工程力学

第9章 弯曲强度

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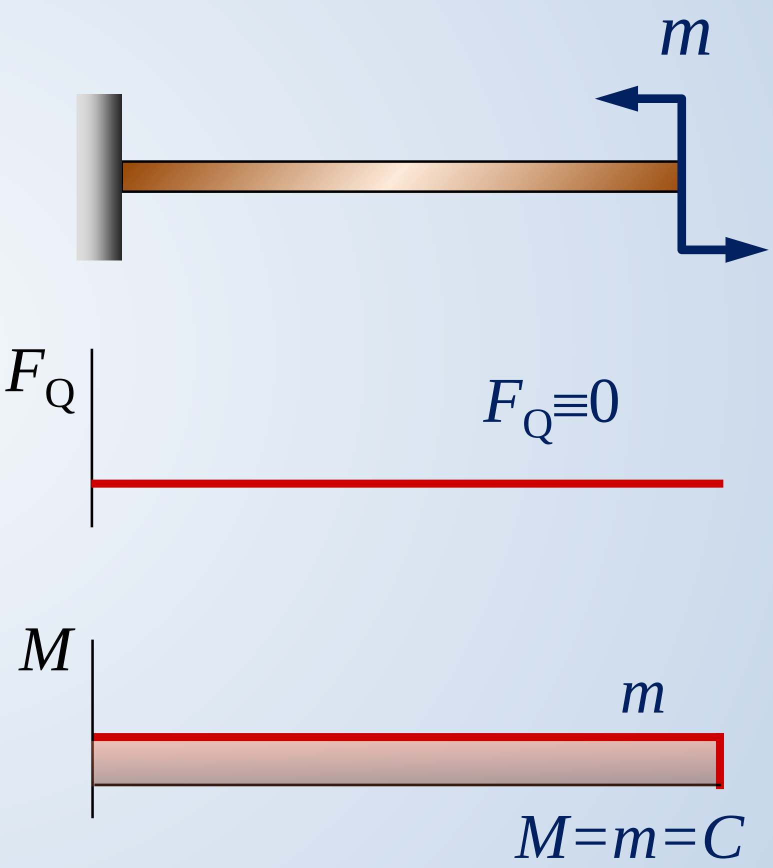
9.1 纯弯曲及其变形

一. 概念:

