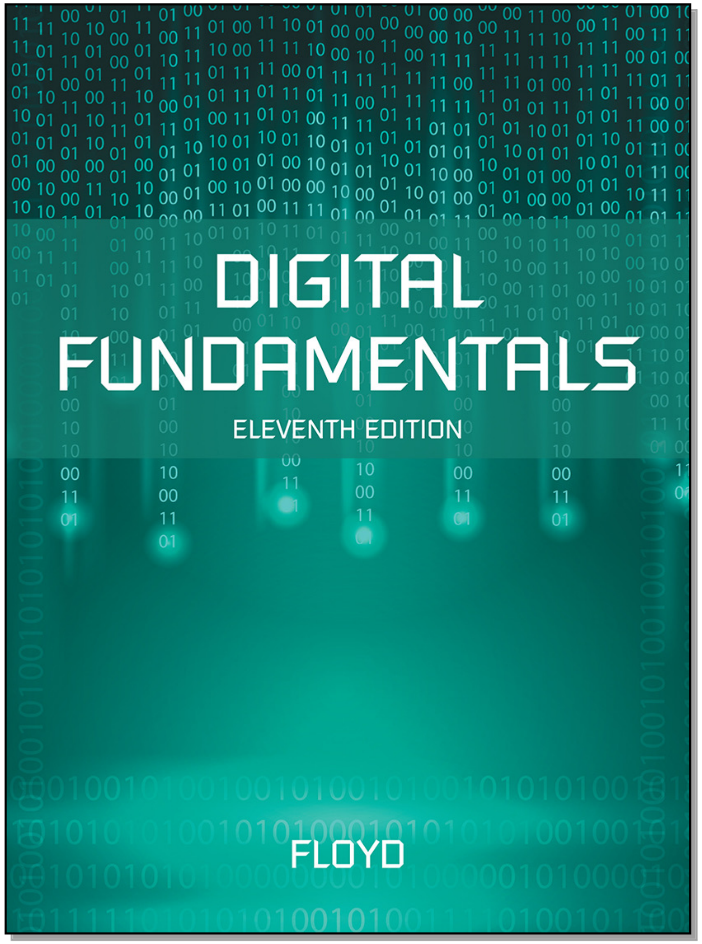
04/Março/2022

Digital Fundamentals

ELEVENTH EDITION

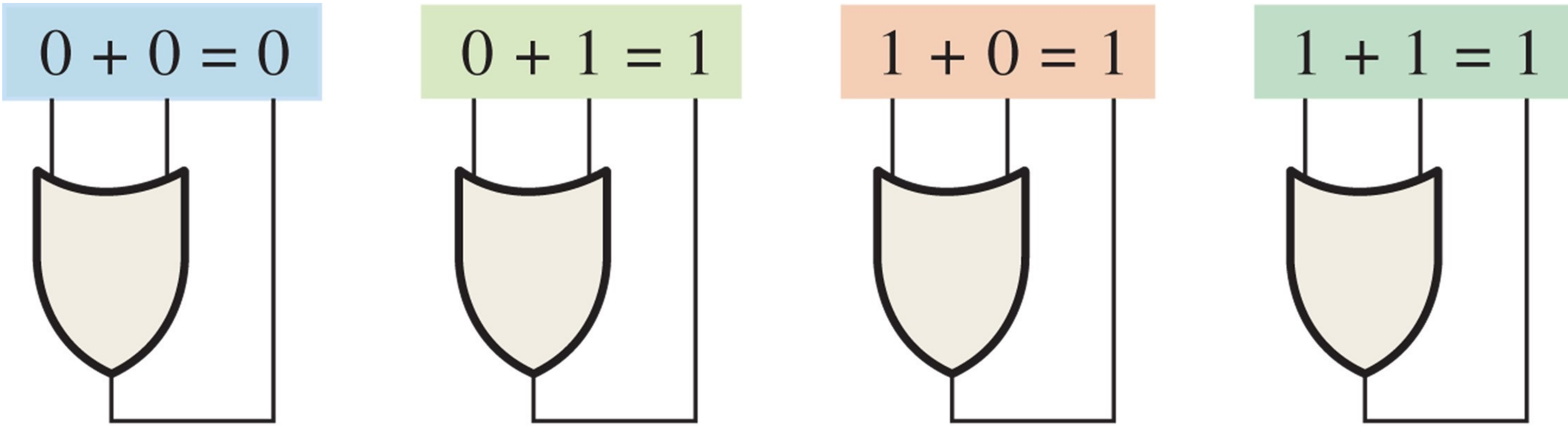


CHAPTER 4

Boolean Algebra and Logic Simplification

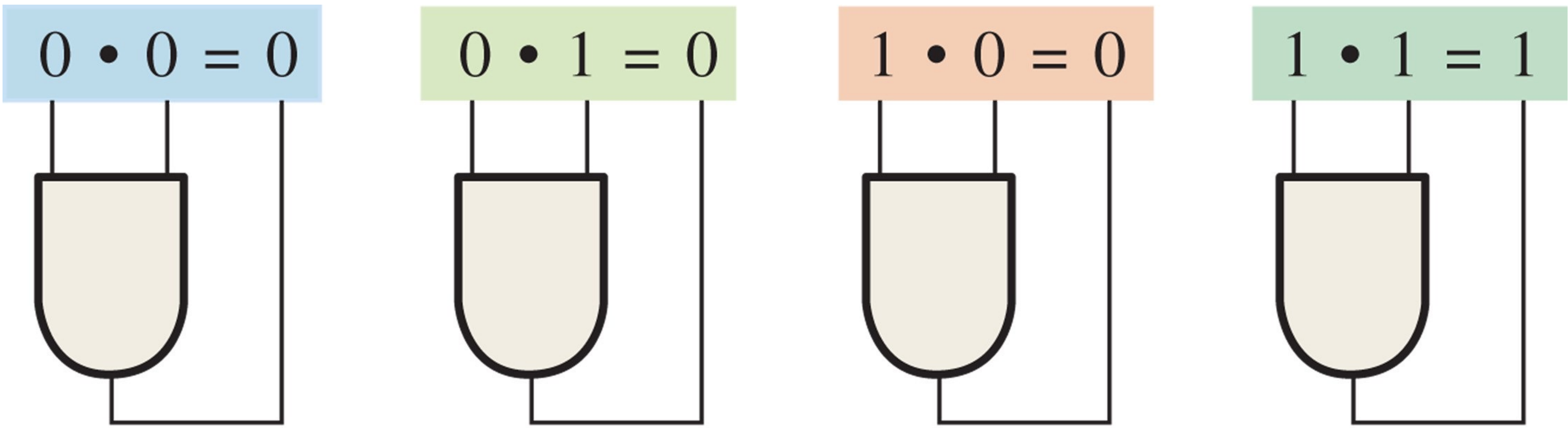
Adição Booleana

- A adição booleana é equivalente à operação OR
- As regras básicas são ilustradas com sua relação com a porta OR



Multiplicação Booleana

- A multiplicação booleana é equivalente à operação AND
- As regras básicas são ilustradas com sua relação com a porta AND

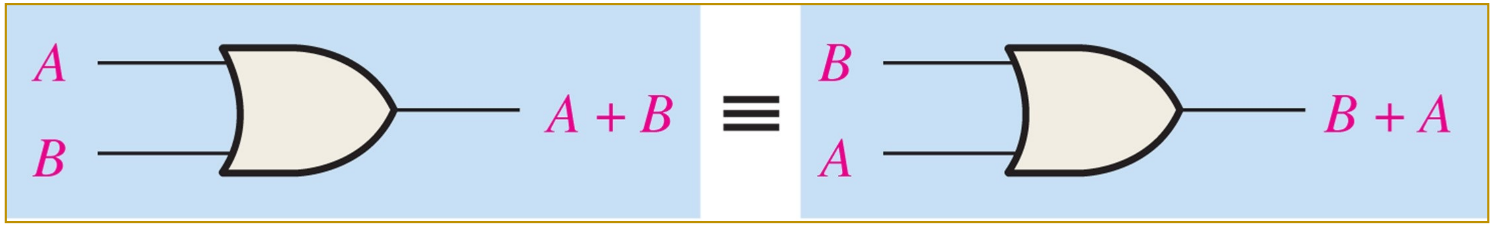


PROPRIEDADE COMUTATIVA

As **propriedades comutativas** são aplicadas à adição e multiplicação.

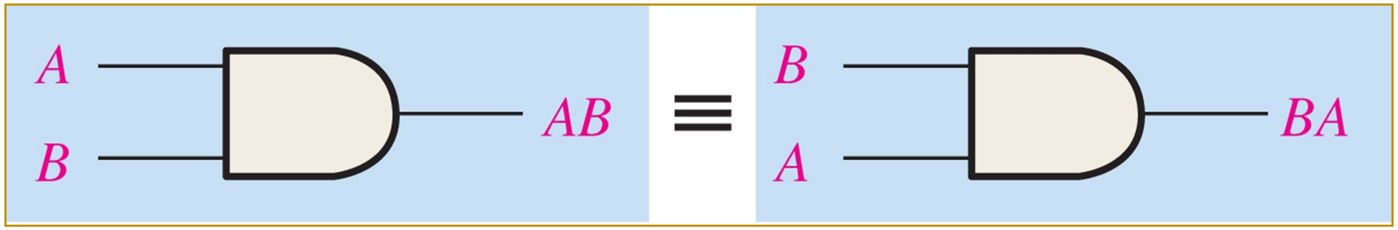
- ✓ Para adição, a propriedade comutativa afirma que a ordem em que as variáveis são combinadas com **OR não fazem diferença**.

$A + B = B + A$



- ✓ Para multiplicação, a propriedade comutativa afirma que a ordem em que as variáveis são colocadas em **AND** também **não fazem diferença**.

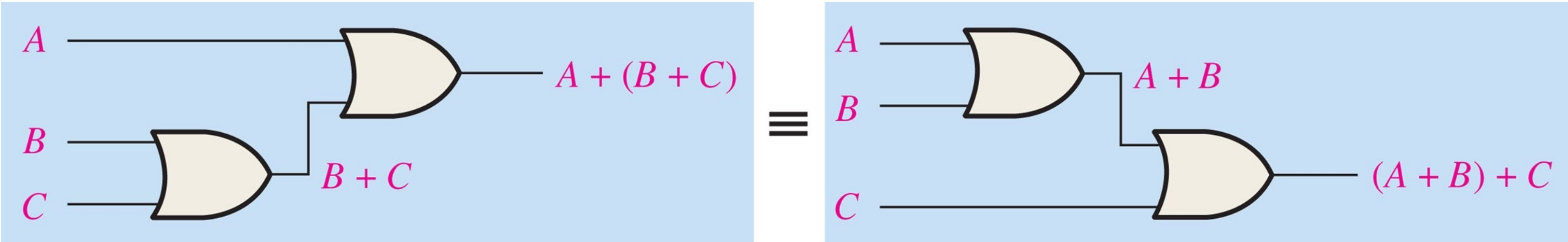
$A.B = B.A$



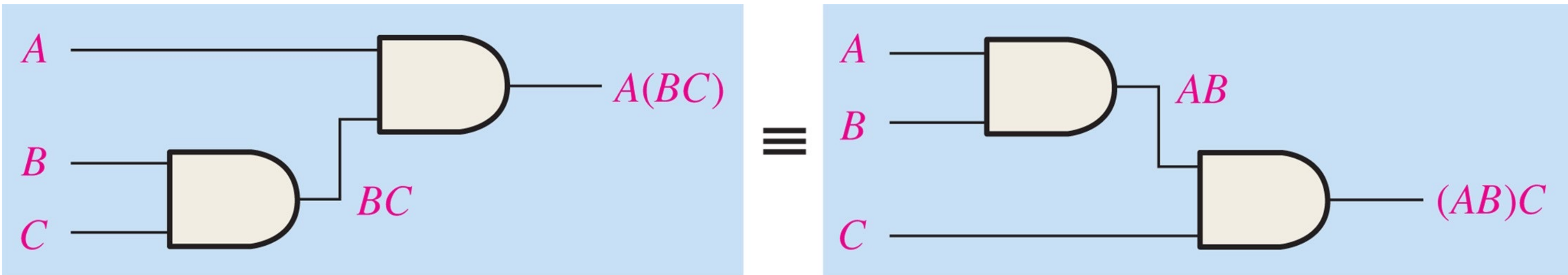
PROPRIEDADE ASSOCIATIVA

As **propriedades associativas** são aplicadas à adição e multiplicação.

- ✓ Na adição, ao usarmos um **OR** em mais de duas variáveis, o resultado é o mesmo, **independentemente do agrupamento das variáveis**.



