

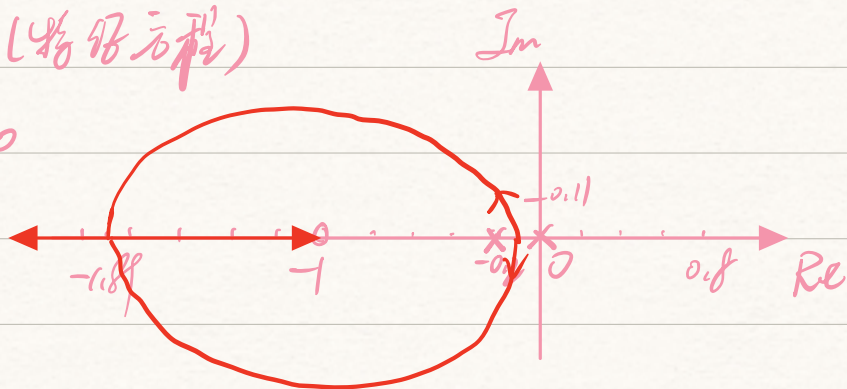
1. $G(s) = \frac{k(s+1)}{s(s+0.2)}$ 单位负反馈

CE: $1 + G(s) = 0$ (特征方程)

$\Rightarrow 1 + k \frac{s+1}{s(s+0.2)} = 0$

$p_1 = 0, p_2 = -0.2$

$z = -1$



13) 渐近线: $\sigma_a = \frac{\sum p_i - \sum z_i}{n-m} = 0.8$

$\varphi_2 = \frac{(2k+1)\pi}{n-m} = \pi$

(5) $\sum_{i=1}^n \frac{1}{s - p_i} = \sum_{j=1}^m \frac{1}{s - z_j}$

$\frac{1}{s} + \frac{1}{s+0.2} = \frac{1}{s+1}$

$s = -1 \pm 0.8j$

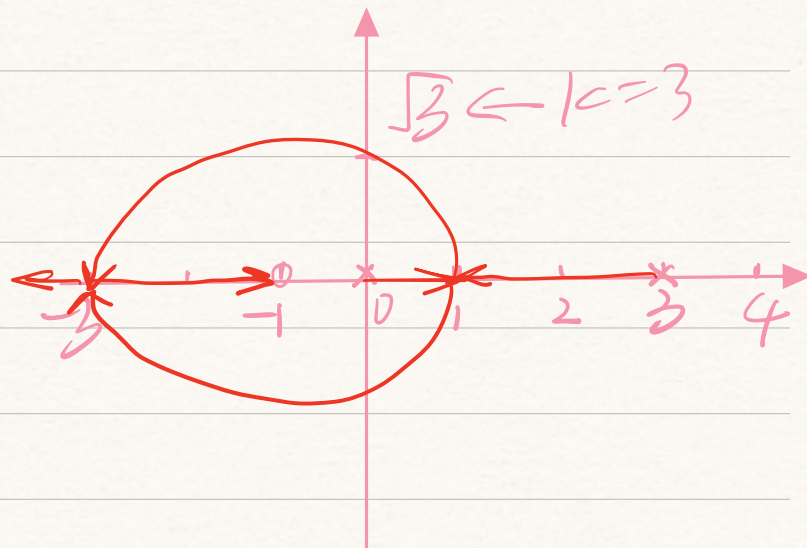
$s = -0.11$ 或 -1.89 即两点均在根轨迹上

2. $G(s) = \frac{k(s+1)}{s(s-3)}$

$p_1 = 0, p_2 = 3$

$z_1 = -1$

2) 2条



$$(3) \quad \varphi_\alpha = \frac{(2k+1)\pi}{n-m} = \pi$$

$$\sigma_\alpha = \frac{3 - (-1)}{1} = 4$$

$$(5) \quad \frac{1}{a-0} + \frac{1}{a-3} = \frac{1}{a+1}$$

$$a = -3, 1$$

间隔为 $\frac{\pi}{2}$

$$(7) \quad 1 + G(s) = 0$$

$$\text{证: } s^2 + (k-3)s + k = 0$$

