

# iSAGE: An Incremental Version of SAGE for Online Explanation on Data Streams

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# Collaboration



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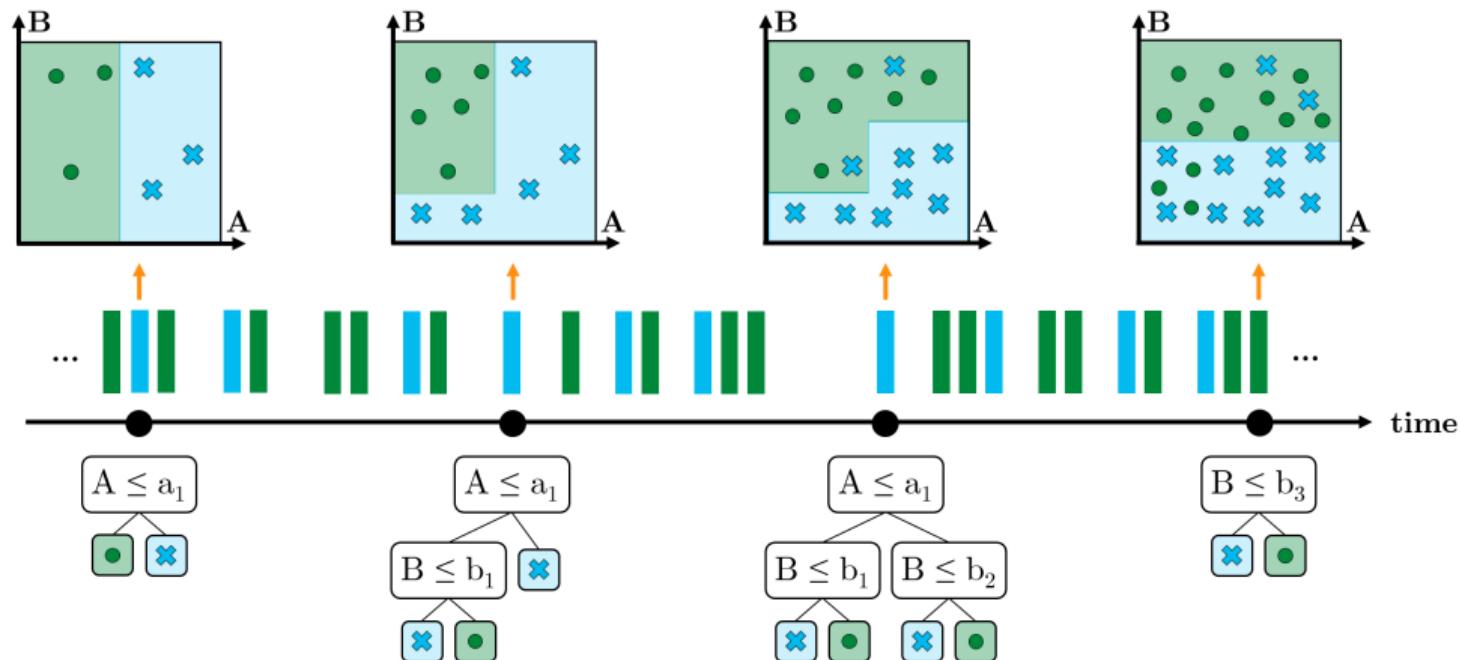


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Hüllermeier



\* denotes equal contribution

# Models in Flux: Incremental Learning from Data Streams



Various applications: Bifet and Gavaldà (2007), Gama et al. (2014), Davari et al. (2021), etc.

# Examples of Models in Flux



Fraud  
Detection



Sensor  
Networks



Automotive  
Industry

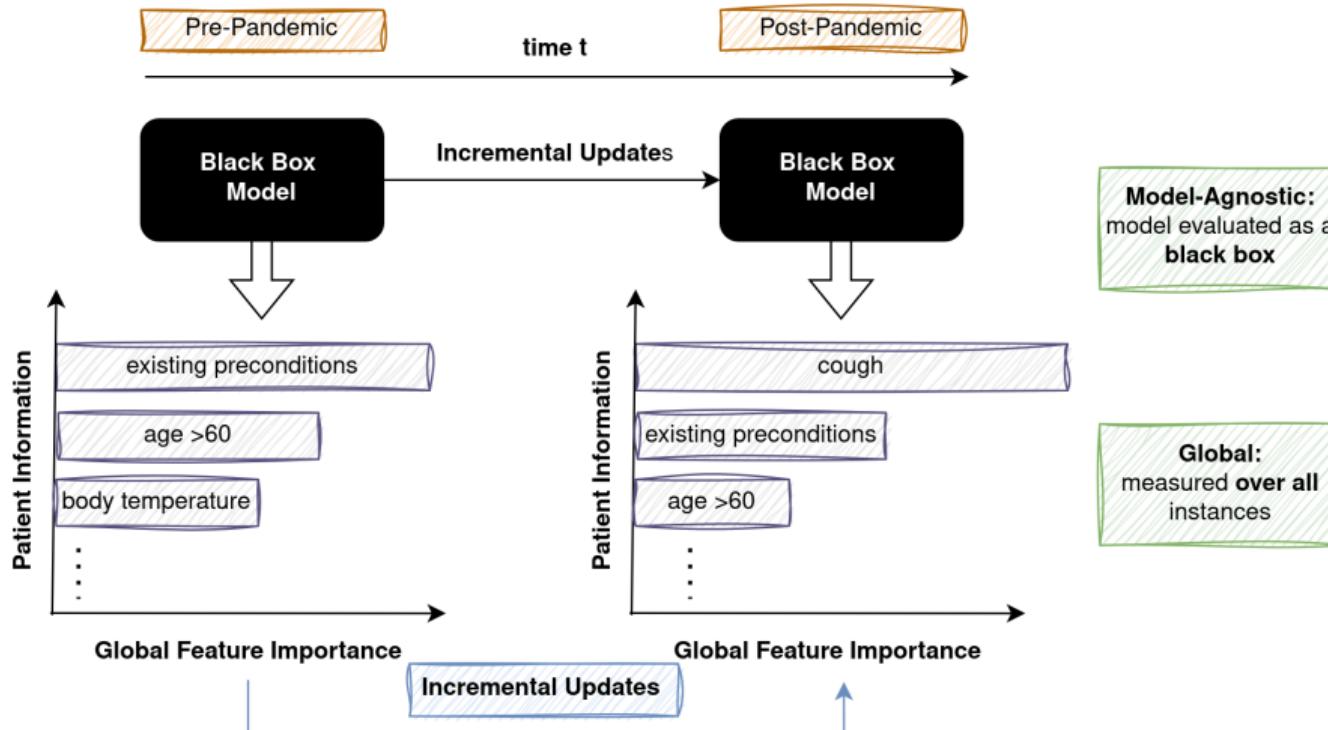


Predictive  
Maintenance

Images generated with Leonardo.ai.

