Lecture 2 Continuous Data Measures	Section A:Continous Data: Useful Summary Statistics
Learning Objectives Upon completion of this lecture you will be able to: Compute a sample mean and standard deviation Interpret the estimated mean, standard deviation, median and various percentiles computed for a sample of data	Summarizing and Describing Continuous Data Measures of the center of data — Mean — Median (50th percentile) Measure of data variability — Standard Deviation Other Measures of Location — Percentiles
Sample Mean: The Average or Arithmetic Mean Add up data, then divide by sample size (n) The sample size n is the number of observations (pieces of data)	Example 1: Mean, on Small SBP Dataset ■ Five systolic blood pressures (mmHg) (n=5) 120, 80, 90, 110, 95 Can be represented with math type notation: x₁= 120, x₂=80,x₅=95 The sample mean is easily computed by adding up the five values and

Mean, Example

■ Five systolic blood pressures (mmHg) (n=5) 120, 80, 90, 110, 95

$$\overline{x} = \frac{120 + 80 + 90 + 110 + 95}{5} = 99 \text{ mmHg}$$

Notes on Sample Mean

■ Generic Formula Representation

$$\overline{c} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Where

 $\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \dots + x_n$

In the formula to find the mean, we use the "summation sign"— Σ : This is just mathematical shorthand for "add up all of the observations"

Notes on Sample Mean

- Also called sample average or arithmetic mean
- Sensitive to extreme values (in smaller samples)
- One data point could make a great change in sample mean
- Why is it called the sample mean?
 - To distinguish it from population mean (an unknown, unknowable value of interest μ , that can be estimated by $\overline{\chi}$

Example 1: Median, on Small SBP Dataset

■ The median is the middle number (also called the 50th percentile)

Sample Median

- The sample median is not sensitive to extreme values
 - $-\,\,$ For example, if 120 became 200, the median would remain the same, but the mean would change from 99 mmHg to 115 mmHg

Sample Median

■ If the sample size is an even number

90 95 110 120 125

Median

$$\frac{95 + 110}{2}$$
 = 102.5 mmHg

Describing Variability

- Sample variance (s²)
- Sample standard deviation (s or SD)
- The sample variance is the average of the square of the deviations about the sample mean

$$s^{2} = \frac{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}{n-1}$$

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Describing Variability

■ The sample standard deviation is the square root of s²

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

..

Example 1: Standard Deviation, on Small SBP Dataset

Recall, the 5 systolic blood pressures (mm Hg) with sample mean (\$\overline{x}\$) of 99 mmHg

Five systolic blood pressures (mmHg) (n=5) 120, 80, 90, 110, 95

$$\sum_{i=1}^{5} (x_i - \overline{x})^2 = (120 - 99)^2 + (80 - 99)^2 + (90 - 99)^2 + (110 - 99)^2 + (95 - 99)^2$$

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Describing Variability

■ Example: n = 5 systolic blood pressures (mm Hg)

$$\sum_{i=1}^{5} (x_i - \bar{x})^2 = (21)^2 + (-19)^2 + (-9)^2 + (11)^2 + (-4)^2$$
$$= (441) + (361) + (81) + (121) + (16)$$
$$= 1020 \text{mmHg}^2$$

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Describing Variability

Sample variance

$$s^{2} = \frac{\sum_{i=1}^{n} (x_{i} - \overline{x})^{2}}{n-1} = \frac{1020}{4} = 255 \text{ mmHg}^{2}$$

■ Sample standard deviation (s)

$$\sqrt{s^2} = \sqrt{255 \text{ mmHg}^2}$$

 $s = 15.97 \approx 16 \text{ (mmHg)}$

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Notes on s

- lacktriangle The bigger s is, the more variability there is
- s measures the spread about the mean
- s can equal 0 only if there is no spread
 All n observations have the same value
- The units of s are the same as the units of the data (for example, mm Hg)
- Often abbreviated SD or sd
- s^2 is the best estimate from the sample of the population variance σ^2 ; s is the best estimate of the population standard deviation σ

Notes on s

■ The formula

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Why divide by n-1 instead of n?

