

Design and Prototyping of Direct Torque Control of Induction Motors in FPGAs

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Abstract

This work presents an implementation of a Direct Torque Control (DTC) strategy, which is used to control induction motors. Following a tendency in the research area, the algorithm proposed is implemented in an unique FPGA substrate, which allows for a faster validation and simplifies the control structure. A binary format is used with a variable word-size approach, which permits to reduce truncation through the calculation processes, resulting in smaller errors without increasing excessively hardware area in the prototyping step. Matlab is used to validate the algorithm. The calculation error between Matlab's double precision representation and the proposed alternative is small and can be neglected in DTC control.

1. Introduction

Induction motors have many advantages when compared to direct current (DC) motors. The formers do not necessarily present mechanical commutators, which restrict power and velocity, greatly increasing machine maintenance costs. Nevertheless, DC drives have dominated the field of adjustable speed electrical drives since the beginning of the last century due to the high dynamic performance which they permit [1].

Induction machine control was only possible after the development of efficient and powerful switching components. Besides, it involves complex equations to be solved during operation whose implementation has always been associated with microelectronic advances. For instance, the first control theory was presented in 1971 [2] and implemented ten years later with the advent of microprocessors.

Direct Torque Control (DTC) was formulated by Takahashi (1986) [3] and Depenbrock (1988) [4] and introduced as a viable alternative for the control of induction machines. Allowing for a performance even

superior to DC machine drives, it was soon accepted by industry and is slowly becoming a standard, together with Field Oriented Control techniques.

DTC has usually been implemented using DSP processors associated with ASIC components [5]. This was certainly due to DSP availability as the most powerful tool at the moment.

The recent evolution in ASIC technology has given rise to the implementation of DTC in an unique integrated component ([6] – [9]), simplifying algorithm development and validation.

The main contribution of this work is an implementation of DTC drive in FPGA using a fixed-point arithmetic with a variable word-size approach. This is proposed as an alternative to 16 or 32-bit choices used before ([6] – [9]). A separation of the algorithm in functional blocks is used to simplify validation task which was accomplished in comparison with Matlab results.

The paper is organized as follows. In the next section, the algorithm is presented and discussed. Section 3 proposes an architecture to implement the algorithm. Section 4 presents an initial validation with architecture simulation results compared with Matlab results. Finally, in section 5, some conclusions are presented.

2. Algorithm

Figure 1 presents the topology of a DTC drive:

- *Induction motor* is a system to be voltage controlled. *The inverter* applies three-phase voltage signals determined by changes on the *switching keys*, which are calculated by the algorithm;
- *Switching table* calculates the switching keys according to a strategy as a function of the torque and flux comparators and the spatial sector in which the stator flux is lying at calculation time;

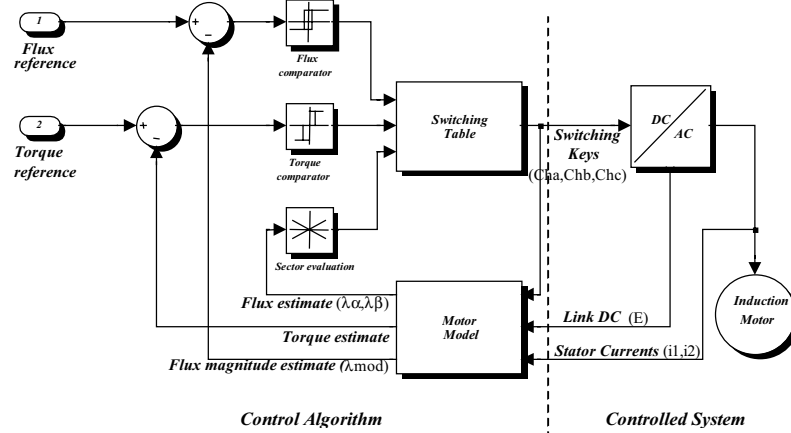


Figure 1. DTC strategy.

