

On the Design of Quantum Communication Systems with non-Gaussian States

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Postulate 1: State Representation

The state of an isolated quantum system is represented by a complex unitary vector in an Hilbert space:

$$|\psi\rangle \in \mathcal{H}$$

Postulate 2: Observables

Every observables of the system is represented by an Hermitian operator acting on the state space:

$$\mathcal{M} : \mathcal{H} \rightarrow \mathcal{H}$$

Postulate 3: Born's Rule

The probability to get the measurement λ_i from the observable \mathcal{M} in the system in state $|\psi\rangle$ is:

$$\mathbb{P}(\lambda_i) = \langle\psi|\mathcal{P}_i|\psi\rangle$$

Postulate 4: Wavefunction Collapse

The state after measurement of λ_i is $\mathcal{P}_i|\psi\rangle$ (with the necessary normalization):

$$|\psi'\rangle = \frac{\mathcal{P}_i|\psi\rangle}{\langle\psi|\mathcal{P}_i|\psi\rangle}.$$

Postulate 5: Time Evolution

The time evolution of an isolated quantum system is given by an unitary operator \mathcal{U} :

$$|\psi(t)\rangle = \mathcal{U}(t_0, t) |\psi(t_0)\rangle .$$

Postulate 6: Composite System

The state space of a system composed of \mathcal{H}_1 and \mathcal{H}_2 is given by

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 .$$

Classical

$$\mathbf{e}(\mathbf{r}, t) = - \sum_n p_n(t) \mathbf{u}_n(\mathbf{r})$$

$$\mathbf{h}(\mathbf{r}, t) = \sum_n q_n(t) \nabla \times \mathbf{u}_n(\mathbf{r})$$

$$\text{with } \mathbf{u}_n(\mathbf{r}) = \mathbf{u}_{n0} e^{i\mathbf{k}_n \cdot \mathbf{r}}$$

$$H_n = \hbar \omega_n |a_n(t)|^2$$

$$a_n(t) = \frac{\omega_n q_n(t) + i p_n(t)}{\sqrt{2\hbar\omega_n}}$$

Quantized

$$p_n(t) \implies P_n(t) : \mathcal{H} \rightarrow \mathcal{H}$$

$$q_n(t) \implies Q_n(t) : \mathcal{H} \rightarrow \mathcal{H}$$

$$[Q_n, P_m] = i\hbar \delta_{n,m} I$$

$$[Q_n, Q_m] = 0$$

$$[P_n, P_m] = 0$$

$$H_n = \hbar \omega_n \mathbf{A}_n^\dagger \mathbf{A}_n$$

$$\mathbf{A}_n(t) = \frac{\omega_n \mathbf{Q}_n(t) - i \mathbf{P}_n(t)}{\sqrt{2\hbar\omega_n}}$$

Fock State

The Fock state $|n\rangle$ represents the quantum state with exactly n photons. It is defined as:

$$\mathbf{N} |n\rangle = n |n\rangle \quad \text{with } \mathbf{N} = \mathbf{A}^\dagger \mathbf{A}$$

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Fock Representation

Every quantum state Ξ can be expressed as

$$\Xi = \sum_{n,m} c_{n,m} |n\rangle \langle m| \quad \text{with } c_{n,m} = \langle n | \Xi | m \rangle$$

Quantum Mechanics Abstract

Wigner Function



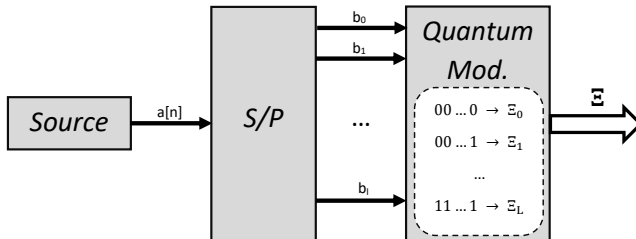
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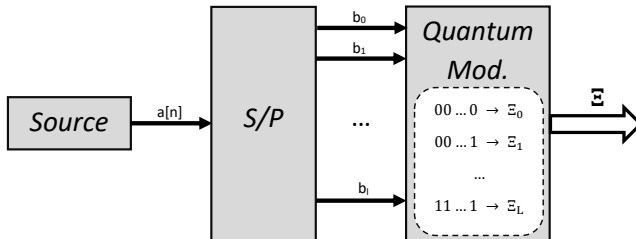
Quantum Mechanics Abstract

Some important states



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OOK

$$\Xi_0 = \Xi_{th}$$

$$\Xi_1 = \Xi_{th}(\mu)$$

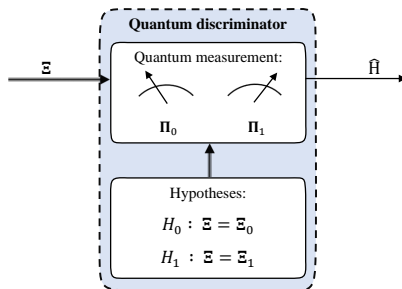
BPSK

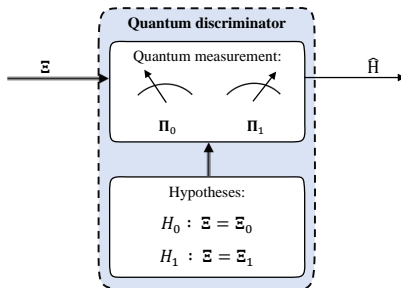
$$\Xi_0 = \Xi_{th}(-\mu)$$

$$\Xi_1 = \Xi_{th}(\mu)$$

Quantum Communication

Quantum Discriminator





Distribution error probability

$$P_e = 1 - (p_0 \text{tr}\{\Xi_0 \Pi_0\} + p_1 \text{tr}\{\Xi_1 \Pi_1\})$$

Helstrom Bound: Minimum distribution error probability

$$\check{P}_e = \frac{1}{2} (1 - \|p_1 \Xi_1 - p_0 \Xi_0\|_1)$$

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$$\check{P}_e = \frac{1}{2} (1 - \|p_1 \Xi_1 - p_0 \Xi_0\|_1)$$

