吉林大学 JILIN UNIVERSITY

工程力学

第3章 任意力系

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3.1 任意力系的简化

3.2 任意力系的平衡

3.3 物体系统的平衡

3.1 任意力系的简化

一、任意力系的概念

二、力的平移定理

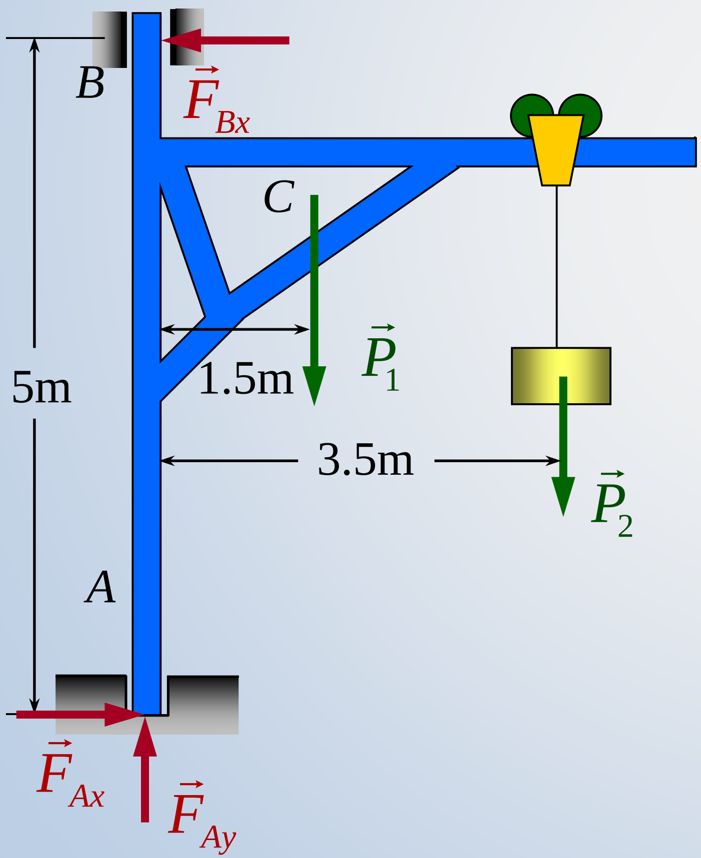
三、力系向一点简化

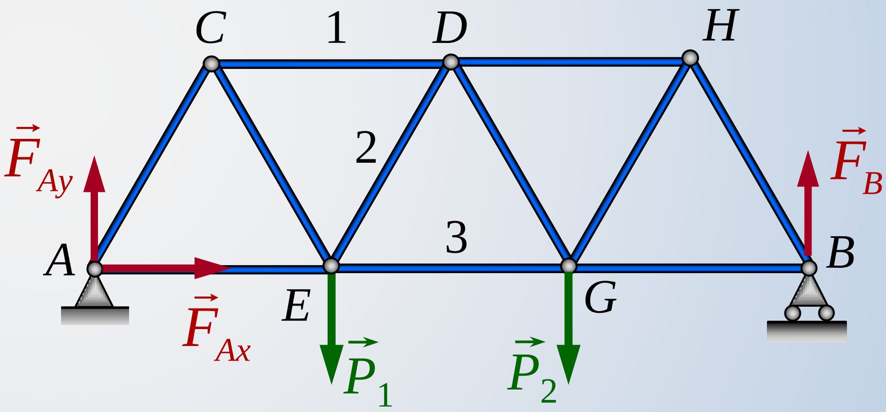
四、任意力系简化结果

3.1 任意力系的简化

一、任意力系的概念

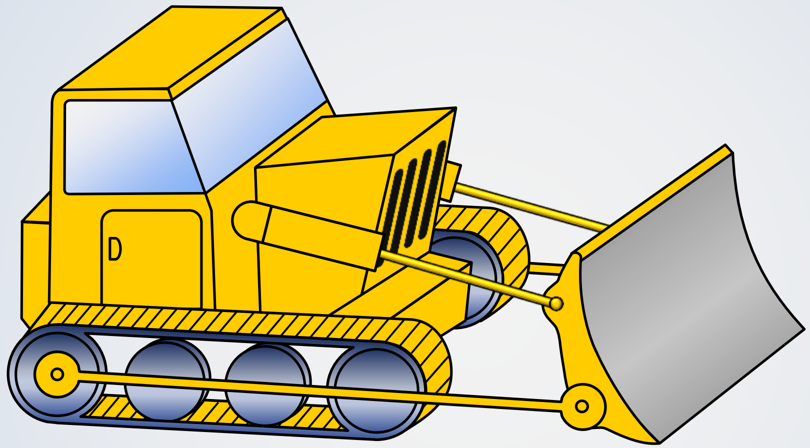
1. 平面任意力系：力线共面且任意分布。

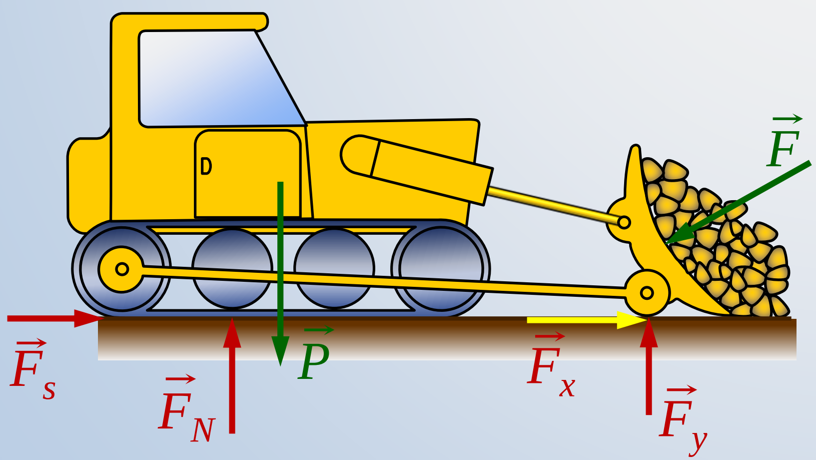


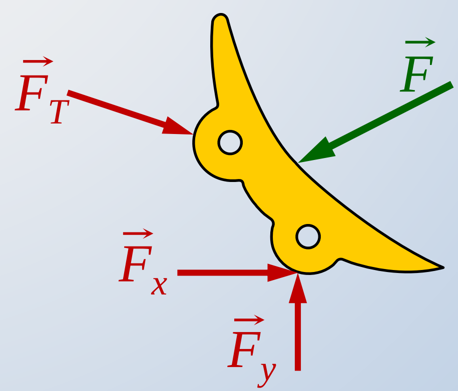


3.1 任意力系的简化

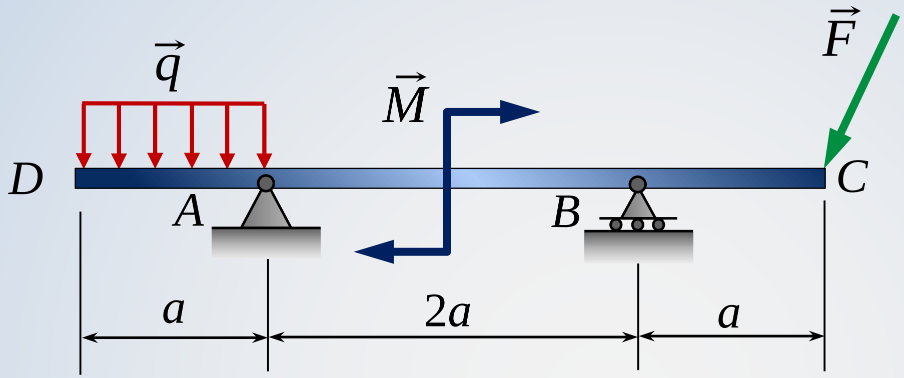
平面任意力系实例

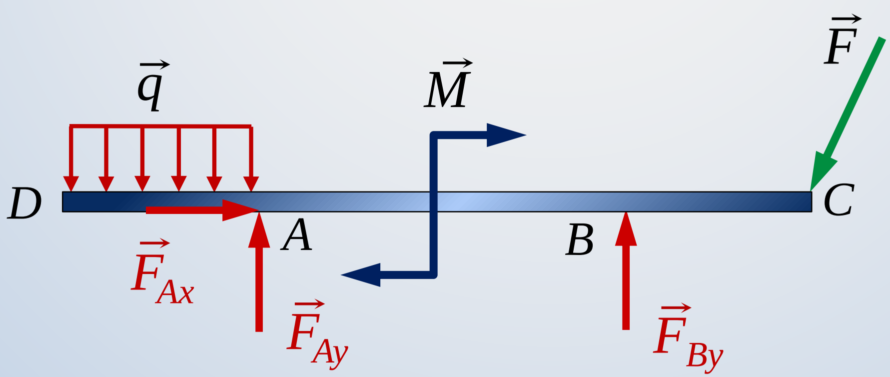






3.1 任意力系的简化

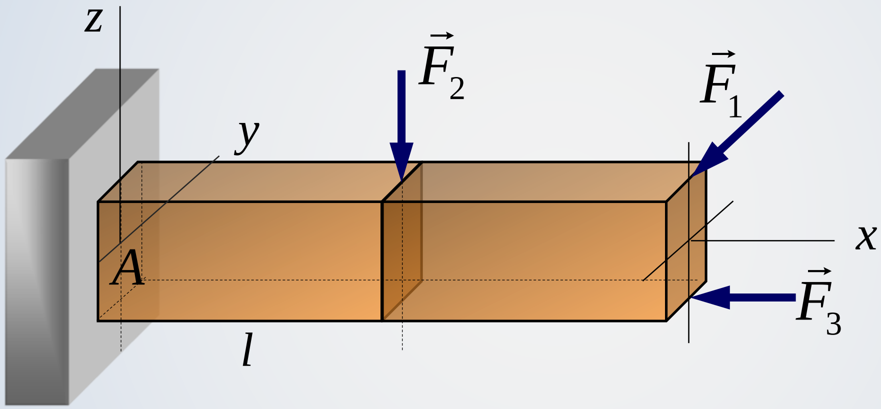


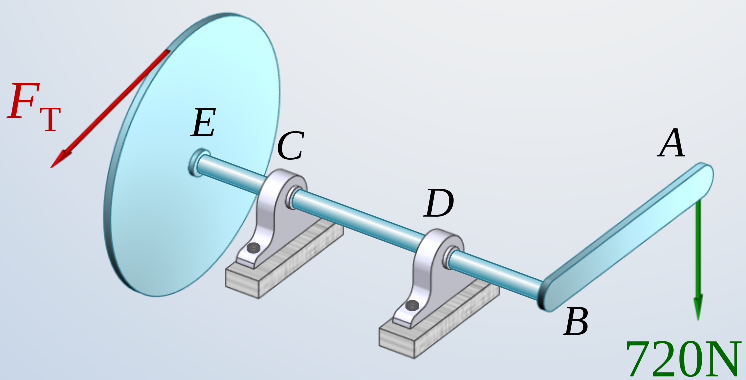


3.1 任意力系的简化

2. 空间任意力系

力线不共面且任意分布。

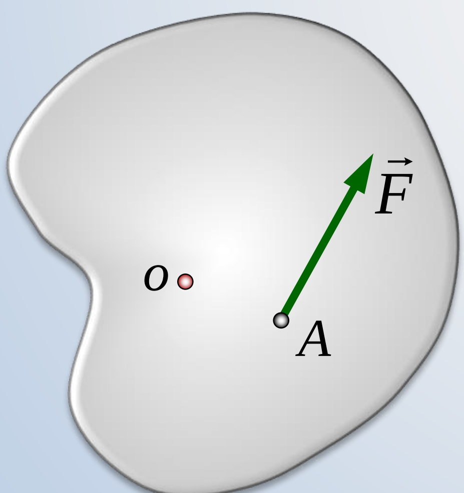




3.1 任意力系的简化