- ✓ Na multiplicação, ao usarmos uma **AND** em mais de duas variáveis, o resultado é o mesmo, **independentemente do agrupamento das variáveis**.

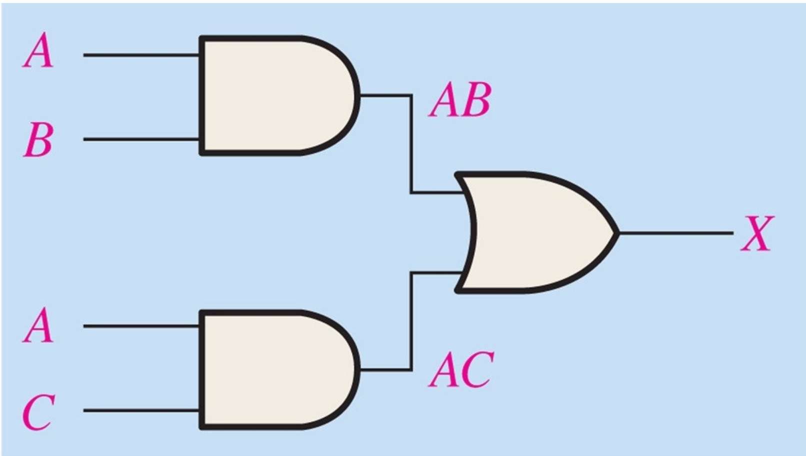


PROPRIEDADE DISTRIBUTIVA

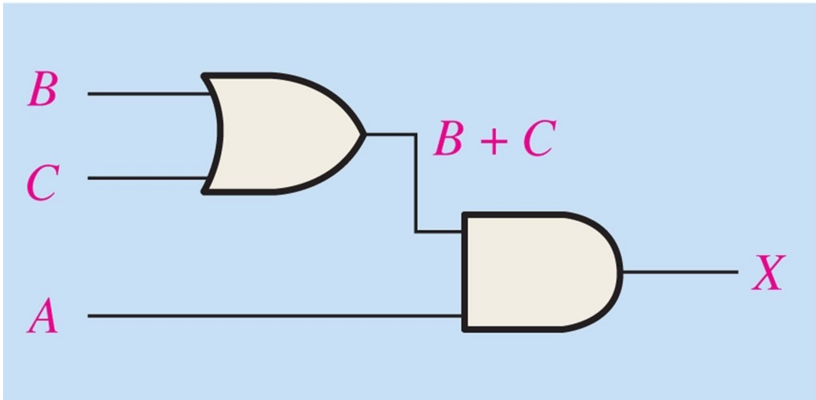
A **propriedade distributiva** é uma **propriedade de fatoramento**. Isto é, uma variável comum pode ser fatorada de uma expressão, assim como encontramos na álgebra.

$$AB + AC = A.(B + C)$$

- ✓ A propriedade distributiva pode ser ilustrada com circuitos equivalentes:



$X = AB + AC$



$X = A(B + C)$

REGRAS BÁSICAS DA ÁLGEBRA BOOLEANA

A, **B**, ou **C** podem representar uma única variável ou uma combinação de variáveis

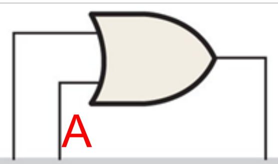
TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Rules of Boolean Algebra (1-2)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$



0	0	0
0	1	1
1	0	1
1	1	1

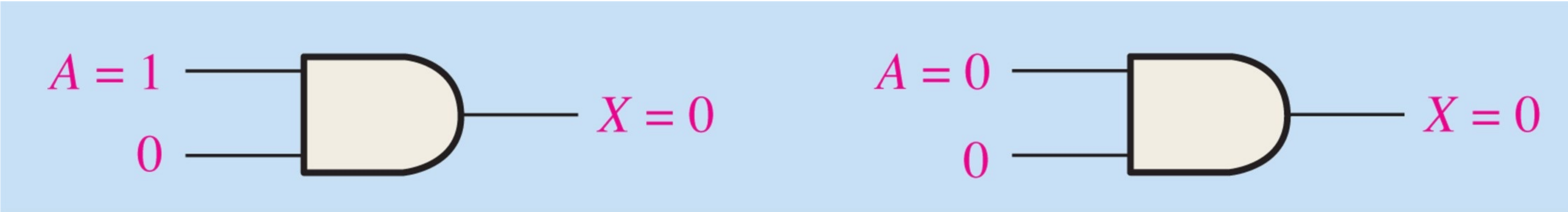
OR

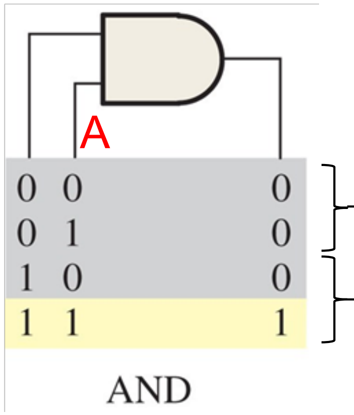
$$X = A + 0 = A$$

$$X = A + 1 = 1$$

Rules of Boolean Algebra (3-4)

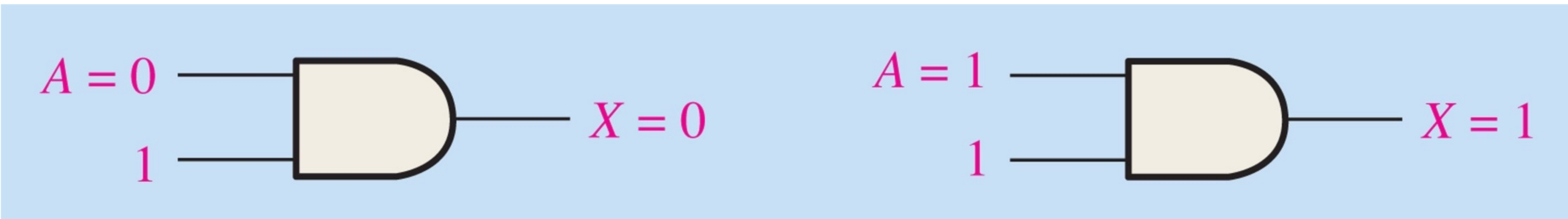
1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$





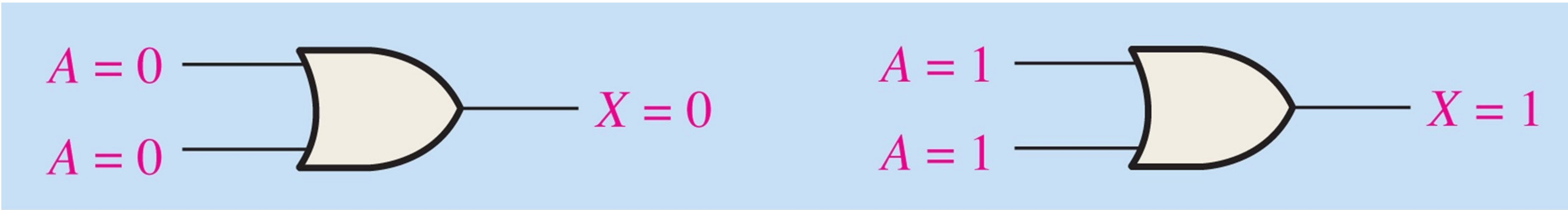
$$X = A \bullet 0 = 0$$

$$X = A \bullet 1 = A$$

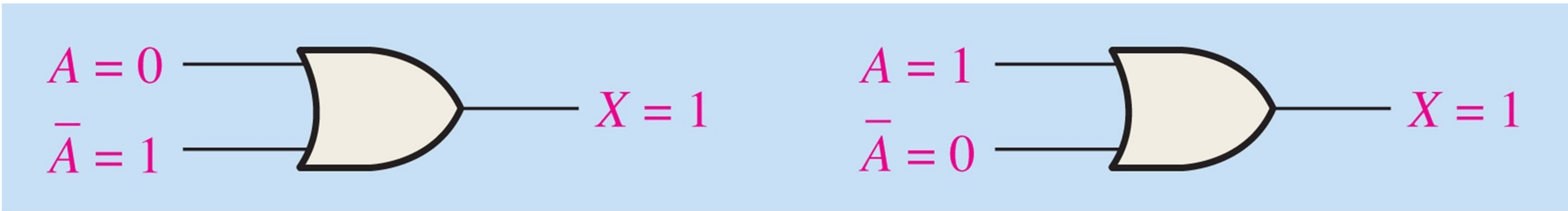


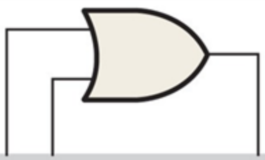
Rules of Boolean Algebra (5-6)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$



$$X = A + A = A$$





0	0	0
0	1	1
1	0	1
1	1	1

OR

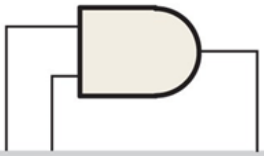
$$X = A + \overline{A} = 1$$

Rules of Boolean Algebra (7-8)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

$$X = A \bullet A = A$$

$$X = A \bullet \overline{A} = 0$$



0	0	0
0	1	0
1	0	0
1	1	1

AND

Rules of Boolean Algebra (9)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\boxed{\overline{\overline{A}} = A}$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

$$\overline{\overline{A}} = A$$

Rules of Boolean Algebra (10)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $\boxed{A + AB = A}$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Regra 10: **A + A.B = A**

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

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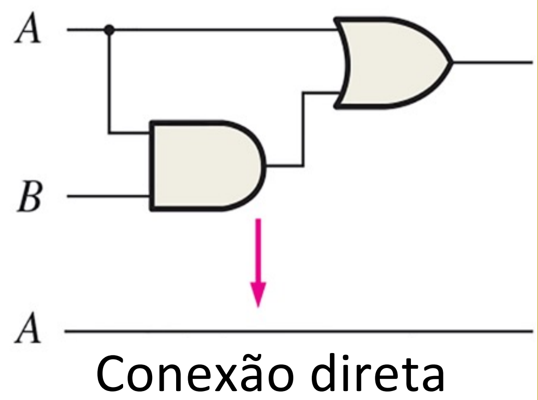
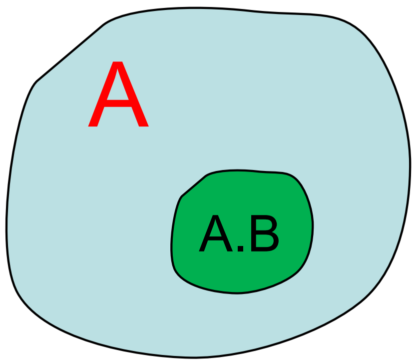


A + A.B = A . 1 + A.B	Regra 4: A . 1 = A
= A.(1 + B)	Propriedade Distributiva
= A . 1	Regra 2: (1 + B) = 1
= A	Regra 4: A . 1 = A



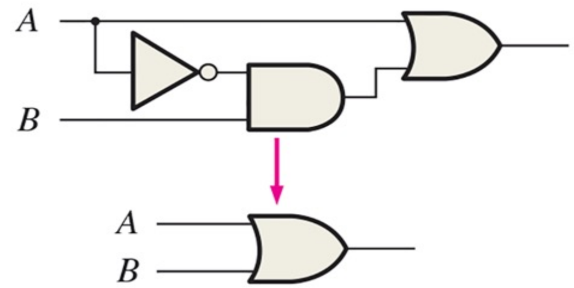
Rules of Boolean Algebra (11)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $\boxed{A + \overline{A}B = A + B}$
12. $(A + B)(A + C) = A + BC$

<u>Regra 11:</u> **$A + \overline{A}.B = A + B$**

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

iguais



$$\mathbf{A + \overline{A}.B = (A + A.B) + \overline{A}.B}$$

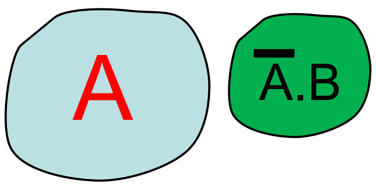
Regra 10: $A = A + A.B$

$$\mathbf{= A + A.B + \overline{A}.B}$$

$$\mathbf{= A + B.\,(A + \overline{A})}$$

Regra 6: $A + \overline{A} = 1$

$$\mathbf{= A + B}$$



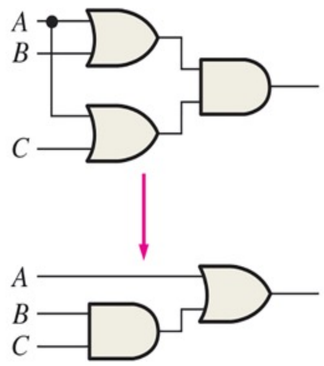
Rules of Boolean Algebra (12)

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $\boxed{(A + B)(A + C) = A + BC}$

Regra 12: **(A + B).(A + C) = A + B.C**

A	B	C	$A+B$	$A+C$	$(A+B)(A+C)$	BC	$A+BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

iguais



(A + B).(A + C) = A.A + A.C + A.B + B.C Propriedade Distributiva

= A + A.C + A.B + B.C Regra 7: A.A = A

= A.(1 + C + B) + B.C Propriedade Distributiva

= A.1 + B.C Regra 2: A + 1 = 1

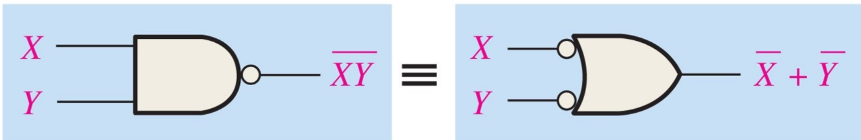
= A + B.C Regra 4: A.1 = A

TEOREMAS DE DEMORGAN

OBJETIVO DE APLICAR DEMORGAN:

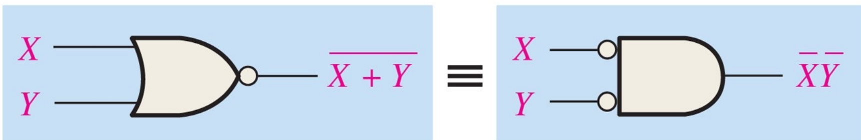
- ✓ Remover a inversão sobre um conjunto de literais.