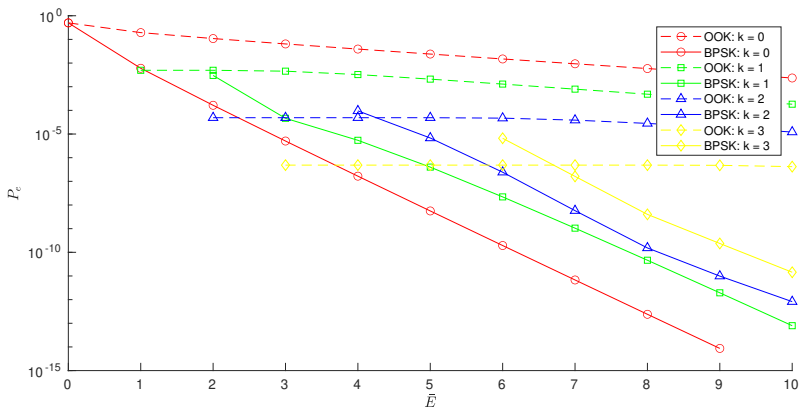
MDEP for pure states

$$\check{P}_e = \frac{1}{2} \left(1 - \sqrt{1 - 4p_0 p_1 |\langle \psi_0 | \psi_1 \rangle|^2} \right)$$

Discrimination of Photon Added States



Noisy PACS discrimination



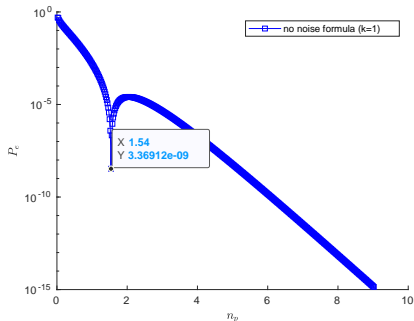
MDEP comparison of a QOOK and QBPSK systems with PACS in terms of the mean energy of the system \bar{E} .

$$N = 45; \bar{n} = 10^{-2}; p_0 = p_1 = 1/2$$

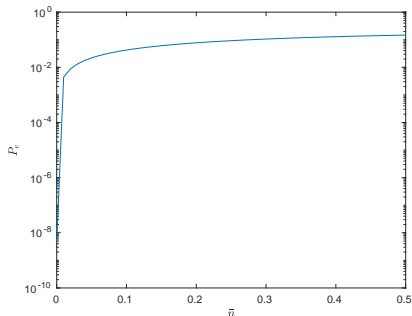
Discrimination of Photon Added States



Noise effect



MDEP of a QBPSK with PACSs, without noise. $k = 1$.

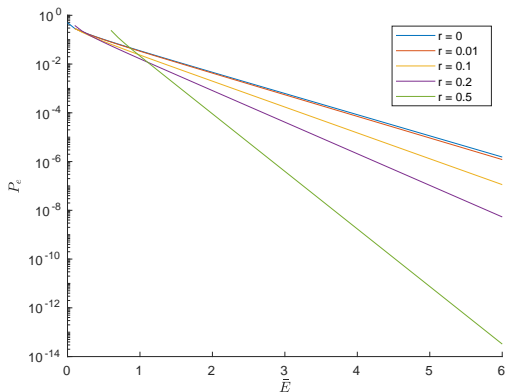


MDEP for $\mu = 0.54$ (value of the zero) in terms of \bar{n} .

Discrimination of Photon Added States



Squeezed States discrimination: BPSK

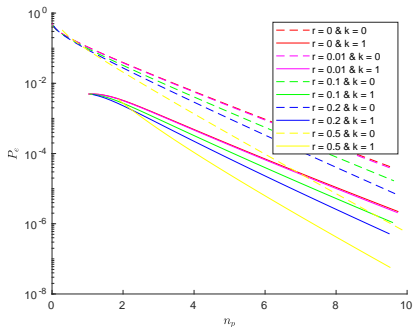


MDEP of a QBPSK system with squeezed states in terms of the mean energy of the system \bar{E} .

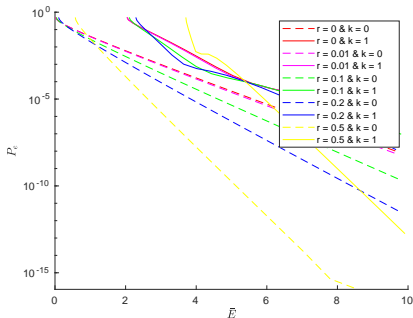
$$N = 30; \bar{n} = 0; p_0 = p_1 = 1/2$$

Discrimination of Photon Added States

Noisy PASS discrimination



MDEP of a QOOK system with PASS in terms of the mean photon number n_p .
 $N = 30$; $\bar{n} = 10^{-2}$; $\theta = \pi$; $p_0 = p_1 = 1/2$



MDEP of a QBPSK system with PASS in terms of the mean energy of the system \bar{E} .
 $N = 30$; $\bar{n} = 10^{-2}$; $\theta = \pi$; $p_0 = p_1 = 1/2$