- *Motor model* is a mathematical model used to calculate flux, currents and voltages referred to the stator in $\alpha\beta$ stationary coordinates and electrical torque. It is the most complex part of the algorithm, involving multiplications and a square root evaluation. These operations are computed serially in order to reduce hardware area in the prototyping step;
- *Link DC (E)* and *stator currents* (i_1 and i_2) are determined by three serial A/D converters, which correspond to the slower part of the algorithm, taking 25 μ s to perform the operation. The conversions are executed simultaneously;
- *Torque and flux comparators* are two and three-level relays with hysteresis – they are used to compare torque and flux references with their actual values;
- *Sector evaluation* – is a function that calculates the position of stator flux vector in $\alpha\beta$ coordinates in a plane. In this case, the plane is divided into 6 regions or *sectors* corresponding to the voltage vectors that can be applied by the inverter.

The motor model is composed by the equations presented in Figure 2.

In Figure 2, V_α , V_β , i_α , i_β , λ_α , λ_β are coordinate components of stator voltage, current and flux respectively. $\lambda_{\alpha_{OLD}}$ and $\lambda_{\beta_{OLD}}$ are values of stator flux calculated in the previous instant. E is the *link DC* voltage; i_1 and i_2 are stator current values of two of the three power lines to which the motor is linked. Ch_a , Ch_b and Ch_c are the switching inverter commands applied in the previous instant. R_s is a stator resistance estimate evaluated off-line. T_s is the A/D converter sampling time.

Equations 1 and 2 transform current and voltage components measured in a 16-bit value by the A/D into $\alpha\beta$ components which are adequate to DTC algorithm. Voltage components need switching commands applied in the previous instant to be calculated. Equation 3, corresponds to an integration using Forward Euler Method. Sampling time (T_s) precision is critical to DTC

implementation since it affects flux estimation. Equation 4 estimates electrical torque, which is directly controlled as a main proposition of DTC technique. Equation 5 calculates flux magnitude, which is used in the flux control loop.

- 1. Stator Voltages in $\alpha\beta$ coordinates**

$$V_\alpha = E/3 \cdot (2 \cdot Ch_a - Ch_b - Ch_c);$$

$$V_\beta = \sqrt{3}/3 \cdot E \cdot (Ch_b - Ch_c);$$
- 2. Stator Currents in $\alpha\beta$ coordinates**

$$i_\alpha = i_1;$$

$$i_\beta = \sqrt{3}/3 \cdot (i_1 + 2 \cdot i_2);$$
- 3. Stator Flux estimation**

$$\lambda_\alpha = \lambda_{\alpha_{OLD}} + T_s \cdot (V_\alpha - R_s \cdot i_\alpha);$$

$$\lambda_\beta = \lambda_{\beta_{OLD}} + T_s \cdot (V_\beta - R_s \cdot i_\beta);$$
- 4. Torque estimation**

$$torque = 3/4 \cdot P \cdot (i_\beta \cdot \lambda_\alpha - i_\alpha \cdot \lambda_\beta);$$
- 5. Stator Flux magnitude determination**

$$\lambda_{mod} = \sqrt{\lambda_\alpha^2 + \lambda_\beta^2};$$

Figure 2. Equations modeling the motor behavior.

3. DTC Architecture

DTC algorithm is implemented in an architecture composed by five main blocks: motor model, flux comparator, sector evaluation, torque comparator and switching table (Figure 3). The available processing time is dictated by the A/D converters, corresponding to 25 μ s. This time interval is partitioned into five time slots, allowing for the processing of input samples in a pipeline structure.

The motor model module uses time slots 1 to 3. In the first time slot (Id_1), 16-bit samples are read from the A/D converters. Modules flux comparator, sector evaluation and torque comparator are processed in parallel in the

The diagram illustrates the proposed control system for a three-phase induction motor. The system components and their interconnections are as follows:

- Motor model:** Receives inputs i_1 , i_2 , i_3 , and E . It outputs i_1 , i_2 , i_3 , and E . It also receives a reference current $Id1$ (indicated by a red arrow).
- Flux comparator:** Receives inputs from the Motor model and the Sector evaluation block. It outputs a signal to the Sector evaluation block.
- Sector evaluation:** Receives inputs from the Motor model, the Flux comparator, and the Torque comparator. It outputs a signal to the Switching table.
- Torque comparator:** Receives inputs from the Motor model and the Sector evaluation block. It outputs a signal to the Sector evaluation block. It also receives a reference current $Id4$ (indicated by a red arrow).
- Switching table:** Receives inputs from the Sector evaluation block and the Torque comparator. It outputs a signal to the Motor model. It also receives a reference current $Id5$ (indicated by a red arrow).

