Protocol
W2 - Stirling Engine

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Contents

1 Objective

In this exercise, a Stirling engine is studied in order to determine some of its thermodynamic properties. Firstly, the engine is run manually as a heat pump, in order to assess the reversibility of the Stirling cycle through temperature measurements at the warm and cold sinks. Secondly, the engine is run by supplying heat through combustion of Ethanol as a fuel. Through measurements of temperature, pressure and volume, p/V diagrams are created and the following values are determined:

1. Efficiency of heat transfer between heat source and engine.
2. Mechanical efficiency of the engine.
3. Total efficiency of the engine.

Additionally, the engine's Frequencies at different temperatures are compared.

2 Theory

The Stirling cycle is a thermodynamic cycle, in which a working fluid (in the case of this experiment air) undergoes four steps:

1. An isothermal (no change in temperature) expansion of the air. Heat is taken up by the system from a hot reservoir, while work is given off to a flywheel.
2. An isochoric (no change in volume) cooling of the air. No work is done by or put into the system.
3. An isothermal contraction of the air. Heat is given off to a cold reservoir, and work is put into the system by the flywheel.
4. An isochoric heating of the air. No work is done by or put into the system.

2.1 Total Work in a Stirling Cycle

The first law of thermodynamics states, that the change of the internal energy of a system can be expressed as a sum of the heat which is added to or removed from the it, as well as the mechanical work, which is done by or put into it.

$$\mathrm{d}U = \delta Q + \delta W = \delta Q - p\,\mathrm{d}V \tag{1}$$

where: U = Internal energy of the system [J]
Q = Heat added to or removed from the system [J]
W = Heat added to or done by the system [J]
p = Pressure of the system [Pa]
V = Volume of the system [m^3]

During the isothermal steps of the process, the internal energy U of the System does not change, meaning $\delta - Q = W = -p\,\mathrm{d}V$. By integrating all terms and using the ideal gas law $pV = nRT$, to substitute p, the work which is carried out during an isothermal state change can be calculated:

$$-Q = W = -nRT \cdot ln\frac{V_2}{V_1} \tag{2}$$

where: n = Amount of substance [mol]
R = Ideal gas constant = 8.3145 [J mol^{-1} K^{-1}]
T = Temperature of the system [K]
V_2 = Volume at the end of isothermal state change [m^3]
V_1 = Volume at the start of isothermal state change [m^3]

During the isochoric steps, dV is equal to zero, meaning no work is carried out. As such the change of internal energy is equal to the heat which is added to or removed from the system.

$$\Delta U = Q \tag{3}$$

Since there is no change in Volume and thus no work during the isochoric steps, the total work which is carried out by the system during the Stirling cycle can be calculated by adding the work of both isothermal steps.

$$W_{pV} = -nRT_1 \cdot \ln \frac{V_2}{V_1} - nRT_2 \cdot \ln \frac{V_1}{V_2} = nR(T2 - T1) \cdot \ln \frac{V_2}{V_1} \tag{4}$$

This work is equal to the enclosed area within the p/V diagram of the cycle.

2.2 Efficiency of the Stirling Cycle

It is known that the idealized Stirling cycle has a thermal efficiency equal to that of the idealized Carnot cycle.

$$\eta = \frac{W_{pV}}{Q_1} = \frac{T_1 - T_2}{T_1} \tag{5}$$

In a real-world Stirling engine however, this efficiency is not reached due to the overlap of the cycle's four steps with each other, along with other losses such as friction or during heat transfer. As such, other efficiency values are used to assess the quality of the real-world engine.

The first value of interest is the efficiency of heat transfer between the heat source (in this case a small lamp, fuelled by Ethanol) and the hot reservoir (bottom side) of the Stirling engine.

$$\eta_H = \frac{Q_1}{Q_H} \tag{6}$$

where: η_H = Efficiency of heat transfer between heat source and hot reservoir
Q_1 = Heat received at the hot reservoir [J]
Q_H = Heat given off by the heat source [J]

Secondly, the mechanical efficiency, which relates the amount of usable work received at the flywheel to the work which is carried out by the system.

$$\eta_m = \frac{W_m}{W_{pV}} \tag{7}$$

where: η_m = Mechanical efficiency of the engine
W_m = Usable work received at the flywheel [J]
W_{pV} = Work given off by the engine [J]

Thirdly, the total efficiency, which relates the amount of usable work received at the flywheel to the heat which is given off by the heat source.

