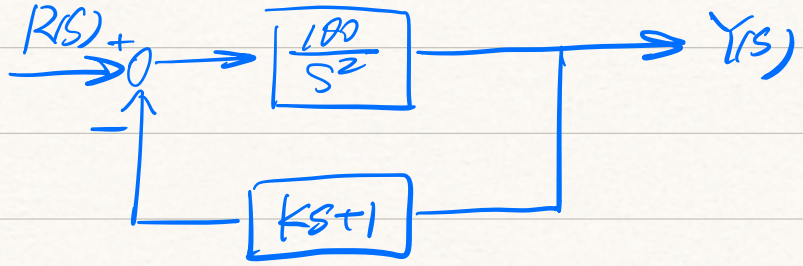
E 5.5



$r(t) = At \qquad R(s) = \frac{A}{s^2}$

$E(s) = R(s) - Y(s)$

$\frac{E(s)}{R(s)} = 1 - \frac{G}{1+GH}$

$E(s) = \frac{A}{s^2}\left(1 - \frac{\frac{100}{s^2}}{1+\frac{100(ks+1)}{s^2}}\right) = \frac{s+100k}{s(s^2+100ks+100)} \cdot A$

$e_{ss} = \lim_{s\to 0} s \cdot \frac{A(s+100k)}{s[s^2+100ks+100)} = kA$

(b) $T(s) = \frac{Y(s)}{R(s)} = \frac{100}{s^2+100ks+100}$

$\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$2\zeta\omega_n = 100k \Rightarrow \zeta = 5k$

$\sigma\% = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$

$= e^{-\frac{5k\pi}{\sqrt{1-25k^2}}} \times 100\% \Rightarrow$

$1 - 25k^2 = 0 \Rightarrow k = 0.2$

无解，故为为 $S_{1,2} = -50k \pm 10\sqrt{1-25k^2}$?

Es.8 $G(s) = \frac{K}{s(s+\sqrt{2K})}$

(a) $1 + GH = s^2 + \sqrt{2K}\,s + K$

$$\begin{cases} \omega_n^2 = K \\ 2\zeta\omega_n = \sqrt{2K} \end{cases} \Rightarrow \begin{cases} \omega_n = \sqrt{K} \\ \zeta = \frac{\sqrt{2}}{2} \end{cases}$$

$$\sigma = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = 4.3\%$$

$$t_s = \frac{4}{\zeta\omega_n} = \frac{8}{\sqrt{2K}}$$

(b) $t_s < 1 \quad K > 32$

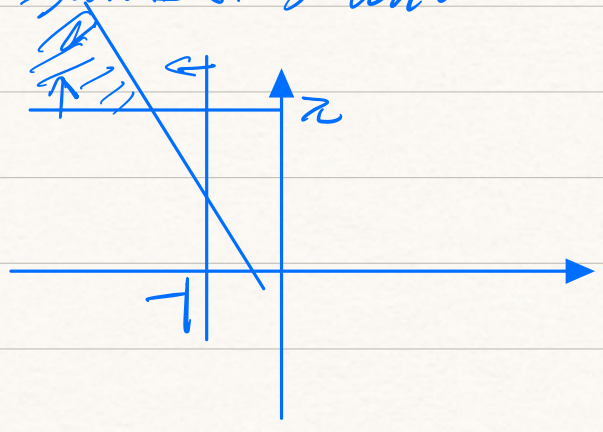
Es.9

$$\sigma\% = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% \le 5\% \Rightarrow -\frac{\sqrt{1-\zeta^2}}{\zeta} \ge -\frac{\pi}{\ln 5} = -1.95$$

$$T_s = \frac{4}{\zeta\omega_n} < 4 \Rightarrow \zeta\omega_n > 1 \Rightarrow -\zeta\omega_n < -1$$

$$T_p = \frac{\pi}{\sqrt{1-\zeta^2}\,\omega_n} < 1 \Rightarrow \sqrt{1-\zeta^2}\,\omega_n > \pi$$