Abaixo é mostrado as **portas equivalentes** e as correspondentes **tabelas verdade** que ilustram os **Teoremas de DeMorgan**. Observe a igualdade das duas colunas de saída em cada tabela. Isso mostra que as portas equivalentes executam a mesma função lógica.



NAND　　　Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



NOR　　　Negative-AND

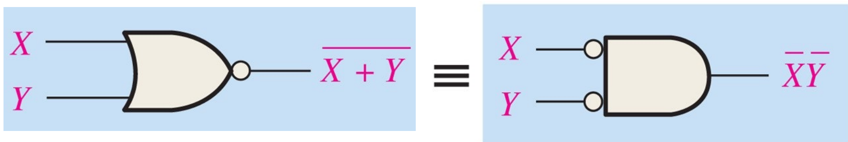
Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

TEOREMAS DE DEMORGAN

Exemplo: Aplique o **Teorema de DeMorgan** para remover a negação que cobre ambos os termos da expressão: $X = \overline{\overline{C} + D}$

✓ Para aplicar o Teorema de DeMorgan à expressão, você pode remover a negação que cobre ambos os termos e alterar o sinal entre os termos:

$$X = \overline{\overline{C} + D}$$



$$X = \overline{\overline{C}} \,.\, \overline{D}$$

Remove negação dupla

$$X = C \,.\, \overline{D}$$

DeMorgan's Theorem

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + Y + Z}$.

Solution

$$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{Y} + \overline{Z}$$
$$\overline{W + X + Y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$$

DeMorgan's Theorem

Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B.\bar{C}} + D.(\overline{E + \bar{F}})}$$

$$(\overline{\overline{A + B.\bar{C}}}).(\overline{D.(\overline{E + \bar{F}})})$$ Substitui por and

$$(A + B.\bar{C}).(\bar{D} + \overline{\overline{(E + \bar{F})}})$$ DeMorgan do segundo termo

$$(A + B.\bar{C}).(\bar{D} + E + \bar{F})$$ Pronto, todos literais isolados

DeMorgan's Theorem

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

SIMPLIFICAÇÃO LÓGICA USANDO ÁLGEBRA BOOLEANA

Simplify this expression AB + A(B + C) + B(B + C)

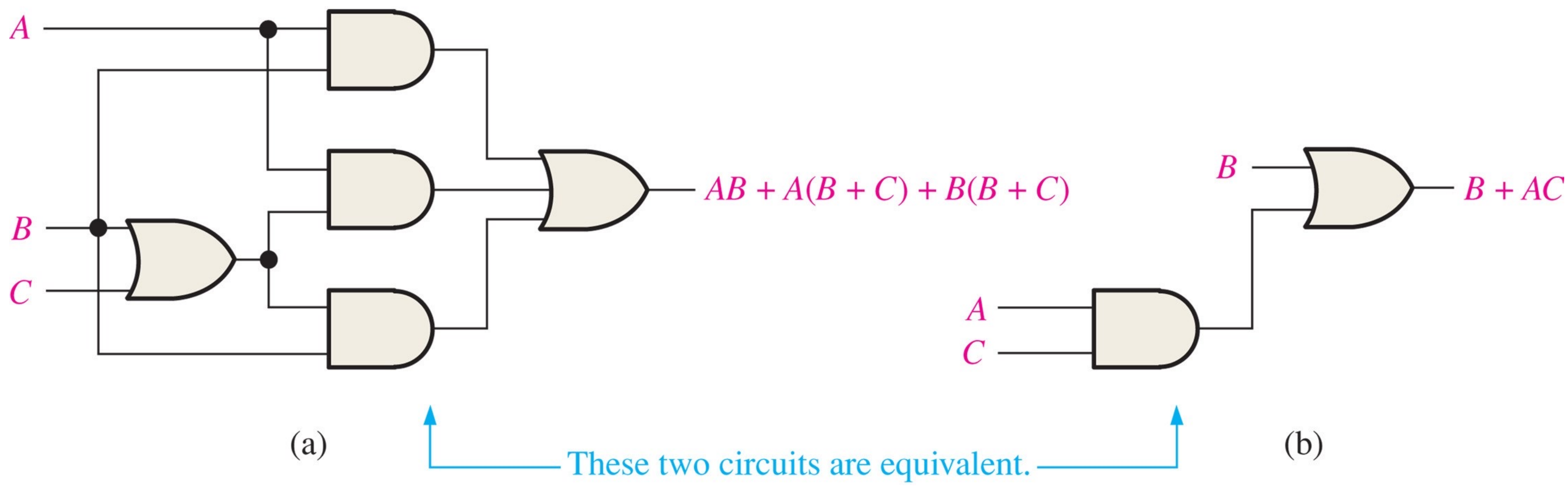


FIGURE 4-20 Gate circuits for Example 4–9.

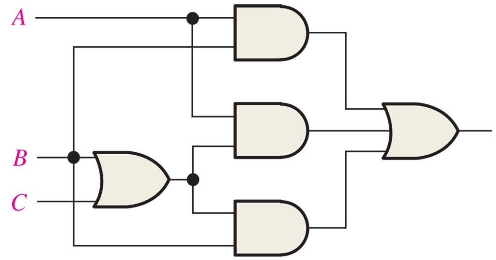
Solução próximo slide

Logic Simplification Using Boolean Algebra

Simplify this expression AB + A(B + C) + B(B + C)

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

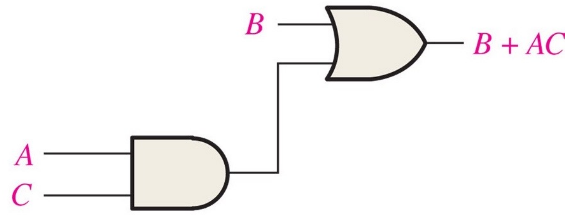


Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

B + B.(qq coisa) = B

B + AC



MINIMIZAR

Simplify this expression [AB'(C +BD) + A '.B '] . C
(p.206)

Solution: B'.C

MINIMIZAR 1/4

$$F = B.C.D.E + B.C.(\overline{D.E}) + (\overline{B.C}).D.E$$

$$F = B.C.D.[B.C + \bar{D}.(C.D + B.D)]$$

$$F = \bar{B}.\bar{C}.D + \overline{(B + C + D)} + \bar{B}.\bar{C}.\bar{D}.E$$

MINIMIZAR 2/4

$$F = C.E + C.(E + F) + \bar{E}.(E + G)$$

$$F = (C + CD).(C + \bar{C}D)(C + E)$$

MINIMIZAR 3/4

Exercise 2.7 Simplify the following Boolean equations using Boolean theorems. Check for correctness using a truth table or K-map.

10. $A + AB = A$

11. $A + \overline{A}B = A + B$

(a) $Y = AC + \overline{A}\,\overline{B}C$ = AC + AB'C + A'B'C = **AC + B'C**

(b) $Y = \overline{A}\,\overline{B} + \overline{A}B\overline{C} + \overline{(A + \overline{C})}$

(c) $Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C} + A\overline{B}C\overline{D} + ABD + \overline{A}\,\overline{B}C\overline{D} + B\overline{C}D + \overline{A}$

(b) = A'B'+ A'BC'+ **A'C** = A' (B'+ BC'+ C) = A' (B'+C'+ C) = A'

(c) $Y = \overline{A}\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C} + A\overline{B}C\overline{D} + ABD + \overline{A}\overline{B}C\overline{D} + B\overline{C}D + \overline{A}$

Tirar o A de onde não precisa

$$Y = \bar{B}.\bar{C} + \bar{B}.C.\bar{D} + B.D + B.\bar{C}.D + \bar{A}$$

$$Y = \bar{B}.\bar{C} + \bar{B}.C.\bar{D} + B.D + B.\bar{C}.D + \bar{A}$$

$$Y = \bar{B}.\bar{C} + \bar{B}.C.\bar{D} + B.D + \bar{A}$$

$$Y = \bar{B}.(\bar{C} + C.\bar{D}) + B.D + \bar{A}$$

$$Y = \bar{B}.(\bar{C} + \bar{D}) + B.D + \bar{A}$$

$$Y = \bar{B}.\bar{C} + \bar{B}.\bar{D} + B.D + \bar{A}$$

MINIMIZAR 4/4

$$F = \overline{A.B + A.C} + A.\overline{B}.C$$

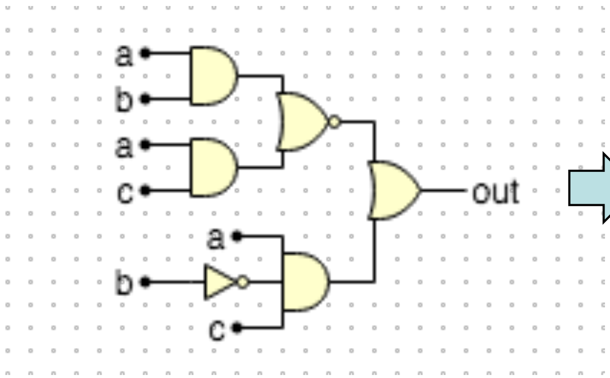
- Simplifique a expressão acima utilizando as regras de álgebra Booleana. Explicar o procedimento.
- Desenhe o circuitos lógicos correspondentes à expressão acima e ao circuito lógico minimizado.
- Apresente a tabela verdade da função, E apresentar a função na forma de um somatório de mintermos.

