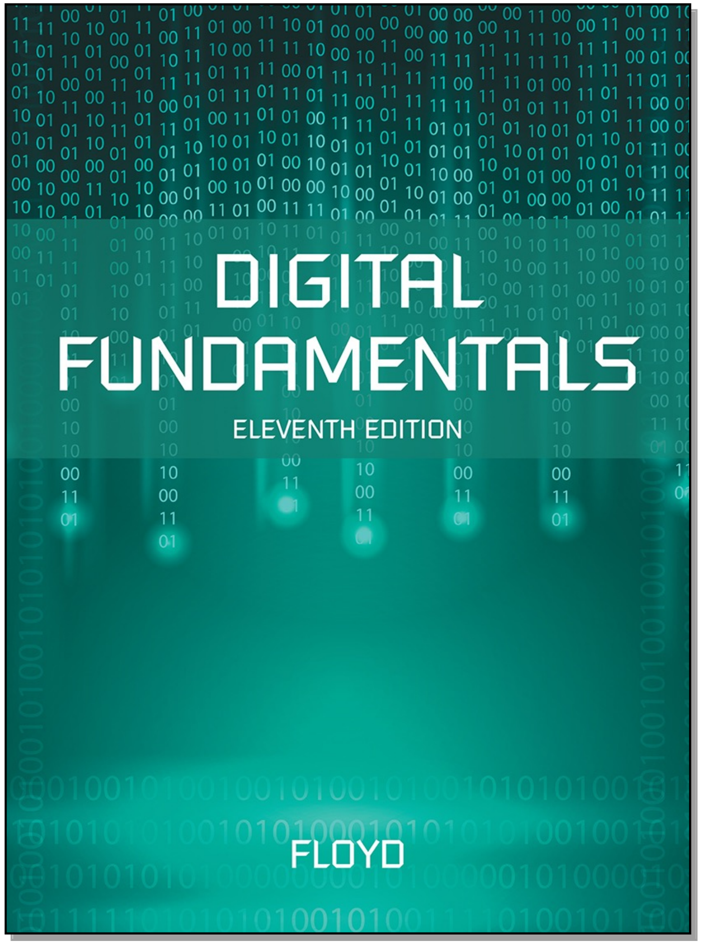
27/julho/2025

Digital Fundamentals

ELEVENTH EDITION

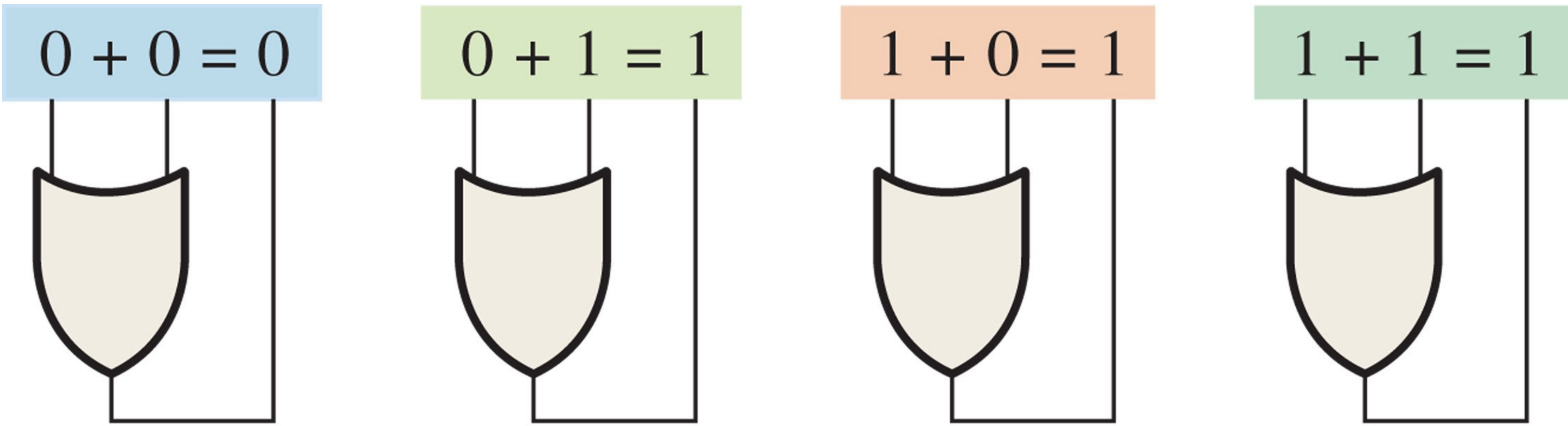


CHAPTER 4

Álgebra Booleana e Simplificação Lógica

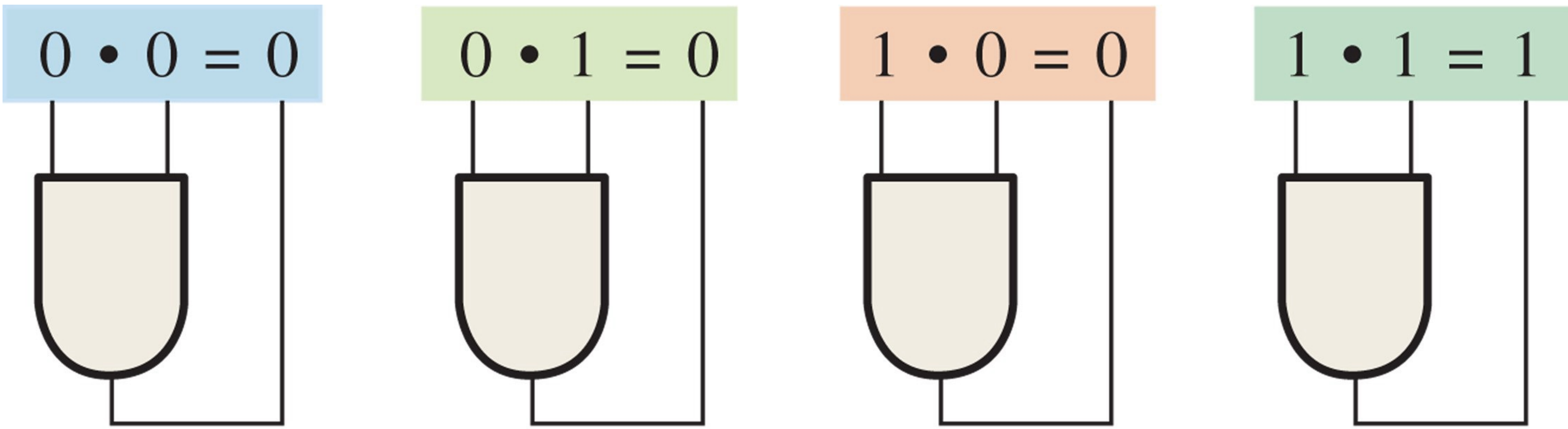
Adição Booleana

- A adição booleana é equivalente à operação OR
- As regras básicas são ilustradas com sua relação com a porta OR



Multiplicação Booleana

- A multiplicação booleana é equivalente à operação AND
- As regras básicas são ilustradas com sua relação com a porta AND

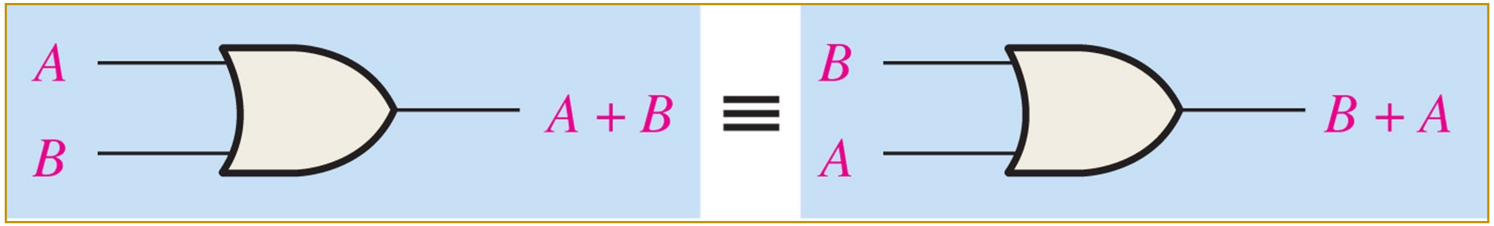


PROPRIEDADE COMUTATIVA

As **propriedades comutativas** são aplicadas à adição e multiplicação.

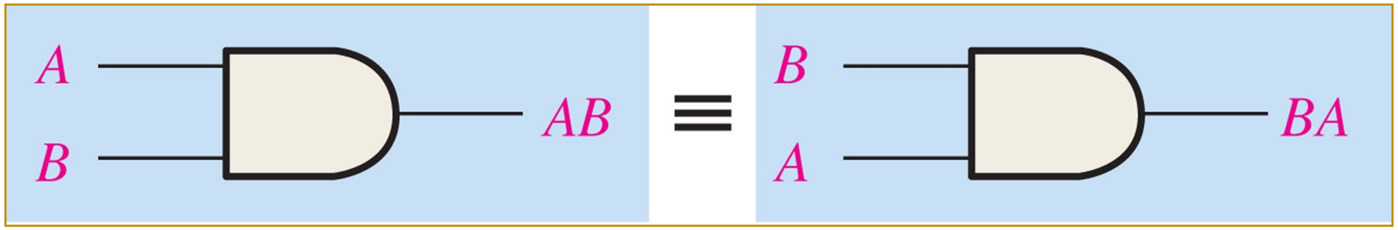
- ✓ Para adição, a propriedade comutativa afirma que a ordem em que as variáveis são combinadas com **OR não fazem diferença**.

$A + B = B + A$



- ✓ Para multiplicação, a propriedade comutativa afirma que a ordem em que as variáveis são colocadas em **AND** também **não fazem diferença**.

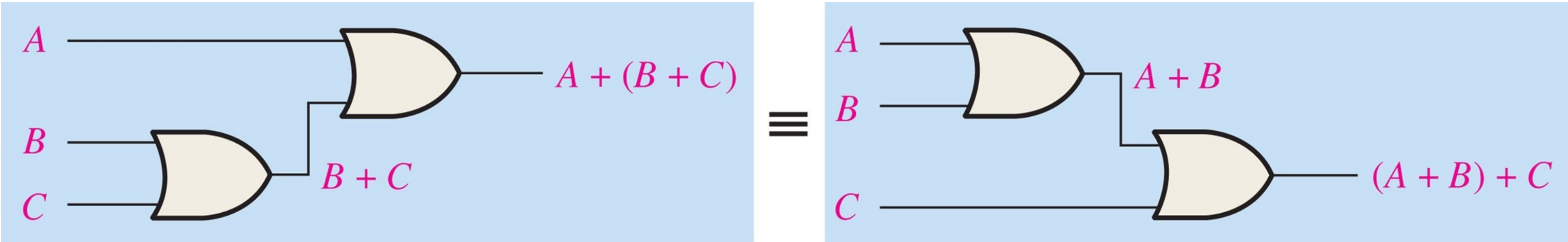
$A.B = B.A$



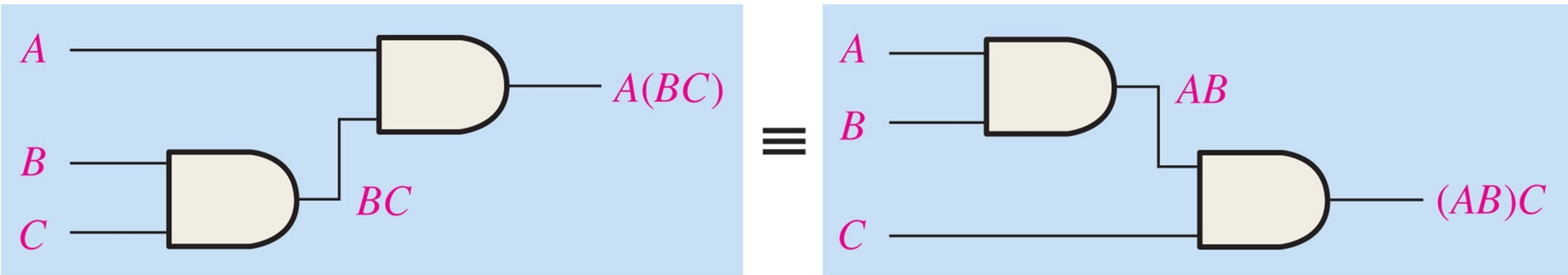
PROPRIEDADE ASSOCIATIVA

As **propriedades associativas** são aplicadas à adição e multiplicação.

- ✓ Na adição, ao usarmos um **OR** em mais de duas variáveis, o resultado é o mesmo, **independentemente do agrupamento das variáveis**.



- ✓ Na multiplicação, ao usarmos uma **AND** em mais de duas variáveis, o resultado é o mesmo, **independentemente do agrupamento das variáveis**.

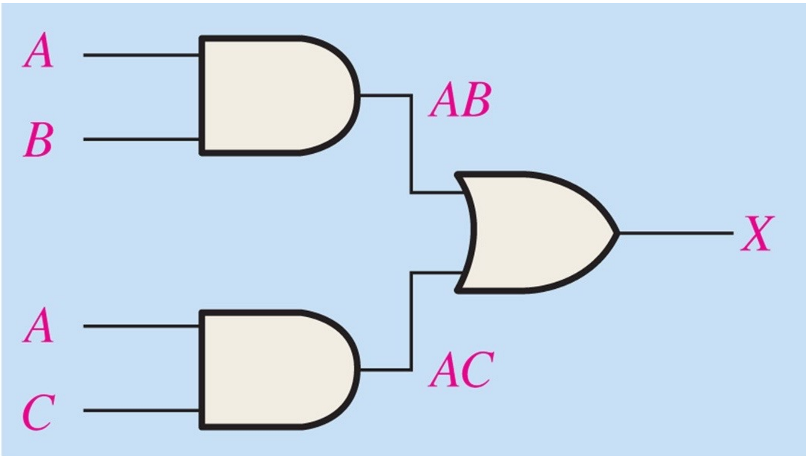


PROPRIEDADE DISTRIBUTIVA

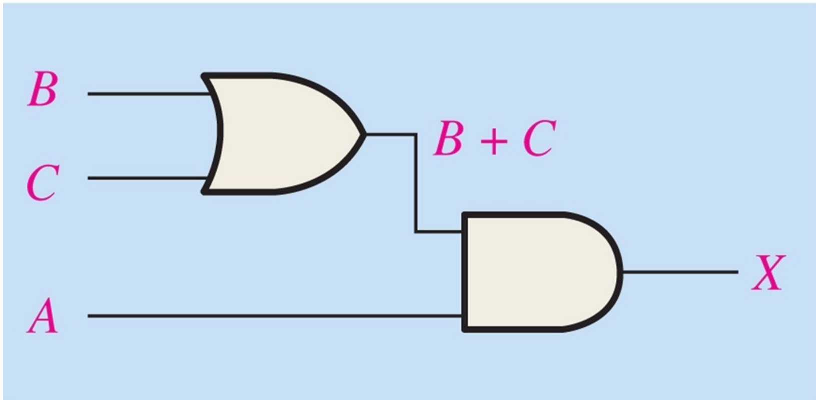
A **propriedade distributiva** é uma **propriedade de fatoramento**. Isto é, uma variável comum pode ser fatorada de uma expressão, assim como encontramos na álgebra.

$$AB + AC = A.(B + C)$$

- ✓ A propriedade distributiva pode ser ilustrada com circuitos equivalentes:



$X = AB + AC$



$X = A(B + C)$

REGRAS BÁSICAS DA ÁLGEBRA BOOLEANA

A, **B**, ou **C** podem representar uma única variável ou uma combinação de variáveis

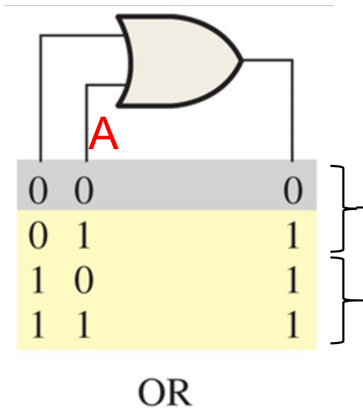
TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Rules of Boolean Algebra (1-2)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

