Review of Hypothesis Testing for Continuous Data

This cheat sheet is meant to help review the various tests we have learned for continuous outcomes. Don't think of this as a cookbook for statistics, but rather a refresher!

Notation: We have multiple independent realizations of continuous random variable.

One-sample:

$$X \sim N(\mu, \sigma^2)$$

The sample mean of multiple realizations of X is $\bar{x}$ and sample standard deviation is s.

We aim to test $H_0 : \mu = \mu_0$ vs. $H_A : \mu \neq \mu_0$.

Two-sample:

$$X_1 \sim N(\mu_1, \sigma_1^2)$$

The sample mean of multiple realizations of X_1 is $\bar{x}_1$ and sample standard deviation is s_1.

$$X_2 \sim N(\mu_2, \sigma_2^2)$$

The sample mean of multiple realizations of X_2 is $\bar{x}_2$ and sample standard deviation is s_2.

Additionally, the pooled variance estimate is:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

We aim to test $H_0 : \mu_1 = \mu_2$ vs. $H_A : \mu_1 \neq \mu_2$.

Paired: If X_1 and X_2 are paired data,

$$X_d = X_1 - X_2 \sim N(\mu_d, \sigma_d^2)$$

The sample mean of multiple realizations of X_d is $\bar{x}_d$ and sample standard deviation is s_d.

We aim to test $H_0 : \mu = 0$ vs. $H_A : \mu \neq 0$.

Review Table

Groups	Assumptions	Test Statistic	95% Confidence Interval
1	Large sample	$Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \overset{H_0}{\sim} N(0,1)$	$\bar{x} \pm Z_{0.975}\sigma/\sqrt{n}$
1	X is normal σ known	$Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \overset{H_0}{\sim} N(0,1)$	$\bar{x} \pm Z_{0.975}\sigma/\sqrt{n}$
1	X is normal σ unknown	$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \overset{H_0}{\sim} t_{n-1}$	$\bar{x} \pm t_{0.975,n-1}s\sqrt{n}$
2	paired, X_d is normal σ unknown	$t = \frac{\bar{x}_d}{s_d/\sqrt{(n}} \overset{H_0}{\sim} t_{n_d-1}$	$\bar{x}_d \pm t_{0.975,n-1}s\sqrt{n_d}$
2	unpaired, X is normal w/in group σ equal between groups	$t = \frac{\bar{x}_1-\bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \overset{H_0}{\sim} t_{n_1+n_2-2}$	$\bar{x}_1 - \bar{x}_2 \ \pm t_{0.975,n-1}s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$
2	unpaired, X is normal w/in group σ unequal between groups	$t = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} \overset{H_0}{\sim} t_{df}$	$\bar{x}_1 - \bar{x}_2 \pm t_{0.975,df}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$
k	unpaired, X is normal w/in group σ equal between groups	$F = \frac{s_B^2}{s_W^2} \sim F_{k-1,n-k}$	

* df are degrees of freedom from two-sample t-test with unequal variances; use Stata to calculate.