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On the Design of Quantum Communication Systems with non-Gaussian States

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Contents

Chapter 1

Quantum Mechanics Abstract

In this chapter, a bief overview of quantum mechanics postulates and of the notation used in this thesis is given. The target of that is to explain to the reader the essential concept [...]

1.1 Postulates

Like every phisics theory, quantum mechanics is builded from few essential postulates. In this section are briefly introduced the six Dirac-Von Newman postulates of Quantum Mechanics [**quantumMec_Dirac**][**quantumMec_Neumann**].

1.1.1 First Postulate

Postulate 1 (State Representation) *The state of an isolated quantum system is represented by a complex unitary vector in an Hilbert space:*

$$|\psi\rangle \in \mathcal{H}$$

The space of possible states of the system is called state space and it is a separable complex Hilbert space.

Observation *Differently from the classical physics, in quantum mechanics the concept of state of system is introduced. In classical mechanics a system is described by his observables, like position or four-wheeled.*

1.1.2 Second Postulate

Postulate 2 (Observables) *Every observables of the system is represented by an Hermitian operator acting on the state space:*

$$\mathcal{M} : \mathcal{H} \to \mathcal{H}$$

The outcomes of the measurement can only be one of the eigenvalue of the operator $\mathcal{M}$.

Observation *The possible outcomes of the measurement are real number because $\mathcal{M}$ is self-andjoint.*

1.1.3 Third Postulate

Postulate 3 (Born's Rule) *The probability to get the measurement λ_i from the observable $\mathcal{M}$ in the system in state $|\psi\rangle$ is:*

$$\mathbb{P}(\lambda_i) = \langle\psi| \mathcal{P}_i |\psi\rangle$$

where $\langle\psi|$ is the correspondent vector of $|\psi\rangle$ in the dual space of $\mathcal{H}$ and where $\mathcal{P}_i$ is the projection operator of λ_i in the correspondent space.

1.1.4 Fourth Postulate

Postulate 4 (Wavefunction Collapse) *The state after measurement of λ_i is $\mathcal{P}_i |\psi\rangle$ (with the necessary normalization):*

$$|\psi'\rangle = \frac{\mathcal{P}_i |\psi\rangle}{\langle\psi| \mathcal{P}_i |\psi\rangle}.$$

1.1.5 Fifth Postulate

Postulate 5 (Time Evolution) *The time evolution of an isolated quantum system is given by an unitary operator $\mathcal{U}$:*

$$|\psi(t)\rangle = \mathcal{U}(t_0, t) |\psi(t_0)\rangle .$$

Observation (Time dependent Shrodinger Equation) *From postulate 5, is possible to found the time dependent Shrodinger Equation:*

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

where $H(t)$ is the Hemiltonian matrix.

1.1.6 Sixth Postulate

Postulate 6 (Composite System) *The state space of a system composite from $\mathcal{H}_1$ and $\mathcal{H}_2$ is given by*

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2.$$

1.2 Combining Systems

The last postulate 6 has very important consequences for composite system. It is possible to descrive two tipe of combined systems:

Definition 1 (Product States) *A state $|\psi\rangle \in \mathcal{H}$ with $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is a pure state if exists $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$ such that:*

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle .$$

A product state represent two states which do not interact; an operation on one of them don't perturb the other.

Definition 2 (Entengled States) *A system that is not in a product state (1), is an entengled state.*

When a system is in an entengled state is not possible to charaterize the two subsystems with the states vector, as the state vector of the composite system is known.

1.2.1 Density Operator

For a more general treatment, the following representation of states is given:

Definition 3 *The state of quantum system is described by a linear operator, called density operator such that:*

$$\Xi : \mathcal{H} \to \mathcal{H};\ \Xi^\dagger = \Xi;\ tr\{\Xi\} = 1.$$

According to the definition 3, the postulates 3, 4, 5 can be reformulate as following.

$$\mathbb{P}(\lambda_i) = tr\{\Xi \mathcal{P}_i\} \tag{1.1}$$

$$\Xi' = \frac{\mathcal{P}_i \Xi \mathcal{P}_i^\dagger}{tr\{\mathcal{P}_i \Xi \mathcal{P}_i^\dagger\}} \tag{1.2}$$

$$\Xi(t) = \mathcal{U}\Xi(t_0)\mathcal{U}^\dagger \tag{1.3}$$

1.3 Quantized Electromagnetic Field

Electromagnetic field is the main means of communication for contemporary application. It is important therefor, to give a quantum representation.

In a volume $\mathcal{V} \in \mathbb{R}^3$ classical magnetic field is determinated from Maxwell's equations as a superposition of the cavity modes

$$\mathbf{e}(\mathbf{r}, t) = \sum_n p_n(t)\mathbf{u}_n(\mathbf{r}) \tag{1.4}$$