On the Design of Quantum Communication Systems with non-Gaussian States

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Postulate 1: State Representation

The state of an isolated quantum system is represented by a complex unitary vector in an Hilbert space:

$$|\psi\rangle \in \mathcal{H}$$

Postulate 2: Observables

Every observables of the system is represented by an Hermitian operator acting on the state space:

$$\mathcal{M} : \mathcal{H} \to \mathcal{H}$$

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Postulate 3: Born's Rule

The probability to get the measurement λ_i from the observable $\mathcal{M}$ in the system in state $|\psi\rangle$ is:

$$\mathbb{P}(\lambda_i) = \langle\psi| \mathcal{P}_i |\psi\rangle$$

Postulate 4: Wavefunction Collapse

The state after measurement of λ_i is $\mathcal{P}_i |\psi\rangle$ (with the necessary normalization):

$$|\psi'\rangle = \frac{\mathcal{P}_i |\psi\rangle}{\langle\psi| \mathcal{P}_i |\psi\rangle}.$$

Postulate 5: Time Evolution

The time evolution of an isolated quantum system is given by an unitary operator $\mathcal{U}$:

$$|\psi(t)\rangle = \mathcal{U}(t_0, t)\,|\psi(t_0)\rangle\,.$$

Postulate 6: Composite System

The state space of a system composed of $\mathcal{H}_1$ and $\mathcal{H}_2$ is given by

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2.$$

Density operator

The state of quantum system is described by a linear operator $\boldsymbol{\Xi}$, called density operator such that:

$$\boldsymbol{\Xi} : \mathcal{H} \to \mathcal{H};$$
$$\boldsymbol{\Xi}^{\dagger} = \boldsymbol{\Xi};$$
$$tr\{\boldsymbol{\Xi}\} = 1.$$

Classical

$$\boldsymbol{e}(\boldsymbol{r},t) = -\sum_n p_n(t)\boldsymbol{u}_n(\boldsymbol{r})$$

$$\boldsymbol{h}(\boldsymbol{r},t) = \sum_n q_n(t)\nabla\times\boldsymbol{u}_n(\boldsymbol{r})$$

$$\text{with } \boldsymbol{u}_n(\boldsymbol{r}) = \boldsymbol{u}_{n0}\ e^{i\boldsymbol{k}_n\cdot\boldsymbol{r}}$$

$$H_n = \hbar\omega_n |a_n(t)|^2$$

$$a_n(t) = \frac{\omega_n q_n(t) + ip_n(t)}{\sqrt{2\hbar\omega_n}}$$

Quantized

$$p_n(t) \implies P_n(t) : \mathcal{H} \to \mathcal{H}$$

$$q_n(t) \implies Q_n(t) : \mathcal{H} \to \mathcal{H}$$

$$[\boldsymbol{Q}_n, \boldsymbol{P}_m] = i\hbar\delta_{n,m}\boldsymbol{I}$$

$$[\boldsymbol{Q}_n, \boldsymbol{Q}_m] = 0$$

$$[\boldsymbol{P}_n, \boldsymbol{P}_m] = 0$$

$$H_n = \hbar\omega_n \boldsymbol{A}_n^\dagger \boldsymbol{A}_n$$

$$\boldsymbol{A}_n(t) = \frac{\omega_n \boldsymbol{Q}_n(t) - i\boldsymbol{P}_n(t)}{\sqrt{2\hbar\omega_n}}$$

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Fock State

The Fock state $|n\rangle$ represents the quantum state with exactly n photons. It is defined as:

$$\boldsymbol{N}\,|n\rangle = n\,|n\rangle \quad \text{with } \boldsymbol{N} = \boldsymbol{A}^{\dagger}\boldsymbol{A}$$

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Fock Representation

Every quantum state $\boldsymbol{\Xi}$ can be expressed as

$$\boldsymbol{\Xi} = \sum_{n,m} c_{n,m}\,|n\rangle\,\langle m| \quad with\ c_{n,m} = \langle n|\,\boldsymbol{\Xi}\,|m\rangle$$

s-order characteristic function

$$\chi(\xi, s) = \exp\left\{\frac{s}{2}|\xi|^2\right\} tr\{\boldsymbol{\Xi}\boldsymbol{D}_\xi\}$$

$$\boldsymbol{D}_\xi = \exp\{\xi \boldsymbol{A}^\dagger - \xi^* \boldsymbol{A}\} \quad \textit{Displacement Operator}$$

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s-order quasi-probability function

$$W(\alpha, s) = \frac{1}{\pi^2} \int_{\mathbb{R}^2} \chi(\xi, s) e^{\alpha\xi^* - \alpha^* \xi} d\xi^2.$$

The quasi-probability function for $s = 0$ is called Wigner W-function.