# Model-Agnostic Explanations with Global Feature Importance

## Prediction of Hospital Admission



# SAGE: Global Feature Importance

$(X, Y) \sim \mathbb{P}$  data distribution on  $\mathcal{X} \times \mathcal{Y}$      $f : \mathcal{X} \rightarrow \mathcal{Y}$  black box model     $\ell : \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R}$  loss function

Explanation Goal: Difference between Model Loss *with* Features and *without*

$$\nu(D) := \underbrace{\mathbb{E}_Y [\ell(\bar{y}, Y)]}_{\text{no feature information}} - \underbrace{\mathbb{E}_{(X, Y)} [\ell(f(X), Y)]}_{\text{with feature information}} \quad \text{with mean prediction } \bar{y} := \mathbb{E}_X[f(X)]$$

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Requirement: Restricted Improvement in Loss given  $S \subset D$

$$\nu(S) := \mathbb{E}_Y[\ell(\bar{y}, Y)] - \mathbb{E}_{(X, Y)} [\ell(f(X, S), Y)] \quad \text{with restricted model } f(x, S)$$

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SAGE values  $\phi$  of feature  $i \in D$ , i.e. Shapley values (Shapley 1953)

$$\phi(i) := \sum_{S \subset D \setminus \{i\}} \frac{1}{d} \binom{d-1}{|S|}^{-1} [\nu(S \cup \{i\}) - \nu(S)]$$

# Restricted Model

→ Requirement: access model with **partial information** (without  $\bar{S} := D \setminus S$ )

## Interventional SAGE

$$f^{\text{int}}(x, S) := \mathbb{E} \left[ f(x^{(S)}, X^{(\bar{S})}) \right]$$



*“true to the model”*

## Observational SAGE

$$f^{\text{obs}}(x, S) := \mathbb{E} \left[ f(x^{(S)}, X^{(\bar{S})}) \mid X^{(S)} = x^{(S)} \right]$$



*“true to the data”*

sampling of replacements

$$\tilde{x}_m^{(\bar{S})}$$



computation in practice

$$\hat{f}(x, S) := \frac{1}{M} \sum_{m=1}^M f(x^{(S)}, \tilde{x}_m^{(\bar{S})})$$

Discussion: Janzing, Minorics, and Blöbaum (2020), Chen et al. (2020), Aas, Jullum, and Løland (2021)

# SAGE: Computation

SAGE estimator by Covert, Lundberg, and Lee (2020)

$$\hat{\phi}^{\text{SAGE}}(i) := \frac{1}{N} \sum_{n=1}^N \underbrace{\ell(\hat{f}(x_n, u_i^-(\pi_n)), y_n) - \ell(\hat{f}(x_n, u_i^+(\pi_n)), y_n)}_{\Delta_n(i)}$$

Illustration of Shapley Permutation Sampling by Castro, Gómez, and Tejada (2009)

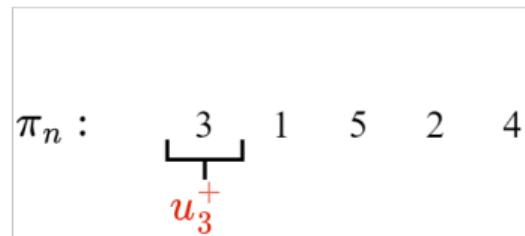
$$\pi_n : \quad 3 \quad 1 \quad 5 \quad 2 \quad 4$$

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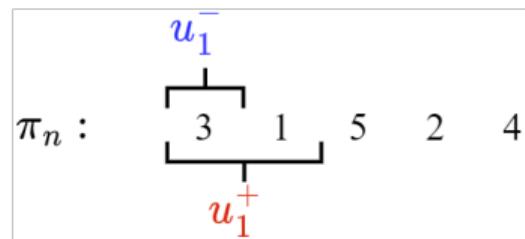
$$\Delta_n(3) = \ell(\hat{f}(x_n, \{\emptyset\}), y_n) - \ell(\hat{f}(x_n, \{3\}), y_n))$$

# SAGE: Computation

SAGE estimator by Covert, Lundberg, and Lee (2020)

$$\hat{\phi}^{\text{SAGE}}(i) := \frac{1}{N} \sum_{n=1}^N \underbrace{\ell(\hat{f}(x_n, u_i^-(\pi_n)), y_n) - \ell(\hat{f}(x_n, u_i^+(\pi_n)), y_n)}_{\Delta_n(i)}$$

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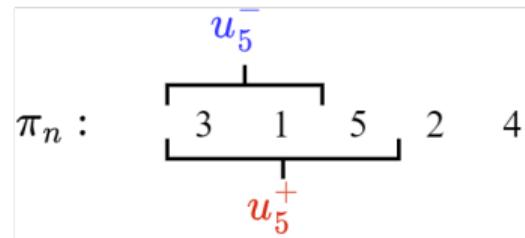
$$\Delta_n(1) = \ell(\hat{f}(x_n, \{3\}), y_n) - \ell(\hat{f}(x_n, \{3, 1\}), y_n))$$

# SAGE: Computation

SAGE estimator by Covert, Lundberg, and Lee (2020)

$$\hat{\phi}^{\text{SAGE}}(i) := \frac{1}{N} \sum_{n=1}^N \underbrace{\ell(\hat{f}(x_n, u_i^-(\pi_n)), y_n) - \ell(\hat{f}(x_n, u_i^+(\pi_n)), y_n)}_{\Delta_n(i)}$$

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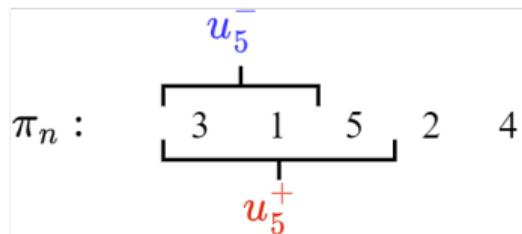
$$\Delta_n(5) = \ell(\hat{f}(x_n, \{3, 1\}), y_n) - \ell(\hat{f}(x_n, \{3, 1, 5\}), y_n))$$

# SAGE: Computation

SAGE estimator by Covert, Lundberg, and Lee (2020)