$$\eta = \frac{W_m}{Q_H} \tag{8}$$

where: η = Total efficiency of the process

The values used in these calculations can be determined as follows:

1. Q_1 is determined from Equation 2 by applying the ideal gas law $pV = nRT$:

$$Q_1 = p_1 V_1 \cdot ln(\frac{V_2}{V_1}) \tag{9}$$

where: p_1 = Pressure at the start of isothermal expansion [Pa]
V_1 = Volume at the start of isothermal expansion [m^3]
V_2 = Volume at the end of isothermal expansion [m^3]

2. Q_H is determined by multiplying the molar heat of combustion of the fuel with the amount of fuel used, which in turn is calculated by dividing the mass of used fuel with its molar mass. The mass of used fuel can be determined by multiplying the fuel consumption rate of the lamp with the length of time over which the fuel was burned.

$$Q_H = \Delta_c H_m \cdot \frac{m_{EtOH}}{M_{EtOH}} = Q_H = \Delta_c H_m \cdot \frac{k \cdot \Delta t}{M_{EtOH}} \tag{10}$$

where: $\Delta_c H_m$ = Molar heat of combustion of burned fuel [J/mol]
m_{EtOH} = Mass of burned fuel [g]
M_{EtOH} = Molar mass of burned fuel [g/mol]
k = Fuel consumption rate of the lamp [g/s]
Δt = Time period [s]

3. W_m is determined by calculating the work required to spin the flywheel up to its regular frequency from a standstill.

$$W_m = 4\pi^2 m r^2 (\frac{f_f - f_0}{\Delta t_{start}}) \tag{11}$$

where: m = Mass of flywheel [kg]
r = Radius of flywheel [m]
f_f = Frequency of the engine after several cycles [Hz]
f_0 = Frequency of the engine during the first cycle [Hz]
Δt_{start} = Time between the start of the cycle used to determine f_0 and end of the cycle used to determine f_f [s]

4. W_{pV} is determined by calculating the area enclosed in the p/V diagram.

3 Experiment and Results

3.1 Determination of the Reversibility of the Stirling Engine

In order to assess the reversibility of the engine, a crank was attached to the flywheel, allowing for manual input of work into the system. The upper part of the engine (cold reservoir) was covered with 100 g of water, while the bottom part (hot reservoir) was left exposed to air. The crank was then spun by hand, first clockwise (the direction in which it would run during operation as a regular heat engine), then counterclockwise, for one minute each. Special attention was paid towards trying to keep a constant angular velocity while cranking the flywheel, both during individual runs as well as from one run to another. The temperatures on both sides of the engine were recorded before and after the runs and are shown in Table 1.[1]

Table 1: Recorded temperatures before and after spinning the flywheel manually

	Clockwise		**Counter-Clockwise**	
	Top	**Bottom**	**Top**	**Bottom**
T_{Start} [°C]	25.2	28	25.2	28
T_{End} [°C]	25.3	27	25.0	29

These results show that, in it's usual configuration as a heat engine, heat is transferred from the bottom of the engine, where normally a heat source would be located, to the top of it. When run in reverse, the opposite phenomenon is observed, and heat is transferred from the top of the engine to the bottom.

The change in heat on each side of the engine was calculated using the specific heat capacities of the adjacent fluids, the mass of the fluid and the difference in temperature.

$$Q = m \cdot c \cdot \Delta T \tag{12}$$

where: Q = Heat added to or removed to the system [J]
m = Mass of the adjacent fluid [kg]
c = Heat capacity of the fluid [J kg^{-1} K^{-1}]
ΔT = Temperature difference [K]

While the fluid at the top of the engine is simply the water covering the piston, there is no similarly contained volume of fluid adjacent to the bottom end of the engine. Instead, for the bottom part of the engine, the air inside the engine cylinder is used for this calculation instead. Assuming its density is approximately equal to its density near sea level of 1.205 g/L[2] and its volume is approximately equal to the 0.748 L (the average of V_2 and V_1 shown in Table 4 at 230 °C), an approximate mass of 0.9 g of air is caluclated. Table 2 shows the used temperature differences, masses and heat capacities, as well as the resulting amounts of heat.