Example 2: Blood Pressure Data, 113 Men

■ Example 2: Systolic blood pressure (SBP) measurements from a random sample of 113 adult men taken from a clinical population

The first 50 measurements:

142	116	137	126	124
123	116	127	115	129
107	103	130	133	116
129	117	131	107	138
114	113	142	120	147
105	122	111	111	129
132	89	134	121	120
128	120	119	112	139
121	124	132	140	120
116	152	123	131	141

Example 2: Blood Pressure Data, 113 Men

■ Example 2: Systolic blood pressure (SBP) measurements from a random sample of 113 adult men taken from a clinical population

This, if course, is too much data to compute summary statistics by hand. The results from a computer are as follows:

(Estimate of μ): $\bar{x} = 123.6 \text{ mmHg}$

(Estimate of σ): s = 12.9 mmHg

(Estimate of population median): $\hat{m} = 123.0 \text{ mmHg}$

Percentiles

 Other values that can help us quantify the distribution of continuous data values include the sample percentiles (as estimates of the underlying population percentiles)

In general, the p^{th} sample percentile is that value in sample of data such that p percent of the sample values are lesser than or equal to this value, and (100-p) percent are greater than this value (example: the median is the 50th percentile)

Percentiles can be computed "by hand", but are generally done via computer

Example 2: Blood Pressure Data, 113 Men

■ Example 2: Systolic blood pressure (SBP) measurements from a random sample of 113 adult men taken from a clinical population

Based on the results from computer:

The 10^{th} percentile for these 113 blood pressure measurements is 107 mmHg, meaning that approximately 10% of the men in the sample have SBP \leq 107 mmHg, and (100-10) = 90% of the men have SBP >107 mmHg

The 75h percentile for these 113 blood pressure measurements is 132 mmHg, meaning that approximately 75% of the men in the sample have SBP \leq 132 mmHg, and (100-75) = 25% of the men have SBP > 132 mmHg

Example 2: Blood Pressure Data, 113 Men

Example 2: Systolic blood pressure (SBP) measurements from a random sample of 113 adult men taken from a clinical population

Some percentiles of these data, based on the results from computer:

Percentile	Sample Estimate
2.5 th	100.7 mmHg
25 th	114 mmHg
50 th	123 mmHg
75 th	132 mmHg
97.5 th	151.2 mmHg

Example 3: Length of Stay Data

 Example 3: Length of stay claims at Heritage Health with an inpatient stay of at least one day in 2011¹ (12,928 claims)

The first 50 measurements:

21	1	1	1	1
1	4	3	2	7
12	1	2	2	7
5	8	3	1	1
1	2	6	1	1
1	1	4	3	20
1	1	2	3	3
1	6	7	1	4
2	11	10	10	17
6	1	1	5	4

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Example 3: Length of Stay Data

 Example 3: Length of stay claims at Heritage Health with an inpatient stay of at least one day in 2011¹ (12,928 claims)

(Estimate of μ): $\bar{x} = 4.3 \text{ days}$

(Estimate of σ): $s = 4.9 \, \mathrm{days}$

(Estimate of population median): $\hat{m} = 2 \text{ days}$

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Example 3: Length of Stay Data

 Example 3: Length of stay claims at Heritage Health with an inpatient stay of at least one day in 2011¹ (12,928 claims)

Some percentiles of these data, based on the results from computer:

Percentile	Sample Estimate
2.5 th	1 day
25 th	1 day
50 th	2 days
75 th	5 days
97.5th	20 days

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Summary

- Summary measures that can be computed on a sample of continuous data include the mean, standard deviation, median (50th percentiles), and other percentiles
- These sample based estimates are the best estimates of unknown, underlying population quantities. For example:
 - \overline{x} is the best estimate of the population mean (μ) s is the best estimate of the population standard deviation (σ)
- (Soon) we will discuss how to address the uncertainty in the estimates of certain sample quantities (ex: mean)

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Section B: Continuous Data: Visual Displays

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Learning Objectives

- Upon completion of this lecture section you will be able to:
 - Utilize histograms and boxplots to visualize the distributions of samples of continuous data
 - Identify key summary statistics on the boxplot
 - Name and describe basic characteristics of some common distribution shapes for continuous data