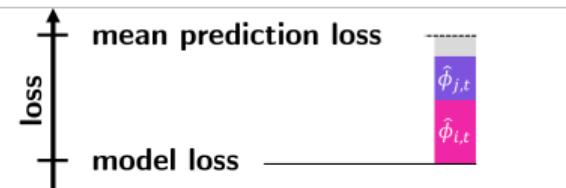
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Illustration of Shapley Permutation Sampling by Castro, Gómez, and Tejada (2009)



$$\Delta_n(5) = \ell(\hat{f}(x_n, \{3, 1\}), y_n) - \ell(\hat{f}(x_n, \{3, 1, 5\}), y_n)$$

SAGE values follow **efficiency** criterion:

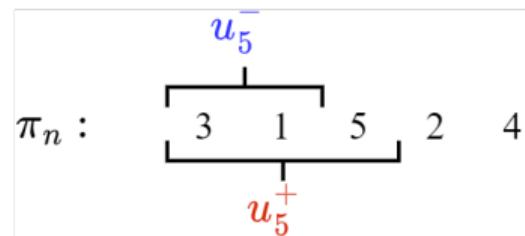


# SAGE: Computation

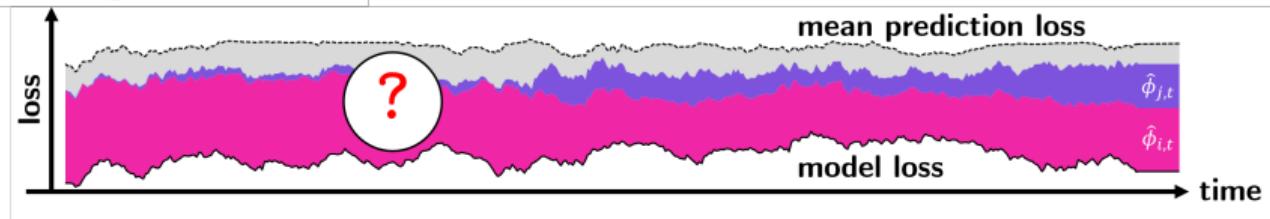
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Illustration of Shapley Permutation Sampling by Castro, Gómez, and Tejada (2009)



$$\Delta_n(5) = \ell(\hat{f}(x_n, \{3, 1\}), y_n) - \ell(\hat{f}(x_n, \{3, 1, 5\}), y_n)$$



# Incremental SAGE (iSAGE) for Explaining Models in Flux

## Online Learning on Data Streams

- unlimited data stream  $(x_0, y_0), \dots, (x_t, y_t), \dots$
- incrementally updated model  $f_{t+1} \leftarrow \text{IncrementalUpdate}(f_t, x_t, y_t)$

calculation at time  $t$

$$\Delta_t(i) := \ell(\hat{f}_t(x_t, u_i^-(\pi_t)), y_t) - \ell(\hat{f}_t(x_t, u_i^+(\pi_t)), y_t)$$

initial computation

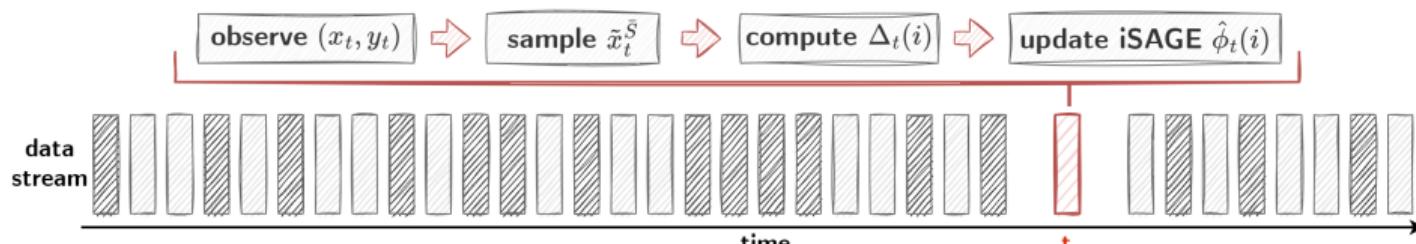
$$\hat{\phi}_{t_0-1}(i) := 0 \text{ for } t \geq t_0 > 0$$

incremental update to iSAGE

$$\text{iSAGE: } \hat{\phi}_t(i) = (1 - \alpha) \cdot \hat{\phi}_{t-1}(i) + \alpha \cdot \Delta_t(i)$$