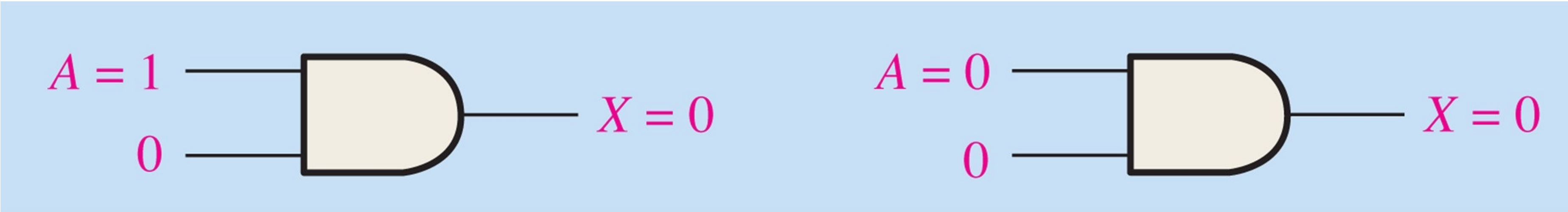


$$X = A + 0 = A$$

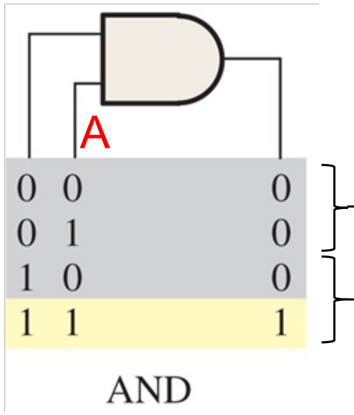
$$X = A + 1 = 1$$

Rules of Boolean Algebra (3-4)

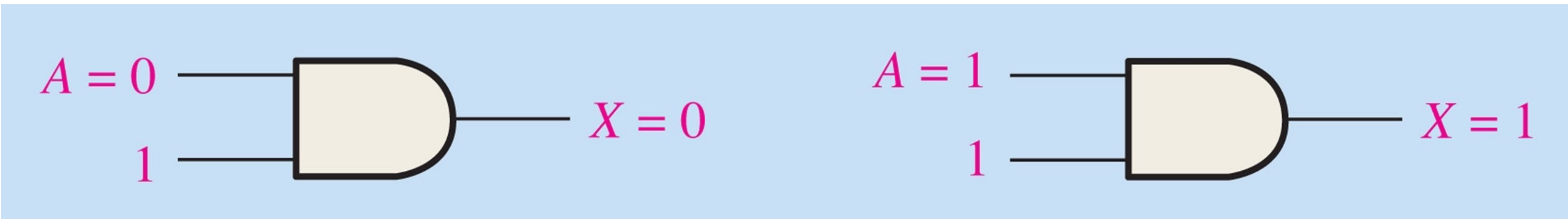
1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$



$$X = A \cdot 0 = 0$$



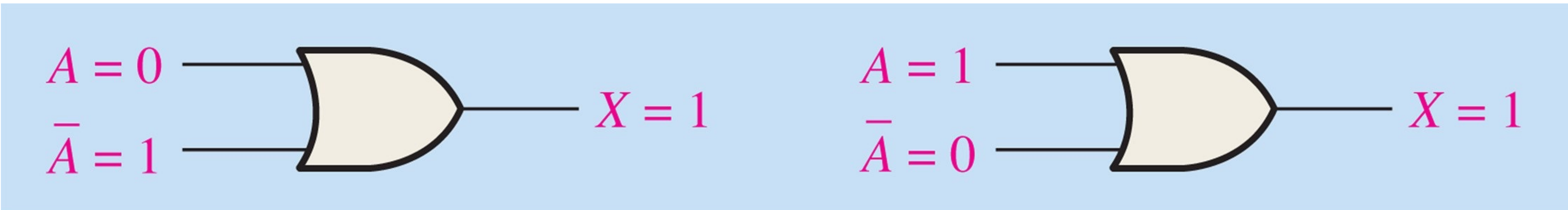
$$X = A \cdot 1 = A$$



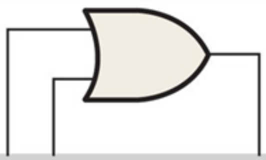
Rules of Boolean Algebra (5-6)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $\boxed{A + A = A}$
6. $\boxed{A + \overline{A} = 1}$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

$$X = A + A = A$$



$$X = A + \overline{A} = 1$$

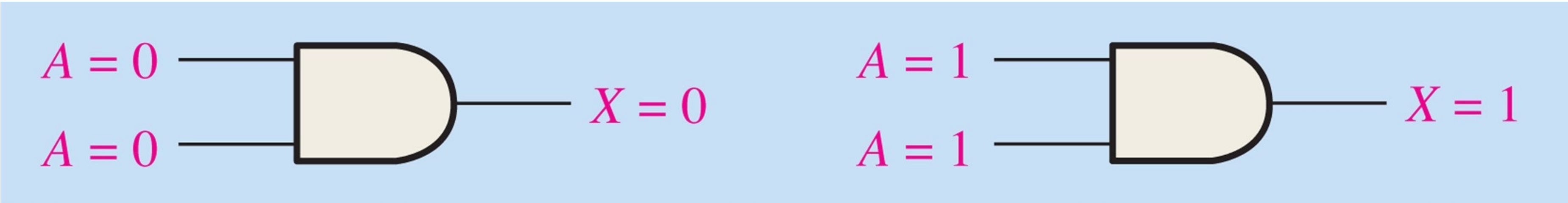


0	0	0
0	1	1
1	0	1
1	1	1

OR

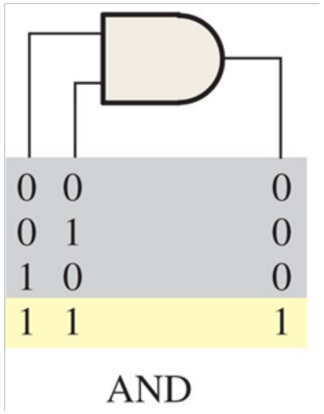
Rules of Boolean Algebra (7-8)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $\boxed{A \cdot A = A}$
8. $\boxed{A \cdot \overline{A} = 0}$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$



$$X = A \cdot A = A$$

$$X = A \cdot \overline{A} = 0$$



Rules of Boolean Algebra (9)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\boxed{\overline{\overline{A}} = A}$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

$$\overline{\overline{A}} = A$$

Rules of Boolean Algebra (10)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $\boxed{A + AB = A}$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Regra 10: **A + A.B = A**

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

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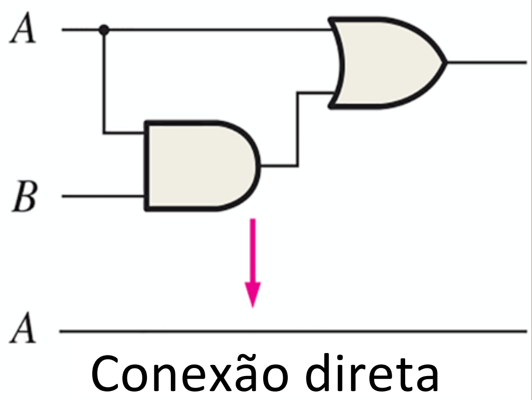
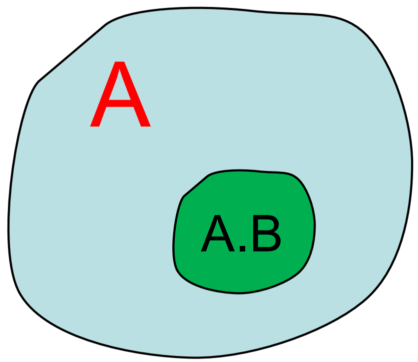


A + A.B = A . 1 + A.B Regra 4: A . 1 = A

= A.(1 + B) Propriedade Distributiva

= A . 1 Regra 2: (1 + B) = 1

= A Regra 4: A . 1 = A



Rules of Boolean Algebra (11)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $\boxed{A + \overline{A}B = A + B}$
12. $(A + B)(A + C) = A + BC$

Regra 11: $A + \overline{A}.B = A + B$

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

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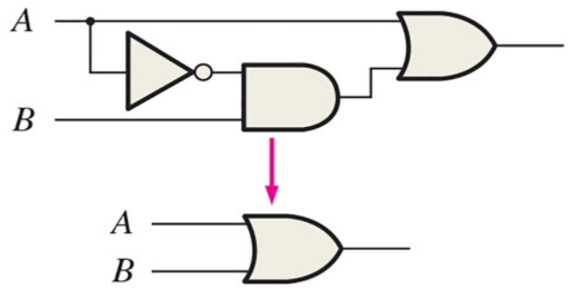


$$A + \overline{A}.B = (A + A.B) + \overline{A}.B$$

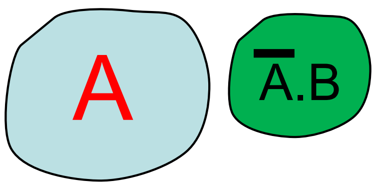
Regra 10: $A = A + A.B$

$$= A + A.B + \overline{A}.B$$

$$= A + B.(A + \overline{A})$$

Regra 6: $A + \overline{A} = 1$

$$= A + B$$



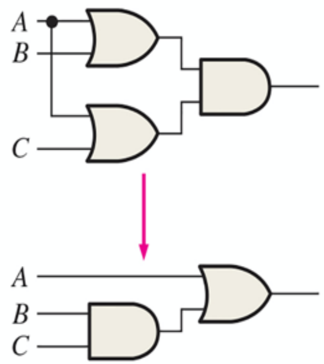
Rules of Boolean Algebra (12)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $\boxed{(A + B)(A + C) = A + BC}$

Regra 12: **(A + B).(A + C) = A + B.C**

A	B	C	$A+B$	$A+C$	$(A+B)(A+C)$	BC	$A+BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

iguais



(A + B).(A + C) = A.A + A.C + A.B + B.C — Propriedade Distributiva

= A + A.C + A.B + B.C — Regra 7: A.A = A

= A.(1 + C + B) + B.C — Propriedade Distributiva

= A.1 + B.C — Regra 2: A + 1 = 1

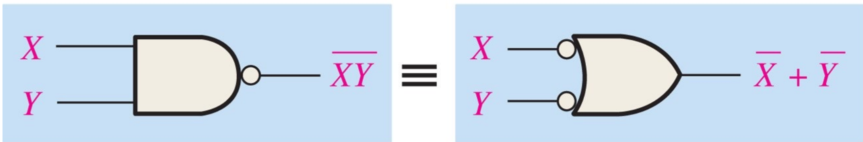
= A + B.C — Regra 4: A.1 = A

TEOREMAS DE DEMORGAN

OBJETIVO DE APLICAR DEMORGAN:

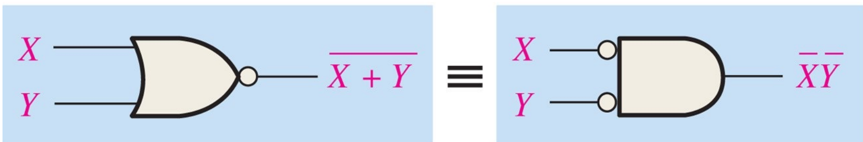
- ✓ Remover a inversão sobre um conjunto de literais.

Abaixo é mostrado as **portas equivalentes** e as correspondentes **tabelas verdade** que ilustram os **Teoremas de DeMorgan**. Observe a igualdade das duas colunas de saída em cada tabela. Isso mostra que as portas equivalentes executam a mesma função lógica.



NAND Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X}+\overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



NOR Negative-AND

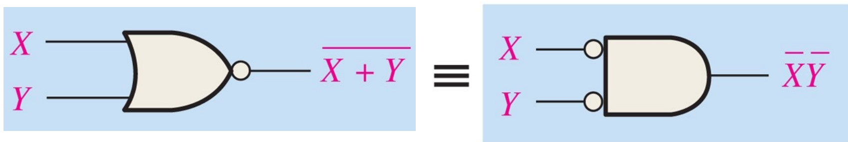
Inputs		Output	
X	Y	$\overline{X+Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

TEOREMAS DE DEMORGAN

<u>Exemplo:</u> Aplique o **Teorema de DeMorgan** para remover a negação que cobre ambos os termos da expressão: $X = \overline{\bar{C} + D}$

- ✓ Para aplicar o Teorema de DeMorgan à expressão, você pode remover a negação que cobre ambos os termos e alterar o sinal entre os termos:

$$X = \overline{\bar{C} + D}$$



$$X = \bar{\bar{C}} \; . \; \bar{D}$$

Remove negação dupla

$$X = C \; . \; \bar{D}$$

DeMorgan's Theorem

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

Solution

$$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{W + X + Y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

DeMorgan's Theorem

Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B.\bar{C}} + D.(\overline{E + \bar{F}})}$$

$$(\overline{\overline{A + B.\bar{C}}}).(\overline{D.(\overline{E + \bar{F}})})$$ Substitui por and

$$(A + B.\bar{C}).(\bar{D} + \overline{\overline{(E + \bar{F})}})$$ DeMorgan do segundo termo

$$(A + B.\bar{C}).(\bar{D} + E + \bar{F})$$ Pronto, todos literais isolados

DeMorgan's Theorem

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

(a) $\overline{(A + B + C)} + \overline{D}$

$\bar{A}.\bar{B}.\bar{C} + \bar{D}$

(b) $\overline{(A.B.C)}\,.\,\overline{(D.E.F)}$

$(\bar{A} + \bar{B} + \bar{C})\,.\,(\bar{D} + \bar{E} + \bar{F})$

(c) $\overline{(A.\bar{B})}.\overline{(\bar{C}.D)}.\overline{(E.F)}$

$(\bar{A} + B)\,.\,(C + \bar{D}).\,(\bar{E} + \bar{F})$

De Morgan

$$F = \overline{A.\overline{B}.(C+\overline{D})}$$

$$F = \overline{A} + \overline{\overline{B}} + \overline{(C+\overline{D})} = \overline{A} + B + (\overline{C} + \overline{\overline{D}}) = \overline{A} + B + \overline{C} + D$$

$$F = \overline{\overline{\left(\overline{A}+B+C+D\right)} . \overline{(A.\overline{B}.\overline{C}.D)}}$$

$$F = \overline{\overline{\left(\overline{A}+B+C+D\right)}} + \overline{\overline{(A.\overline{B}.\overline{C}.D)}}$$

$$F = \overline{A} + B + C + D + A.\overline{B}.\overline{C}.D$$

$$F = \overline{A + \overline{B.\overline{C}} + C.D} + \overline{\overline{B.C}}$$

$$F = \overline{A}.\overline{\overline{B.\overline{C}}}.\overline{C.D} + B.C$$

$$F = \overline{A}. B.\overline{C}.(\overline{C}+\overline{D}) + B.C$$

$$F = \overline{A}.B.\overline{C}.\overline{C} + \overline{A}.B.\overline{C}.\overline{D} + B.C$$

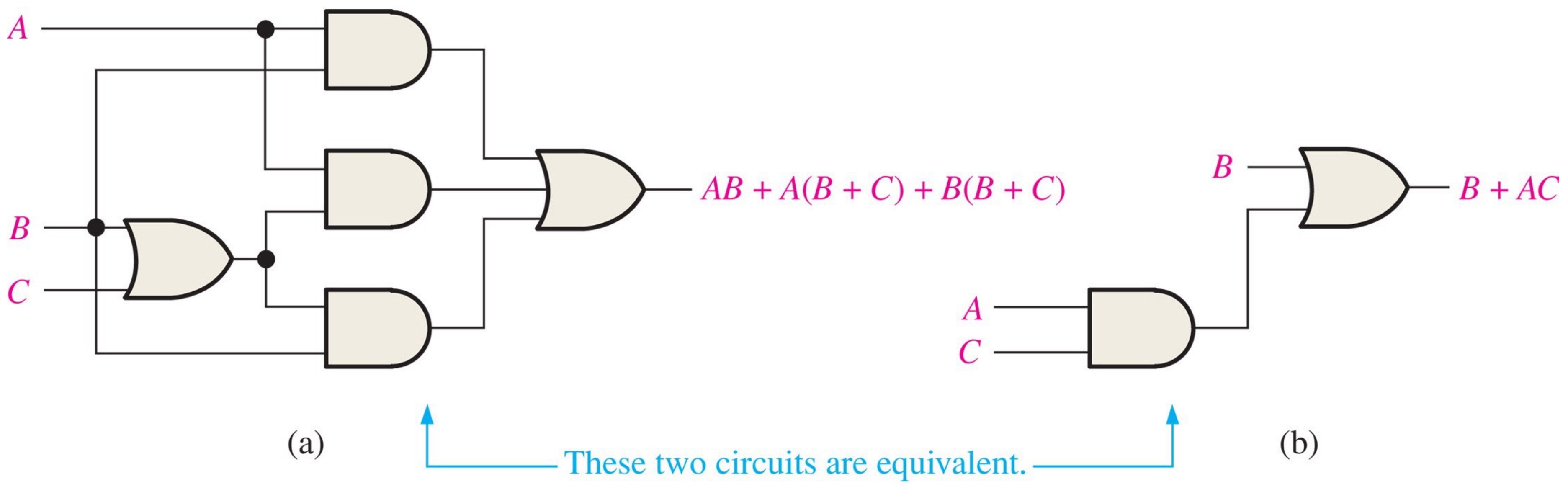
$$F = \overline{A}.B.\overline{C} + \overline{A}.B.\overline{C}.\overline{D} + B.C$$

$$F = \overline{A}.B.\overline{C}.(1+\overline{D}) + B.C$$

$$F = \overline{A}.B.\overline{C} + B.C$$

SIMPLIFICAÇÃO LÓGICA USANDO ÁLGEBRA BOOLEANA

Simplify this expression AB + A(B + C) + B(B + C)