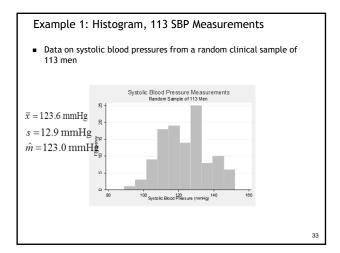
Pictures of Data: Continuous Variables

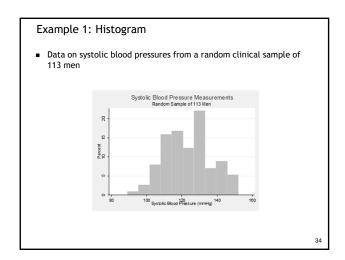
- Histograms and Boxplots
 - Means, standard deviations and percentile values do not tell whole story of data distributions
 - Differences in shape of the distribution
 - Histograms are a way of displaying the distribution of a set of data by charting the number (or percentage) of observations whose values fall within pre-defined numerical ranges
 - Boxplots are graphics that display key characteristics of a dataset: these are especially nice tools for comparing data from multiple samples visually

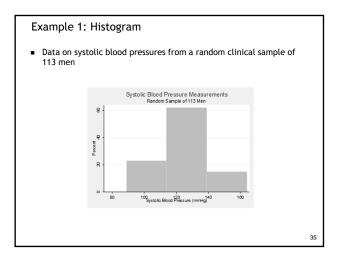
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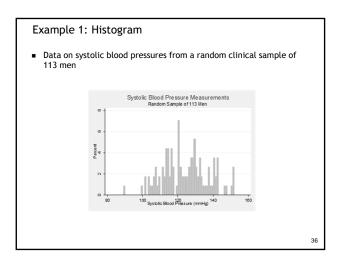
Example 1: Histogram, 113 SBP Measurements

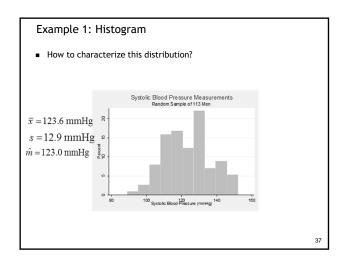
- Data on systolic blood pressures from a random clinical sample of 113 men
- A histogram can be created by:
 - Breaking the data (blood pressure) range into bins of equal width
 - Counting the number of the 113 observations whose blood pressure values fall within each bin
 - Plotting the number (or relatively frequency) of observations that fall within each bin as a bar graph