Table 2: Measured temperature differences of the manual tests, masses and heat capacities of the respective fluids at the top and bottom of the engine as well as resulting heat values

	Clockwise		**Counter-Clockwise**	
	Top	**Bottom**	**Top**	**Bottom**
ΔT [K]	0.1	-1	-0.2	1
m [kg]	0.1	$9 \cdot 10^{-4}$	0.1	$9 \cdot 10^{-4}$
c [J kg^{-1} K^{-1}]	4.2	1.0	4.2	1.0
Q [J]	$4.2 \cdot 10^{-2}$	$-9 \cdot 10^{-4}$	$-8.4 \cdot 10^{-2}$	$9 \cdot 10^{-4}$

[1] The inconsistent amount of decimal places is due to the lower accuracy of the bottom thermometer.
[2] NIST. *Composition of AIR, DRY (NEAR SEA LEVEL).* 2024. URL: `https://physics.nist.gov/cgi-bin/Star/compos.pl?refer=ap%7B%5C&%7Dmatno=104`.

3.2 Determination of Efficiency Values of the Stirling Engine

In order to calculate the efficiency values of the engine, the crank was removed from the flywheel and replaced by a gear, which was connected to a pV-sensor via belt drive. The pV-sensor as well as the temperature-probes were in turn connected to a pVnT-measuring device. Temperature values at specific times in the experiment were obtained directly from the displays of this device, while pressure and volume data was measured over a continuous timespan using an oscilloscope connected to it. The engine frequency was determined at specific points of the experiment as the reciprocal value of the time of a single cycle, which was determined from the pressure and volume data.

In order to calculate Q_H as shown in Equation 10, the fuel consumption rate of the used lamp was determined by letting the flame burn for a specific duration and weighing the lamp before and afterwards.

$$k = \frac{\Delta m}{\Delta t} \tag{13}$$

where: k = Fuel consumption rate of the lamp [g/s]
Δm = Difference in mass of the lamp [g]
Δt = Time the lamp was lit for [s]

The experiment was conducted several times and the recorded mass differences and durations as well as the resulting fuel consumption rates and their average are shown in Table 3.

Table 3: Measured mass differences and burn times of the lamp and resulting fuel consumption ratios as well as their average, standard deviation and error margin

Δm [g]	Δt [s]	k [g/s]
0.78	60	0.013
0.70	60	0.012
0.70	60	0.012
0.76	60	0.013
	$\overline{x}$	0.012

In the three[3] runs of the main experiment - each at a different temperature - the lamp was put underneath the engine and ignited. Once the engine reached the desired temperature, the flywheel was put in motion by hand and the starting temperature was recorded. After the first few cycles, the recording of the oscilloscope was stopped and the data exported onto a USB-drive. Another recording was made and exported once the engine reached its full operating frequency and another temperature measurement was taken before the torch was extinguished and the engine came to a halt.

In order to convert the voltages measured by the oscilloscope into pressures and volumes, the following pre-determined conversions were used:

$$V = \frac{U + 41}{56} \cdot 10^{-3} \tag{14}$$

where: V = Volume [m^3]
U = Measured current [V]

[3]An additional fourth run was conducted at a temperature of 210 °C, for which data seems to have been incorrectly exported from the oscilloscope and which can thus not be evaluated.

$$p = (\frac{U + 1.05}{3.1} + p_0) \cdot 10^5 \tag{15}$$

where: p = Pressure [Pa]
U = Measured current [V]
p_0 = Ambient pressure [bar]

For this calculation, an ambient pressure of 1.013 bar was assumed.

In Figure 1, the volume and pressure of the working fluid at different temperatures are shown plotted against time, while in Figure 2, the same plots are shown at constant motion of the engine, in addition to the corresponding p/V-diagrams.