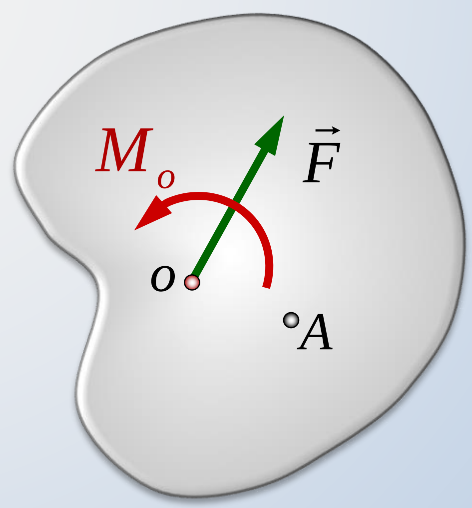
二、力的平移定理

作用在刚体上点A 的力F 可以平行移到任意一点o，但必须附加一个力偶，这个附加力偶的矩等于原来的力F对新作用点o的矩。

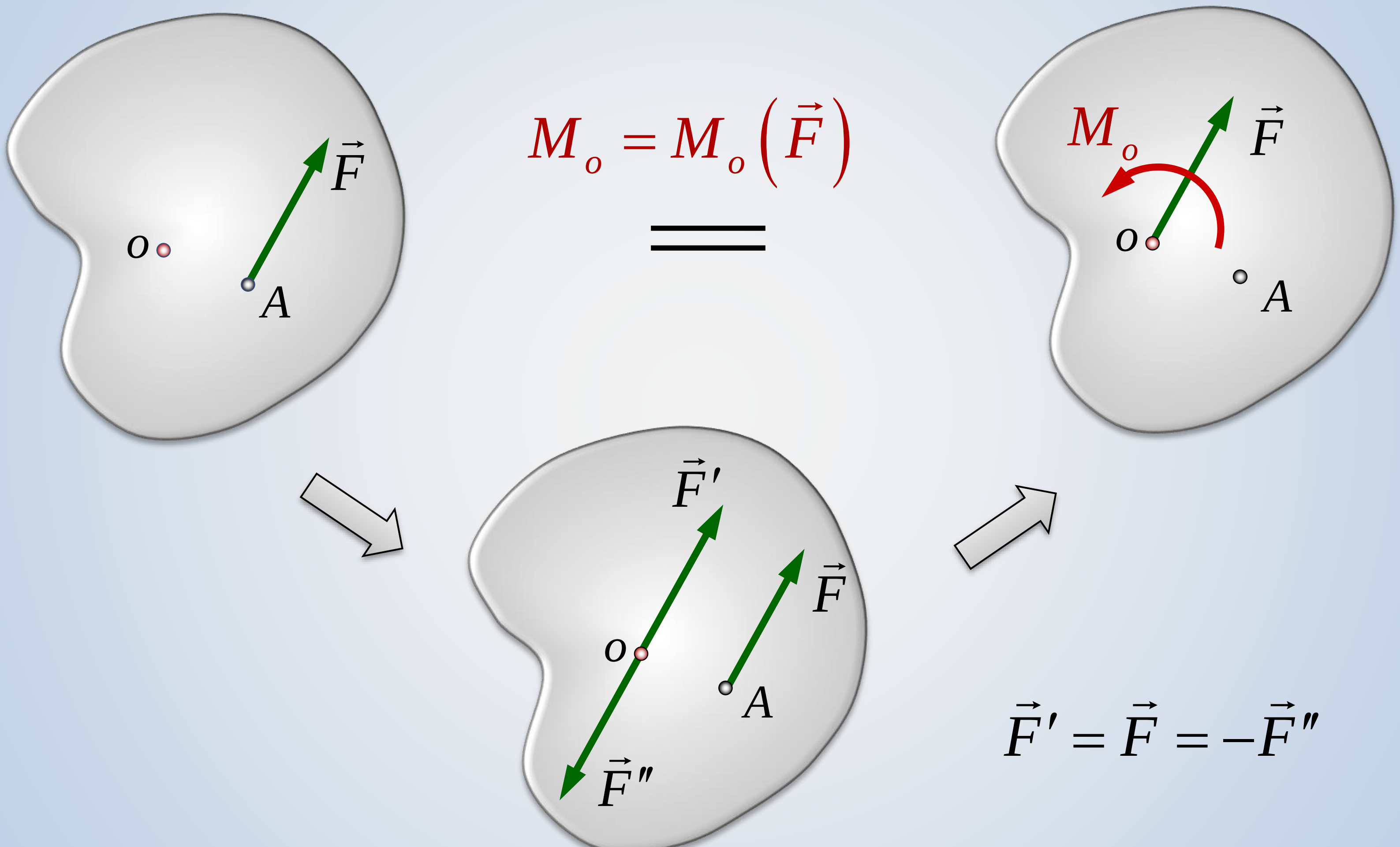


$$M_o = M_o(\vec{F})$$

$$=$$



3.1 任意力系的简化



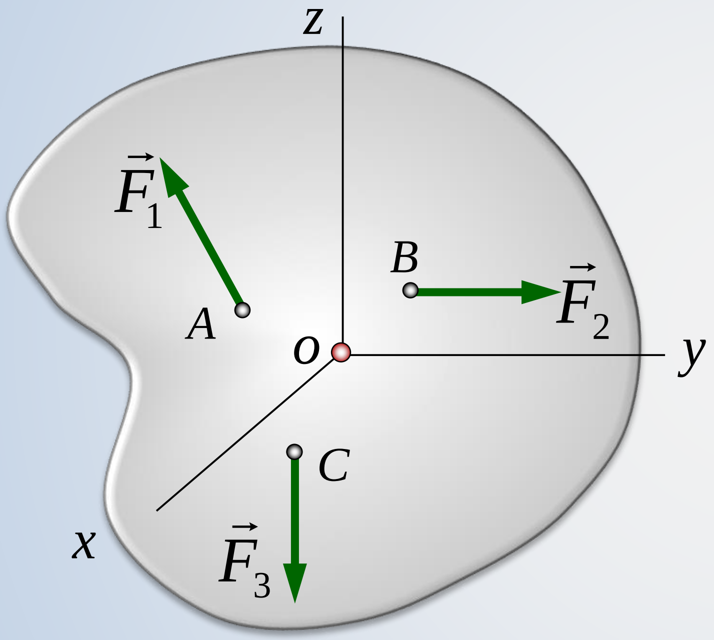
$$M_o = M_o\left(\vec{F}\right)$$

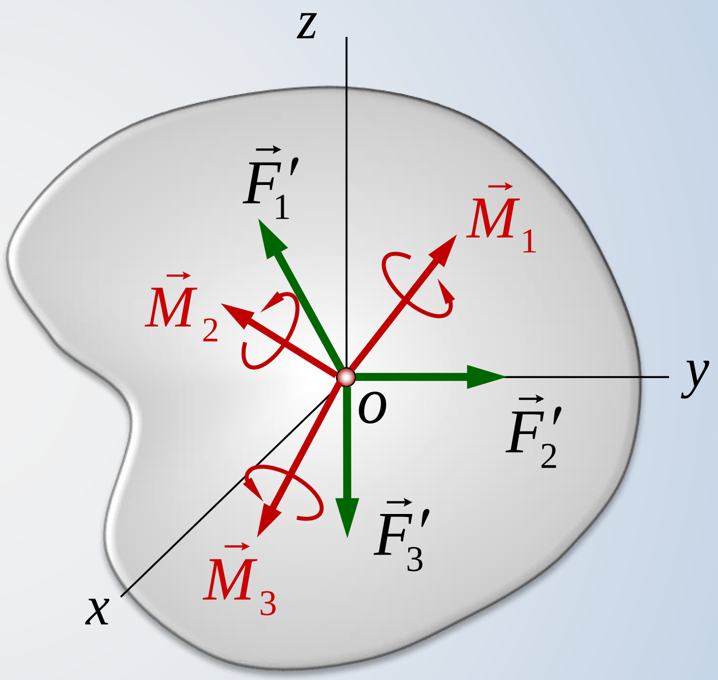
$$\vec{F}' = \vec{F} = -\vec{F}''$$

O —简化中心

3.1 任意力系的简化

三、力系向一点的简化

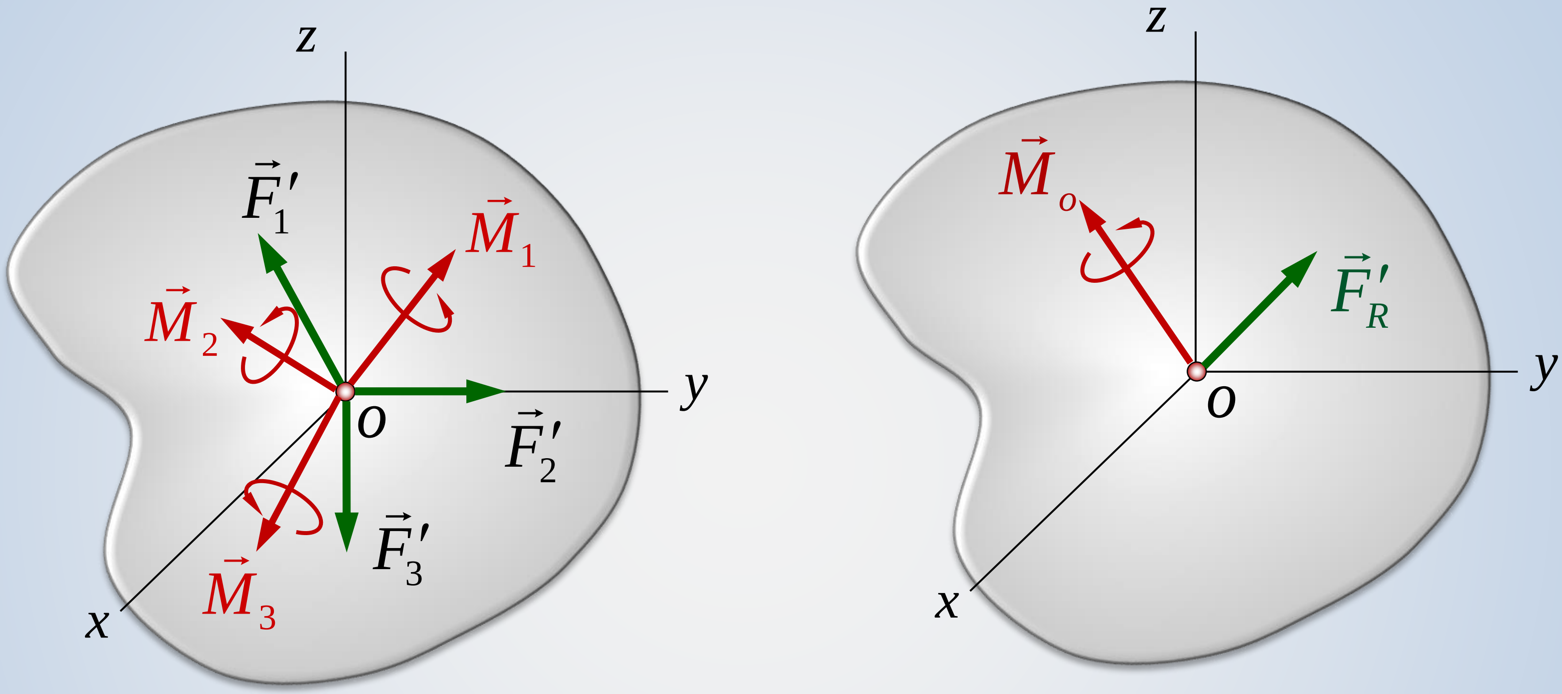




工具：力的平移定理

$$M_1 = M_o(\vec{F}_1) \quad M_2 = M_o(\vec{F}_2) \quad M_3 = M_o(\vec{F}_3)$$

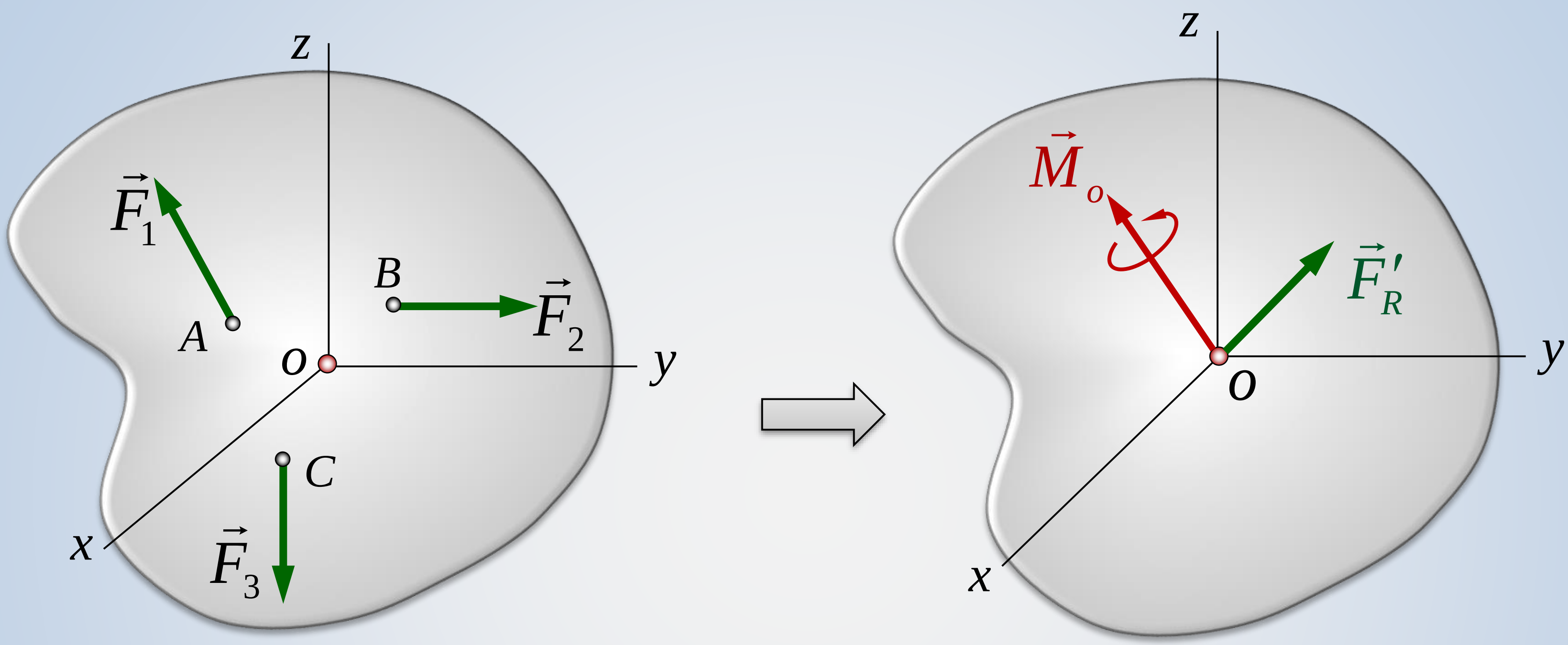
3.1 任意力系的简化



汇交力系 $\vec{F}_R' = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \sum \vec{F}_i$

力偶系 $\vec{M}_o = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 = \sum \vec{M}_o\left(\vec{F}_i\right)$

