

Math Reference

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1 Equations of Motion

Let (X,Y,Z) be EME2000 fixed frame Cartesian co-ordinates. The orbit of an object is governed by the following equations of motion:

$$\begin{aligned}\ddot{X} &= f_{Kep,X}(X, Y, Z) + f_{J2,X}(X, Y, Z) + f_{C22,X}(X, Y, Z, t) + f_{S22,X}(X, Y, Z, t) \\ &+ f_{Moon,X}(X, Y, Z, t) + f_{Sun,X}(X, Y, Z, t) + f_{SRP,X}(X, Y, Z, t) \\ \ddot{Y} &= f_{Kep,Y}(X, Y, Z) + f_{J2,Y}(X, Y, Z) + f_{C22,Y}(X, Y, Z, t) + f_{S22,Y}(X, Y, Z, t) \\ &+ f_{Moon,Y}(X, Y, Z, t) + f_{Sun,Y}(X, Y, Z, t) + f_{SRP,Y}(X, Y, Z, t) \\ \ddot{Z} &= f_{Kep,Z}(X, Y, Z) + f_{J2,Z}(X, Y, Z) + f_{C22,Z}(X, Y, Z, t) + f_{S22,Z}(X, Y, Z, t) \\ &+ f_{Moon,Z}(X, Y, Z, t) + f_{Sun,Z}(X, Y, Z, t) + f_{SRP,Z}(X, Y, Z, t)\end{aligned}\tag{1}$$

1.1 KepComponent

$$\begin{aligned}f_{Kep,X}(X, Y, Z) &= -\frac{GM_E X}{(X^2+Y^2+Z^2)^{3/2}} \\ f_{Kep,Y}(X, Y, Z) &= -\frac{GM_E Y}{(X^2+Y^2+Z^2)^{3/2}} \\ f_{Kep,Z}(X, Y, Z) &= -\frac{GM_E Z}{(X^2+Y^2+Z^2)^{3/2}}\end{aligned}\tag{2}$$

1.1.1 Implementation details

First calculate shared factor

$$d = \frac{1}{(X^2+Y^2+Z^2)^{3/2}}\tag{3}$$

After that calculate

$$\begin{aligned}f_{Kep,X}(X, Y, Z) &= -GM_E dX \\ f_{Kep,Y}(X, Y, Z) &= -GM_E dY \\ f_{Kep,Z}(X, Y, Z) &= -GM_E dZ\end{aligned}\tag{4}$$

1.2 J2Component

$$\begin{aligned} f_{J2,X}(X, Y, Z) &= \frac{GM_E R_E^2 \sqrt{5} C_{20} X}{2(X^2 + Y^2 + Z^2)^{1/2}} \left(\frac{3}{(X^2 + Y^2 + Z^2)^2} - \frac{15Z^2}{(X^2 + Y^2 + Z^2)^3} \right) \\ f_{J2,Y}(X, Y, Z) &= \frac{GM_E R_E^2 \sqrt{5} C_{20} Y}{2(X^2 + Y^2 + Z^2)^{1/2}} \left(\frac{3}{(X^2 + Y^2 + Z^2)^2} - \frac{15Z^2}{(X^2 + Y^2 + Z^2)^3} \right) \\ f_{J2,Z}(X, Y, Z) &= \frac{GM_E R_E^2 \sqrt{5} C_{20} Z}{2(X^2 + Y^2 + Z^2)^{1/2}} \left(\frac{9}{(X^2 + Y^2 + Z^2)^2} - \frac{15Z^2}{(X^2 + Y^2 + Z^2)^3} \right) \end{aligned} \quad (5)$$

1.2.1 Implementation details

Precalculate constant factor

$$f = \frac{GM_E R_E^2 \sqrt{5} C_{20}}{2} \quad (6)$$

Per (X,Y,Z) calculate

$$\begin{aligned} d_1 &= f \frac{1}{\sqrt{X^2 + Y^2 + Z^2}} \\ d_2 &= \frac{1}{(X^2 + Y^2 + Z^2)^2} \\ s &= \frac{15Z^2}{(X^2 + Y^2 + Z^2)^3} \end{aligned} \quad (7)$$