$F_Q=0$

$M= C$ → 纯弯曲

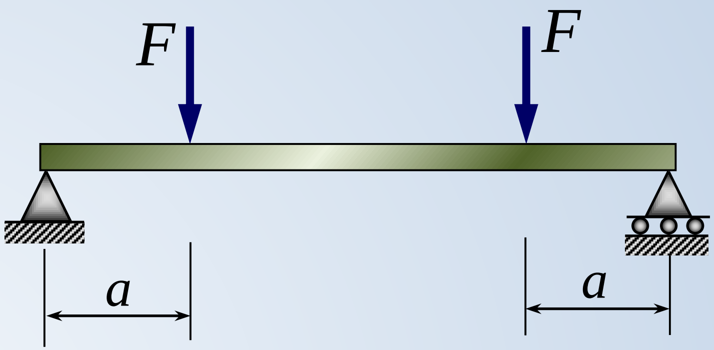


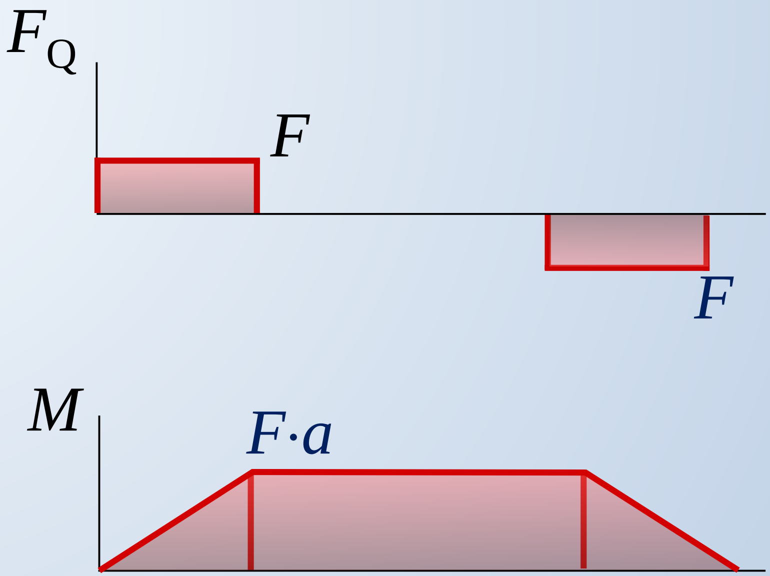
9.1 纯弯曲及其变形

$$F_Q \neq 0$$

$$M \neq C$$

→ 横力弯曲

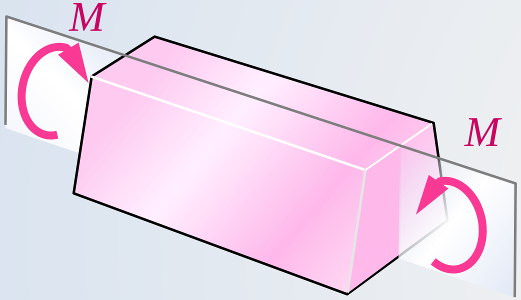


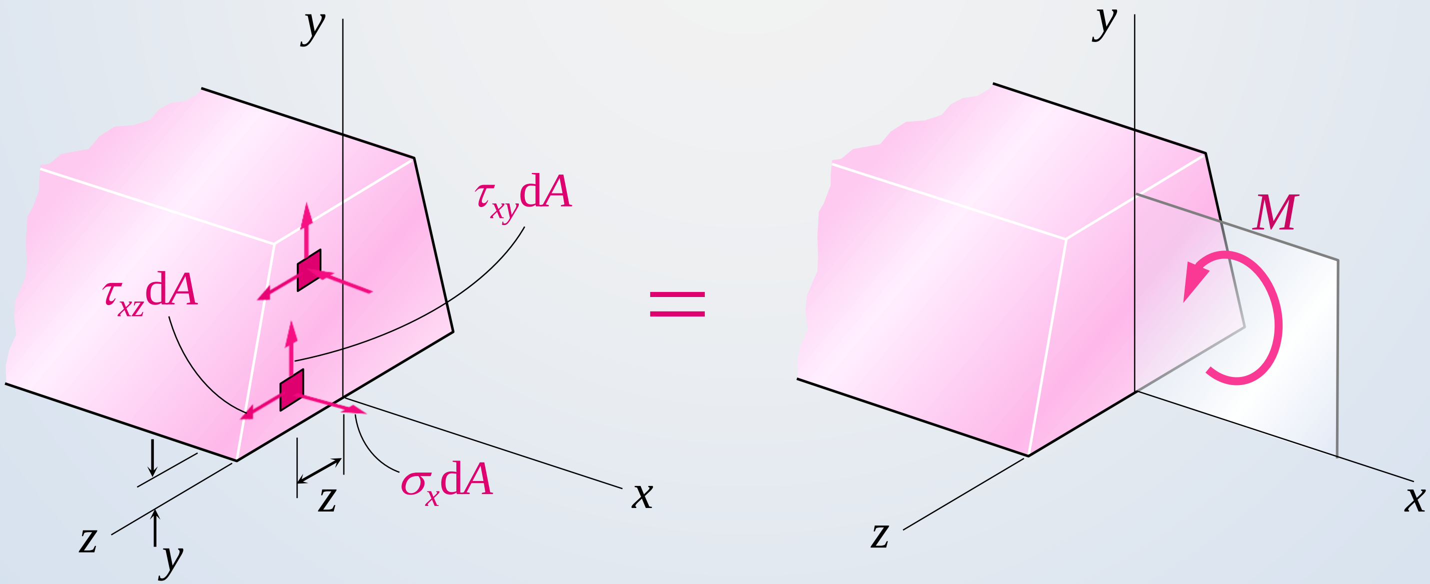


9.1 纯弯曲及其变形

首先研究**纯弯曲**时**横截面**上的应力问题

已知是横截面上的**正应力组成了M**（**切应力组成F_Q**），但如何分布、大小都是未知，所以求解应力的问题属**超静定问题**。

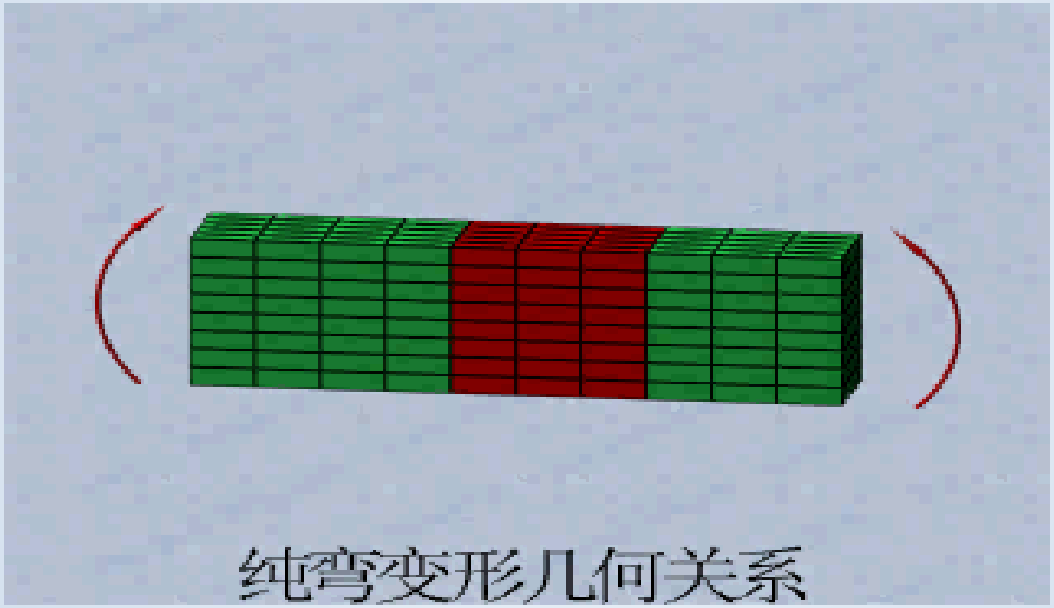


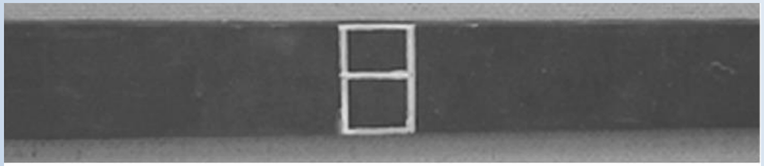


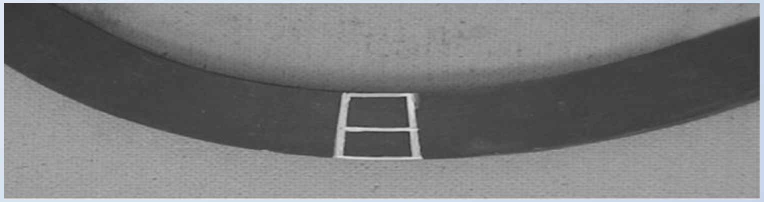
9.1 纯弯曲及其变形

二. 变形协调方程-几何方程

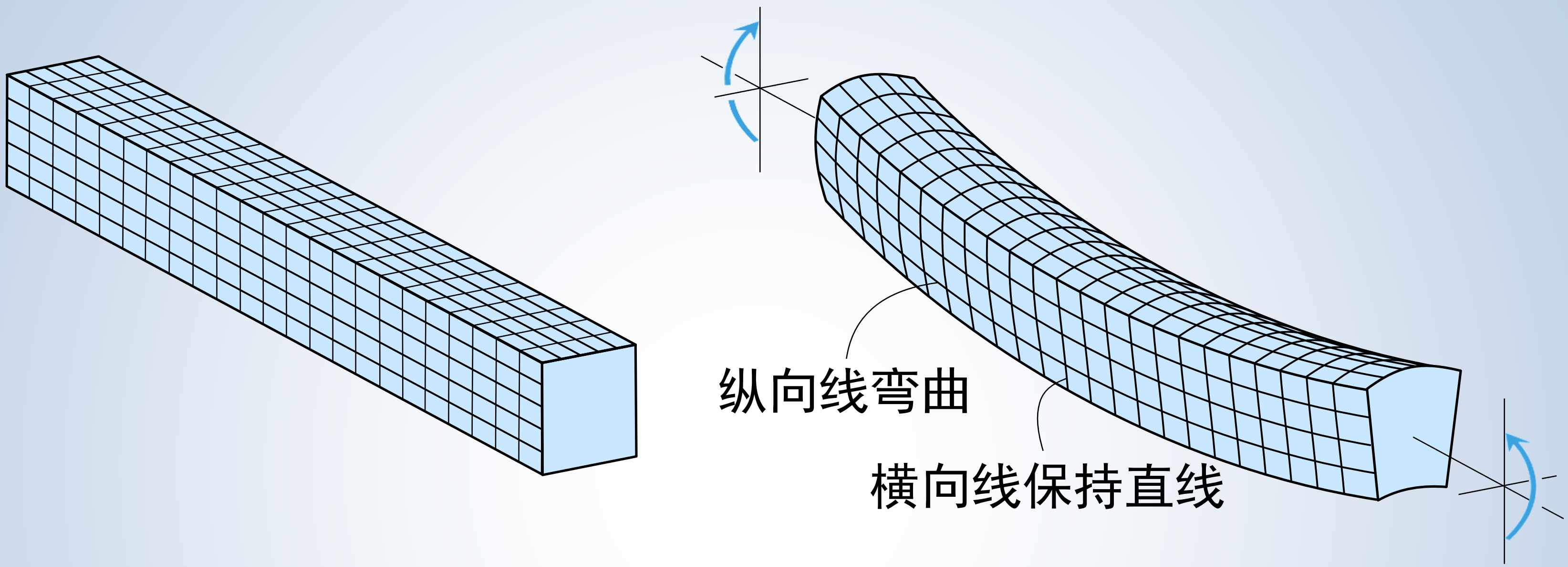
1. 实验观察







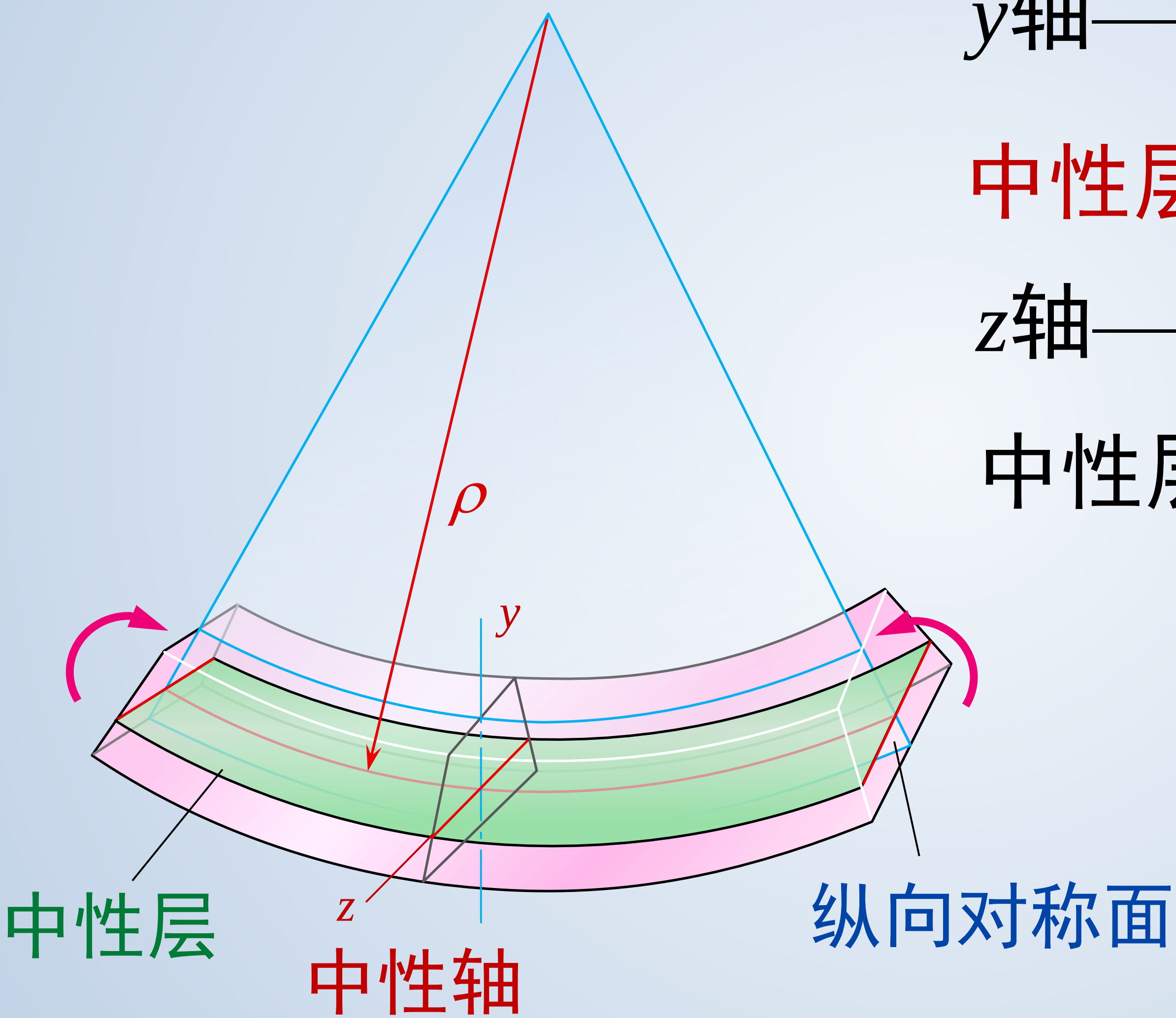
9.1 纯弯曲及其变形



横向线——偏转——夹角$\mathrm{d}\theta$

$$
\text{纵向线——弯曲}\begin{cases}\text{缩短 } \varepsilon < 0 \\ \text{中性层 } \varepsilon = 0 \\ \text{伸长 } \varepsilon > 0\end{cases}
$$

9.1 纯弯曲及其变形



y轴——纵向对称轴

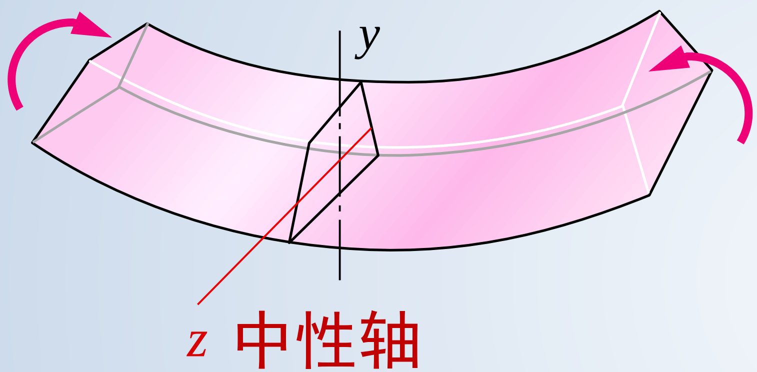
中性层曲率——$1/\rho$

z轴——中性轴

中性层与横截面的交线

9.1 纯弯曲及其变形

2.推理假设



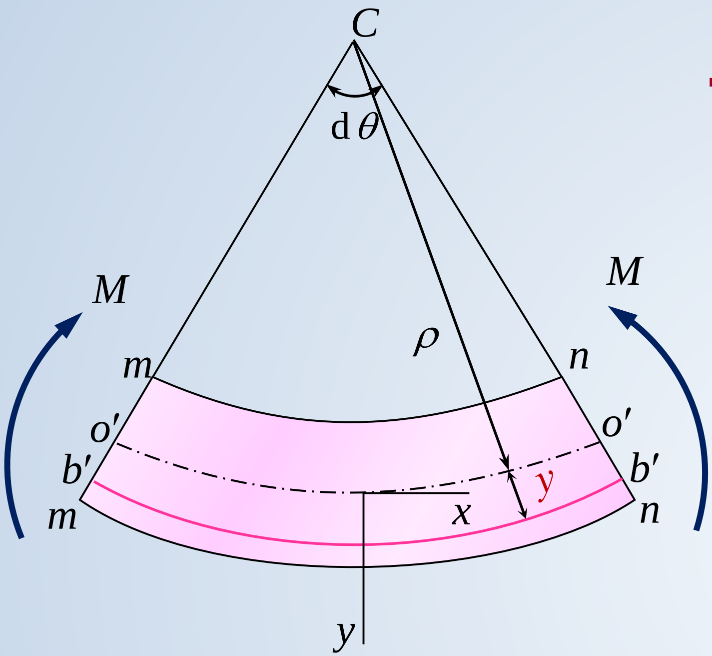
1）平面假设---变形前为平面的横截面变形后仍为平面,且垂直于变形后的轴线

$\gamma=0$ 得 $\tau=0$ $\varepsilon \rightarrow \sigma$

2）纵向纤维互不挤压(纵向纤维间无σ)

结 论 等截面直梁在纯弯时,横截面上只产生正应力σ.

9.2 纯弯曲时梁横截面上的正应力

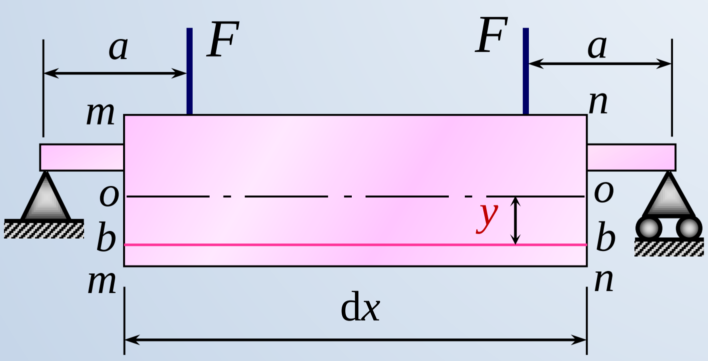


一. 变形几何关系（应变–位移）

$$\varepsilon=\frac{\Delta(\mathrm{d}x)}{\mathrm{d}x}=\frac{b'b'-bb}{bb}=\frac{b'b'-o'o'}{o'o'}$$

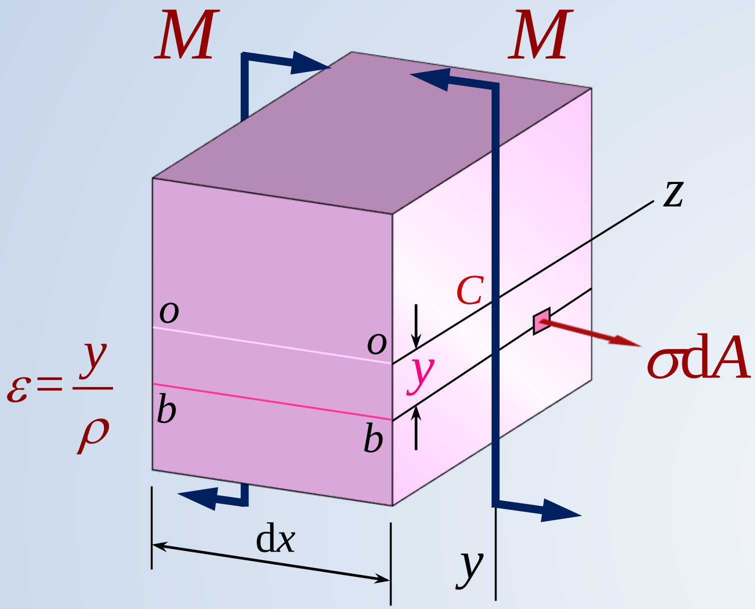
$$=\frac{(\rho+y)\mathrm{d}\theta-\rho\,\mathrm{d}\theta}{\rho\,\mathrm{d}\theta}=\frac{y}{\rho}$$

$$\varepsilon=\frac{y}{\rho}$$



结 论 纵向纤维的线应变与它到中性轴的距离成正比,沿y轴线性分布。

9.2 纯弯曲时梁横截面上的正应力

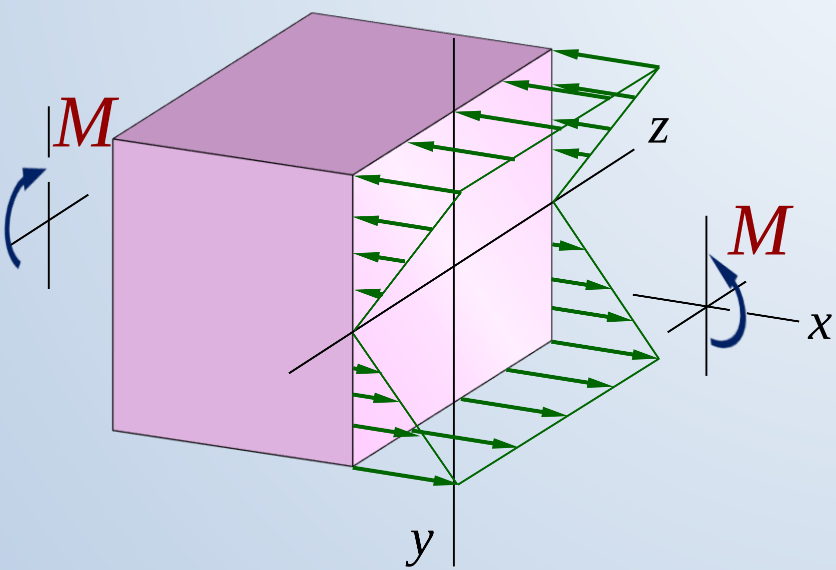


二. 物理关系($\sigma \sim \varepsilon$)

设 $\sigma \le \sigma_p$ $E_t=E_c=E$

把 $\varepsilon=\dfrac{y}{\rho}$ 代入胡克定律

$$\sigma = E\varepsilon = E\frac{y}{\rho}$$



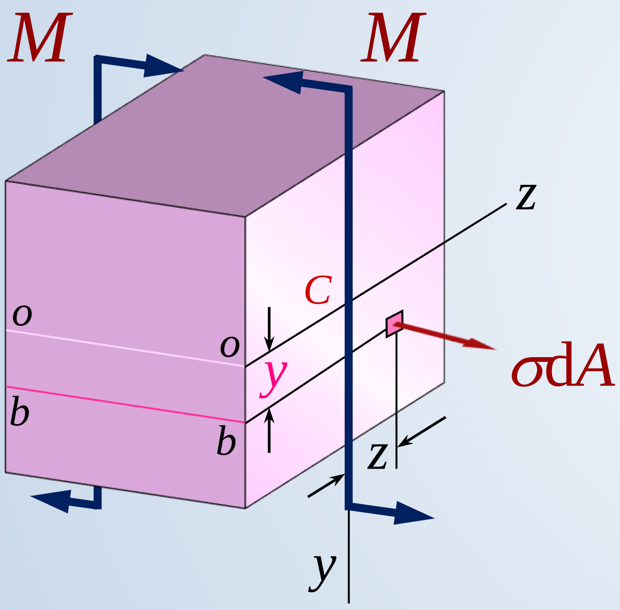
结 论

横截面上σ沿y轴线性分布，中性轴上$\sigma=0$.