3.1 任意力系的简化



结论：简化得到一个力和一个力偶

$$\vec{F}'_R = \sum \vec{F}_i$$ 主矢

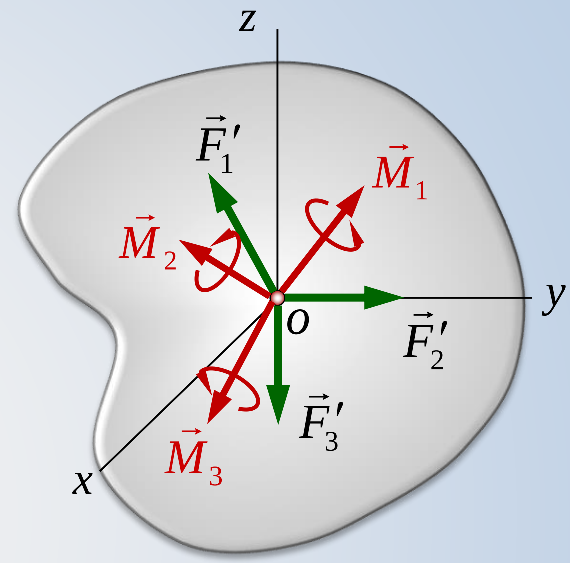
$$\vec{M}_o = \sum \vec{M}_o\left(\vec{F}_i\right)$$ 主矩

3.1 任意力系的简化

主矢：(Principal vector)

1. 所有力的大小和方向的总和。

$$\vec{F}_R' = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n = \sum \vec{F}_i$$

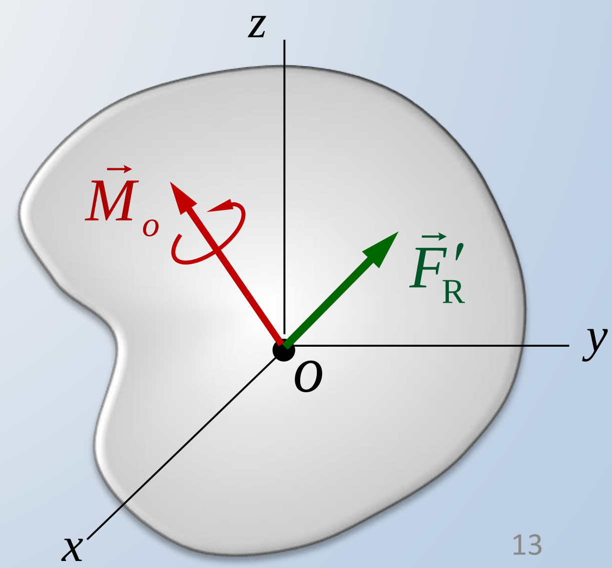


2. 主矢的计算

$$F_{Rx}' = \sum F_x \quad F_{Ry}' = \sum F_y \quad F_{Rz}' = \sum F_z$$

$$F_R' = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2 + \left(\sum F_z\right)^2}$$

$$\cos\alpha = \frac{F_{Rx}'}{F_R'}, \quad \cos\beta = \frac{F_{Ry}'}{F_R'}, \quad \cos\gamma = \frac{F_{Rz}'}{F_R'}$$



主矢的大小和方向与简化中心位置无关。

3.1 任意力系的简化

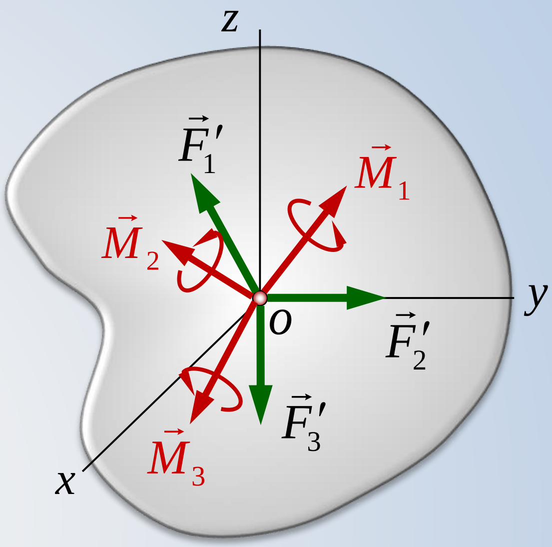
主矩： (Principal moment)

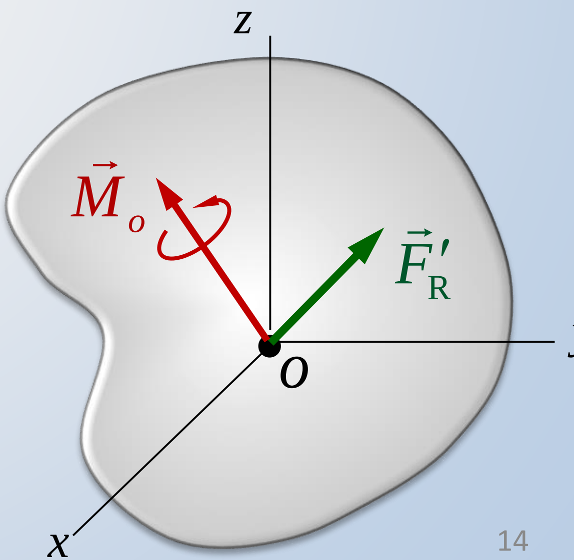
所有力对o点矩的矢量和。

$$\vec{M}_o = \vec{M}_o(\vec{F}_1) + \vec{M}_o(\vec{F}_2) + \cdots + \vec{M}_o(\vec{F}_n)$$

$$= \sum \vec{M}_o(\vec{F})$$

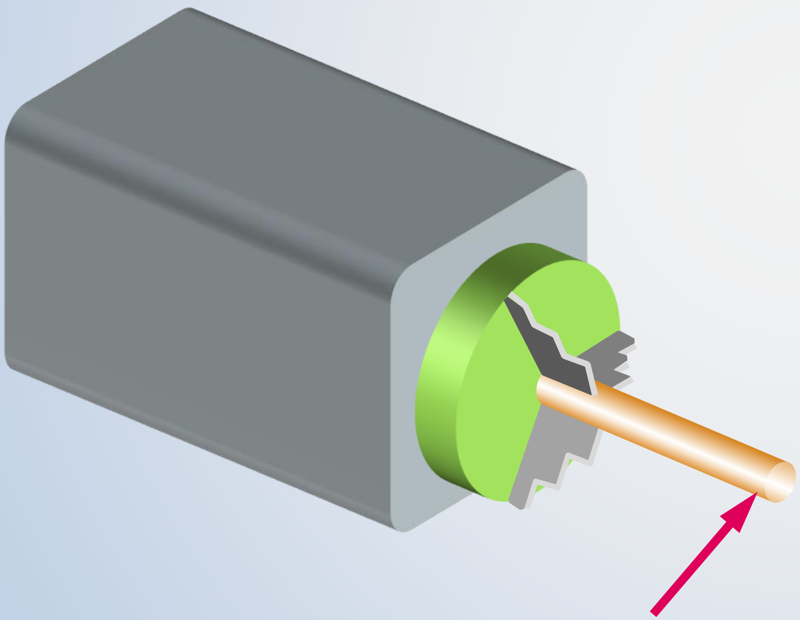
当 $\vec{F}_R' \neq 0$，主矩与简化中心位置有关

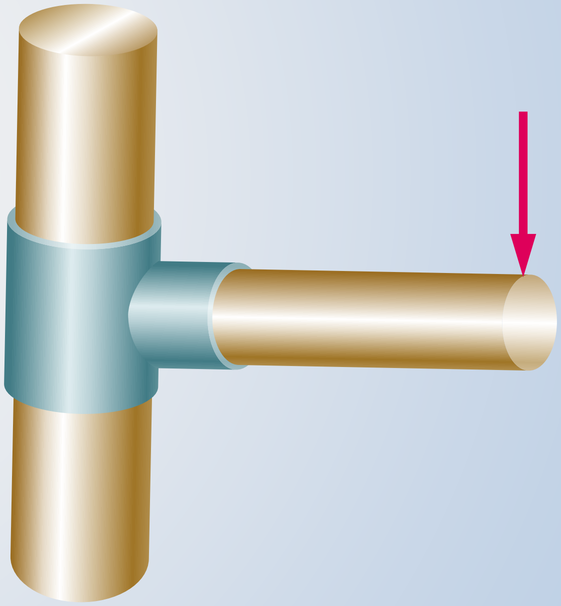




3.1 任意力系的简化

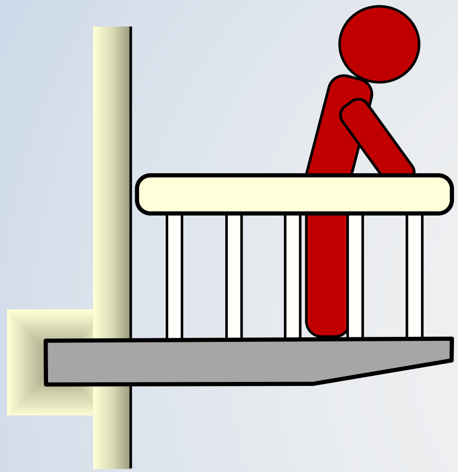
固定端约束　Built-in support

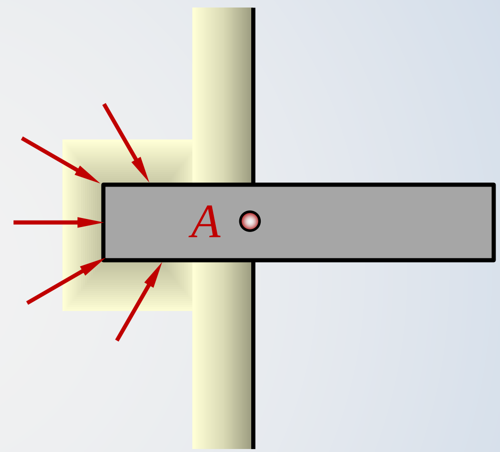


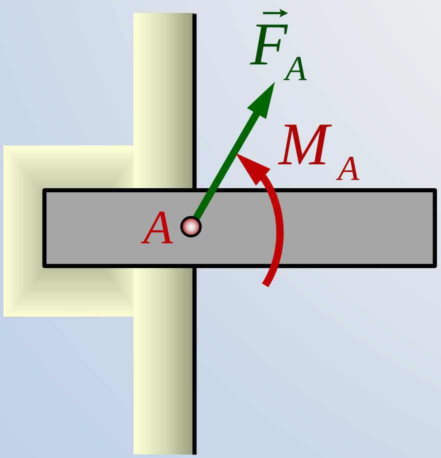


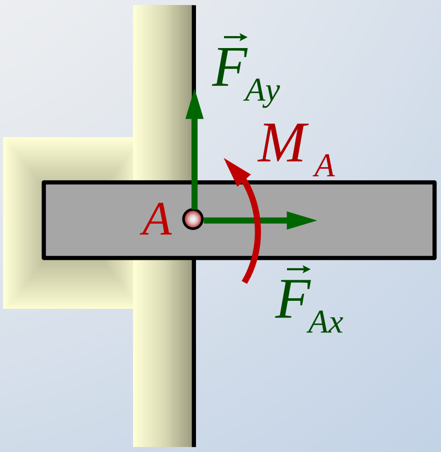
3.1 任意力系的简化

平面固定端约束









3.1 任意力系的简化

例1 已知：正方板受平面力系作用，边长$2a$

$$F_1 = F_2 = F_3 = F_4 = F,\ F_5 = \sqrt{2}F$$

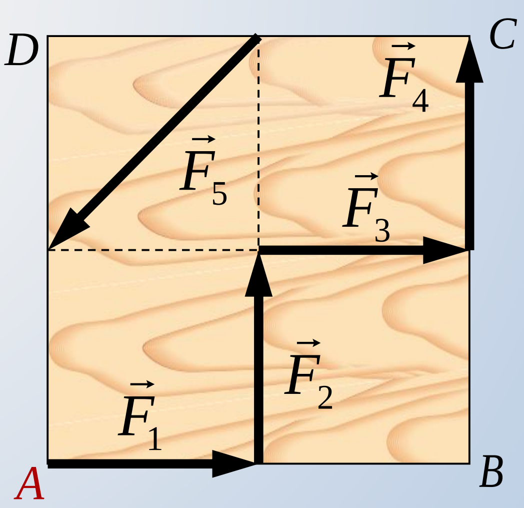
求： 1.力系向A 处简化, 主矢与主矩的大小。

2.力系简化的最后结果。

解：$F'_{Rx} = \sum F_x = F \qquad F'_{Ry} = \sum F_y = F$

$$F'_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{2}F$$

$$M_A = \sum m_A(\vec{F}_i) = 3Fa$$



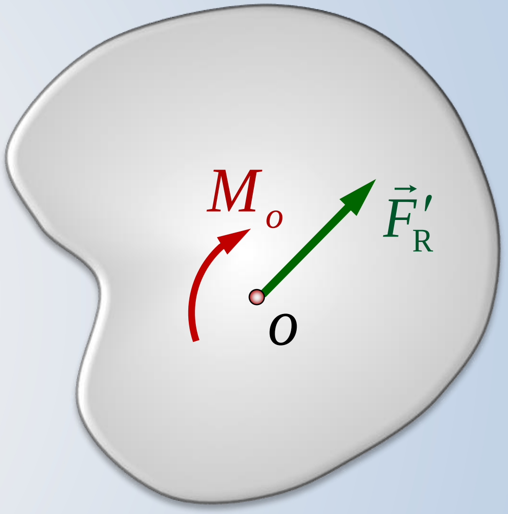
简化为合力：$R = \sqrt{2}F$

距离：$d = \dfrac{\sum m_A(\vec{F}_i)}{R} = \dfrac{3\sqrt{2}a}{2}$

四、任意力系简化结果

平面力系简化结果

主矢	主矩	简化结果
$\vec{F}_R' = 0$	$M_O = 0$	平　衡
$\vec{F}_R' = 0$	$M_O \neq 0$	合力偶
$\vec{F}_R' \neq 0$	$M_O = 0$	合力o
$\vec{F}_R' \neq 0$	$M_O \neq 0$	合力o'