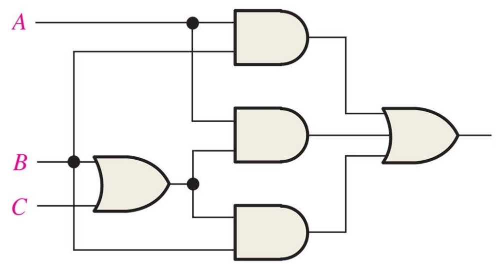
Solução próximo slide

Logic Simplification Using Boolean Algebra

Simplify this expression AB + A(B + C) + B(B + C)

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

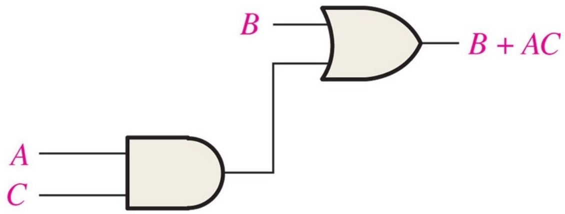


Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

B + B.(qq coisa) = B

B + AC



MINIMIZAR

Simplify this expression (p.206)

$$F = (A.\bar{B}.(C + B.D) + \bar{A}.\bar{B}).C$$

$$F = (A.\bar{B}.C + \cancel{A.\bar{B}.B.D} + \bar{A}.\bar{B}).C$$

$\bar{B}.B$ = 0

$$F = A.\bar{B}.C.C + \bar{A}.\bar{B}.C$$

$C.C = C$

$$F = A.\bar{B}.C \; + \bar{A}.\bar{B}.C$$

$$F = \bar{B}.C.(A + \bar{A})$$

$A + \bar{A}$ = 1

$$F = \bar{B}.C$$

LISTA DE MINIMIZAÇÃO (questão 7)

7. Usando técnicas da álgebra booleana, simplifique as seguintes expressões o máximo possível:

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

a) A.(A + B)

A.A + A.B

A + A.B

A

b) A.($\overline{\text{A}}$ + A.B)

A.A’ + A.A.B

A.B

c) B.C + $\overline{\text{B}}$.C

C.(B + B’)

C

d) A.(A + $\overline{\text{A}}$.B)

A.A + A.A’.B

A

e) A.$\overline{\text{B}}$.C + $\overline{\text{A}}$.B.C + $\overline{\text{A}}$.$\overline{\text{B}}$.C

C.(A.B’ + A’.B + A’.B’)

C.(A’.(B +B’) + B’.(A + A’))

C.(A’ + B’)

A’.C + B’.C

MINIMIZAR 1/6

$$\boldsymbol{F = C.E + C.(E + F) + \bar{E}.(E + G)}$$

L- 8a

$$F = C.E + C.F + \bar{E}.G$$

$$\boldsymbol{F = \bar{B}.\bar{C}.D + \overline{(B + C + D)} + \bar{B}.\bar{C}.\bar{D}.E}$$

L- 8b

$$F = \bar{B}.\bar{C}.D + \bar{B}.\bar{C}.\bar{D} + \bar{B}.\bar{C}.\bar{D}.E$$

$$F = \bar{B}.\bar{C}.(D + \bar{D} + E)$$

$$F = \bar{B}.\bar{C}$$

MINIMIZAR 2/6

$$F = (C + C.D).(C + \overline{C}.D).(C + E)$$

L- 8c

C.(C + C'.D).(C + E)

C.(C.C + C.E + ~~C'CD~~ + C'.D.E)

C + C.E + ~~C.C'.D.E~~

C + C.E

C

$$F = B.C.D.E + B.C.(\overline{D.E}) + (\overline{B.C}).D.E$$

L- 8d

$$F = B.C(D.E + \overline{D} + \overline{E}) + (B.C + \overline{B} + \overline{C}).D.E$$

$C + \overline{B}$ (prop 11)

$$C + \overline{C} = 1$$

$$F = B.C + D.E$$

$$ou\ F = B.C(D.E + \overline{D.E}) + DE(B.C + \overline{B.C})$$

$$F = B.C.D.[B.C + \overline{D}.(C.D + B.D)]$$

L- 8e

$$F = B.C.D.B.C + \cancel{B.C.D.\overline{D}.C.D} + \cancel{B.C.D.\overline{D}.B.D}$$

$$F = B.C.D$$

MINIMIZAR 3/6

Exercise 2.7 Simplify the following Boolean equations using Boolean theorems. Check for correctness using a truth table or K-map.

10. $A + AB = A$
11. $A + \overline{A}B = A + B$

(a) $Y = AC + \overline{A}\,\overline{B}C$ = C.(A + A'B') = C.(A + B') = **AC + B'C**

(b) $Y = \overline{A}\,\overline{B} + \overline{A}B\overline{C} + \overline{(A + \overline{C})}$

(c) $Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C} + A\overline{B}C\overline{D} + ABD + \overline{A}\,\overline{B}C\overline{D} + B\overline{C}D + \overline{A}$

(b) = A'B'+ A'BC'+ **A'C** = A' (B'+ BC'+ C) = A' (B'+C'+ C) = A'

(c) $Y = \cancel{\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}} + \cancel{A}\overline{B}\,\overline{C} + \cancel{A}\overline{B}C\overline{D} + \cancel{A}BD + \cancel{\overline{A}\,\overline{B}C\overline{D}} + B\overline{C}D + \overline{A}$

Corta os termos redundantes com A’

Remove o A dos termos, pois temos A’

$$Y = \bar{B}.\bar{C} + \bar{B}.C.\bar{D} + B.D + B.\bar{C}.D + \bar{A}$$

$$Y = \bar{B}.(\bar{C} + \bar{D}) + B.(D + \cancel{\bar{C}.D}) + \bar{A}$$

$$Y = \bar{B}.\bar{C} + \bar{B}.\bar{D} + B.D + \bar{A}$$

MINIMIZAR 4/6

$$F = \overline{A.B + A.C} + A.\overline{B}.C$$

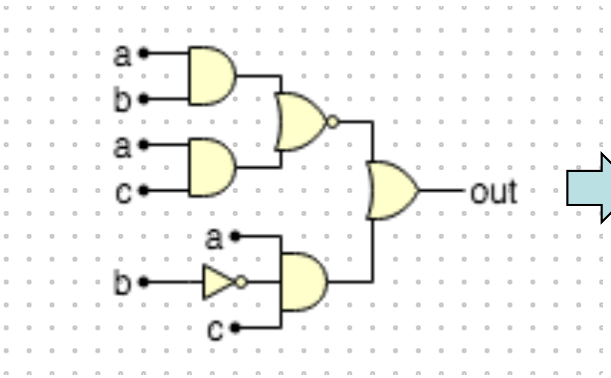
- Simplifique a expressão acima utilizando as regras de álgebra Booleana. Explicar o procedimento.
- Desenhe o circuitos lógicos correspondentes à expressão acima e ao circuito lógico minimizado.
- Apresente a tabela verdade da função, E apresentar a função na forma de um somatório de mintermos.

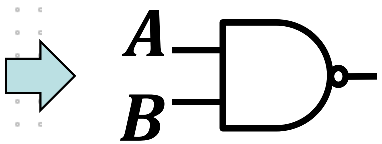
$$F = \overline{A.B + A.C} + A.\overline{B}.C = (\overline{A} + \overline{B}).(\overline{A} + \overline{C}) + A.\overline{B}.C$$

$$= \overline{A} + \overline{B}.\overline{C} + A.\overline{B}.C$$

$$= \overline{A} + \overline{B}.\overline{C} + \overline{B}.C$$

$$= \overline{A} + \overline{B}$$





A	B	C	F	minitermos
0	0	0	1	0
0	0	1	1	1
0	1	0	1	2
0	1	1	1	3
1	0	0	1	4
1	0	1	1	5
1	1	0		
1	1	1		

$$\boldsymbol{F}(A, B, \mathrm{C}) = \sum (0, 1, 2,3,4,5)$$

MINIMIZAR 5/6

Mininizar: f(a,b,c,d) = $\sum(5, 6, 10, 11, 13, 14, 15)$

$(\mathbf{5}) = (\mathbf{0101})_2$ $\bar{a}.b.\bar{c}.d$ somatório de mintermos

$(\mathbf{6}) = (\mathbf{0110})_2$

$(\mathbf{10}) = (\mathbf{1010})_2$

$(\mathbf{11}) = (\mathbf{1011})_2$

$(\mathbf{13}) = (\mathbf{1101})_2$

$(\mathbf{14}) = (\mathbf{1110})_2$

$(\mathbf{15}) = (\mathbf{1111})_2$

$\bar{a}$ + a

f(a,b,c,d) = $\bar{a}.\boldsymbol{b}.\bar{c}.\boldsymbol{d} + \bar{a}.\boldsymbol{b}.\boldsymbol{c}.\bar{d} + \boldsymbol{a}.\bar{b}.\boldsymbol{c}.\bar{d} + \boldsymbol{a}.\bar{b}.\boldsymbol{c}.\boldsymbol{d} + \boldsymbol{a}.\boldsymbol{b}.\bar{c}.\boldsymbol{d} + \boldsymbol{a}.\boldsymbol{b}.\boldsymbol{c}.\bar{d} + \boldsymbol{a}.\boldsymbol{b}.\boldsymbol{c}.\boldsymbol{d}$

5 6 10 11 13 14 15

$\bar{d}$ + d $\bar{d}$ + d

$\bar{a}$ + a

f(a,b,c,d) = $b.\bar{c}.d + b.c.\bar{d} + a.\bar{b}.c + a.b.c$

$\bar{b}$ + b

f(a,b,c,d) = $\boldsymbol{b}.\bar{\boldsymbol{c}}.\boldsymbol{d} + \boldsymbol{b}.\boldsymbol{c}.\bar{\boldsymbol{d}} + \boldsymbol{a}.\boldsymbol{c}$

MINIMIZAR 6/6

$$F = B.\overline{C}.D \ + \ \cancel{\overline{A}}.B.D \ + \ \cancel{\overline{A}}.C.D + \underbrace{A.D \ + \ A.\overline{D}}_{D \ + \ \overline{D}} \ + \ B.\overline{C}.\overline{D}$$

$$F = B.\underbrace{\overline{C}.D \ + \ B.D \ + \ C.D + \ A \ + \ B.\overline{C}.\overline{D}}_{D + \overline{D}}$$

$$F = B.\overline{C} + \ B.D \ + \ C.D + \ A$$

TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Propriedade 11: remove $\overline{A}$

FORMATO DAS EXPRESSÕES BOOLEANAS

- As expressões booleanas podem ser escritas na forma de **soma de produtos** (**SOP**) ou na forma de **produto de somas** (**POS**)
  - Essas formas podem simplificar a implementação de lógica combinacional

- Uma expressão está na forma **SOP** (Soma de Produtos) quando dois ou mais produto são somados

$$\overline{A}\,\overline{B}\,\overline{C} + A\,B \qquad A\,B\,\overline{C} + \overline{C}\,\overline{D} \qquad C\,D + \overline{E}$$

- Uma expressão está na forma **POS** (Produto de Somas) quando dois ou mais termos são multiplicados

$$(A + B)(\overline{A} + C) \qquad (A + B + \overline{C})(B + D) \qquad (\overline{A} + B)C$$

FIGURE 4-22 Implementation of the SOP expression $AB + BCD + AC$.

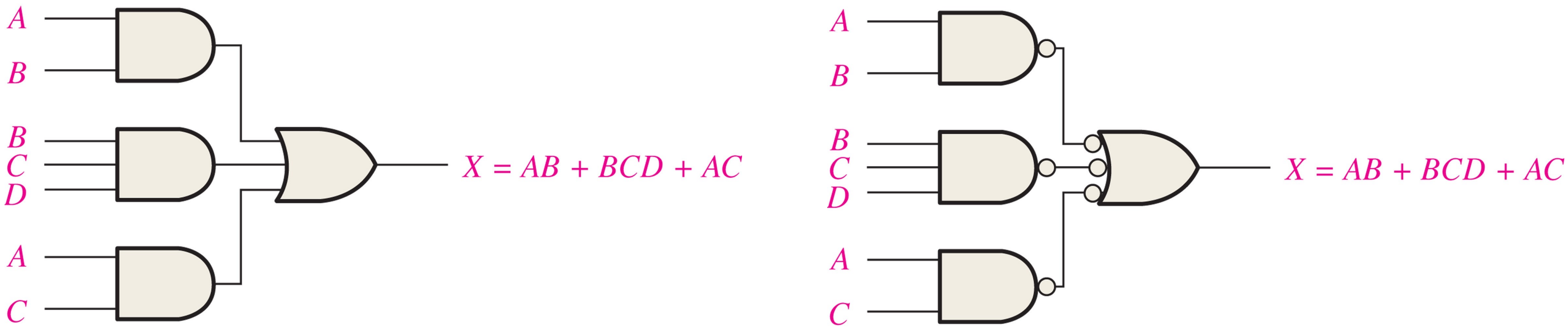
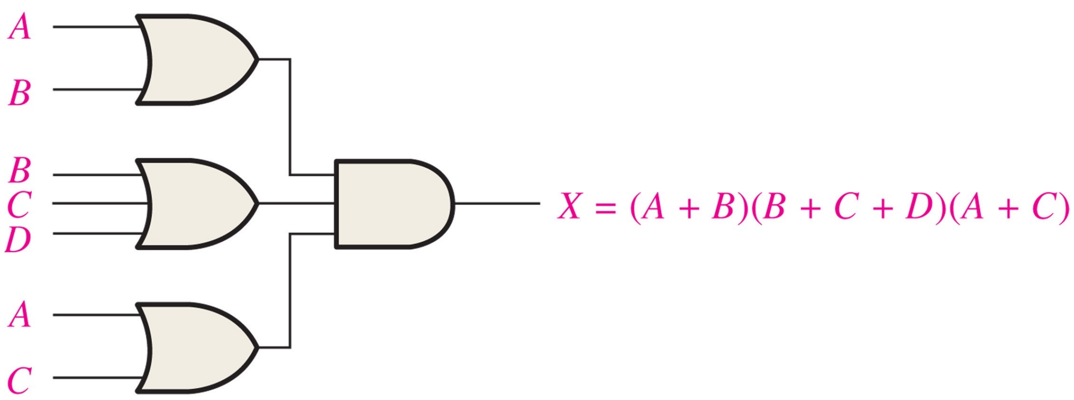


FIGURE 4-24 Implementation of the POS expression $(A + B)(B + C + D)(A + C)$.