9.2 纯弯曲时梁横截面上的正应力

三. 静力关系

将平行力系$\sigma \mathrm{d}A$向形心C简化，得到F_{N}, M_y , M_z



$$F_{\mathrm{N}}=\int_A \sigma \mathrm{d}A=0$$

$$M_y=\int_A z\cdot\sigma \mathrm{d}A=0$$

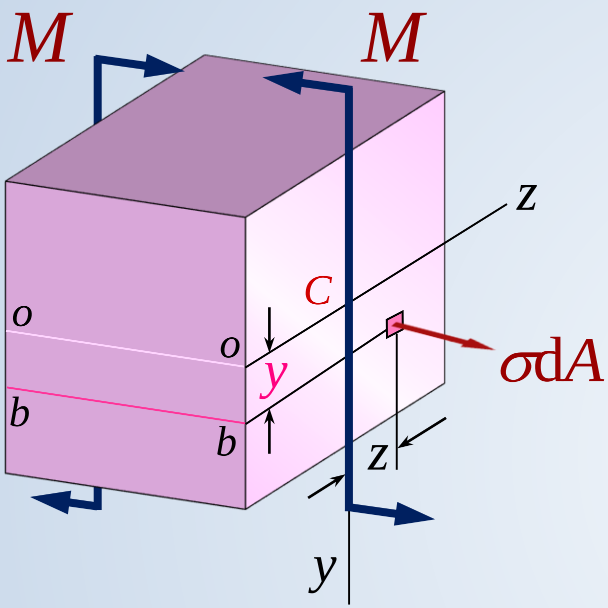
$$M_z=\int_A y\cdot\sigma \mathrm{d}A=M$$

将 $\sigma=\dfrac{E}{\rho}y$ 代入 $F_{\mathrm{N}}=\int_A \sigma \mathrm{d}A=0$

$$\sigma=\frac{E}{\rho}y$$

$$F_{\mathrm{N}}=\int_A \frac{E}{\rho}y\mathrm{d}A=0 \qquad F_{\mathrm{N}}=\frac{E}{\rho}\int_A y\mathrm{d}A=0$$

9.2 纯弯曲时梁横截面上的正应力



令 $S_z = \int_A y \mathrm{d}A$

S_z 为 A 对 z 轴的静矩；

又可表示为 $S_z = \bar{Y}A$

因为 $F_\mathrm{N} = \dfrac{E}{\rho}\int_A y\,\mathrm{d}A = \dfrac{E}{\rho}S_z = 0$

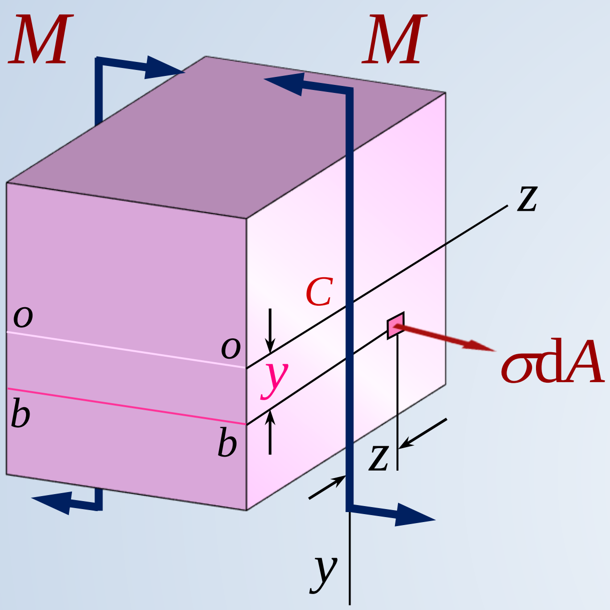
$$F_\mathrm{N} = \frac{E}{\rho}\int_A y\,\mathrm{d}A = 0$$

故 $S_z = \bar{Y}A = 0$

得 $\bar{Y} = 0$

结 论 z轴中性轴过横截面形心C。

9.2 纯弯曲时梁横截面上的正应力



将 $\sigma = \dfrac{E}{\rho}y$ 代入 $M_y = \int_A z \cdot \sigma \mathrm{d}A = 0$

$$M_y = \int_A z \cdot \frac{E}{\rho} y \mathrm{d}A = \frac{E}{\rho}\int_A yz\mathrm{d}A = 0$$

令 $I_{yz} = \int_A yz\mathrm{d}A$

I_{yz}为 A 对 y, z 轴的惯性积

$$F_N = \int_A \sigma \mathrm{d}A = 0$$

$$M_y = \int_A z\sigma \mathrm{d}A = 0$$

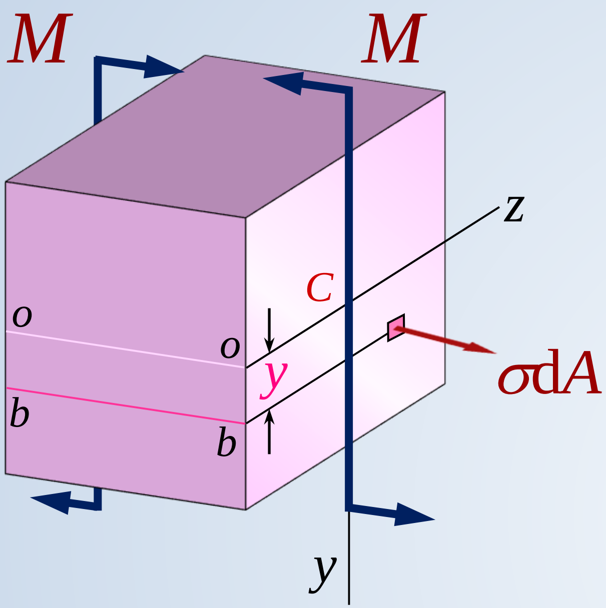
$$M_z = \int_A y\sigma \mathrm{d}A = M$$

显然若y,z 轴中有一个为对称轴

则 $I_{yz} = 0$

结 论 式 (b)自然满足。

9.2 纯弯曲时梁横截面上的正应力

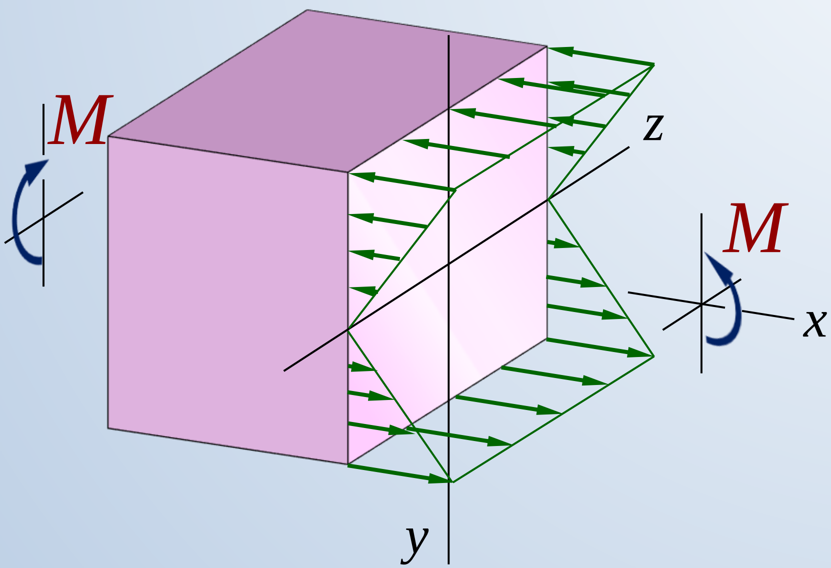


将 $\sigma = \frac{E}{\rho} y$ 代入 $M_z = \int_A y \cdot \sigma \mathrm{d}A = M$

$$M_z = \int_A y \cdot \frac{E}{\rho} y \mathrm{d}A = \frac{E}{\rho} \int_A y^2 \mathrm{d}A = M$$

令 $I_z = \int_A y^2 \mathrm{d}A$

I_z 为A 对z 轴的惯性矩

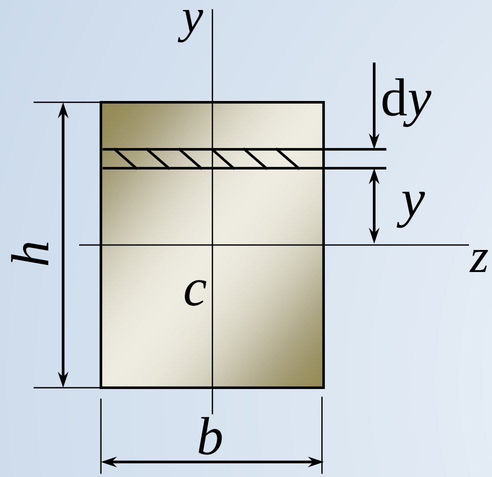


于是 $\frac{E}{\rho} I_z = M$ 得 $\frac{1}{\rho} = \frac{M}{EI_z}$

代入 $\sigma = \frac{E}{\rho} y$ 得 $\boxed{\sigma = \frac{M \cdot y}{I_z}}$

9.2 纯弯曲时梁横截面上的正应力

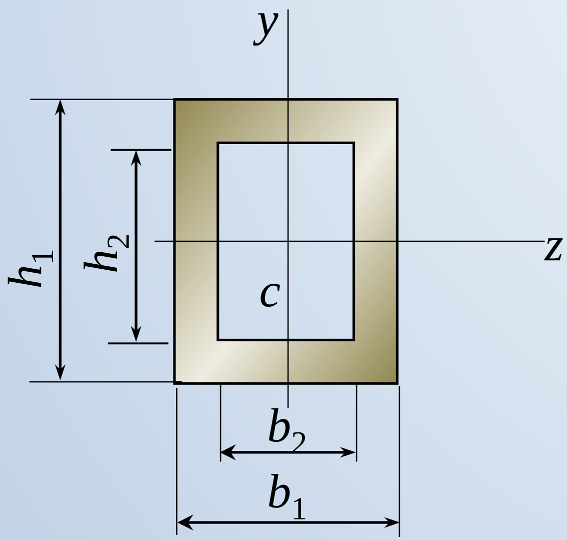
常用图形I_y、I_z



1. 矩形

$$I_z = \int_A y^2 \mathrm{d}A = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 b\mathrm{d}y = \frac{bh^3}{12}$$

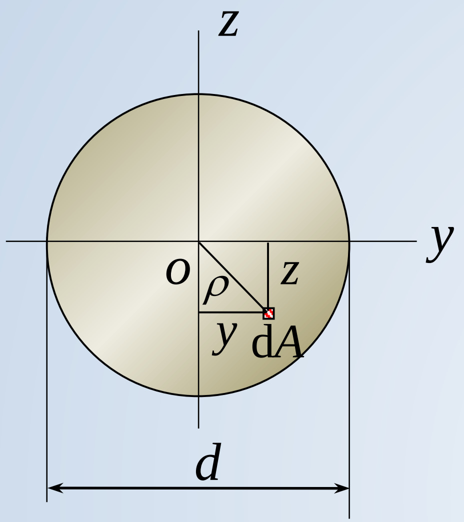
同理： $I_y = \dfrac{hb^3}{12}$



$$I_z = \frac{b_1 h_1^{\ 3}}{12} - \frac{b_2 h_2^{\ 3}}{12}$$

同理： $I_y = \dfrac{h_1 b_1^{\ 3}}{12} - \dfrac{h_2 b_2^{\ 3}}{12}$

9.2 纯弯曲时梁横截面上的正应力



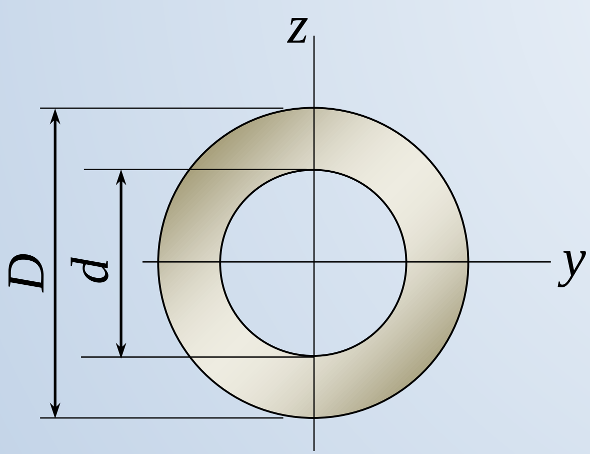
2.圆形

由定义知：

$$I_{\mathrm{P}} = \int_A \rho^2 \mathrm{d}A = \int_A (y^2 + z^2)\mathrm{d}A$$

$$= I_z + I_y$$

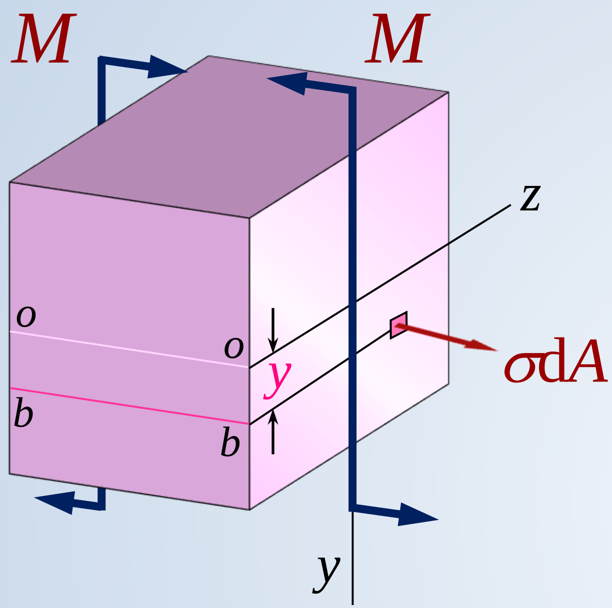
$$I_z = I_y = \frac{I_{\mathrm{P}}}{2} = \frac{\pi d^4}{64}$$