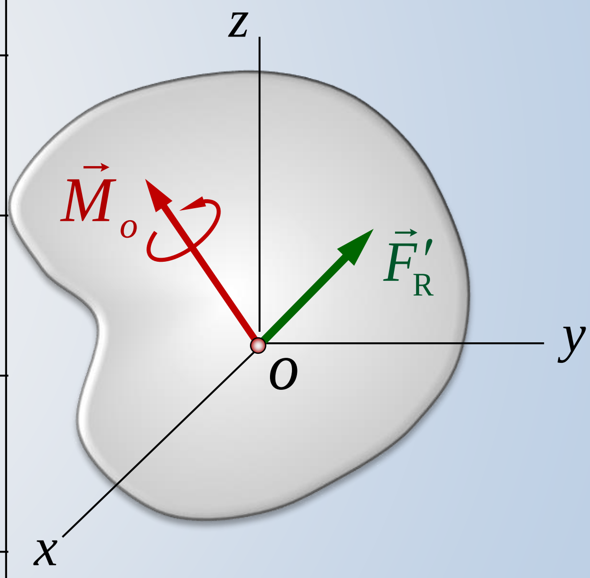


$$\vec{F}_R' \perp \vec{M}_O$$

3.1 任意力系的简化

空间力系简化结果

主矢	主矩	简化结果
$\vec{F}_R' = 0$	$\overleftarrow{M}_O = 0$	平衡
$\vec{F}_R' = 0$	$\vec{M}_O \neq 0$	合力偶
$\vec{F}_R' \neq 0$	$\vec{M}_O = 0$	合力 o
$\vec{F}_R' \neq 0$	$\vec{M}_O \neq 0$	力螺旋 o'



3.2 任意力系的平衡

一、平衡条件

二、平衡方程

三、平衡方程的不同形式

3.2 任意力系的平衡

一、平衡条件 Condition of Equilibrium

力系平衡 $\Longleftrightarrow$ $\begin{cases} \vec{F}_R' = 0 \\ \vec{M}_o = 0 \end{cases}$

主矢和主矩同时为零。

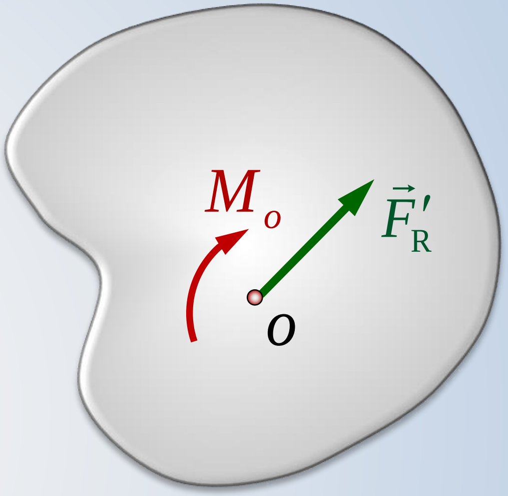
$$\begin{cases} \vec{F}_R' = \sum \vec{F}_i \\ \vec{M}_o = \sum M_o\left(\vec{F}_i\right) \end{cases} \Longleftrightarrow \begin{cases} \sum \vec{F}_i = 0 \\ \sum \vec{M}_o\left(\vec{F}_i\right) = 0 \end{cases}$$

二、平衡方程

1. 平面力系平衡方程

$$F_R' = \sum F_i = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = 0$$

$$M_o = \sum M_o(F_i) = 0$$



平衡方程基本形式

$$\begin{cases} \vec{F}_R' = \sum \vec{F}_i = 0 \\ \vec{M}_o = \sum \vec{M}_o\left(\vec{F}_i\right) = 0 \end{cases}$$

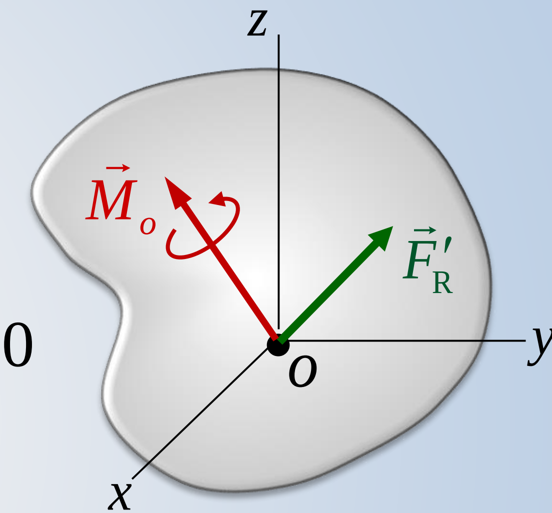
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_o = 0 \end{cases}$$

3.2 任意力系的平衡

2. 空间力系平衡方程

$$F_R' = \sum F = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2 + \left(\sum F_z\right)^2} = 0$$

$$M_o = \sum M = \sqrt{\left(\sum M_x\right)^2 + \left(\sum M_y\right)^2 + \left(\sum M_z\right)^2} = 0$$



平衡方程基本形式

$$\begin{cases} \sum F_x = 0 & \sum M_x = 0 \\ \sum F_y = 0 & \sum M_y = 0 \\ \sum F_z = 0 & \sum M_z = 0 \end{cases}$$

$$\begin{cases} \vec{F}_R' = \sum \vec{F}_i = 0 \\ \vec{M}_o = \sum \vec{M}_o\left(\vec{F}_i\right) = 0 \end{cases}$$

三、解题规范

1、 取研究对象

2、 画受力图

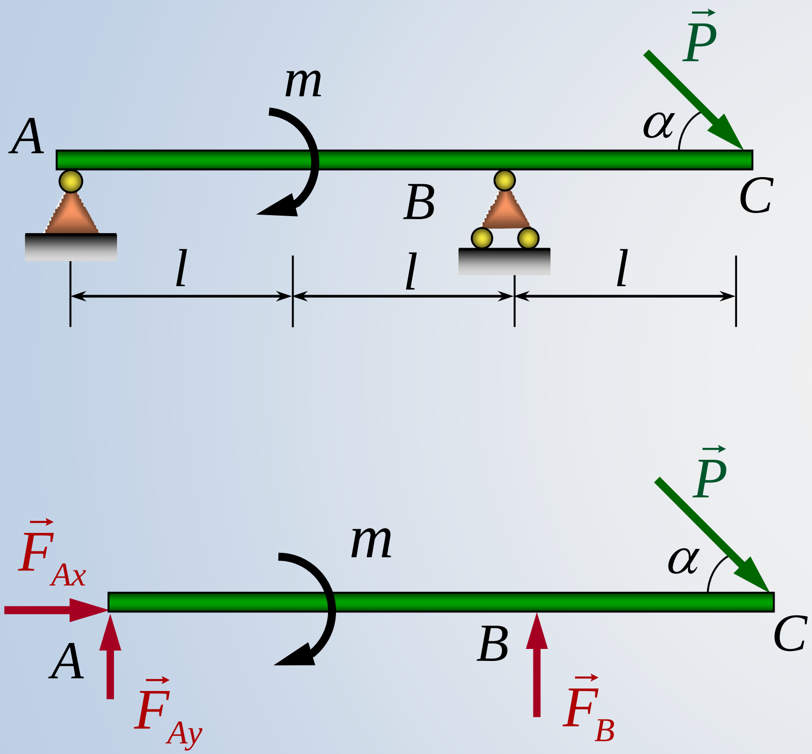
3、 列平衡方程

4、 求解

四、平面任意力系的平衡例题

3.2 任意力系的平衡

例2 已知 $AC=3l$，P，m，$\alpha=45^\circ$ 求：A、B 处约束反力



解：对AC杆

$$\sum F_x=0 \qquad F_{Ax}+P\cos\alpha=0$$

$$\sum F_y=0 \qquad F_B+F_{Ay}-P\sin\alpha=0$$

$$\sum M_o=0 \qquad F_B 2l-m-3Pl\sin\alpha=0$$

解得：$F_{Ax}=-P\cos\alpha$

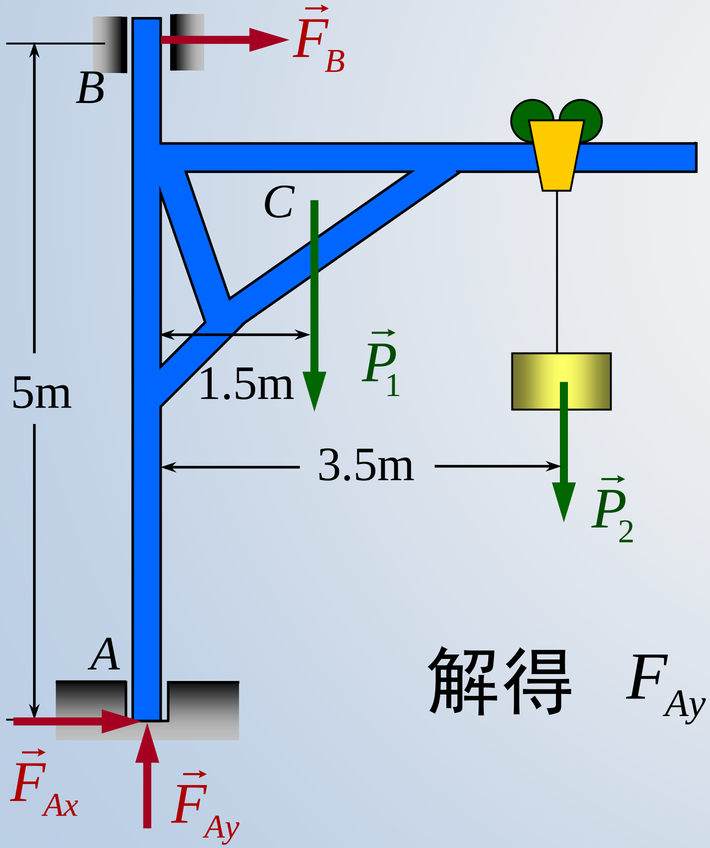
$$F_{Ay}=P\sin\alpha-F_B$$

$$F_B=\frac{m+3Pl\sin\alpha}{2l}$$

3.2 任意力系的平衡

例3　已知：$P_1 = 10\text{kN}$, $P_2 = 40\text{kN}$, 尺寸如图；

求：轴承A，B处的约束力。



解：　对整体

$$\sum F_x = 0 \qquad F_{Ax} + F_B = 0$$

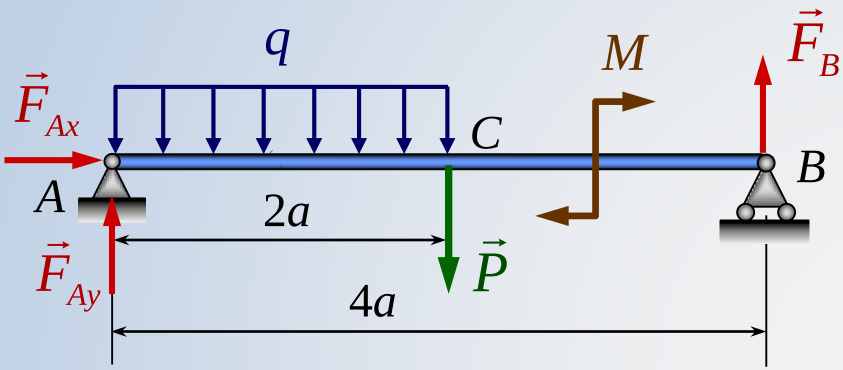
$$\sum F_y = 0 \qquad F_{Ay} - P_1 - P_2 = 0$$

$$\sum M_A = 0 \quad -F_B \cdot 5 - 1.5 \cdot P_1 - 3.5 \cdot P_2 = 0$$

解得　$F_{Ay} = 50\,\text{kN}$　$F_B = -31\text{kN}$ ←　$F_{Ax} = 31\text{kN}$

3.2 任意力系的平衡

例4 已知：$P,q,a,M=Pa$ 求：支座A、B处的约束力。



解：取AB梁，画受力图

$$\sum F_x = 0 \qquad \text{解得} \quad F_{Ax} = 0$$

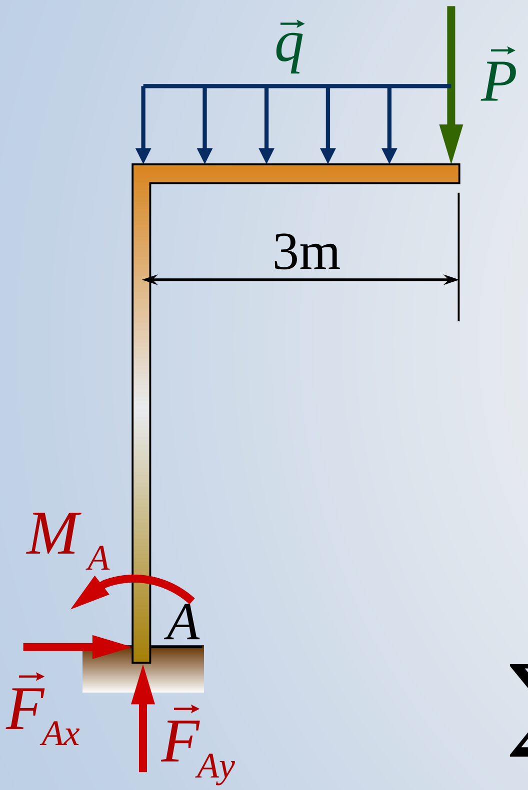
$$\sum F_y = 0 \quad F_{Ay} - q\cdot 2a - P + F_B = 0$$

$$\sum M_A = 0 \quad F_B\cdot 4a - M - P\cdot 2a - q\cdot 2a\cdot a = 0$$

解得 $F_B = \dfrac{3}{4}P + \dfrac{1}{2}qa \qquad F_{Ay} = \dfrac{P}{4} + \dfrac{3}{2}qa$

