Protocol
W2 - Stirling Engine

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Contents

1 Objective

In this exercise, a Stirling engine is studied in order to determine some of its thermodynamic properties. Firstly, the engine is run manually as a heat pump, in order to assess the reversibility of the Stirling cycle through temperature measurements at the warm and cold sinks. Secondly, the engine is run by supplying heat through combustion of Ethanol as a fuel. Through measurements of temperature, pressure and volume, a p/V diagram is created and different efficiency values of the process are calculated.

2 Theory

A Stirling engine is a thermodynamic cycle, in which a working fluid (in the case of this experiment air) undergoes four steps:

1. An isothermal (no change in temperature) expansion of the air. Heat is taken up by the system from a hot reservoir, while work is given off to a flywheel.
2. An isochoric (no change in volume) cooling of the air. No work is done by or put into the system.
3. An isothermal contraction of the air. Heat is given off to a cold reservoir, and work is put into the system by the flywheel.
4. An isochoric heating of the air. No work is done by or put into the system.

2.1 Total Work in a Stirling Cycle

The first law of thermodynamics states, that the change of the internal energy of a system can be expressed as a sum of the heat which is added to or removed from the it, as well as the mechanical work, which is done by or put into it.

$$\mathrm{d}U = \delta Q + \delta W = \delta Q - p\,\mathrm{d}V \tag{1}$$

where: U = Internal energy of the system [kJ]
Q = Heat added to or removed from the system [kJ]
W = Heat added to or done by the system [kJ]
p = Pressure of the system [Pa]
V = Volume of the system [Pa]

The Stirling cycle is made up of four steps, alternating between isothermal and isochoric. During the isothermal steps of the process, the internal energy U of the System does not change, meaning $\delta - Q = W = -p\,\mathrm{d}V$. By integrating all terms and using the ideal gas law $pV = nRT$, to substitute p, the work which is carried out during an isothermal state change can be calculated:

$$-Q = W = -nRT \cdot ln\frac{V_2}{V_1} \tag{2}$$

where: n = Amount of substance [mol]
R = Ideal gas constant = 8.3145 [J mol^{-1} K^{-1}]
T = Temperature of the system [K]
V_2 = Volume at the end of isothermal state change [m^3]
V_1 = Volume at the start of isothermal state change [m^3]

During the isochoric steps, $\mathrm{d}V$ is equal to zero, meaning no work is carried out. As such the change of internal energy is equal to the heat which is added to or removed from the system.

$$\Delta U = Q \tag{3}$$

Since there is no change in Volume and thus no work during the isochoric steps, the total work which is carried out by the system during the Stirling cycle can be calculated by adding the work of both isothermal steps.

$$W_{pV} = -nRT_1 \cdot \ln\frac{V_2}{V_1} - nRT_2 \cdot \ln\frac{V_1}{V_2} = nR(T2 - T1) \cdot \ln\frac{V_2}{V_1} \tag{4}$$

This work is equal to the enclosed area within the p/V diagram of the cycle.

2.2 Efficiency of the Stirling Cycle

It is known that the idealized Stirling cycle has a thermal efficiency equal to that of the idealized Carnot cycle.

$$\eta = \frac{W_{pV}}{Q_1} = \frac{T_1 - T_2}{T_1} \tag{5}$$

In a real-world Stirling engine however, this efficiency is not reached, due to the overlap of the cycle's four steps with each other, along with other losses, such as friction or during heat transfer. As such, other efficiency values are used to assess the quality of the real-world engine.

The first value of interest is the efficiency of heat transfer between the heat source (in this case a small lamp, fuelled by Ethanol) and the hot reservoir of the Stirling engine.

$$\eta_H = \frac{Q_1}{Q_H} \tag{6}$$

where: η_H = Efficiency of heat transfer between heat source and hot reservoir
Q_1 = Heat received at the hot reservoir [kJ]
Q_H = Heat given off by the heat source [kJ]

Secondly, the mechanical efficiency, which relates the amount of usable work received at the flywheel to the work which is carried out by the system.

$$\eta_m = \frac{W_m}{W_{pV}} \tag{7}$$

where: η_m = Mechanical efficiency of the engine
W_m = Usable work received at the flywheel [kJ]
W_{pV} = Work given off by the engine [kJ]

Thirdly, the total efficiency, which relates the amount of usable work received at the flywheel to the heat which is given off by the heat source.

$$\eta = \frac{W_m}{Q_H} \tag{8}$$

where: η = Total efficiency

The values used in these calculations can be determined as follows:

1. Q_1 is determined from Equation 2 by applying the ideal gas law $pV = nRT$:

$$Q_1 = p_1 V_1 \cdot ln(\frac{V_2}{V_1}) \tag{9}$$

where: p_1 = Pressure at the start of isothermal expansion [Pa]
V_1 = Volume at the start of isothermal expansion [m^3]
V_2 = Volume at the end of isothermal expansion [m^3]