And

$$\begin{aligned} f_{J2,X}(X, Y, Z) &= d_1 X (3d_2 - s) \\ f_{J2,Y}(X, Y, Z) &= d_1 Y (3d_2 - s) \\ f_{J2,Z}(X, Y, Z) &= d_1 Z (9d_2 - s) \end{aligned} \quad (8)$$

1.3 C22S22Component

$$\begin{aligned} f_{C22,X}(X, Y, Z, t) &= f_{C22,x}(x, y, z) \cos(\theta_G + \nu_E t) - f_{C22,y}(x, y, z) \sin(\theta_G + \nu_E t) \\ f_{C22,Y}(X, Y, Z, t) &= f_{C22,x}(x, y, z) \sin(\theta_G + \nu_E t) + f_{C22,y}(x, y, z) \cos(\theta_G + \nu_E t) \\ f_{C22,Z}(X, Y, Z, t) &= f_{C22,z}(x, y, z) \\ f_{S22,X}(X, Y, Z, t) &= f_{S22,x}(x, y, z) \cos(\theta_G + \nu_E t) - f_{S22,y}(x, y, z) \sin(\theta_G + \nu_E t) \\ f_{S22,Y}(X, Y, Z, t) &= f_{S22,x}(x, y, z) \sin(\theta_G + \nu_E t) + f_{S22,y}(x, y, z) \cos(\theta_G + \nu_E t) \\ f_{S22,Z}(X, Y, Z, t) &= f_{S22,z}(x, y, z) \end{aligned} \quad (9)$$

with

$$\begin{aligned} x &= X \cos(\theta_G + \nu_E t) + Y \sin(\theta_G + \nu_E t) \\ y &= -X \sin(\theta_G + \nu_E t) + Y \cos(\theta_G + \nu_E t) \\ z &= Z \end{aligned} \quad (10)$$

and

$$\begin{aligned}
f_{C22,x}(x, y, z) &= \frac{5GM_E R_E^2 \sqrt{15} C_{22} x (y^2 - x^2)}{2(x^2 + y^2 + z^2)^{7/2}} + \frac{GM_E R_E^2 \sqrt{15} C_{22} x}{(x^2 + y^2 + z^2)^{5/2}} \\
f_{C22,y}(x, y, z) &= \frac{5GM_E R_E^2 \sqrt{15} C_{22} y (y^2 - x^2)}{2(x^2 + y^2 + z^2)^{7/2}} - \frac{GM_E R_E^2 \sqrt{15} C_{22} y}{(x^2 + y^2 + z^2)^{5/2}} \\
f_{C22,z}(x, y, z) &= \frac{5GM_E R_E^2 \sqrt{15} C_{22} z (y^2 - x^2)}{2(x^2 + y^2 + z^2)^{7/2}} \\
f_{S22,x}(x, y, z) &= -\frac{5GM_E R_E^2 \sqrt{15} S_{22} x^2 y}{(x^2 + y^2 + z^2)^{7/2}} + \frac{GM_E R_E^2 \sqrt{15} S_{22} y}{(x^2 + y^2 + z^2)^{5/2}} \\
f_{S22,y}(x, y, z) &= -\frac{5GM_E R_E^2 \sqrt{15} S_{22} x y^2}{(x^2 + y^2 + z^2)^{7/2}} + \frac{GM_E R_E^2 \sqrt{15} S_{22} x}{(x^2 + y^2 + z^2)^{5/2}} \\
f_{S22,z}(x, y, z) &= -\frac{5GM_E R_E^2 \sqrt{15} S_{22} x y z}{(x^2 + y^2 + z^2)^{7/2}}
\end{aligned} \tag{11}$$