3.2 任意力系的平衡

例5　已知：$P=5\,\text{kN}, q=4\,\text{kN/m}$　求：A处受的力



解：对构架

$$\sum F_x=0 \qquad F_{Ax}=0$$

$$\sum F_y=0 \qquad F_{Ay}-P-3q=0$$

解得：　$F_{Ay}=17\,\text{kN}$

$$\sum m_A\left(\vec{F}\right)=0 \quad M_A-3q\times1.5-3P=0$$

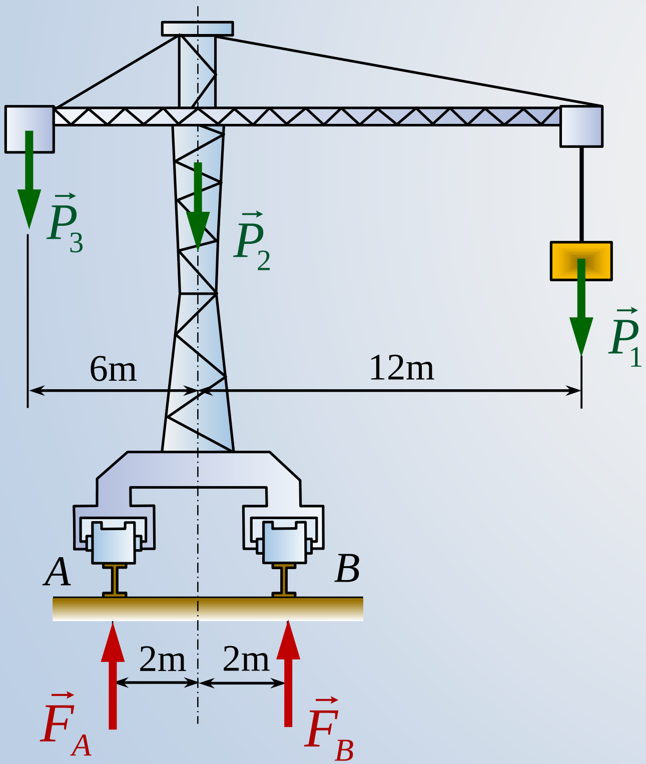
解得：　$M_A=33\,\text{kN}\cdot\text{m}$

3.2 任意力系的平衡

例6 已知：$P_1 = 200\text{kN}$，$P_2 = 700\text{kN}$, 尺寸如图；

求：1.起重机满载和空载时不翻倒，平衡载重P_3；

2.轨道压力不小于30kN，平衡载重P_3。



解：1.取起重机，画受力图。

满载时，$F_A = 0$ 为不安全状况

$$\sum M_B = 0 \quad P_{3\min} \cdot 8 + 2P_2 - 10P_1 = 0$$

解得 $P_{3\min} = 75\text{kN}$

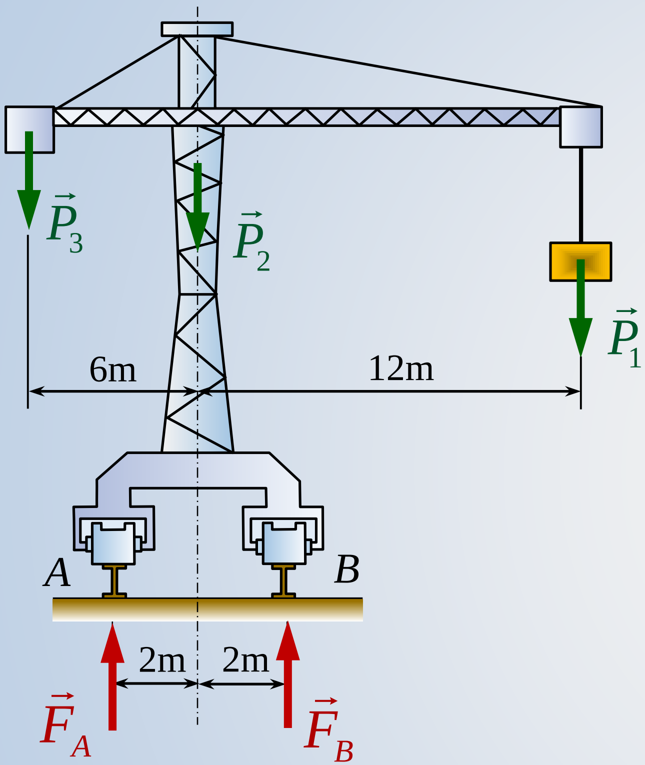
空载时，$\vec{F}_B = 0$, 为不安全状况

$$\sum M_A = 0 \quad 4P_{3\max} - 2P_2 = 0$$

解得 $P_{3\max} = 350\text{kN}$

$$75\text{kN} \le P_3 \le 350\text{kN}$$

3.2 任意力系的平衡



2.轨道压力不小于30kN，平衡载重P_3。

满载时，$F_A = 30\,\text{kN}$ 为不安全状况

$$\sum M_B = 0 \quad P_{3\min} \cdot 8 + 2P_2 - 10P_1 - 4 \times F_A = 0$$

解得 $P_{3\min} = 90\text{kN}$

空载时，$F_B = 30\,\text{kN}$ 为不安全状况

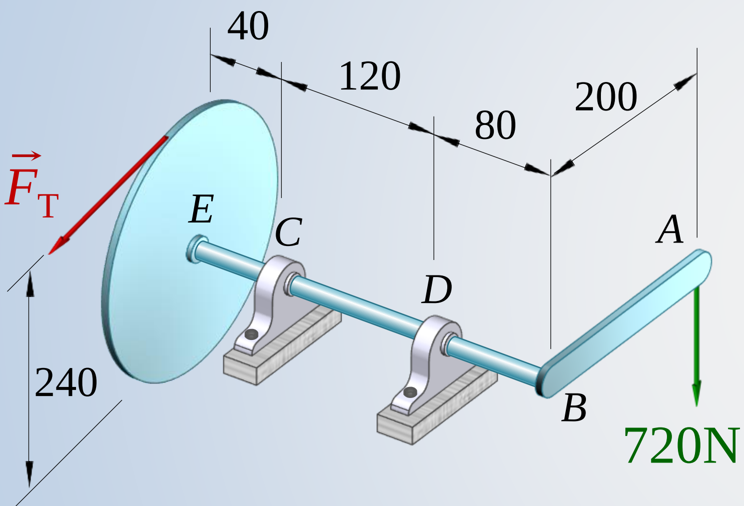
$$\sum M_A = 0 \quad 4P_{3\max} - 2P_2 + 4F_B = 0$$

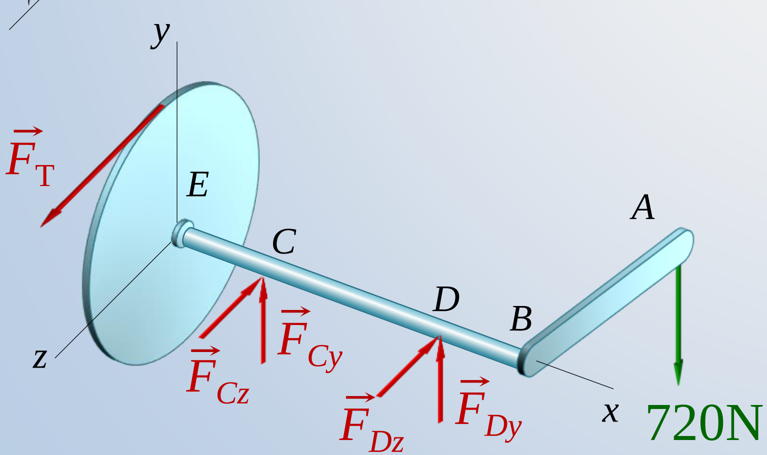
解得 $P_{3\max} = 320\text{kN}$

$$90\text{kN} \le P_3 \le 320\text{kN}$$

3.2 任意力系的平衡

例7 如图, 当摇杆处于水平位置时，轮受到720N的垂直载荷，试求（1）F_T；（2）轴承C、D的约束力。





解：取整体，画受力图

$$\sum M_x = 0 \quad F_T \cdot 120 - 720\times 200 = 0$$

$$\sum M_y = 0 \quad F_T \cdot 40 + F_{Dz} \cdot 120 = 0$$

$$\sum M_z = 0 \quad F_{Dy} \cdot 120 - 720\times(120+80) = 0$$

$$\sum F_y = 0 \quad F_{Cy} + F_{Dy} - 720 = 0$$

$$\sum F_z = 0 \quad F_T - F_{Cz} - F_{Dz} = 0$$

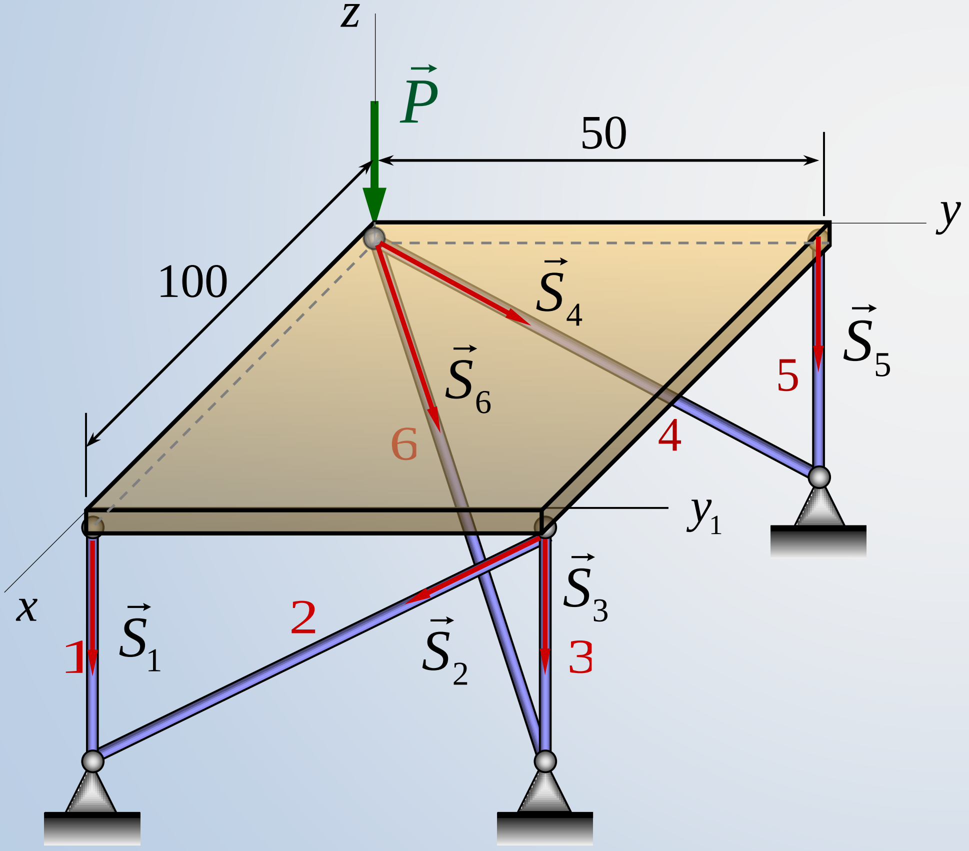
解得 F_T=1200N, F_{Dy}= 1200N,

F_{Dz}= – 400N, $F_{Cy} = -480$N, $F_{Cz} = 1600$N

3.2 任意力系的平衡

例8 已知：水平板，受六杆支撑，$\vec{P}$，不计板重，

求：各杆的内力。



解： 对板，

$$\sum M_z = 0 \qquad S_2 = 0$$

$$\sum F_x = 0, \qquad S_6 = 0$$

$$\sum F_y = 0, \qquad S_4 = 0$$

$$\sum M_{y_1} = 0 \qquad S_5 = -P$$

$$\sum M_x = 0 \qquad S_3 = P$$

$$\sum F_z = 0, \qquad S_1 = -P$$

3.3 物体系统的平衡

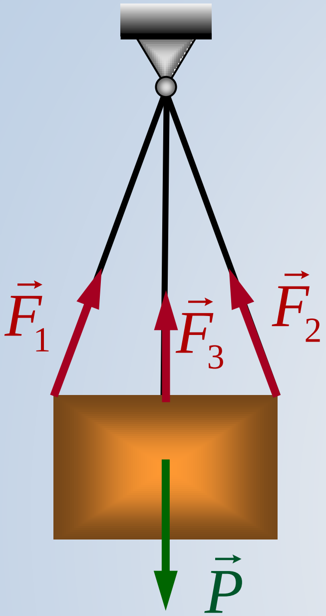
一、静定与超静定问题的概念

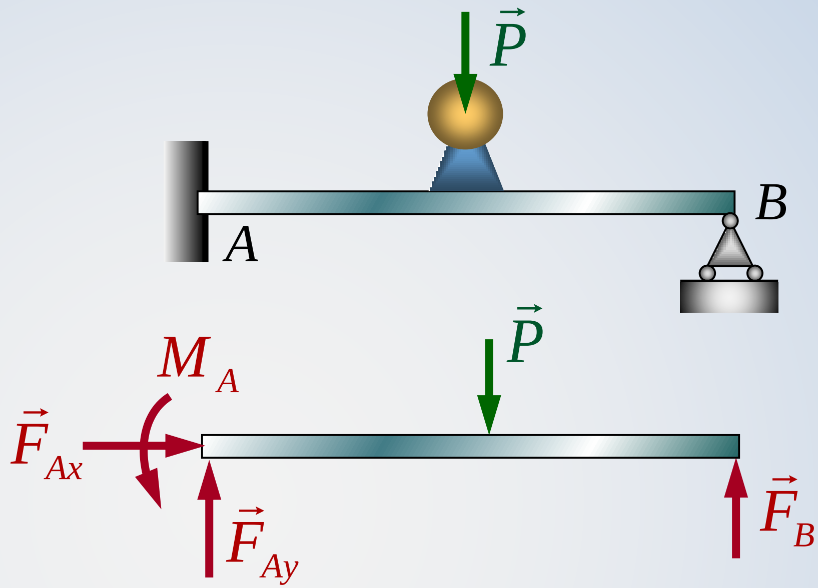
	独立方程数	未知量数
平面汇交力系	2	2
平面力偶系	1	1
平面平行力系	2	2
平面任意力系	3	3