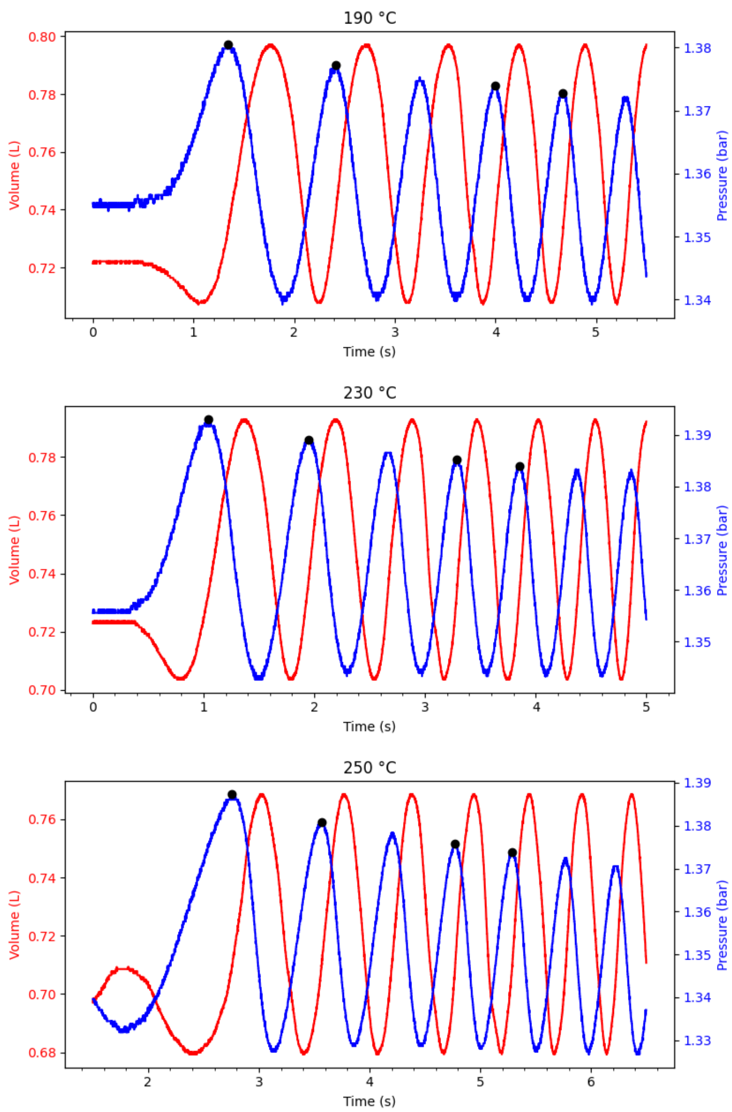

Figure 1: Volume (shown in red) and pressure (shown in blue) of the working fluid, plotted against time, at the start of each experiment. The start and end of the first and fourth cycle, which are used to determine f_0 and f_f, were selected to coincide with the local maxima of the pressure graph, and are marked with black dots.