二阶系统所有实数根 > 0

$$s^2 \quad 1 \quad k$$

$$s^1 \quad k-3 \quad 0$$

$$s^0 \quad k$$

$$k=3$$

$$s^2 + 3 = 0$$

$$s = \pm \sqrt{3}j$$

$$\text{证: } \text{令 } s = j\omega$$

$$-\omega^2 - 3j\omega + k + k = 0$$

$$\begin{cases} -\omega^2 + k = 0 \\ -3\omega + k\omega = 0 \end{cases} \Rightarrow \begin{cases} k=3 \\ \omega = \sqrt{3} \end{cases}$$

$$3. G(s) = \frac{K}{s^4 + 128s^3 + 64s^2 + 128s}$$

$$= \frac{K}{s(s+4)(s+4+j)(s+4-j)}$$

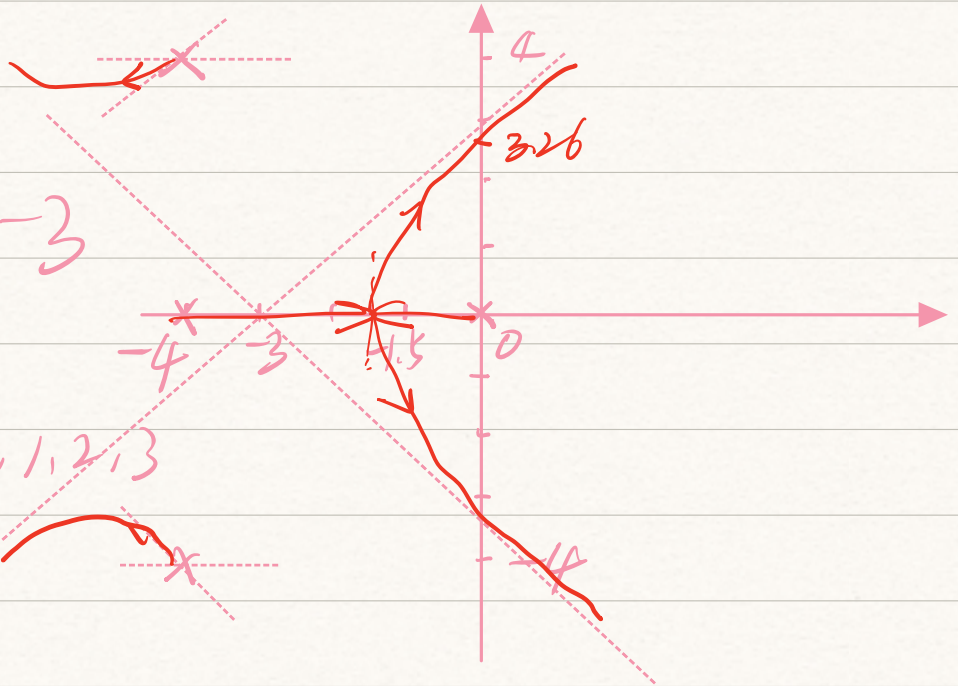
$$p_1=0, p_2=-4, p_3=-4-4j, p_4=-4+4j$$

2 roots

$$(3) \sigma_a = \frac{-12}{4} = -3$$

$$\varphi_a = \frac{2k+1}{4} \pi, k=0,1,2,3$$

$$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$



$$(5) s^4 + 12s^3 + 64s^2 + 128s + K = 0$$

$$4s^3 + 26s^2 + 128s + 128 = 0$$

$$(6) 0 - \left(\frac{3\pi}{4} + \frac{\pi}{2} + \frac{\pi}{2} + \theta \right) = (2k+1)\pi$$

