Università degli Studi di Ferrara

Università degli Studi di Ferrara

DEPARTMENT OF ENGINEERING
Bachelor's degree in Electronic and Computer Science Engineering

COMMUNICATION NETWORKS

On the Design of Quantum Communication Systems with non-Gaussian States

Candidate:
Federico Forzano

Supervisor:
Chiar.mo Prof. Andrea Conti

Co-Supervisor:
Dott. Ing. Stefano Guerrini

Academic Year 2020-2021

Sommario

La scienza dell'informazione quantistica sta aprendo le porte a nuove tecnologie per l'elaborazione e la trasmissione delle informazioni. Queste tecnologie fondano le proprie radici in alcune caratteristiche peculiari della teoria quantistica quali la superposition, l'entanglement e il principio di indeterminazione. La capacità di maneggiare le proprietà della meccanica quantistica potrà portare, in un futuro molto vicino, a realizzare e progettare tecnologie di prossima generazione in grado di soddisfare le esigenze di un mondo sempre più informatizzato e connesso. Le comunicazioni in tutto ciò rivestiranno un ruolo chiave; studiare ed ottimizzare un sistema di comunicazione quantistico efficiente e affidabile risulterà essenziale.

In questa tesi vengono analizzati sistemi di comunicazioni quantistici basati su stati non-classici non-Gaussiani. In particolare vengono studiati sistemi con modulazioni binarie di tipo on-off keying (OOK) e binary phase-shifting keying (BPSK) in presenza di rumore termico. Tale studio evidenzia come l'utilizzo di stati non-Gaussiani PACSs e PASSs permetta di raggiungere prestazioni più elevate.

Abstract

Quantum information science is opening doors to new technologies for the elaboration and the transmission of the informations. Those technologies are based on some peculiarities of the quantum mechanics such as the superposition, the entanglement and the indeterminancy principle. The ability to handle quantum mechanic properties will lead, in the foreseeable future, to design and implement innovative technologies. That is a very significant fact if we consider the increasingly computerization and necessity to be connected in our society. Communication will play a key role in this scenario, so it is essential to study and optimize an efficient and reliable quantum communication system.

In this thesis are analyzed quantum communication systems based on non-classical and non-Gaussian states. In particulare we study systems with on-off keying (OOK) binary modulation and binary phase-shifting keying modulation in presence of the thermal noise. This study highlights how the use of PACSs and PASSs improves the performance.

Contents

Chapter 1

Introduction

The evolution of information and communication technologies (ICT) founds its roots in the mathematical theory of communications developed in the last century by Claude Shannon [1]. Against that background, quantum mechanics is a the key enabler for the next generation communication systems and networks. The possibility to engineer the properties of quantum systems [2] is essential for the design of an optimized quantum communication systems.

Quantum technologies can be classified into two categories [3]: discrete variables systems (DVs) and continuous variables systems (CVs). DV technologies are based on discrete quantum states, such as qubits, which are the quantum equivalents of digital signals. CV technologies are based on continuous values quantum states, such as coherent states, which are the equivalents of analog signals. The use of CVs offers the possibility to use the existing classical network infrastructure by just adapting the apparatuses in the network nodes; and, on the other side, CVs states are more easy to generate and manage [3]. Communication systems are furthermore well described by the use of CVs. For this reasons, in this thesis we will describe and analyze the second category of quantum communication system.

Quantum communication is the task of transferring classical or quantum information ([4, 5, 6, 7]) from one place to another one, by using a quantum carrier. The use of a quantum carrier allows to overcome the limits of classical communication systems. In particular, the use of non-Gaussian states can improves significantly the performance of communication systems. PACSs and PASSs are two important classes of non-Gaussian states that can be easily generated from Gaussian states by using off-the-shelf devices. In this thesis we will analyze the performance of quantum communication systems using this states [8, 4, 5, 6].

The goal of this thesis is to analyze the performance of quantum communication systems using PACSs and PASS. remainder of this paper is organized as follows. Chapter 2 provides a brief introduction of the quantum theory for CVs with particular emphasis on the characterization of non-Gaussian photon-added states. Chapter 3 describes a quantum communication system with CVs. Chapter 4 characterizes a quantum communication system using PACSs and PASSs.

Chapter 2

Elements of quantum information with continuous variables

This chapter gives a brief overview of quantum mechanics postulates, of the notation and of the essential concept used in this thesis. The target of that is to explain to the reader the essential concept of quantum mechanics and of quantum continuous-variable states, in order to give him the possibility to understand the obtained result.

2.1 Preliminaries on quantum mechanics

For understand the important results about the communication with continuos states, it is essential to give a brief introduction about the main aspects of quantum mechanics theory. This theory is based on a solid mathematic framework presented in this section. It is impossible to discuss about quantum mechanics without its mathematical formalism.

2.1.1 Postulates

Like every phisics theory, quantum mechanics is builded from few essential postulates. In this section are briefly introduced the six Dirac-Von Newman postulates of Quantum Mechanics [9, 10].

Postulate 1 (State Representation) *The state of an isolated quantum system is represented by a complex unitary vector $|\psi\rangle$ in an Hilbert space $\mathcal{H}$. The space of possible states of the system is called state space and it is a separable complex Hilbert space.*

Every ket-vector $|\psi\rangle \in \mathcal{H}$ can be represented as a column vector $|\psi\rangle = [a_1, a_2, \ldots, a_N]$, where N is the dimension of the state space $\mathcal{H}$. The bra-vector $\langle\psi|$ is the correspondent vector of $|\psi\rangle$ in the dual-space of $\mathcal{H}$ and can be represented as the transposed conjugate of $|\psi\rangle$.

Differently from the classical physics, in quantum mechanics the concept of state of system is introduced. In classical mechanics a system is described by his observables, like position or four-wheeled.

Postulate 2 (Observables) *Every observables of the system is represented by an Hermitian operator $\boldsymbol{M} : \mathcal{H} \rightarrow \mathcal{H}$ acting on the state space. The outcomes of the measurement can only be one of the eigenvalue of the operator $\boldsymbol{M}$.*

The possible outcomes of the measurement are real number because $\boldsymbol{M}$ is self-andjoint.

Postulate 3 (Born's Rule) *The probability to get the measurement λ_i from the observable $\boldsymbol{M}$ in the system in state $|\psi\rangle$ is:*

$$\mathbb{P}(\lambda_i) = \langle\psi| \boldsymbol{P}_i |\psi\rangle$$

where $\langle\psi|$ is the correspondent vector of $|\psi\rangle$ in the dual space of $\mathcal{H}$ and where $\boldsymbol{P}_i$ is the projection operator of λ_i in the correspondent space.

Postulate 4 (Wavefunction Collapse) *The state $|\psi'\rangle$ after measurement of λ_i is $\boldsymbol{P}_i |\psi\rangle$ (with the necessary normalization):*

$$|\psi'\rangle = \frac{\boldsymbol{P}_i |\psi\rangle}{\langle\psi| \boldsymbol{P}_i |\psi\rangle}.$$

Postulate 5 (Time Evolution) *The time evolution $|\psi(t)\rangle$ of an isolated quantum system is given by an unitary operator $\boldsymbol{U}$:*

$$|\psi(t)\rangle = \boldsymbol{U}(t_0, t) |\psi(t_0)\rangle .$$

From postulate 5, it is possible to obtain the *time dependent Shrodinger Equation*:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \boldsymbol{H}(t) |\psi(t)\rangle \tag{2.1}$$

where $\boldsymbol{H}(t)$ is the Hamiltonian matrix, $\hbar$ is the reduced Planck's constant and $i = \sqrt{-1}$ is the immaginary unit.

Postulate 6 (Composite System) *The state space $\mathcal{H}$ of a system composed of $\mathcal{H}_1$ and $\mathcal{H}_2$ is given by*

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2.$$

2.1.2 The density operator

The last postulate 6 has very important consequences for composite system. It is possible to describe two tipes of combined systems:

Definition 1 (Product states) *A state $|\psi\rangle \in \mathcal{H}$ with $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is a pure state if exists $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$ such that:*

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle .$$

A product state represents two systems which do not interact; an operation on one of them does not perturb the other.

Definition 2 (entengled states) *A system that is not in a product state (1), is in an entengled state.*

When a system is in an entengled state it is not possible to characterize the two subsystems with the states vector, although the state vector of the composite system is known.

Density operator

For a more general treatment, the following representation of states is given:

Definition 3 *The state of quantum system is described by a linear operator* $\boldsymbol{\Xi} : \mathcal{H} \to \mathcal{H}$*, called density operator such that* $\boldsymbol{\Xi}^\dagger = \boldsymbol{\Xi}$ *and* $\mathrm{tr}\{\boldsymbol{\Xi}\} = 1$.

According to the definition 3, the postulates 3, 4, 5 can be reformulate as following.