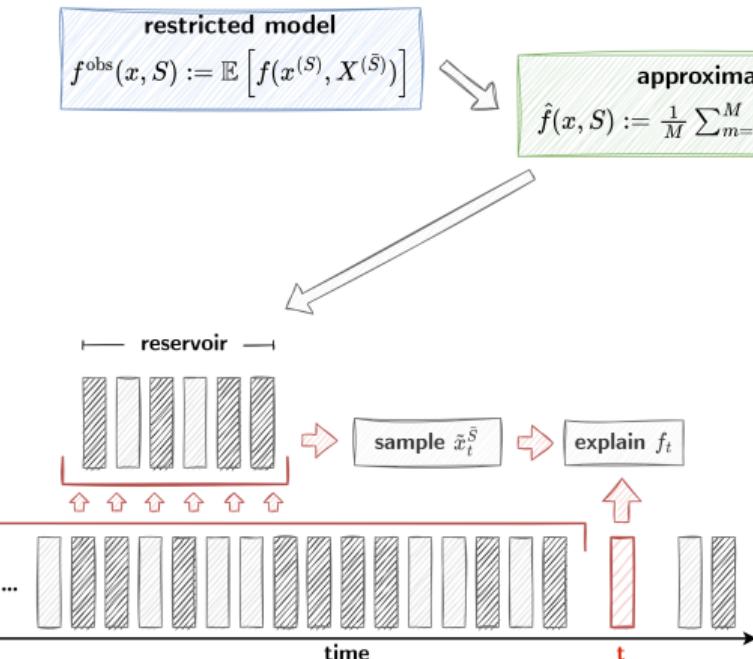
smoothing parameter

$$\alpha \in (0, 1)$$

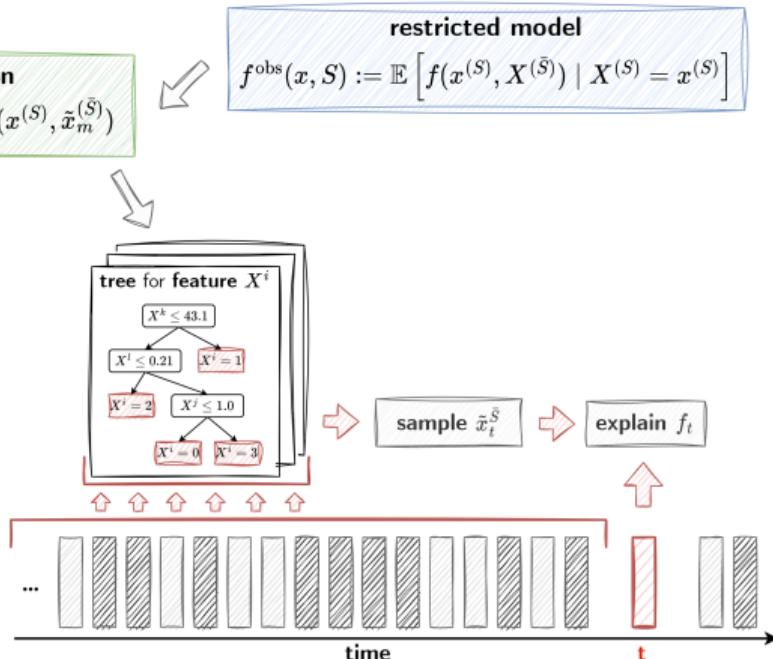


# Incremental Sampling Mechanisms

## Interventional iSAGE



## Observational iSAGE



Reservoir Sampling

Incremental Partition Trees

# Theoretical Guarantees

**Assumptions:** static model  $f_t \equiv f$  and data generating process  $(X_t, Y_t) \sim \mathbb{P}_t \equiv \mathbb{P}$

## Theorem (Convergence)

For iSAGE  $\hat{\phi}_t(i) \rightarrow \phi_t(i)$  for  $M \rightarrow \infty$  and  $t \rightarrow \infty$ .

## Theorem (Variance)

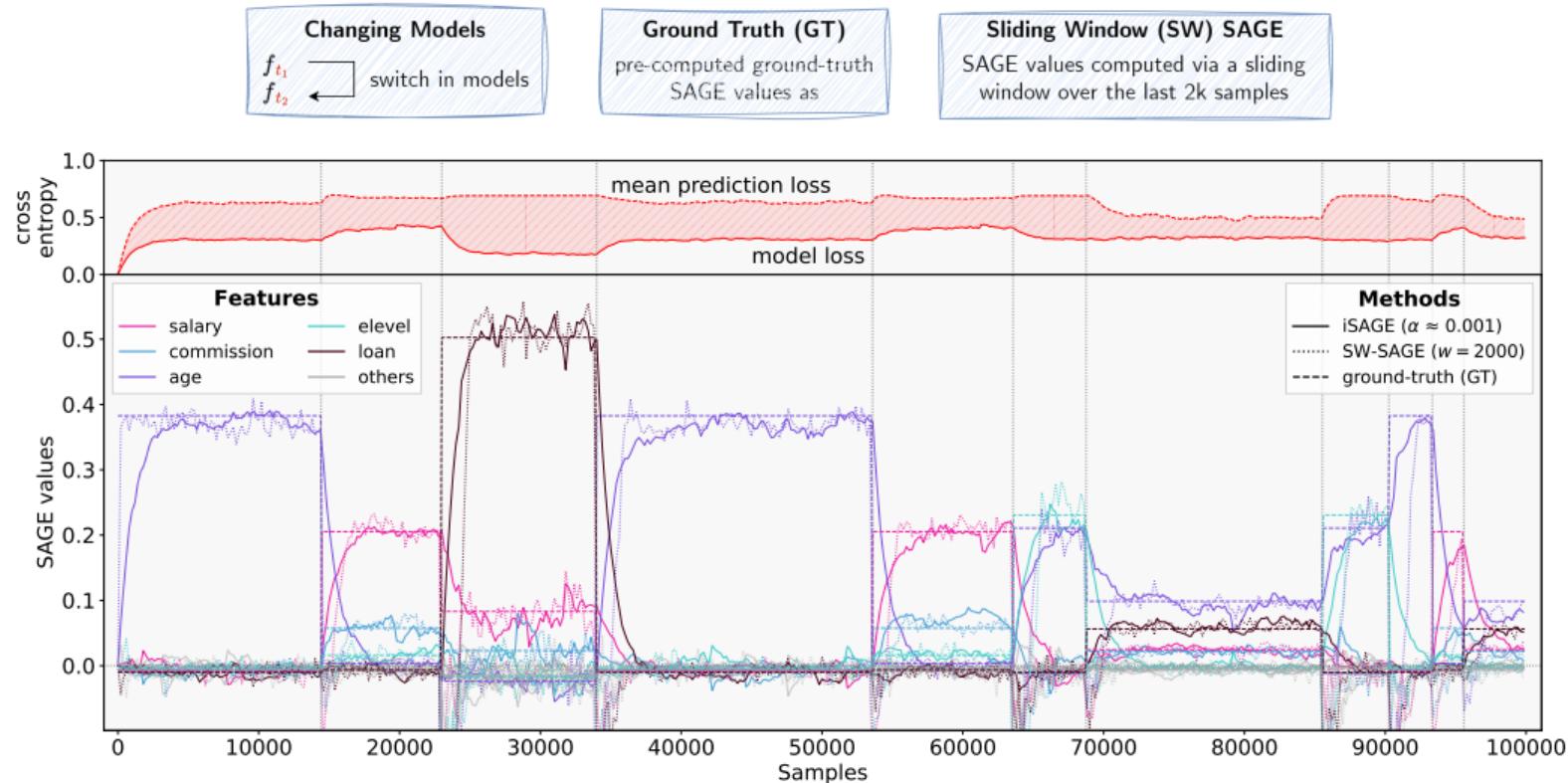
The variance of iSAGE is controlled by  $\alpha$ , i.e.  $\mathbb{V}[\hat{\phi}_t(i)] = \mathcal{O}(\alpha)$ .