Coherent state

$$\boldsymbol{A}\left|\mu\right\rangle = \mu\left|\mu\right\rangle$$
$$\left|\mu\right\rangle = \boldsymbol{D}_\mu\left|0\right\rangle$$

Squeezed state

$$\left|\mu,\zeta\right\rangle = \boldsymbol{D}_\mu\boldsymbol{S}_\zeta\left|0\right\rangle$$

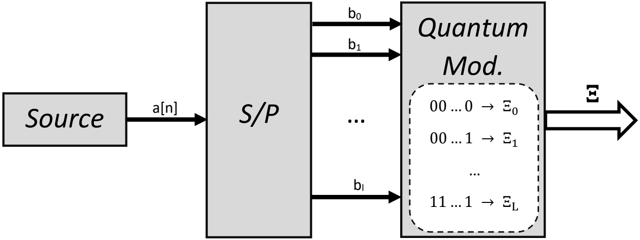
Thermal noise state

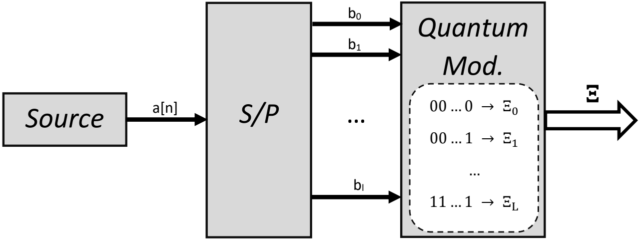
$$\boldsymbol{\Xi}_{th} = (1-v)\sum_{n=0}^{\infty} v^n\left|n\right\rangle\left\langle n\right|$$

$$v = \frac{\bar{n}}{\bar{n}+1}; \qquad \bar{n} = \left(\exp\left\{\frac{\hbar\omega}{k_B T}\right\} - 1\right)^{-1}$$

Photon added states

$$\boldsymbol{\Xi}^{(k)} = \frac{(\boldsymbol{A}^\dagger)^k\boldsymbol{\Xi}\boldsymbol{A}^k}{tr\{(\boldsymbol{A}^\dagger)^k\boldsymbol{\Xi}\boldsymbol{A}^k\}}$$


Source
a[n]
S/P
b0
b1
...
bl
Quantum Mod.
00 ... 0 → Ξ0
00 ... 1 → Ξ1
...
11 ... 1 → ΞL
Ξ

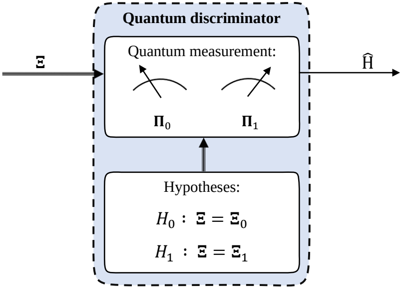


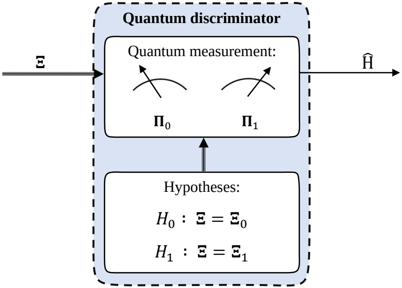
OOK

$$\Xi_0 = \Xi_{th}$$
$$\Xi_1 = \Xi_{th}(\mu)$$

BPSK

$$\Xi_0 = \Xi_{th}(-\mu)$$
$$\Xi_1 = \Xi_{th}(\mu)$$


Quantum discriminator
$\boldsymbol{\Xi}$
Quantum measurement:
$\boldsymbol{\Pi}_0$
$\boldsymbol{\Pi}_1$
$\hat{H}$
Hypotheses:
$H_0 : \boldsymbol{\Xi} = \boldsymbol{\Xi}_0$
$H_1 : \boldsymbol{\Xi} = \boldsymbol{\Xi}_1$



Distribution error probability

$$P_e = 1 - (p_0 tr\{\Xi_0 \Pi_0\} + p_1 tr\{\Xi_1 \Pi_1\})$$

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Helstrom Bound: Minium distribution error probability

$$\breve{P}_e = \frac{1}{2}\left(1 - \left\| p_1 \mathbf{\Xi}_1 - p_0 \mathbf{\Xi}_0 \right\|_1\right)$$

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Helstrom Bound: Minium distribution error probability