1.3.1 Implementation details

Precalculate constant factors

$$f = GM_E R_E^2 \sqrt{15} \tag{12}$$

$$\begin{aligned}
f_{C22_2} &= GM_E R_E^2 \sqrt{15} C_{22} \\
&= f_{C22} \\
f_{C22_1} &= \frac{5GM_E R_E^2 \sqrt{15} C_{22}}{2} \\
&= f_{C22_2} \left(\frac{5}{2} \right)
\end{aligned} \tag{13}$$

$$\begin{aligned}
f_{S22_2} &= GM_E R_E^2 \sqrt{15} S_{22} \\
&= f_{S22} \\
f_{S22_1} &= -5GM_E R_E^2 \sqrt{15} S_{22} \\
&= -5f_{S22_2}
\end{aligned} \tag{14}$$

Per time step calculate

$$\begin{aligned}
f_{sin} &= \sin(\theta_G + \nu_E t) \\
f_{cos} &= \cos(\theta_G + \nu_E t)
\end{aligned} \tag{15}$$

Per (X,Y,Z) calculate

$$\begin{aligned}
x &= X f_{cos} + Y f_{sin} \\
y &= -X f_{sin} + Y f_{cos} \\
z &= Z
\end{aligned} \tag{16}$$

Shared terms

$$\begin{aligned}
n_{C22} &= f_{C22_1} (y^2 - x^2) \\
n_{S22} &= f_{S22_1} xy
\end{aligned} \tag{17}$$

$$\begin{aligned}
d_1 &= \frac{1}{\sqrt{(x^2 + y^2 + z^2)^7}} \\
d_2 &= \frac{1}{\sqrt{(x^2 + y^2 + z^2)^5}}
\end{aligned} \tag{18}$$

Calculate

$$\begin{aligned}
f_{C22,x}(x,y,z) &= n_{C22}xd_1 + f_{C22_2}xd_2 \\
f_{C22,y}(x,y,z) &= n_{C22}yd_1 + f_{C22_2}yd_2 \\
f_{C22,z}(x,y,z) &= n_{C22}zd_1 \\
f_{S22,x}(x,y,z) &= n_{S22}xd_1 + f_{S22_2}yd_2 \\
f_{S22,y}(x,y,z) &= n_{S22}yd_1 + f_{S22_2}xd_2 \\
f_{S22,z}(x,y,z) &= n_{S22}zd_1
\end{aligned} \tag{19}$$

$$\begin{aligned}
f_{C22,X}(X,Y,Z,t) &= f_{C22,x}(x,y,z)f_{cos} - f_{C22,y}(x,y,z)f_{sin} \\
f_{C22,Y}(X,Y,Z,t) &= f_{C22,x}(x,y,z)f_{sin} + f_{C22,y}(x,y,z)f_{cos} \\
f_{C22,Z}(X,Y,Z,t) &= f_{C22,z}(x,y,z) \\
f_{S22,X}(X,Y,Z,t) &= f_{S22,x}(x,y,z)f_{cos} - f_{S22,y}(x,y,z)f_{sin} \\
f_{S22,Y}(X,Y,Z,t) &= f_{S22,x}(x,y,z)f_{sin} + f_{S22,y}(x,y,z)f_{cos} \\
f_{S22,Z}(X,Y,Z,t) &= f_{S22,z}(x,y,z)
\end{aligned} \tag{20}$$