Minterm

- **Minterm** → termo representando por um **produto** (E) de **todas** as variáveis de uma função Booleana, que resulta em verdadeiro (1) para uma única combinação de valores das variáveis

- Origem do termo "termo mínimo": a forma mais básica e indivisível de representar uma combinação específica de variáveis que leva a função a ser **verdadeira**

- Exemplo:

$$\boldsymbol{m_2(\mathbf{a},\mathbf{b},\mathbf{c}) = \bar{a}.b.\bar{c}}$$

Maxterm

- **Maxterm →** termo máximo em álgebra Booleana, representando uma **soma** (OU) de **todas** as variáveis de uma função Booleana que resulta em falso (0) para uma única combinação de valores das variáveis
- Origem do termo "termo máximo”: indica uma expressão que engloba o máximo de variáveis em uma soma lógica, representando a condição para que a função seja **falsa**
- Exemplo:

$$\boldsymbol{M_2}(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \overline{\boldsymbol{a}} + \boldsymbol{b} + \overline{\boldsymbol{c}}$$

Standard Forms of Boolean Expressions

- Para **SOP** na **forma padrão** cada produto é um **minterm**
- Expansão de um termo não padrão para minterm: para cada variável ***x*** ausente multiplicar o termo por ***(x+x')***
- Converter $X = \bar{A}.\bar{B} + A.B.C$ para a forma padrão:

O primeiro termo não inclui a variável C. Portanto, multiplique-o por $(\bar{C} + C)$:

$$X = \bar{A}.\bar{B}.(\bar{C} + C) + A.B.C$$

$$X = \bar{A}.\bar{B}.\bar{C} + \bar{A}.\bar{B}.C + A.B.C$$

Standard Forms of Boolean Expressions

EXAMPLE 4–15

Convert the following Boolean expression into standard SOP form:

$$A\bar{B}C + \bar{A}\,\bar{B} + AB\bar{C}D$$

EXAMPLE 4–15

Convert the following Boolean expression into standard SOP form:

$$A\bar{B}C + \bar{A}\,\bar{B} + AB\bar{C}D$$

Solution

The domain of this SOP expression is A, B, C, D. Take one term at a time. The first term, $A\bar{B}C$, is missing variable D or $\bar{D}$, so multiply the first term by $D + \bar{D}$ as follows:

$$A\bar{B}C = A\bar{B}C(D + \bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$$

In this case, two standard product terms are the result.

The second term, $\bar{A}\,\bar{B}$, is missing variables C or $\bar{C}$ and D or $\bar{D}$, so first multiply the second term by $C + \bar{C}$ as follows:

$$\bar{A}\,\bar{B} = \bar{A}\,\bar{B}(C + \bar{C}) = \bar{A}\,\bar{B}C + \bar{A}\,\bar{B}\,\bar{C}$$

The two resulting terms are missing variable D or $\bar{D}$, so multiply both terms by $D + \bar{D}$ as follows:

$$\begin{aligned}\bar{A}\,\bar{B} &= \bar{A}\,\bar{B}C + \bar{A}\,\bar{B}\,\bar{C} = \bar{A}\,\bar{B}C(D + \bar{D}) + \bar{A}\,\bar{B}\,\bar{C}(D + \bar{D}) \\ &= \bar{A}\,\bar{B}CD + \bar{A}\,\bar{B}C\bar{D} + \bar{A}\,\bar{B}\,\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D}\end{aligned}$$

In this case, four standard product terms are the result.

The third term, $AB\bar{C}D$, is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\bar{B}C + \bar{A}\,\bar{B} + AB\bar{C}D = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\,\bar{B}CD + \bar{A}\,\bar{B}C\bar{D} + \bar{A}\,\bar{B}\,\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + AB\bar{C}D$$

8 4 2 1

$$Y = A.\bar{B}.C.x + \bar{A}.\bar{B}.x.x + A.B.\overline{C}.D$$

10/11 0/1/2/3 13

8 4 2 1 | 8 4 2 1 | 8 4 2 1 | 8 4 2 1 | 8 4 2 1 | 8 4 2 1 | 8 4 2 1

$$= A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\,\bar{B}CD + \bar{A}\,\bar{B}C\bar{D} + \bar{A}\,\bar{B}\,\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + AB\bar{C}D$$

11 10 3 2 1 0 13

$$Y = \sum(0, 1, 2, 3, 10, 11, 13)$$

Truth Table → SOP

TABLE 4–8

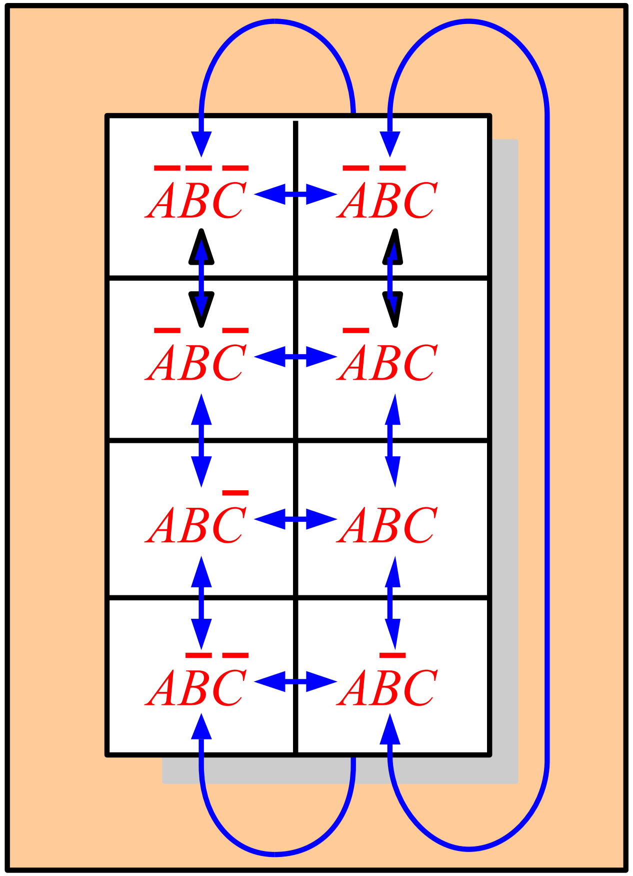
Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

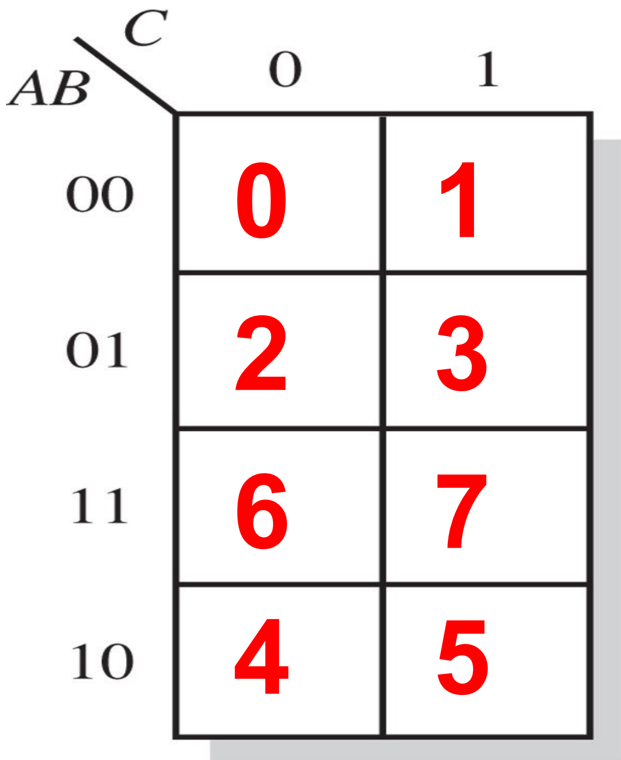
$$011 \longrightarrow \overline{A}BC$$
$$100 \longrightarrow A\overline{B}\,\overline{C}$$
$$110 \longrightarrow AB\overline{C}$$
$$111 \longrightarrow ABC$$

X=A'BC+AB'C'+ABC'+ABC

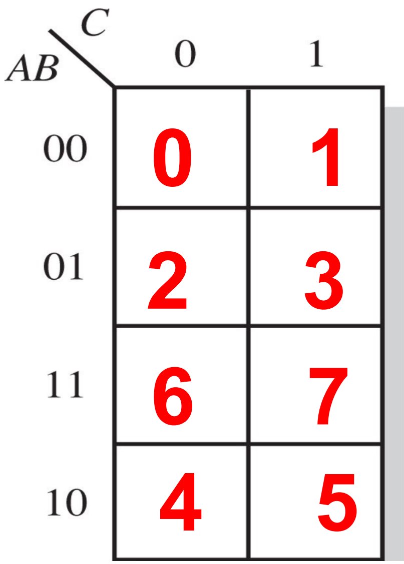
Karnaugh maps

- O mapa de Karnaugh é um método para simplificar a lógica combinacional com até 6 variáveis. Para 3 variáveis, 8 células são necessárias (2^3)
- **Cada célula difere de uma célula adjacente por apenas uma variável**
- O mapa mostrado abaixo é para três variáveis denominadas como A, B e C. Cada célula representa **mintermo**





Karnaugh maps



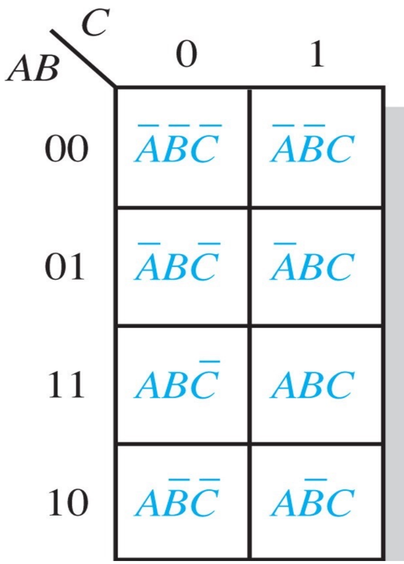


TABELA VERDADE E MAPA DE KARNAUGH

Inputs			Output
A	*B*	*C*	*X*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

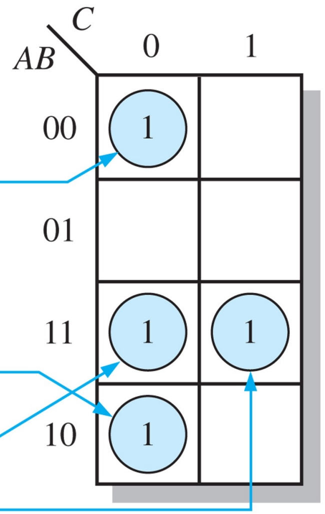
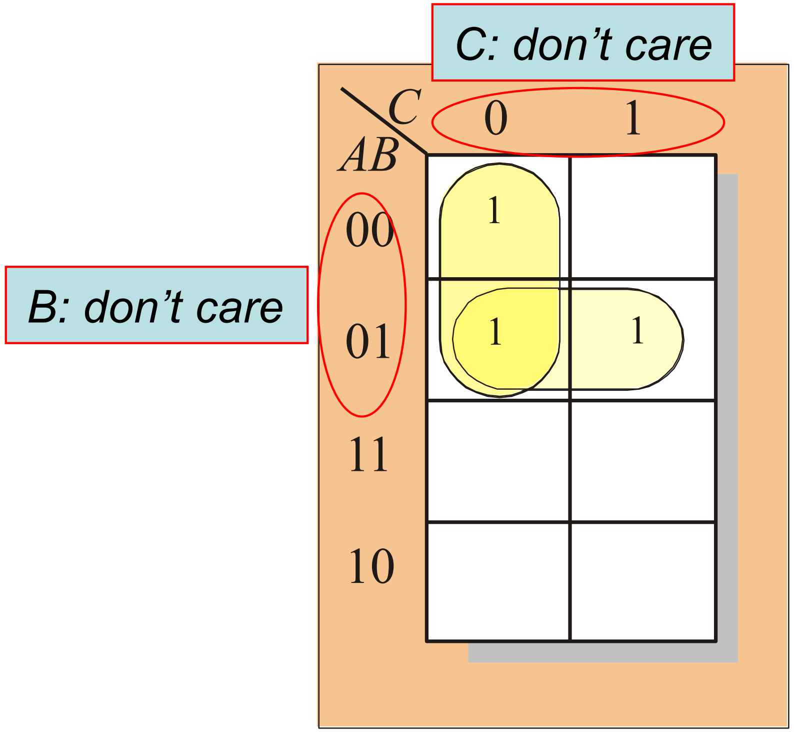


FIGURE 4-25 A 3-variable Karnaugh map showing Boolean product terms for each cell.

Karnaugh maps

K-maps podem simplificar a lógica combinacional agrupando células e ***eliminando variáveis redundantes***

$$A.B + A.\overline{B} = A$$



1. Agrupar os 1's em grupos de 2/4/8 mintermos
2. Eliminar as variáveis redundantes
3. O grupo vertical é lido como: $\overline{A}.\overline{C}$
4. O grupo horizontal é lido como: $\overline{A}.B$

$$X = \overline{A}.\overline{C} + \overline{A}.B$$

Minimize (1/5)

$$X = \overset{0}{\overline{A}\,\overline{B}\,\overline{C}} + \overset{4}{A\overline{B}\,\overline{C}} + \overset{6}{AB\overline{C}} + \overset{7}{ABC}$$

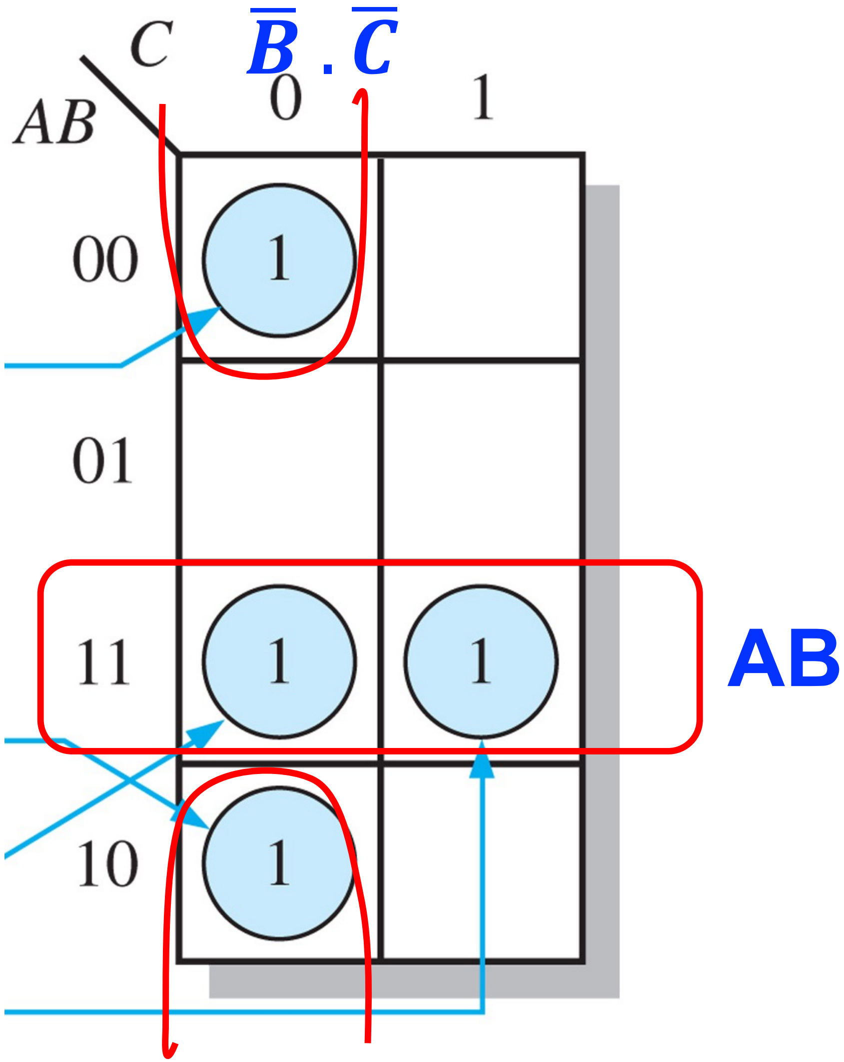
	Inputs A	B	C	Output X
0	0	0	0	1
	0	0	1	0
	0	1	0	0
	0	1	1	0
4	1	0	0	1
	1	0	1	0
6	1	1	0	1
7	1	1	1	1

$AB \backslash C$	0	1
00	1	
01		
11	1	1
10	1	

FIGURE 4-39 Example of mapping directly from a truth table to a Karnaugh map.

Minimize (1/5)

$$X = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + AB\overline{C} + ABC$$



$$X = \overline{B}\,.\,\overline{C} + A.B$$

FIGURE 4-39 Example of mapping directly from a truth table to a Karnaugh map.