The probability to get the measurement λ_i from the observable $\boldsymbol{M}$ in the system in state $\boldsymbol{\Xi}$ is:

$$\mathbb{P}(\lambda_i) = \mathrm{tr}\{\boldsymbol{\Xi}\boldsymbol{P}_i\}. \tag{2.2}$$

The state $\boldsymbol{\Xi'}$ after measurement of λ_i is given by

$$\boldsymbol{\Xi'} = \frac{\boldsymbol{P}_i\boldsymbol{\Xi}\boldsymbol{P}_i^\dagger}{\mathrm{tr}\left\{\boldsymbol{P}_i\boldsymbol{\Xi}\boldsymbol{P}_i^\dagger\right\}}. \tag{2.3}$$

The time evolution $\boldsymbol{\Xi}(t)$ of an isolated quantum system is given by an unitary operator $\boldsymbol{U}$ as:

$$\boldsymbol{\Xi}(t) = \boldsymbol{U}\boldsymbol{\Xi}(t_0)\boldsymbol{U}^\dagger. \tag{2.4}$$

2.2 Continuous-Variables Quantum Systems

A quantum system is called a continuous-variable system when it has an infinite-dimensional Hilbert space described by observables with continuous eigenspectra [11]. Continuous-variables systems play a very important role in communications. this section presents the key aspects for the representation of this systems.

2.2.1 Hilbert space

Let consider a single-mode bosonic continuous-variable system, corresponding to a single mode radiation of electromagnetic field, i.e. a single mode quantum harmonic oscillator. Its space of states is an infinite dimensional Hilbert space $\mathcal{H}$ in which it is possible to define a pair of bosonic field operators $\{\boldsymbol{A}, \boldsymbol{A}^\dagger\}$ called annihilation and creation operators [11] satisfying the canonical commutation relation $\left[\boldsymbol{A}, \boldsymbol{A}^\dagger\right] = \boldsymbol{I}$.

In this space $\mathcal{H}$ it is possible to define a number operator $\boldsymbol{N}$, defined as

$$\boldsymbol{N} = \boldsymbol{A}^\dagger\boldsymbol{A}. \tag{2.5}$$

The eigenstates of $\boldsymbol{N}$, i.e. the vector $|n\rangle$ for which $\boldsymbol{N}|n\rangle = n|n\rangle$, are countable and form a countable basis of $\mathcal{H}$ called Fock basis: $\{|n\rangle\}_{n=0}^{\infty}$.

The action over this states of the bosonic operators is determinated by [11]

$$\boldsymbol{A}|0\rangle = 0; \quad \boldsymbol{A}|n\rangle = \sqrt{n}\,|n-1\rangle \quad (for\ n \geq 1), \tag{2.6}$$

and

$$\boldsymbol{A}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle \quad (for\ n \geq 0).$$

Every quantum state $\boldsymbol{\Xi} : \mathcal{H} \to \mathcal{H}$ can be represented as:

$$\boldsymbol{\Xi} = \sum_{n,m} c_{n,m}\,|n\rangle\langle m| \tag{2.7}$$

where

$$c_{n,m} = \langle n|\,\boldsymbol{\Xi}\,|m\rangle\,. \tag{2.8}$$

This representation is called Fock Representation.

2.2.2 Phase space

As seen before in 2.1.2, a quantum system can be completely described by a density operator $\boldsymbol{\Xi}$ defined in an infinite-dimensional Hilbert space $\mathcal{H}$, and this operator can be expressed by the Fock representation (2.7). Sometimes, however, it is convenient to give another representation of state Ξ by means of a complex function introduced by Wigner [12]: the quasi-probability distribution. In this thesis, this representation will be introduced and it will be used to classify the possible states.

Definition 4 (Quantum characteristic function) *The s-order characteristic function $\chi(\xi, s)$, with $\xi, s \in \mathbb{C}$, associated to the quantum state Ξ is defined as:*

$$\chi(\xi, s) = \exp\left\{\frac{s}{2}|\xi|^2\right\} \operatorname{tr}\{\Xi \boldsymbol{D}_\xi\} \tag{2.9}$$

where $\boldsymbol{D}_\xi$ is the displacement operator of parameter ξ, defined as:

$$\boldsymbol{D}_\xi = \exp\left\{\xi \boldsymbol{A}^\dagger - \xi^* \boldsymbol{A}\right\}. \tag{2.10}$$

The quantum characteristic function is the Fourier-Weyl transform of the density operator associated to the state $\boldsymbol{\Xi}$. We can notice that, in contrast to classical probability theory, there is an infinite number of quantum characteristic functions, indexed by the parameter $s \in \mathbb{C}$, representing the same quantum state.

The quasi-probability function is obtained as the inverse Fourirer transform of the quantum characteristic function.

Definition 5 (Quasi-probability distribution) *The s-order quasi-probability distribution $W(\alpha, s)$, with $s \in \mathbb{C}$, associated to the quantum state $\boldsymbol{\Xi}$ is given by:*

$$W(\alpha, s) = \frac{1}{\pi^2} \int_{\mathbb{R}^2} \chi(\xi, s) e^{\alpha\xi^* - \alpha^*\xi} d\xi^2.$$

The quasi-probability distribution, for $s = 0$ ($W(\alpha) = W(\alpha, 0)$) is called Wigner W-function.

2.3 Gaussian States

Gaussian quantum states are an important class of quantum states of continuous-variables systems. They are defined as ([3] quoting [13, 14, 11, 15, 16]):

Definition 6 (Gaussian state) *A quantum state $\boldsymbol{\Xi_G}$ is a Gaussian state if its Wigner W-function $W_G(\alpha)$ is Gaussian, i.e*

$$W_G(\alpha) = \frac{1}{\pi\sqrt{\det\{\check{\boldsymbol{C}}_0\}}} \exp\left\{-\frac{1}{2}(\check{\boldsymbol{\alpha}} - \check{\boldsymbol{\mu}})^H \check{\boldsymbol{C}}_0^{-1} (\check{\boldsymbol{\alpha}} - \check{\boldsymbol{\mu}})\right\}. \tag{2.11}$$

where $\check{\boldsymbol{\mu}}$ is the augmented displacement vector, and $\check{\boldsymbol{C}}_0$ is the augmented covariance matrix.

We remark that if $\boldsymbol{\mu} \in \mathbb{R}^2$ is the displacement vector and $\boldsymbol{C}_0$ is the covariance matrix, the augmented displacement vector and covariance matrix are given by the following transformation:

$$\check{\boldsymbol{\mu}} = \frac{1}{\sqrt{2}} \boldsymbol{J} \boldsymbol{\mu} \tag{2.12a}$$

$$\check{\boldsymbol{C}}_0 = \frac{1}{2} \boldsymbol{J} \boldsymbol{C}_0 \boldsymbol{J}^H \tag{2.12b}$$

where

$$\boldsymbol{J} = \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}.$$

Two important types of Gaussian states will be analyzed now: the coherent state and the squeezed state. For each one of these states is presented the noisy version too.

2.3.1 Coherent state

A coherent state is the state of a quantum armonic oscillator of amplitude μ. It is defined ([3] seen [17, 18]) as the eigenvector $|\mu\rangle$ of $\boldsymbol{A}$ associated to the eigenvalue μ; i.e

$$\boldsymbol{A}\,|\mu\rangle = \mu\,|\mu\rangle\,. \tag{2.13}$$

It is possible to obtain a coherent state of parameter μ, appliying the displacement operator to the ground state:

$$|\mu\rangle = \boldsymbol{D}_\mu\,|0\rangle\,. \tag{2.14}$$

As mentioned before, it is possible to characterize a state with the Fock representation and, equivalently, with the Wigner W-function. The last one is given, for a coherent state, by [19]:

$$W(\alpha) = \frac{2}{\pi}\exp\left\{-2|\alpha-\mu|^2\right\}. \tag{2.15}$$

It is easy to proof that $W(\alpha)$ is gaussian, with $\check{\boldsymbol{\mu}} = [\mu\ \mu^*]^T$ and

$$\check{\boldsymbol{C}}_0 = \frac{1}{2}\boldsymbol{I}.$$

The Fock representation is given by [20]:

$$|\mu\rangle = e^{-\frac{|\mu|}{2}^2}\sum_{n=0}^{\infty}\frac{\mu^n}{\sqrt{n}}\,|n\rangle\,. \tag{2.16}$$

Noisy coherent states

It is possible to characterize the state of a noisy armonic oscillator introducing the thermal state, i.e the state of an electromagnetic cavity in thermal equilibrium. The Fock representation of the thermal state Ξ_{th} is given by [3]

$$\boldsymbol{\Xi}_{\text{th}} = (1-v)\sum_{n=0}^{\infty}v^n\,|n\rangle\,\langle n| \tag{2.17}$$

where

$$v = \frac{\bar{n}}{\bar{n}+1}$$

and $\bar{n}$ is the well-known Plank distribution

$$\bar{n} = \left(\exp\left\{\frac{\hbar\omega}{k_B T}\right\}-1\right)^{-1}.$$

A noisy coherent states $\boldsymbol{\Xi}_{\text{th}}(\mu)$ of parameter μ can be obtained by appling the displacement operator $\boldsymbol{D}_\mu$ to the thermal state $\boldsymbol{\Xi}_{\text{th}}$, as follow:

$$\boldsymbol{\Xi}_{\text{th}}(\mu) = \boldsymbol{D}_\mu^\dagger\boldsymbol{\Xi}_{\text{th}}\boldsymbol{D}_\mu. \tag{2.18}$$