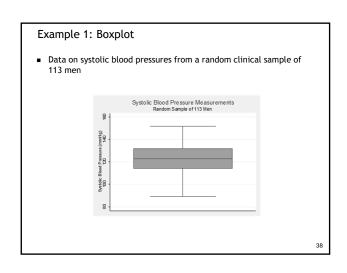


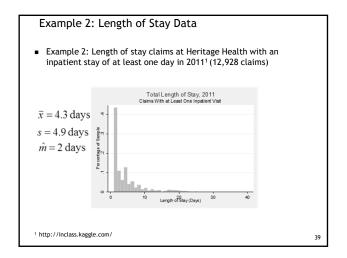


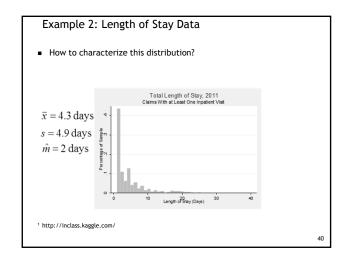


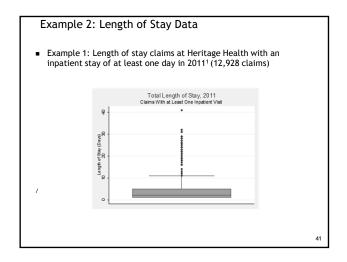


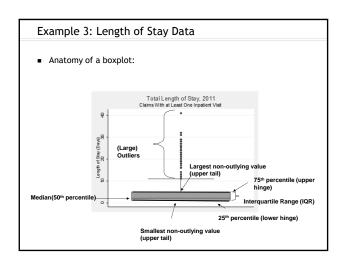


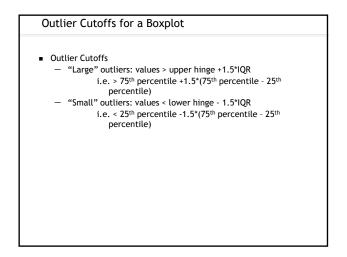


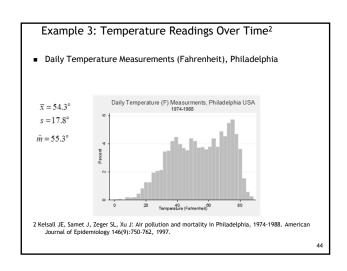


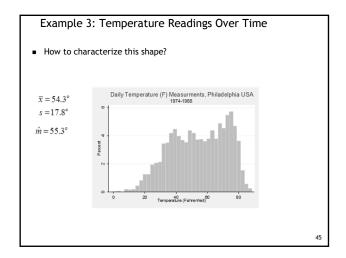


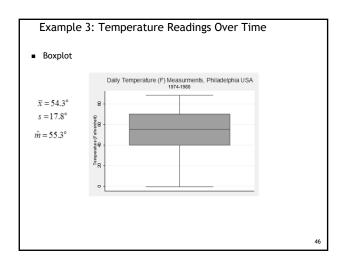












Some "Common" Distribution Shapes

Symmetric and "bell shaped"

Right (positively) skewed

Some "Common" Distribution Shapes

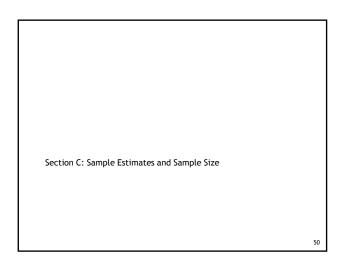
Left (negatively) skewed

Uniform

Summary

- Histograms and Boxplots are useful visuals tools for characterizing the shape of a data distribution above and beyond the information given by summary statistics
- Relatively common shapes for samples of continuous data measures include symmetric and "bell" shaped, right skewed, left skewed and uniform

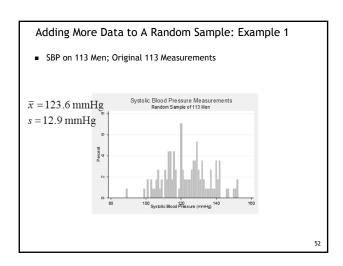
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Learning Objectives

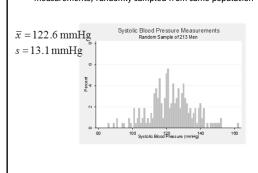
- Upon completion of this lecture section you will be able to:
 - Understand that a random sample taken from a larger population will (imperfectly) mimic the characteristics of the larger population
 - Understand that the distribution of values in a random sample should reflect the distribution of the values in the population from which the sample was taken
 - Understand and explain that sample size does not systematically decrease or decrease sample summary statistic estimates
 - Begin to understand that while sample size does not systematically decrease or decrease sample summary statistic estimates, the estimates become less variable with larger samples

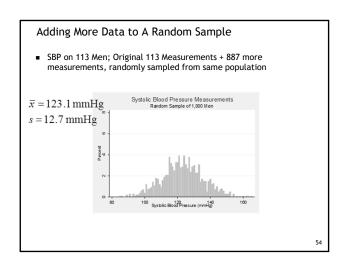
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Adding More Data to A Random Sample

■ SBP on 113 Men; Original 113 Measurements + 100 more measurements, randomly sampled from same population





Summary Statistics and Sample Size

■ Sample means from 5 random samples of men, of sizes n=113, 213, and 1,000 respectively

Sample	n=113	n=213	n=1,000
1	124.4	123.5	122.6
2	121.7	123.4	123.6
3	122.6	123.1	122.6
4	123.1	123.4	122.8
5	120.5	123.1	123.1

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Summary Statistics and Sample Size

■ Sample standard deviations from 5 random samples of men, of sizes n=113, 213, and 1,000 respectively

Sample	n=113	n=213	n=1,000
1	12.9	13.1	12.4
2	14.4	11.6	13.3
3	12.0	13.1	13.1
4	14.5	12.9	12.9
5	13.6	13.6	13.1

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Random Samples of Various Sizes: Example 2

 Length of stay claims at Heritage Health with an inpatient stay of at least one day in 2011¹ (12,928 claims)

(Estimate of μ): $\bar{x} = 4.3 \text{ days}$

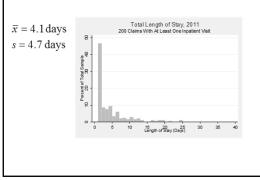
(Estimate of σ): $s = 4.9 \, days$

(Estimate of population median): $\hat{m} = 2 \text{ days}$

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Random Samples of Various Sizes: Example 2

Random Sample of 200 Patients, Heritage Health Data



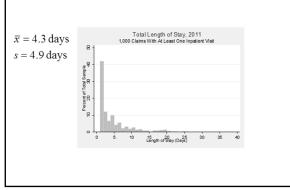
Random Samples of Various Sizes: Example 2