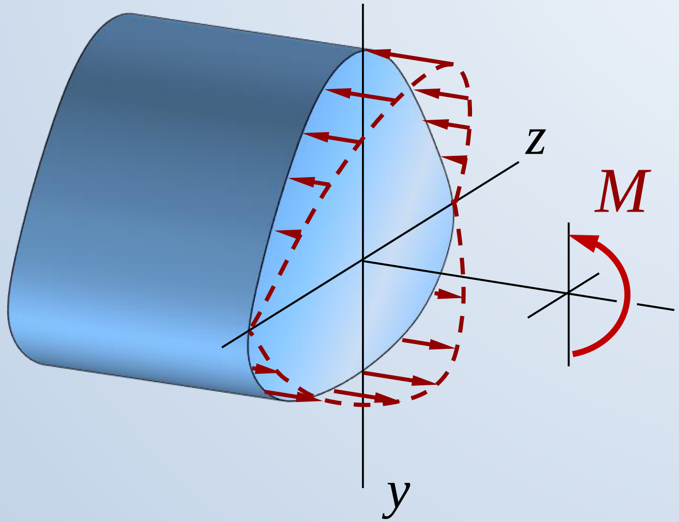


$$\alpha = \frac{d}{D}$$

$$I_z = I_y = \frac{\pi D^4}{64} - \frac{\pi d^4}{64} = \frac{\pi D^4}{64}(1-\alpha^4)$$

9.2 纯弯曲时梁横截面上的正应力





$$\boxed{\sigma = \frac{M \cdot y}{I_z}}$$

适用条件:

1.平面弯曲;

2.纯弯曲;

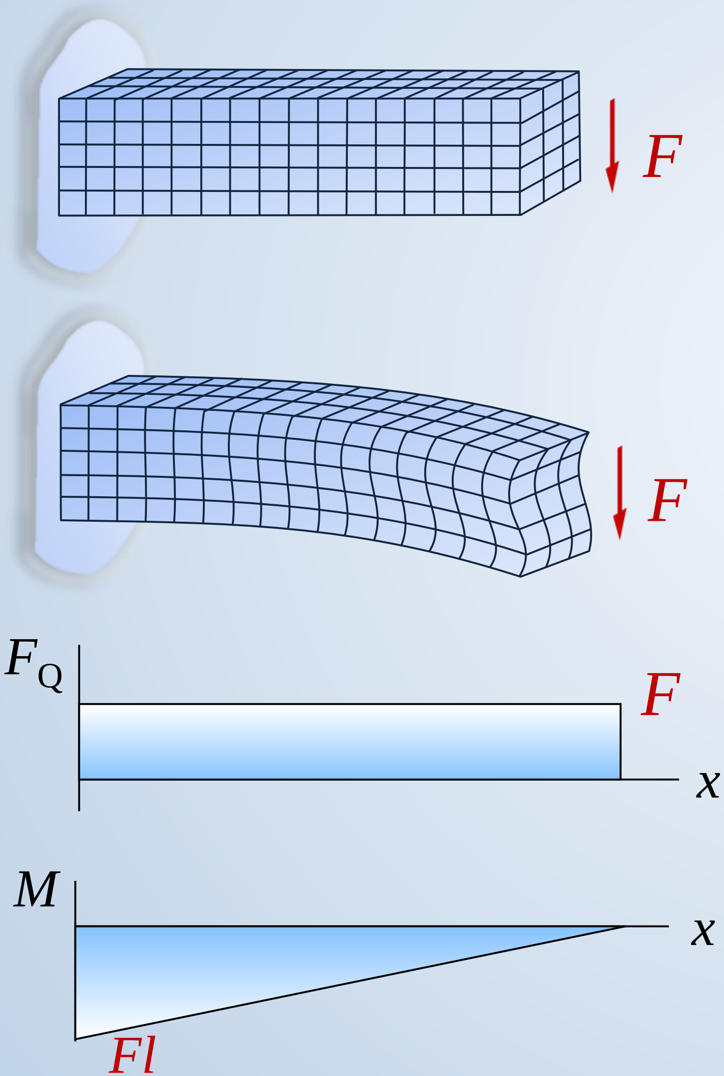
3. $\sigma \le \sigma_{\mathrm{p}}$, $E_{\mathrm{t}} = E_{\mathrm{c}}$;

4.等截面直梁;

5.截面形状任意.

9.3 横力弯曲时梁横截面上的正应力　弯曲正应力强度条件

一. 横力弯曲



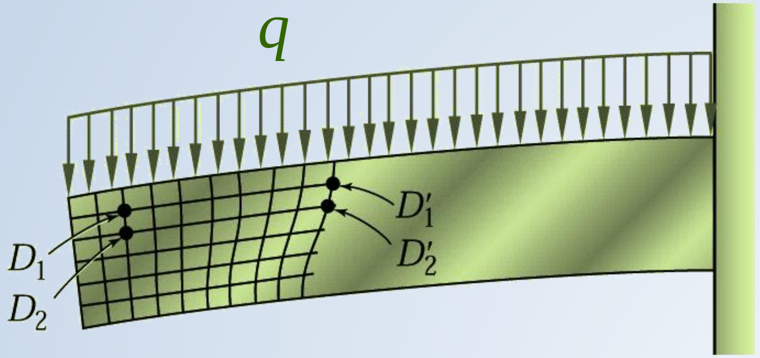
M —σ— ε

F_Q —τ —γ 横截面翘曲

当F_Q=C, 各横截面翘曲相同

结　论　用公式 $\sigma\dfrac{M\cdot y}{I_z}$

计算仍是完全正确的



当$F_{\mathrm{Q}} \neq C$,各横截面翘曲不相同

理论分析与实验表明

结 论 当 $l/h \geq 5$ 用公式 $\sigma = \dfrac{M \cdot y}{I_z}$ 计算，其影响小于1.7%，工程上是完全允许的。

结 论

纯弯曲 等截面 直梁	条件放松 → 公式推广	横力弯曲 变截面梁 折梁 曲梁

$$\sigma = \frac{M \cdot y}{I_z} \longrightarrow \sigma = \frac{M(x) \cdot y}{I_z(x)}$$

二. 弯曲正应力强度条件(1. 塑性　2. 脆性)

1. 塑性材料

$$\sigma_{\max} = \frac{|M|_{\max} |y|_{\max}}{I_z}$$

令 $W_z = \dfrac{I_z}{y_{\max}}$

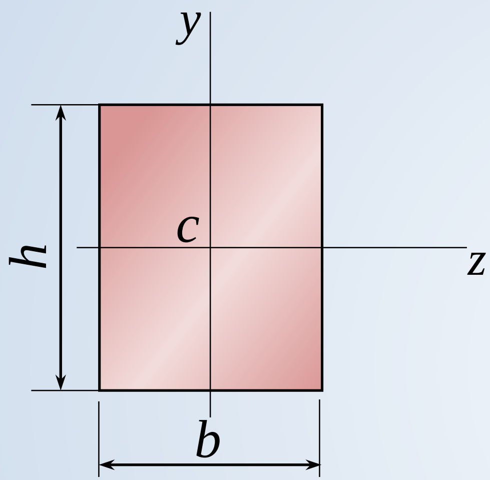
W_z —— 抗弯截面系数

等截面梁

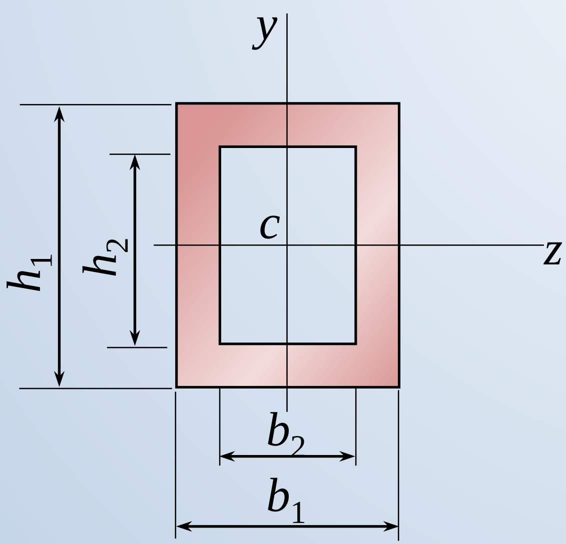
$$\boxed{\sigma_{\max} = \frac{|M|_{\max}}{W_z} \le [\sigma]}$$

注意　当梁为变截面梁时，$\sigma_{\max}$ 并不一定发生在 $|M|_{\max}$ 所在面上.

常用图形W_z

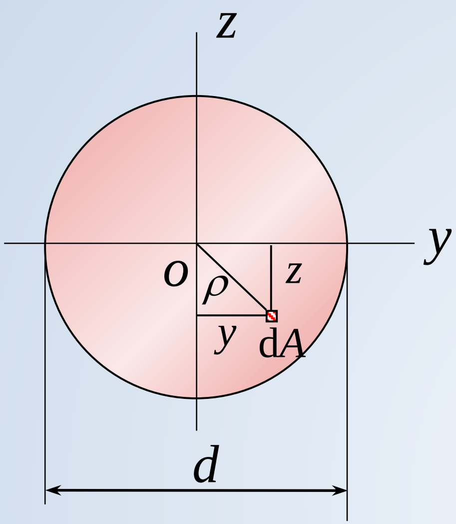


$$W_z=\frac{I_z}{\frac{1}{2}h}=\frac{bh^3}{12}\cdot\frac{2}{h}=\frac{bh^2}{6}$$

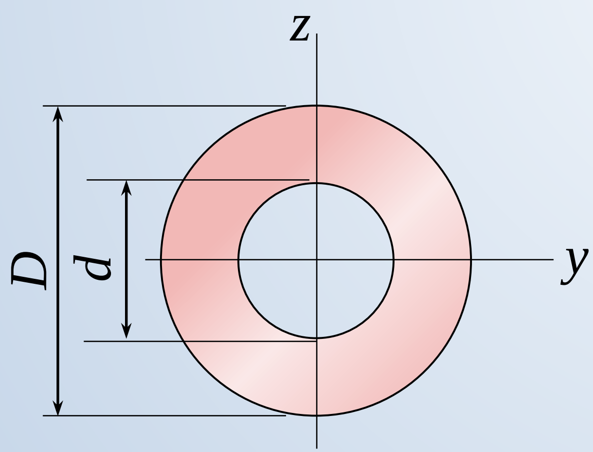


$$W_z=\frac{I_z}{\frac{1}{2}h_1}=\frac{1}{h_1}\left(\frac{b_1h_1^3}{6}-\frac{b_2h_2^3}{6}\right)$$

9.3 横力弯曲时梁横截面上的正应力　弯曲正应力强度条件



$$W_z = \frac{I_z}{\frac{1}{2}d} = \frac{\pi d^4}{64} \cdot \frac{2}{d} = \frac{\pi d^3}{32}$$



$$\alpha = \frac{d}{D}$$

$$W_z = \frac{I_z}{\frac{1}{2}D} = \frac{\pi D^4 (1-\alpha^4)}{64} \cdot \frac{2}{D}$$

$$= \frac{\pi D^3}{32}(1-\alpha^4)$$

2.脆性材料

因为: $[\sigma_t] < [\sigma_c]$ 所以分别建立强度条件

$$\sigma_{\mathrm{t\,max}} = \frac{|M|_{\max} y_{\mathrm{t}}}{I_z} \le [\sigma_{\mathrm{t}}]$$

$$\sigma_{\mathrm{c\,max}} = \frac{|M|_{\max} y_{\mathrm{c}}}{I_z} \le [\sigma_{\mathrm{c}}]$$

注意

当截面中性轴不对称时，最大正弯矩和最大负弯矩所在截面，都是危险截面。

三. 强度计算

强度条件解决三类问题 $\left\{\begin{array}{l}\text{1.校核}\\ \text{2.设计}\\ \text{3.确载}\end{array}\right.$

塑性材料 步骤

1.求外力、内力（画M 图, 确定危险截面 $|M|_{\max}$ ）

2.应力计算（危险点）

3.强度计算 $$\sigma_{\max}=\frac{|M|_{\max}}{W_z}\leq[\sigma]$$

脆性材料 步骤

1.求内力（画M 图, 确定危险截面）

二个危险截面 $+M_{\max}$ $\left|-M\right|_{\max}$

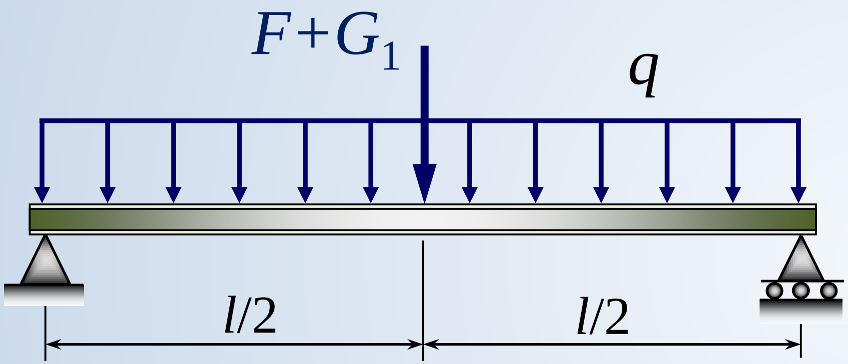
*2. 确定危险点（画危险截面σ 分布）

3.强度计算（先计算 $y_{\mathrm{t}}, y_{\mathrm{c}}, I_z$）

$$\sigma_{\mathrm{t\,max}} = \frac{\left|M\right|_{\max} y_{\mathrm{t}}}{I_z} \le [\sigma_t]$$

$$\sigma_{\mathrm{c\,max}} = \frac{\left|M\right|_{\max} y_{\mathrm{c}}}{I_z} \le [\sigma_{\mathrm{c}}]$$

例1　已知 F=50kN, G_1=6.5kN, q为梁自重，l=10m, $[\sigma]$=140MPa，试选择工字钢截面.