The Wigner W-function is given by [19]

$$W_{th}(\alpha) = \frac{1}{\pi(\bar{n}+\frac{1}{2})}\exp\left\{-\frac{|\alpha-\mu|^2}{\bar{n}+\frac{1}{2}}\right\} \tag{2.19}$$

and it can be proved that it is a Gaussian function with $\check{\boldsymbol{\mu}} = [\mu\ \mu^*]^T$ and

$$\check{\boldsymbol{C}}_0 = \left(\bar{n}+\frac{1}{2}\right)\boldsymbol{I}.$$

The Fock representation is given by

$$\langle n|\,\Xi_{\text{th}}(\mu)\,|m\rangle = (1-v)e^{-(1-v)|\mu|^2}\sqrt{\frac{n!}{m!}}v^n[(1-v)\mu^*]^{m-n}L_n^{m-n}\left(\frac{-(1-v)^2|\mu|^2}{v}\right) \tag{2.20}$$

2.3.2 Squeezed state

A squeezed state with amplitude μ and squeezing parameter ζ, is a defined as [3, 21, 22]

$$|\mu, \zeta\rangle = \boldsymbol{D}_\mu \boldsymbol{S}_\zeta |0\rangle \tag{2.21}$$

where $\boldsymbol{S}_\zeta$ is the squeezing operator, defined as

$$\boldsymbol{S}_\zeta = \exp\left\{\frac{1}{2}\left(\zeta\left(\boldsymbol{A}^\dagger\right)^2 + \zeta^* \boldsymbol{A}^2\right)\right\}. \tag{2.22}$$

It can be proven that a squeezed state is a Gaussian state with $\check{\boldsymbol{\mu}} = [\mu\ \mu^*]^T$ and

$$\check{\boldsymbol{C}}_0 = \frac{1}{2}\begin{bmatrix} \cosh(2r) & \sinh(2r)e^{-i\phi} \\ \sinh(2r)e^{-i\phi} & \cosh(2r) \end{bmatrix}$$

with $\zeta = re^{i\phi}$. The Wigner W-function of a squeezed state, differently from the one of a coherent state, has not a circular symmetry.

Noisy squeezed states

The representation of a noisy squeezed state $\boldsymbol{\Xi}_{\text{th}}(\mu, \zeta)$ is obtained, similarly to a noisy coherent state, as:

$$\boldsymbol{\Xi}_{\text{th}}(\mu, \zeta) = \boldsymbol{D}_\mu \boldsymbol{S}_\zeta \boldsymbol{\Xi}_{\text{th}} \boldsymbol{S}_\zeta^\dagger \boldsymbol{D}_\mu^\dagger. \tag{2.23}$$

The Gaussian Wigner W-function is obtained with $\check{\boldsymbol{\mu}} = [\mu\ \mu^*]^T$ and

$$\check{\boldsymbol{C}}_0 = \left(\bar{n} + \frac{1}{2}\right)\begin{bmatrix} \cosh(2r) & \sinh(2r)e^{-i\phi} \\ \sinh(2r)e^{-i\phi} & \cosh(2r) \end{bmatrix}. \tag{2.24}$$

The Fock representation is given by [23]

$$\begin{aligned} \langle n| \boldsymbol{\Xi}_{\text{th}}(\mu, \zeta) |m\rangle = \frac{\pi Q(0)}{(n!m!)^{1/2}} \sum_{k=0}^{min(n,m)} k! \binom{n}{k}\binom{m}{k} \tilde{A}^k \left(\frac{1}{2}\tilde{B}\right)^{(n-k)/2} \\ \left(\frac{1}{2}\tilde{B}^*\right)^{(m-k)/2} H_{n-k}((2\tilde{B})^{-1/2}\tilde{C}) H_{m-k}((2\tilde{B}^*)^{-1/2}\tilde{C}^*) \end{aligned} \tag{2.25}$$

where $H_n(x)$ is the Hermite polynomial with parameter n, $Q(0)$ is a constant defined by

$$Q(0) = \frac{1}{\pi}[(1+A)^2 - |B|^2]^{-1/2} \exp\left\{-\frac{(1+A)|C|^2 + \frac{1}{2}[B(C^*)^2 + B^*C^2]}{(1+A)^2 - |B|^2}\right\} \tag{2.26}$$

and

$$\tilde{A} = \frac{A(1+A) - |B|^2}{(1+A)^2 - |B|^2} \tag{2.27a}$$

$$\tilde{B} = \frac{B}{(1+A)^2 - |B|^2} \tag{2.27b}$$

$$\tilde{C} = \frac{(1+A)C + BC^*}{(1+A)^2 - |B|^2}. \tag{2.27c}$$

The parameter A, B and C are defined as:

$$A = \bar{n} + (2\bar{n}+1)(\sinh(r))^2 \tag{2.28a}$$

$$B = -(2\bar{n}+1)e^{i\phi}\sinh(r)\cosh(r) \tag{2.28b}$$

$$C = \mu. \tag{2.28c}$$

2.4 Non-Gaussian States

A state that does not fulfill the definition 2.3 is a non-Gaussian state. An important and useful for communications class of non-Gaussian states, are the photon added states, examined in this thesis. Lastly will be mentioned another type of non-Gaussian state: the photon subtracted state.

2.4.1 Photon added states

The photon added state $\boldsymbol{\Xi}^{(1)}$, obtained from the quantum state $\boldsymbol{\Xi}$, is given by:

$$\boldsymbol{\Xi}^{(1)} = \frac{\boldsymbol{A}^{\dagger}\boldsymbol{\Xi}\boldsymbol{A}}{\operatorname{tr}\left\{\boldsymbol{A}^{\dagger}\boldsymbol{\Xi}\boldsymbol{A}\right\}}. \tag{2.29}$$

The name *photon addition* could lead to believe that the mean photon number of the photon added state is icreased by one compared to the previous non photon added state. However, that is incorrect. In general, its mean number of photons could be the same, more or less than the starting state. Only if $\boldsymbol{\Xi} = |n\rangle\langle n|$, i.e $\boldsymbol{\Xi}$ is the density operator corresponding to the Fock state $|n\rangle$, the result of the photon addition is a state with one more photon.

The photon added state $\boldsymbol{\Xi}^{(k)}$ (with k photon addition) is given by

$$\boldsymbol{\Xi}^{(k)} = \frac{(\boldsymbol{A}^{\dagger})^k\boldsymbol{\Xi}\boldsymbol{A}^k}{\operatorname{tr}\left\{(\boldsymbol{A}^{\dagger})^k\boldsymbol{\Xi}\boldsymbol{A}^k\right\}}. \tag{2.30}$$

The Fock representation of a photon added state, can be obteined as:

$$\boldsymbol{\Xi}^{(k)} = \frac{\tilde{\boldsymbol{\Xi}}^{(k)}}{\operatorname{tr}\left\{\tilde{\boldsymbol{\Xi}}^{(k)}\right\}} \tag{2.31}$$

and

$$\langle n|\,\tilde{\boldsymbol{\Xi}}^{(k)}\,|m\rangle = \begin{cases}\sqrt{\frac{n!m!}{(n-k)!(m-k)!}}\,\langle n-k|\,\boldsymbol{\Xi}\,|m-k\rangle & \text{if } n,m \geq k\\ 0 \text{ otherwise.}\end{cases}$$

The Wigner W-function of a photon added state is not Gaussian (2.3).

Photon added coherent states

Let $\boldsymbol{\Xi}$ be a coherent state of amplitude $\mu \in \mathbb{C}$:

$$\boldsymbol{\Xi} = |\mu\rangle\langle\mu|$$

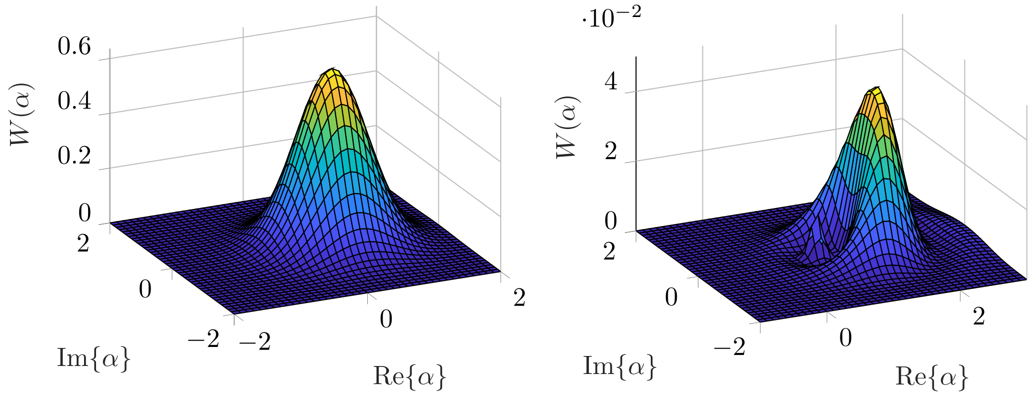
the photon added state $|\mu^{(k)}\rangle$ is called photon added coherent state (PACS). The Wigner W-function of this state is given by [3]

$$W_+^{(k)}(\alpha) = B_+^{(k)}(\alpha)W_c(\alpha) \tag{2.32}$$

where $W_c(\alpha)$ is the Wigner function of the coherent state 2.15 and

$$B_+^{(k)}(\alpha) = (-1)^k\frac{2L_k\left(|2\alpha-\mu|^2\right)}{\pi L_k\left(-|\mu|^2\right)}. \tag{2.33}$$

In figure 2.1 are plotted the Wigner W-function of a coherent state and of a PACS without noise, for $k = 2$. It is evident that the Wigner W-function of the photon added state is not Gaussian.



