frac{0.026}{0.974}\right)} \approx 0.92$$

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## Example 3:

- Odds Ratio: Interpretation
  - The aspirin group has 0.92 times the odds (of developing CHD) of the placebo group
  - The aspirin group has 8% lower odds of developing CHD than the placebo group
- Relative Risk versus Odds Ratio: in this example the relative risk and odds ratio estimates are identical in value, unlike the previous two examples

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## Example 3:

- Odds Ratio: Interpretation
  - The aspirin group has 0.92 times the odds (of developing CHD) of the placebo group
  - The AZT group has 8% lower odds of developing CHD than the placebo group
- Relative Risk versus Odds Ratio: in this example the relative risk and odds ratio estimates are identical in value, unlike the previous two examples

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## Why Even Bother With The Odds Ratio?

- In many ways, the odds ratio is less intuitive and less direct measure of association than the relative risk
- However:
  - In some types of studies (case control, more details coming in term 2), the odds ratio is the only measure of association that can be estimate
  - In logistic regression (also coming in term 2), the results are initially presented as odds ratios, and hence frequently presented as odds ratios in publications

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## Comparing More Than Two Groups

- With more than 2 categories, common practice is to designate one of the categories as he "reference group", and present comparisons of all other categories to this reference

  While the choice of reference group is arbitrary, in many cases it is purposely chosen to highlight the substantive emphasis

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## Example 4: Obesity and Depression

- From Abstract

Data from the Third National Health and Nutrition Examination Survey (1988–1994) were used to examine the relation between obesity and depression. Past-month depression was defined using criteria from the *Diagnostic and Statistical Manual of Mental Disorders*, Third Edition, and was measured with the Diagnostic Interview Schedule. Obesity was defined as a body mass index (weight (kg)/height (m)$^2$) of 30 or higher. The authors compared risks of depression in obese and normal-weight (body mass index 18.5–24.9) persons. Obesity was

[4] Onyike C, et al. Is Obesity Associated with Major Depression? Results form the Third National Health and Nutrition Examination Survey. *American Journal of Epidemiology* (2003). 158(11); 1138-1304

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## Example 4: Obesity and Depression

- Table 4

**TABLE 4. Unadjusted odds ratios (from logistic regression) for the association between relative body weight and different definitions of DIS †DSM-III‡ major depression, by gender, Third National Health and Nutrition Examination Survey, 1988–1994§**