Minimize (2/5)

$Y.Z$

	X \ YZ	00	01	11	10
	0	0	0	1	0
$X.\bar{Z}$	1	1	0	1	1

$$F = \sum(3,4,6,7)$$

$$F = Y.Z + X.\bar{Z}$$

X \ YZ	00	01	11	10
0	m0	m1	m3	m2
1	m4	m5	m7	m6

m2 + m3 + m6 + m7

$\bar{X}Y \qquad XY = (\bar{X} + X)\,Y = Y$

É o que há de comum entre os 4 mintermos

Minimize (3/5)

$$F = \sum(0,1,2,3,4,5,6)$$

$$F = \bar{A} + \bar{B} + \bar{C}$$

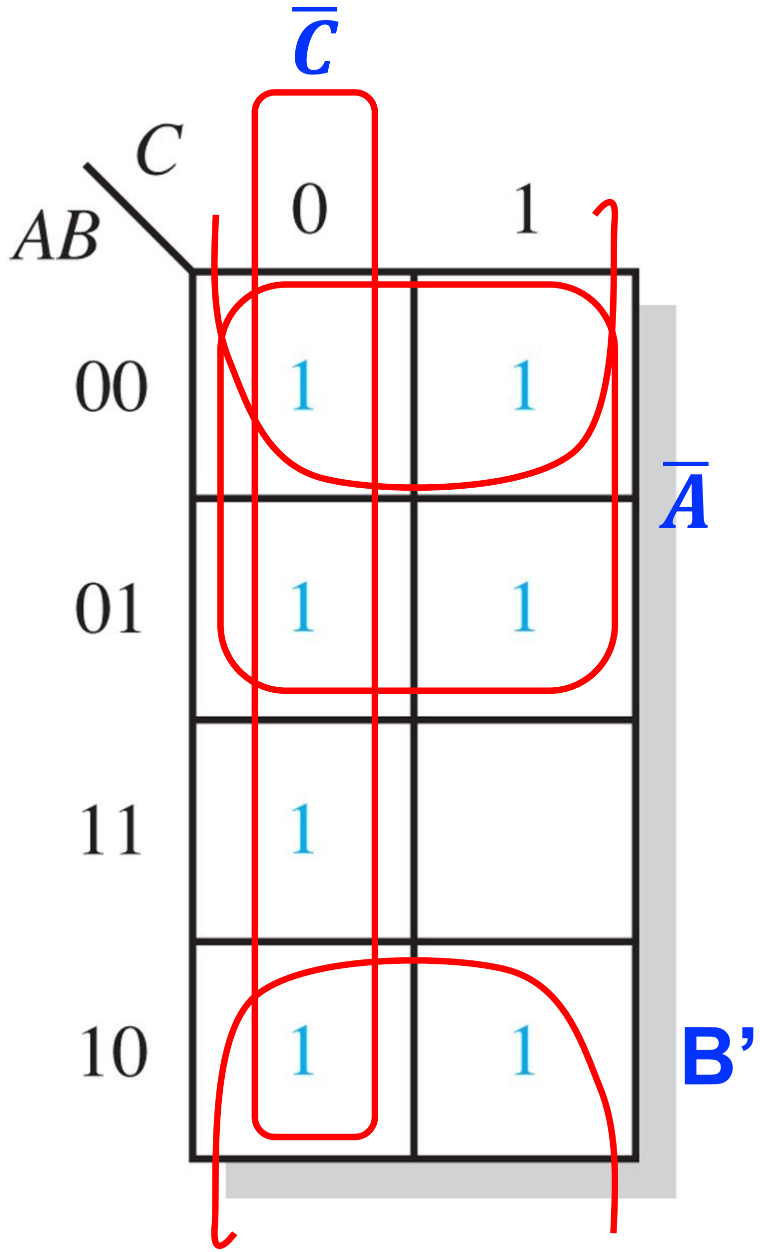
Using maxterms:

$$\bar{F} = \prod(7)$$

$$\bar{F} = A\,.\,B\,.\,C$$

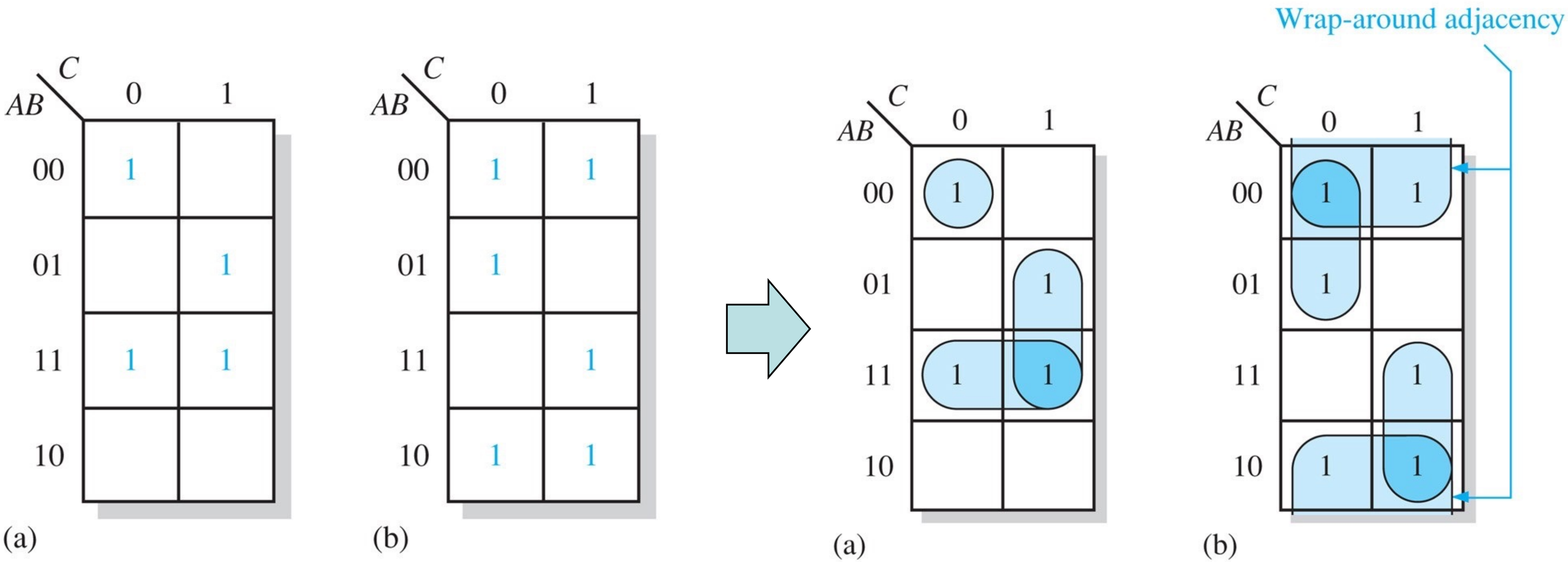
De Morgan:

$$F = \bar{A} + \bar{B} + \bar{C}$$



Minimize (4/5)

Como agrupar? Qual a função minimizada resultante?

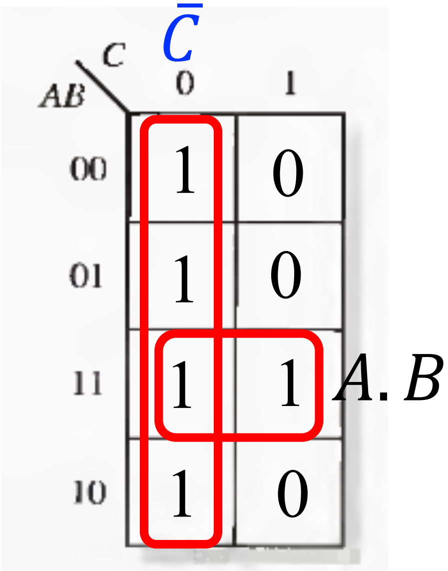


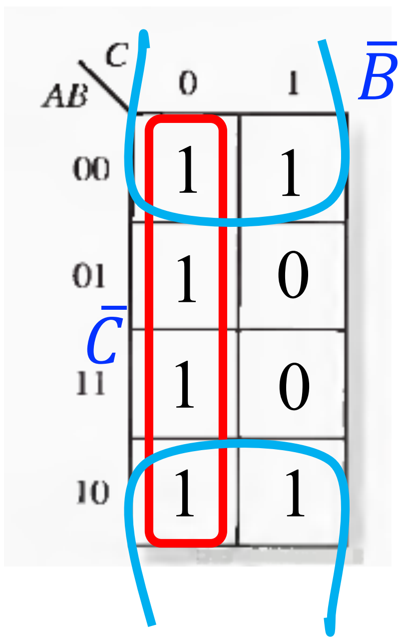
$$(a)\ F = AB + BC + \bar{A}.\bar{B}.\bar{C}$$

$$(b)\ F = \bar{A}.\bar{C} + A.C + \bar{B}$$

Minimize (4/5)

Como agrupar? Qual a função minimizada resultante?





Minimize (5/5)

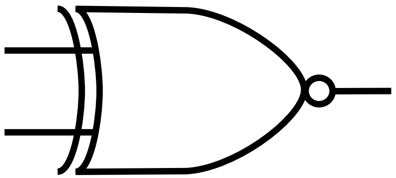
Determine the functions:

- F1 = ∑ (0,1,7,6)
- F2 = ∑ (0,2,3,4,6,7)

A \ BC	00	01	11	10
0				
1				

A \ BC	00	01	11	10
0	1	1		
1			1	1

F1= $\bar{A}.\bar{B} + A.B$



A \ BC	00	01	11	10
0	1		1	1
1	1		1	1

F2= $B + \bar{C}$

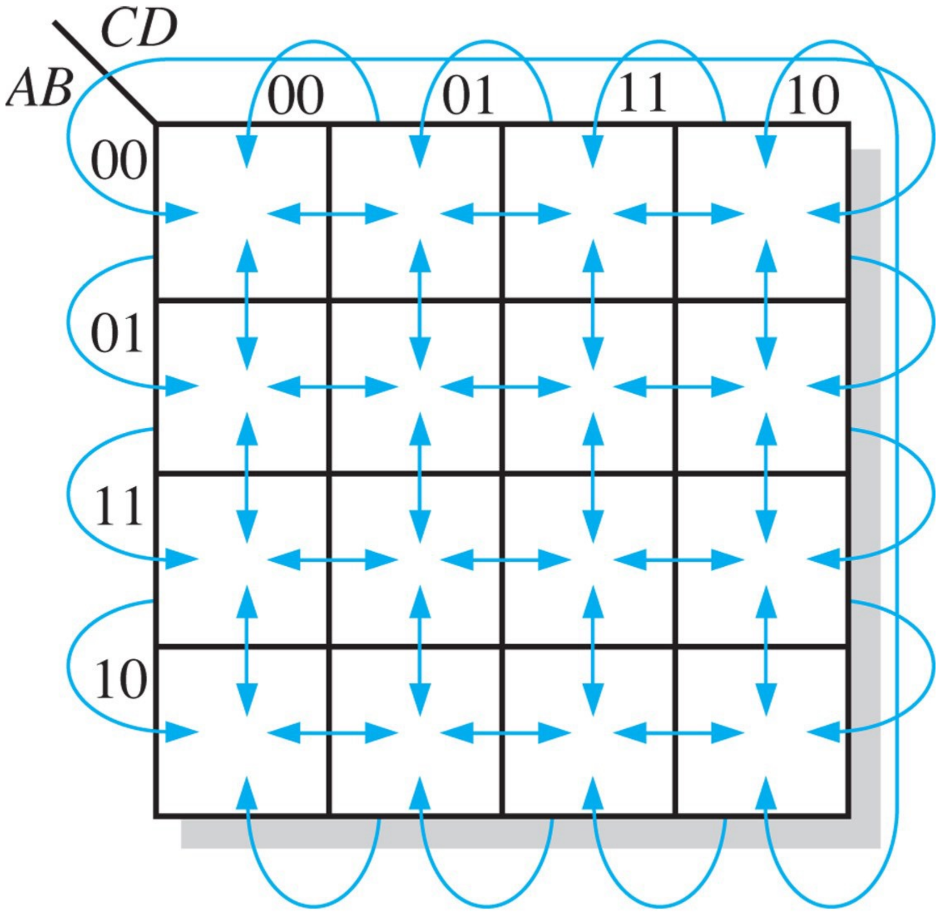
$\overline{F2} = \bar{B}.C$

(aplica de Morgam e obtém F2)

Karnaugh maps – 4 variáveis

Um mapa de **4 variáveis** tem uma célula adjacente em cada um dos seus quatro limites

A	*B*	*C*	*D*
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



Cada célula difere de sua célula adjacente por apenas uma variável

FIGURE 4-26 A 4-variable Karnaugh map

$AB \backslash CD$	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

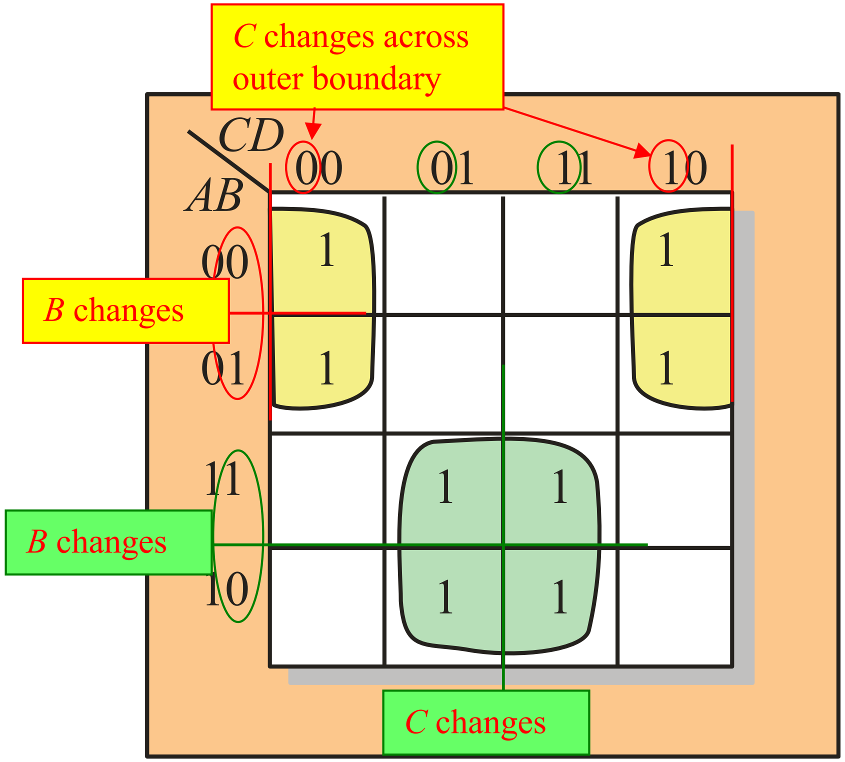
(a)

$AB \backslash CD$	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}\,\overline{B}CD$	$\overline{A}\,\overline{B}C\overline{D}$
01	$\overline{A}B\overline{C}\,\overline{D}$	$\overline{A}B\overline{C}D$	$\overline{A}BCD$	$\overline{A}BC\overline{D}$
11	$AB\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
10	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

(b)

Karnaugh maps – 4 variables

Agrupar os 1's no K-map de forma a minimizar a função.