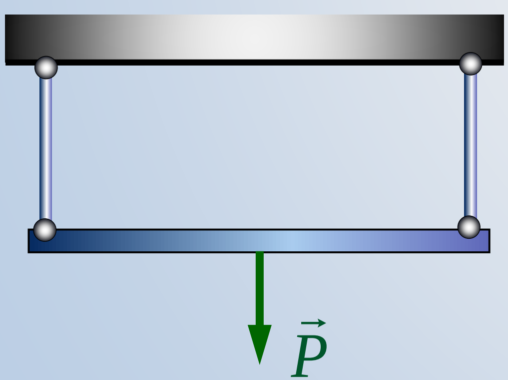
静定：　　未知力数 ＝ 独立平衡方程数

超静定：　未知力数 ＞ 独立平衡方程数

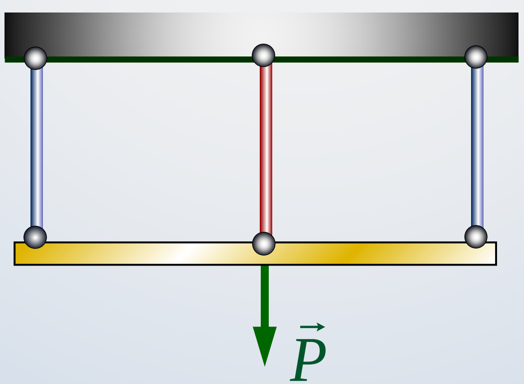
3.3 物体系统的平衡



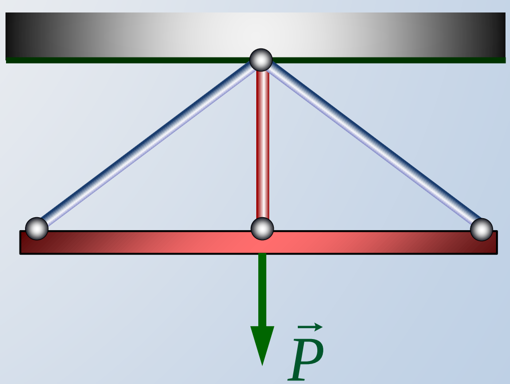




静定



超静定



超静定

3.3 物体系统的平衡

二、物体系统的平衡

整体平衡 $\Longleftrightarrow$ 分离体平衡

1、恰当选取研究对象

整体　　某一部分　　某个分离体

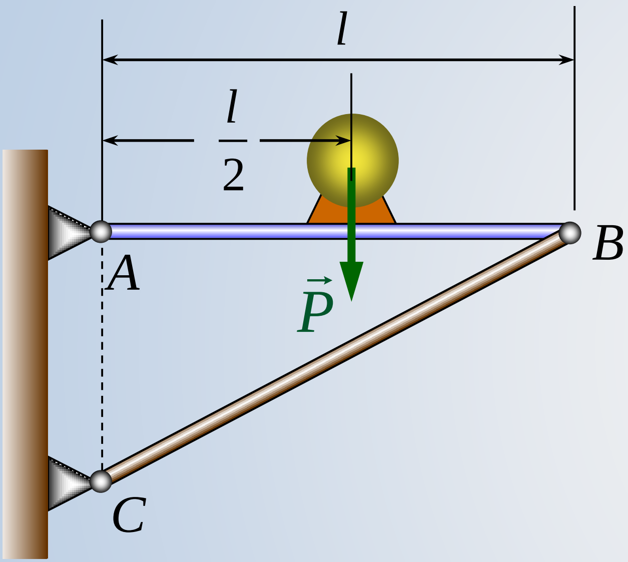
2、灵活选取平衡方程

投影方程　　投影轴的选取

矩方程　　矩心的选取

3.3 物体系统的平衡

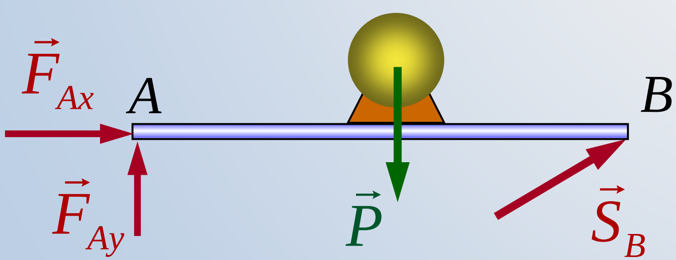
例9 已知：$P=50\mathrm{kN}$ $\angle ABC=30^0$ 求：杆BC所受的力

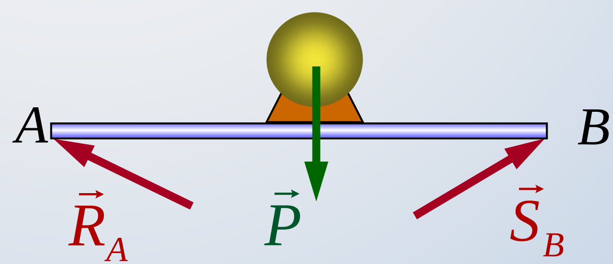


解： 方法一 对AB杆及电机

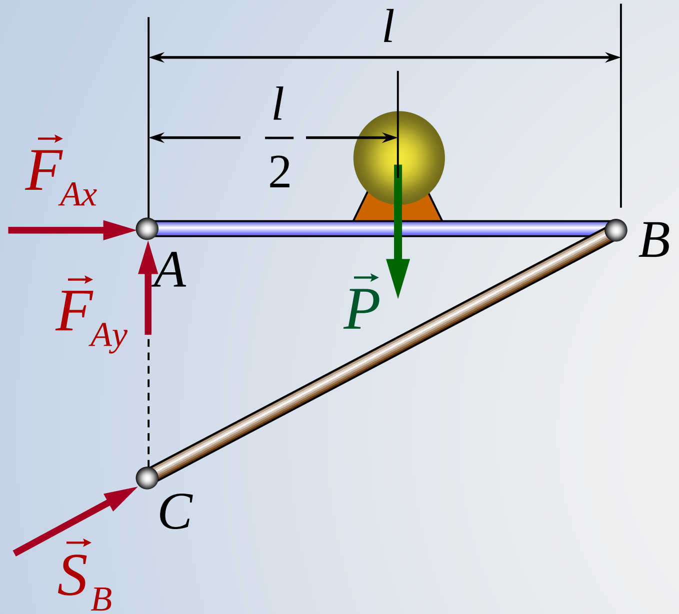
$$\sum M_A=0 \qquad S_B\sin 30^\circ\cdot l-P\frac{l}{2}=0$$

解得 $S_B=P=50\mathrm{kN}$





3.3 物体系统的平衡



方法二　　对整体

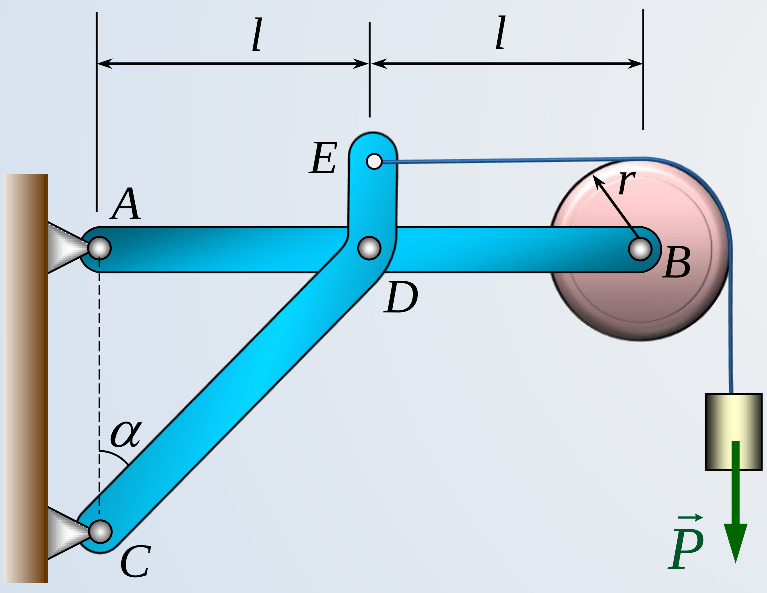
$$\sum M_A = 0 \quad S_B l \cos 30^\circ - P\frac{l}{2} = 0$$