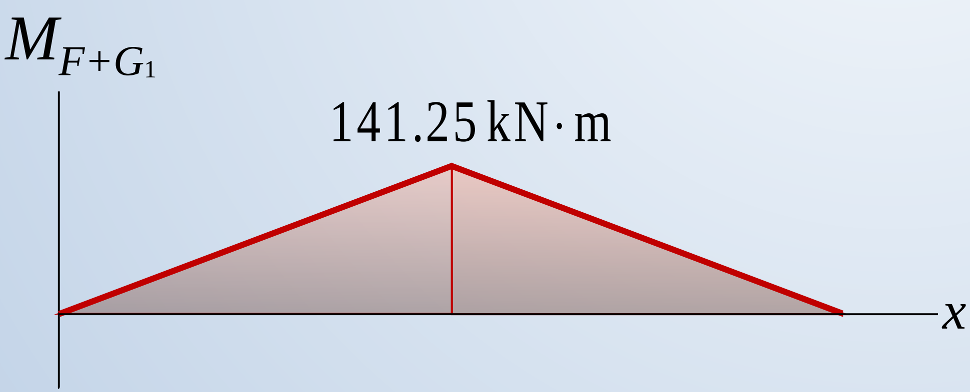


分析• 先按$F+G_1$选截面

• 查表　• 检验

解：1. 画M 图— M_{max}

$$M_{max} = 141.25\,\text{kN}\cdot\text{m}$$

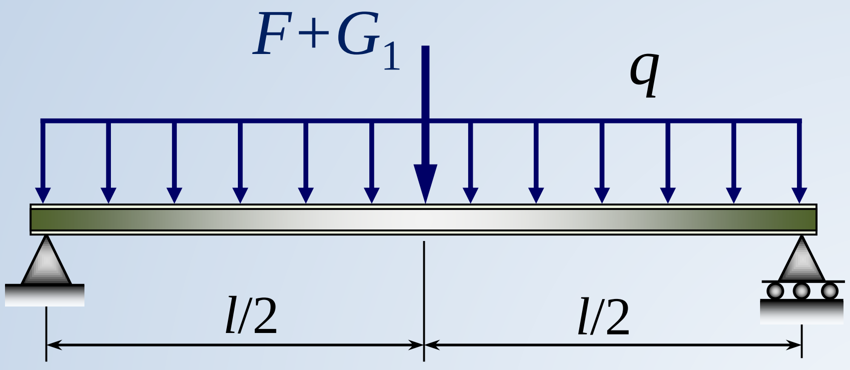


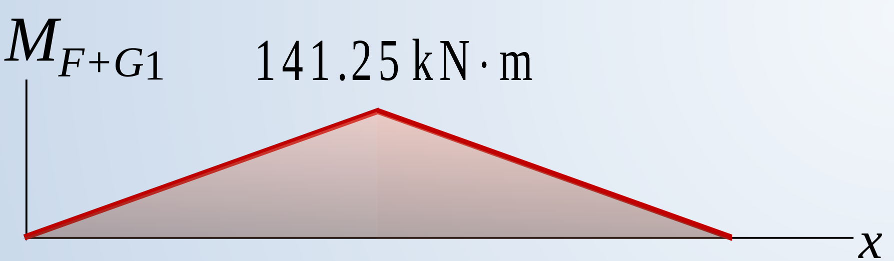
2. 应用强度条件计算

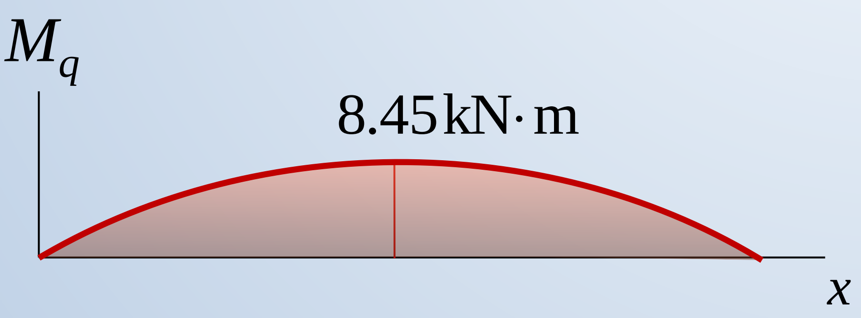
$$\sigma_{max} = \frac{|M|_{max}}{W_z} \le [\sigma]$$

$$W_z \ge \frac{M_{max}}{[\sigma]} = \frac{141.25\times 10^3}{140\times 10^6} = 1009\,\text{cm}^3$$

9.3 横力弯曲时梁横截面上的正应力　弯曲正应力强度条件







3.查表

40a工字钢　$W_z = 1090\ \mathrm{cm}^3$

4.检验

$q = 67.6\ \mathrm{kg/m}.\quad M_q = 8.45\,\mathrm{kN\cdot m}$

$$M_{总} = M_{\max} + M_q = 149.7\,\mathrm{kN\cdot m}$$

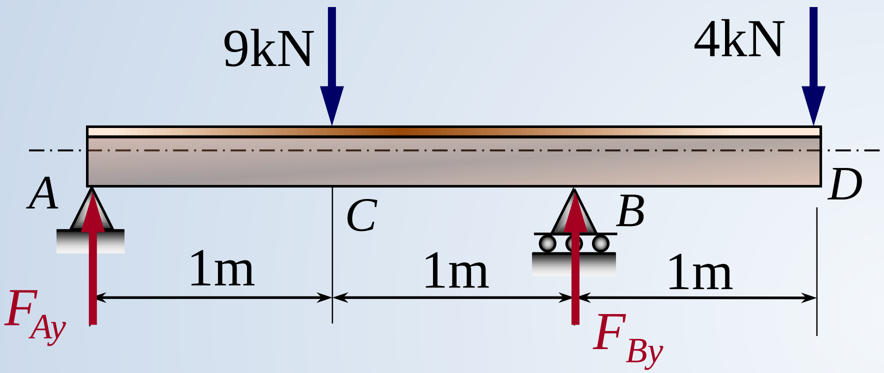
$$\sigma_{\max} = \frac{M_{总}}{W_z} = \frac{149.7\times 10^3}{1090\times 10^{-6}}$$

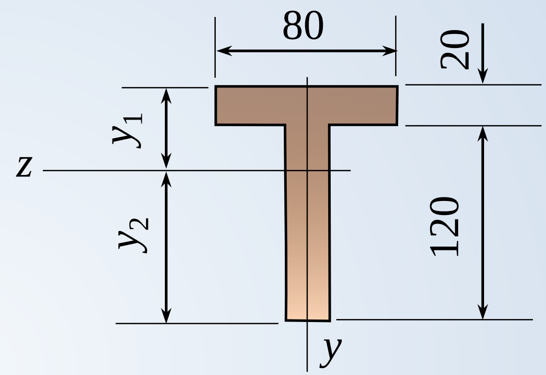
$$= 137.34\,\mathrm{MPa} < [\sigma]$$

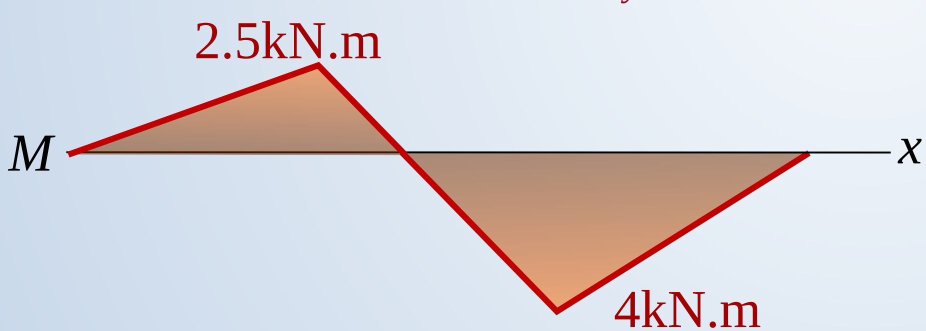
40a工字钢满足强度条件

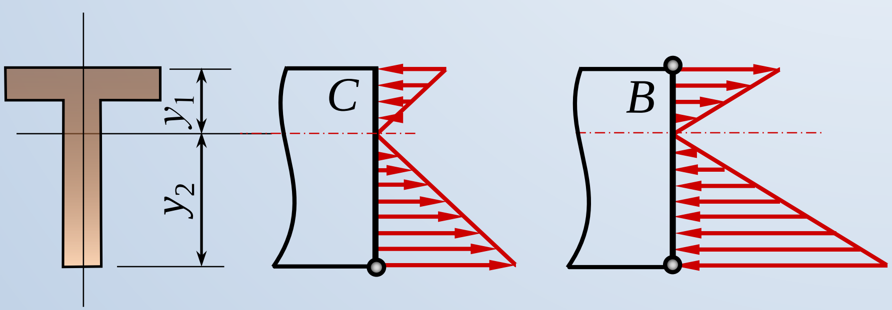
例2　已知$[\sigma_t]$=30MPa，$[\sigma_c]$=160MPa，I_z=763cm^4, y_1=52mm

试校核梁的强度.









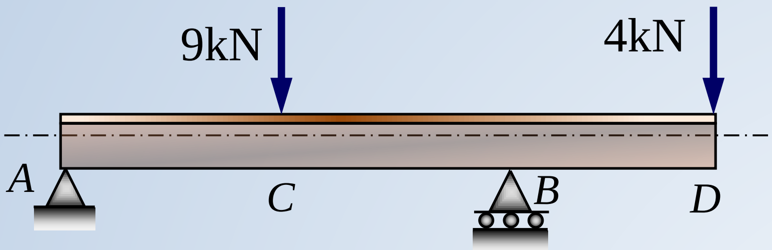
解：1. 求内力（画M 图）

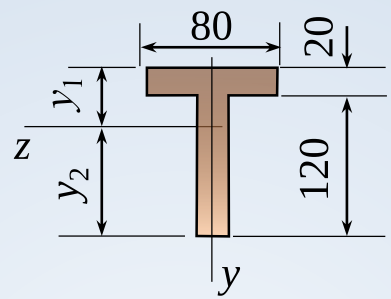
$F_{Ay} = 2.5\text{kN}$　　$F_{By} = 10.5\text{kN}$

危险截面：B, C

$M_C = 2.5\text{kN·m}$　　$M_B = -4\text{kN·m}$

2.　画σ分布，确定危险点(如图)

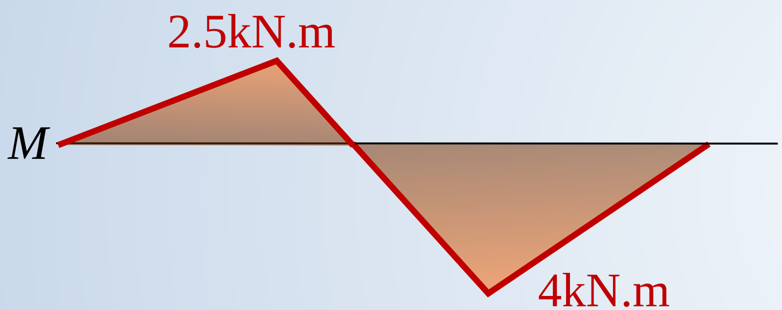




3.强度计算

$y_1=52\text{mm}$　　$I_z=763\text{cm}^4$，

$y_2=120+20-52=88\text{mm}$

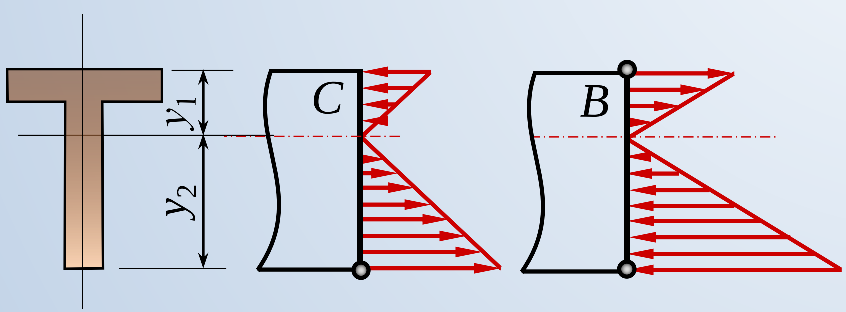


B截面:

$$\sigma_{\text{t}\max}=\frac{|M_B|y_1}{I_z}=\frac{4\times10^3\times52\times10^{-3}}{763\times10^{-8}}=27.2\,\text{MPa}$$

$$\sigma_{\text{c}\max}=\frac{|M_B|y_2}{I_z}=46.7\,\text{MPa}$$

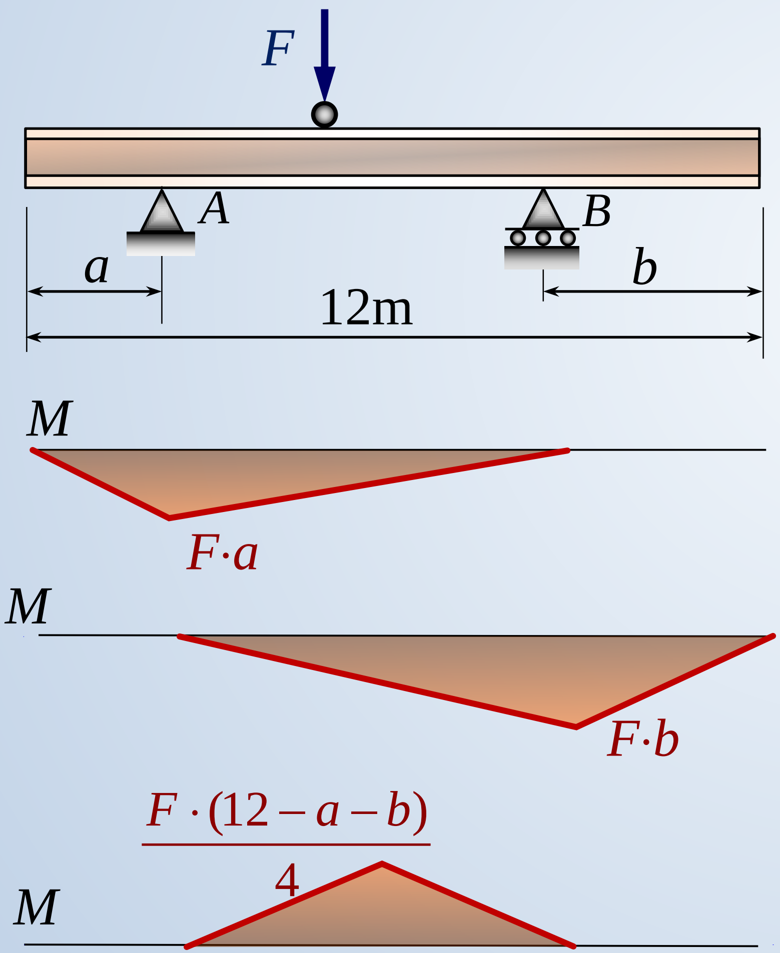
C截面:　$$\sigma_{\text{t}\max}=\frac{M_C y_2}{I_z}=28.8\,\text{MPa}$$



$$\because\ \sigma_{\text{t}\max}<[\sigma_\text{t}]\,,\quad \sigma_{\text{c}\max}<[\sigma_\text{c}]$$

