Symbolic 2D Nested Convolution Fast-Convolution Python Library June 20, 2024

 $s = (a_1^t \otimes a_2^t) \{ [(q_1 \odot b_1)g(q_2 \odot b_2)^t] \odot (c_1^t \otimes c_2^t) \} d$ $G = (q_1 \odot b_1)g(q_2 \odot b_2)^t$

		$\begin{bmatrix} D_0 \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \\ D_{13} \\ D_{14} \\ D_{15} \\ D_{16} \\ D_{17} \\ D_{18} \\ D_{19} \\ D_{20} \\ D_{21} \\ D_{22} \\ D_{23} \\ D_{24} \end{bmatrix}$				
	$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{bmatrix} =$	$-4d_1$			G_{10} G_{15}	
	$=igg _{S_{10}}$	$a_{1} + 2d_{12}$			G_1 G_2 G_6 G_7 G_{11} G_1 G_{16} G_2 G_{21} G_2	
C =	$S_{11} + S_{11}$	$4d_{11} + 4d_{13} + 4d_{13} + 2d_{10} + 6d_{10} + 3d_{10} + 3d_{$			G_{1} G_{3} G_{8} G_{13} G_{18} G_{23}	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$12 + 4S_{13}$	$-2d_{12} - 2d_{12} - 2d_{13} - 2d_{14} - 2d_{11} + 2d_{11} + 2d_{11} - 2d_{11} - 2d_{11} + 2d_$		C =	$G_{4} \ G_{9} \ G_{14} \ G_{19} \ $	$\begin{bmatrix} \gamma_0 \\ \gamma_5 \\ \gamma_{10} \end{bmatrix}$
	$S_{12} + 2$ $S_{13} + S_{14} + 2$ $S_{11} - S_{11} + 3$ $S_{12} - 4S_{13} + 4S_{12} + 2S_{13} + 4S_{14} + 2S_{15} + 2S_{1$	$\begin{array}{c} 2d_{13} - 2 \\ 2d_{13} + 2 \\ 2d_{13} + 2 \\ 2d_{13} + 2 \\ 2d_{14} - 2a \\ + 2d_{16} - \\ d_{12} - d_{1} \\ + d_{12} - \\ + 3d_{12} - \\ d_{13} - d_{1} \\ d_{13} - d_{1} \\ - 3d_{12} - \\ + 9d_{12} - \\ 3d_{13} - 3a \\ 2d_{15} - d_{1} \\ - 2a \\ 2d_{16} \\ 2d_{16} - d_{1} \\ d_{15} + 2d_{1} \\ 3 + 4d_{16} \\ d_{13} - d_{13} - d_{14} \\ - d_{13} - d_{13} \end{array}$	A	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		γ_1 γ_6 γ_{11}
1 2 ² 2 ² 2 1	$2S_{13} + R_{16} + S_{16} + S_{12} - S_{12} + S_{12} - S_{13} - S_{13} - S_{15} + 4R_{13} + 4S_{13} + 4S$	$d_{16} - d_{16} - d_{16} - d_{16} - d_{16} - d_{16} - d_{16} - d_{17} - d_{13} + d_{13} - d_{13} + d_{13} - d_{13} + d_{14} + d_{16} - d_{16} - d_{16} - d_{16} - d_{17} - d_{16} - d_{17} - d$	$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$			$\gamma_2 \gamma \\ \gamma_7 \gamma \\ \gamma_{12} \gamma_1$
$A = \begin{bmatrix} 2^1 & 0 & 0 \\ 2^1 & 0 & 0 \\ 2^1 & 0 & 0 \\ 2^1 & 0 & 0 \\ 2^2 & 2^1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 &$	$S_{16} - S_{17} + S_{13} + 2S_{13} + 2S_{14} + 2S_{16} + 4S_{16} + 4S_{16} - 4S_{16}$	$egin{array}{ll} l_{17} + d_{1} \\ d_{17} + d_{1} \\ d_{17} + d_{18} \\ 2d_{18} + d_{16} \\ 2d_{18} + d_{15} \\ 5 - d_{16} \\ d_{16} - d_{16} \\ d_{16} - d_{17} \\ d_{15} - d_{11} \\ 2d_{16} - d_{16} \\ d_{16} - d_{17} \\ d_{16} - d_{17} \\ d_{16} - d_{18} \\ d_{17} + d_{18} \\ d_{17} + d_{18} \\ d_{18} + d_{19} \\ d_{17} - 2d_{18} \\ d_{18} - 2d_{1$	0 0 0 0 0 0	3 4 2 0 2 0 2 0 2 0 2 0 0 0 0 0 0 0 0 0 0 0		$ \begin{array}{ccc} $