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{1-\zeta^2}\,\omega_n i$$



Es.10

$$\frac{E(s)}{R(s)} = \frac{1}{1+G} = \frac{s(s+1)(s+3)(s+10)}{s(s+1)(s+3)(s+10)+6(s+5)}$$

① $R(s) = \frac{A}{s}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{A}{s} \cdot \frac{s(s+1)(s+3)(s+10)}{s(s+1)(s+3)(s+10)+6(s+5)} = 0$$

② $R(s) = \frac{A}{s^2}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{A}{s^2} \cdot \frac{s(s+1)(s+3)(s+10)}{s(s+1)(s+3)(s+10)+6(s+5)} = A$$

ES.13 (a) $\frac{E(s)}{R(s)} = 1 - \frac{G}{1+GH}$

$$= 1 - \frac{\frac{0.4}{s(s+2)}}{1+\frac{0.4}{s(s+2)}\cdot\frac{(s+3)}{(s+0.1)}}$$

$$= 1 - \frac{0.4(s+0.1)}{s(s+2)(s+0.1)+0.4(s+3)}$$

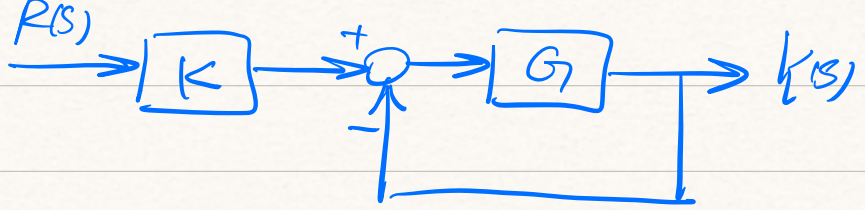
$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \left(1 - \frac{0.4(s+0.1)}{s(s+2)(s+0.1)+0.4(s+3)}\right)$$

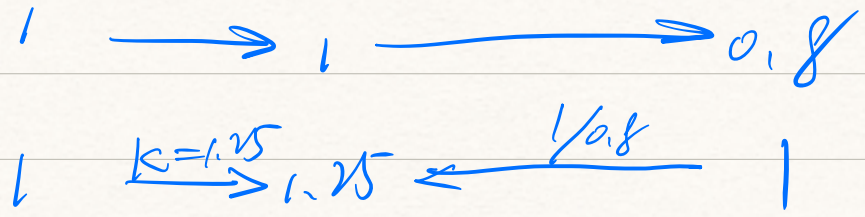
$$= \frac{29}{30}$$

(b) $e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \left(1 - \frac{\frac{K}{s(s+2)}}{1+\frac{K}{s(s+2)}\frac{(s+3)}{(s+0.1)}} \cdot G_p(s)\right) = 0$

$\Rightarrow G_p(s) = 30$

ES.17





E6.1 $S^3 + 3KS^2 + (2+K)S + 5 = 0$

S^3	1	$(2+K)$
S^2	$3K$	5
S^1	$\frac{3K^2+6K-5}{3K}$	0
S^0	5	

$$\begin{cases} 3K > 0 \\ \frac{3K^2+6K-5}{3K} > 0 \end{cases} \Rightarrow K > 0.63$$

E6.2 $S^3 + 9S^2 + 26S + 24 = 0$

S^3	1	26
S^2	9	24
S^1	23	0
S^0	24	

E6.5 $GH(s) = \frac{20}{(S+1)(S+3)(S+6)}$

$1 + GH(s) = 0 \Rightarrow S^3 + 10S^2 + 27S + 38 = 0$

$S_1 = -6.9 \quad S_{2,3} = -1.6 \pm 1.8j$

E6.6 $s_{1,2} = \pm a\hat{j}$

$s^3 + 10s^2 + 27s + 18 + k\big|_{s=s_{1,2}} = 0$

$\begin{cases} -a^3\hat{j} - 10a^2 + 27a\hat{j} + 18 + k = 0 \\ a^3\hat{j} - 10a^2 - 27a\hat{j} + 18 + k = 0 \end{cases}$