(a) Coherent state (b) PACS

Figure 2.1: Comparison between the Wigner W-function of a coherent state and of a PACS with $k = 2$.

If Ξ is a noisy coherent state of amplitude μ ($\Xi = \Xi_{\text{th}}(\mu)$), the photon added state $\Xi_{\text{th}}^{(k)}(\mu)$ is called noisy photon added coherent state (noisy PACS). The Fock representation can be obtained by 2.31 and can be given in closed form by [8]

$$\langle n| \, \Xi_{th}^{(k)}(\mu) \, |m\rangle = \begin{cases} c_{n,m}^{(k)} \text{ for both } n, m \geq k \\ 0 \text{ otherwise} \end{cases} \tag{2.34}$$

where

$$c_{n,m}^{(k)} = \frac{(1-v)^{k+1} e^{-(1-v)|\mu|^2}}{v^k} \sqrt{n! m!} \binom{m}{k} v^n \left[(1-v)\,\mu^*\right]^{m-n} \frac{L_{n-k}^{m-n}\left(\frac{-(1-v)|\mu|^2}{v}\right)}{L_k\left(-|\mu|^2(1-v)\right)}.$$

The Wigner W-function is given by:

$$W(\alpha) = \frac{(-1)^k}{(2\bar{n}+1)^k} \frac{L_k\left(\frac{|2\alpha(\bar{n}+1)-\mu|^2}{(2\bar{n}+1)(\bar{n}+1)}\right)}{L_k\left(-\frac{|\mu|^2}{\bar{n}+1}\right)} W_{th}(\alpha) \tag{2.35}$$

where $W_{\text{th}}(\alpha)$ is the Wigner W-function of a noisy coherent state 2.19.

Photon added squeezed states

Let Ξ be a squeezed state of amplitude μ and squeezing parameter ζ, with $\mu, \zeta \in \mathbb{C}$:

$$\Xi = |\mu, \zeta\rangle \, \langle \mu, \zeta| \, .$$

The Wigner W-function of this state is given by

$$W_+^{(k)}(\alpha) = B_+^{(k)}(\alpha) W_s(\alpha), \tag{2.36}$$

where $W_s(\alpha)$ is the Wigner function of the squeezed state, as given in 2.24, and $B_+^{(k)}(\alpha)$ is given in 2.33. The Wigner function of the PASS, as that of PACS, is not Gaussian.

if Ξ is a noisy squeezed state with amplitude μ and squeezing factor ζ ($\Xi = \Xi_{\text{th}}(\mu, \zeta)$), the photon added state $\Xi_{\text{th}}^{(k)}(\mu, \zeta)$ is called noisy photon added squeezed state (PASS).

2.4.2 Photon subtracted states

Another class of non-Gaussian states are the photon subtracted states. Similarly to PASs, a PSS $|\psi_-\rangle$ can be obtained from a generic quantum state $|\psi\rangle$ as

$$|\psi_-\rangle = \frac{\boldsymbol{A}^k |\psi\rangle}{\sqrt{\langle\psi| (\boldsymbol{A}^\dagger)^k |\psi\rangle}}. \tag{2.37}$$

As in PASs the name *photon subtraction* can not be interpreted in deterministic sense. It is also important to notice that if $|\psi\rangle = |\mu\rangle$, i.e. the state $|\psi\rangle$ is a coherent state, the photon subtraction has not effect, that is:

$$|\psi_-\rangle = |\mu\rangle .$$

Chapter 3

Quantum Communication Systems with non-Gaussian States

Thanks to quantum mechanics, it will be possible to overcome the limits of classical communication systems. In the last decades the research in this field has led to very intresting results that could significantly improve the performance of communication systems. This chapter gives a brief overview of quantum communication tools: in the first section we give an overview of a quantum communication system, in the second we present the equivalents of classical modulation for quantum communication systems; in the last section we report the concept of quantum states discriminator (QSD), included when it can be considered optimal.

3.1 Quantum Communication System

A quantum communication system can be described similarly to a classical one, as we can see in figure 3.1. An information source emits a unknown classical symbol $\mathsf{a} \in \mathcal{A}$ which have to be modulated with a quantum modulator that emits on the communication channel a corresponding quantum state $\boldsymbol{\Xi} \in \mathcal{A}_p$. The channel can distort the state and deliver the state $\boldsymbol{\Upsilon} \in \mathcal{B}_p$ that is in general different from $\boldsymbol{\Xi}$. The receiver have to recognize the unknown transmitted $\hat{\mathsf{a}}$ as well as possible, i.e. with the minimum possible error probability.

3.2 Quantum Modulations

As in a classical system, it is possible to define the concept of modulation for a quantum communication system. The transmitted information will be associated to a quantum state of the electromagnetic field, so it can be transmitted on the communication channel.

It is possible to think about the quantum transmitter as a device composed by a bit source and a quantum modulator. The bit source emits a bit sequence $\mathsf{a}[n]$ and sends them to the quantum modulator which associates one quantum state to every bit or group of bit. The operation of quantum state creation, in real cases, is affected by noise.

The sequence of operations is very close to a classical transmitter: the main difference is that the modulator maps the bits into quantum states instead of classical modulation. Therefore, it is possible to achieve the quantum equivalent of classical modulation, with several states. After that, the impact on performance can be tested. This thesis only considers and assesses the binary cases, in the OOK and BPSK configuration.

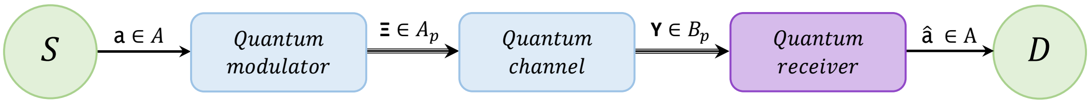


Figure 3.1: Block-chain of a quantum communication system

3.2.1 OOK modulations

The OOK (on-off keying) is the most simple possible configuration for a communication system. The quantum implementation of that is realized associating the low-energy state to the ground state $|0\rangle$ and the high-energy state to another state. It is important to consider that the physical realization of these states are not free-noise; this issue will be considered using noisy states 3.1b.

$$\Xi_0 = \Xi_{\text{th}} \tag{3.1a}$$
$$\Xi_1 = \Xi_{\text{th}}(\mu) \tag{3.1b}$$

In the equation 3.1b, the high-energy state is associated to a coherent state. This configuration has been widely analyzed in [24, 25, 26, 27, 28, 29] but this is not the only possible way. The use of PACS states $\Xi_{\text{th}}^{(k)}(\mu)$ is analyzed in [8, 3]; the use of PASS are briefly assessed in the following chapter of this thesis. The constellation of an OOK system with PACSs is given by:

$$\Xi_0 = \Xi_{\text{th}}^{(0)}(0) \tag{3.2a}$$
$$\Xi_1 = \Xi_{\text{th}}^{(k)}(\mu). \tag{3.2b}$$

3.2.2 BPSK modulations

BPSK quantum systems are implemented using two states with opposite amplitude, like

$$\Xi_0 = \Xi_{\text{th}}(-\mu) \tag{3.3a}$$
$$\Xi_1 = \Xi_{\text{th}}(\mu). \tag{3.3b}$$

There is no guarantee that the use of a BPSK solution in a quantum system will improve its performance. The effect depends on which are the used states. In the next chapter some configuration are assessed. The constellation of a BPSK system with PACSs are given by

$$\Xi_0 = \Xi_{\text{th}}^{(k)}(-\mu) \tag{3.4a}$$
$$\Xi_1 = \Xi_{\text{th}}^{(k)}(\mu). \tag{3.4b}$$

3.3 Quantum Receiver

The problem of quantum receiver is one of the most important aspects of quantum communication. As in classical communication, the ability to distinguish between two or more states in presence of noise can be decisive in order to determine the performance of the communication system. The problem of the receiver can be, therefor, reformulated as a problem of discrimination between M states. However, differently from the classical situation, the discrimination can be done using a custom-designed quantum discriminator, overcoming the classical physics limits.

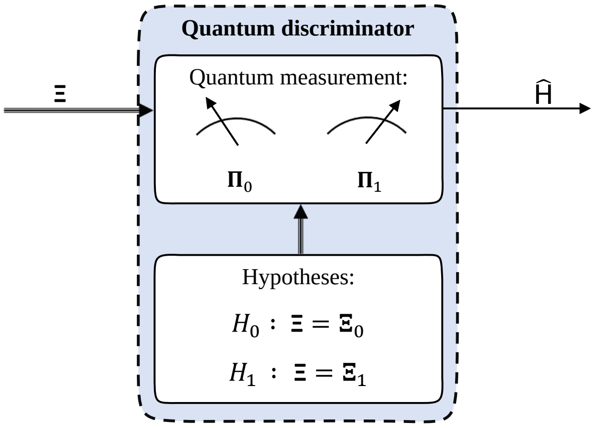


Figure 3.2: Binary quantum state discriminator.