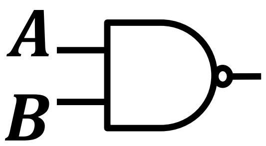
$$F = \overline{A.B + A.C} + A.\overline{B}.C = (\overline{A} + \overline{B}).(\overline{A} + \overline{C}) + A.\overline{B}.C$$

$$= \overline{A} + \overline{B}.\overline{C} + A.\overline{B}.C$$

$$= \overline{A} + \overline{B}.\overline{C} + \overline{B}.C$$

$$= \overline{A} + \overline{B}$$





A	B	C	F	minitermos
0	0	0	1	0
0	0	1	1	1
0	1	0	1	2
0	1	1	1	3
1	0	0	1	4
1	0	1	1	5
1	1	0		
1	1	1		

$$F(A, B, C) = \sum(0, 1, 2,3,4,5)$$

De Morgan

$$F = \overline{\boldsymbol{A}.\overline{\boldsymbol{B}}\,.(\boldsymbol{C}+\overline{\boldsymbol{D}})}$$

$$\overline{A\overline{B}(C+\overline{D})} = \overline{A\overline{B}} + \overline{(C+\overline{D})} = \boldsymbol{\overline{A}+B+\overline{C}D}$$

$$F = \overline{\overline{(\overline{\boldsymbol{A}}+\boldsymbol{B}+\boldsymbol{C}+\boldsymbol{D})}\;.\;\overline{(\boldsymbol{A}.\overline{\boldsymbol{B}}\,.\overline{\boldsymbol{C}}\,.\boldsymbol{D})}}$$

$$\overline{\overline{(\overline{A}+B+C+D)}\,\overline{(A\overline{B}\,\overline{C}D)}} = \overline{(A\overline{B}\,\overline{C}\,\overline{D})(\overline{A}+B+C+\overline{D})}$$

$$\overline{A\overline{B}\,\overline{C}\,\overline{D}} + \overline{\overline{A}+B+C+\overline{D}} = \boldsymbol{\overline{A}+B+C+D+A\overline{B}\,\overline{C}D}$$

$$F = \overline{\boldsymbol{A} + \overline{\boldsymbol{B}.\overline{\boldsymbol{C}}} + \boldsymbol{C}.\boldsymbol{D}} + \overline{\overline{\boldsymbol{B}.\boldsymbol{C}}}$$

$$\overline{(A+\overline{B\overline{C}}+CD)} + \overline{\overline{BC}} = \overline{A}(\overline{\overline{B\overline{C}}})(\overline{CD}) + BC = \overline{A}(B\overline{C})(\overline{CD}) + BC$$

$$\overline{A}B\overline{C}(\overline{C}+\overline{D}) + BC = \overline{A}B\overline{C} + \overline{A}B\overline{C}\,\overline{D} + BC = \overline{A}B\overline{C}(1+\overline{D}) + BC$$

$$\boldsymbol{\overline{A}B\overline{C} + BC}$$

FORMATO DAS EXPRESSÕES BOOLEANAS

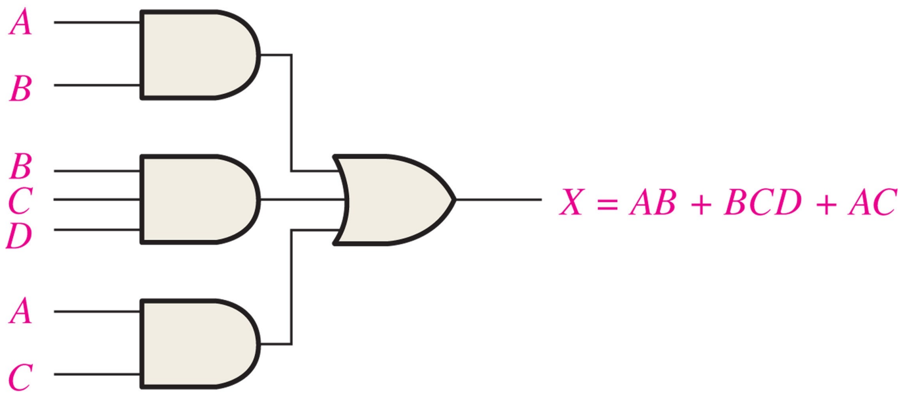
- Boolean expressions can be written in the **sum-of-products** form (**SOP**) or in the **product-of-sums** form (**POS**)
  - These forms can simplify the implementation of combinational logic
  - In both forms, an overbar cannot extend over more than one variable
- An expression is in SOP form when two or more product terms are summed as in the following examples

$$\overline{A}\,\overline{B}\,\overline{C} + A\,B \qquad A\,B\,\overline{C} + \overline{C}\,\overline{D} \qquad C\,D + \overline{E}$$

- An expression is in POS form when two or more sum terms are multiplied as in the following examples:

$$(A + B)(\overline{A} + C) \qquad (A + B + \overline{C})(B + D) \qquad (\overline{A} + B)C$$

FIGURE 4-22 Implementation of the SOP expression $AB + BCD + AC$.



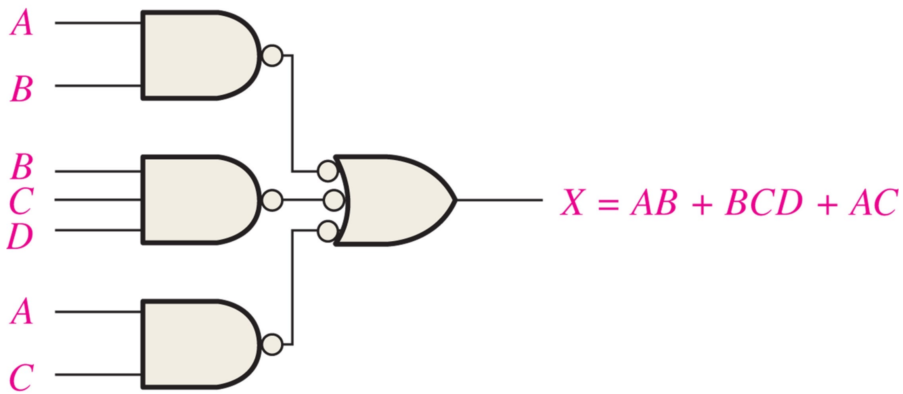
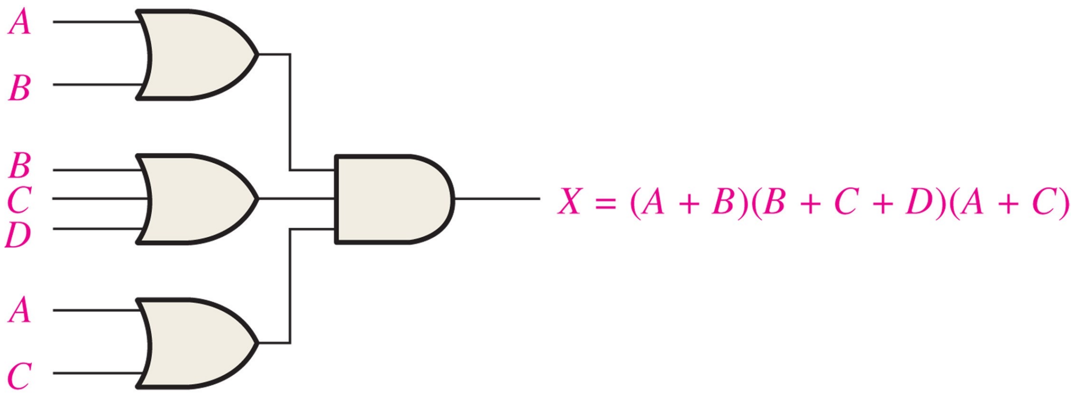


FIGURE 4-24 Implementation of the POS expression $(A + B)(B + C + D)(A + C)$.



Standard Forms of Boolean Expressions

- In **SOP standard form**, every variable in the domain must appear in each term
- You can expand a **nonstandard** term to **standard** form by multiplying the term by a term consisting of the sum of the missing variable and its complement

- Convert $X = \overline{A}\,\overline{B} + A\,B\,C$ to standard form:

The first term does not include the variable C. Therefore, multiply it by the $(C + \overline{C})$, which = 1:

$$X = \overline{A}\,\overline{B}\,(C + \overline{C}) + A\,B\,C$$
$$= \overline{A}\,\overline{B}\,C + \overline{A}\,\overline{B}\,\overline{C} + A\,B\,C$$

Standard Forms of Boolean Expressions

EXAMPLE 4–15

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

EXAMPLE 4–15

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Solution

The domain of this SOP expression is A, B, C, D. Take one term at a time. The first term, $A\overline{B}C$, is missing variable D or $\overline{D}$, so multiply the first term by $D + \overline{D}$ as follows:

$$A\overline{B}C = A\overline{B}C(D + \overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

In this case, two standard product terms are the result.

The second term, $\overline{A}\,\overline{B}$, is missing variables C or $\overline{C}$ and D or $\overline{D}$, so first multiply the second term by $C + \overline{C}$ as follows:

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C + \overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $D + \overline{D}$ as follows:

$$\begin{aligned}\overline{A}\,\overline{B} &= \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} = \overline{A}\,\overline{B}C(D + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D + \overline{D})\\ &= \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\end{aligned}$$

In this case, four standard product terms are the result.

The third term, $AB\overline{C}D$, is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

8 4 2 1

$$Y = A.\overline{B}.C.x + \overline{A}.\overline{B}.x.x + A.B.\overline{C}.D$$

10/11 0/1/2/3 13

8 4 2 1 8 4 2 1 8 4 2 1 8 4 2 1 8 4 2 1 8 4 2 1 8 4 2 1

$$= A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

11 10 3 2 1 0 13

$$Y = \sum(0, 1, 2, 3, 10, 11, 13)$$

Truth Table → SOP

TABLE 4–8

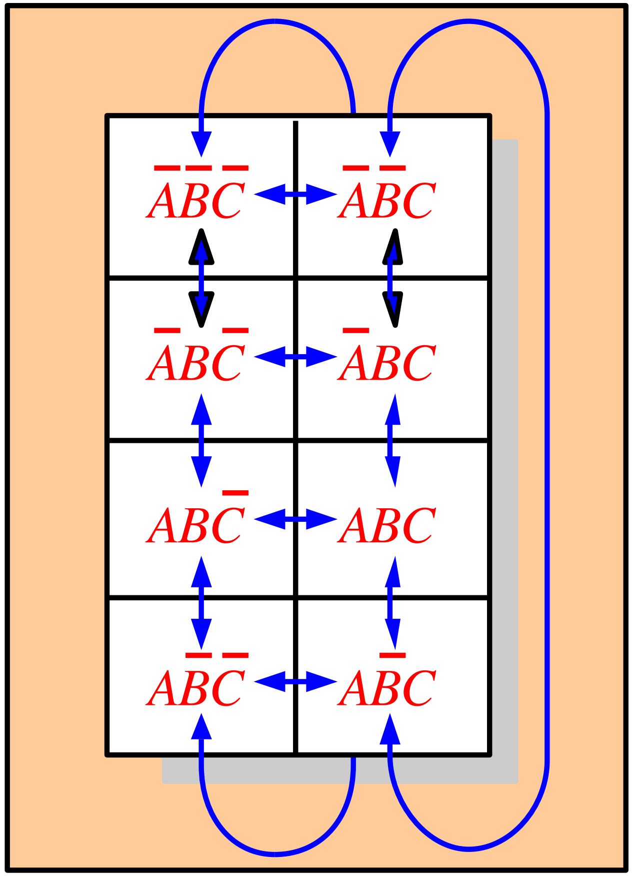
Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

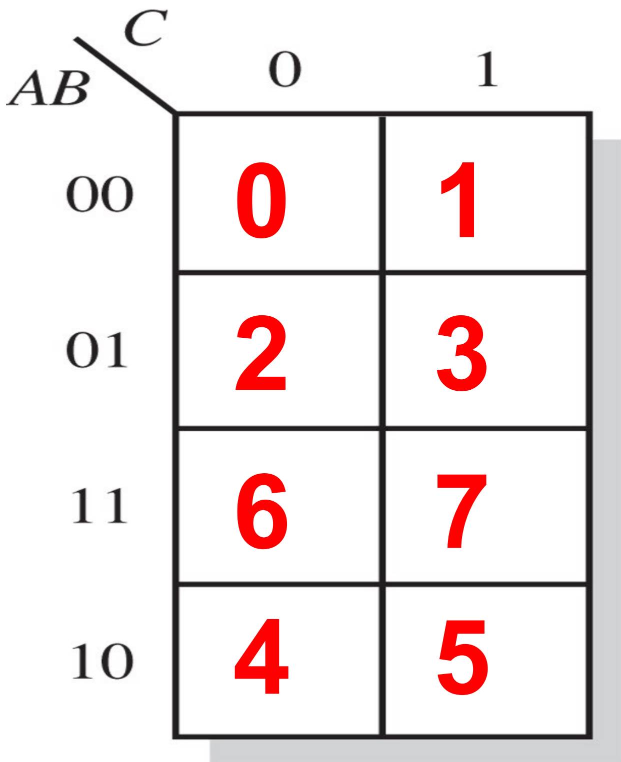
$$011 \longrightarrow \overline{A}BC$$
$$100 \longrightarrow A\overline{B}\,\overline{C}$$
$$110 \longrightarrow AB\overline{C}$$
$$111 \longrightarrow ABC$$

X=A’BC+AB’C’+ABC’+ABC

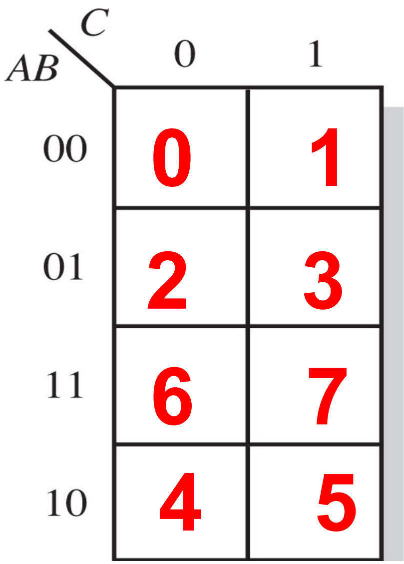
Karnaugh maps

- O mapa de Karnaugh é um método para simplificar a lógica combinacional com até 6 variáveis. Para 3 variáveis, 8 células são necessárias (2^3)
- **Cada célula difere de uma célula adjacente por apenas uma variável**
- O mapa mostrado abaixo é para três variáveis denominadas como A, B e C. Cada célula representa minitermo