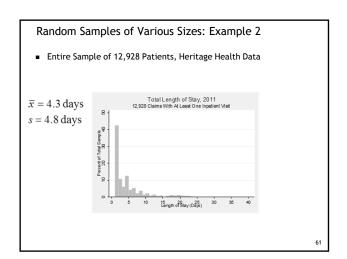
■ Random Sample of 500 Patients, Heritage Health Data

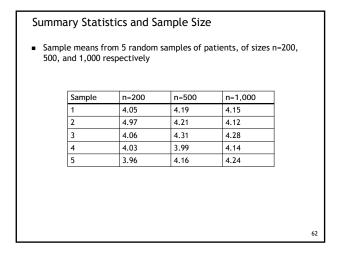
 $\overline{x} = 4.2 \text{ days}$ s = 4.6 days Total Length of Stay, 2011 soo Claims With At Least One Inpatient Visit of Stay and the Stay of Stay (Days) and the Stay (Days) and the

Random Samples of Various Sizes: Example 2

■ Random Sample of 1,000 Patients, Heritage Health Data

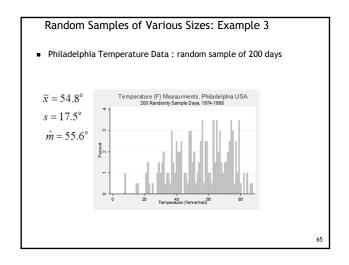


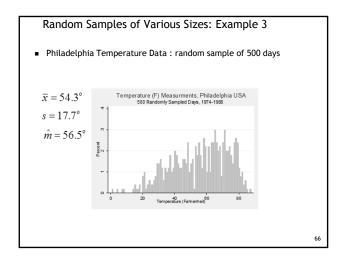




Summary Statistics and Sample Size Sample standard deviations from 5 random samples of patients, of sizes n=200, 500, and 1,000 respectively Sample n=200 n=500 n=1,000 4.68 4.78 5.02 5.67 4.84 4.70 3 5.10 5.10 4.90 4.50 4.78 4.74 4.45 4.73 4.65

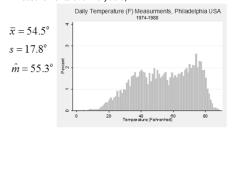
Random Samples of Various Sizes: Example 3	
 Philadelphia Temperature Data (total of 5,471 measurements ove 15 years) 	er
(Estimate of μ): $\bar{x} = 54.3^{\circ}$	
(Estimate of σ): $s = 17.8^{\circ}$	
(Estimate of population median): $\hat{m} = 55.3^{\circ}$	
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Random Samples of Various Sizes: Example 3

Philadelphia Temperature Data: entire sample (total of 5,471 measurements over 15 years)



Summary Statistics and Sample Size: Temperature Data

 Sample means from 5 random samples of days, of sizes n=200, 500, and 1,000 respectively

Sample	n=200	n=500	n=1,000
1	56.1	54.3	53.9
2	52.3	53.2	55.0
3	54.0	54.4	54.7
4	52.7	54.3	54.3
5	55.0	53.7	53.7

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Summary Statistics and Sample Size

■ Sample standard deviations from 5 random samples of days, of sizes n=200, 500, and 1,000 respectively

Sample	n=200	n=500	n=1,000
1	18.1	18.1	17.9
2	18.4	17.8	18.0
3	17.8	17.1	17.7
4	17.3	16.9	17.8
5	17.4	18.1	18.0

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Summary Statistics and Sample Size

 Sample medians from 5 random samples of days, of sizes n=200, 500, and 1,000 respectively

Sample	n=200	n=500	n=1,000
1	56.2	56.5	54.3
2	53.5	52.9	55.5
3	55.6	55.5	56.6
4	51.8	54.0	55.0
5	55.5	54.5	54.1

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Wrap Up

- The distribution of sample values of continuous value should (imperfectly) mimic the distribution of the values in the population from which the sample was taken
- With regards to the a distribution of sample values, increased sample size:
 - Will not systematically alter the shape of the sample distribution
 - Will result in a more "filled out" distribution

Wrap Up

- With regards to sample summary statistics, increased sample size:
 - Will not systematically alter the values of the sample statistic

 ► The sample statistic estimates with vary from random
 sample to random sample but will not systematically get
 larger (or smaller) with increasing sample size
 - Will increase the precision of the summary statistics as estimates of the unknown (population level) true values (more to come shortly..)