Let $\mathcal{A} = \{\boldsymbol{\Xi}_i\}_{i=0}^{M-1}$ be the set of possible states of the quantum system. If the state of the system is unknown, there are M hypotheses about the state $\boldsymbol{\Xi}$ of the quantum system given by

$$H_j : \boldsymbol{\Xi} = \boldsymbol{\Xi}_j \text{ for } j = 0, \ldots, M-1. \tag{3.5}$$

The decision among the M hypotheses is done by mean of a quantum measurement described by the POVM $\mathcal{P} = \{\boldsymbol{\Pi}_0, \boldsymbol{\Pi}_1, \ldots, \boldsymbol{\Pi}_{M-1}\}$ so that the probability that the hypothesis H_j is choosen when H_k is true is given by [3]

$$\mathbb{P}\{H_j|H_k\} = \operatorname{tr}\{\boldsymbol{\Xi}_k \boldsymbol{\Pi}_j\}. \tag{3.6}$$

3.3.1 Binary case

If $M = 2$ we are talking about binary systems. In this case, as shown in figure 3.2, there are two hypotheses about the state $\boldsymbol{\Xi}$, given by

$$H_0 : \boldsymbol{\Xi} = \boldsymbol{\Xi}_0 \tag{3.7a}$$
$$H_1 : \boldsymbol{\Xi} = \boldsymbol{\Xi}_1. \tag{3.7b}$$

The POVM is reduced to

$$\mathcal{P} = \{\boldsymbol{\Pi}_0, \boldsymbol{\Pi}_1\} \tag{3.8}$$

and the probability that the hypothesis H_0 is choosen if H_1 is the right choose (or vice-versa) is given by using the equation 3.6

$$\mathbb{P}\{H_0|H_1\} = \operatorname{tr}\{\boldsymbol{\Xi}_0 \boldsymbol{\Pi}_1\}. \tag{3.9}$$

The distribution error probability (DEP) in the discrimination process, if p_0 and p_1 are respectively the probabilty of symbols 0 and 1, is so given by

$$P_e = \mathbb{P}\{\mathsf{a} \neq \hat{\mathsf{a}}\} = 1 - (p_0 \operatorname{tr}\{\boldsymbol{\Xi}_0 \boldsymbol{\Pi}_0\} + p_1 \operatorname{tr}\{\boldsymbol{\Xi}_1 \boldsymbol{\Pi}_1\}). \tag{3.10}$$

3.3.2 Optimal binary receiver

The issue of finding the optimal POVM that minimizes the DEP was exhaustively discuss by Helstrom in [2, 30]. The minimum distribution error probability (MDEP) for a binary communication system is given by the well-known Helstrom bound

$$\breve{P}_e = \frac{1}{2}\left(1 - \|p_1 \boldsymbol{\Xi}_1 - p_0 \boldsymbol{\Xi}_0\|_1\right), \tag{3.11}$$

where p_0, p_1 are the probability that the states $\boldsymbol{\Xi}_0$, $\boldsymbol{\Xi}_1$ are trasmitted and the operator $\|\cdot\|_1$ represents the trace norm. The MDEP 3.11 is obtained with the POVM defined as

$$\breve{\boldsymbol{\Pi}}_0 = \sum_{\substack{i \\ \lambda_i < 0}} |\lambda_i\rangle \langle\lambda_i| \,, \tag{3.12}$$

$$\breve{\boldsymbol{\Pi}}_1 = 1 - \breve{\boldsymbol{\Pi}}_0 = \sum_{\substack{i \\ \lambda_i \geq 0}} |\lambda_i\rangle \langle\lambda_i| \,;$$

where $|\lambda_i\rangle$ is the eigenvector of $p_1\boldsymbol{\Xi}_1 - p_0\boldsymbol{\Xi}_0$ associated to the eigenvalue λ_i. For pure states, i.e $\boldsymbol{\Xi}_0 = |\psi_0\rangle\langle\psi_0|$ and $\boldsymbol{\Xi}_1 = |\psi_1\rangle\langle\psi_1|$, the equation 3.11 begin

$$\breve{P}_e = \frac{1}{2}\left(1 - \sqrt{1 - 4p_0p_1|\langle\psi_0|\psi_1\rangle|^2}\right). \tag{3.13}$$

It is possible to observe that, for pure states, the MDEP is equal to 0 if $\langle\psi_0|\psi_1\rangle$, that is $|\psi_0\rangle$ and $|\psi_1\rangle$ are orthogonal states.

Chapter 4

Performance Analysis

This chapter characterizes two systems with different types of photon added state in term of their discrimination MDEP 3.11: a photon added coherent state system (from [8])and a photon added squeezed state. For each system, we consider the OOK configuration and the BPSK. The use of photon added states, as it will be shown, can improve significantly the performance of the communication.

The analyzed situation does not consider the channel effect on the transmitted information: it has been supposed that the noisy states reach the discriminator as they were created. The channel effects for a PACS system are described in [8].

4.1 Quantum communication systems with PACSs

The effect of the use of PACS in an OOK communication system was extensively discussed in [8]. In this section the most important result will be reported and a BPSK system will be tested.

4.1.1 Quantum OOK

The use of PACS in an OOK system can significantly improve the performance. The MDEP, with thermal noise, is given by the Helstrom bound 3.11, where, for an OOK PACS system,

$$\boldsymbol{\Xi}_0 = \boldsymbol{\Xi}_{\mathrm{th}}^{(0)}(0) \tag{4.1a}$$

$$\boldsymbol{\Xi}_1 = \boldsymbol{\Xi}_{\mathrm{th}}^{(k)}(\mu). \tag{4.1b}$$

It is useful, in order to evaluate the performance of the system, to introduce the mean number of photon n_p in a quantum state $\boldsymbol{\Xi}$, which is given by:

$$n_p = \mathrm{tr}\Big\{\boldsymbol{\Xi}\boldsymbol{A}^\dagger\boldsymbol{A}\Big\}. \tag{4.2}$$

For a photon added coherent state, the equation 4.2 becomes [8]:

$$n_p(\mu,\bar{n}) = \frac{N_{k+1}(\mu,\bar{n})}{N_k(\mu,\bar{n})} - 1, \tag{4.3}$$

where

$$N_k(\mu,\bar{n}) = \mathrm{tr}\Big\{(\boldsymbol{A}^\dagger)^k\boldsymbol{\Xi}_{\mathrm{th}}(\mu)\boldsymbol{A}^k\Big\}. \tag{4.4}$$

It is possible to observe that the minimum of n_p is given by:

$$n_p(0,\bar{n}) = (k+1)(\bar{n}+1) - 1. \tag{4.5}$$

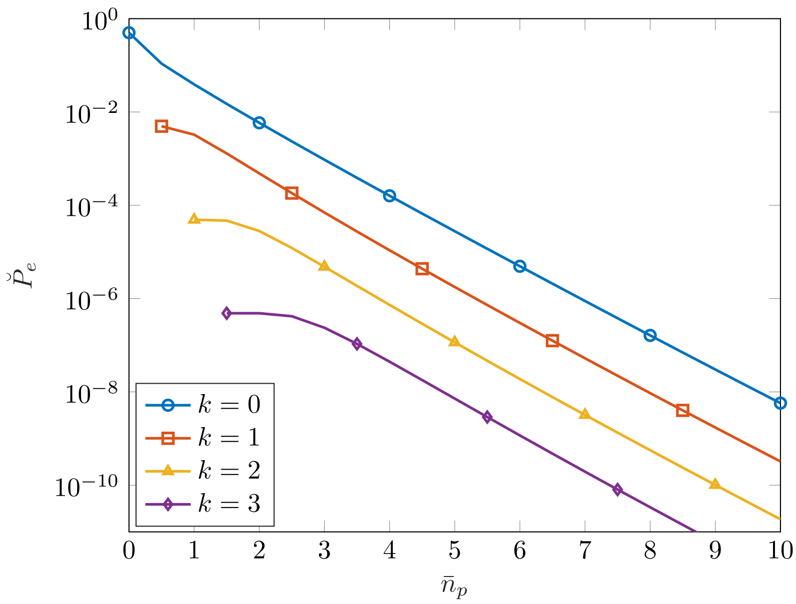


Figure 4.1: MDEP for PACS QOOK as function of $\bar{n}_p$ with: $k = 0, 1, 2, 3$; $\bar{n} = 10^{-2}$; $p_0 = p_1 = 1/2$

The MDEP of a quantum OOK system with PACS in function of $\bar{n}_p$ is represented in the figure 4.1. The parameter $\bar{n}_p$ represents the mean number of photons in the system and it is defined as

$$\bar{n}_p = \frac{1}{2}\left(n_{p_0} + n_{p_1}\right)$$

where n_{p_i} is the mean number of photons in the state $\boldsymbol{\Xi}_i$. The parameter $\bar{n}_p$ is equal to $n_p/2$ for OOK systems and to the average of the n_p for each state for the BPSK system. In the y-axes there are the MDEP ($\breve{P}_e$). The plot was obtained for equiprobable symbols and mean number of thermal photons $\bar{n} = 10^{-2}$. The argument of the trace norm $\|\cdot\|_1$ in the Helstrom bound 3.11, is an operator in an infinite dimensional Hilbert space; for the simulation, it has been approximated in $N = 30$ dimension. We can observe that the photon addition improves significantly the performance in terms of error probability. Incrasing the value of the parameter k the MDEP of the system transate; the error probability, for the same energy-level, is lower if k is bigger. We can notice too that the graphs do not start all from 0. This is because the minimum mean number of photons in a photon added state is not always 0 as it possible to see in equation 4.5.