The diagram shows the flow of signals between these components, with specific labels for inputs and outputs. The red arrows indicate reference currents $Id1$, $Id4$, and $Id5$.

Each model module has three 16-bit inputs supplied by converters: i_1 , i_2 and E ; and produces four outputs: torque, $\lambda\alpha$, $\lambda\beta$ and λ_{mod} (Figure 4).

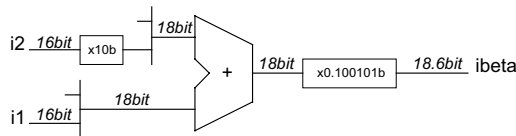
Equations modeling motor behavior are implemented according to the architecture presented in Figure 4. As can be observed, complex mathematical operations are performed such as multiplications and a square root. These operations are implemented using serial algorithms to reduce the final circuit area. Between the second and the third time slots (ld2 and ld3), four multiplications are executed in parallel. The same parallelism can be observed through the rest of the architecture. This inherent hardware parallelism would not be possible in software implementations showing the superior performance of the hardware solution when compared to the software one.

In the algorithm implementation, many digital properties have to be considered. Characteristics such as quantization, sampling, and adopted binary format are key performance factors in the control process.

In ASIC implementation, the word size is critical. A larger word reduces quantization effects but increases silicon area and affects costs. On the other hand, a smaller word may affect precision, increasing control error or even destabilizing the system. A huge silicon area may also implicate in technological constraints.

In Figure 4, $f1 = ca \oplus (cb.cc)$ and $f2 = cb \oplus cc$.

Figure 5 is an extract of Figure 4 containing only the second equation of the motor model (*ibeta*). Variable word sizing can be easily observed. For instance, *i2* starts with 16 bits and is multiplied by 2 (shift operation). To avoid overflow, the result is saved in an 18-bit word. This product is added to *i1* and the result is multiplied by $\sqrt{3}/3$ ($\approx 0.578125_{10}$, $\approx 0.100101_2$) which results in an 18.6 bit word (18-bit integer part and 6-bit fractional part). In this case, the 6 fractional bits are not discarded to avoid loss of precision, resulting in a fixed-point word with 24 bits.



Equivalent VHDL code:

```

architecture calc_is of calc_is is
    signal sm, i2_1, i1_1: STD_LOGIC_VECTOR (17 downto 0);
    signal i2_2, i2_3, i2_4: STD_LOGIC_VECTOR (23 downto 0);
begin
    -- i2*2 multiplied by 2 with signal extension
    i2_1 <= i2(15) & i2 & '0';

    -- i1 with signal extension
    i1_1 <= i1(15) & i1(15) & i1;

    -- sum
    sm <= i2_1 + i1_1;

    -- multiplication by 0.100101
    i2_2 <= sm(17) & sm(17) & sm(17) & sm(17) & sm(17) & sm(17) & sm;
    i2_3 <= sm(17) & sm(17) & sm(17) & sm(17) & sm(17) & sm & "00";
    i2_4 <= sm(17) & sm & "000000";
    ibeta <= i2_2 + i2_3 + i2_4;

    -----
end calc_is;

```

Figure 5. Variable word-size implementation example.

A critical part in the architecture is the $\lambda\alpha$, $\lambda\beta$ calculation, since a feedback is executed to perform the integration. This operation can induce errors and overflow can easily occur if T_s value is not properly scaled. Scaling factor is necessary because representing 25 μ s with good precision would need a minimum of 24 bits. In the first implementation, T_s was assumed to be 0.01_2 (equals to 0.25), which is 10000 bigger than its correct value. This scaling factor proved to be inefficient because flux calculation was overflowed due to the integration process, which involves accumulation of values. Therefore, another scale factor was adopted.