Section D: Comparing Distributions of Continuous Data

Learning Objectives

- Upon completion of this lecture section you will be able to:
 Suggest graphical approaches to comparing distributions
 - Suggest graphical approaches to comparing distributions of continuous data between two or more samples
 - Explain why a difference in sample means can be used to quantify, in a single number summary, differences in distributions of continuous data

Motivation

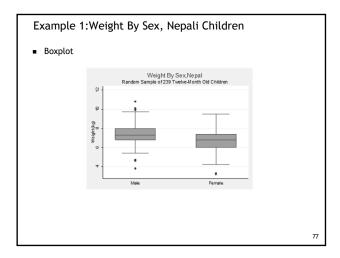
- Frequently, in public health/medicine/science etc.., researchers/practitioners are interested in comparing two (or more) populations via data collected on samples from these populations
- Such comparisons can be used to investigate questions such as:
 - How does weight change differ between those who are on a low-fat diet compared to those on a low-carbohydrate diet?
 - How do salaries differ between males and females?
 - How do cholesterol levels differ across weight groups?

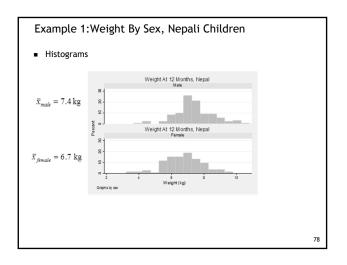
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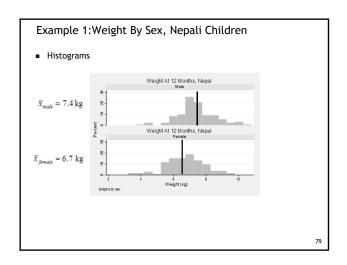
Motivation

- While these comparisons can be done visually, it is also useful to have a numerical summary
- Theoretically, this numerical summary could be many things:
 - Difference in medians
 - Ratio of means
 - Difference in 95th percentiles
 - Ratio of standard deviations
 - Etc.

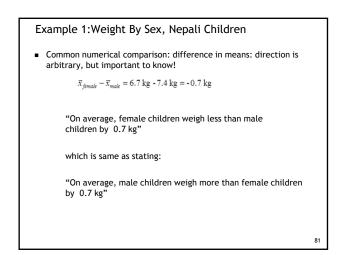
However, what is commonly used (for reasons that we will elaborate on shortly) is a difference in sample means: When comparing sample distributions this can be a reasonable measure of the overall differences in these distributions (as an estimate of the underlying difference in the population distributions)

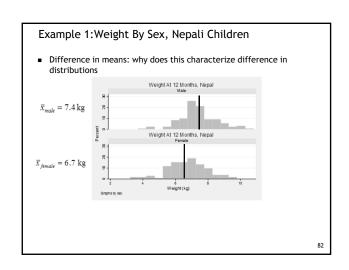


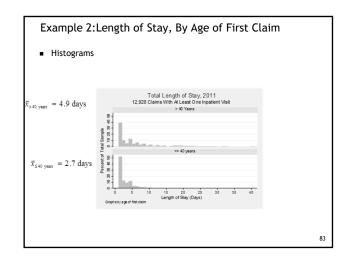


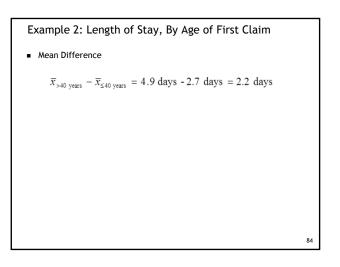


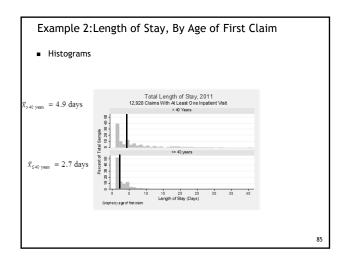
■ Common numerical comparison: difference in means $\bar{x}_{male} - \bar{x}_{female} = 7.4 \, \text{kg} - 6.7 \, \text{kg} = 0.7 \, \text{kg}$ "On average, male children weigh more than female children by $0.7 \, \text{kg}$ "