1. Agrupe os 1's em dois grupos separados, conforme indicado.
2. Para cada grupo eliminar os literais redundantes.

O grupo amarelo é $\bar{A}\,.\,\bar{D}$

O grupo verde é **A.D**

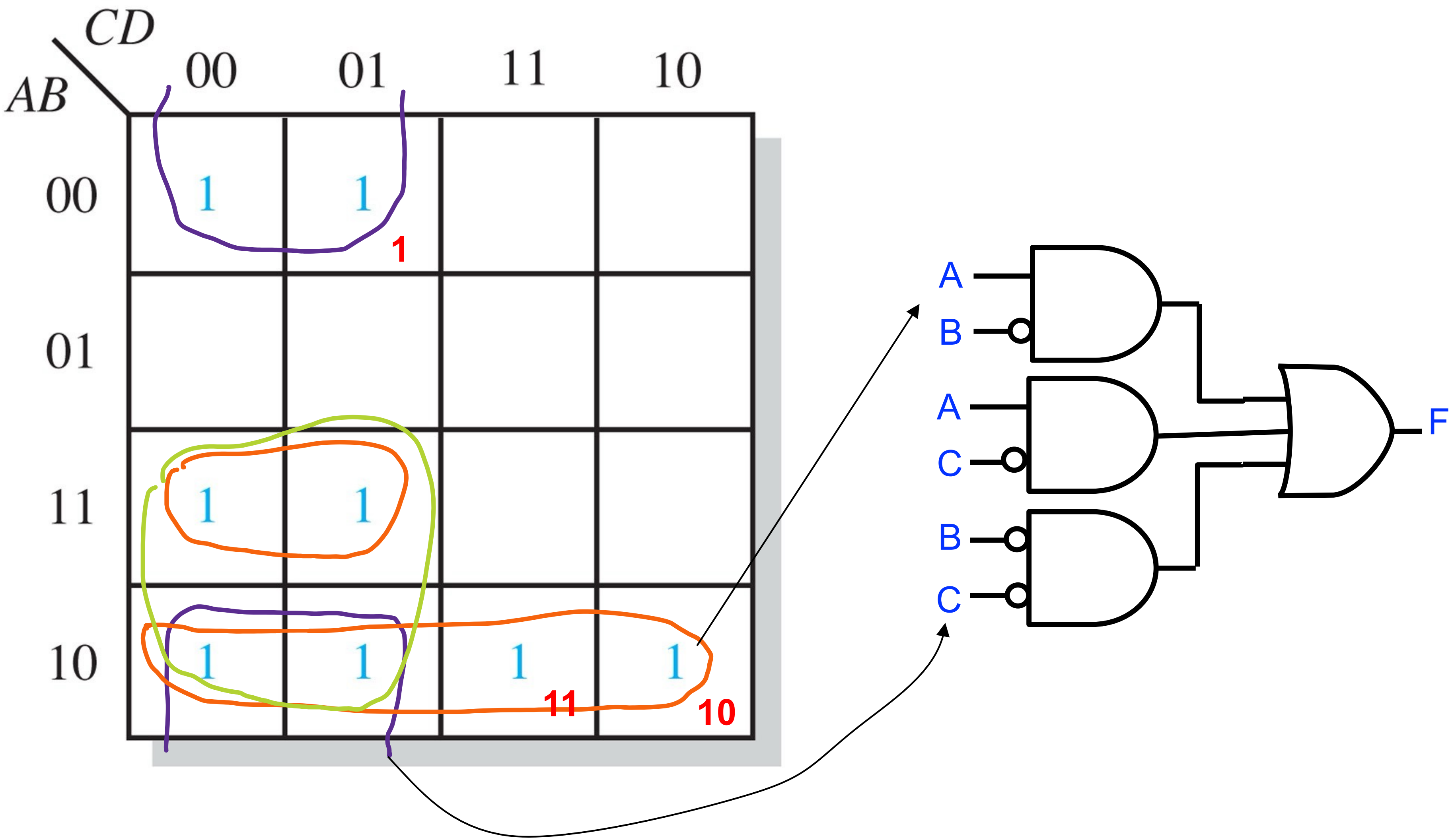
$$F = \bar{A}\,.\,\bar{D} + A.\,D$$

Minimizar (1/3)

Minimize a seguinte expressão em um mapa de Karnaugh:

$$F = \bar{B}.\bar{C} + A.\bar{B} + A.B.\bar{C} + A.\bar{B}.C.\bar{D} + \bar{A}.\bar{B}.\bar{C}.D + A.\bar{B}.C.D$$

10 1 11



Minimizar (2/3)

F2 = ∑ (2, 3, 4, 5, 6, 7, 9, 12, 13, 14, 15)

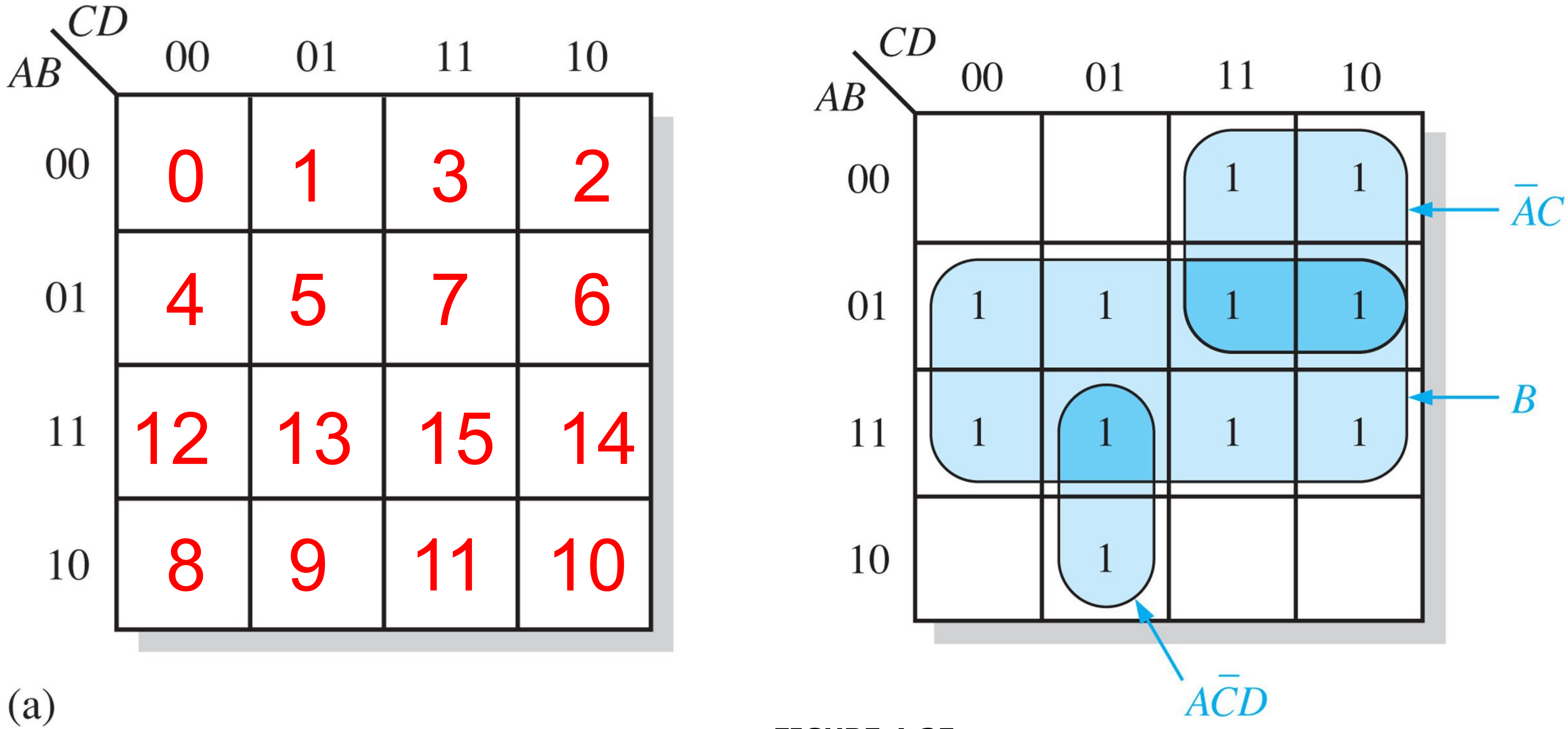


FIGURE 4-35

Minimizar (3/3)

Mapa de Karnaugh para as seguintes expressões, apresentando o circuito resultante

1. A’BC + AB’C + AB’C’
2. A’BCD’ + ABCD’ + ABC’D’ + ABCD

A/BC	00	01	11	10
0			1	
1	1	1		

F1 = AB’+ A’BC

AB/CD	00	01	11	10
00				
01				1
11	1		1	1
10				

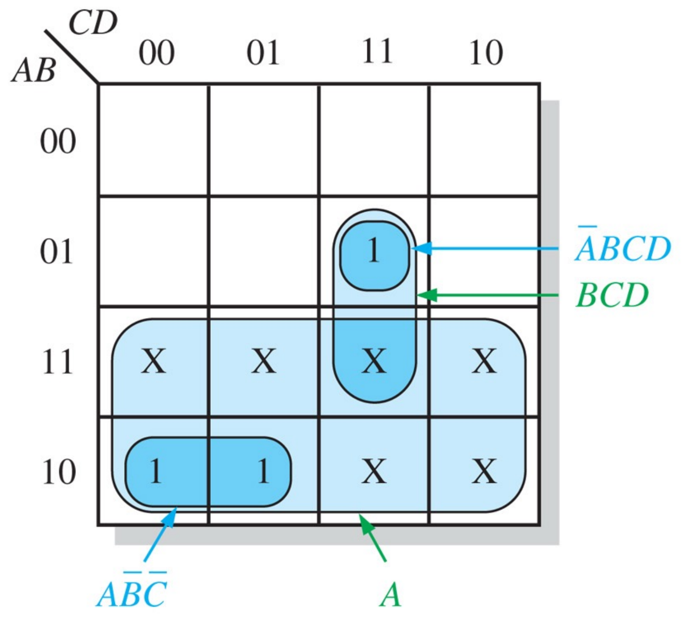
F2 = ABC + ABD’ + BCD’

Uso de don’t cares

Inputs				Output
A	*B*	*C*	*D*	*Y*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don’t cares

(a) Truth table



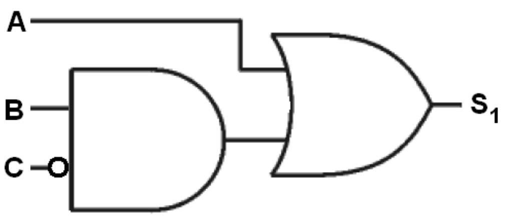
(b) Without “don’t cares” $Y = A\overline{B}\,\overline{C} + \overline{A}BCD$
With “don’t cares” $Y = A + BCD$

FIGURE 4-40 Example of the use of “don’t care” conditions to simplify an expression.

Minimize (4/4) – com don't cares

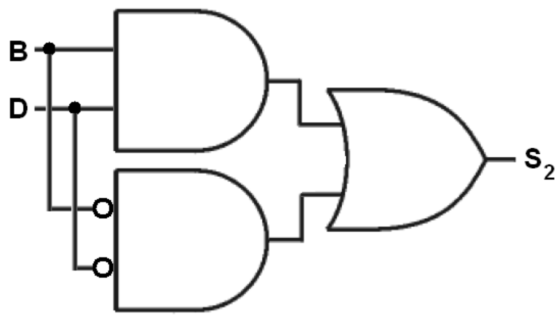
S_1 AB \ CD	00	01	11	10
00	0	X	0	0
01	1	X	X	0
11	1	1	X	1
10	1	1	X	1

$$S_1 = B\overline{C} + A$$



S_2 AB \ CD	00	01	11	10
00	1	0	0	X
01	0	1	X	0
11	0	X	1	0
10	X	0	0	1

$$S_2 = BD + \overline{B}\,\overline{D}$$



USO PRÁTICO DE DON'T CARE - BCD para 7 segementos

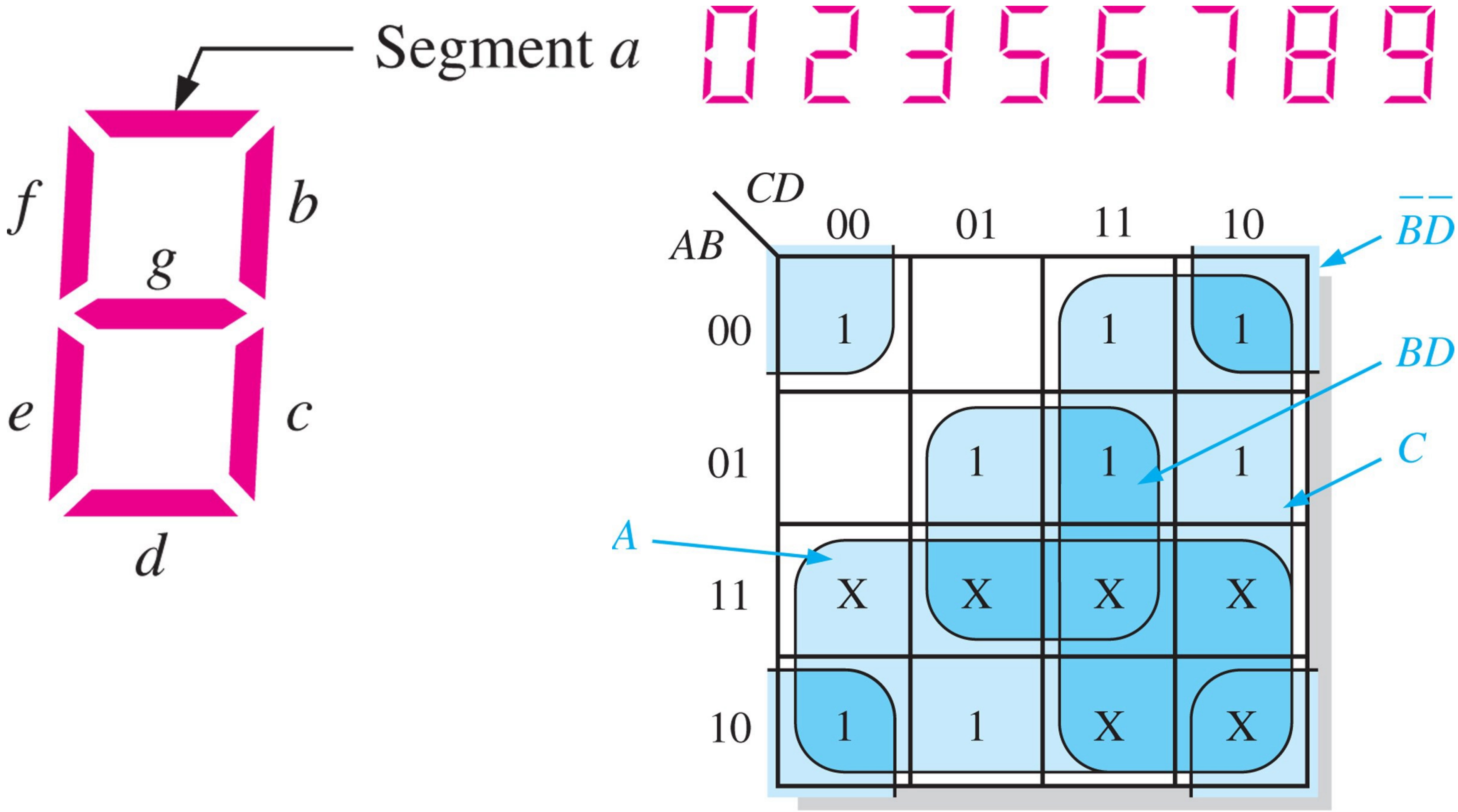


FIGURE 4-41 7-segment display.