The adopted scale factor was 0.025. Figure 6 presents a comparison between flux estimate errors. Relative errors obtained when compared to Matlab double precision estimates can be seen in figures (B) and (C). Two representations were tested for T_s : 0.0251464844375 (0.000001100110) and 0.02499961853027 (0.000001100110011011). After the accumulation, results were truncated to 33 bits (19.14b) to avoid excessive increase of fixed-point representation. It

corresponds to a reduction of 10 and 17 bits respectively (Figure 4).

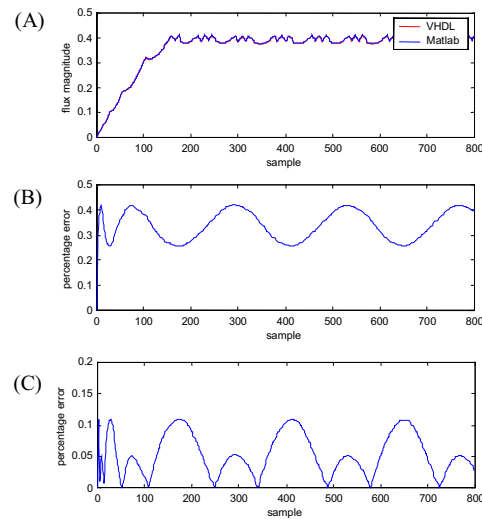


Figure 6. (A) Comparison between Matlab (double precision) and VHDL (fixed point) estimation of stator flux magnitude; (B) error with $T_s=0.000001100110b$; (C) error with $T_s=0.000001100110011011b$.

3.2. Sampling

According to [3][4], sampling time (T_s) is of crucial importance to the algorithm, specially when flux estimation process is concerned. The sampling time was limited to 25 μ s due to the A/D characteristics. All the mathematical operations involved in the model were performed within the sampling time.

3.3. Binary format

Two-complement fixed-point format was used since it simplifies implementation of operations like sum, subtraction and multiplication by scalar.

4. Function Validation

Function validation was performed comparing the VHDL simulation results (Active HDL – www.aldec.com) to Matlab results. Modeling equations presented in Figure 2 were simulated using Matlab with double precision (64-bit float arithmetic) and compared with VHDL simulated results using variable size words.

In Figures 7 to 9, percentage error graphs showing Matlab and VHDL comparative results are presented. Error peaks occasioned by small values of flux and torque in the error function denominator can be observed. Even in these cases, error is smaller than 0.3 per cent.

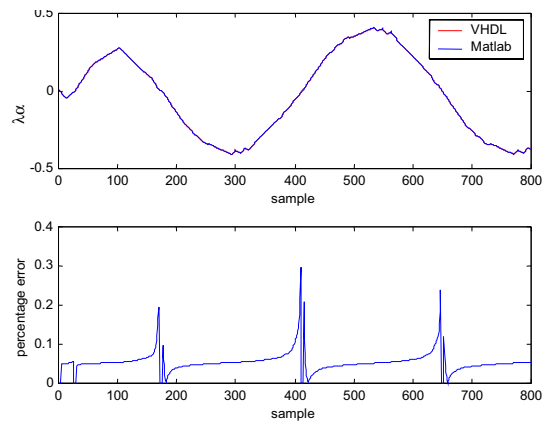


Figure 7. Comparison between full-precision $\lambda\alpha$ Matlab and variable word-size VHDL estimation.

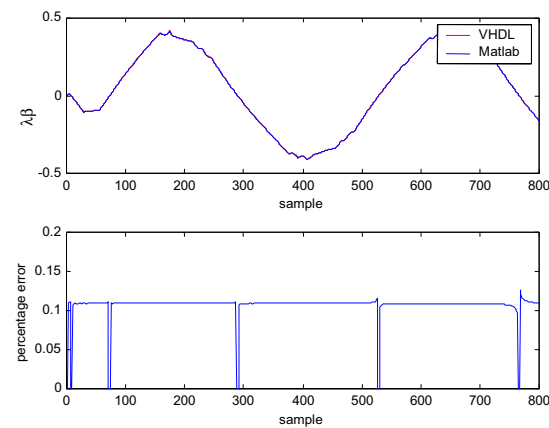


Figure 8. Comparison between full-precision $\lambda\beta$ Matlab estimation and variable word-size VHDL estimation.