## Theorem (Confidence Bounds)

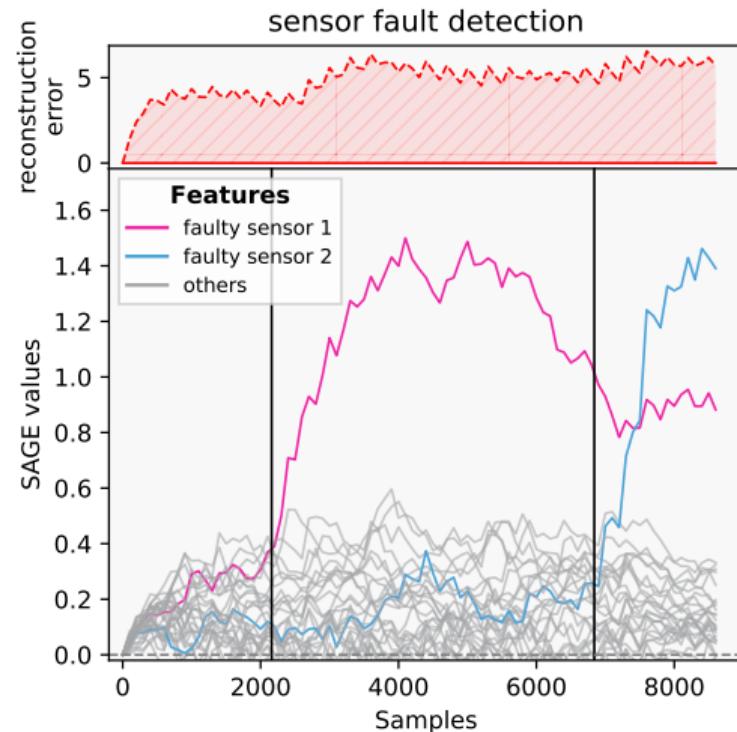
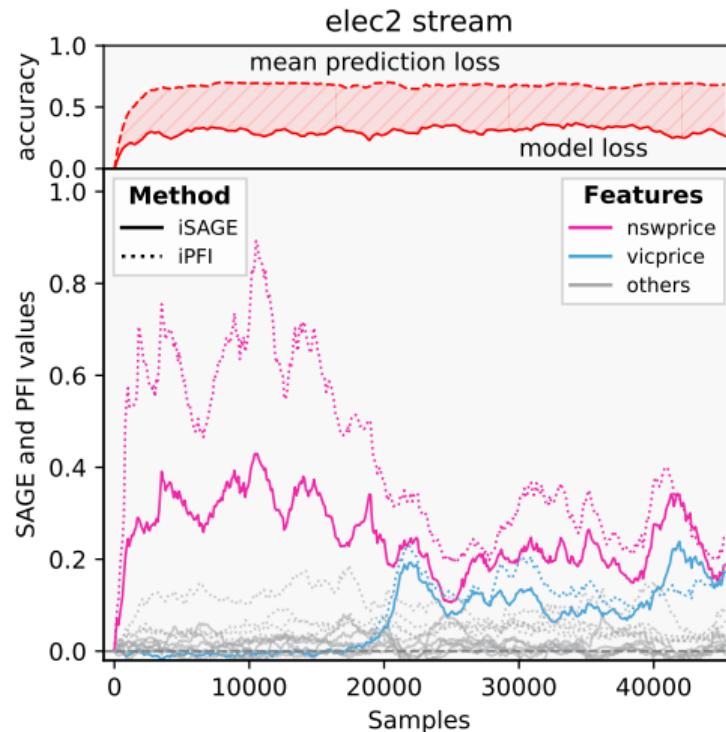
Given the SAGE estimator  $\hat{\phi}_t^{SAGE}(i)$  computed at time  $t$  over all previously observed data points, it holds for iSAGE with  $M \rightarrow \infty$ ,  $\alpha = \frac{1}{t}$  and every  $\epsilon > (1 - \alpha)^{t-t_0+1}$  that

$$\mathbb{P}\left(|\hat{\phi}_t(i) - \hat{\phi}_t^{SAGE}(i)| > \epsilon\right) = \mathcal{O}\left(\frac{1}{t}\right).$$

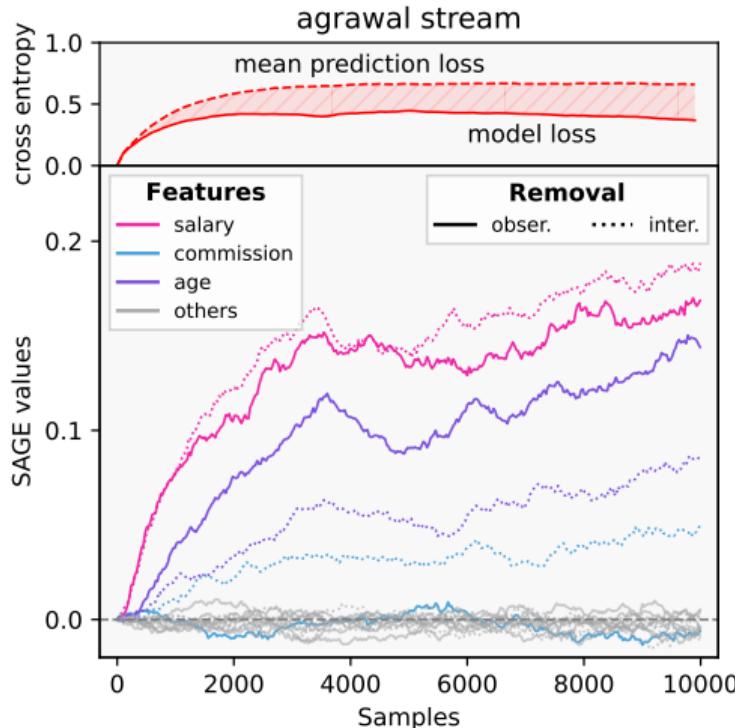
# iSAGE recovers Ground Truth SAGE Values for Models in Flux



# Example Applications: Concept Drift Detection



# Observational vs. Interventional iSAGE



## Setting:

- $X^{\text{com.}}$  depends on  $X^{\text{salary}}$
- knowledge about  $X^{\text{salary}}$  allows perfect reconstruction of  $X^{\text{com.}}$
- target depends indirectly on  $X^{\text{com.}}$

**observational and interventional iSAGE retrieve different FI scores**

- observational iSAGE shows that  $X^{\text{com.}}$  is not important
- interventional iSAGE shows that the model has learned to use  $X^{\text{com.}}$  (i.e. decision splits exist for  $X^{\text{com.}}$ )

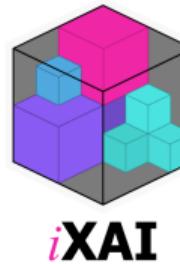
# The Road Ahead and Open Source Implementation

Towards Explaining Change.

- iSAGE is a **model-agnostic XAI** method to compute **global SAGE** values for ML models **in flux**.
- Other online XAI methods include **iPFI** (ECMLPKDD'23) and **iPDP** (xAI'23).

