

Example Problem: HIV prevalence in South Africa

According to UN AIDS*, HIV prevalence in South Africa was 17.8% among adults 15 to 49 years old in 2009. Assume this prevalence estimate is accurate today, and we randomly sample 500 individuals in South Africa. Suppose X is the number of HIV positive individuals in the sample.

Model X using the binomial distribution.

1. How many individuals do we expect to be HIV positive in the sample.

$$E(X) = np = 500 \cdot 0.178 = 89$$

2. What is the standard deviation of the number of HIV positive individuals in the sample?

$$sd(X) = \sqrt{np(1-p)} = 8.553245$$

3. What is the probability of observing more than 100 HIV positive individuals?

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. di 1 - binomial(500, 100, 0.178)
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```
.09089616
```

```
. di binomialtail(500, 101, 0.178)
```

```
.09089616
```

4. What is the probability of observing between 85 and 95 HIV positive individuals?

```
. di binomial(500, 95, 0.178) - binomial(500, 84, 0.178)
```

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.47533949
```

```
. di binomialtail(500, 85, 0.178) - binomialtail(500, 96, 0.178)
```

```
.47533949
```

Now, model X using the normal distribution instead.

1. What is $E(X)$?

$$E(X) = np = 500 \cdot 0.178 = 89$$

2. What is $sd(X)$?

$$sd(X) = \sqrt{np(1-p)} = 8.553245$$

3. What is the probability of observing more than 100 HIV positive individuals?

$$P(X > 100) = P(Z > (100 - 89) / 8.55) = P(Z > 1.286)$$

```
. di 1-normal(1.286)
```

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.09922153
```

4. What is the probability of observing between 85 and 95 HIV positive individuals?

$P(85 < X < 95)$

$= P(X < 95) - P(X < 85)$

$= P(Z < (95 - 89)/ 8.55) - P(Z < (85 - 89)/ 8.55)$

$= P(Z < .702) - P(Z < -.468)$

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. di normal(0.702) - normal(-0.468)
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.43876812
```

5. Do the normal and binomial models give similar results?

What is the probability of observing more than 100 HIV positive individuals?

Binomial: .09089616

Normal: .09922153

What is the probability of observing between 85 and 95 HIV positive individuals?

Binomial: .47533949

Normal: .43876812

Yes, they give similar results. Approximation is better “in the tails”, i.e. for calculating the probability of observing more than 100 HIV+ individuals; than in the center of the distribution (between 85 and 95 HIV+).

*<http://www.unaids.org/en/regionscountries/countries/southafrica/>