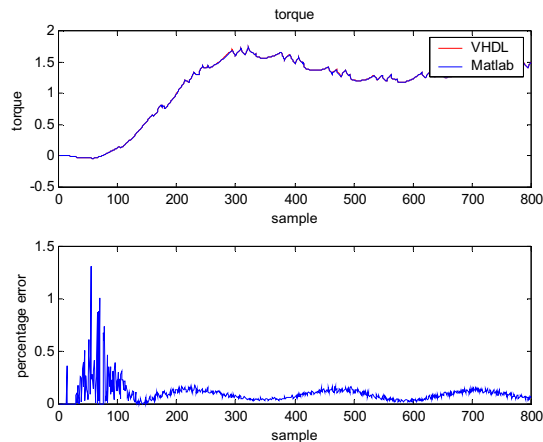


Figure 9. Comparison between double torque Matlab estimation and variable word-size VHDL estimation.

Figures 10 to 12 present the outcome of DTC algorithm, or else the switching keys that are applied to

the induction motor (cha, chb, chc). Absolute error figures show the good approximation between Matlab and VHDL results.

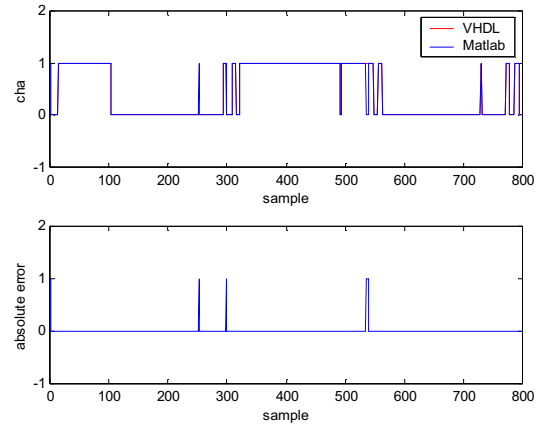


Figure 10. Comparison between Matlab and VHDL estimates of Cha command.

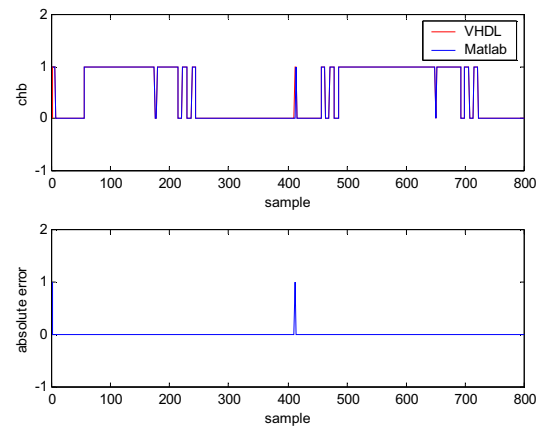


Figure 11. Comparison between Matlab and VHDL estimates of Chb command.

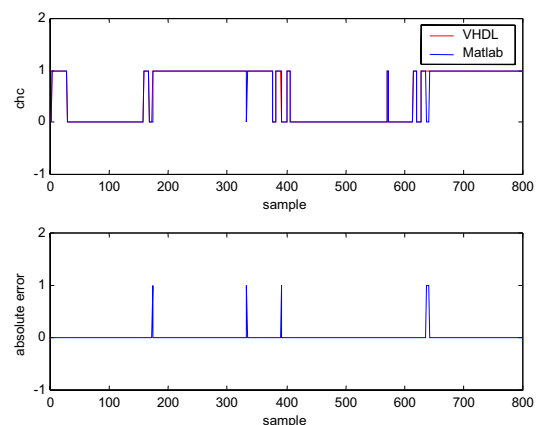


Figure 12. Comparison between Matlab and VHDL estimates of Chc command.

5. Conclusions

DTC implementation in VHDL is presented using fixed-point arithmetic to minimize quantization errors. The architecture proposed is written in synthesizable VHDL (RTL style). The resulting area usage for an EP20K200 device (Altera) is 4100 logic cells. A minimum error was obtained between the fixed-point representation proposed and Matlab's double representation used. The integrated solution does not utilize microcontrollers, which allows new algorithms to be quickly implemented, simplifying test and validation of new control strategies.

6. References

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