Karnaugh maps



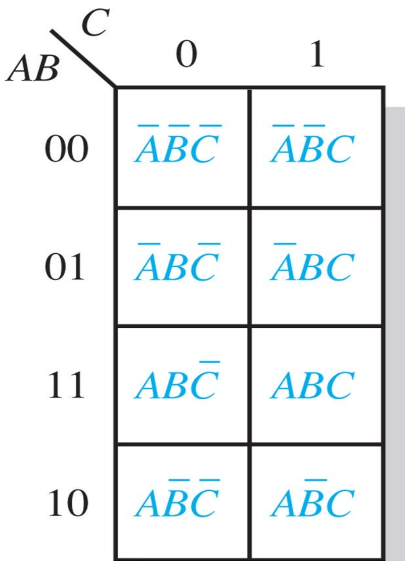


TABELA VERDADE E MAPA DE KARNAUGH

Inputs			Output
A	*B*	*C*	*X*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

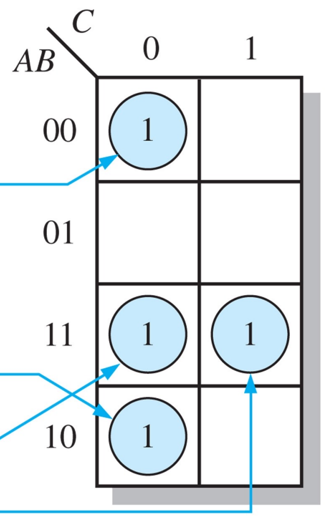
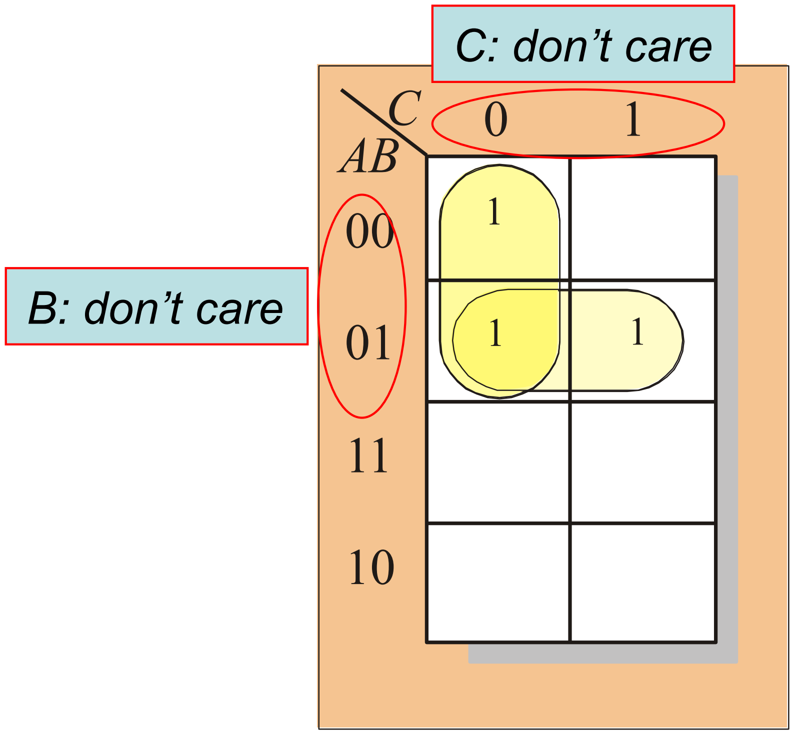


FIGURE 4-25 A 3-variable Karnaugh map showing Boolean product terms for each cell.

Karnaugh maps

K-maps podem simplificar a lógica combinacional agrupando células e ***eliminando variáveis redundantes***

$$\boxed{A.B + A.\overline{B} = A}$$



1. Agrupar os 1's em grupos de 2/4/8 mintermos
2. Eliminar as variáveis redundantes
3. O grupo vertical é lido como: $\overline{A}.\overline{C}$
4. O grupo horizontal é lido como: $\overline{A}.B$

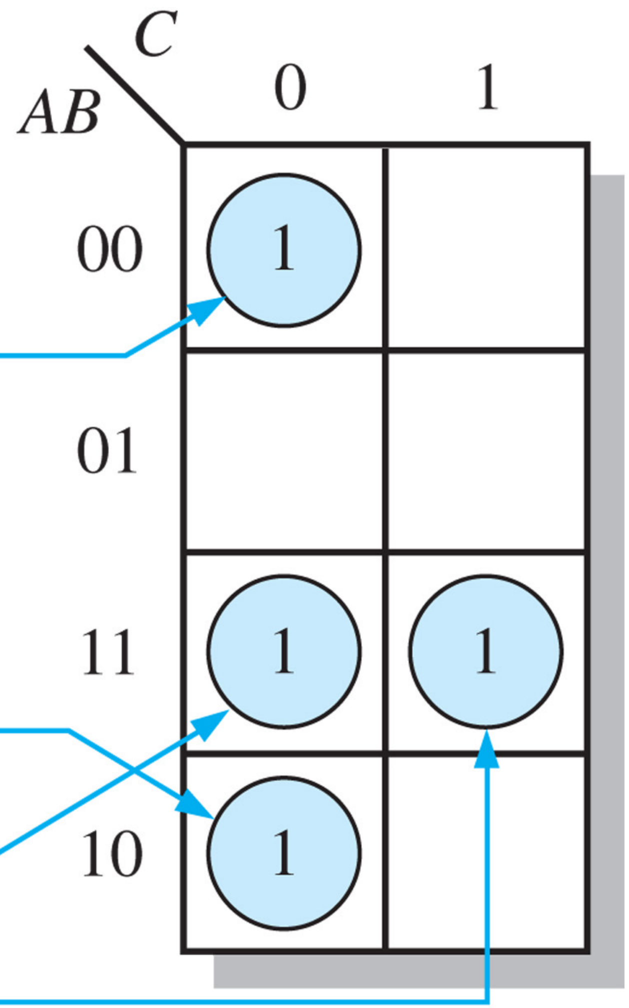
$$X = \overline{A}.\overline{C} + \overline{A}.B$$

Minimize (1/5)

FIGURE 4-39 Example of mapping directly from a truth table to a Karnaugh map.

$$X = \overset{0}{\overline{A}\,\overline{B}\,\overline{C}} + \overset{4}{A\overline{B}\,\overline{C}} + \overset{6}{AB\overline{C}} + \overset{7}{ABC}$$

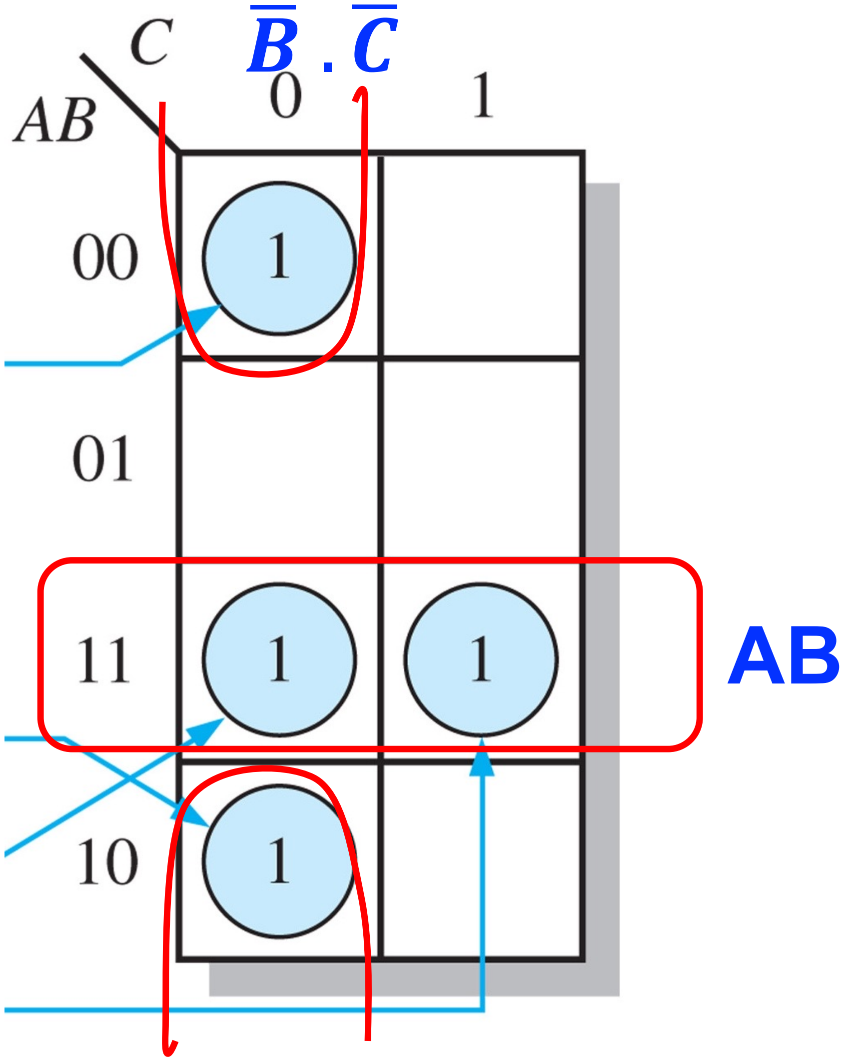
	Inputs			Output
	A	*B*	*C*	*X*
0	0	0	0	1
	0	0	1	0
	0	1	0	0
	0	1	1	0
4	1	0	0	1
	1	0	1	0
6	1	1	0	1
7	1	1	1	1



Minimize (1/5)

FIGURE 4-39 Example of mapping directly from a truth table to a Karnaugh map.

$$X = \overline{A}\overline{B}\overline{C} + A\overline{B}\overline{C} + AB\overline{C} + ABC$$



$$X = \overline{B}.\overline{C} + A.B$$

Minimize (2/5)

$Y.Z$

X \ YZ	00	01	11	10
0	0	0	1	0
$X.\bar{Z}$ 1	1	0	1	1

$$F = \sum(3, 4, 6, 7)$$

$$F = Y.Z + X.\bar{Z}$$

X \ YZ	00	01	11	10
0	m0	m1	m3	m2
1	m4	m5	m7	m6

m2 + m3 + m6 + m7

$\bar{X}Y$ $\quad$ $XY = (\bar{X} + X)\,Y = Y$

É o que há de comum entre os 4 mintermos

Minimize (3/5)

