1. 解：对误差e，$\mu_{ZE}(0.6)=0.7$，$\mu_{PS}(0.6)=0.3$

$e'=\frac{1}{0.6}$，根据Mamdani推理，

由规则1：

$U_1'=\alpha_{e_1}\wedge ZE=0.7\wedge ZE$

$$\text{即}\ \mu_{U_1'}(u)=\begin{cases}\frac{1}{2}(u-2), & 2<u\le 3.4\\ 0.7, & 3.4<u\le 4.6\\ -\frac{1}{2}(u-6), & 4.6<u\le 6\\ 0, & \text{其它}\end{cases}$$

由规则2：

$U_2'=\alpha_{e_2}\wedge NS=0.3\wedge NS$

$$\text{即}\ \mu_{U_2'}(u)=\begin{cases}\frac{1}{2}u, & 0<u\le 0.6\\ 0.3, & 0.6<u\le 3.4\\ -\frac{1}{2}(u-4), & 3.4<u\le 4\\ 0, & \text{其它}\end{cases}$$

$U'=U_1'\vee U_2'$

$$\text{即}\ \mu_{U'}(u)=\begin{cases}\frac{1}{2}u, & 0<u\le 0.6\\ 0.3, & 0.6<u\le 3.2\\ \frac{1}{2}(u-2), & 3.2<u\le 3.4\\ 0.7, & 3.4<u\le 4.6\\ -\frac{1}{2}(u-6), & 4.6<u\le 6\\ 0, & \text{其它}\end{cases}$$

$$u=\frac{\int_0^8 u\mu_{u'}(u)\,du}{\int_0^8 \mu_{u'}(u)\,du}$$

$$=\frac{7.79}{2.33}=3.34$$

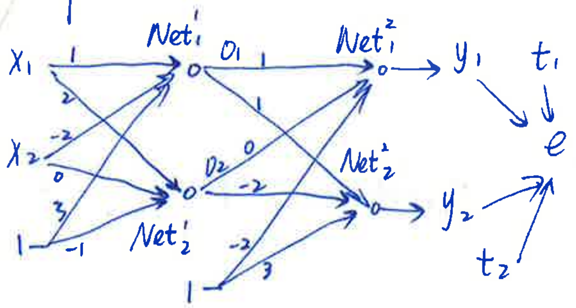
∴ 当输入误差为$e=0.6$时，电压$u=3.34$

2. 答：模糊控制器的主要组成部分为：
① 模糊化过程；
② 知识库；
③ 推理决策逻辑；
④ 精确化计算。

3. 答：可分为：
① 前向网络。
② 反馈网络。
③ 相互结合型网络。
④ 混合型网络。

4. 答：BP算法基本步骤：
① 从训练样例集中取一样例，把输入信息输入到网络中；
② 由网络分别计算各层节点的输出；
③ 计算网络的实际输出与期望输出的误差；
④ 从输出层反向计算到第一个隐含层，按一定原则（必须确定如何沿减少误差的方向调整连接权值）向减少误差方向调整网络的各个连接权值；
⑤ 对训练样例集中的每一个样例重复以上步骤，直到对整个训练样例集的误差达到要求为止。

解：



$Net_1^1 = w_{10} + w_{11}x_1 + w_{12}x_2 = 1 - 2\times 3 + 3 = -2$

$Net_2^1 = w_{20} + w_{21}x_1 + w_{22}x_2 = -1 + 2 = 1$

$O_1 = \sigma(Net_1^1) = 0.119$

$O_2 = \sigma(Net_2^1) = 0.731$

$Net_1^2 = w_{11}O_1 + w_{12}O_2 + w_{10} = O_1 - 2 = -1.881$

$Net_2^2 = w_{21}O_1 + w_{22}O_2 + w_{20} = 0.119 - 2\times 0.731 + 3 = 1.657$

$y_1 = \sigma(Net_1^2) = 0.132$

$y_2 = \sigma(Net_2^2) = 0.840$

$e = \frac{1}{2}\left[(y_{d1}-y_1)^2 + (y_{d2}-y_2)^2\right] = 0.441$

若第一次迭代条件 $e<\varepsilon$ 不满足，则有

$\delta_1^2 = -\frac{\partial e}{\partial Net_1^2} = -\frac{\partial e}{\partial y_1}\frac{\partial y_1}{\partial Net_1^2} = (y_{d1}-y_1)y_1(1-y_1) = 0.0880$