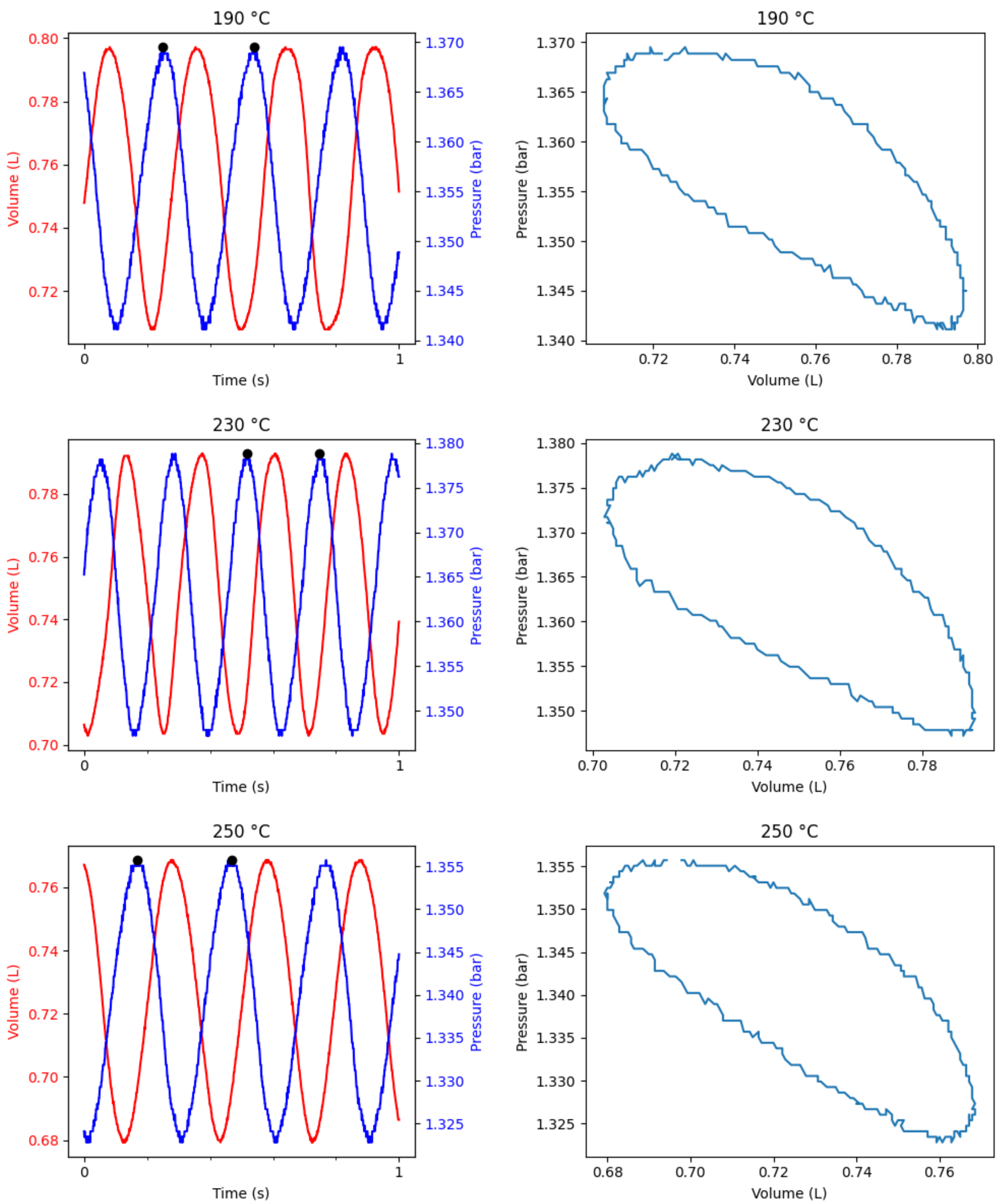

Figure 2: Volumes (shown in red) and pressures (shown in blue) of the working fluid, plotted against time, at constant motion of the engine, are shown on the left for each temperature. The start and end of the first cycle is marked with black dots. The period in between these is used to plot the pressure against volume, which is shown on the right. In addition, the minimum and maximum pressures during the cycle are similarly marked and used for later calculations.

Table 4 shows relevant data for the calculation of the Stirling engine's efficiency values. Values for Q_1 are calculated using Equation 9, with V_1 and V_2 taken as the maximum and minimum volumes during the marked cycle in Figure 2. Since Q_1 is calculated during isothermic expansion, the higher volume is used as V_2. p_1 is thus taken as the pressure at the time at which V_1 is found. Values for Q_H are calculated using Equation 10, using a molar heat of combustion $\Delta_c H_m$(EtOH) of 1360 kJ/mol. Values for W_m are calculated using Equation 11, using a mass of 4.7 kg and a radius of 0.17 m for the flywheel. W_{pV} is calculated as the enclosed area in the p/V diagram, using a supplied excel sheet.

Since Q_H, unlike the other calculated values, is not determined for a single cycle, it needs to be normalized to the cycle duration as follows:

$$Q_{H,corrected} = Q_{H,calculated} \cdot \frac{\Delta t}{\Delta t_{start}} \tag{16}$$

where: $Q_{H,corrected}$ = Q_H for once cycle [J]
$Q_{H,calculated}$ = Q_H as calculated using Equation 9 [J]
Δt = Duration of one cycle [s]
Δt_{start} = Time difference between start of first and end of fourth cycle, as used in Equation 9 [s]

Table 4: Measured data during experiment runs of the efficiency determination part of the experiment, as well as thermodynamic values calculated from it

$T_{start,bottom}$ [°C]	**190**	**230**	**250**
$T_{start,top}$ [°C]	31.2	34.9	36.5
f_0 [Hz]	0.93	1.10	1.23
f_f [Hz]	1.48	1.79	1.95
Δt_{start} [s]	3.33	2.81	2.53
f_{const} [Hz]	3.41	4.35	3.36
Δt [s]	0.29	0.23	0.30
V_1 [m^3]	$7.08 \cdot 10^{-4}$	$7.03 \cdot 10^{-4}$	$7.03 \cdot 10^{-4}$
V_2 [m^3]	$7.97 \cdot 10^{-4}$	$7.93 \cdot 10^{-4}$	$7.93 \cdot 10^{-4}$
p_1 [Pa]	$1.37 \cdot 10^5$	$1.37 \cdot 10^5$	$1.35 \cdot 10^5$
Q_1 [J]	11.48	11.62	11.45
Q_H [J]	105.96	83.18	107.77
W_m [J]	0.0391	0.0533	0.0891
W_{pV} [J]	0.123	0.138	0.137