| Population and BMI¶ category | No. of participants | Past-month major depression | | Past-year major depression | | Lifetime major depression | | Recurrent major depression | |
|---|---|---|---|---|---|---|---|---|---|
| | | OR* | 95% CI* | OR | 95% CI | OR | 95% CI | OR | 95% CI |
| All respondents | | | | | | | | | |
| BMI (continuous variable) | 8,410 | 1.05 | 1.01, 1.09 | 1.03 | 0.99, 1.06 | 1.02 | 0.99, 1.05 | 1.01 | 0.97, 1.05 |
| Normal weight (BMI 18.5–24.9) | 4,154 | 1.00# | | 1.00# | | 1.00# | | 1.00# | |
| Underweight (BMI <18.5) | 301 | 1.17 | 0.49, 2.80 | 1.39 | 0.66, 2.95 | 1.35 | 0.73, 2.48 | 1.21 | 0.54, 2.70 |
| Overweight (BMI 25.0–29.9) | 2,297 | 0.86 | 0.53, 1.41 | 0.84 | 0.53, 1.34 | 0.93 | 0.65, 1.35 | 0.84 | 0.54, 1.29 |
| Obese (BMI ≥30) | 1,658 | 1.88 | 1.02, 3.46 | 1.41 | 0.86, 2.36 | 1.22 | 0.81, 1.82 | 1.13 | 0.72, 1.78 |
| Obesity class 1 (BMI 30–34.9) | 981 | 1.26 | 0.64, 2.56 | 1.01 | 0.54, 1.86 | 0.87 | 0.54, 1.4 | 0.79 | 0.46, 1.31 |
| Obesity class 2 (BMI 35–39.9) | 410 | 1.76 | 0.78, 3.95 | 1.67 | 0.91, 3.09 | 1.39 | 0.77, 2.49 | 1.41 | 0.71, 2.81 |
| Obesity class 3 (BMI ≥40) | 267 | 4.98 | 2.07, 11.99 | 2.92 | 1.28, 6.67 | 2.60 | 1.38, 4.91 | 2.29 | 0.93, 5.60 |
| Females | | | | | | | | | |
| BMI (continuous variable) | 4,561 | 1.05 | 1.01, 1.09 | 1.02 | 0.99, 1.06 | 1.02 | 1.03, 1.06 | 1.03 | 0.97, 1.03 |
| Normal weight (BMI 18.5–24.9) | 2,160 | 1.00# | | 1.00# | | 1.00# | | 1.00# | |
| Underweight (BMI <18.5) | 202 | 1.0 | 0.38, 2.64 | 1.36 | 0.60, 3.05 | 1.2 | 0.59, 2.46 | 1.03 | 0.41, 2.60 |
| Overweight (BMI 25.0–29.9) | 1,095 | 1.05 | 0.64, 1.74 | 0.81 | 0.54, 1.22 | 0.94 | 0.65, 1.35 | 0.71 | 0.45, 1.13 |
| Obese (BMI ≥30) | 1,084 | 1.82 | 1.01, 3.3 | 1.29 | 0.8, 2.09 | 1.12 | 0.77, 1.62 | 0.97 | 0.63, 1.51 |
| Obesity class 1 (BMI 30–34.9) | 597 | 1.28 | 0.60, 2.87 | 0.9 | 0.44, 1.81 | 0.74 | 0.42, 1.29 | 0.68 | 0.36, 1.27 |
| Obesity class 2 (BMI 35–39.9) | 285 | 1.75 | 0.71, 4.73 | 1.66 | 0.78, 3.5 | 1.41 | 0.73, 2.74 | 1.43 | 0.66, 2.98 |
| Obesity class 3 (BMI ≥40) | 202 | 3.76 | 1.64, 8.69 | 2.19 | 0.97, 4.94 | 2.15 | 1.17, 3.92 | 1.95 | 0.59, 3.15 |
| Males | | | | | | | | | |
| BMI (continuous variable) | 3,849 | 1.06 | 0.98, 1.15 | 1.04 | 0.96, 1.1 | 1.02 | 0.97, 1.07 | 1.03 | 0.96, 1.10 |
| Normal weight (BMI 18.5–24.9) | 1,974 | 1.00# | | 1.00# | | 1.00# | | 1.00# | |
| Underweight (BMI <18.5) | 99 | 1.09 | 0.21, 5.61 | 0.57 | 0.11, 2.98 | 1.06 | 0.27, 4.15 | 1.12 | 0.17, 7.36 |
| Overweight (BMI 25.0–29.9) | 1,202 | 0.82 | 0.34, 1.96 | 1.08 | 0.56, 2.1 | 1.16 | 0.64, 2.09 | 1.25 | 0.63, 2.49 |

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## Example 4: Obesity and Depression

- Table 4

| Population and BMI¶,‡ category | No. of participants | Past-month major depression | |
|---|---|---|---|
| | | OR* | 95% CI* |
| All respondents | | | |
| BMI (continuous variable) | 8,410 | 1.05 | 1.01, 1.09 |
| Normal weight (BMI 18.5–24.9) | 4,154 | 1.00§ | |
| Underweight (BMI <18.5) | 301 | 1.17 | 0.49, 2.80 |
| Overweight (BMI 25.0–29.9) | 2,297 | 0.86 | 0.53, 1.41 |
| Obese (BMI ≥30) | 1,658 | 1.88 | 1.02, 3.46 |
| Obesity class 1 (BMI 30–34.9) | 981 | 1.26 | 0.64, 2.56 |
| Obesity class 2 (BMI 35–39.9) | 410 | 1.76 | 0.78, 3.95 |
| Obesity class 3 (BMI ≥40) | 267 | 4.98 | 2.07, 11.99 |