$\delta_2^2 = (y_{d2}-y_2)y_2(1-y_2) = -0.0726$

$\delta_1^1 = -\frac{\partial e}{\partial Net_1^1} = -\frac{\partial e}{\partial O_1}\frac{\partial O_1}{\partial Net_1^1}$

$= -\left(\frac{\partial e}{\partial y_1}\frac{\partial y_1}{\partial Net_1^2}\frac{\partial Net_1^2}{\partial O_1} + \frac{\partial e}{\partial y_2}\frac{\partial y_2}{\partial Net_2^2}\frac{\partial Net_2^2}{\partial O_1}\right)\frac{\partial O_1}{\partial Net_1^1}$

$= \left[(y_{d1}-y_1)y_1(1-y_1)w_{11} + (y_{d2}-y_2)y_2(1-y_2)w_{21}\right]O_1(1-O_1)$

$= 0.00161$

$\delta_2^1 = -2\times(-0.0726)\times \cancel{0.84}\ 0.731\times(1-0.731)$

$= 0.0286$

对于输出层到隐藏层，权重更新如下：

$\Delta w_{11} = \eta\delta_1^2 O_1 = 0.0105\eta$　$\Delta w_{21} = \eta\delta_2^2 O_1 = 0.00864\eta$

$\Delta w_{12} = \eta\delta_1^2 O_2 = 0.0643\eta$　$\Delta w_{22} = \eta\delta_2^2 O_2 = -0.0531\eta$

$\Delta w_{10} = \eta\delta_1^2 = 0.088\eta$　$\Delta w_{20} = \eta\delta_2^2 = -0.0726\eta$

~~对于输隐~~

$w_{11}^{(2)} = w_{11}^{(1)} + \Delta w_{11} = 1 + 0.0105\eta$　$w_{21}(2) = 1 + 0.00864\eta$

$w_{12}^{(2)} = w_{12}^{(1)} + \Delta w_{12} = 0.0643\eta$　$w_{22}(2) = -2 - 0.0531\eta$

$w_{10}(2) = w_{10}^{(1)} + \Delta w_{10} = -2 + 0.088\eta$　$w_{20}(2) = 3 - 0.0726\eta$

对于隐藏层到输入层，权重更新如下：

$\Delta w_{11} = \eta\delta_1^1 x_1 = 0.00161\eta$　$\Delta w_{21} = 0.0286\eta$

$\Delta w_{12} = \eta\delta_1^1 x_2 = \cancel{0.0858\eta}\ 0.00483\eta$　$\Delta w_{22} = 0.0858\eta$

$\Delta w_{10} = \eta\delta_1^1 = 0.00161\eta$　$\Delta w_{20} = 0.0286\eta$

$w_{11}(2) = 1 + 0.00161\eta$　$w_{21}(2) = 2 + 0.0286\eta$

$w_{12}(2) = -2 + 0.00483\eta$　$w_{22}(2) = 0.0858\eta$

$w_{10}(2) = 3 + 0.00161\eta$　$w_{20}(2) = -1 + 0.0286\eta$

（未设定学习步长）

$Net_1^1 = W_{10} + W_{11}X_1 + W_{12}X_2 = 3 + 1\cdot 1 \ -2\times 3 = -2$

$Net_2^1 = W_{20} + W_{21}X_1 + W_{22}X_2 = -1 + 2\cdot 1 = 1$

$O_1 = f(Net_1^1) = \frac{1}{1+e^2} = 0.119$

$O_2 = f(Net_2^1) = \frac{1}{1+e^{-1}} = 0.731$

$Net_1^2 = W_{10} + W_{11}O_1 + W_{12}O_2 = -2 + 0.119 = -1.881$

$Net_2^2 = W_{20} + W_{21}O_1 + W_{22}O_2 = 3 + 0.119 - 2\times 0.731 = 1.657$

$y_1 = f(Net_1^2) = 0.132$

$y_2 = f(Net_2^2) = 0.840$

$e = \frac{1}{2}\left[(t_1-y_1)^2 + (t_2-y_2)^2\right] = 0.647$

$$\delta_1^2 = -\frac{\partial e}{\partial Net_1^2} = -\frac{\partial e}{\partial y_1}\frac{\partial y_1}{\partial Net_1^2} = (t_1-y_1)y_1(1-y_1) = 0.0937$$