## Workshop Friday Afternoon Slot

- Time: 14:00-18:00
- Room: PoliTo Room 10i
- Title: *Explainable Artificial Intelligence: From Static to Dynamic*



[docs](#) [passing](#) [pypi](#) v0.1.3 [status](#) alpha [License](#) MIT

### Installation

```
pip install ixai
```

### Quickstart

```
>>> for (n, (x, y)) in enumerate(stream, start=1)
...     accuracy.update(y, model.predict_one(x))      # inference
...     incremental_pfi.explain_one(x, y)            # explaining
...     model.learn_one(x, y)                         # learning
```

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-  Covert, Ian, Scott M. Lundberg, and Su-In Lee (2020). "Understanding Global Feature Contributions With Additive Importance Measures". In: *Advances in Neural Information Processing Systems* 33: (NeurIPS 2020), pp. 17212–17223.

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-  Davari, Narjes et al. (2021). "Predictive Maintenance Based on Anomaly Detection Using Deep Learning for Air Production Unit in the Railway Industry". In: *8th IEEE International Conference on Data Science and Advanced Analytics (DSAA 2021)*. IEEE, pp. 1–10. DOI: 10.1109/DSAA53316.2021.9564181.
-  Gama, João et al. (2014). "A Survey on Concept Drift Adaptation". In: *ACM Comput. Surv.* 46.4, 44:1–44:37. DOI: 10.1145/2523813.
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-  Shapley, L. S. (1953). "A Value for n-Person Games". In: *Contributions to the Theory of Games (AM-28), Volume II*. Princeton University Press, pp. 307–318. ISBN: 9781400881970. DOI: 10.1515/9781400881970-018.

# Complete iSAGE Algorithm

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**Algorithm 1** Incremental SAGE (iSAGE)

---

**Require:** stream  $\{x_t, y_t\}_{t=1}^{\infty}$ , feature indices  $D = \{1, \dots, d\}$ , model  $f_t$ , loss function  $\ell$ , and inner samples  $m$

- 1: Initialize  $\hat{\phi}^1 \leftarrow 0, \hat{\phi}^2 \leftarrow 0, \dots, \hat{\phi}^d \leftarrow 0$ , and smoothed mean prediction  $y_{\emptyset} \leftarrow 0$
- 2: **for all**  $(x_t, y_t) \in$  stream **do**
- 3:   Sample  $\pi$ , a permutation of  $D$
- 4:    $S \leftarrow \emptyset$
- 5:    $y_{\emptyset} \leftarrow (1 - \alpha) \cdot y_{\emptyset} + \alpha \cdot f(x_t)$  {Update mean prediction}
- 6:    $\text{lossPrev} \leftarrow \ell(y_{\emptyset}, y_t)$  {Compute mean prediction loss}
- 7:   **for**  $j = 1$  to  $d$  **do** {Iterate over  $\pi$ }
- 8:      $S \leftarrow S \cup \{\pi[j]\}$
- 9:      $y \leftarrow 0$
- 10:    **for**  $k = 1$  to  $m$  **do** {Marginalize prediction with  $S$ }
- 11:     Sample  $x_k^{(S)} \sim \mathbb{Q}_t^{(x, S)}$  {interventional (Appendix, Algorithm 2) or observational (Appendix, Algorithm 3)}
- 12:      $y \leftarrow y + f_t(x_t^{(S)}, x_k^{(\bar{S})})$
- 13:    **end for**
- 14:     $\bar{y} \leftarrow \frac{y}{m}$
- 15:     $\text{loss} \leftarrow \ell(\bar{y}, y_t)$
- 16:     $\Delta \leftarrow \text{lossPrev} - \text{loss}$
- 17:     $\hat{\phi}^{\pi[j]} \leftarrow (1 - \alpha) \cdot \hat{\phi}^{\pi[j]} + \alpha \cdot \Delta$
- 18:     $\text{lossPrev} \leftarrow \text{loss}$
- 19:   **end for**
- 20: **end for**
- 21: **return**  $\phi^1, \phi^2, \dots, \phi^d$

---

# Explanation Procedure

## General Explanation Algorithm

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**Algorithm 6** Incremental explanation procedure

---

**Require:** stream  $\{x_t, y_t\}_{t=1}^{\infty}$ , model  $f(\cdot)$ , loss function  $\mathcal{L}(\cdot)$

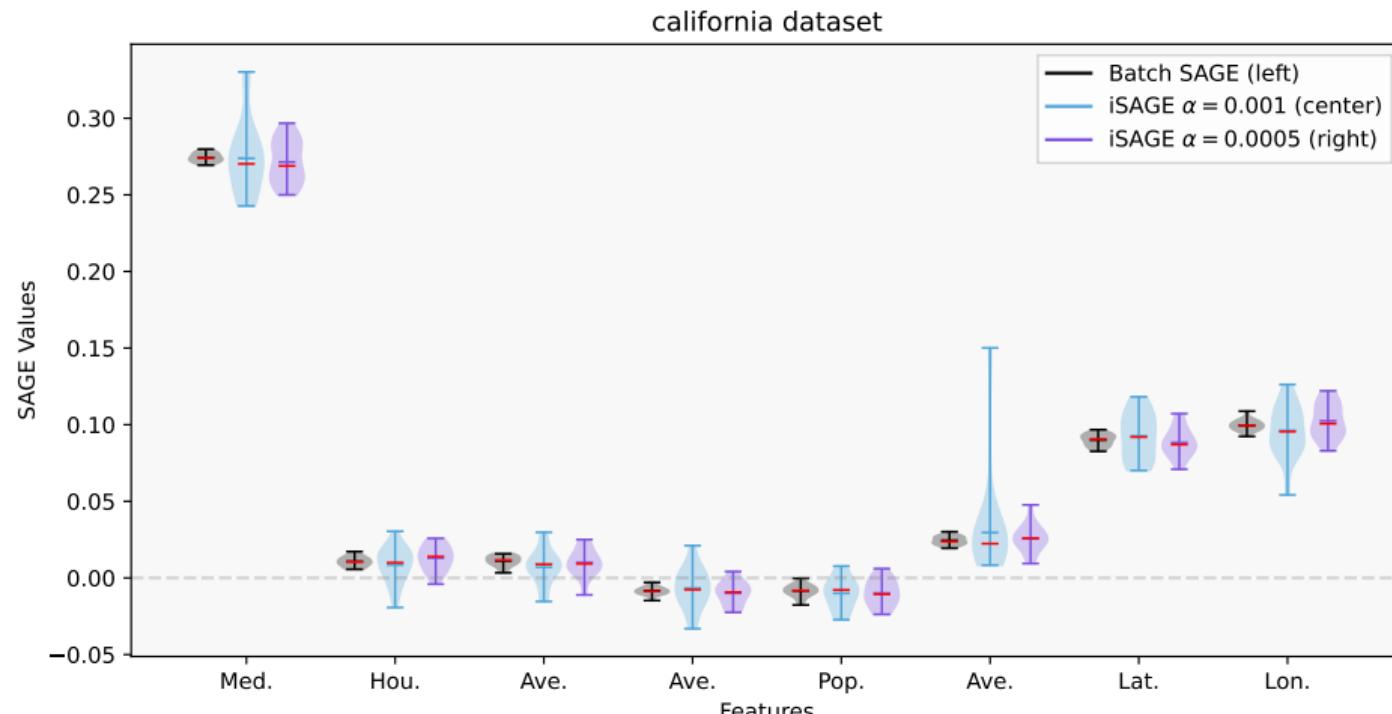
```
1: for all  $(x_t, y_t) \in$  stream do
2:    $\hat{y}_t \leftarrow f_t(x_t)$ 
3:    $\hat{\phi}_t \leftarrow \text{explain\_one}(x_t, y_t)$ 
4:    $f_{t+1} \leftarrow \text{learn\_one}(\mathcal{L}(\hat{y}_t, y_t))$ 
5: end for
```