故此梁满足强度条件

例3 可动载荷F作用于18号工字钢梁，$[\sigma]=160\text{MPa}$, a,b多长时梁的强度最好？并确定许可载荷。



解：•画弯矩图

当 $+M_{\max}=|-M|_{\max}$ 时，$\sigma_{\max}$最小，此时梁的强度最好

$$Fa=Fb=\frac{F\cdot(12-a-b)}{4}$$

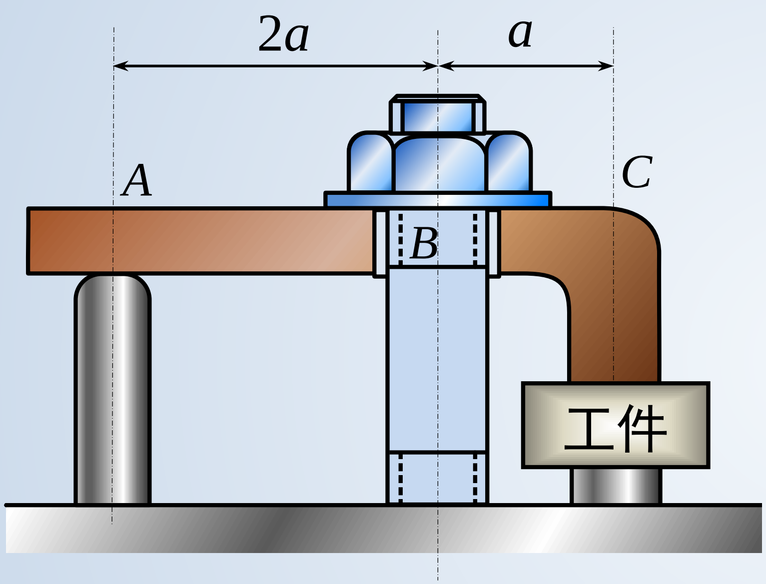
解得 ： $a=b=2\text{m}$ $M_{\max}=2F$

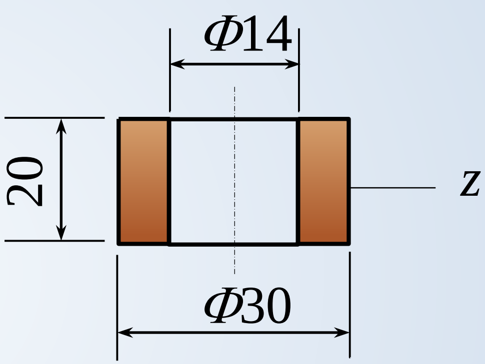
•由 $\sigma_{\max}=\dfrac{|M|_{\max}}{W_z}\le[\sigma]$ 确定许可载荷

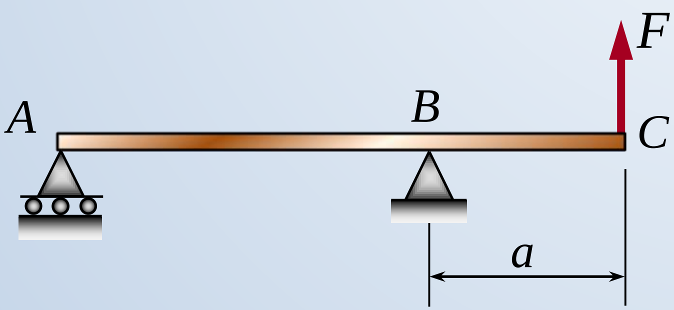
$$\frac{2F}{185\times10^{-6}}\le160\times10^{6}$$

$\boxed{[F]=14.8\text{kN}}$

例4　已知 $3a=150\text{mm}$, $[\sigma]=140\text{MPa}$.　求$[F]$

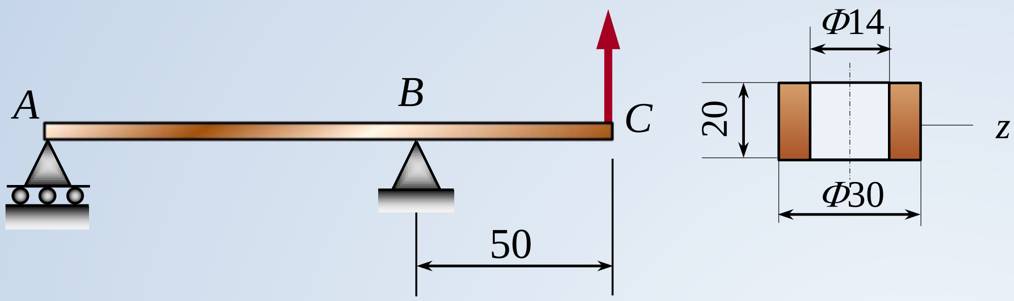


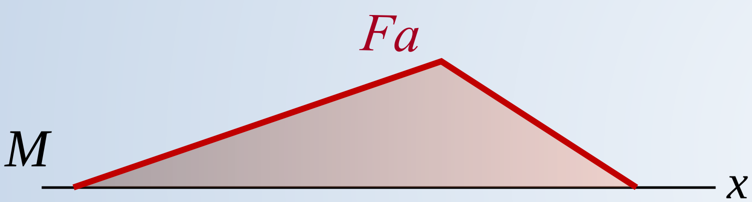




解：先建模

9.3 横力弯曲时梁横截面上的正应力　弯曲正应力强度条件





1.画M 图— $M_{\max}$

$$M_{\max} = M_B = F\cdot a$$

2. 应用强度条件计算

$$I_z = \frac{3\times 2^3}{12} - \frac{1.4\times 2^3}{12} = 1.07\,\text{cm}^4$$

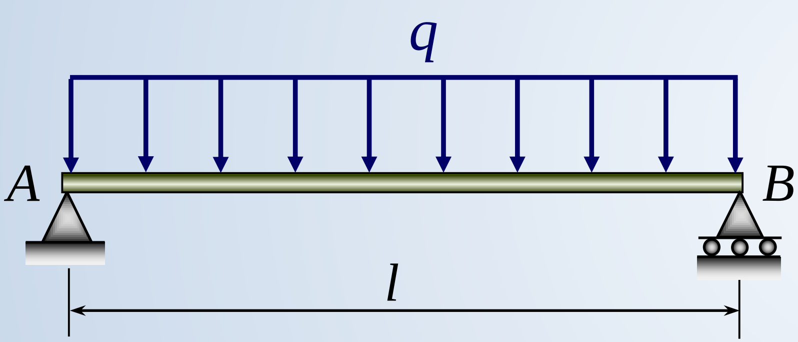
$$W_z = \frac{I_z}{2/2} = 1.07\,\text{cm}^3 = 1.07\times 10^{-6}\,\text{mm}^3$$

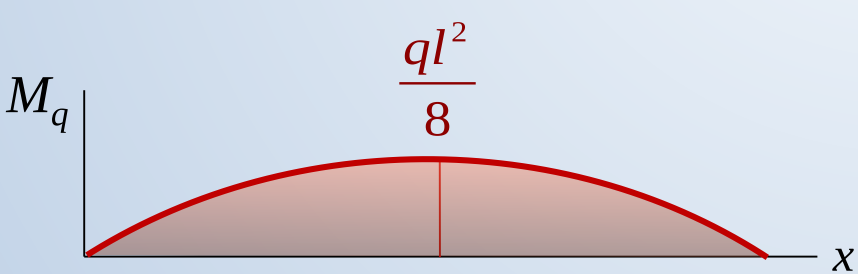
$$\sigma_{\max} = \frac{|M|_{\max}}{W_z} = \frac{Fa}{W_z} \le [\sigma]$$

$$F \le \frac{W_z[\sigma]}{a} = \frac{1.07\times 10^{-6}\times 140\times 10^{6}}{50\times 10^{-3}} = 3000\,\text{N}$$

$$\boxed{[F] = 3\text{kN}}$$

例5　已知 q=2kN/m, l=2m,分别采用截面面积相等的实心和空心圆截面, D_1=40mm, d_2/D_2=3/5, 求：1.$\sigma_{实}$, $\sigma_{空}$ 2.$(\sigma_{空}-\sigma_{实})/\sigma_{实}$





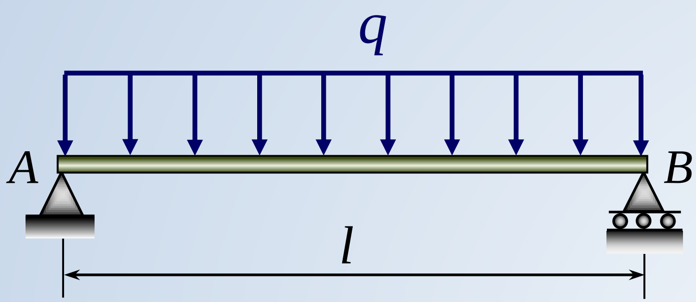
解：1. 画M图

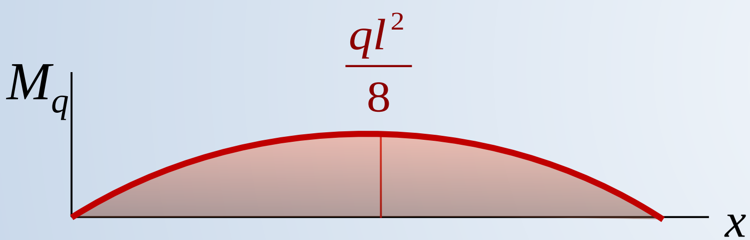
$$M_{\max}=\frac{ql^2}{8}=\frac{2\times10^3\times2^2}{8}=1\text{kN}\cdot\text{m}$$

2. 应力计算

实心圆截面

$$\sigma_{实\max}=\frac{M_{\max}}{W_z}=\frac{32M_{\max}}{\pi D^3}=159\,\text{MPa}$$





空心圆截面

$$A_{空} = A_{实},\ 且\ \alpha = \frac{d_2}{D_2},$$

即 $$\frac{\pi}{4}\left[D_2^2 - \left(\tfrac{3}{5}D_2\right)^2\right] = \frac{\pi}{4}D_1^2$$

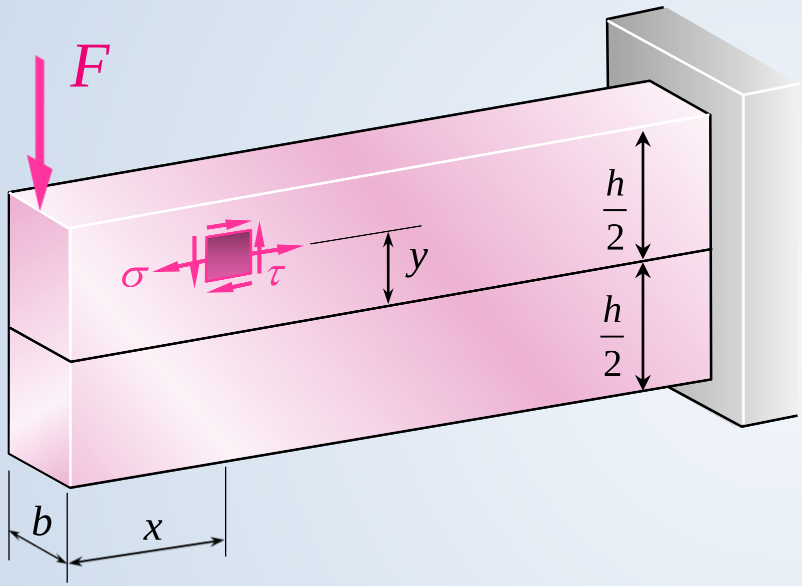
得 $$D_2 = \frac{5}{4}D_1 = 50\,\mathrm{mm}$$

$$\sigma_{空\max} = \frac{M_{\max}}{W_z} = \frac{M_{\max}}{\frac{1}{32}\pi D^3(1-\alpha^4)} = 93.6\,\mathrm{MPa}$$

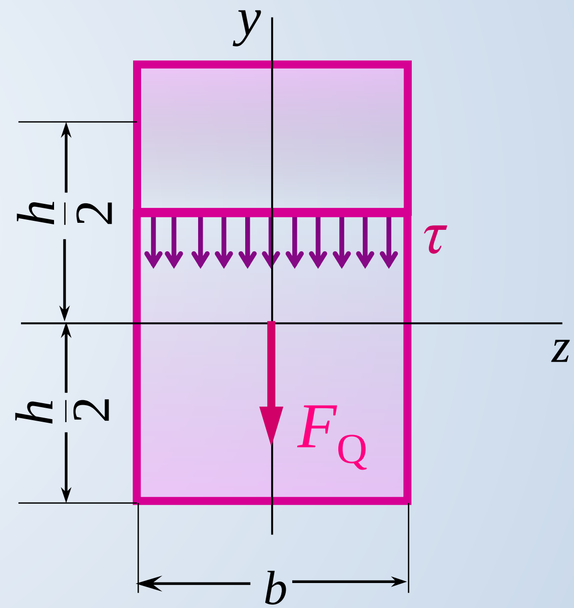
$$\frac{(\sigma_{实\max} - \sigma_{空\max})}{\sigma_{实\max}} \times 100\% = 41.2\%$$