$$\delta_2^2 = -\frac{\partial e}{\partial Net_2^2} = -\frac{\partial e}{\partial y_2}\frac{\partial y_2}{\partial Net_2^2} = (t_2-y_2)y_2(1-y_2) = -0.106$$

$$\delta_1^1 = -\frac{\partial e}{\partial Net_1^1} = -\frac{\partial e}{\partial O_1}\frac{\partial O_1}{\partial Net_1^1} = -\left(\frac{\partial e}{\partial y_1}\frac{\partial y_1}{\partial Net_1^2}\frac{\partial Net_1^2}{\partial O_1} + \frac{\partial e}{\partial y_2}\frac{\partial y_2}{\partial Net_2^2}\frac{\partial Net_2^2}{\partial O_1}\right)\cdot\frac{\partial O_1}{\partial Net_1^1}$$

$$= \left[(t_1-y_1)y_1(1-y_1)W_{11} + (t_2-y_2)y_2(1-y_2)W_{21}\right]O_1(1-O_1)$$

$$= \cancel{0.00982} \ -0.00129$$

$$\delta_2^1 = -\frac{\partial e}{\partial Net_2^1} = -2\times(-0.106)\ 0.731(1-0.731) = 0.0417$$

Wtj

~~W10~~

$W_{11} = \eta\delta_1^2\cdot O_1 = 0.0112 \overset{+W_{11}}{} = 1.0112$

$W_{12} = \eta\delta_1^2 O_2 = 0.0685 \overset{+W_{12}}{}$

$W_{21} = \eta\delta_2^2 O_1 \overset{+W_{21}}{=} 0.987$

$W_{22} = \eta\delta_2^2 O_2 \overset{+W_{22}}{=} -2.0775$

能量函数计算公式为:

$$E = -w_{12}y_1y_2 - w_{13}y_1y_3 - w_{23}y_2y_3 - \theta_1y_1 - \theta_2y_2 - \theta_3y_3$$

当分别计算000, 001, 010, 011, 100, 101, 110, 111 8种状态，得011状态下

$$E = -0.5\times0\times1 - 0.2\times0\times1 - 0.6\times1\times1 - 0.1\times0 - 0\times1 - 0\times1$$

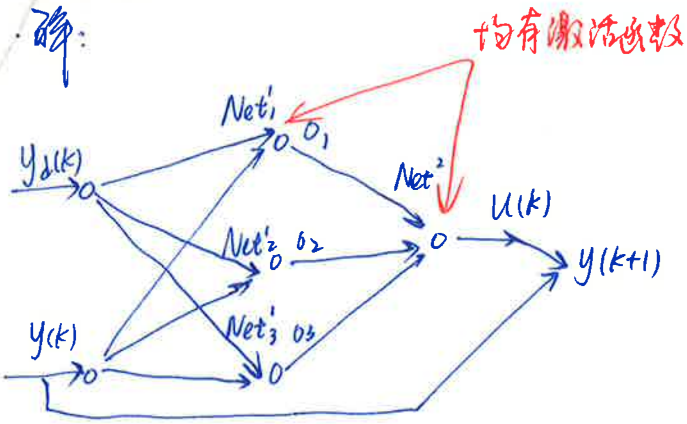
$$= -0.6 \quad (最小)$$

即 $y_1y_2y_3 = 011$ 为此神经网络能量井

8. 遗传算法的步骤:

① 群体的初始化.
② 评价群体中每一个体的性能
③ 选择下一代个体.
④ 执行简单的操作算子(如交叉、变异).
⑤ 评价下一代群体的性能.
⑥ 判断终止条件满足否? 若不, 则转③; 若满足, 则结束.

