Homework Assignment 2

Due Date: January 31, 2021, 23:59

Note. Please note that this semester all assignments are group assignments. Further note that for the grading we will apply a "10%" rule, i.e. the maximum number of points for this assignments is 55, but 50 will be counted as 100%. Points that exceed 50 will be stored in a separate counter and used later for compensation of lost points in other assignments or (if not used up this way) the final exam.

EXERCISE 1.

(i) Generalise the mergesort algorithm splitting a list into k sublists instead of just 2. Show that the complexity remains the same.

(ii) Modify the mergesort algorithm adding a threshold value t such that for lists with a length at most t an elementary sorting algorithm (bubblesort, insertion sort, selection sort) is used instead of mergesort. Implement the modified algorithm (choose one of the three elementary sorting algorithms).

(iii) Provide a theoretical and experimental analysis to determine a good value for the threshold t, which optimises performance.

total points: 18

EXERCISE 2. Suppose you have to process n advance bookings of rooms for a hotel with k identical rooms. Bookings contain an arrival date and a departure date. You have to find out whether there are enough rooms in the hotel to satisfy the demands.

(i) Design an algorithm that solves this problem in time $O(n \log n)$.

Hint. Sort the set of all arrivals and departures and process it in sorted order. Choose the most appropriate sorting algorithm for this problem.

(ii) Implement your algorithm.

total points: 13

EXERCISE 3. Explain how to implement a FIFO queue using two stacks so that each FIFO operation takes amortised constant time.

total points: 8

EXERCISE 4. It is easy to check to check whether an algorithm produces a sorted output. It is less easy to check whether the output is also a permutation of the input. However, for integers there exists a fast and simple algorithm:

(i) Show that $[e_1, \dots, e_{n_i}]$ is a permutation of $[e'_1, \dots, e'_{n_i}]$ iff the polynomial

$$P(x) \; = \; \prod_{i=1}^{n} (x - e_i) - \prod_{i=1}^{n} (x - e'_i)$$

in the variable x is identically zero.

(ii) For any $\varepsilon > 0$ let p be a prime with $p > \max\{n/\varepsilon, e_1, \dots, e_{n_i}, e'_1, \dots, e'_{n_i}\}$. The idea is to evaluate the above polynomial $P(x)$ modulo p for a random value $x \in [0, p-1]$.

Show that if $[e_1, \dots, e_{n_i}]$ is not a permutation of $[e'_1, \dots, e'_{n_i}]$, then the result of the evaluation is zero with probability at most ε.

Hint. A non-zero polynomial of degree n has at most n zeroes.

total points: 16