$\Rightarrow \begin{cases} a = 3\sqrt{3} \\ k + 18 = 270 \end{cases}$

$s^3 + 10s^2 + 27s + 270 = 0$

$s_{1,2} = \pm 3\sqrt{3}\hat{j}, \quad s_3 = -10$

E6.7 (a) $1 + GH = 0$

$s^2 + (k-1)s + 2k = 0$

$\begin{cases} \zeta = 0.707 \\ 2\zeta\omega_n = k - 1 \\ \omega_n^2 = 2k \end{cases} \Rightarrow \begin{cases} \omega_n = 3.4 \\ k = 5.78 \end{cases}$

(b) $s_{1,2} = \pm a\hat{j}$

$\begin{cases} -a^2 + (k-1)a\hat{j} + 2k = 0 \\ -a^2 - (k-1)a\hat{j} + 2k = 0 \end{cases} \Rightarrow \begin{cases} a = \sqrt{2} \\ k = 1 \end{cases}$

$k = 1, \quad s_{1,2} = \pm\sqrt{2}\hat{j}$

E6.9 $s^3 + 3s^2 + (k+1)s + 4 = 0$

s^3	1	$(k+1)$
s^2	3	4
s^1	$k - \frac{1}{3}$	0

s^0 4

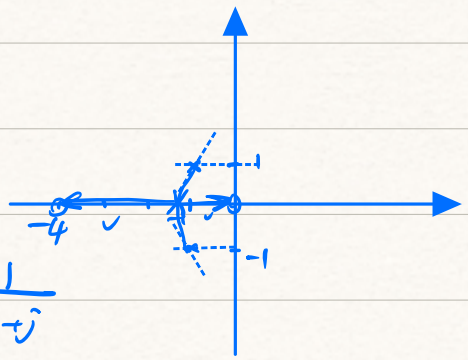
$k-\frac{1}{3}>0 \Rightarrow \quad k>\frac{1}{3}$

E7.1 $GH(s)=\frac{Ks(s+4)}{s^2+2s+2}=\frac{Ks(s+4)}{(s+1-j)(s+1+j)}$

① $n=2 \quad m=2$

$p_1=-1+j \quad p_2=-1-j$

$z_1=0 \quad z_2=-4$



② $\frac{1}{d+4}+\frac{1}{d}=\frac{1}{d+1-j}+\frac{1}{d+1+j}$

$d=-1.2$

$\frac{(2k+1)\pi}{2}=\frac{\pi}{2},\frac{3\pi}{2}$

$\Sigma\varphi-\Sigma\theta=\pi$

③ $\Sigma\varphi-\Sigma\theta=\pi$

$\frac{3}{4}\pi+\arctan\frac{1}{3}-\frac{\pi}{2}-\theta=\pi$

$\theta=-116.6^\circ$

(b) $s_{1,2}=-1.2$

$(1+k)s^2+(2+4k)s+2=0$

$k=0.3$

(c) $s_{1,2}=-1.2$

(d) $1.3s^2+3.2s+2=0$

$2\zeta\omega_n=3.2$

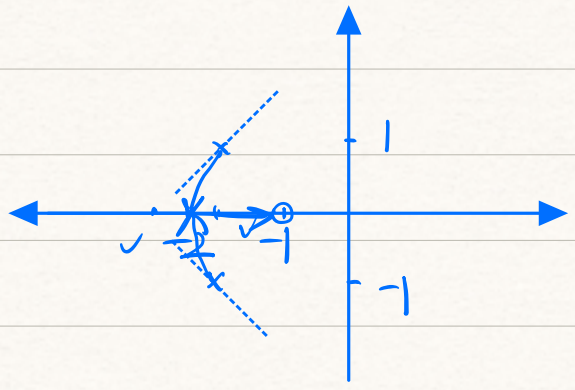
$$T_s = \frac{4}{\zeta \omega_n} = 2.5\,s$$