From footnotes section:

* DIS, Diagnostic Interview Schedule; DSM-III, *Diagnostic and Statistical Manual of Mental Disorders*, Third Edition; BMI, body mass index; OR, odds ratio; CI, confidence interval.
† For past-year, past-month, and recurrent major depression, noncases included persons with lifetime major depression.
‡ Weight (kg)/height (m)$^2$.
§ Reference category.

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## Example 4: Obesity and Depression

- Table 4

| Population and BMI¶,‡ category | No. of participants | Past-month major depression | |
|---|---|---|---|
| | | OR* | 95% CI* |
| All respondents | | | |
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| Normal weight (BMI 18.5–24.9) | 4,154 | 1.00§ | |
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† For past-year, past-month, and recurrent major depression, noncases included persons with lifetime major depression.
‡ Weight (kg)/height (m)$^2$.
§ Reference category.

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## Summary

- The odds ratio, $\hat{OR}$, provides an alternative to the relative risk, $\hat{RR}$, for quantifying the association between a binary outcome between groups
  - The odds ratio Is ratio of odds between two groups: odds is related to risk (probability, proportion)
- The odds ratio and relative risk both estimate the association between a binary outcome between groups at the individual level
  - These two measures will agree in terms of direction, but not always magnitude
  - The smaller the risk in the groups being compared, the more similar $\hat{OR}$ and $\hat{RR}$

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Section D: A Brief Note About Ratios

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## Learning Objectives

- Upon completion of this lecture you will be able to:
  - Understand that the scaling of ratios is not symmetric around the value of 1 (which would indicate equal values in the numerator and denominator)
  - Consider the implications of the previous point when interpreting size of association
  - Understand the on the log scale (we'll use natural log, ln) the values of ln(ratios) are symmetric about the value 0

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## Example 1 : Maternal/Infant HIV Transmission

- Recall the Results

*Results.* From April 1991 through December 20, 1993, the cutoff date for the first interim analysis of efficacy, 477 pregnant women were enrolled; during the study period, 409 gave birth to 415 live-born infants. HIV-infection status was known for 363 births (180 in the zidovudine group and 183 in the placebo group). Thirteen infants in the zidovudine group and 40 in the placebo group were HIV-infected.

| (at 18 mos) | AZT | Placebo | |
|---|---|---|---|
| HIV+ | 13 | 40 | 53 |
| HIV- | 167 | 143 | 310 |
| | 180 | 183 | 363 |

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## Example 1

- Summary Measures:

$$\hat{p}_{AZT} - \hat{p}_{Placebo} = 0.07 - 0.22 = -0.15 \text{ (-15\%)}$$

$$\hat{RR} = \frac{\hat{p}_{AZT}}{\hat{p}_{Placebo}} = \frac{0.07}{0.22} \approx 0.32$$

$$\hat{OR} = \frac{\left(\frac{\hat{p}_{AZT}}{1-\hat{p}_{AZT}}\right)}{\left(\frac{\hat{p}_{Placebo}}{1-\hat{p}_{Placebo}}\right)} = \frac{\left(\frac{0.07}{0.93}\right)}{\left(\frac{0.22}{0.78}\right)} \approx 0.27$$

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## Example 1

- Interpretations: AZT associated with:
  - (risk difference)15% (absolute) decrease in HIV transmission risk
  - (relative risk) 68% (relative) decrease in HIV transmission risk
  - (odds ratio) 73% (relative) reduction in HIV transmission odds

As compared to No Treatment (Placebo)