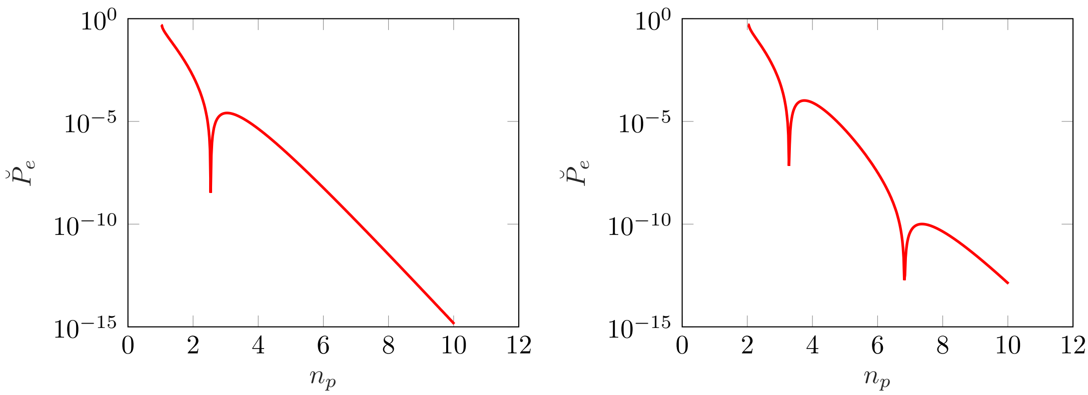
4.1.2 Quantum BPSK

It can be interesting to assess the effect of photon addition in a quantum BPSK system. The constellation is given, for a PACS BPSK, by:

$$\boldsymbol{\Xi}_0 = \boldsymbol{\Xi}_{\text{th}}^{(k)}(-\mu) \tag{4.6a}$$
$$\boldsymbol{\Xi}_1 = \boldsymbol{\Xi}_{\text{th}}^{(k)}(\mu). \tag{4.6b}$$

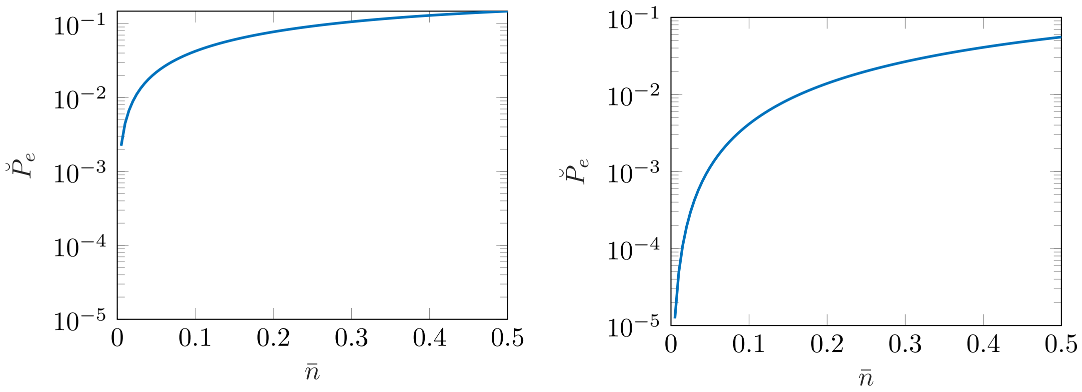
In absence of noise ($\bar{n} = 0$), the MDEP is given by formula 3.13 where

$$|\psi_0\rangle = |-\mu^{(k)}\rangle\,, \tag{4.7a}$$
$$|\psi_1\rangle = |\mu^{(k)}\rangle\,. \tag{4.7b}$$



(a) $k = 1$ (b) $k = 2$

Figure 4.2: MDEP of quantum BPSK as function of n_p, in absence of noise with $N = 30$.



(a) $k = 1$ and $\mu = 0.54$ (b) $k = 2$ and $\mu = 1.58$

Figure 4.3: MDEP in corrispondence of MDEP zeros as function of $\bar{n}$ ($N = 40$).

The inner product is given, in closed form, by [8]:

$$\langle -\mu^{(k)} | \mu^{(k)} \rangle = \frac{L_k(|\mu|^2)}{L_k(-|\mu|^2)} e^{-2|\mu|^2}, \tag{4.8}$$

where $L_k(x)$ is the Laguerre polynomial of parameter k, evaluate in x. In figure 4.2 the MDEP in absence of noise, for QBPSK with PACS, is plotted for $k = 1$ and for $k = 2$, in function of n_p, with $N = 30$. It can be noticed that exist k zeros in the MDEP plot, where k is the number of photon additions, that corresponds to the zeros of $L_k(|\mu|^2)$ in equation 4.8. The existence of these zeros is not really useful for the design of a quantum communication system because their selectivity factors are too high for a phisical implementation. It is, nevertheless, possible to use that in order to evaluate the effect of the thermal noise. In figure 4.3, the sluggish performance due to the thermal noise is clear. The plot shows the trend of MDEP, for zeros value of μ, in function of $\bar{n}$; the used approximation is $N = 30$. The MDEP in presence of noise is given using the expression 3.11. We can shown, using the formula 2.3.1, that $\bar{n}$ in a real cases in near to zero, the plot show the general trend of the MDEP.

The figure 4.4 show a comparison between a QBPSK system and a QOOK system in terms of MDEP as function of $\bar{n}_p$, i.e. the mean number of photons in the system. The plots are given with $\bar{n} = 10^{-2}$, $N = 45$ and equiprobable symbols. The obtained result is really intresting: the quantum BPSK system has a sluggish performance due two the photon addition process. At an equal level of energy $\bar{n}_p$, for PACS systems, it is possible to find an OOK configuration that

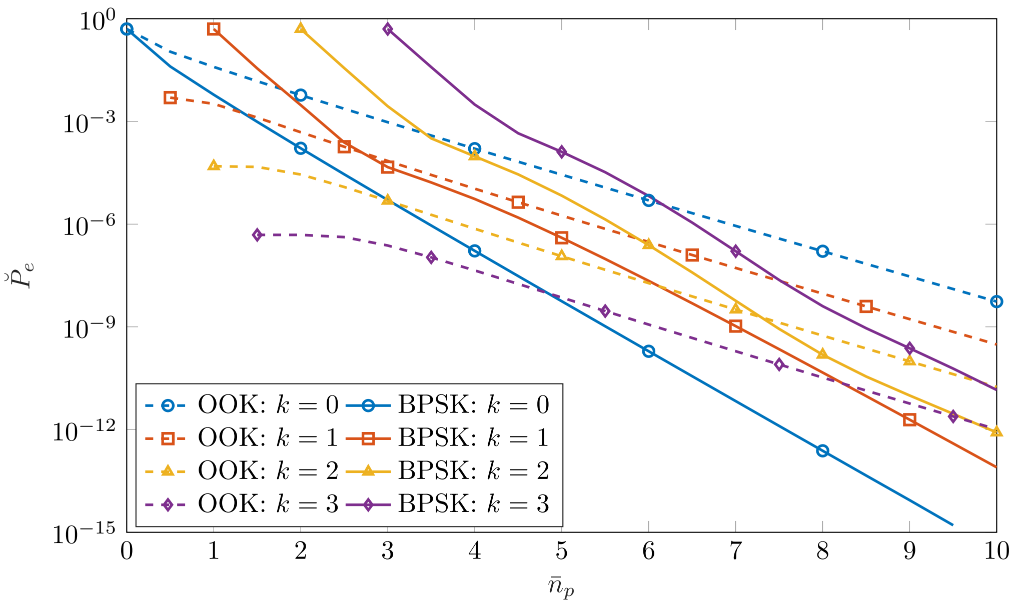


Figure 4.4: BPSK and OOK comparison in terms of MDEP as function of $\bar{n}_p$ with $N = 45$, $\bar{n} = 10^{-2}$, $p_0 = p_1 = 1/2$

maximizes the performance.

4.2 Quantum communication systems with squeezed states

In this section we assess the effect of squeezing on the performance in absence of photon addition and thermal noise. The representation of squeezed states are given in 2.3.2. As for PACS systems, it can be useful to define the mean number of photon n_p in a squeezed state, which is given by

$$n_p(\mu, r) = |\mu|^2 + (\sinh r)^2 \, ; \tag{4.9}$$

where μ is the amplitude of the starter coherent state and the squeezing factor is $\zeta = re^{i\theta}$. The minimum value of n_p is given by $n_p(0, r) = \sinh r^2$.

4.2.1 Quantum OOK

A quantum OOK system with squeezed states is a system with the following constellation

$$\boldsymbol{\Xi}_0 = \boldsymbol{\Xi}(0, 0) \tag{4.10a}$$

$$\boldsymbol{\Xi}_1 = \boldsymbol{\Xi}(\mu, \zeta) \tag{4.10b}$$

where $\boldsymbol{\Xi}_{\text{th}}(\mu, \zeta)$ is the squeezed state with amplitude μ and squeezing parameter ζ.