1.4 SolComponent

$$\begin{aligned}
f_{Sun,X}(X,Y,Z,t) &= -GM_{\odot} \left(\frac{(X-X_{\odot})}{[(X-X_{\odot})^2+(Y-Y_{\odot})^2+(Z-Z_{\odot})^2]^{3/2}} + \frac{X_{\odot}}{(X_{\odot}^2+Y_{\odot}^2+Z_{\odot}^2)^{3/2}} \right) \\
f_{Sun,Y}(X,Y,Z,t) &= -GM_{\odot} \left(\frac{(Y-Y_{\odot})}{[(X-X_{\odot})^2+(Y-Y_{\odot})^2+(Z-Z_{\odot})^2]^{3/2}} + \frac{Y_{\odot}}{(X_{\odot}^2+Y_{\odot}^2+Z_{\odot}^2)^{3/2}} \right) \\
f_{Sun,Z}(X,Y,Z,t) &= -GM_{\odot} \left(\frac{(Z-Z_{\odot})}{[(X-X_{\odot})^2+(Y-Y_{\odot})^2+(Z-Z_{\odot})^2]^{3/2}} + \frac{Z_{\odot}}{(X_{\odot}^2+Y_{\odot}^2+Z_{\odot}^2)^{3/2}} \right)
\end{aligned} \tag{21}$$

where

$$\begin{pmatrix} X_{\odot} \\ Y_{\odot} \\ Z_{\odot} \end{pmatrix} = \begin{pmatrix} r_{\odot} \cos \lambda_{\odot} \\ r_{\odot} \sin \lambda_{\odot} \cos \varepsilon \\ r_{\odot} \sin \lambda_{\odot} \sin \varepsilon \end{pmatrix} \tag{22}$$

with

$$\begin{aligned}
\lambda_{\odot} &= \Omega_{\odot} + \omega_{\odot} + \ell_{\odot} + \left(\frac{6892}{3600} \sin \ell_{\odot} + \frac{72}{3600} \sin 2\ell_{\odot} \right) \\
r_{\odot}[10^6 \text{km}] &= 149.619 - 2.499 \cos \ell_{\odot} - 0.021 \cos 2\ell_{\odot} \\
\ell_{\odot} &= \varphi_{\odot,0} + \nu_{\odot} t
\end{aligned} \tag{23}$$

1.4.1 Implementation details

Per time step calculate

$$\begin{aligned}
\lambda_{\odot} &= \Omega_{\odot} + \omega_{\odot} + \ell_{\odot} + \left(\frac{6892}{3600} \sin \ell_{\odot} + \frac{72}{3600} \sin 2\ell_{\odot} \right) \\
r_{\odot}[10^6 \text{km}] &= 149.619 - 2.499 \cos \ell_{\odot} - 0.021 \cos 2\ell_{\odot} \\
\ell_{\odot} &= \varphi_{\odot,0} + \nu_{\odot} t
\end{aligned} \tag{24}$$

$$\begin{aligned}
f_{X_{\odot}} &= \cos \lambda_{\odot} \\
f_{Y_{\odot}} &= \sin \lambda_{\odot} \cos \varepsilon \\
f_{Z_{\odot}} &= \sin \lambda_{\odot} \sin \varepsilon
\end{aligned} \tag{25}$$

$$\begin{aligned}
X_{\odot} &= f_{X_{\odot}} r_{\odot} \times 10^6 \\
Y_{\odot} &= f_{Y_{\odot}} r_{\odot} \times 10^6 \\
Z_{\odot} &= f_{Z_{\odot}} r_{\odot} \times 10^6
\end{aligned} \tag{26}$$

$$d_1 = \frac{1}{\sqrt{(X_\odot^2 + Y_\odot^2 + Z_\odot^2)^3}} \quad (27)$$

$$\begin{aligned} P_{X_\odot} &= d_1 X_\odot \\ P_{Y_\odot} &= d_1 Y_\odot \\ P_{Z_\odot} &= d_1 Z_\odot \end{aligned} \quad (28)$$

Per (X,Y,Z) calculate

$$d_2 = \frac{1}{\sqrt{((X-X_\odot)^2 + (Y-Y_\odot)^2 + (Z-Z_\odot)^2)^3}} \quad (29)$$

$$\begin{aligned} f_{Sun,X}(X,Y,Z,t) &= -GM_\odot(d_2(X - X_\odot) + P_{X_\odot}) \\ f_{Sun,Y}(X,Y,Z,t) &= -GM_\odot(d_2(Y - Y_\odot) + P_{Y_\odot}) \\ f_{Sun,Z}(X,Y,Z,t) &= -GM_\odot(d_2(Z - Z_\odot) + P_{Z_\odot}) \end{aligned} \quad (30)$$