$$\theta = -\frac{7}{4}\pi - (2k+1)\pi$$

$$= -\frac{3}{4}\pi$$

$$s^4 + 12s^3 + 64s^2 + 128s + k = 0$$

$$s^4 \quad 1 \quad 64 \quad k$$

$$s^3 \quad 12 \quad 128 \quad 0$$

$$s^2 \quad \frac{160}{3} \quad k$$

$$s \quad 128 - \frac{36k}{160} \quad 0$$

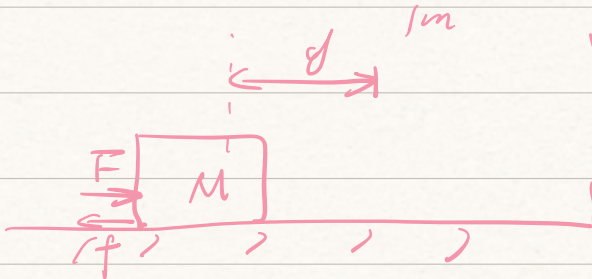
$$s^0 \quad k$$

$$128 - \frac{36k}{160} = 0 \Rightarrow k =$$

$$\frac{160}{3} s^2 + k = 0$$

$$s = \pm \sqrt{\frac{3k}{160}} j$$

4.



$$f = B\dot{x} = B \cdot \dot{y}$$

$$M = 1 \text{ kg} \quad B = 1$$

$$M\ddot{a} = F - f = F - B\dot{y}$$

$$M\ddot{y} = F(t) - B\dot{y}$$

$$M\ddot{y} + B\dot{y} = F(t)$$

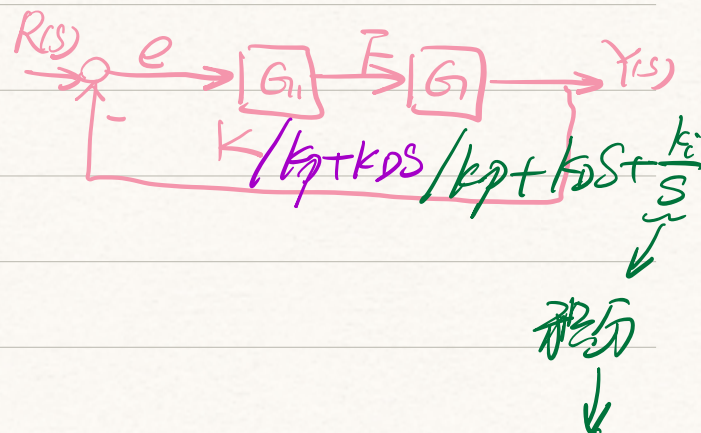
$$Ms^2 Y(s) + BS Y(s) = F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + BS}$$