The MDEP as function of $\bar{n}_p$, i.e the mean number of photon in the system, is plotted in figure 4.5 with: $\theta = \pi$, $N = 45$, equiprobable symbols and $\bar{n} = 0$. It can be noticed that the optimal configuration of r depends on the energy in the system. For low energy levels the squeezing has not a positive effect.

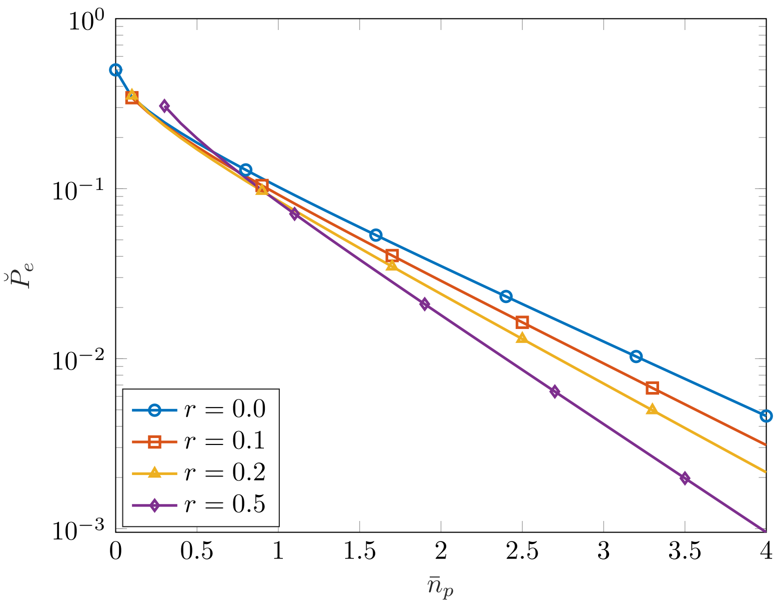


Figure 4.5: MDEP of OOK squeezed states system as function of $\bar{n}_p$ without thermal noise, $N = 45$, $\theta = \pi$ and equiprobable symbols.

4.2.2 Quantum BPSK

The effect of the squeezing in a BPSK system is similar to that in the OOK. The constellation of this type of system is given by

$$\boldsymbol{\Xi}_0 = \boldsymbol{\Xi}(-\mu, \zeta) \tag{4.11a}$$

$$\boldsymbol{\Xi}_1 = \boldsymbol{\Xi}(\mu, \zeta). \tag{4.11b}$$

Figure 4.6 shows the MDEP of the system as function of $\bar{n}_p$ with $\theta = \pi$, $N = 30$, equiprobable symbols and $\bar{n} = 0$. We can notice that for values of $\bar{n}_p$ large enough, the squeezing improves the performance of the system. For a given energy in the system it is so possible to find an optimal squeezing configuration.

4.3 Quantum communication systems with PASSs

The use of squeezed states instead of coherent states allows us to overcome the limits of PACSs. The representation of a noisy squeezed state $\boldsymbol{\Xi}_{\text{th}}(\mu, \zeta)$ is given in 2.3.2 and the performance analysis without thermal noise is given in 4.2. The noisy PASSs $\boldsymbol{\Xi}_{\text{th}}^{(k)}(\mu, \zeta)$ correspondent to the squeezed state $\boldsymbol{\Xi}_{\text{th}}(\mu, \zeta)$ is obtained using the equation 2.29, as it is described in 2.4.1. This section discusses the advantages of using PASSs in quantum OOK and BPSK systems.

The MDEP for PASSs systems is found again with the Helstrom bound 3.11. As for the other systems, it is useful to define the mean number of photons n_p for noisy photon added squeezed states, that is given by:

$$n_p(\mu, \zeta, \bar{n}) = \frac{N_{k+1}(\mu, \zeta, \bar{n})}{N_k(\mu, \zeta, \bar{n})} - 1, \tag{4.12}$$

where

$$\begin{aligned} N_k(\mu, \zeta, \bar{n}) &= \operatorname{tr}\Big\{(\boldsymbol{A}^\dagger)^k \boldsymbol{\Xi}_{\text{th}}(\mu, \zeta) \boldsymbol{A}^k\Big\} \\ &= \operatorname{tr}\Big\{(\boldsymbol{A}^\dagger)^k \boldsymbol{D}_\mu \boldsymbol{S}_\zeta \boldsymbol{\Xi}_{\text{th}} \boldsymbol{S}_\zeta^\dagger \boldsymbol{D}_\mu^\dagger \boldsymbol{A}^k\Big\}. \end{aligned} \tag{4.13}$$

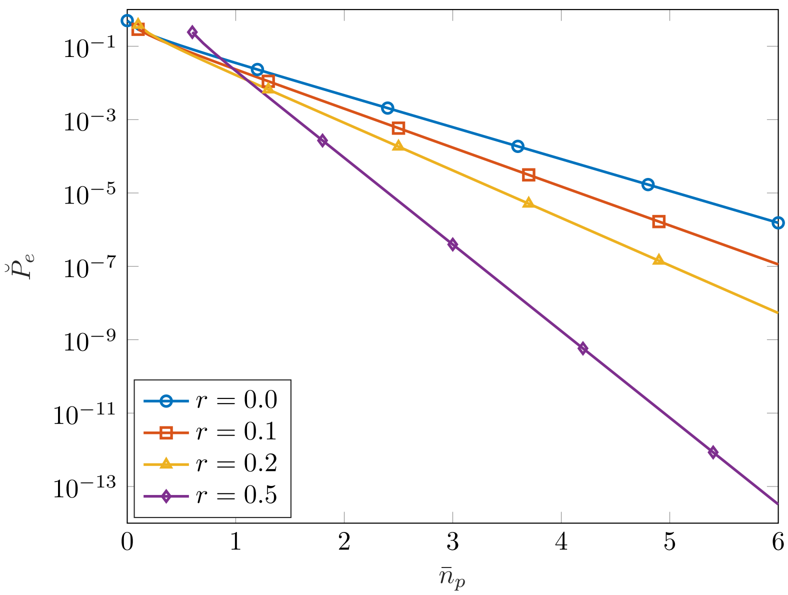


Figure 4.6: MDEP of squeezed state BPSK system as function of $\bar{n}_p$. $N = 30$, $\bar{n} = 0$, $\theta = \pi$, $p_0 = p_1 = 1/2$

4.3.1 Quantum OOK

The constellation of a quantum OOK system with noisy PASS, is given by:

$$\boldsymbol{\Xi}_0 = \boldsymbol{\Xi}_{\text{th}}^{(0)}(0, 0) \tag{4.14a}$$

$$\boldsymbol{\Xi}_1 = \boldsymbol{\Xi}_{\text{th}}^{(k)}(\mu, \zeta). \tag{4.14b}$$

In figure 4.7, the MDEP of a quantum OOK noisy PASS system is plotted in function of the mean number of photon $\bar{n}_p$ in the system. For the simulation are used $N = 30$, $\bar{n} = 10^{-2}$, $\theta = \pi$ and equiprobable symbols. It can be noticed that the photon addition significantly improves the performance of the system, at least for the plotted energy level.

4.3.2 Quantum BPSK

Similary to the PACS BPSK, the constellation of PASS BPSK is given by:

$$\boldsymbol{\Xi}_0 = \boldsymbol{\Xi}_{\text{th}}^{(k)}(-\mu, \zeta) \tag{4.15a}$$

$$\boldsymbol{\Xi}_1 = \boldsymbol{\Xi}_{\text{th}}^{(k)}(\mu, \zeta). \tag{4.15b}$$

The figure 4.8 shows the effects of the photon addition in a quantum BPSK system, in terms of the mean photon number in the system $\bar{n}_p$, given by the average of the mean photon number n_p of $\boldsymbol{\Xi}_0$ and $\boldsymbol{\Xi}_1$. The parameters used for the simulation are $N = 40$, $\bar{n} = 10^{-2}$, $\theta = \pi$ and equiprobable symbols. As in PACS case, it is evident that the photon addition, in a BPSK system, has not the positive effect that has in an OOK system.

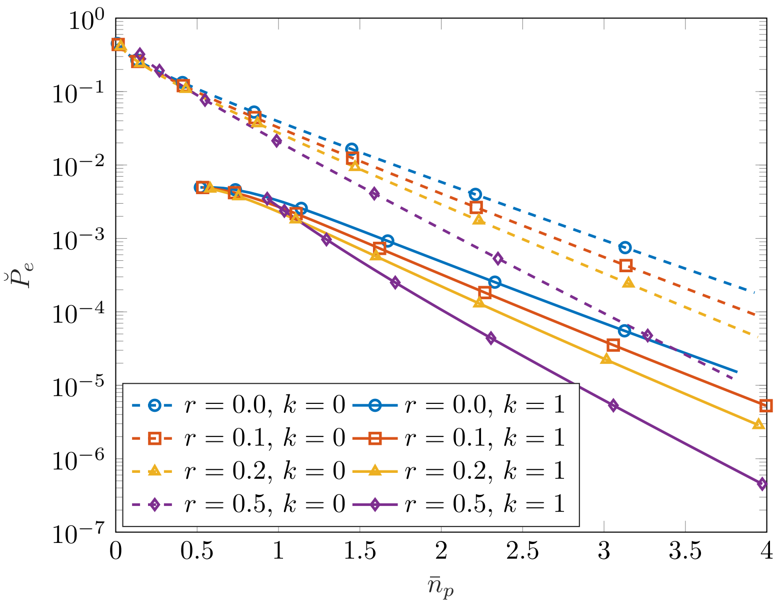


Figure 4.7: MDEP of noisy PASS quantum OOK system as function of $\bar{n}_p$ with $N = 30$, $\bar{n} = 10^{-2}$, $\theta = \pi$ and $p_0 = p_1 = 1/2$.