$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ -2^2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2^2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2^2 \end{array}$	$egin{array}{l} f_{17} + 2S \\ 4S_{18} + R \\ f_{15} + 2S \\ 2S_{16} - f_{16} + 2S \\ S_{17} + 4 \\ f_{17} + 8S \end{array}$	$8 - 4d_1$ $18 + 4d_1$ $- 2d_1$	4 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0			$= \begin{bmatrix} rac{g_0}{2} \\ rac{g_1}{2} \\ rac{g_0}{2} \end{bmatrix}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 &$	$\begin{array}{c} -2d_2 - \\ -2d_3 + \\ +2d_3 + \\ +d_1 - 2d_2 \\ +d_{18} - \\ -3 + 4d_6 \\ -2d_6 - 2d_1 \\ -7 + d_{18} - \\ -4d_{18} - 4d_1 \\ -4d_{18} - 4d_1 \\ -4d_{18} + 4d_1 \\ -3d_7 - \\ -d_8 \\ +d_7 + 2d_1 \\ -d_{22} + \\ -3d_{22} \\ -d_{23} - 2d_1 - d_{22} \end{array}$	0 0 1 0 1 1 0 0 1	6 7 1 2 2 1 1 -2 3 1 0 -2 1 2 4 4 2 -4 6 2 0 -4 2 4 -6 -2 0 4 -2 1 2 2 1 -2 3 1 0 -2 1 -2 1 -2 0 4 -2 1 -2 0 4 -2 1 -2 1 -2 1 -2 1 4 -2 1 -2 1 -2 1 -2 1 -2 1 -2 1 -2 1 -2 1		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	S_5 S S_{10} S_5 S_{15} S_{15} S_{20} S_2 S_2 S_3 S_4 S_4 S_5 S_5 S_5 S_6 S_7 S_8	$+2d_3 + +2d_3 - d_6 - d_8 2 - 4d_5 + 2 + 2d_7 - d_6 + 6d_7 d_8 - 4d_6 + 2 + 4d_5 - 3 - 2d_7 + d_6 - 6d_7 2d_8 + 4d_6 - 2d_7 - d_8 d_8 d_8 - 2d_2 - - 2d_2 - + d_23 - 4 + d_23 + 2d_6 + 2d_6 + 2d_7 - 2d_2 - - 2d_2 - + d_23 - 4 + d_23 + - 2d_6 + 2d_7 - d_8 - 2d_8 - 4d_8 - d_9 - 2d_8 - d_8 - d_9 - 2d_8 - d_9 - d_9 $	1		$-\frac{\gamma_{12}}{2} \\ -\frac{\gamma_{12}}{2} \\ -\frac{\gamma_{12}}{6} \\ \frac{2\gamma_{12}}{3}$	$-\frac{g_1}{2}$ - $-\frac{g_4}{2}$ - $-\frac{g_7}{2}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$2d_{6} + d_{7}$ $2d_{8} - 2d_{8}$ $-2d_{8}$ $-2d_{8}$ $-2d_{7} - d_{8}$		9 10 0		$\frac{g_2}{2}$ — $\frac{g_5}{2}$ — $\frac{g_8}{2}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	S_{13} S_{18} S_{23} S_{18} S_{23} S_{18} S_{23} S_{18} S_{23} S_{18} S_{23} S_{18} S_{23} S_{18} S_{18} S_{23} S_{18} S_{23} S_{18} S_{23} S	$d_{7} - d_{8}$ $3d_{7} - d_{8}$ $3d_{7} - d_{6}$ $-2d_{6} + d_{7} - 2d_{6}$ $d_{8} - 2d_{4}$ $d_{4} - 2d_{5}$ $d_{4} - 2d_{5}$ $d_{5} - 2d_{6}$ $d_{7} + 2d_{8}$ $d_{6} - 2d_{7}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\frac{g_0}{6} + \frac{g_1}{6}$ $\frac{g_3}{6} + \frac{g_4}{6}$ $\frac{g_6}{6} + \frac{g_7}{6}$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		d_{8} $d_{7} + 2d_{8}$ d_{9} d_{8} d_{9} d_{2} $d_{6} - 4$ d_{8} d_{2} d_{8}	1 -1 1 -1 1 1 1 -1 1 -1 1			$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