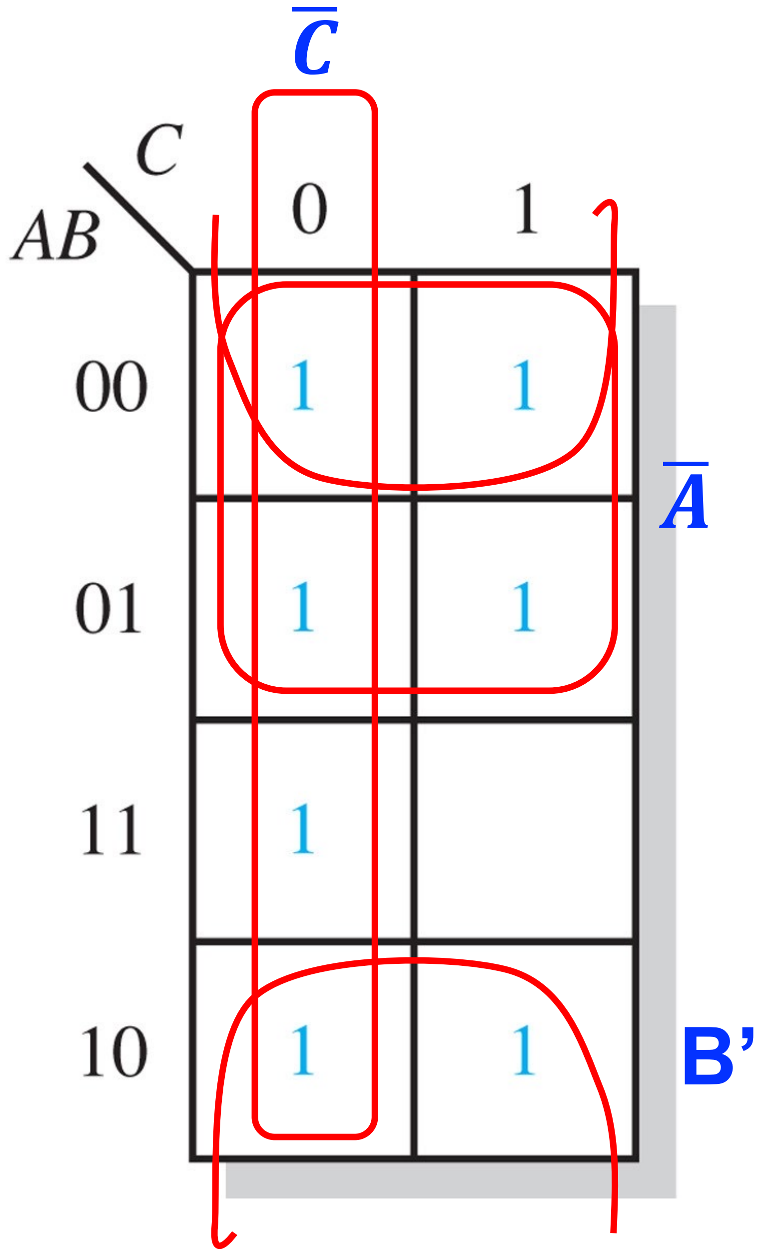
$$F = \sum(0,1,2,3,4,5,6) = \prod(7)$$

$$F = \bar{A} + \bar{B} + \bar{C}$$

Using maxterms:

$$\bar{F} = A \,.\, B \,.\, C$$

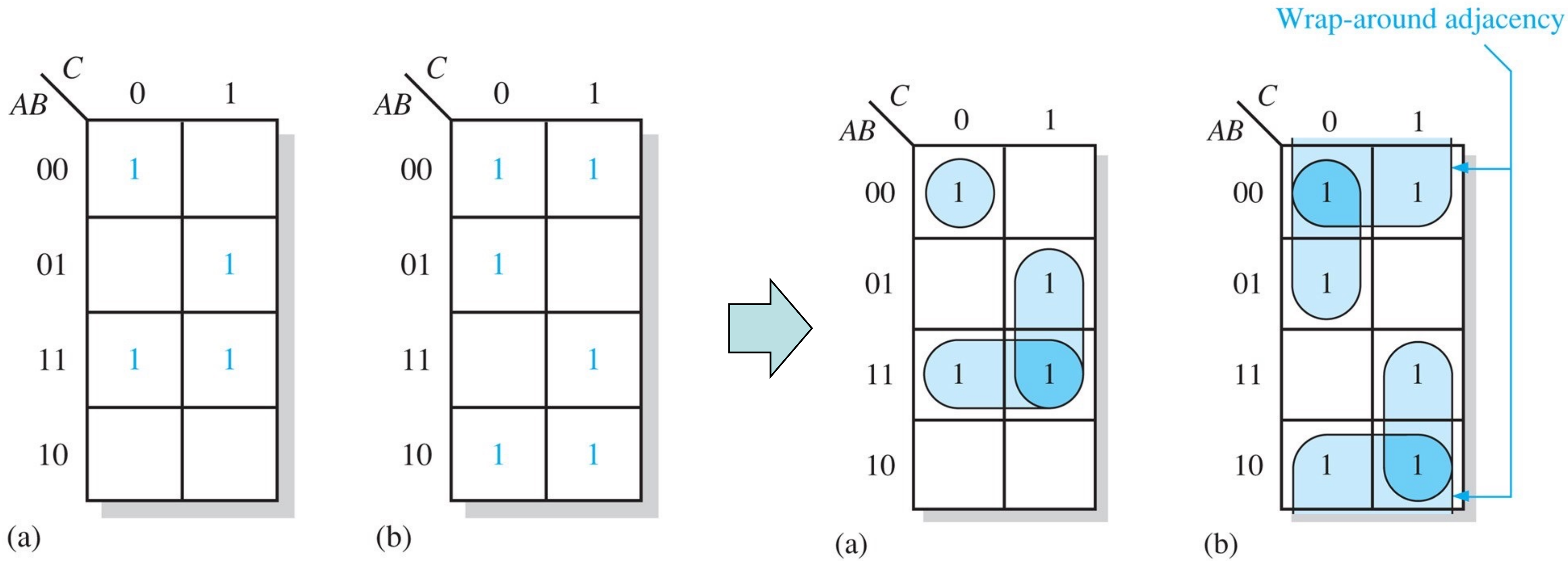
De Morgan: $F = \bar{A} + \bar{B} + \bar{C}$



Minimize (4/5)

FIGURE 4-33

Como agrupar? Qual a função minimizada resultante?



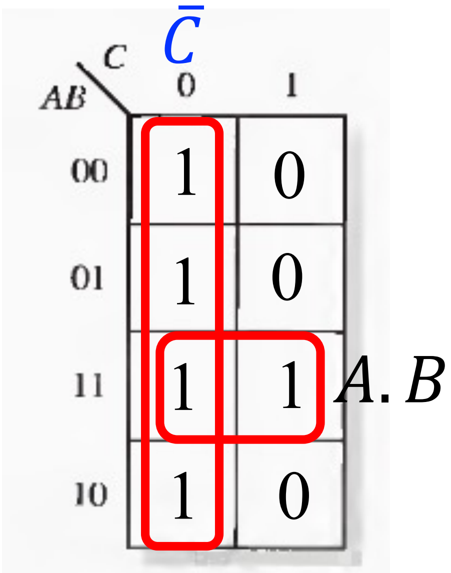
$$(a)\ F = AB + BC + \bar{A}.\bar{B}.\bar{C}$$

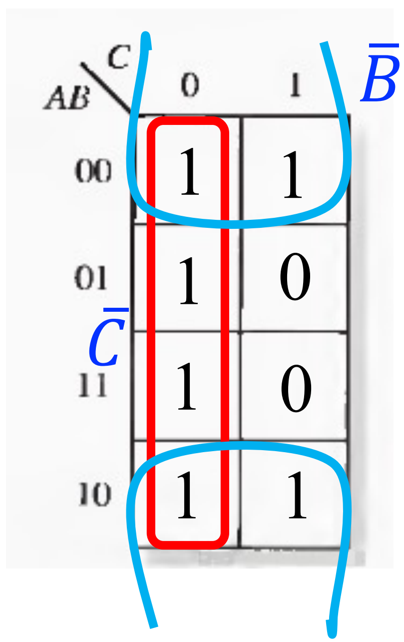
$$(b)\ F = \bar{A}.\bar{C} + A.C + \bar{B}$$

Minimize (4/5)

FIGURE 4-33

Como agrupar? Qual a função minimizada resultante?





Minimize (5/5)

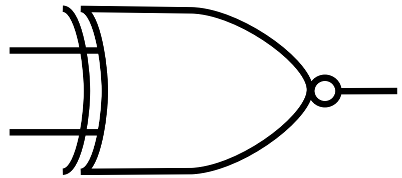
Determine the functions:

- F1 = ∑ (0,1,7,6)
- F2 = ∑ (0,2,3,4,6,7)

A \ BC	00	01	11	10
0				
1				

A \ BC	00	01	11	10
0	1	1		
1			1	1

F1= $\bar{A}.\bar{B} + A.B$



A \ BC	00	01	11	10
0	1		1	1
1	1		1	1

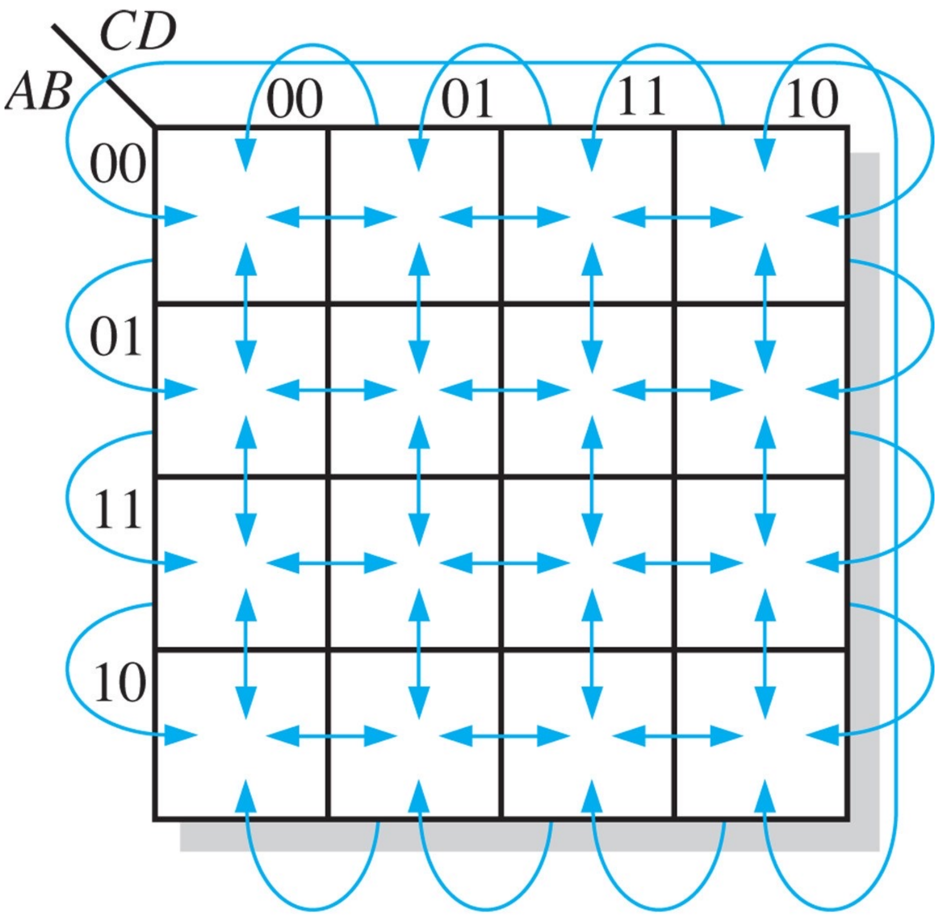
F2= $B + \bar{C}$

$\overline{F2} = \bar{B}.C$

(aplica de Morgam e obtém F2)

Karnaugh maps – 4 variables

A 4-variable map has an adjacent cell on each of its four boundaries



Each cell is different only by one variable from an adjacent cell

FIGURE 4-26 A 4-variable Karnaugh map.

$AB \backslash CD$	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

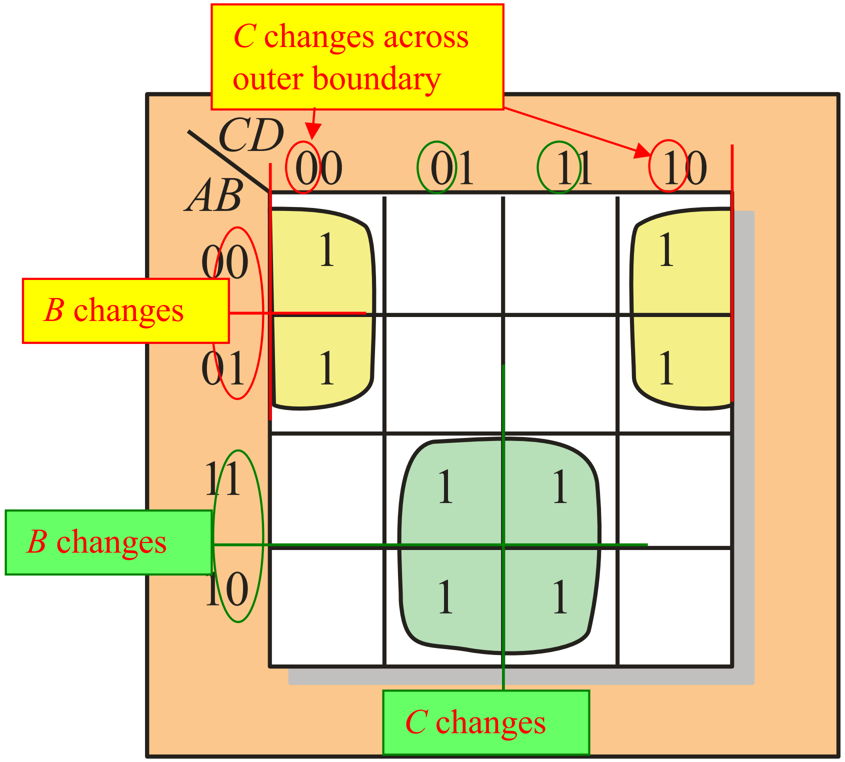
(a)

$AB \backslash CD$	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}\,\overline{B}CD$	$\overline{A}\,\overline{B}C\overline{D}$
01	$\overline{A}B\overline{C}\,\overline{D}$	$\overline{A}B\overline{C}D$	$\overline{A}BCD$	$\overline{A}BC\overline{D}$
11	$AB\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
10	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

(b)

Karnaugh maps – 4 variables

Agrupar os 1's no K-map de forma a minimizar a função.