E7.4 $G(s) = \dfrac{K(s+1)}{s^2+4s+5} = \dfrac{K(s+1)}{(s+2-i)(s+2+i)}$

① $n=2 \quad m=1$

$p_1 = -2+i \quad p_2 = -2-i$

$z_1 = -1$



② $\sum \varphi - \sum \theta = \pi$

$\frac{3}{4}\pi - \frac{\pi}{2} - \theta = \pi$

$\theta = -\frac{3}{4}\pi$

③ $\sigma = \dfrac{-2+i-2-i+1}{2-1} = -3$

$\varphi = \dfrac{(2k+1)\pi}{2-1} = \pi, \quad k=0$

④ $\dfrac{1}{d+1} = \dfrac{1}{d+2-i} + \dfrac{1}{d+2+i}$

$d^2+2d-1=0$

$d = -2.4$

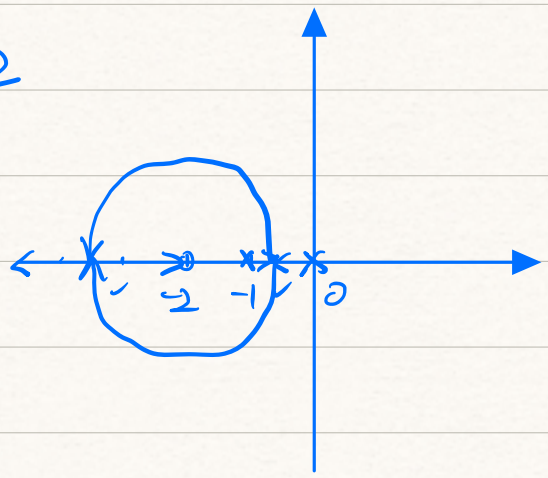
$\frac{\pi}{2}, \frac{3\pi}{2}$

E7.10 $KG(s) = \dfrac{K(s+4)}{s(s+1)}$

$s(s+1)$

① $n=2 \quad m=1$

$p_1=0 \quad p_2=-1 \quad z_1=-2$



② $\frac{1}{d+1}+\frac{1}{d}=\frac{1}{d+2}$

$d^2+4d+2=0$

$d_1=-0.6 \quad d_2=-3.4$

③ $\sigma=\frac{-1-(-2)}{2-1}=1$

$\varphi=\frac{(2k+1)2}{2-1}=\pi, \quad k=0$

④ $s^2+(1+k)s+2k=0$

写成 $1+k\frac{s}{\cdots}$ 之类

$s_{1,2}=-2\pm aj$

取 $s_1=-2+aj$

$(-2+aj)^2+(1+k)(-2+aj)+2k=0$

$(2-a^2)+a(k-3)j=0$

$\begin{cases} a(k-3)=0 \\ 2-a^2=0 \end{cases} \Rightarrow \begin{cases} a=\sqrt{2} \\ k=3 \end{cases}$