$$G = \frac{1}{Ms^2 + BS}$$

$$G_1 = k$$

$$Y(s) \quad G_1 G$$



$$\frac{1}{R(s)} = \frac{1}{1+G_1 G_2}$$

$$G_1 G_2 = \frac{k}{Ms^2 + Bs} = \frac{k}{s(s+1)}$$

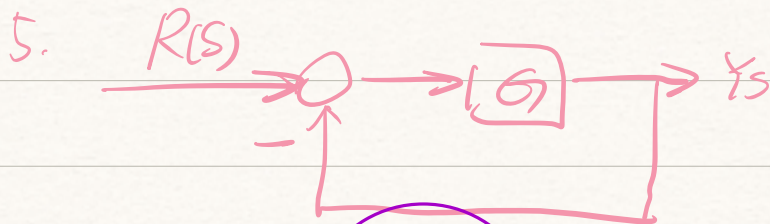
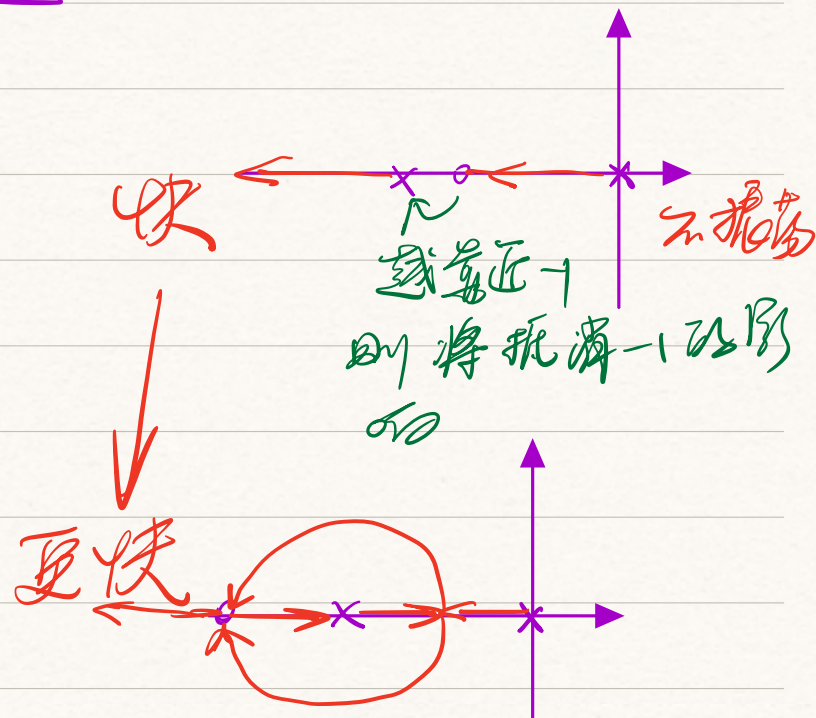
$$\frac{E(s)}{R(s)} = \frac{1}{1+G_1 G_2}$$

$$e(\infty) = \lim_{s \rightarrow 0} s \cdot \frac{1}{1+G_1 G_2} \cdot \frac{1}{s} = 0$$

稳态误差



$$G_1 G_2 = \frac{k_p + k_d s}{s(1+s)}$$



$$G = \frac{k}{s(s+1)}$$

$$1+G=0$$

只有根轨迹相同
交叉点用来分析
系统

$$s^2 + as + k = 0 \Rightarrow 1 + \frac{as}{s^2 + k} = 0$$

表 4-1 根轨迹图绘制法则

序号	内 容	法 则
1	根轨迹的起点和终点	根轨迹起于开环极点(包括无限极点),终于开环零点(包括无限零点)
2	根轨迹的分支数、对称性和连续性	根轨迹的分支数等于开环极点数 $n(n > m)$, 或开环零点 $m(m > n)$ 根轨迹对称于实轴
3	根轨迹的渐近线	$n-m$ 条渐近线与实轴的交角和交点为 $\varphi_a = \frac{(2k+1)\pi}{n-m} \quad (k=0, 1, \dots, n-m-1)$ $\sigma_a = \frac{\sum_{i=1}^n p_i - \sum_{j=1}^m z_j}{n-m}$
4	根轨迹在实轴上的分布	实轴上某一区域, 若其右方开环实数零、极点个数之和为奇数, 则该区域必是根轨迹
5	根轨迹的分离点和分离角	l 条根轨迹分支相遇, 其分离点坐标由 $\sum_{j=1}^m \frac{1}{d-z_j} = \sum_{i=1}^n \frac{1}{d-p_i}$ 确定; 分离角等于 $(2k+1)\pi/l$
6	根轨迹的起始角与终止角	起始角: $\theta_{p_i} = (2k+1)\pi + \left(\sum_{j=1}^m \varphi_{z_j p_i} - \sum_{\substack{j=1 \\ (j \neq i)}}^n \theta_{p_j p_i} \right)$ 终止角: $\varphi_{z_i} = (2k+1)\pi - \left(\sum_{\substack{j=1 \\ (j \neq i)}}^m \varphi_{z_j z_i} - \sum_{j=1}^n \theta_{p_j z_i} \right)$
7	根轨迹与虚轴的交点	根轨迹与虚轴交点的 K^* 值和 ω 值, 可利用劳斯判据确定
8	根之和	$\sum_{i=1}^n s_i = -\sum_{i=1}^n p_i$

有零极点再算

没有零极点时, 求特征方程的根

有 n 个根

(根之和)