1. Agrupe os 1's em dois grupos separados, conforme indicado.
2. Para cada grupo eliminar os literais redundantes.

O grupo amarelo é $\overline{\boldsymbol{A}}.\ \overline{\boldsymbol{D}}$

O grupo verde é $\boldsymbol{A}.\boldsymbol{D}$

$$F = \overline{A}.\ \overline{D} + A.D$$

Minimizar (1/3)

Map the following SOP expression on a Karnaugh map:

F = B'C' + AB' + ABC' + AB'CD' + A'B'C'D + AB'CD

10 01 11

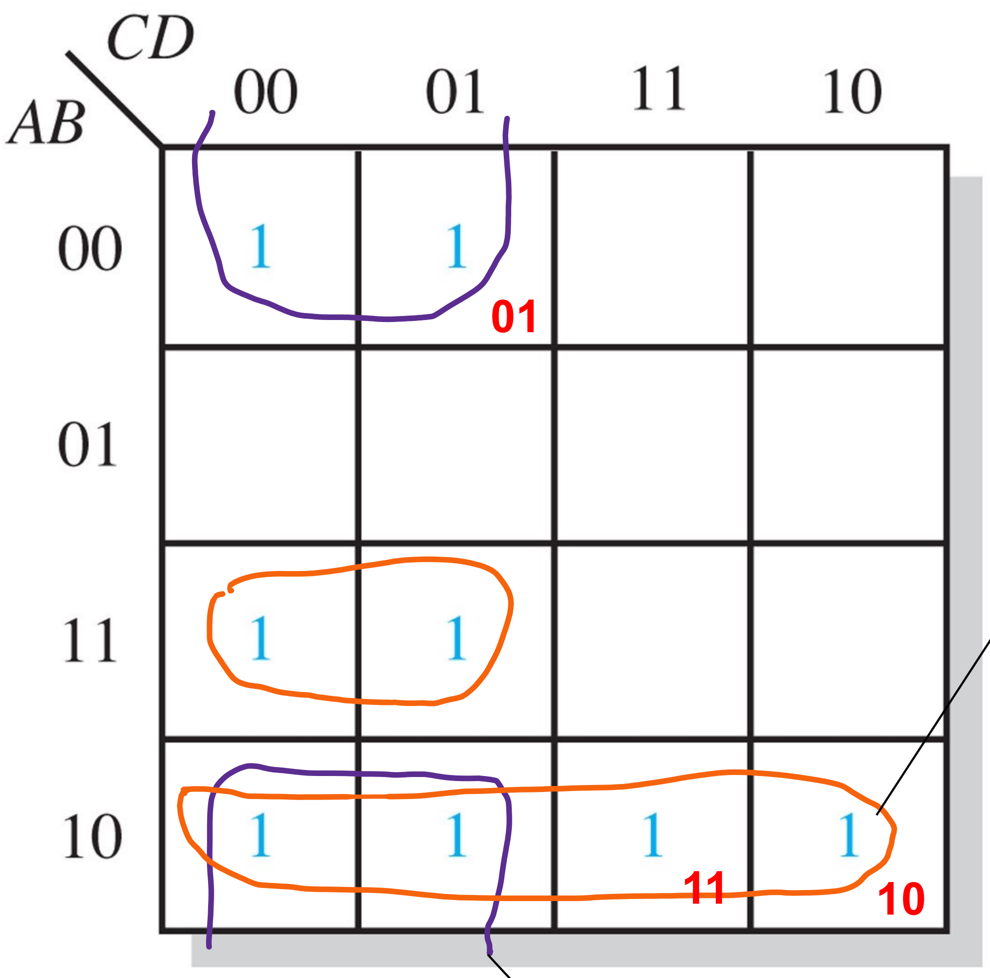
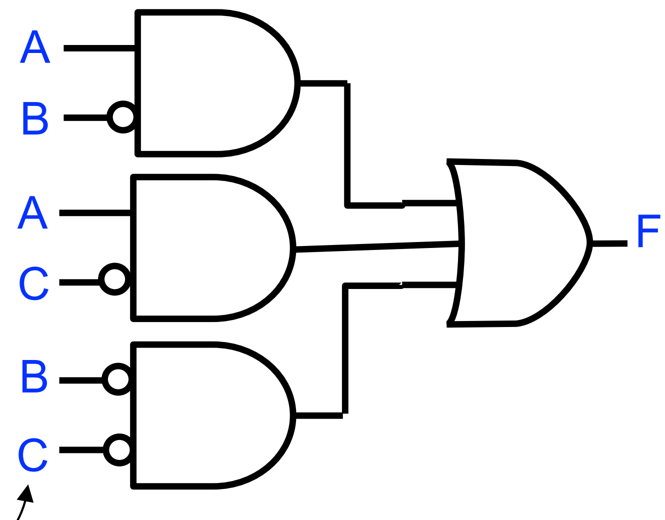


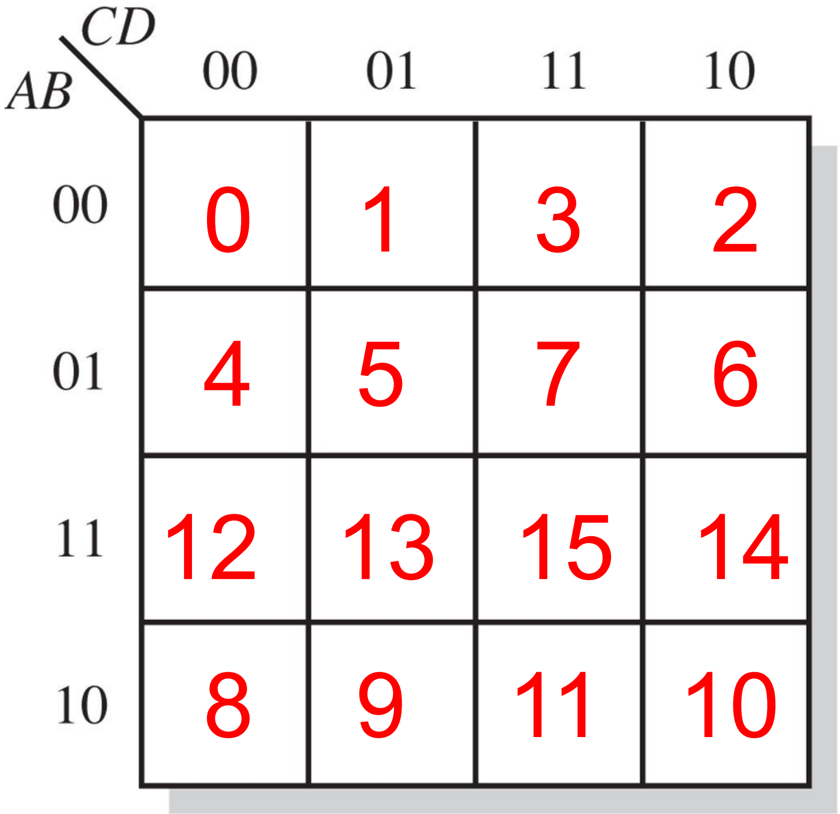
FIGURE 4-32

Which is the minimized function?



Minimizar (2/3)

$F2 = \sum (2, 3, 4, 5, 6, 7, 9, 12, 13, 14, 15)$



(a)

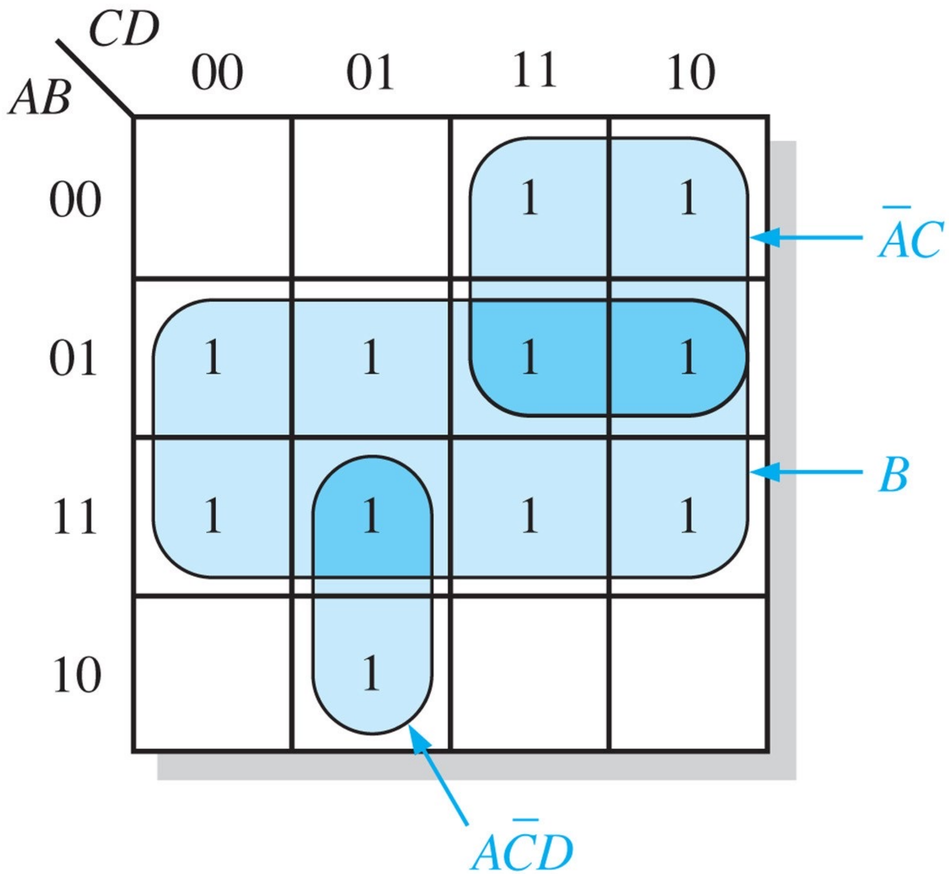


FIGURE 4-35

Minimizar (3/3)

Mapa de Karnaugh para as seguintes expressões, apresentando o circuito resultante

1. A’BC + AB’C + AB’C’
2. A’BCD’ + ABCD’ + ABC’D’ + ABCD

A/BC	00	01	11	10
0			1	
1	1	1		

F1 = AB’+ A’BC

AB/CD	00	01	11	10
00				
01				1
11	1		1	1
10				

F2 = ABC + ABD’ + BCD’

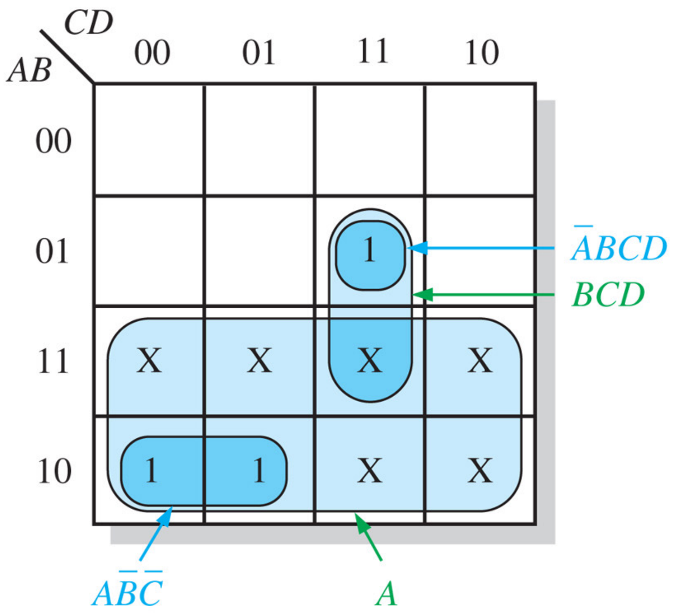
Uso de don’t cares

FIGURE 4-40 Example of the use of “don’t care” conditions to simplify an expression.