$K=3$, $s_{1,2}=-2\pm\sqrt{2}j$

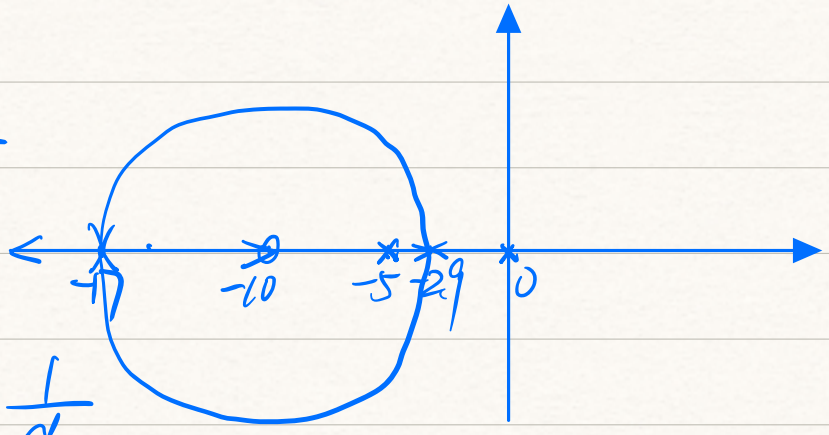
E7.14

$$G(s)=\frac{K(s+10)}{s(s+5)}$$

① $n=2$ $m=1$

$p_1=0$, $p_2=-5$

$z_1=-10$



② $\frac{1}{d+10}=\frac{1}{d+5}+\frac{1}{d}$

$d^2+20d+50=0$

$d_1=-2.9$ $d_2=-17$

$\frac{\pi}{2}$, $\frac{3\pi}{2}$

③ $\sigma=\frac{-5-(-10)}{2-1}=5$

$\varphi=\frac{(2l+1)\pi}{2-1}=\pi$

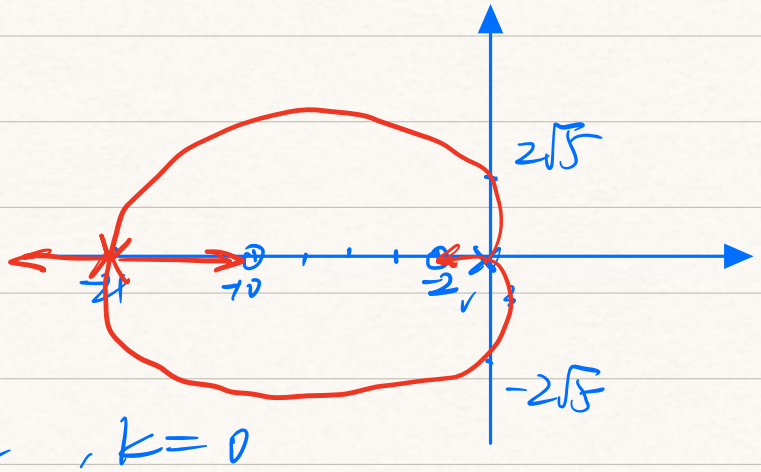
④ $s^2+(5+K)s+10K=0$ 与虚轴无交点

(b) $s^2+(5+K)s+10K=0$

$$\begin{cases}\zeta=\frac{1}{\sqrt{2}}\\ 2\zeta\omega_n=5+K\\ \omega_n^2=10K\end{cases}\Rightarrow\begin{cases}K=5\\ \omega_n=5\sqrt{2}\end{cases}$$

E7.15

(a) $G(s)H(s) = \dfrac{K(s+10)(s+2)}{s^3}$



① $n=3,\ m=2$

$p_{1,2,3}=0$

$z_1=-10,\ z_2=-2$

② $\sigma = \dfrac{(2+10)}{3-2} = 12,\ K=0$

$\sigma = \dfrac{0+10+2}{3-2} = 12$

③ $\dfrac{2}{d} = \dfrac{1}{d+2} + \dfrac{1}{d+10}$

$d^2+24d+60=0$

$d=-21$

$\frac{12}{2},\ \frac{288}{2}$

④ $s^3 + Ks^2 + 12Ks + 20K = 0$

s^3	1	$12k$
s^2	k	$20k$
s^1	$12k-20$	0
s^0	$20k$	

$12k-20=0 \Rightarrow k=\frac{5}{3}$

$\frac{5}{3}s^2 + \frac{100}{3} = 0$

$$s = \pm j 2\sqrt{5}$$

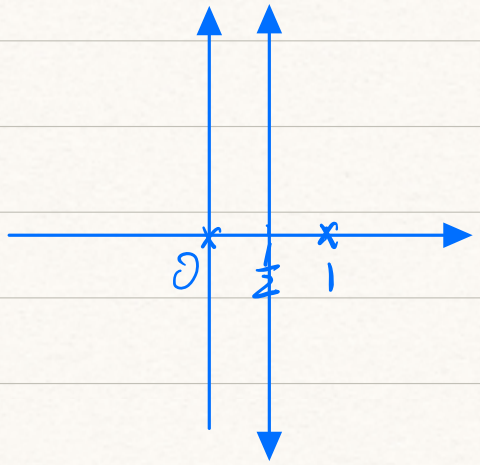
(b) $12k - 20 > 0 \quad k > \frac{5}{3}$