解:



~~(1) $y_d(0)=0$, $y(0)=0 \Rightarrow u(0)=0 \Rightarrow y(1)=0$~~ $y_d(1)$

目标函数 $J(e(k)) = \frac{1}{2}e^2(k)$

$e(k) = y_d(k) - y(k)$

~~对于输出层到隐藏层~~

(1) $\left.\begin{matrix} y_d(0)=0 \\ y(0)=0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} u(0)=0 \\ y(0)=0 \end{matrix}\right\} \Rightarrow y(1)=0$

$y_d(1) = \sin\frac{2\pi}{25} + \sin\frac{2\pi}{10} = \cancel{0.0154}$ 0.8365

$Net'_1(1) = -1.7502 \cdot y_d(1) = -0.0270$

$Net'_2(1) = -0.0128$

$Net'_3(1) = -0.0178$

$O_1(1) = \sigma(Net'_1(1)) = -0.0270$

$O_2(1) = \sigma(Net'_2(1)) = -0.0128$

$O_3(1) = \sigma(Net'_3(1)) = -0.0178$

$u(1) = -2.0026 \cdot O_1(1) + 0.9642 \cdot O_2(1) + 0.5201 O_3(1)$

$= 0.0325$

$y(2) = u^3(1) = 3.433 \times 10^{-5}$

$y_d(2) = 0.0307$

对于~~隐含~~输出层到隐藏层，有 $\frac{\partial u}{\partial Net^2}\frac{\partial Net^2}{\partial w_{11}}$

$w_{11}(2) = w_{11}(1) - \eta \frac{\partial J}{\partial e}\frac{\partial e}{\partial y}\frac{\partial y}{\partial u}\frac{\partial u}{\partial w_{11}}$ O_1 $-(y_d(k)-y(k))$ $(0.0307-3.433\times10^{-5})$

~~$w_{12}(2)=$~~ $= -2.0026 + 0.01 \times (0.0307 - 3.433\times10^{-5}) \times (0.0307-3.433\times10^{-5}) / 0.0325 \cdot O_1$

~~$w_{13}(2)=$~~ $J = \frac{1}{2}e^2(k)$ ~~$e(k) =$~~ $y_d(k) - y(k)$

$= -2.0026$

$w_{12}(2) = 0.9642 + 0.01 \times (0.0307 - 3.433\times10^{-5})^2 / 0.0325 \times O_2$

$= 0.9642$

$w_{13}(2) = 0.5201$

对于隐藏层到输入层 $\frac{\partial u}{\partial Net^2}\frac{\partial Net^2}{\partial O_1}$

$w_{11}(2) = w_{11}(1) - \eta \frac{\partial J}{\partial e}\frac{\partial e}{\partial y}\frac{\partial y}{\partial u}\frac{\partial u}{\partial O_1}\frac{\partial O_1}{\partial Net'_1}\frac{\partial Net'_1}{\partial w_{11}}$ y_d w_{11}

$= -1.7502 - 0.01 \times (0.0307 - 3.433\times10^{-5})^2 / 0.0325 \times (-2.0026)$ $\times(-2.0026)(1+2.0026)$ $1-(-0.027)^2$ $y_d(1)$ $\frac{\partial O_1}{\partial Net'_1}$

$= -1.7502$

$w_{12}(2) = -0.2857$

$w_{21}(2) = -0.8314$

$w_{22}(2) = -0.9792$

$w_{31}(2) = -1.1564$

$w_{32}(2) = -0.5336$

(2) $k=2$ 时,

~~$Net'_1(2) = -1.7502 y_d(2) = -0.0537$~~

~~$Net'_2(2) =$~~

$Net'_1(2) = -1.7502\, y_d(2) - 0.2857\, y(2) = -0.0537$

$Net'_2(2) = -0.0255$

$Net'_3(2) = -0.0355$

$O_1(2) = \sigma(Net'_1(2)) = -0.0536$

$O_2(2) = -0.0255$

$O_3(2) = -0.0355$

$\Rightarrow u(2) = -2.0026 \cdot O_1(2) + 0.9642\, O_2(2) + 0.5201\, O_3(2)$

$= 0.0643$

1、

$$\text{“很年轻”}(e)=\mu_W(e)=\begin{cases}1, & 0\le e\le 25\\ \left[1+\left(\frac{e-25}{5}\right)^2\right]^{-2}, & 25<e\le 200\end{cases}$$

$$\text{“不年老”}(e)=\mu_{\bar{O}}(e)=1-\mu_O(e)=\begin{cases}1, & 0\le e\le 50\\ 1-\left[1+\left(\frac{e-50}{5}\right)^{-2}\right]^{-1}, & 50<e\le 200\end{cases}$$