From the data compiled in Table 4, using Equations 6, 7 and 8, the resulting efficiency values can then be caluclated:

Table 5: Efficiency values determined from measurement of the Stirling engine

T_{start} [°C]	**190**	**230**	**250**
η_H	0.108	0.140	0.106
η_m	0.319	0.386	0.651
η	$3.69 \cdot 10^{-4}$	$6.41 \cdot 10^{-4}$	$8.27 \cdot 10^{-4}$

4 Discussion of Results and Uncertainty of Measurements

4.1 Reversibility of the Stirling Engine

As the data in Table 2 confirms, the Stirling cycle is thermodynamically reversible. As was expected, in the clockwise experiment, heat was taken from the bottom end of the engine and moved to the top, while in the counterclockwise experiment, the opposite effect was observed.

While qualitatively, the experiment shows results that align with expectations, the quantitative measurements require closer scrutiny due to the following factors:

1. Due to the low precision of the taken temperature measurements (especially for the bottom of the engine, where the resolution of the thermometer was only 1 °C), only very small changes in temperature during the experiment (with measurements often only changing by a single unit on the respective scale), the temperature differences shown in Table 2 could deviate substantially from those which were observed. This issue could be counteracted either by using eqipment which can measure the temperature at greater resolution, or by running the experiment for a longer time and/or at a higher frequency, allowing for a higher change in temperature at both ends of the engine.

2. Since the engine is not perfectly insulated, its components as well as the surrounding air will take up or give off heat to or from the engine. Since the water at the top of the piston has a high heat capacity in comparison to air or steel and is present in considerable amounts, it can act as a reservoir, meaning that it will form a thermal equilibrium with its surroundings rather slowly. In comparison however, the air at the bottom of the engine, which is used to calculate the heat added or removed during the experiment, has a lower heat capacity as well as a lower mass than the water, it will much more quickly gain or lose heat. This can be seen in Table 2, where, despite a bigger change in temperature, the calculated heat values are two orders of magnitude lower than those at the top of the engine.

3. While attention was paid towards keeping a constant frequency while spinning the flywheel, as well as running the experiment for precisely one minute, it is still to be expected that the exact values will deviate between the two experiments. While important to keep in mind, these deviations are assumed to be much less significant than the two previously mentioned sources for inaccuracies.

4.2 Efficiency of the Stirling Engine

Table 5, shows a heat transfer efficiency η_H of about 10-15 %. This states the portion of heat from the combustion of ethanol which is used by the engine and converted into work. The mechanical efficiency η_m in turn shows that between 32 % and 65 % of the work produced by the working gas is transferred to the flywheel. Finally, the overall efficiency values η show that only between 0.037 % and 0.083 % of the produced heat are transferred into usable work.

While η_H does not show a clear trend with temperature[4] , η_m and especially η show increased values with a higher starting temperature, which matches with expectations, since the theoretical efficiency η_{Carnot} also increases with a higher temperature of the hot reservoir.

Just like with the prevous experiment, several possible sources of uncertainty were identified for the determined values:

1. Since the heat source was unable to supply enough heat to keep the engine at a constant temperature, and the measurements for constant engine motion could only be taken after exporting those for the start of an experiment from the oscilloscope to the USB-drive, with time, less heat was available to the engine at its bottom end. Additionally, since the water at its top was only cooled by the surrounding air, it heated up over time, meaning less heat could be transferred from the working gas to the cold reservoir, further lowering the engine's efficiency. As such, values calculated from data recorded at the end of the respective runs will tend towards similar values (this is especially apparent for Q_1 but also for W_{pV}).

2. In order to counteract the previously stated issues, measurements for constant engine motion were taken as soon as possible after exporting the starting values. Thus, the time between start and constant value measurements almost certianly differed between experiments.

3. Since there was no barometer available at the time of experiment, the ambient pressure used in Equation 15 was arbitrarily chosen as 1 atm (1.013 bar).

[4] In fact, the value at 230 °C is higher than that at the other two temperatures. Since η_H is a measure of how much heat is lost to the surroundings between the flame and the engine body, it is assumed that its value does not or only slightly change with temperature, meaning that the observed variation could have arisen out of other measurement or calculation uncertainties.