---

- similarly to the **sequential** training: models are explained sequentially.
- data points are used first for explanations (model has not seen the observation, line 3) and then the model is allowed to use it for training (line 4)

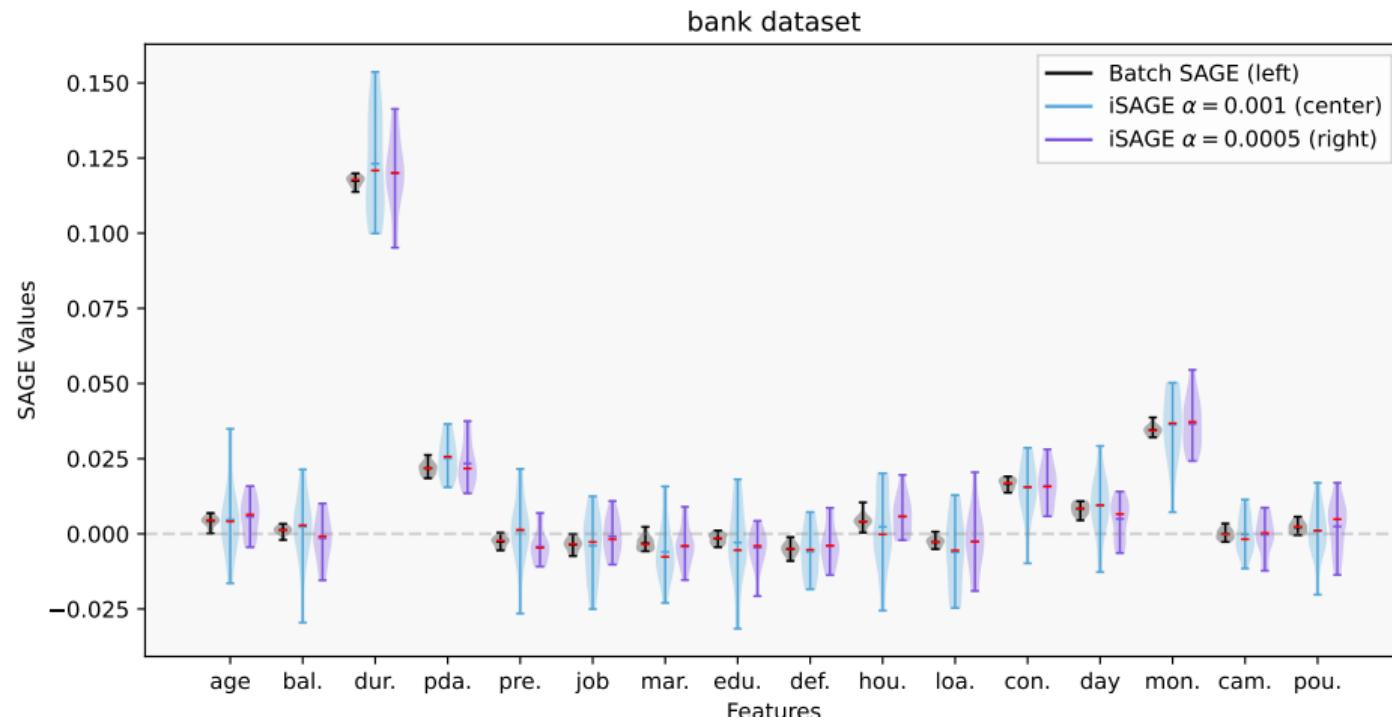
# iSAGE retrieves SAGE values in Static Learning Environments

## California Housing Dataset (Regression) for a Neural Network

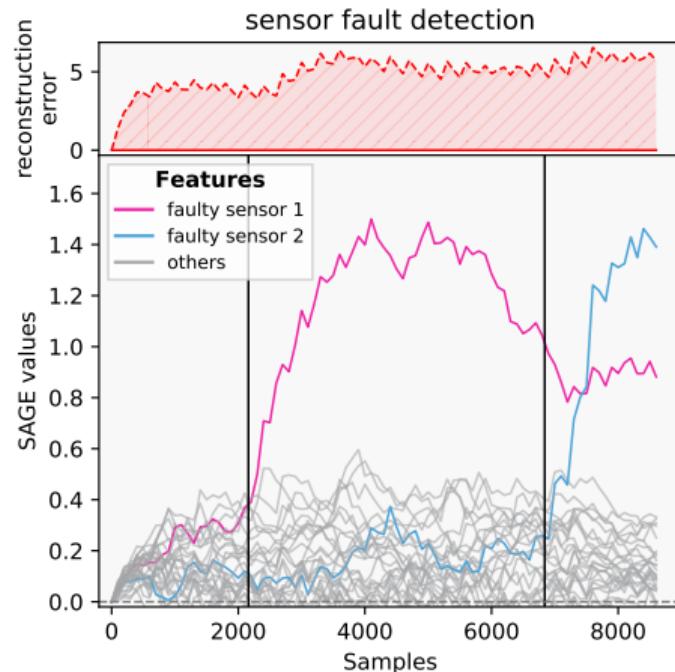


# iSAGE retrieves SAGE values in Static Learning Environments

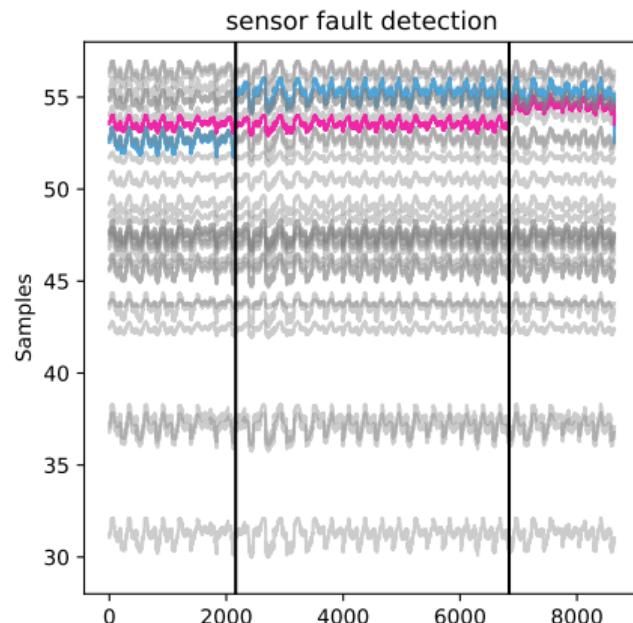
## Bank Dataset (Classification) for a Neural Network