20 0 0 20 0 0 20 21 0 0 21 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1	$= \begin{bmatrix} G_{1} \\ G_{1} \\ G_{2} \end{bmatrix}$ $+ S_{7} + S_{8} + S_{8} + S_{9} + S_{6} + S_{8} + S_{8$	$d_8 - d_9$ $4d_7 + 2$	$ \begin{array}{c} 1 \\ 2 \\ 4 \\ -1 \\ -2 \\ -4 \\ 1 \\ 2 \\ 4 \end{array} $	3 14 2 0 2 0 2 0 2 0 2 0 2 0 1 0 1 0 1 0 1 0 3 0 3 0 3 0 3 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 0 0 0 0 0		
$\begin{array}{c} 0 \\ 0 \\ 2^0 \\ 0 \\ 0 \\ \end{array}$	S_{8} S_{8} $S_{7} + S_{8}$	$= \frac{1}{2} d_8$	0 1 0 0 1 0 0 2 0 0 -1 0 0 4 0 0	0		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$G = G_1$ G_1 G_2 G_3 G_4 G_5 G_7 G_8 G_8 G_8 G_8 G_9	$\begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	-4		
$\begin{array}{c} 2^{0} \\ -2^{0} \\ 2^{0} \\ -2^{0} \\ 2^{0} \\ -2^{0} \\ 2^{0} \\ -2^{0} \\ 2^{0} \\ 2^{1} \\ 2^{1} \\ 2^{1} \\ 2^{0} \\ 2^{0} + 2^{1} \\ 2^{0} \\ 2^{0} + 2^{1} \\ 0 \\ 2^{0} + 2^{1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 2^{0} \\ 2^{0} + 2^{1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 2^{0} \\ 2^{0} \\ 2^{0} + 2^{1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 2^{0} \\ 0 \\ 0 \\ 2^{0} \\ 0 \\ 0 \\ 2^{0} \\ 0 \\ 0 \\ 0 \\ 2^{0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$	$ \begin{array}{cccc} 7 & G_8 \\ 2 & G_{13} \\ 7 & G_{18} \\ 2 & G_{23} \end{array} $ $ \begin{array}{c} 2 & G_{13} \\ 6 & G_{23} \end{array} $	0	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	0 -		$(q_1)g(q_2)$
-2^{0} -2^{0} -2^{0} -2^{0} -2^{0} -2^{0} -2^{0} -2^{0} -2^{0} -2^{0} -2^{0}	G_{3} G_{1} G_{2} G_{2} G_{3} G_{2} G_{2} G_{3} G_{2} G_{3} G_{4} G_{5} G_{5	0 0 0	2 0 4 1 2 0 4 0 8 2 4 0 8 0			
	$\begin{bmatrix} 0 & 4 & 0 & 0 \\ 4 & 9 & 4 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 &$	2 0 2 0 2 0 2 0 2 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1	3 20 0 0 0 0 0 0 0 0 0 0 0 0 0 0		
0	$\begin{array}{cccc} D_5 \\ D_{10} \\ D_{15} \\ D_{20} \\ \end{array}$ $\begin{array}{ccccc} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ \end{array}$	0 0	$0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1$			$\odot \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \end{bmatrix}$
$ 2^{0} $ $ 2^{0} $ $ 2^{0} $ $ 2^{1} $ $ 2^{2} $ $ 2^{1} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{2} $ $ 2^{3} $ $ 2^{1} $ $ 2^{$	$\begin{array}{c} D_{6} \\ D_{11} \\ D_{16} \\ D_{21} \\ \end{array}$ $\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \end{array}$	-2 4	0 0 0 0 0 0 0 0 0 0 1 0 2 0	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$-\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$