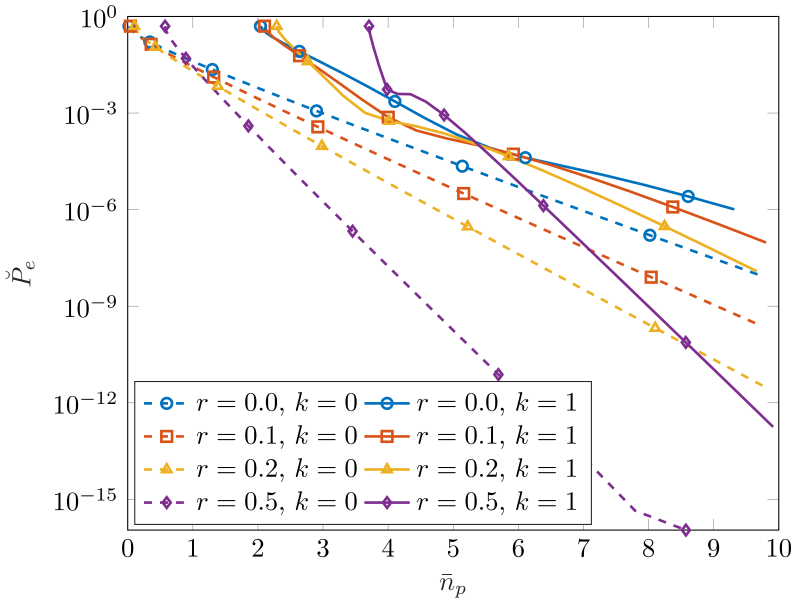


Figure 4.8: MDEP of noisy PASS quantum BPSK system as function of $\bar{n}_p$ with $N = 40$, $\bar{n} = 10^{-2}$, $\theta = \pi$ and $p_0 = p_1 = 1/2$.

Chapter 5

Conclusion

The aim of this thesis was the characterization of the perfomance of binary communication systems with non-Gaussian states. This perfomance was evaluated in terms of error probability relating to symbol recognition. In the first place, we have presented the quantum mechanics postulates, formulated by Dirac and Von Neumann and generalized with the use of density operators. So we have introduced non-Gaussian quantum states and, in particular, PACSs and PASSs. Then, we have described the quantum modulation and quantum receiver concepts. Finally, we have analyzed some systems perfomance in terms of MDEP. All evaluations were made assuming the absence of effects associated to the communication channel, then supposing that the recieved state does not present any difference compared to the emitted one.

The findings of this thesis is to highlights the fact that the use of non-Gaussian photon added states instead of gaussian states in OOK systems can ameliorate the QSD. In particular, the combination of squeezing and photon addition turns out extremely effective. Instead, in BPSK quantum systems, the photon addition effect manifests itself as negative.

The obtained results can be significantly important in a quantum communication system design, in which the use of quantum mechanics can provide a significant advantage to the system performance.

Bibliography

[1] C. E. Shannon, "A mathematical theory of communication," *The Bell System Technical Journal*, vol. 27, no. 3, pp. 379–423, 1948.

[2] C. W. Helstrom, *Quantum detection and estimation theory.* Academic Press New York, 1976.

[3] S. Guerrini, "Quantum communications: State characterization and system design," Ph.D. dissertation, Department of Engineering, University of Ferrara, Ferrara, Italy, Sep. 2021, thesis advisor: Professor Andrea Conti.

[4] S. Guerrini, M. Chiani, M. Z. Win, and A. Conti, "Quantum pulse position modulation with photon-added squeezed states," in *Proc. IEEE Workshop on Quantum Commun. and Inf. Technol. (QCIT), Global Telecomm. Conf.*, Taipei, Taiwan, Dec. 2020, pp. 1–5.

[5] ——, "Quantum pulse position modulation with photon-added coherent states," in *Proc. IEEE Workshop on Quantum Commun. and Inf. Technol. (QCIT), Global Telecomm. Conf.*, Waikoloa, HI, USA, Dec. 2019, pp. 1–5.

[6] S. Guerrini, M. Chiani, and A. Conti, "Secure key throughput of intermittent trusted-relay QKD protocols," in *Proc. IEEE Workshop on Quantum Commun. and Inf. Technol. (QCIT), Global Telecomm. Conf.*, Abu Dhabi, UAE, Dec. 2018, pp. 1–5.

[7] M. Chiani, A. Conti, and M. Z. Win, "Piggybacking on quantum streams," *Phys. Rev. A*, vol. 102, p. 012410, Jul. 2020.

[8] S. Guerrini, M. Z. Win, M. Chiani, and A. Conti, "Quantum discrimination of noisy photon-added coherent states," *IEEE J. Sel. Areas Inf. Theory*, vol. 1, no. 2, pp. 469–479, Aug. 2020, special issue on *Quantum Information Science.*

[9] P. A. M. Dirac, *The Principles of Quantum Mechanics.* Clarendon Press, 1930.

[10] J. von Neumann, *Mathematical Foundations of Quantum Mechanics: New Edition*, N. A. Wheeler, Ed. Princeton University Press, 2018.

[11] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, "Gaussian quantum information," *Rev. Mod. Phys.*, vol. 84, pp. 621–669, May 2012.

[12] E. Wigner, "On the quantum correction for thermodynamic equilibrium," *Phys. Rev.*, vol. 40, pp. 749–759, Jun 1932.

[13] S. L. Braunstein and P. van Loock, "Quantum information with continuous variables," *Rev. Mod. Phys.*, vol. 77, no. 2, p. 513–577, Jun 2005.

[14] A. Ferraro, S. Olivares, and M. Paris, *Gaussian states in continuous variable quantum information*, ser. Napoli Series on physics and Astrophysics. Bibliopolis, 2005.

[15] G. Adesso, S. Ragy, and A. R. Lee, "Continuous variable quantum information: Gaussian states and beyond," *Open Systems & Information Dynamics*, vol. 21, no. 01n02, p. 1440001, Mar 2014.

[16] A. Serafini, *Quantum Continuous Variables: A Primer of Theoretical Methods*, 07 2017.

[17] R. J. Glauber, "The quantum theory of optical coherence," *Phys. Rev.*, vol. 130, pp. 2529–2539, Jun 1963.

[18] ——, "Coherent and incoherent states of the radiation field," *Phys. Rev.*, vol. 131, 9 1963.

[19] C. W. Gardiner and P. Zoller, *Quantum Noise*, 2nd ed. Springer, 2000.

[20] J. Dowling, *Schrodinger's Killer App: Race to Build the World's First Quantum Computer*, 05 2013.

[21] H. P. Yuen, "Two-photon coherent states of the radiation field," *Phys. Rev. A*, vol. 13, pp. 2226–2243, Jun 1976.

[22] C. M. Caves, "Quantum-mechanical noise in an interferometer," *Phys. Rev. D*, vol. 23, pp. 1693–1708, Apr 1981.

[23] P. Marian and T. A. Marian, "Squeezed states with thermal noise. i. photon-number statistics," *Phys. Rev. A*, vol. 47, pp. 4474–4486, May 1993.

[24] C. W. Helstrom, "Detection theory and quantum mechanics," *Information and Control*, vol. 10, no. 3, pp. 254–291, 1967.

[25] C. W. Helstrom, "Fundamental limitations on the detectability of electromagnetic signals," *International Journal of Theoretical Physics*, vol. 1, no. 1, pp. 37–50, May 1968.

[26] R. Yoshitani, "On the detectabillity limit of coherent optical signals in thermal radiation," *J. Stat. Phys.*, vol. 2, no. 4, pp. 347–378, Dec. 1970.

[27] M. Takeoka and M. Sasaki, "Discrimination of the binary coherent signal: Gaussian-operation limit and simple non-gaussian near-optimal receivers," *Phys. Rev. A*, vol. 78, no. 2, Aug 2008.

[28] N. Dalla Pozza and G. Pierobon, "Optimality of square-root measurements in quantum state discrimination," *Phys. Rev. A*, vol. 91, p. 042334, Apr 2015.

[29] R. Yuan, M. Zhao, S. Han, and J. Cheng, "Kennedy receiver using threshold detection and optimized displacement under thermal noise," *IEEE Communications Letters*, vol. 24, no. 6, pp. 1313–1317, 2020.

[30] A. S. Holevo, *Probabilistic and Statistical Aspects of Quantum Theory; 2nd ed.*, ser. Publications of the Scuola Normale Superiore. Monographs. Dordrecht: Springer, 2011.