# Example Applications: Concept Drift Detection

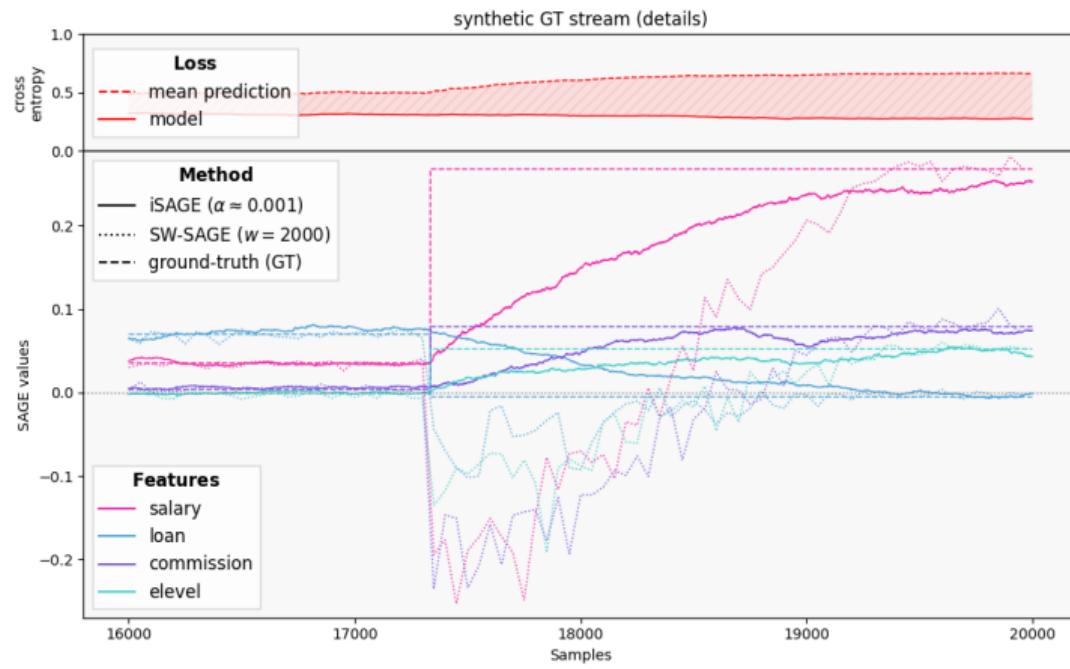


Concept Drift Detection



Raw Sensor Data

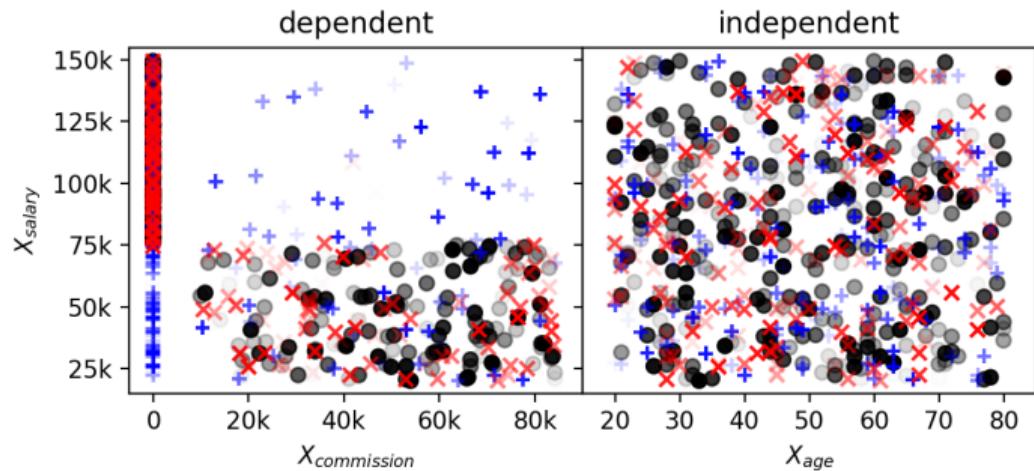
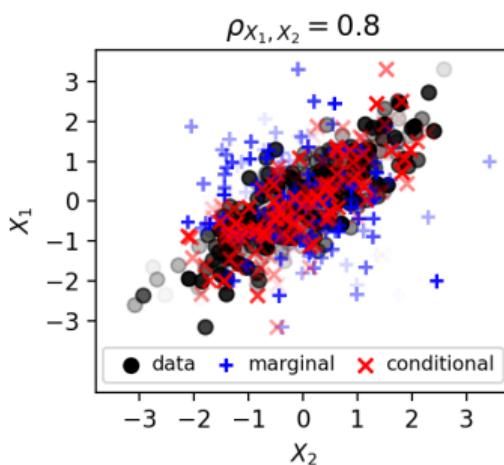
# The Problem with Sliding Window (SW) Explanations



detail view of a ground-truth (GT) data stream

- change point: 17 335
  - before: iSAGE and SW-SAGE approximate the GT well
  - after: SW-SAGE recovers more slowly with a high approximation error
- ! after the change in the model **no previous information is useful anymore**

# Observational and Interventional Feature Removal



## Feature Distribution:

- $X_1 \sim \mathcal{N}(0, 1)$ ,  $X_2 \sim \mathcal{N}(0, 1)$ ,  $X_{\text{age}} \sim \text{unif}([20, 80])$
- $X_{\text{salary}} \sim \text{unif}([20k, 150k])$ , and  $X_{\text{commission}} = 1(X_{\text{salary}} \leq 75k) \cdot Q$  with  $Q \sim \text{unif}([10k, 75k])$