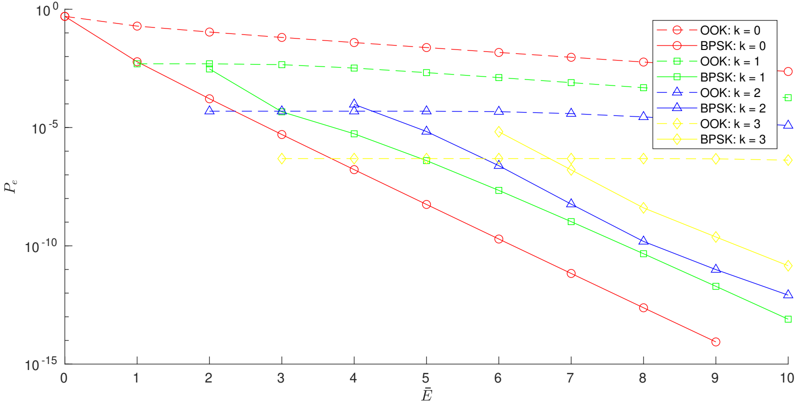
$$\breve{P}_e = \frac{1}{2}\left(1 - \left\| p_1 \boldsymbol{\Xi}_1 - p_0 \boldsymbol{\Xi}_0 \right\|_1\right)$$

MDEP for pure states

$$\breve{P}_e = \frac{1}{2}\left(1 - \sqrt{1 - 4 p_0 p_1 \left|\langle \psi_0 | \psi_1 \rangle\right|^2}\right)$$

Discrimination of Photon Added States

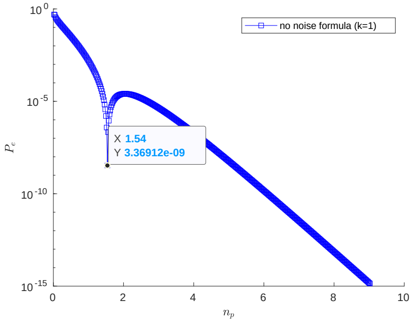
Noisy PACS discrimination



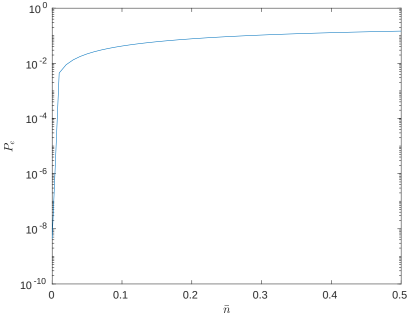
MDEP comparison of a QOOK and QBPSK systems with PACS in terms of the mean energy of the system $\bar{E}$.
$N = 45$; $\bar{n} = 10^{-2}$; $p_0 = p_1 = 1/2$

Discrimination of Photon Added States

Noise effect



MDEP of a QBPSK with PACSs, without noise. $k = 1$.



MDEP for $\mu = 0.54$ (value of the zero) in terms of $\bar{n}$.

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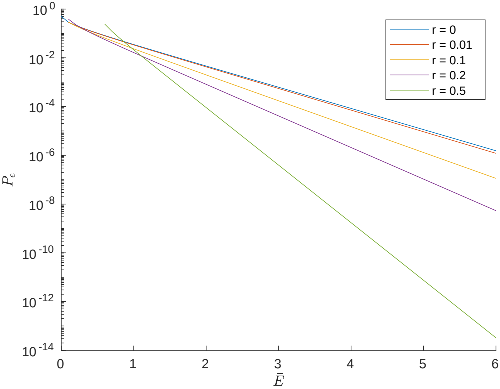
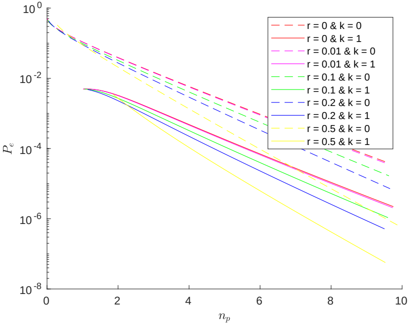


MDEP of a QBPSK system with squeezed states in terms of the mean energy of the system $\bar{E}$.

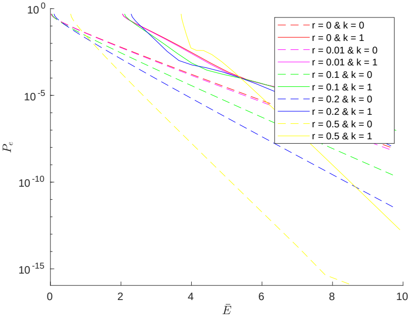
$N = 30$; $\bar{n} = 0$; $p_0 = p_1 = 1/2$

Discrimination of Photon Added States

Noisy PASS discrimination



MDEP of a QOOK system with PASS in terms of the mean photon number n_p. $N = 30$; $\bar{n} = 10^{-2}$; $\theta = \pi$; $p_0 = p_1 = 1/2$



MDEP of a QBPSK system with PASS in terms of the mean energy of the system $\bar{E}$. $N = 30$; $\bar{n} = 10^{-2}$; $\theta = \pi$; $p_0 = p_1 = 1/2$