2-6 variáveis

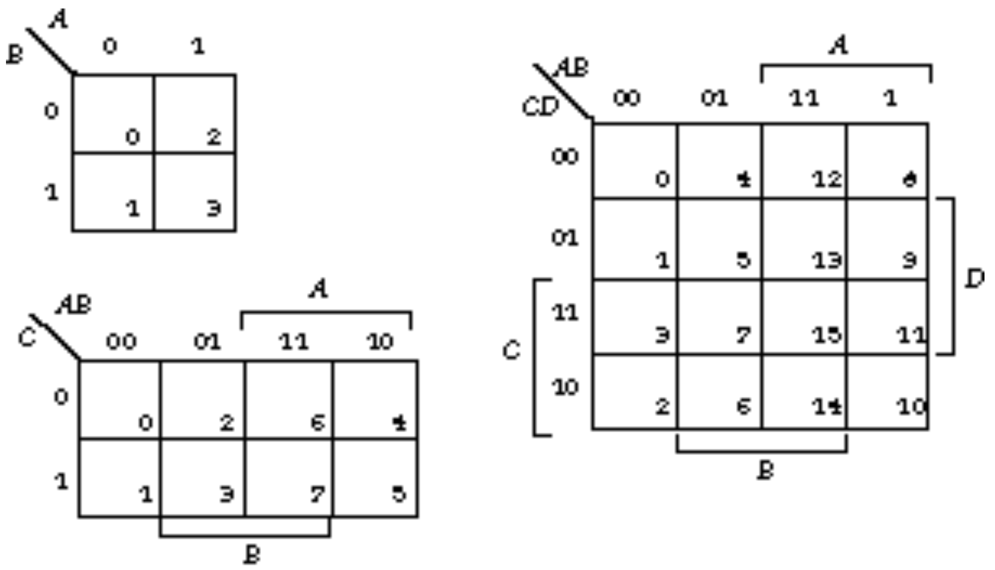
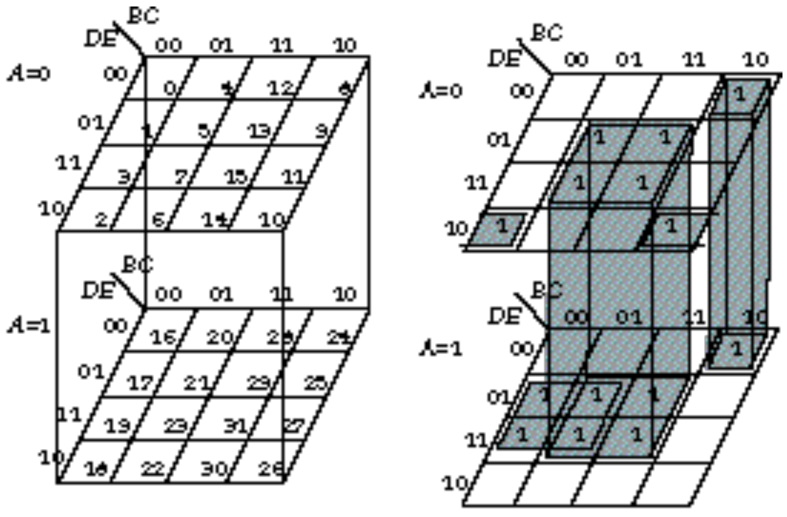
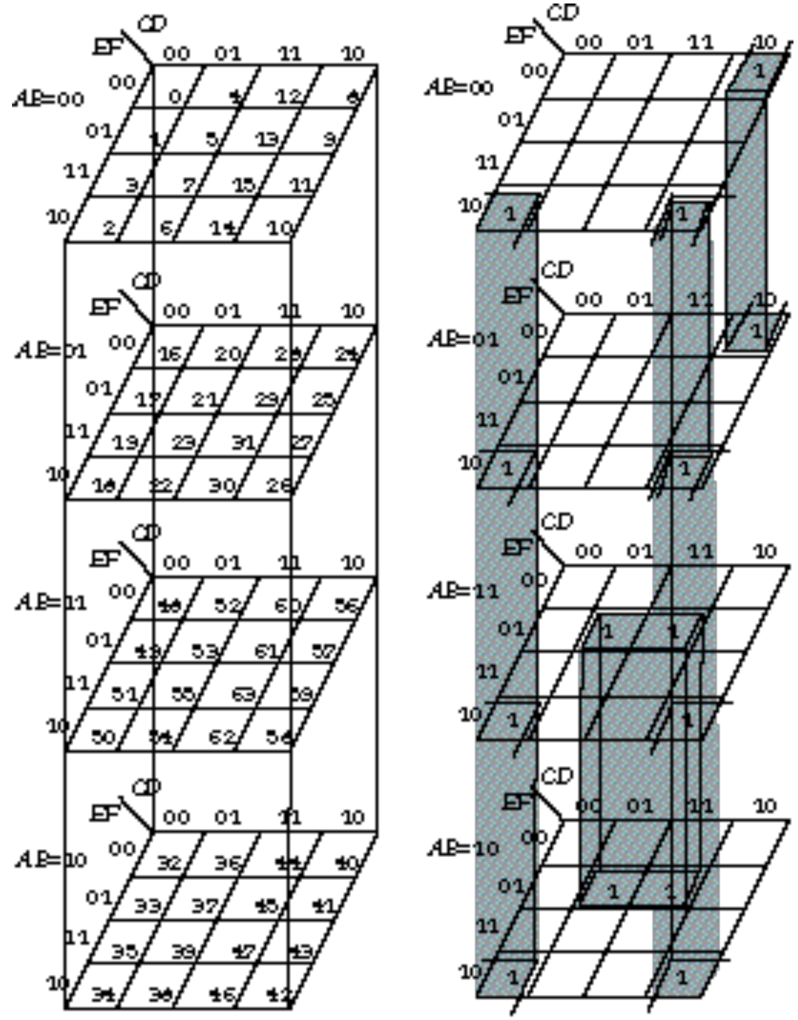


Figure 2.29 Two-, three-, and four-variable K-map templates.



(a) Five-variable K-map (b) Example

Figure 2.51 Five-variable K-map and example.



(a) Six-variable K-map (b) Example

Figure 2.52 Six-variable K-map and example.

Solução da lista

EXERCÍCIOS SOBRE MAPAS DE KARNAUGH E ÁLBEGRA BOOLEANA

1. Minimize as seguintes funções utilizando Mapa de Karnaugh:

a. F(a,b,c) = a.b + b’.c + a’.b’+ b.c’

b. F(a,b,c,d) = a.b.d + a.b’.c + b.c’.d + b.c.d’

c. F(a,b,c,d) = ∑ (0001, 0101, 1111, 1010, 1001)

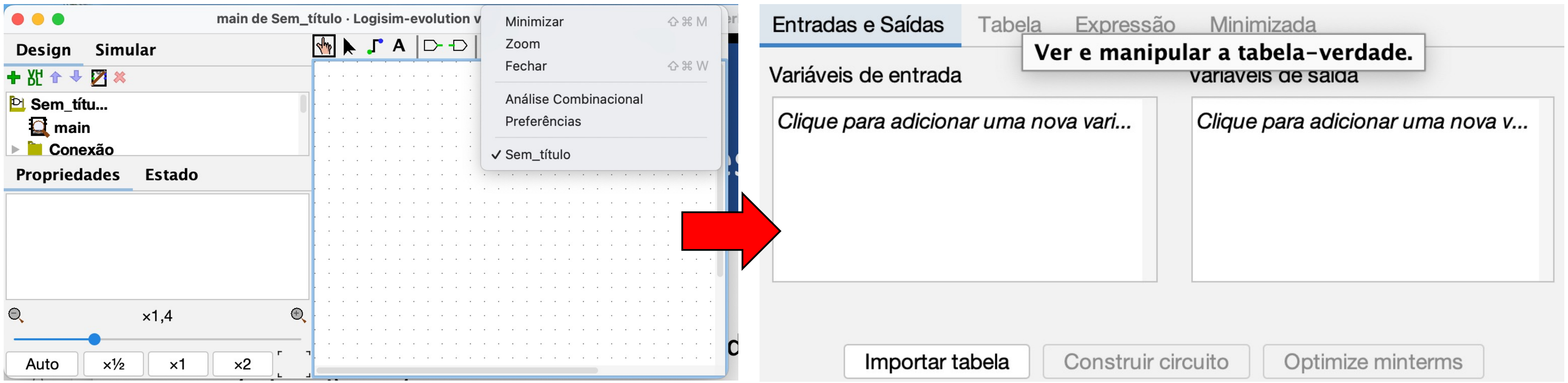
d. F(a,b,c,d) = ∑ (2, 4, 5, 6, 7, 11, 15)

e. F = (a + b) . (b’ + c + d)

f. F = a.b.c.(a’+c’+d’)

https://github.com/logisim-evolution/logisim-evolution/releases/tag/v3.8.0

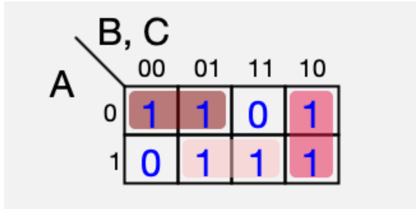
para instalaçao no Windows usar o logisim-evolution-3.8.0-x86.msi



(a) F(a,b,c) = a.b + b'.c + a'.b'+ b.c'

No logisim: A B|~B C|~A ~B|B ~C

A \ BC	00	01	11	10
0				
1				



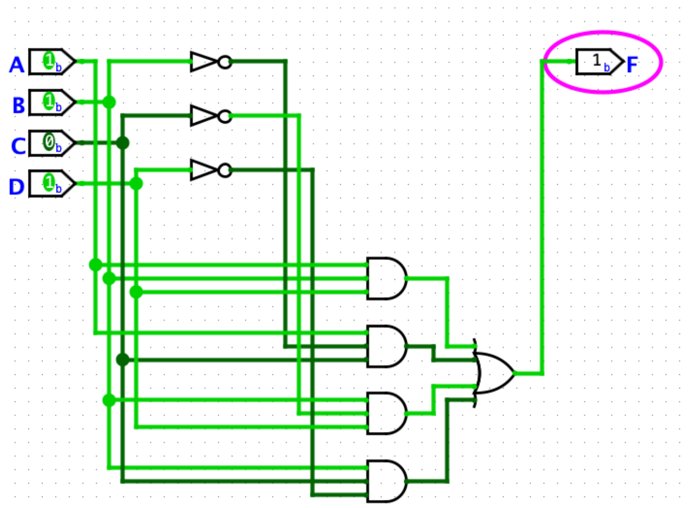
F = A'B' + AC + BC'

(b) F(a,b,c,d) = a.b.d + a.b'.c + b.c'.d + b.c.d'

A B D |A ~B C| B ~C D| B C ~D

F — A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F — A, B \ C, D	00	01	11	10
00	0	0	0	0
01	0	1	0	1
11	0	1	1	1
10	0	0	1	1



$$F = B \cdot \overline{C} \cdot D + B \cdot C \cdot \overline{D} + A \cdot C$$

(c) F(a,b,c,d) = ∑ (0001, 0101, 1111, 1010, 1001)

~A ~B~C D | ~A B ~C D | A B C D | A ~B C ~D | A ~B ~C D

F A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F A, B \ C, D	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	0	0	1	0
10	0	1	0	1

$$F = \overline{A} \cdot \overline{C} \cdot D + \overline{B} \cdot \overline{C} \cdot D + A \cdot \overline{B} \cdot C \cdot \overline{D} + A \cdot B \cdot C \cdot D$$

(d) F(a,b,c,d) = ∑ (2, 4, 5, 6, 7, 11, 15)

~A ~B **C** ~D | ~A **B** ~C ~D | ~A **B** ˜C **D** | ~A **B C** ~D | ~A **B C D** | **A** ~B **C D** | A B C D

F A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F A, B \ C, D	00	01	11	10
00	0	0	0	1
01	1	1	1	1
11	0	0	1	0
10	0	0	1	0

$$F = \overline{A} \cdot C \cdot \overline{D} + \overline{A} \cdot B + A \cdot C \cdot D$$

(e) F = (a + b) . (b’ + c + d)

F = AB’ + AC + AD + ~~BB’~~ + BC + BD

A ~B | A C | A D | B C | B D

F: A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F: A, B \ C, D	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	0	1	1	1
10	1	1	1	1

$$F = B \cdot D + B \cdot C + A \cdot \overline{B}$$

(f) F = a.b.c.(a’+c’+d’) → não usar karnaugh

F = ABCA’ + ABCC’ + ABCD’

F = ABCD’

2. Utilize Mapa de Karnaugh para simplificar as seguintes expressões:

a. A'.B'.C' + A'.B'.C + A.B'.C

A \ BC	00	01	11	10
0	1	1	0	0
1	0	1	0	0

F = A'B' + B'C

b. A.C.(B'+C) **→ ACB' + ACC**

A \ BC	00	01	11	10
0	0	0	0	0
1	0	1	1	0

F = AC

c. A’.(B.C + B.C’) + A.(B.C + B.C’) → **A’BC + A’BC’ + ABC + ABC’**

A \ BC	00	01	11	10
0	0	0	1	1
1	0	0	1	1

F = B

d. A’.B’.C’ + A.B’.C’ + A’.B.C’ + A.B.C’

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	1

F = C’

3. Expanda cada expressão para o formato padrão SOP e então utilize Mapa de Karnaugh para simplificá-las:

a. A.B + A.B'.C + A.B.C → **ABC' + ABC + AB'C + ABC**

A \ BC	00	01	11	10
0	0	0	0	0
1	0	1	1	1

F = AB + AC

b. A + B.C → **AB'C' + AB'C + ABC' + ABC + A'BC + ABC**

A \ BC	00	01	11	10
0	0	0	1	0
1	1	1	1	1

X = A + BC

Exercícios (c) e (d) recomenda-se verificar com o software logisim

4. Utilize Mapa de Karnaugh para simplificar as seguintes expressões:

a. F = $A + B.\overline{C} + C.D$

A + B ~C + C D

A, B \ C, D	00	01	11	10
00	0	0	1	0
01	1	1	1	0
11	1	1	1	1
10	1	1	1	1

b. F = $\overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.\overline{C}.D + A.B.C.D + A.B.C.\overline{D}$

~A ~B ~C ˜D + ~A ~B ~C D + A B C D + A B C ~D

F — A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F — A, B \ C, D	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	0	0

$$F = \overline{A} \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot C$$

c. F = $\overline{A}.B.(\overline{C}.\overline{D} + \overline{C}.D) + A.B.(\overline{C}.\overline{D} + \overline{C}.D) + A.\overline{B}.\overline{C}.D$

~A B ~C + A B ˜C + A ~B ~C D

F — A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F — A, B \ C, D	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	1	1	0	0
10	0	1	0	0

$$F = B \cdot \overline{C} + A \cdot \overline{C} \cdot D$$

d. $F = (\bar{A}.\bar{B} + A.\bar{B}).(C.D + C.\bar{D})$

~A ~B C D + ~A ~B C ~D +
A ~B C D + A ~B C ~D

F: A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F: A, B \ C, D	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	1	1

$$F = \overline{B} \cdot C$$

e. $F = \bar{A}.\bar{B} + A.\bar{B} + \bar{C}.\bar{D} + C.\bar{D}$

~A ~B + A ~B + ~C ~D + C ~D

$f =$

F: A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F: A, B \ C, D	00	01	11	10
00	1	1	1	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$$F = \overline{B} + \overline{D}$$

5. Reduza a função especificada pelas tabelas verdades abaixo utilizando Mapa de Karnaugh:

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

F — A \ B, C	00	01	11	10
0				
1				

F — A \ B, C	00	01	11	10
0	1	1	1	0
1	1	1	1	0

$$F = \overline{B} + C$$

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

F — A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F — A, B \ C, D	00	01	11	10
00	0	1	0	1
01	0	0	1	1
11	1	1	1	0
10	1	0	0	1

$$F = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D + \overline{A} \cdot B \cdot C + \overline{B} \cdot C \cdot \overline{D} + A \cdot \overline{C} \cdot \overline{D} + A \cdot B \cdot D$$

Tabela verdade com don’t cares (- ou X)

A	B	C	D	F
0	0	0	0	1
0	0	0	1	–
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	–
1	0	1	1	1
1	1	0	0	–
1	1	0	1	1
1	1	1	0	–
1	1	1	1	0

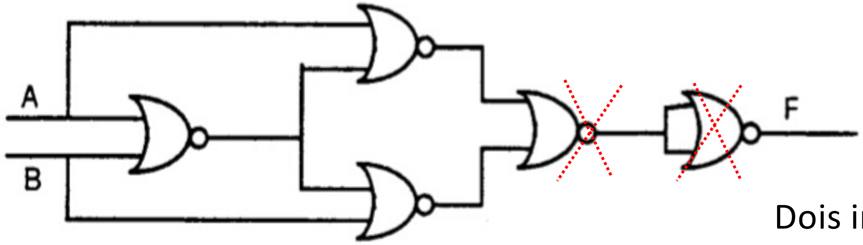
F — A, B \ C, D	00	01	11	10
00				
01				
11				
10				

F — A, B \ C, D	00	01	11	10
00	1	-	0	1
01	1	1	0	1
11	-	1	0	-
10	1	0	1	-

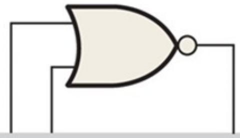
$$F = \overline{D} + A \cdot \overline{B} \cdot C + B \cdot \overline{C}$$

6. Qual a operação lógica realizada pelo circuito abaixo?

Qual a operação lógica realizada pelo circuito abaixo?