1.5 LunComponent

$$\begin{aligned} f_{Moon,X}(X,Y,Z,t) &= -GM_{\mathcal{M}} \left(\frac{(X-X_{\mathcal{M}})}{[(X-X_{\mathcal{M}})^2 + (Y-Y_{\mathcal{M}})^2 + (Z-Z_{\mathcal{M}})^2]^{3/2}} + \frac{X_{\mathcal{M}}}{(X_{\mathcal{M}}^2 + Y_{\mathcal{M}}^2 + Z_{\mathcal{M}}^2)^{3/2}} \right) \\ f_{Moon,Y}(X,Y,Z,t) &= -GM_{\mathcal{M}} \left(\frac{(Y-Y_{\mathcal{M}})}{[(X-X_{\mathcal{M}})^2 + (Y-Y_{\mathcal{M}})^2 + (Z-Z_{\mathcal{M}})^2]^{3/2}} + \frac{Y_{\mathcal{M}}}{(X_{\mathcal{M}}^2 + Y_{\mathcal{M}}^2 + Z_{\mathcal{M}}^2)^{3/2}} \right) \\ f_{Moon,Z}(X,Y,Z,t) &= -GM_{\mathcal{M}} \left(\frac{(Z-Z_{\mathcal{M}})}{[(X-X_{\mathcal{M}})^2 + (Y-Y_{\mathcal{M}})^2 + (Z-Z_{\mathcal{M}})^2]^{3/2}} + \frac{Z_{\mathcal{M}}}{(X_{\mathcal{M}}^2 + Y_{\mathcal{M}}^2 + Z_{\mathcal{M}}^2)^{3/2}} \right) \end{aligned} \quad (31)$$

where

$$\begin{pmatrix} X_{\mathcal{M}} \\ Y_{\mathcal{M}} \\ Z_{\mathcal{M}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon & -\sin \varepsilon \\ 0 & \sin \varepsilon & \cos \varepsilon \end{pmatrix} \cdot \begin{pmatrix} r_{\mathcal{M}} \cos \lambda_{\mathcal{M}} \cos \beta_{\mathcal{M}} \\ r_{\mathcal{M}} \sin \lambda_{\mathcal{M}} \cos \beta_{\mathcal{M}} \\ r_{\mathcal{M}} \sin \beta_{\mathcal{M}} \end{pmatrix} \quad (32)$$

with

$$\begin{aligned} r_{\mathcal{M}}[\text{km}] &= 385000 - 20905 \cos(l_{\mathcal{M}}) - 3699 \cos(2D_{\mathcal{M}} - l_{\mathcal{M}}) \\ &\quad - 2956 \cos(2D_{\mathcal{M}}) - 570 \cos(2l_{\mathcal{M}}) \\ &\quad + 246 \cos(2l_{\mathcal{M}} - 2D_{\mathcal{M}}) - 205 \cos(l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \\ &\quad - 171 \cos(l_{\mathcal{M}} + 2D_{\mathcal{M}}) \\ &\quad - 152 \cos(l_{\mathcal{M}} + l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \end{aligned} \quad (33)$$