(c) $\frac{E(s)}{R(s)} = \frac{1}{1+G}$, $R(s) = \frac{A}{s^2}$

$$E(s) = \frac{A}{s^2} \cdot \frac{s^3}{s^3 + k(s+10)(s+2)}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{A}{s^2} \cdot \frac{s^3}{s^3 + k(s+10)(s+2)} = 0$$

E7.17 (a) $G(s)\,G(s) = \frac{K}{s(s-1)}$



① $n = 2$, $m = 0$

$p_1 = 0$, $p_2 = 1$

② $\sigma = \frac{1}{2-0} = \frac{1}{2}$

$\gamma = \frac{(2k+1)\pi}{2-0} = \frac{\pi}{2}, \frac{3\pi}{2}$, $k = 0, 1$

③ $s^2 - s + k = 0$

$2s - 1 = 0$

$s = \frac{1}{2}$

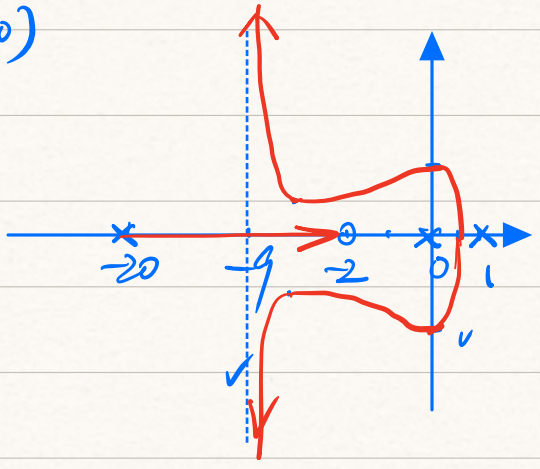
$\frac{\pi}{2}, \frac{3\pi}{2}$

④ $S^2 - S + k = 0$

S^2	1	k
S	-1	
S^0	k	

⑤ 沒有根在右半平面

(b) $G_c \cdot G(s) = \dfrac{K(s+2)}{s(s-1)(s+20)}$



① $n = 3 \quad m = 1$

$p_1 = 0, \ p_2 = 1, \ p_3 = -20$

$z_1 = -2$

② $\sigma = \dfrac{1 - 20 + 2}{3 - 1} = -9$

$\varphi = \dfrac{(2k+1)\pi}{3-1} = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

③ $\dfrac{1}{d} + \dfrac{1}{d-1} + \dfrac{1}{d+20} = \dfrac{1}{d+2}$

$2d^3 + 35d^2 + 76d - 40 = 0$

$d_1 = 0.46 \quad d_2 = -6.5 + 1.4j \quad d_3 = -6.5 - 1.4j$

$\dfrac{(2k+1)\pi}{1} = \pi, 3\pi$

④ $S(S-1)(S+20) + K(S+2) = 0$

$S^3 + 19S^2 + (K-20)S + 2K = 0$

S^3	1	$K-20$
S^2	19	$2K$

s^1 $\frac{17k}{19}-20$ 0

s^0 $2k$

$\frac{17}{19}k-20=0 \Rightarrow k=\frac{380}{17}$

$19s^2+\frac{760}{17}=0$

$s=\pm j4.5$

$\frac{17k}{19}-20>0 \Rightarrow k>\frac{380}{17}$ 稳定

$k=\frac{380}{17}$