Inputs				Output
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don’t cares

(a) Truth table

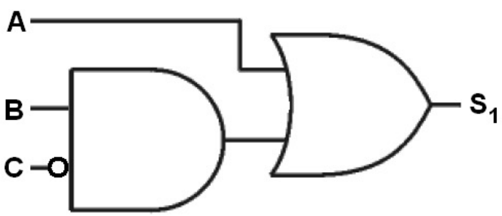


(b) Without “don’t cares” $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With “don’t cares” $Y = A + BCD$

Minimize (4/4) – com don't cares

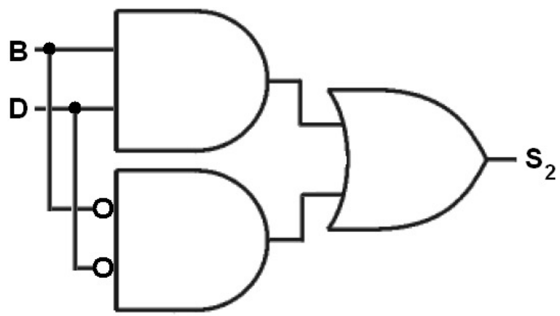
S_1 AB \ CD	00	01	11	10
00	0	X	0	0
01	1	X	X	0
11	1	1	X	1
10	1	1	X	1

$S_1 = B\bar{C} + A$

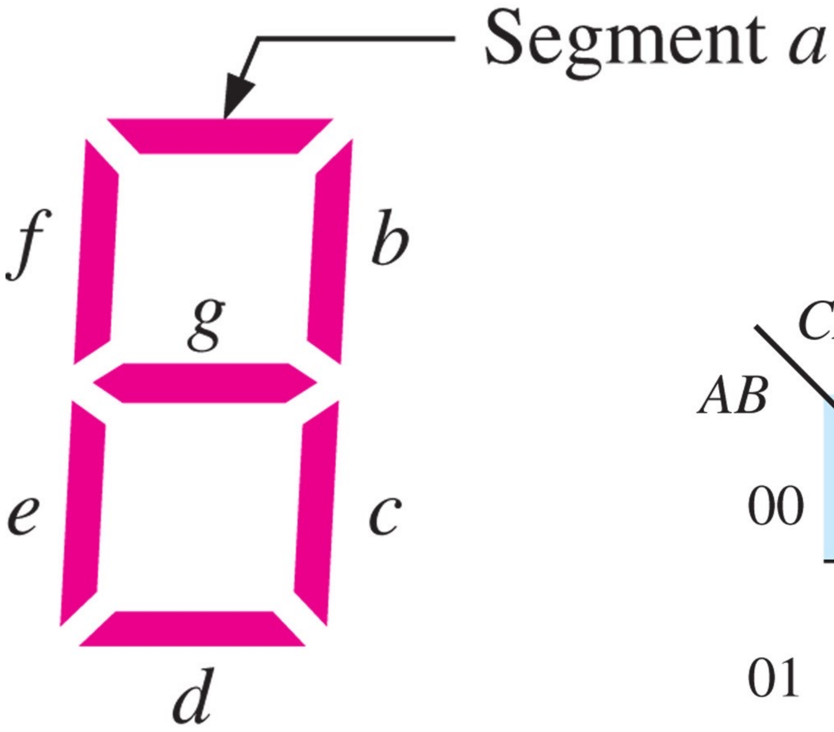


S_2 AB \ CD	00	01	11	10
00	1	0	0	X
01	0	1	X	0
11	0	X	1	0
10	X	0	0	1

$S_2 = BD + \bar{B}\bar{D}$



USO PRÁTICO DE DON'T CARE - BCD para 7 segementos



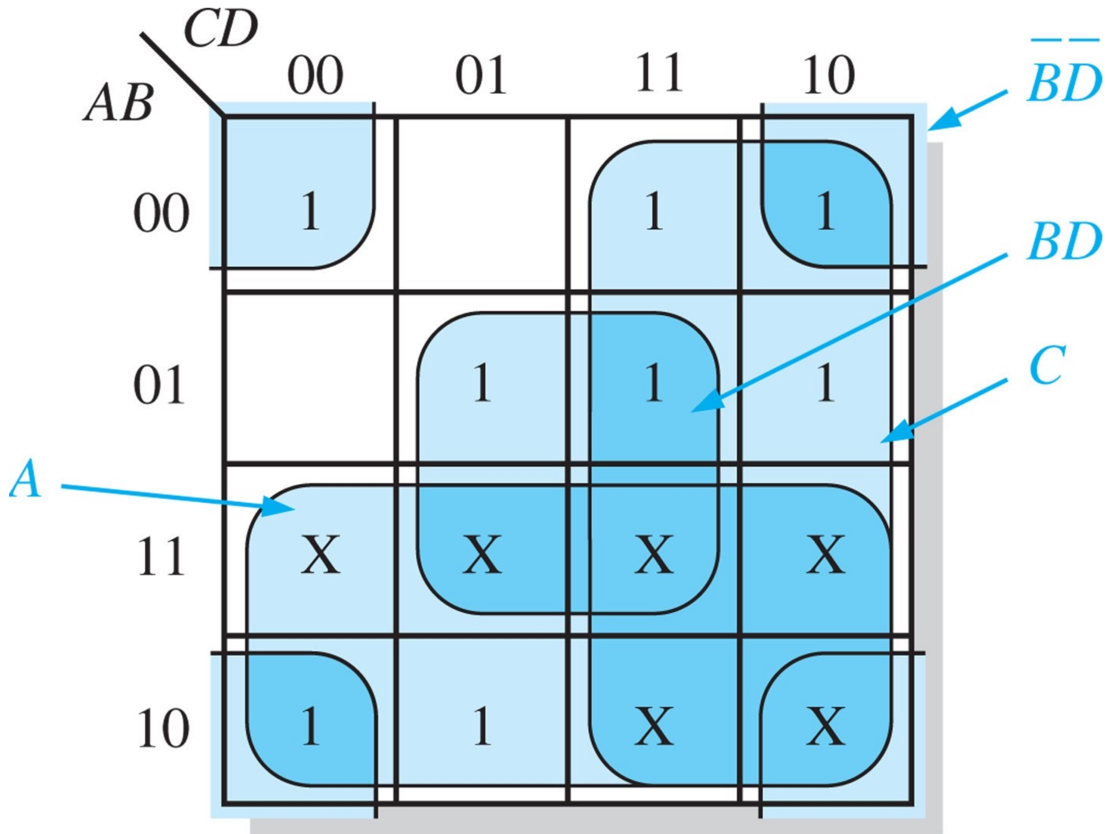


FIGURE 4-41 7-segment display.

FIGURE 4-42

2-6 variables

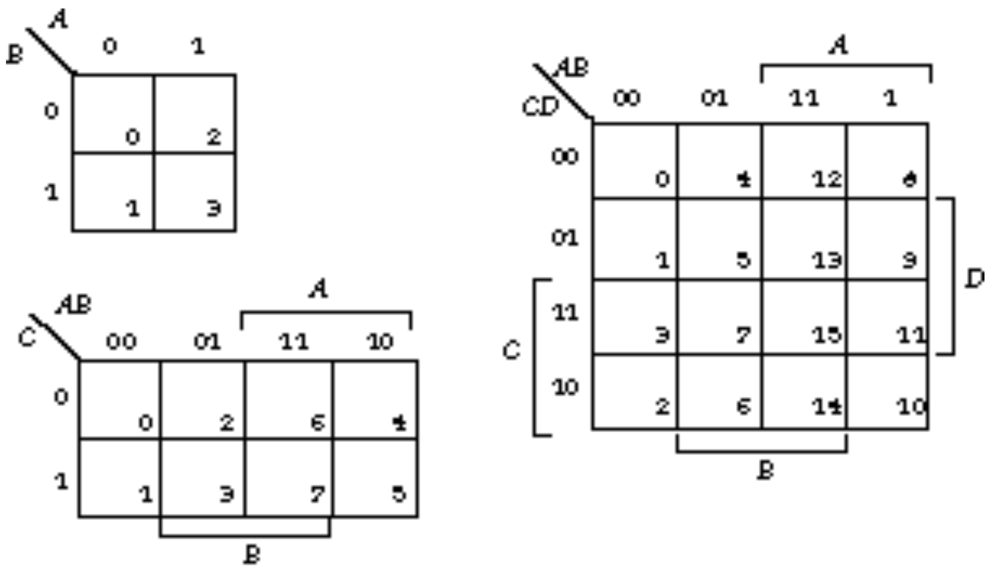
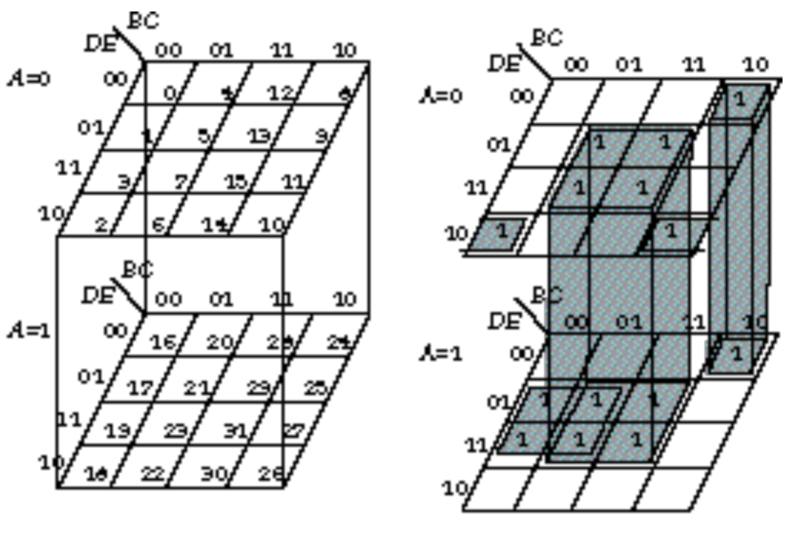
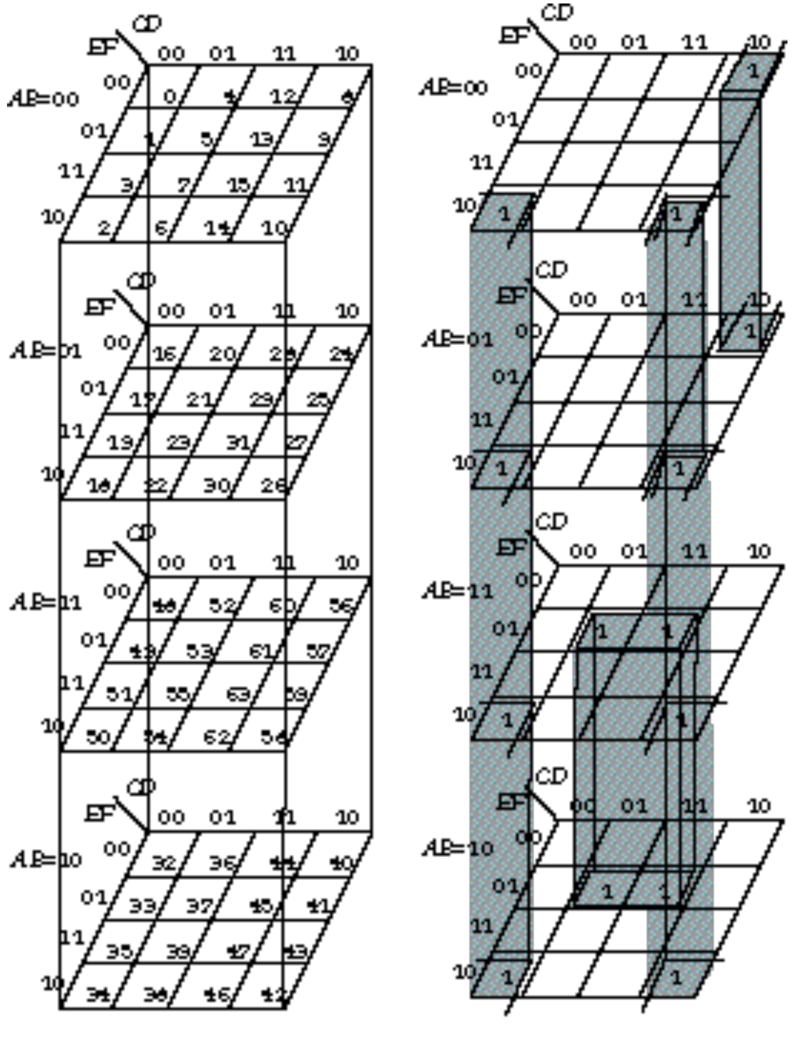


Figure 2.29 Two-, three-, and four-variable K-map templates.



(a) Five-variable K-map (b) Example

Figure 2.51 Five-variable K-map and example.



(a) Six-variable K-map (b) Example

Figure 2.52 Six-variable K-map and example.