$egin{array}{cccc} 2^2 & 2^2 & 2^3 & 2^2 & 2^3 & 2^2 & 2^3 & 2^2 & 2^3 & 2^2 & 2^3 & 2^2 & 2^3 & 2^2 & 2^3$	D_7 D_{12} D_{17}	$ \begin{array}{cccc} -6 & & & \\ 0 & & \\ -2 & - & \\ \end{array} $	=			$-\frac{1}{6}$ $\frac{1}{6}$ $-\frac{1}{6}$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$D_8 \\ D_{13} \\ D_{18}$	2 2				161322
0 2	$ \begin{array}{c c} D_9 \\ D_{14} \\ D_{19} \end{array} $	$ \begin{array}{ccc} 0 & 0 \\ 2 & 0 \end{array} $	l 1 l –1 l 1	$\begin{bmatrix} 2 & - \\ 0 & - \\ 0 & 2 \\ 0 & - \\ 0 & 2 \end{bmatrix}$		$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	l 1 l -1 l 1 -1 l 1 -1 l 1	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		1 0		$\begin{array}{ccc} . & & & & & & & & & & & & & & & & & & $
$\begin{array}{c} 0\\0\\0\\0\\0\\2^0\\2^0\\2^0\\2^0\\2^0\\2^0\\2^0\\2^$	$ \begin{array}{c} 1 \\ 2 \\ 4 \\ -1 \\ -2 \\ -4 \\ 1 \\ 2 \\ 4 \end{array} $			1		$\begin{bmatrix} g_2 \\ g_5 \\ g_8 \end{bmatrix}$
$ \begin{array}{c c} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 &$	$ \begin{array}{cccc} 0 & & & \\ 0 & & & \\ 1 & & & \\ 0 & & & \\ -1 & & & \\ 0 & & & \\ 1 & & & \\ \end{array} $		1 1 - 1	$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \otimes$		$\left(\begin{bmatrix} \frac{1}{2} \\ -\frac{1}{6} \\ -\frac{1}{6} \end{bmatrix}\right)$
) (0) (0) (0) (0) (0) (0) (0) (0) (0) (0	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			0 -		\odot
)))))))))))))))))) 20 21 1 —	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		-1 - -2 - 2 - -1 (2 -		1 0 1 1 1 - 1 2
$egin{array}{cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $	4 0 8 0	$\begin{array}{c} -2 \\ -1 \\ -3 \\ 0 \\ -1 \\ -2 \\ -1 \\ -3 \\ 0 \\ -1 \\ -2 \\ -1 \\ -3 \\ 0 \\ -1 \\ -3 \\ 0 \\ -1 \\ 4 \\ 2 \\ 6 \\ 0 \\ 2 \end{array}$				$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$
$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $		$\begin{array}{cccccccccccccccccccccccccccccccccccc$		$\begin{bmatrix} 0\\0\\0\\0\\2&1 \end{bmatrix}$		\int_{0}^{t}
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0		-			γ_1 γ_6 γ_{11}	
	0 0 0 0 0 0 0 1 1 0 2 4	$egin{array}{cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $			γ_2 γ_7 γ_{12}	
1		$egin{array}{cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 $			γ_3 γ_8 γ_{13}	
$egin{array}{c} S_{22} \ S_{23} \ S_{24} \ \end{bmatrix}$	$egin{array}{c} S_0 \ S_1 \ S_2 \ S_3 \ S_4 \ S_5 \ S_6 \ S_7 \ S_8 \ S_9 \ S_{10} \ S_{11} \ S_{12} \ S_{13} \ S_{14} \ S_{15} \ S_{16} \ S_{17} \ S_{18} \ S_{20} \ S_{21} \ \end{array}$	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0			$egin{array}{c} \gamma_4 \ \gamma_9 \ \gamma_{14} \ \end{bmatrix}$	
		d_0 d_1 d_2 d_3 d_4 d_5 d_6 d_7 d_8 d_{10} d_{11} d_{12} d_{13} d_{14} d_{15} d_{16} d_{17} d_{20} d_{21} d_{22} d_{23} d_{24}				