Dois inversores – se anulam



0	0	1
0	1	0
1	0	0
1	1	0

NOR

$$\boldsymbol{F = \overline{A + (\overline{A + B})} + \overline{B + (\overline{A + B})}}$$

De Morgan em cada parte

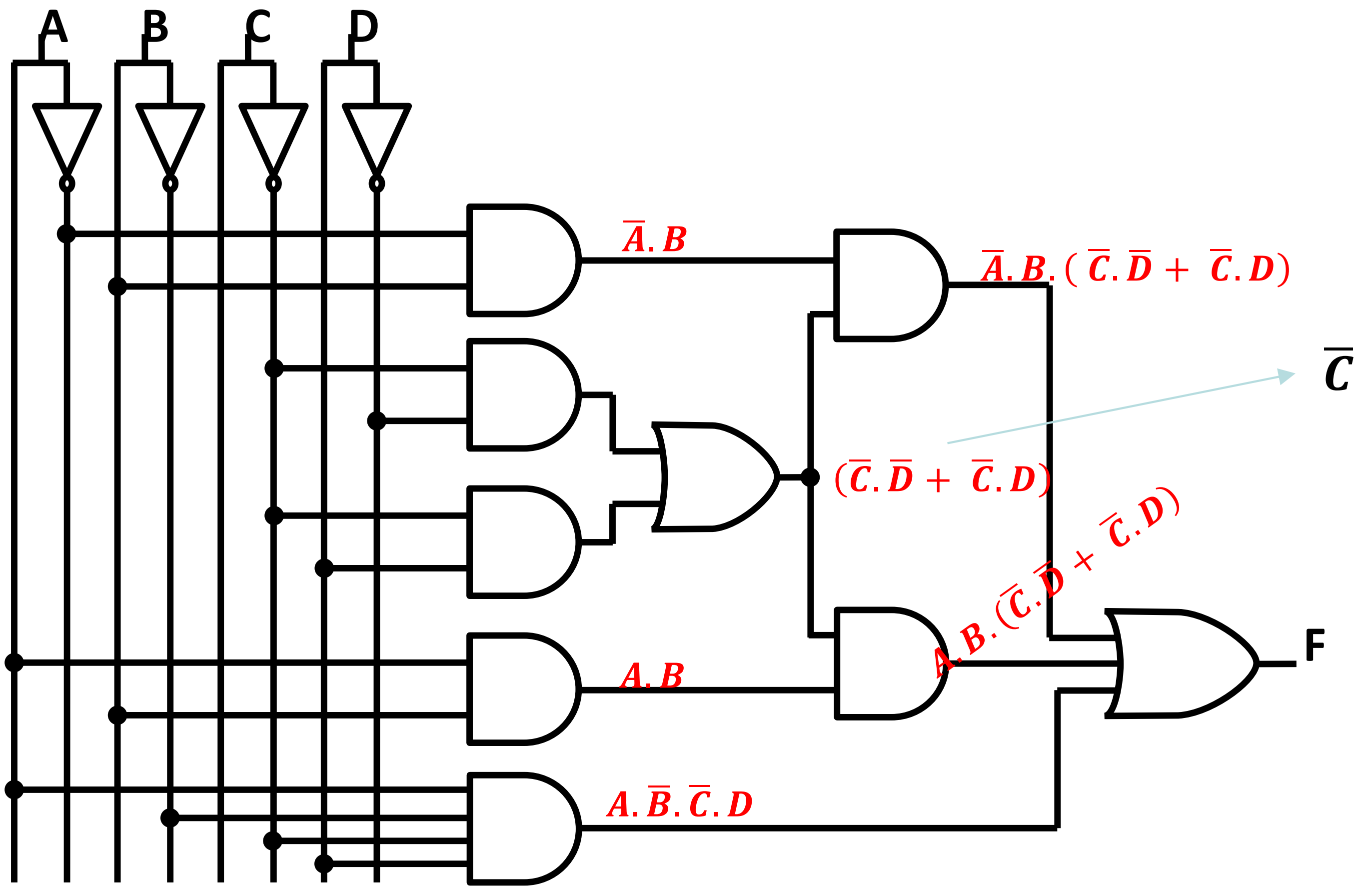
$$\boldsymbol{F = \left(\bar{A}.(A + B)\right) + (\bar{B}.(A + B))}$$

$$\boldsymbol{F = (\bar{A}.B\ ) + (\bar{B}.A)}$$

$$\boldsymbol{F = A\ \ xor\ B}$$

7. Minimizar por álgebra booleana e mapa de Karnaugh

(a) Diagrama de Portas Lógicas



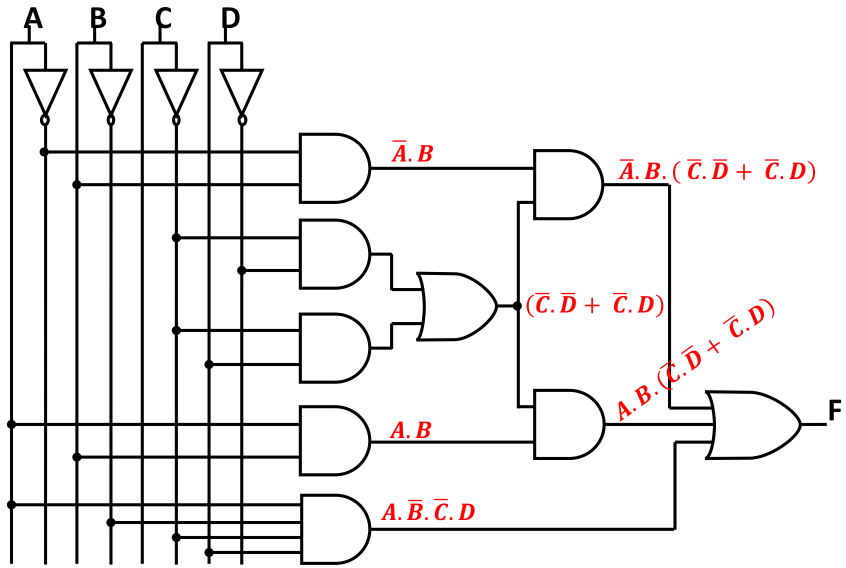
$$F = \overline{A}.B.(\overline{C}.\overline{D} + \overline{C}.D) + A.B.(\overline{C}.\overline{D} + \overline{C}.D) + A.\overline{B}.\overline{C}.D$$

(a) Diagrama de Portas Lógicas (cont)

$$\boldsymbol{F = \bar{A}.B.(\,\bar{C}.\bar{D} + \bar{C}.D) + A.B.(\bar{C}.\bar{D} + \bar{C}.D) + A.\bar{B}.\bar{C}.D}$$

$$F = \bar{A}.B.\bar{C} + A.B.\bar{C} + A.(\bar{B} + B).\bar{C}.\mathrm{D}$$

$$F = B.\bar{C} + A.\bar{C}.\mathrm{D}$$



0100 → 4
0101 → 5
1100 → 12
1101 → 13
100**1** → 9

AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$F = \sum(\,4,5,9,12,13)$$

$B.\bar{C}$

$A.\bar{C}.\mathrm{D}$

F = $\boldsymbol{B.\bar{C} + A.\bar{C}.\mathrm{D}}$

(b) Expressão booleana

$$F = \underbrace{\overline{A}.\overline{B} + A.\overline{B}}_{\overline{B}} + \underbrace{\overline{C}.\overline{D} + C.\overline{D}}_{\overline{D}}$$

AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$\overline{B}$ $\qquad\qquad$ $\overline{D}$

$$F = \overline{B} + \overline{D}$$

00xx → 0, 1, 2, 3
10xx → 8, 9, 10,11
xx00 → 0, 4, 8, 12
xx10 → 2, 6, 10, 14

$$F = \sum (0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)$$

(c) Expressão booleana

$$\boldsymbol{F = B.\overline{C}.D \; + \; \overline{A}.B.D \; + \; \overline{A}.C.D + A.D \; + \; A.\overline{D} \; + \; B.\overline{C}.\overline{D}}$$

$$\boldsymbol{A}$$

$$\boldsymbol{B.\overline{C}}$$

$\boldsymbol{F = A + B.\overline{C} + \; \overline{A}.B.D \; + \; \overline{A}.C.D}$ $\quad (\,\boldsymbol{\overline{A}}\, pode\ ser\ removido)$

$\boldsymbol{F = A + B.\overline{C} + \; B.D \; + \; C.D}$ $\quad (BD = B\bar{C}D + BCD, logo\ pode\ ser\ também\ removido)$

$\boldsymbol{F = A + B.\overline{C} + C.D}$

X101 → m13, m5
01x1 → m5, m7
0x11 → m7, m3
1xx1 → m9, m11, 13, 15
1xx0 → m8, m10, m12, m14
x100 → m4, m12

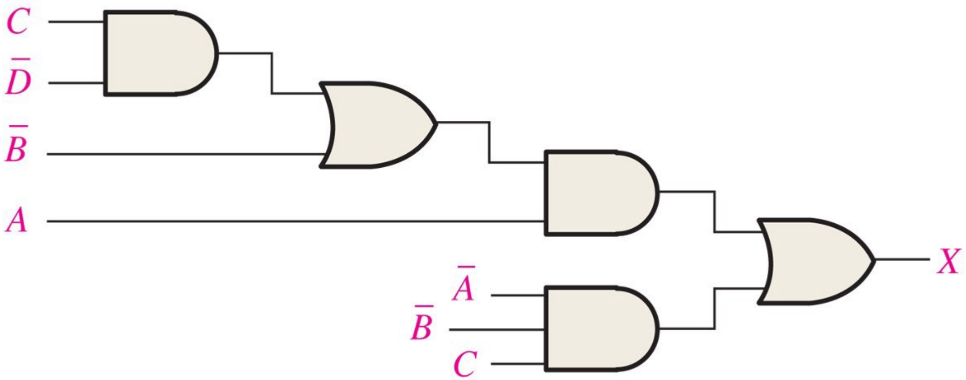
$$F = \sum (3, 4, 5, 7, 8, 9, 10, 11,12, 13, 14, 15)$$

AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

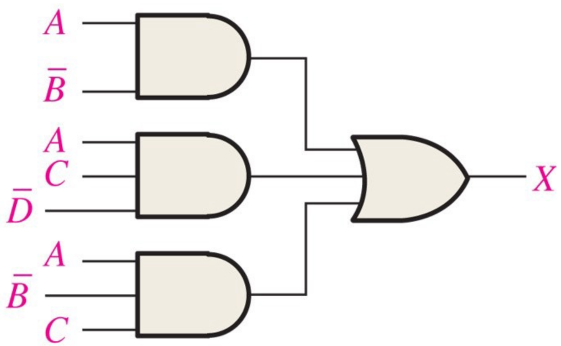
$B.\bar{C}$ $\quad A$ $\quad CD$

$$\boldsymbol{F = A + B.\overline{C} + C.D}$$

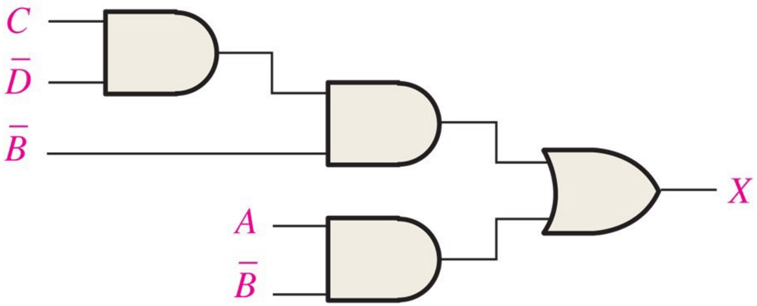
8. Determinar na figura abaixo os circuitos equivalentes



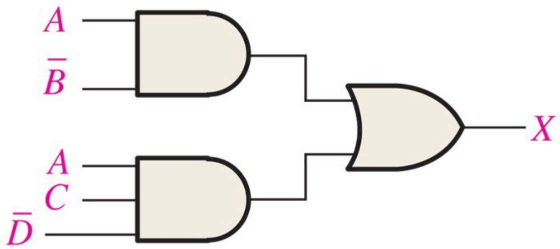
(a)



(b)



(c)



(d)

SOLUÇÃO:

(a)AB’ + B’C + ACD’
(b)AB’+ ACD’
(c)AB’+ B’CD
(d)AB’+ ACD’

B e D

9. Questão POSCOMP

QUESTÃO 14 – Resolva a identidade $\overline{\overline{E} * \overline{B}}$, aplicando as leis da álgebra de Boole.

A) E
B) E*B
C) B
D) E+B
E) $\overline{E}$

Recomenda-se verificar com o software logisim a resposta

10. Questão POSCOMP

675_POSCOMP_NS_6/9/202211:41:36

QUESTÃO 42 – Uma expressão lógica do tipo soma de produtos está na forma canônica se cada um de seus mintermos (termos produto) contêm todas as variáveis, seja na forma direta ou na forma complementada. Assinale a alternativa que contém a expressão lógica, representada pela soma dos seus mintermos, cuja simplificação pela álgebra booleana fornece a expressão:

$$x = A\overline{B} + \overline{B}C$$

A) $f(A, B, C) = \sum m(1,3,5)$
B) $f(A, B, C) = \sum m(1,4,5)$
C) $f(A, B, C) = \sum m(2,3,5)$
D) $f(A, B, C) = \sum m(2,4,6)$
E) $f(A, B, C) = \sum m(2,5,6)$

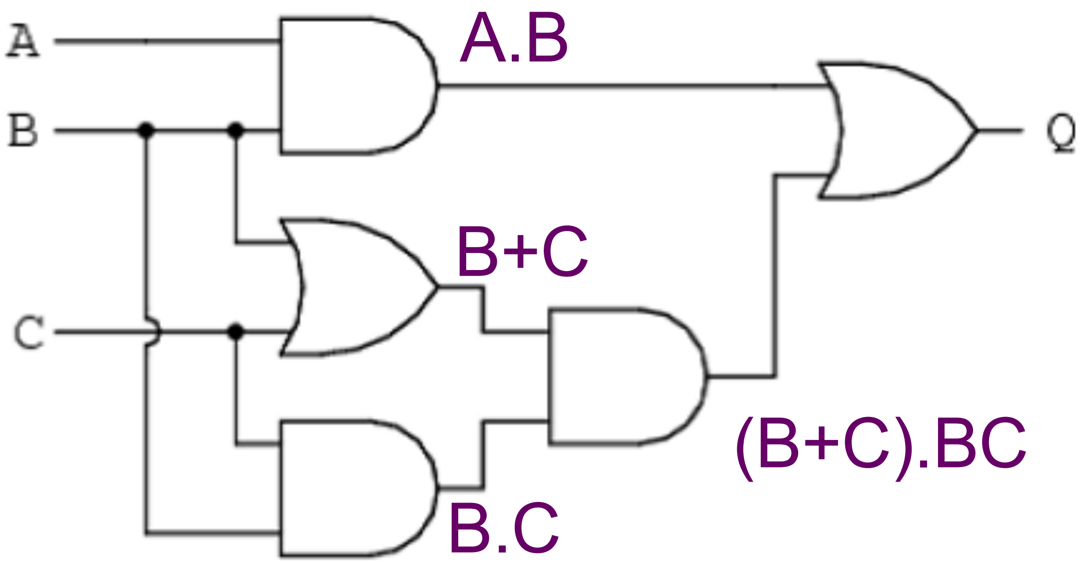
100 / **10**1 + 0**01** / 1**01**

4 / **5** + **1 / 5**

{1, 4, 5} → letra b

11. Questão POSCOMP

(**POSCOMP 2009 - 39**) Considerando o circuito digital abaixo, qual o valor de Q?



a) A + BC
b) B (A + B + C)
c) C (A + B)
d) A (B + C)
e) B (A + C)

A.B + (B+C).(B.C)

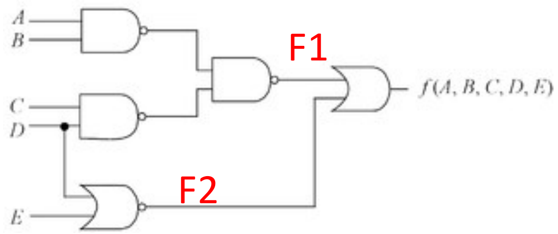
A.B + B.C + B.C

A.B + B.C

B.(A+C) → letra E

12. Questão ENADE

8. (ENADE 2008-CC - 38) No circuito apresentado nesta questão, que possui cinco entradas — A, B, C, D e E — e uma saída f(A, B, C, D, E), qual opção apresenta uma expressão lógica equivalente à função f?



- A) F = (AB)' + (CD)' + DE
- B) (A + B).(C+D) + D.E
- C) A.B + C.D + D'E'
- D) F = (AB)' + (CD)' + D + E
- E) F = AB + CD + D + E

$$F1 = AB + CD$$

$$F2 = \overline{D + E} = \bar{D}.\bar{E}$$

$$saída = AB + CD + \bar{D}.\bar{E}$$

Letra C

13. Fazer por álgebra booleana e Karnaugh

QUESTÃO 16 – Simplificando f=a’bc+abc+abc’, utilizando o diagrama de Karnaugh, temos:

A) f = ab+bc
B) f = ab+b’c
C) f = abc
D) f = ab
E) f = bc

$$A'\cdot B\cdot C + A\cdot B\cdot C + A\cdot B\cdot C'$$

$$AB + BC$$

Letra a

Recomenda-se verificar com o software logisim a resposta