$$\text{“不年轻”}(e)=\mu_{\bar{Y}}(e)=1-\mu_Y(e)=\begin{cases}0, & 0\le e\le 25\\ 1-\left[1+\left(\frac{e-25}{5}\right)^2\right]^{-1}, & 25<e\le 200\end{cases}$$

$$\text{“不年老也不年轻”}(e)=\mu_V(e)=\mu_{\bar{O}\wedge\bar{Y}}(e)$$

$$=\min\{\mu_{\bar{O}}(e),\mu_{\bar{Y}}(e)\}$$

$$=\begin{cases}0, & 0\le e\le 25\\ 1-\left[1+\left(\frac{e-25}{5}\right)^2\right]^{-1}, & 25<e\le \frac{75+\sqrt{725}}{2}\\ \cancel{\min\left\{1-\left[1+\left(\frac{e-50}{5}\right)^{-2}\right]^{-1},1-\left[1+\left(\frac{e-25}{5}\right)^2\right]^{-1}\right\}}, & \cancel{50<e\le 200}\\ 1-\left[1+\left(\frac{e-50}{5}\right)^{-2}\right]^{-1}, & \frac{75+\sqrt{725}}{2}<e\le 200\end{cases}$$

2、

(1)

$$\mu_{ZE}(e)\wedge\mu_{PS}(e)=\frac{0}{-30}+\frac{0}{-20}+\frac{0}{-10}+\frac{0.3}{0}+\frac{0.4}{10}+\frac{0}{20}+\frac{0}{30}$$

(2)

$$\mu_{ZE}(e)\vee\mu_{PS}(e)=\frac{0}{-30}+\frac{0}{-20}+\frac{0.4}{-10}+\frac{1}{0}+\frac{1}{10}+\frac{0.3}{20}+\frac{0}{30}$$

3、

(1)

$$P\circ Q=\begin{bmatrix}0.6 & 0.9\\0.2 & 0.7\end{bmatrix}\circ\begin{bmatrix}0.5 & 0.7\\0.1 & 0.4\end{bmatrix}$$

$$=\begin{bmatrix}0.5\vee0.1 & 0.6\vee0.4\\0.2\vee0.1 & 0.2\vee0.4\end{bmatrix}=\begin{bmatrix}0.5 & 0.6\\0.2 & 0.4\end{bmatrix}$$

$$\Rightarrow (P\circ Q)\circ R=\begin{bmatrix}0.5 & 0.6\\0.2 & 0.4\end{bmatrix}\circ\begin{bmatrix}0.2 & 0.3\\0.7 & 0.7\end{bmatrix}=\begin{bmatrix}0.6 & 0.6\\0.4 & 0.4\end{bmatrix}$$

(2)

$$P\vee Q=\begin{bmatrix}0.6 & 0.9\\0.2 & 0.7\end{bmatrix}$$

$$\Rightarrow (P\vee Q)\circ S=\begin{bmatrix}0.6 & 0.9\\0.2 & 0.7\end{bmatrix}\circ\begin{bmatrix}0.1 & 0.2\\0.6 & 0.5\end{bmatrix}=\begin{bmatrix}0.6 & 0.5\\0.6 & 0.5\end{bmatrix}$$

(3)

$$P\circ S=\begin{bmatrix}0.6 & 0.5\\0.6 & 0.5\end{bmatrix}$$

$$Q\circ S=\begin{bmatrix}0.6 & 0.5\\0.4 & 0.4\end{bmatrix}$$

$$\Rightarrow (P\circ S)\vee(Q\circ S)=\begin{bmatrix}0.6 & 0.5\\0.6 & 0.5\end{bmatrix}$$

4.