得　　$S_B = P = 50\mathrm{kN}$

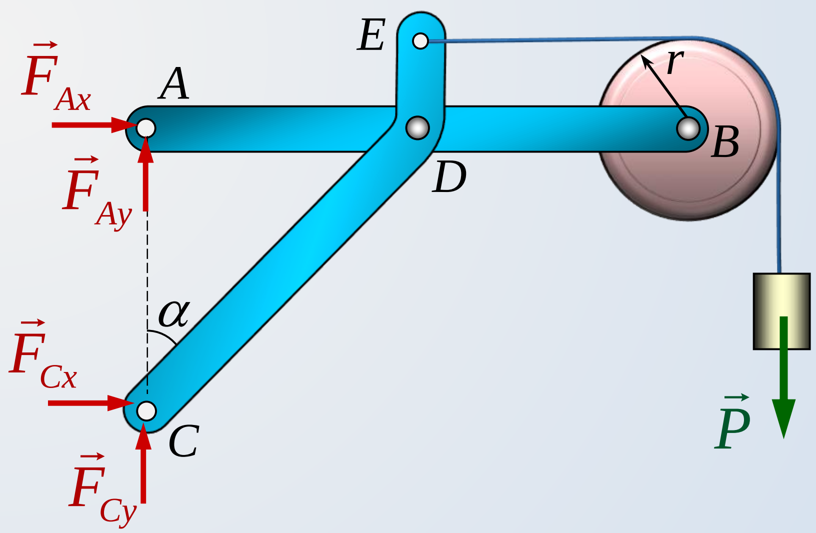
3.3 物体系统的平衡

例10　已知：l，P，r，$\alpha = 45^\circ$，A、C为固定铰支座

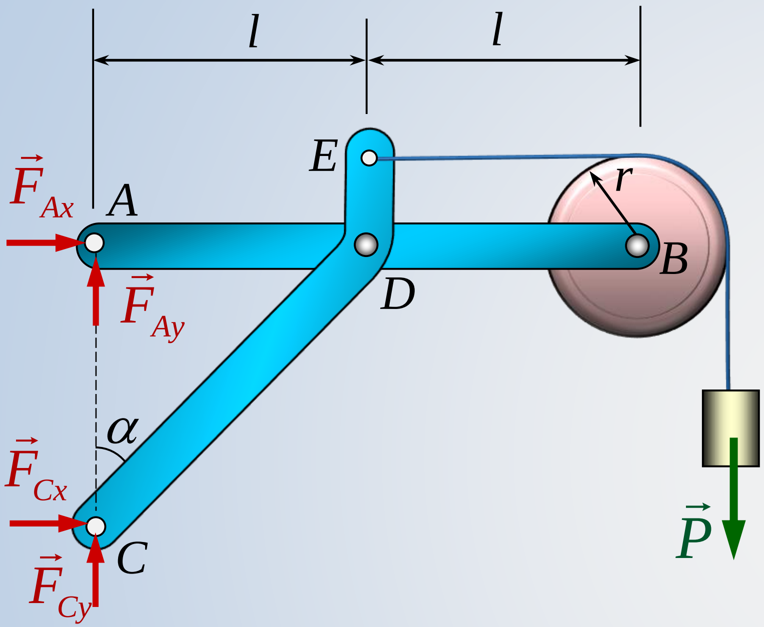
求：A、C 处约束反力



解：1.对整体：



3.3 物体系统的平衡



$$\sum M_A = 0 \qquad F_{Cx} \cdot l - P(2l + r) = 0$$

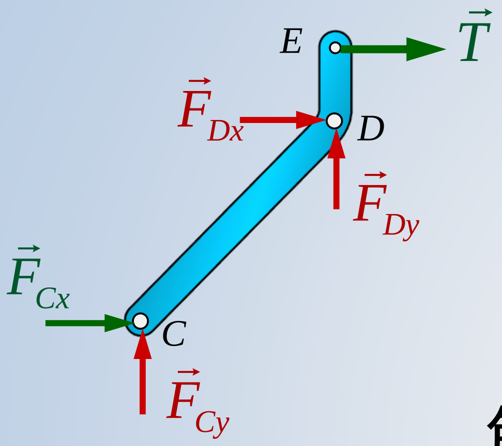
解得： $$F_{Cx} = \frac{P(2l + r)}{l}$$

$$\sum F_x = 0 \qquad F_{Ax} + F_{Cx} = 0$$

解得： $$F_{Ax} = -\frac{P(2l + r)}{l}$$

$$\sum F_y = 0 \qquad F_{Ay} + F_{Cy} - P = 0$$

3.3 物体系统的平衡

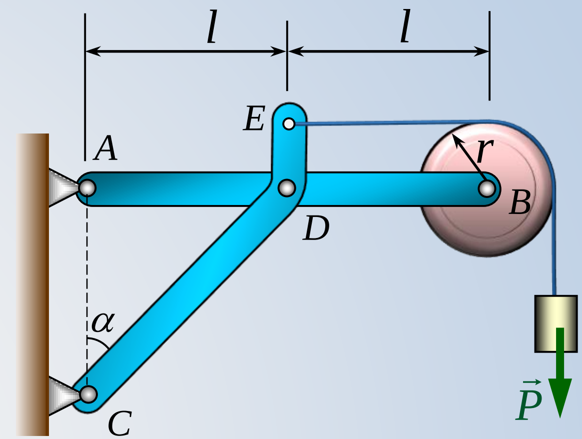


2.对CDE杆：

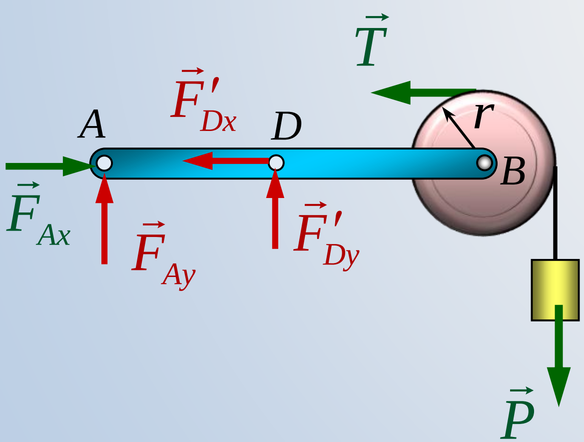
$$\sum M_D\left(\vec{F}\right)=0$$

$$F_{Cx}\cdot l-F_{Cy}\cdot l-T\cdot r=0$$

解得 $F_{Cy}=2P \quad F_{Ay}=-P$



$$F_{Ay}+F_{Cy}-P=0$$



或 对AB杆及轮：

$$\sum M_D\left(\vec{F}\right)=0$$

$$T\cdot r-P\cdot\left(l+r\right)-F_{Ay}\cdot l=0$$

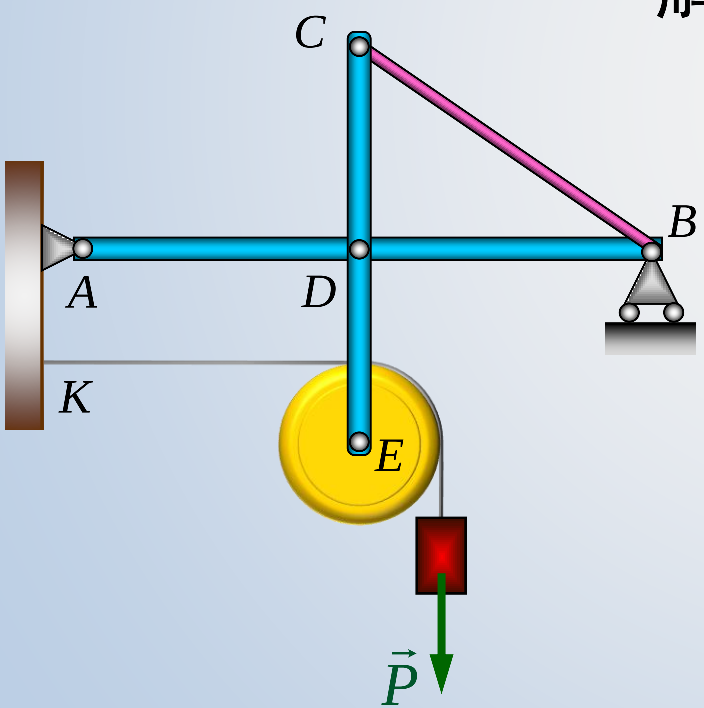
解得 $F_{Ay}=-P \qquad F_{Cy}=2P$

3.3 物体系统的平衡

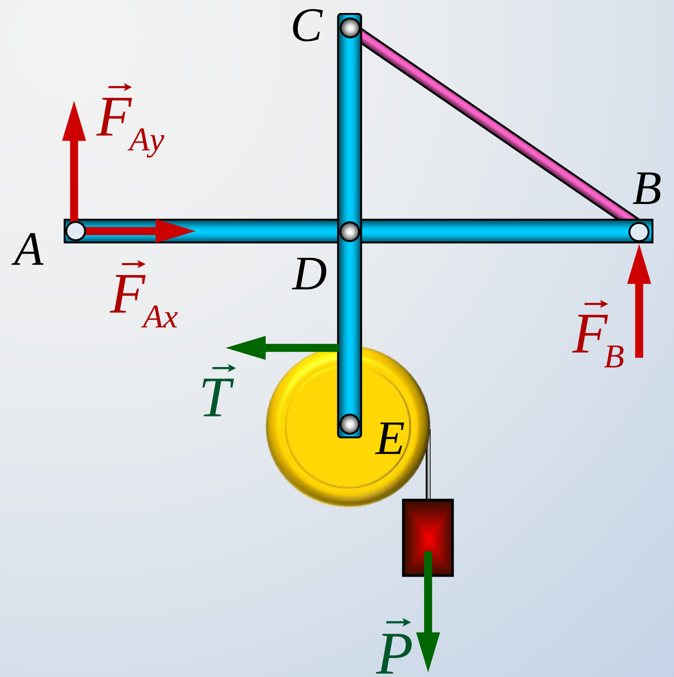
例11 已知：滑轮半径为r，$P=1200\text{N}$ $CD=DE=1.5\text{m}$

$AD=DB=2\text{m}$

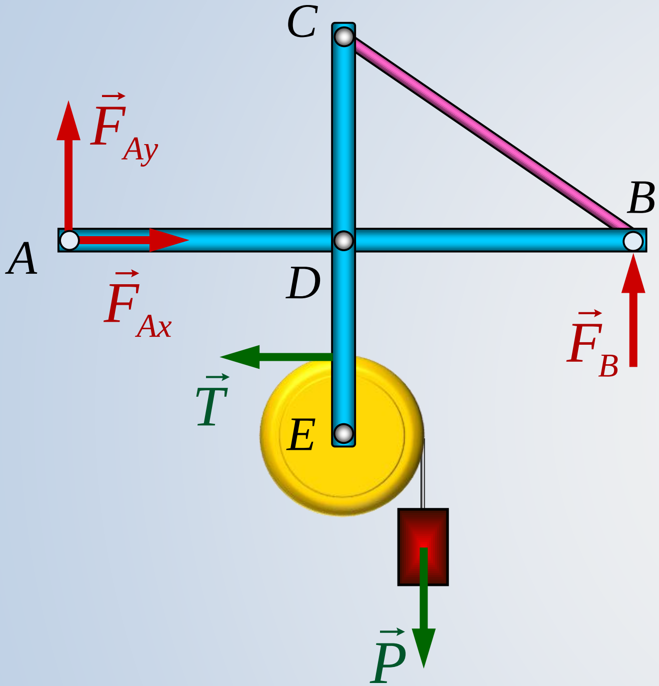
求：A、B 处约束反力和BC杆的内力。



解：1. A, B, M 处约束反力 对整体



3.3 物体系统的平衡



$$\sum F_x = 0,$$

$$F_{Ax} - T = 0 \qquad T = P$$

$$\sum F_y = 0,$$

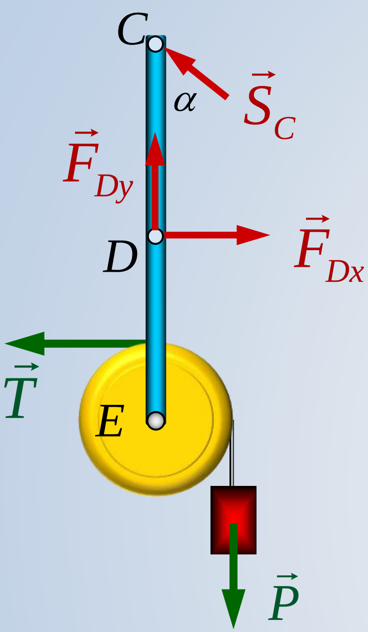
$$F_{Ay} + F_B - P = 0$$

$$\sum M_A = 0,$$

$$4F_B - P(2+r) - P(1.5-r) = 0$$

解得 $F_{Ax} = 1200\text{N},\ F_{Ay} = 150\text{N},\ F_B = 1050\text{N}$

3.3 物体系统的平衡

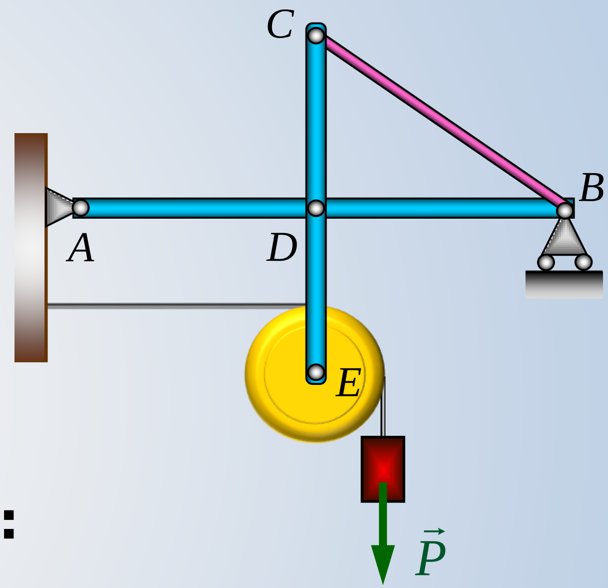


2. 求BC杆的内力 对杆CE及滑轮

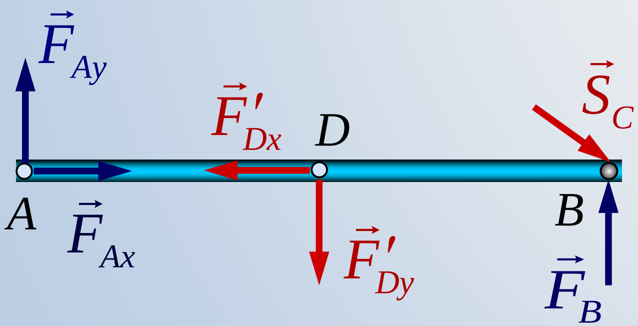
$$\sum M_D\left(\vec{F}\right)=0$$

$$1.5S_C\sin\alpha-1.5P=0$$

解得 $S_C=1500\text{N}$



或 对杆AB及销钉B：



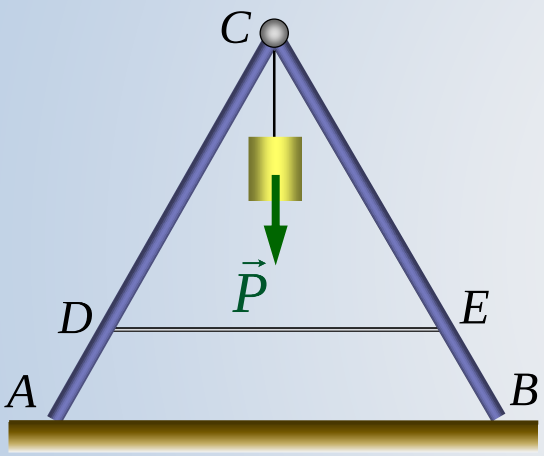
$$\sum M_D\left(\vec{F}\right)=0$$

$$2F_B-2S_C\cos\alpha-2F_{Ay}=0$$

解得 $S_C=1500\text{N}$

3.3 物体系统的平衡

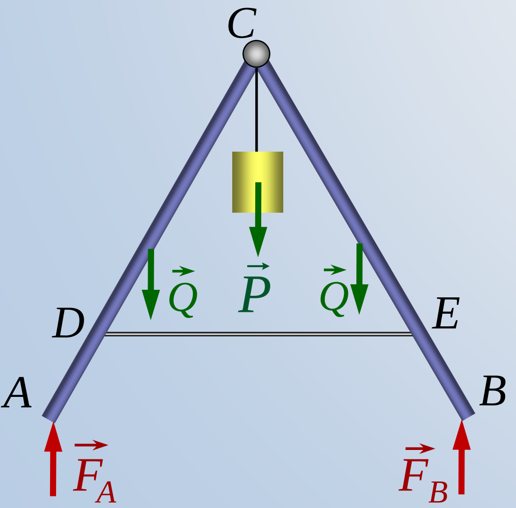
例12 AC、BC各重100N，P=500N,∠CAB=60°, AC=BC=4m, DC=EC=3m, 求杆AC、BC在C处受力