$$\begin{aligned} \lambda_{\mathcal{M}} &= L_0 + \left(\frac{22640}{3600} \sin(l_{\mathcal{M}}) + \frac{769}{3600} \sin(2l_{\mathcal{M}}) \right. \\ &\quad - \frac{4856}{3600} \sin(l_{\mathcal{M}} - 2D_{\mathcal{M}}) + \frac{2370}{3600} \sin(2D_{\mathcal{M}}) \\ &\quad - \frac{668}{3600} \sin(l'_{\mathcal{M}}) - \frac{412}{3600} \sin(2F_{\mathcal{M}}) \\ &\quad - \frac{212}{3600} \sin(2l_{\mathcal{M}} - 2D_{\mathcal{M}}) - \frac{206}{3600} \sin(l_{\mathcal{M}} + l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \\ &\quad + \frac{192}{3600} \sin(l_{\mathcal{M}} + 2D_{\mathcal{M}}) - \frac{165}{3600} \sin(l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \\ &\quad + \frac{148}{3600} \sin(l_{\mathcal{M}} - l'_{\mathcal{M}}) - \frac{125}{3600} \sin(D_{\mathcal{M}}) \\ &\quad \left. - \frac{110}{3600} \sin(l_{\mathcal{M}} + l'_{\mathcal{M}}) - \frac{59}{3600} \sin(2F_{\mathcal{M}} - 2D_{\mathcal{M}}) \right) \end{aligned} \quad (34)$$

$$\begin{aligned}
\beta_{\mathcal{M}} = & \left(\frac{18520}{3600} \sin(F_{\mathcal{M}} + \lambda_{\mathcal{M}} - L_0 + (\frac{412}{3600} \sin(2F_{\mathcal{M}}) + \frac{541}{3600} \sin(l'_{\mathcal{M}}))) \right. \\
& - \frac{526}{3600} \sin(F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& + \frac{44}{3600} \sin(l_{\mathcal{M}} + F_{\mathcal{M}} - 2D_{\mathcal{M}}) - \frac{31}{3600} \sin(-l_{\mathcal{M}} + F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& - \frac{25}{3600} \sin(-2l_{\mathcal{M}} + F_{\mathcal{M}}) - \frac{23}{3600} \sin(l'_{\mathcal{M}} + F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& \left. + \frac{21}{3600} \sin(-l_{\mathcal{M}} + F_{\mathcal{M}}) + \frac{11}{3600} \sin(-l'_{\mathcal{M}} + F_{\mathcal{M}} - 2D_{\mathcal{M}}) \right)
\end{aligned} \tag{35}$$

and

$$\begin{aligned}
\varphi_M &= \nu_{\odot} t \\
\varphi_{M_a} &= \nu_{M_a} t \\
\varphi_{M_p} &= \nu_{M_p} t \\
\varphi_{M_s} &= \nu_{M_s} t \\
L_0 &= \varphi_{M_p} + \varphi_{M_a} + (218.31617) \\
l_{\mathcal{M}} &= \varphi_{M_a} + (134.96292) \\
l'_{\mathcal{M}} &= \ell_{\odot} = \varphi_M + (357.52543) \\
F_{\mathcal{M}} &= \varphi_{M_p} + \varphi_{M_a} + \varphi_{M_s} + (93.27283) \\
D_{\mathcal{M}} &= \varphi_{M_p} + \varphi_{M_a} - \varphi_M + (297.85027)
\end{aligned} \tag{36}$$

1.5.1 Implementation details

Per time step calculate

$$\begin{aligned}
\varphi_M &= \nu_{\odot} t \\
\varphi_{M_a} &= \nu_{M_a} t \\
\varphi_{M_p} &= \nu_{M_p} t \\
\varphi_{M_s} &= \nu_{M_s} t \\
L_0 &= \varphi_{M_p} + \varphi_{M_a} + (218.31617) \\
l_{\mathcal{M}} &= \varphi_{M_a} + (134.96292) \\
l'_{\mathcal{M}} &= \ell_{\odot} = \varphi_M + (357.52543) \\
F_{\mathcal{M}} &= \varphi_{M_p} + \varphi_{M_a} + \varphi_{M_s} + (93.27283) \\
D_{\mathcal{M}} &= \varphi_{M_p} + \varphi_{M_a} - \varphi_M + (297.85027)
\end{aligned} \tag{37}$$