$A:\ \mu_{小}(X)=[1, 0.7, 0.3, 0, 0]$

$B:\ \mu_{大}(y)=[0, 0, 0.4, 0.7, 1]$

$A':\ \mu_{较小}(X)=[1, 0.6, 0.4, 0.2, 0]$

$B'=A'\circ(A\to B)$

由Mamdani推理法有

$B'=A'\circ(A\times B)$

$$=\begin{bmatrix}1 & 0.6 & 0.4 & 0.2 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & 0.4 & 0.7 & 1\\0 & 0 & 0.4 & 0.7 & 0.7\\0 & 0 & 0.3 & 0.3 & 0.3\\0 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0.4 & 0.7 & 1\end{bmatrix}$$

∴当X较小时，y是B'，其隶属度函数为

$$\mu_{B'}(y)=\frac{0}{1}+\frac{0}{2}+\frac{0.4}{3}+\frac{0.7}{4}+\frac{1}{5}$$

$[\text{非常重}] = B^3 = \frac{0.008}{v_1} + \frac{0.064}{v_2} + \frac{0.216}{v_3} + \frac{0.512}{v_4} + \frac{1}{v_5}$

$C = [\text{不非常重}] = 1 - B^3 = \frac{0.992}{v_1} + \frac{0.936}{v_2} + \frac{0.784}{v_3} + \frac{0.488}{v_4} + \frac{0}{v_5}$

$R = (A \to B) \vee (\bar{A} \to C) = (A \times B) \vee (\bar{A} \times C)$

$$= \begin{bmatrix} 0.2 & 0.4 & 0.6 & 0.8 & 1 \\ 0.2 & 0.4 & 0.6 & 0.8 & 0.8 \\ 0.2 & 0.4 & 0.6 & 0.6 & 0.6 \\ 0.2 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0 \\ 0.6 & 0.6 & 0.6 & 0.488 & 0 \\ 0.8 & 0.8 & 0.784 & 0.488 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0.4 & 0.6 & 0.8 & 1 \\ 0.2 & 0.4 & 0.6 & 0.8 & 0.8 \\ 0.4 & 0.4 & 0.6 & 0.6 & 0.6 \\ 0.6 & 0.6 & 0.6 & 0.488 & 0.4 \\ 0.8 & 0.8 & 0.784 & 0.488 & 0.2 \end{bmatrix}$$

$A'_1 = [\text{非常轻}] = \frac{1}{u_1} + \frac{0.512}{u_2} + \frac{0.216}{u_3} + \frac{0.064}{u_4} + \frac{0.008}{u_5}$

$A'_2 = [\text{非常重}] = \frac{0.008}{u_1} + \frac{0.064}{u_2} + \frac{0.216}{u_3} + \frac{0.512}{u_4} + \frac{1}{u_5}$

$\cancel{A' = A^3 \vee B^3 = \frac{1}{u_1} + \frac{0.512}{u_2} + \frac{0.216}{u_3} + \frac{0.512}{u_4} + \frac{1}{u_5}}$

$\cancel{D = A' \circ R}$

$= [0.8 \ \ 0.8 \ \ 0.784 \ \ 0.8 \ \ 1]$

$D_1 = A_1' \circ R = (0.216, 0.4, 0.6, 0.8, 1)$

$D_2 = A_2' \circ R = (0.8, 0.8, 0.784, 0.488, 0.4)$

∴ 当x为非常轻或非常重下y为 $D = \frac{0.8}{v_1} + \frac{0.8}{v_2} + \frac{0.784}{v_3} + \frac{0.8}{v_4} + \frac{1}{v_5}$

$D = D_1 \vee D_2 = (0.8, 0.8, 0.784, 0.8, 1)$

6. (1)

$D = A \times B = \begin{bmatrix} 0.1 & 0.5 & 0.5 \\ 0.1 & 1 & 0.6 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}$

$DT = \begin{bmatrix} 0.1 \\ 0.5 \\ 0.5 \\ 0.1 \\ 1 \\ 0.6 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$

$R = DT \times C$

$$= \begin{bmatrix} 0.1 & 0.1 \\ 0.4 & 0.5 \\ 0.4 & 0.5 \\ 0.1 & 0.1 \\ 0.4 & 1 \\ 0.4 & 0.6 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

(2)

$D' = A' \times B' = \begin{bmatrix} 0.1 & 0.5 & 1 \\ 0.1 & 0.5 & 0.5 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}$

$DT' = \begin{bmatrix} 0.1 \\ 0.5 \\ 1 \\ 0.1 \\ 0.5 \\ 0.5 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$