2. Q_H is determined by multiplying the molar heat of combustion of the used fuel with the amount of fuel used, which in turn is calculated by dividing the combustion rate of the burner with the molar mass of the fuel.

$$Q_H = \Delta_c H_m \cdot m_{EtOH} \cdot \frac{1}{M_{EtOH}} \tag{10}$$

where: $\Delta_c H_m$ = Molar heat of combustion of burned fuel [kJ/mol]
m_{EtOH} = Mass of burned fuel [g]
M_{EtOH} = Molar mass of burned fuel [g/mol]

3. W_m is determined by calculating the work required to spin the flywheel up to its regular frequency from a standstill.

$$W_m = 4\pi^2 m r^2 (\frac{f_f - f_0}{\Delta t}) \tag{11}$$

where: m = Mass of flywheel [kg]
r = Radius of flywheel [m]
f_f = Frequency of the engine after several cycles [Hz]
f_0 = Frequency of the engine during the first cycle [Hz]
Δt = Time between the start of the cycle used to determine f_0 and end of the cycle used to determine f_f [s]

4. W_{pV} is determined by calculating the area enclosed in the p/V diagram.

3 Experiment and Results

3.1 Reversibility of the Stirling Engine

In order to assess the reversibility of the engine, a crank was attached to the flywheel, allowing for manual input of work into the system. The upper part of the engine (cold reservoir) was covered with 100 g of water, while the bottom part (hot reservoir) was left exposed to air. The crank was then spun by hand, first clockwise (the direction in which it would run during operation as a regular heat engine), then counterclockwise, for one minute each. Special attention was paid towards trying to keep a constant angular velocity while cranking the flywheel, both during individual runs as well as from one run to another. The temperatures on both sides of the engine were recorded before and after the runs, and are shown in Table 1.

	Clockwise		**Counter-Clockwise**	
	Top	**Bottom**	**Top**	**Bottom**
Start	25.2	28	25.2	28
End	25.2	27	25.0	29

Table 1: Recorded temperatures (in °C) before and after spinning the flywheel

These results show that, in it's usual configuration as a heat engine, heat is transferred from the bottom of the engine, where normally a heat source would be located, to the top of it. When run in reverse, the opposite phenomenon is observed, and heat is moved from the top of the engine to the bottom.

The change in heat on each side of the engine can be calculated using the specific heat capacities of the adjacent fluids, the mass of the fluid and the difference in temperature.

$$Q = m \cdot c \cdot \Delta T \tag{12}$$

where: Q = Heat added to or removed to the system [J]
m = Mass of the adjacent fluid [g]
ΔT = Change in temperature [°C]

However, since the bottom side of the engine is exposed to the surrounding atmosphere, the corresponding mass is very large, and as such the change in temperature on the bottom end of the engine would be expected to be negligible. The temperature change which is observed however, is due to the heat being taken up or given off by the metal body of the engine itself.

3.2 Efficiency Values of the Stirling Engine

In order to calculate the efficiency values of the engine, the crank was removed from the flywheel and replaced by a gear, which was connected to a pV-sensor via belt drive. The pV-sensor as well as the temperature-probes were in turn connected to a pVnT-measuring device. Temperature values at specific times in the experiment were obtained directly from the displays of this device, while pressure and volume data was measured over a continuous timespan using an oscilloscope connected to it. The engine frequency was determined at specific points of the experiment as the reciprocal value of the time of a single cycle, which was determined from the pressure and volume data.

In order to calculate Q_H as shown in Equation 10, the fuel consumption rate of the used lantern was determined by letting the flame burn for a specific duration and weighing the lantern before and afterwards.

$$x = \frac{\Delta m}{\Delta t} \tag{13}$$

where: k = Fuel consumption rate of the lantern [g/s]
Δm = Difference in mass of the lantern [g]
Δt = Time the lantern was lit for [s]

Table 2 shows the recorded mass differences and durations as well as the resulting fuel consumption rates, their average and standard deviation.

Δm [g]	Δt [s]	k [g/s]
0.78	60	0.013
0.7	60	0.012
0.7	60	0.013
0.76	60	0.013
	$\overline{x}$	0.012
	s	0.0006
	S	0.0009

Table 2: Starting temperatures and durations of the experiment runs as well as frequencies of the first and fourth cycle and at constant frequency

In the four runs of the experiment which were conducted - each at a different temperature - the lantern was put underneath the engine and ignited. Once the engine reached the desired temperature, the flywheel was put in motion by hand and the starting temperature was recorded. After the first few cycles, the recording of the oscilloscope was stopped and the data exported onto a USB-drive. Another recording was made and exported once the engine reached its full operating frequency and another temperature measurement was taken before the torch was extinguished and the engine came to a halt.

T_{start} [°C]	**190**	**210**	**230**	**250**
Duration [s]	82	256	65	141
f_0 [Hz]	x	x	x	x
f_f [Hz]	x	x	x	x
f_{const} [Hz]	x	x	x	x

Table 3: Starting temperatures and durations of the experiment runs as well as frequencies of the first and fourth cycle and at constant frequency