Rearrangement of terms clarifies order of executed calculations

$$\begin{aligned}
r_{\mathcal{M}} = & -152 \cos(l_{\mathcal{M}} + l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& -171 \cos(l_{\mathcal{M}} + 2D_{\mathcal{M}}) \\
& -205 \cos(l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& +246 \cos(2l_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& -2956 \cos(2D_{\mathcal{M}}) \\
& -3699 \cos(2D_{\mathcal{M}} - l_{\mathcal{M}}) \\
& -20905 \cos(l_{\mathcal{M}}) \\
& +385000
\end{aligned} \tag{38}$$

$$\begin{aligned}
\lambda_{\mathcal{M}} = & -\frac{55}{3600} \sin(2F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& -\frac{116}{3600} \sin(l_{\mathcal{M}} + l'_{\mathcal{M}}) \\
& -\frac{125}{3600} \sin(D_{\mathcal{M}}) \\
& +\frac{148}{3600} \sin(l_{\mathcal{M}} - l'_{\mathcal{M}}) \\
& -\frac{165}{3600} \sin(l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& +\frac{192}{3600} \sin(l_{\mathcal{M}} + 2D_{\mathcal{M}}) \\
& -\frac{206}{3600} \sin(l_{\mathcal{M}} + l'_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& -\frac{212}{3600} \sin(2l_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& -\frac{412}{3600} \sin(2F_{\mathcal{M}}) \\
& -\frac{668}{3600} \sin(l'_{\mathcal{M}}) \\
& +\frac{769}{3600} \sin(2l_{\mathcal{M}}) \\
& +\frac{2370}{3600} \sin(2D_{\mathcal{M}}) \\
& -\frac{4856}{3600} \sin(l_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& +\frac{22640}{3600} \sin(l_{\mathcal{M}}) \\
& +L_0
\end{aligned} \tag{39}$$

$$\begin{aligned}
\beta_{\mathcal{M}} = & \frac{11}{3600} \sin(-l'_{\mathcal{M}} + F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& +\frac{21}{3600} \sin(-l_{\mathcal{M}} + F_{\mathcal{M}}) + F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& -\frac{23}{3600} \sin(l'_{\mathcal{M}}) \\
& -\frac{25}{3600} \sin(-2l_{\mathcal{M}} + F_{\mathcal{M}}) \\
& -\frac{31}{3600} \sin(-l_{\mathcal{M}} + F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& +\frac{44}{3600} \sin(l_{\mathcal{M}} + F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& -\frac{526}{3600} \sin(F_{\mathcal{M}} - 2D_{\mathcal{M}}) \\
& +\frac{18520}{3600} \sin(F_{\mathcal{M}} + \lambda_{\mathcal{M}} - L_0 + (\frac{412}{3600} \sin(2F_{\mathcal{M}} + \frac{541}{3600} \sin(l'_{\mathcal{M}})))
\end{aligned} \tag{40}$$

$$\begin{aligned}
f_{\mathcal{M},1} &= \cos \lambda_{\mathcal{M}} \cos \beta_{\mathcal{M}} \\
f_{\mathcal{M},2} &= \sin \lambda_{\mathcal{M}} \cos \beta_{\mathcal{M}} \\
f_{\mathcal{M},3} &= \sin \beta_{\mathcal{M}}
\end{aligned} \tag{41}$$

$$\begin{aligned}
X_{\mathcal{M}} &= f_{\mathcal{M},1} r_{\mathcal{M}} \\
Y_{\mathcal{M}} &= (f_{\mathcal{M},2} \cos \varepsilon - f_{\mathcal{M},3} \sin \varepsilon) r_{\mathcal{M}} \\
Z_{\mathcal{M}} &= (f_{\mathcal{M},2} \sin \varepsilon + f_{\mathcal{M},3} \cos \varepsilon) r_{\mathcal{M}}
\end{aligned} \tag{42}$$

$$d_1 = \frac{1}{\sqrt{(X_{\mathcal{M}}^2 + Y_{\mathcal{M}}^2 + Z_{\mathcal{M}}^2)^3}} \tag{43}$$

$$\begin{aligned}
P_{X_{\mathcal{M}}} &= d_1 X_{\mathcal{M}} \\
P_{Y_{\mathcal{M}}} &= d_1 Y_{\mathcal{M}} \\
P_{Z_{\mathcal{M}}} &= d_1 Z_{\mathcal{M}}
\end{aligned} \tag{44}$$