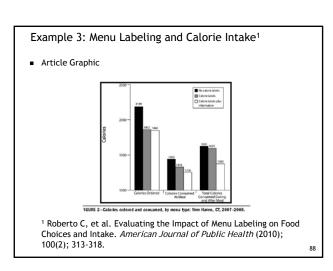






■ From Abstract Coljectives. We assessed the impact of restaurant menu calorie labels on food choices and intake. Methods, Participants in a study dinner (n=303) were randomly assigned to either (1) a menu without calorie labels (no calorie labels, (2) a menu with calorie labels (no calorie labels, (2) a menu with calorie labels and label stating the recommended daily calorie intake for an average solut (calorie labels plus information). Food choices and intake during and later the study dinner were combined, that group consumed 13% fewer calories than the no calorie labels group. Individuals in the calorie labels condition where calories than the study dinner were combined, that group consumed 13% fewer calories than the no calorie labels group. Individuals in the calorie labels condition consumed more calories after the study dinner than those in both other conditions. When calories consumed during and after the study dinner were combined, participants in the calorie labels condition. Calorie labels condition consumed more calories labels in the other group. Conditions. Calorie labels on restaurant menue inspected food choices and intake; adding a recommended daily calorie requirement label increased this effect, suggesting menu label legislation should require such a label. Future research should evaluate menu labelings impact on children's food choices and consumption. (Am J Pubble Health, 2010;100312-318, doc10, 2166ALJPH2009. 1 Roberto C, et al. Evaluating the Impact of Menu Labeling on Food Choices and Intake. American Journal of Public Health (2010); 100(2); 313-318.

Example 3: Menu Labeling and Calorie Intake¹ • From Abstract Objectiver. We assessed the impact of restaurant menu calorie labels on food choices and intake. **Bethods, **Parcicipants in a study dinner (ma 300) were randomly assigned to the study of the compact of the study of the study



■ From abstract Results The mean salary within our cohort was \$167 669 (95% CI, \$158 417-\$176 922) for women and \$200 433 (95% CI, \$194 249-\$206 617) for men. Male gender was associated with higher salary (+513 939, P= 0.01) veen after adjustment in the final model for specially, exademic rank, leadership positions, publications, and research time. Peters-Belson analysis use of coefficients derived from regression model for men applied to women) indicated that the expected mean salary for women, if they retained their other measured characteristics but their gender was male, would be \$12 194 higher than observed. Conclusion Cender difference in salary exist in this select, homogeneous cohort of mid-caneer academic physicians, even after adjustment for differences in specialty, institutional characteristics, academic productivity, academic rank, work hours, and other factors. JAMA. 2012;307(2):2410-2417 www.jama.com

Example 4: Academic Physician Salaries

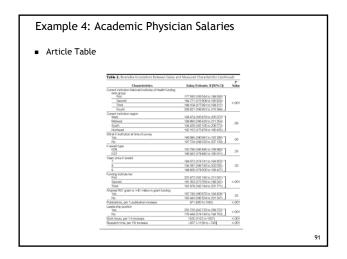
From abstract

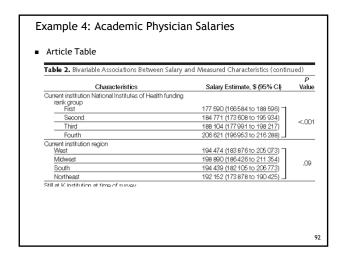
Results: The mean salary within our cohort was \$167,669 (95%, CL, \$158,417-\$176,922) for roomen and \$200.433 (95%, CL, \$154,249-\$206,617) for men. Male gender was associated with higher salary (+\$13.399, P=.001) even after adjustment in the final model for specially, academic rank, loadership positions, publications, and iresearch time. Peters-Beton analysis (use of coefficients derived from regression model for men applied to women) indicated that the expected mean salary for women, if they retained their other measured characteristics but their gender was male, would be \$12 194 higher than observed.

Conclusion: Gender differences in salary exist in this select, homogeneous cohort of mid-career academic physicians, even after adjustment for differences in specialty, institutional characteristics, academic productivity, academic rank, work hours, and other factors.

JAMA. 2012;807(22):2410-2417

www.juma.com





Example 4: Academic Physician Salaries

• Mean differences that can be reported, for example:

While the distributions of continuous data can be compared between samples in many ways, some key approaches include:
 Visual comparisons, such as side-by-side boxplots
 Numerical comparisons, mainly the mean difference between any group of two samples