$C' = (DT')^T \cdot R$

$= [0.4 \ \ 0.5]$

7. 使用削顶法

$A \wedge A' = [0.5 \ \ 0.5 \ \ 0.1]$

$\alpha_A = \vee(A \wedge A') = 0.5$

$B \wedge B' = [0.1 \ \ 0.5 \ \ 0.6]$

$\alpha_B = \vee(B \wedge B') = 0.6$

$C' = (\alpha_A \wedge \alpha_B) \wedge C = 0.5 \wedge [0.4 \ \ 1]$

$= [0.4 \ \ 0.5]$

8. ① 基于矩阵运算

$D_1 = A_1 \times B_1 = \begin{bmatrix} 1 & 0.6 & 0.2 \\ 0.5 & 0.5 & 0.2 \\ 0 & 0 & 0 \end{bmatrix}$

$DT_1 = \begin{bmatrix} 1 \\ 0.6 \\ 0.2 \\ 0.5 \\ 0.5 \\ 0.2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$R_1 = DT_1 \times C_1 = \begin{bmatrix} 1 & 0.4 & 0 \\ 0.6 & 0.4 & 0 \\ 0.2 & 0.2 & 0 \\ 0.5 & 0.4 & 0 \\ 0.5 & 0.4 & 0 \\ 0.2 & 0.2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$D' = A' \times B' = \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.1 & 0.1 & 0.1 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}$

$DT' = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$

$C_1' = (DT')^T \circ R_1 = [0.5 \ \ 0.4 \ \ 0]$

$$A_2 \times B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0.2 & 0.5 & 0.5 \\ 0.2 & 0.6 & 1 \end{bmatrix}$$

$$DT_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \\ 0.5 \\ 0.5 \\ 0.2 \\ 0.6 \\ 1 \end{bmatrix} \qquad R_2 = DT_2 \times C_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.2 & 0.2 \\ 0 & 0.4 & 0.5 \\ 0 & 0.4 & 0.5 \\ 0 & 0.2 & 0.2 \\ 0 & 0.4 & 0.6 \\ 0 & 0.4 & 1 \end{bmatrix}$$

$$C_2' = (DT')^T \circ R_2 = [0 \quad 0.4 \quad 0.5]$$

$$\Rightarrow C' = C_1' \vee C_2' = [0.5 \quad 0.4 \quad 0.5]$$

即推理结论之为 $C' = \frac{0.5}{c_1} + \frac{0.4}{c_2} + \frac{0.5}{c_3}$

② 基于削顶法

$$A_1 \wedge A' = [0.5 \quad 0.1 \quad 0]$$

$$\alpha_{A_1} = \vee (A_1 \wedge A') = 0.5$$

$$B_1 \wedge B' = [0.6 \quad 0.6 \quad 0.2]$$

$$\alpha_{B_1} = \vee (B_1 \wedge B') = 0.6$$

$$C_1' = (\alpha_{A_1} \wedge \alpha_{B_1}) \wedge C_1 = [0.5 \quad 0.4 \quad 0]$$

$$A_2 \wedge A' = [0 \quad 0.1 \quad 0.5]$$

$$\alpha_{A_2} = 0.5$$

$$B_2 \wedge B' = [0.2 \quad 0.6 \quad 0.6]$$

$$\alpha_{B_2} = 0.6$$

$$C_2' = (\alpha_{A_2} \wedge \alpha_{B_2}) \wedge C_2 = [0 \quad 0.4 \quad 0.5]$$

$$\Rightarrow C' = C_1' \wedge C_2' = [0.5 \quad 0.4 \quad 0.5]$$

即之为 $C' = \frac{0.5}{c_1} + \frac{0.4}{c_2} + \frac{0.5}{c_3}$

9. 将方程按合成运算展开

$$(0.6 \wedge r_1) \vee (0.2 \wedge r_2) \vee (0.4 \wedge r_3) = 0.4$$

$$[r_1] = 0.4 \quad [r_2] = \phi \quad [r_3] = [0.4, 1]$$

$$(r_1) = [0, 0.4] \quad (r_2) = [0, 1] \quad (r_3) = [0, 1]$$

$$\left.\begin{aligned} R_1 &= ([r_1], (r_2), (r_3)) \\ R_2 &= ((r_1), [r_2], (r_3)) = \phi \\ R_3 &= ((r_1), (r_2), [r_3]) \end{aligned}\right\}$$

$$\Rightarrow R = R_1 \cup R_3$$

$$= (0.4, [0, 1], [0, 1]) \cup ([0, 0.4], [0, 1], [0.4, 1])$$