Per (X,Y,Z) calculate

$$d_2 = \frac{1}{\sqrt{((X-X_{\mathcal{M}})^2 + (Y-Y_{\mathcal{M}})^2 + (Z-Z_{\mathcal{M}})^2)^3}} \tag{45}$$

$$\begin{aligned}
f_{Moon,X}(X,Y,Z,t) &= -GM_{\mathcal{M}}(d_2(X - X_{\mathcal{M}}) + P_{X_{\mathcal{M}}}) \\
f_{Moon,Y}(X,Y,Z,t) &= -GM_{\mathcal{M}}(d_2(Y - Y_{\mathcal{M}}) + P_{Y_{\mathcal{M}}}) \\
f_{Moon,Z}(X,Y,Z,t) &= -GM_{\mathcal{M}}(d_2(Z - Z_{\mathcal{M}}) + P_{Z_{\mathcal{M}}})
\end{aligned} \tag{46}$$

1.6 SRPComponent

$$\begin{aligned}
f_{SRP,X}(X,Y,Z,t) &= AOM \frac{P_{SRP} a_{\odot}^2 (X-X_{\odot})}{[(X-X_{\odot})^2+(Y-Y_{\odot})^2+(Z-Z_{\odot})^2]^{3/2}} \\
f_{SRP,Y}(X,Y,Z,t) &= AOM \frac{P_{SRP} a_{\odot}^2 (Y-Y_{\odot})}{[(X-X_{\odot})^2+(Y-Y_{\odot})^2+(Z-Z_{\odot})^2]^{3/2}} \\
f_{SRP,Z}(X,Y,Z,t) &= AOM \frac{P_{SRP} a_{\odot}^2 (Z-Z_{\odot})}{[(X-X_{\odot})^2+(Y-Y_{\odot})^2+(Z-Z_{\odot})^2]^{3/2}}
\end{aligned} \tag{47}$$

1.6.1 Implementation details

1.7 DragComponent

1.7.1 Implementation details

2 Constants

$$GM_E = 3.986004407799724 \times 10^5 km^3 sec^{-2} \tag{48}$$

$$GM_{\odot} = 1.32712440018 \times 10^{11} km^3 sec^{-2} \tag{49}$$

$$GM_{\mathcal{M}} = 4.9028 \times 10^3 km^3 sec^{-2} \tag{50}$$

$$R_E = 6378.1363 km \tag{51}$$

$$C_{20} = -4.84165371736 \times 10^{-4} \tag{52}$$

$$C_{22} = 2.43914352398 \times 10^{-6} \tag{53}$$

$$S_{22} = -1.40016683654 \times 10^{-6} \tag{54}$$

$$\theta_G = (280.4606) \tag{55}$$

$$\nu_E = (4.178074622024230 \times 10^{-3}) \tag{56}$$

$$\nu_{\odot} = (1.1407410259335311 \times 10^{-5}) \tag{57}$$

$$\nu_{M_a} = (1.512151961904581 \times 10^{-4}) \tag{58}$$

$$\nu_{M_p} = (1.2893925235125941 \times 10^{-6}) \tag{59}$$

$$\nu_{M_s} = (6.128913003523574 \times 10^{-7}) \tag{60}$$

$$a_{\odot} = 1.49619 \times 10^8 km \tag{61}$$

$$\varepsilon = (23.4392911) \tag{62}$$

$$\varphi_{\odot,0} = (357.5256) \tag{63}$$

$$\Omega_{\odot} + \omega_{\odot} = (282.94) \tag{64}$$

$$P_{SRP} = 4.56 \times 10^{-6} \tag{65}$$