空心圆截面梁更合理

一. 矩形截面

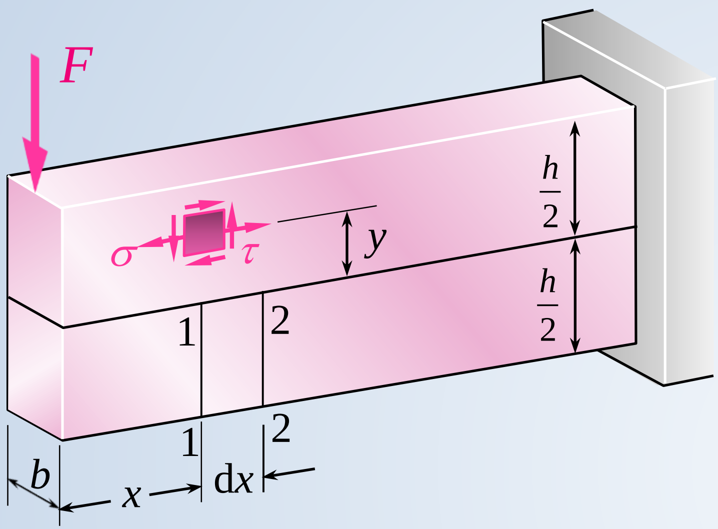


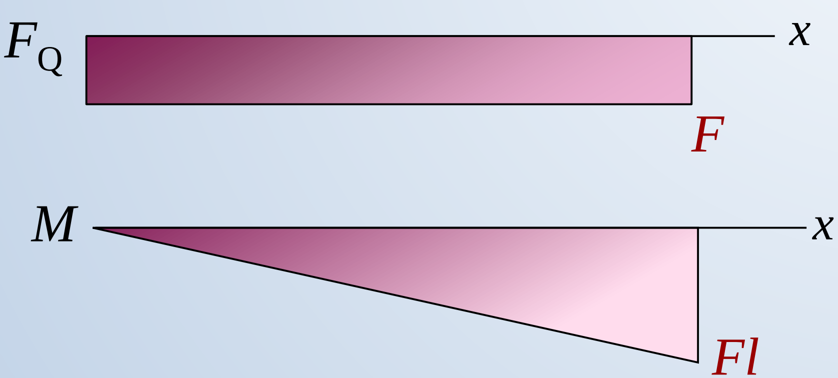
1. 假设τ 的分布:



τ平行于F_Q, 且方向同F_Q，τ沿b均布

9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

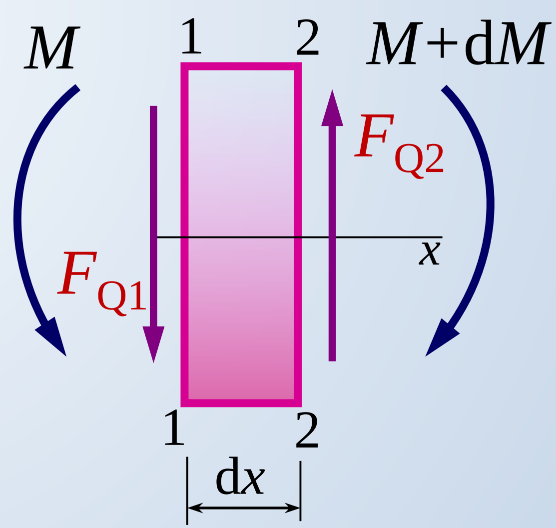




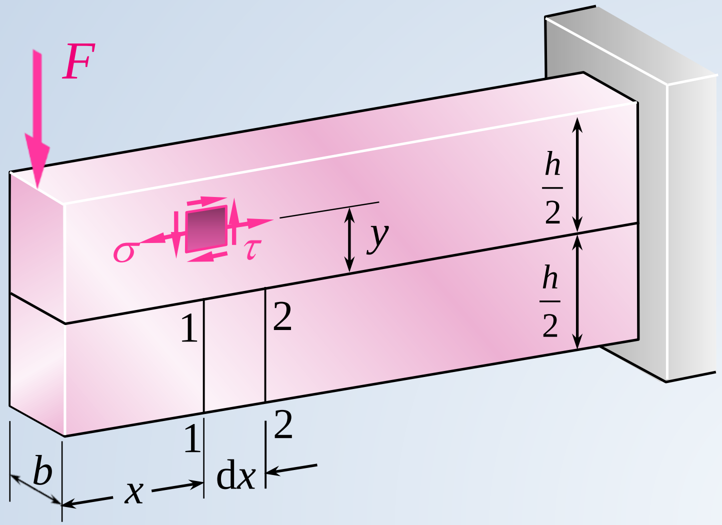
2. τ 的公式推导

F_Q图 M图

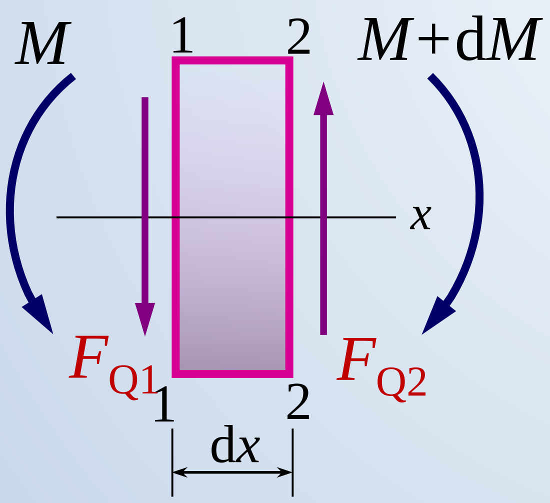
切dx段，

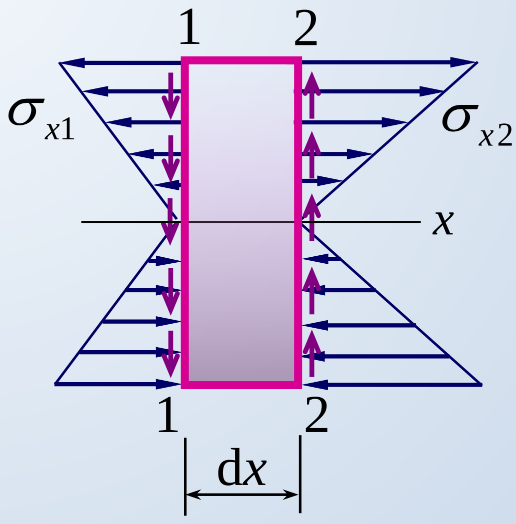


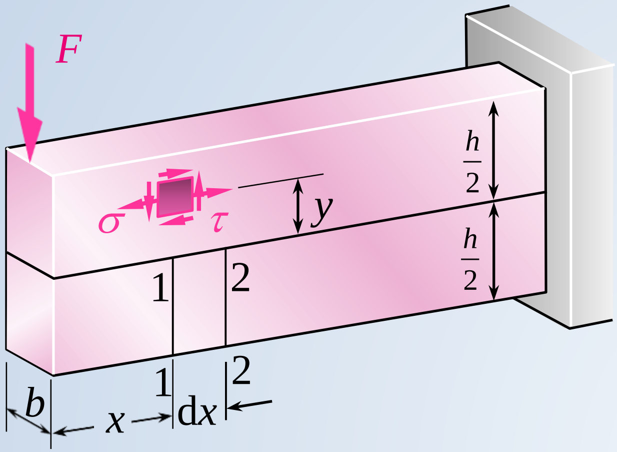
9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件



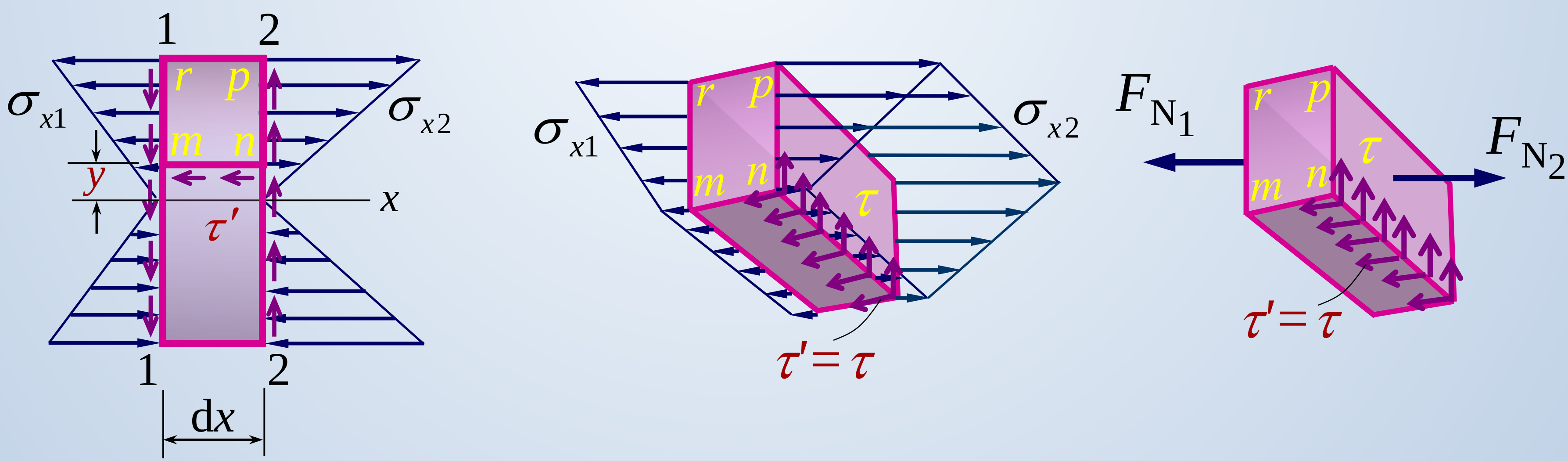
$F_Q \rightarrow \tau$ $M \rightarrow \sigma$



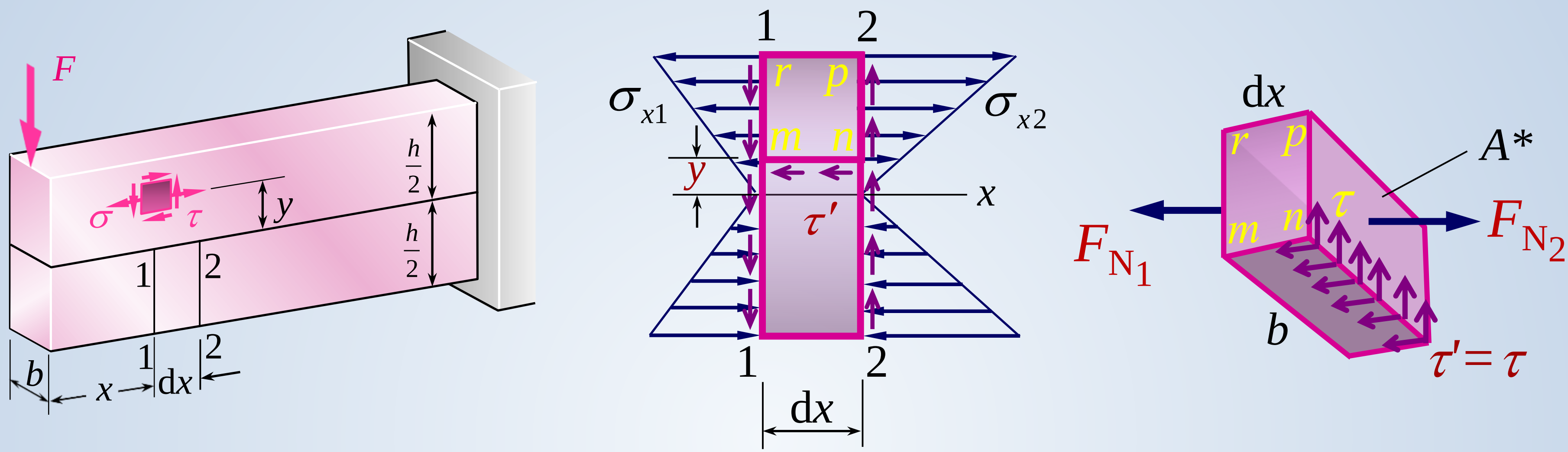




为求出横截面上任意点的τ，在距中性轴y处取出r,p,m,n



9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

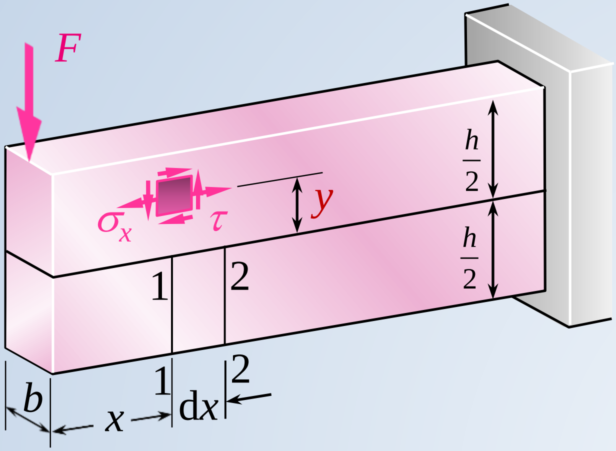


$$\sum F_x = 0 \qquad F_{N2} - F_{N1} = \tau' \cdot \mathrm{d}x \cdot b$$

$$F_{N1} = \int_{A^*} \sigma_{x1} \mathrm{d}A = \int_{A^*} \frac{M\ y_1}{I_z} \mathrm{d}A = \frac{M}{I_z} \int_{A^*} y_1 \mathrm{d}A$$