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## Example 1

- Direction is arbitrary: supposed we instead compared placebo to AZT:

$$\hat{p}_{Placebo} - \hat{p}_{AZT} = 0.22 - 0.07 = 0.15\ (15\%)$$

$$\hat{RR} = \frac{\hat{p}_{Placebo}}{\hat{p}_{AZT}} = \frac{0.22}{0.07} = \frac{1}{0.32} \approx 3.1$$

$$\hat{OR} = \frac{1}{0.27} \approx 3.7$$

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## Example 1

- Interpretations: No Treatment (Placebo) associated with:
  - (risk difference) 15% (absolute) *increase* in HIV transmission risk
  - (relative risk) 210% (relative) *increase* in HIV transmission risk
  - (odds ratio) 270 % (relative) *increase* in HIV transmission odds

As compared to AZT

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## Example 1

- Recap

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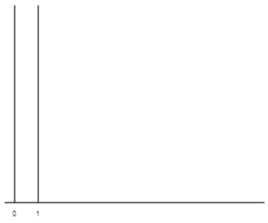
## Scale of Ratios is Not Symmetric

- So why do these associations seem to differ in magnitude if the direction of comparison is reversed??
- The range of possible values for "positive" and "negative" associations are very different

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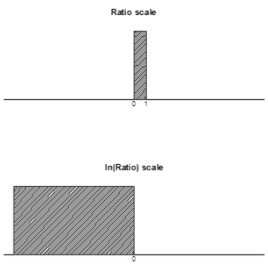
## Scale of Ratios is Not Symmetric

- The ranges are equal on the ln(Ratio) scale

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## Scale of Ratios is Not Symmetric

- For example, with relative risk:

AZT to Placebo

$$\hat{RR}_{AZT\,/\,Placebo} = \frac{\hat{p}_{AZT}}{\hat{p}_{Placebo}} = \frac{0.07}{0.22}$$

$$\ln(\hat{RR}_{AZT\,/\,Placebo}) = \ln\left(\frac{\hat{p}_{AZT}}{\hat{p}_{Placebo}}\right) = \ln(\hat{p}_{AZT}) - \ln(\hat{p}_{Placebo})$$

Placebo to AZT

$$\hat{RR}_{Placebo\,/\,AZT} = \frac{\hat{p}_{Placebo}}{\hat{p}_{AZT}} = \frac{0.22}{0.07}$$

$$\ln(\hat{RR}_{Placebo\,/\,AZT}) = \ln\left(\frac{\hat{p}_{Placebo}}{\hat{p}_{AZT}}\right) = \ln(\hat{p}_{Placebo}) - \ln(\hat{p}_{AZT})$$

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## Scale of Ratios is Not Symmetric

- For example, with relative risk:

AZT to Placebo

$$\hat{RR}_{AZT/Placebo} \approx 0.32$$

$$\ln(\hat{RR}_{AZT/Placebo}) = \ln(0.32) = -1.11$$

Placebo to AZT

$$\hat{RR}_{Placebo/AZT} = \frac{1}{\hat{RR}_{AZT/Placebo}} = 3.1$$

$$\ln(\hat{RR}_{Placebo/AZT}) = \ln(3.1) = 1.11$$

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## Summary

- On the ratio scale (relative risk or odds ratio), the range of possible values is
  - $0 \leq ratio < 1$: for "negative" associations, i.e. where the group in the numerator has lower risk (and hence odds) than the group in the denominator
  - $1 < ratio \leq \infty$ : for "positive" associations, i.e. where the group in the numerator has greater risk (and hence odds) than the group in the denominator
- On the ln(ratio) scale, the range of possible values is:
  - $-\infty < ln(ratio) < 0$: for "negative" associations, i.e. where the group in the numerator has lower risk (and hence odds) than the group in the denominator
  - $0 < \ln(ratio) \leq \infty$ : for "positive" associations, i.e. where the group in the numerator has greater risk (and hence odds) than the group in the denominator

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## Summary

- These properties of ratios and ln(ratios) have potential implications for:
  - Displaying associations for different group comparisons
  - Performing statistical inference on ratios

STAY TUNED: more to come on these properties

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