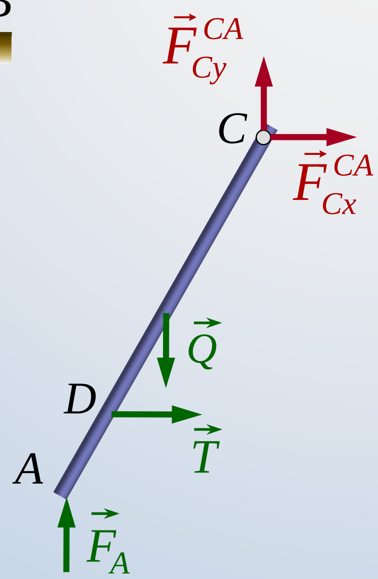


解：1.对整体

$$\sum F_y=0 \quad F_A+F_B-500-100-100=0$$

解得 $F_A=F_B=350\text{N}$





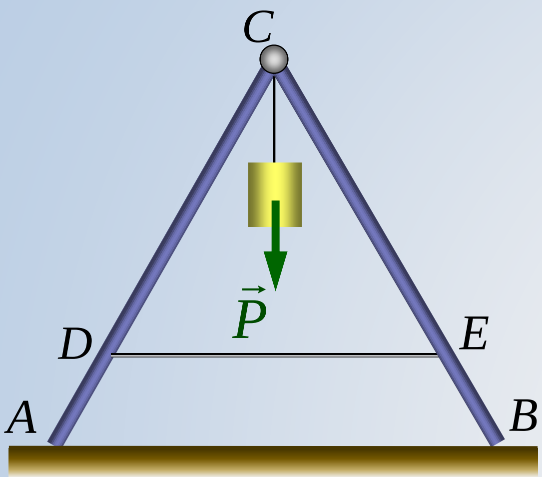
2.对杆AC

$$\sum M_C=0,$$

$$F_A\cdot 4\cdot\frac{1}{2}-T\cdot 3\cdot\frac{\sqrt{3}}{2}-Q\cdot 2\cdot\frac{1}{2}=0$$

解得 T= 231N

3.3 物体系统的平衡

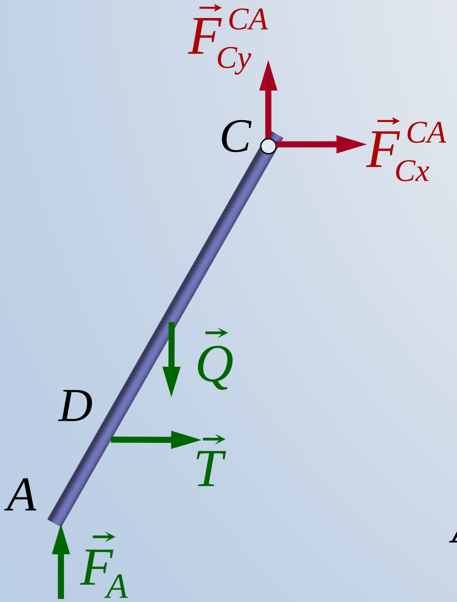


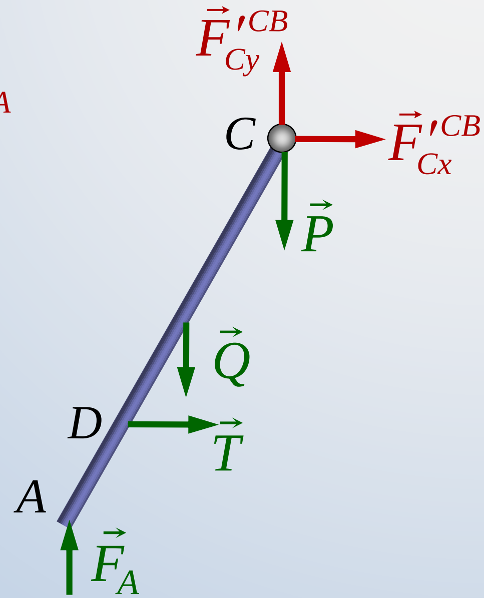
$$\sum F_x = 0, \quad 得 \quad F_{Cx}^{CA} = -T = -231\text{N} \leftarrow$$

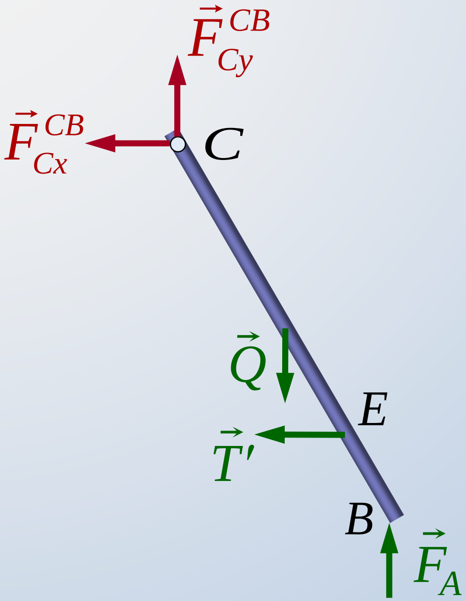
$$\sum F_y = 0, \quad 得 \quad F_{Cy}^{CA} = -F_A + Q = -250\text{N} \downarrow$$

由对称得 $F_{Cx}^{CB} = 231\text{N} \rightarrow$，$F_{Cy}^{CB} = 250\text{N} \downarrow$

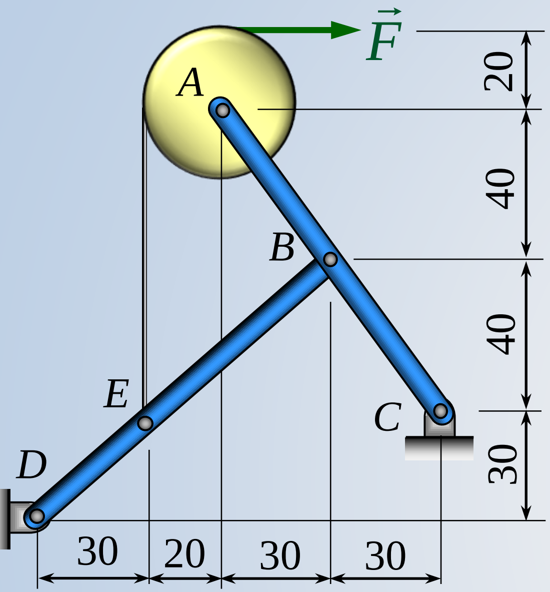
如果对杆AC和销钉







3.3 物体系统的平衡



例13 已知: F=2000N,不计各件自重.

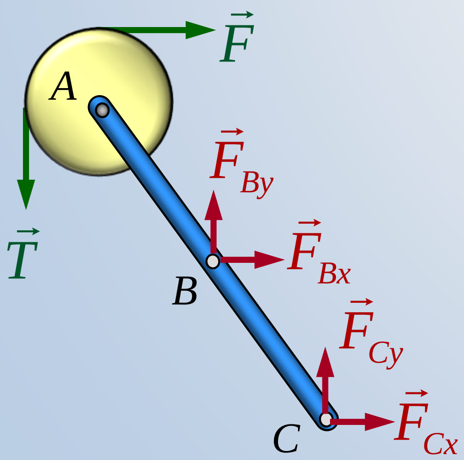
求: 杆AC在B处受力。

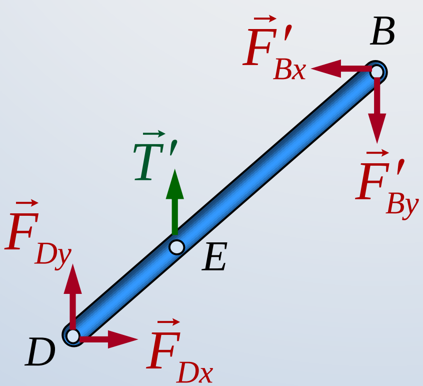
解: 1. 取杆AC和滑轮

$$\sum M_C = 0,\ -F \cdot 100 - F_{Bx} \cdot 40 - F_{By} \cdot 30 + F \cdot 80 = 0$$

2. 取杆DB

$$\sum M_D = 0,\quad F'_{Bx} \cdot 70 - F'_{By} \cdot 80 + F \cdot 30 = 0$$



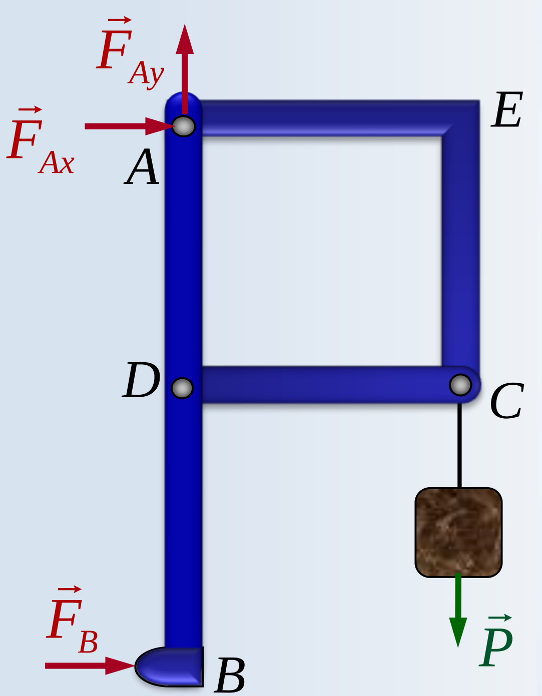


解得 $F_{By} = -75.47\text{N}$

$F_{Bx} = -943.4\text{N}$

3.3 物体系统的平衡

例14 已知$AE=AD=DC=EC$，求支座A的约束力，及杆AC、杆AB在A处受力。



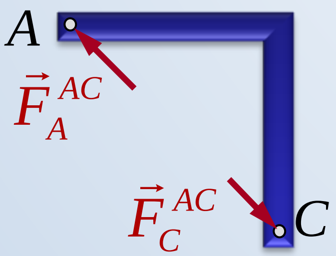
解：1.对整体

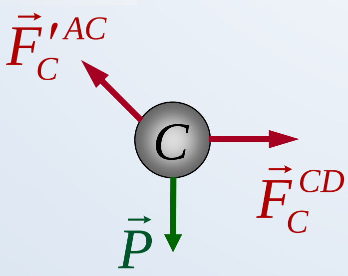
$$\sum M_A = 0 \qquad -P\cdot a + F_B\cdot 2a = 0$$

$$\sum F_x = 0 \qquad -F_{Ax} + F_B = 0$$

$$\sum F_y = 0 \qquad -F_{Ay} + P = 0$$

解得 $F_B = P/2,\ \ F_{Ax} = -P/2,\ \ F_{Ay} = P$



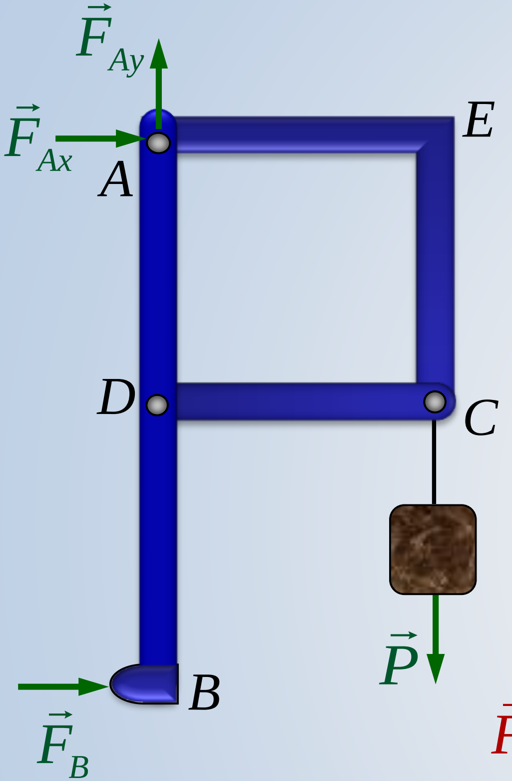


2.杆AC在A处受力（对销C）

$$\sum F_y = 0 \quad -F_C'^{AC}\cos 45^\circ + P = 0$$

解得 $F_C'^{AC} = F_C^{AC} = F_A^{AC} = \sqrt{2}P$

3.3 物体系统的平衡



3.求AB杆A处受力（对销A）

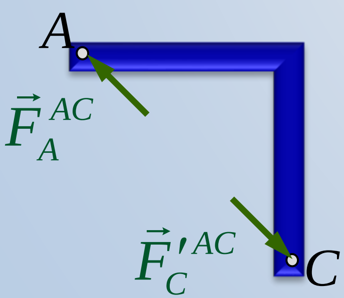
$$\sum F_x = 0,\quad F_{Ax}^{AB} + F_{Ax} + F_A'^{AC}\cos 45^\circ = 0$$

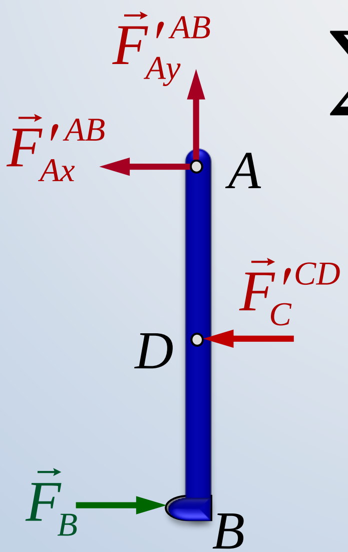
$$\sum F_y = 0,\quad F_{Ay}^{AB} + F_{Ay} - F_A'^{AC}\sin 45^\circ = 0$$

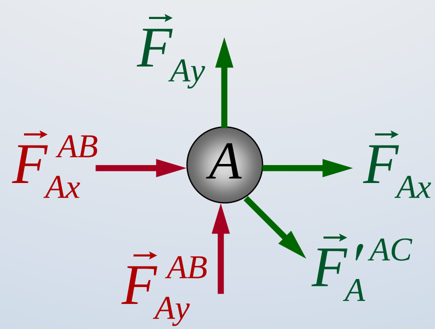
解得 $F_{Ax}^{AB} = -\dfrac{P}{2},\quad F_{Ay}^{AB} = 0$ 即 $F_A'^{AB} = -\dfrac{P}{2} \rightarrow$

亦可取AB杆

$$\sum M_D = 0,\quad F_{Ax}'^{AB}\cdot AD + F_B\cdot DB = 0$$







解得 $F_{Ax}'^{AB} = -\dfrac{P}{2} \rightarrow$

$\sum F_y = 0$，得 $F_{Ay}'^{AB} = 0$

谢 谢！