令 $S^* = \int_{A^*} y_1 \mathrm{d}A$ S^*为A^*对z 轴的静矩

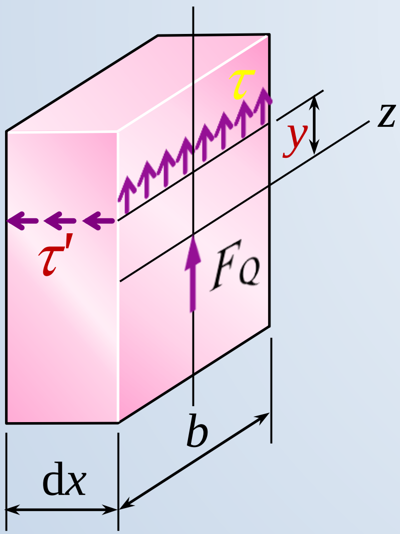
9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件



$$F_{N1}=\frac{M}{I_z}S^* \qquad S^*=\int_{A^*}y_1\,\mathrm{d}A$$

同理 $$F_{N2}=\frac{M+\mathrm{d}M}{I_z}S^*$$

将 F_{N_1}, F_{N_2} 代入 $\sum F_x=0$



$$F_{N2}-F_{N1}=\tau'\cdot\mathrm{d}x\cdot b$$

得 $$\tau'=\frac{\mathrm{d}M}{\mathrm{d}x}\cdot\frac{S^*}{I_z b}=\frac{F_Q S^*}{I_z b}$$

$$\boxed{\tau=\frac{F_Q S^*}{I_z b}}$$

矩形截面梁横截面上任意点τ

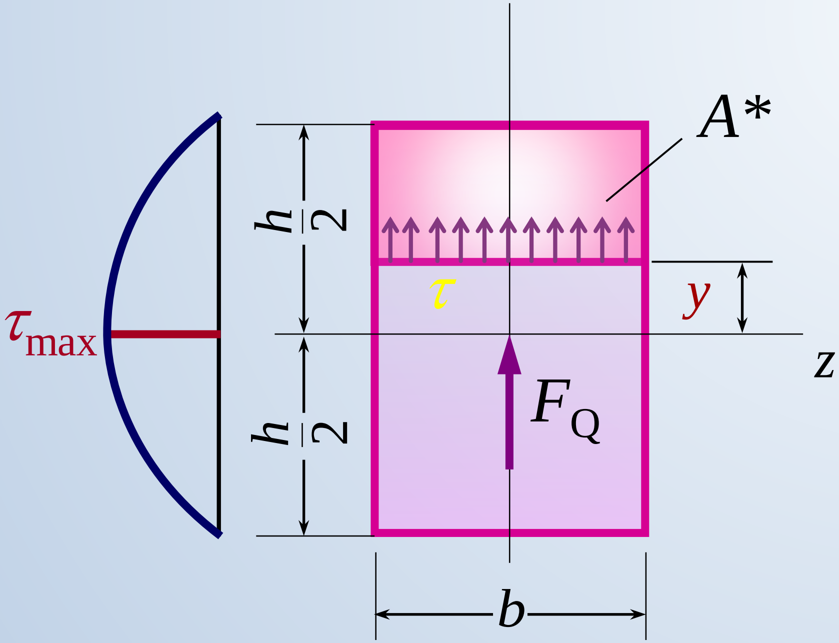
$$\tau = \frac{F_Q S^*}{I_z b}$$

$$S^* = \int_{A^*} y_1 \mathrm{d}A = A^* \cdot \bar{Y} = \frac{b}{2}(\frac{h}{2} - y)(\frac{h}{2} + y)$$

$$\tau = \frac{F_Q}{2I_z}(\frac{h^2}{4} - y^2)$$

τ 随 S_z^* 变

τ 沿 y 轴抛物线分布

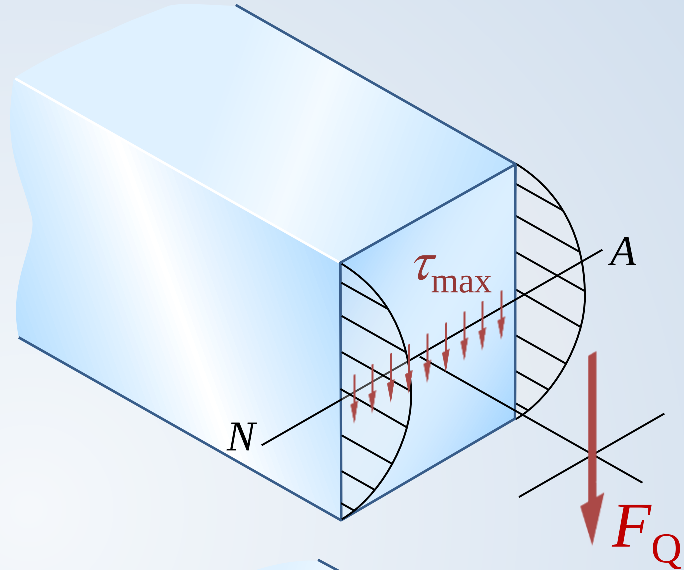


当 $y = 0$ 时 $\tau_{\max} = \frac{3F_Q}{2bh} = 1.5\tau_{平均}$

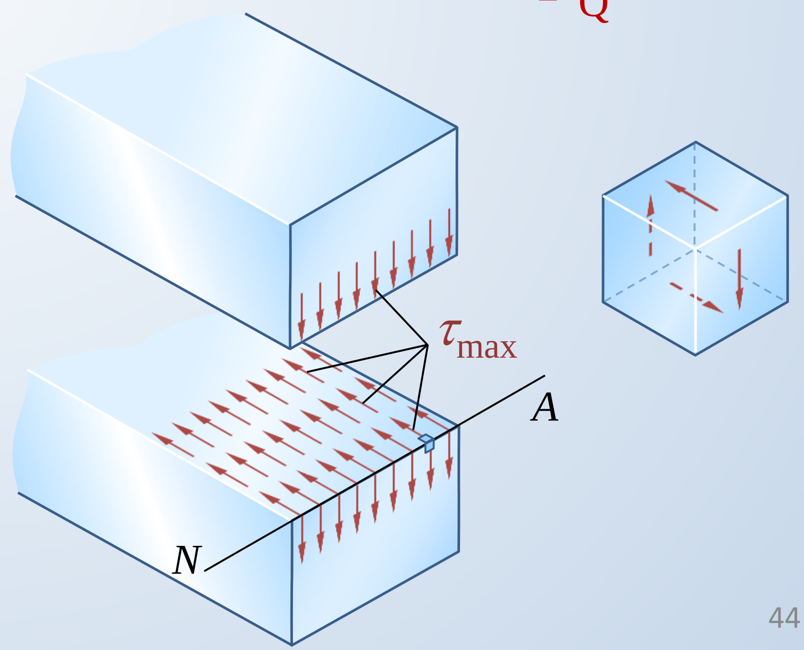
$$\tau_{\max} = \frac{3F_Q}{2A}$$

9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

切应力的分布

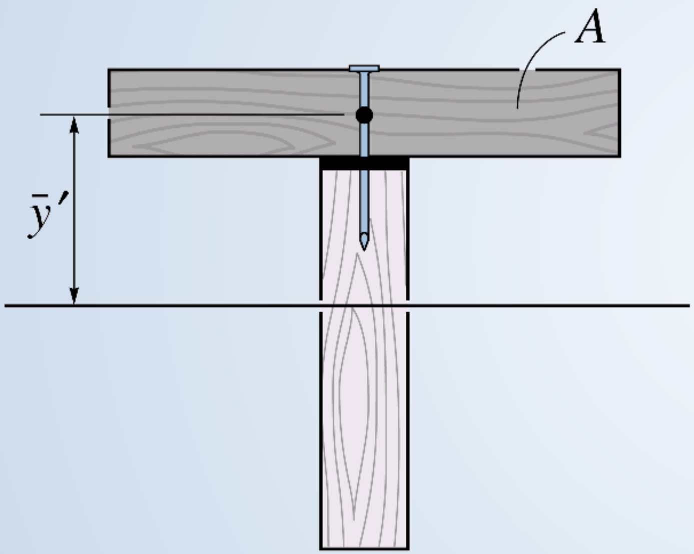


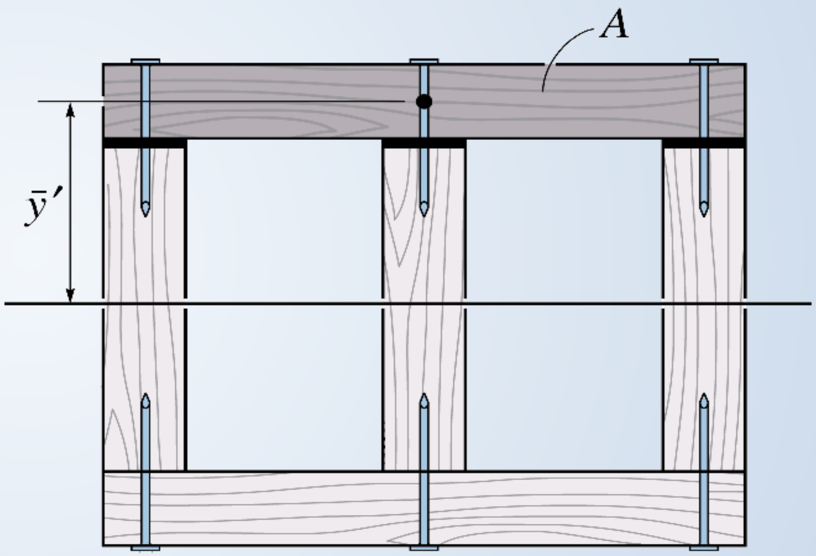
切应力互等定理



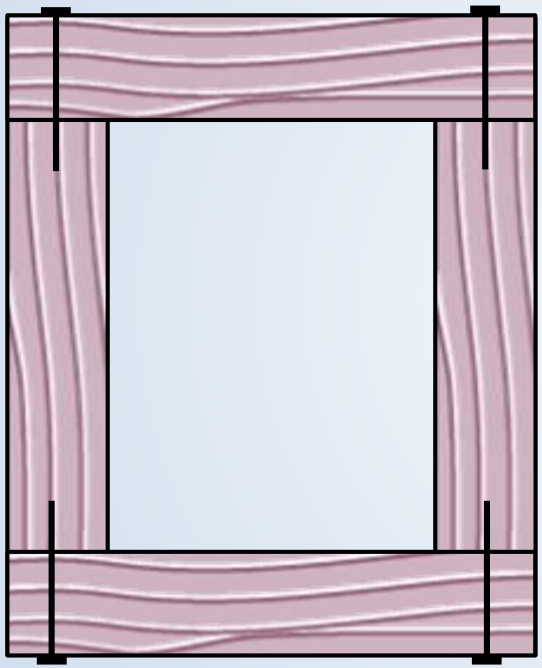
9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

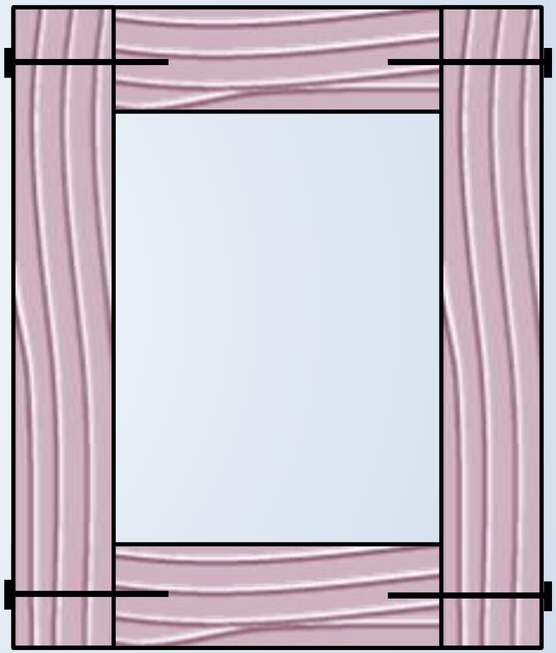
9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件



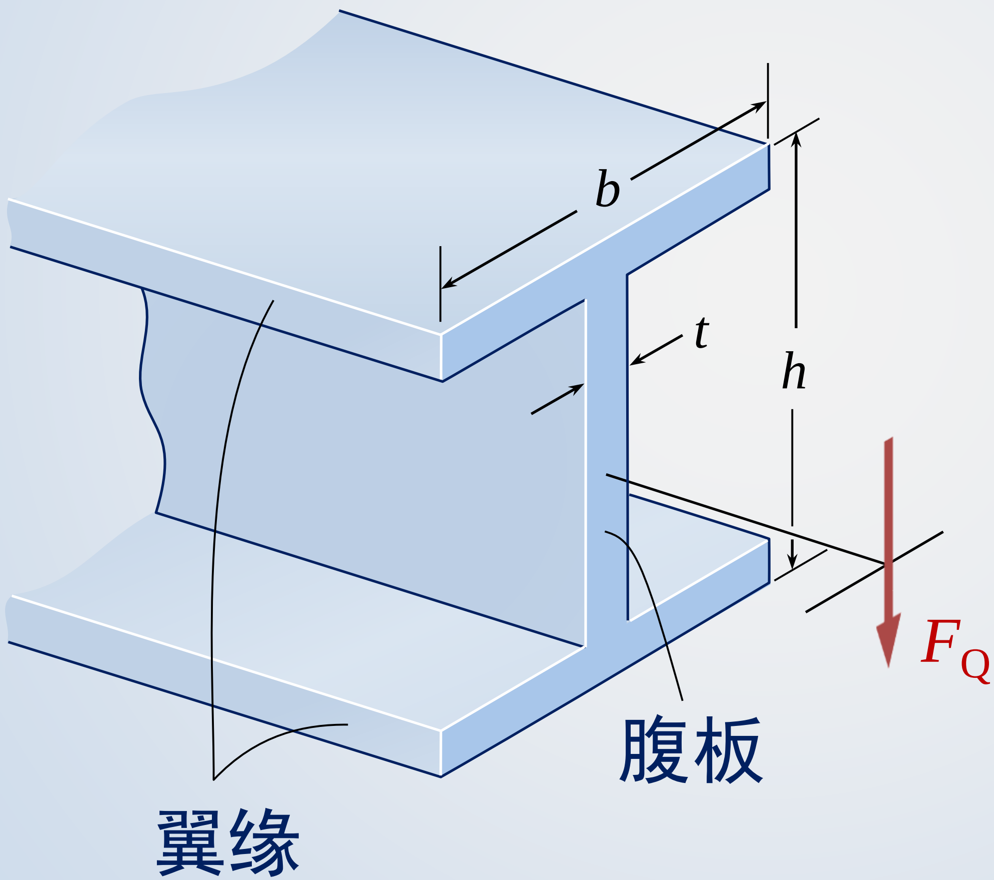


9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件





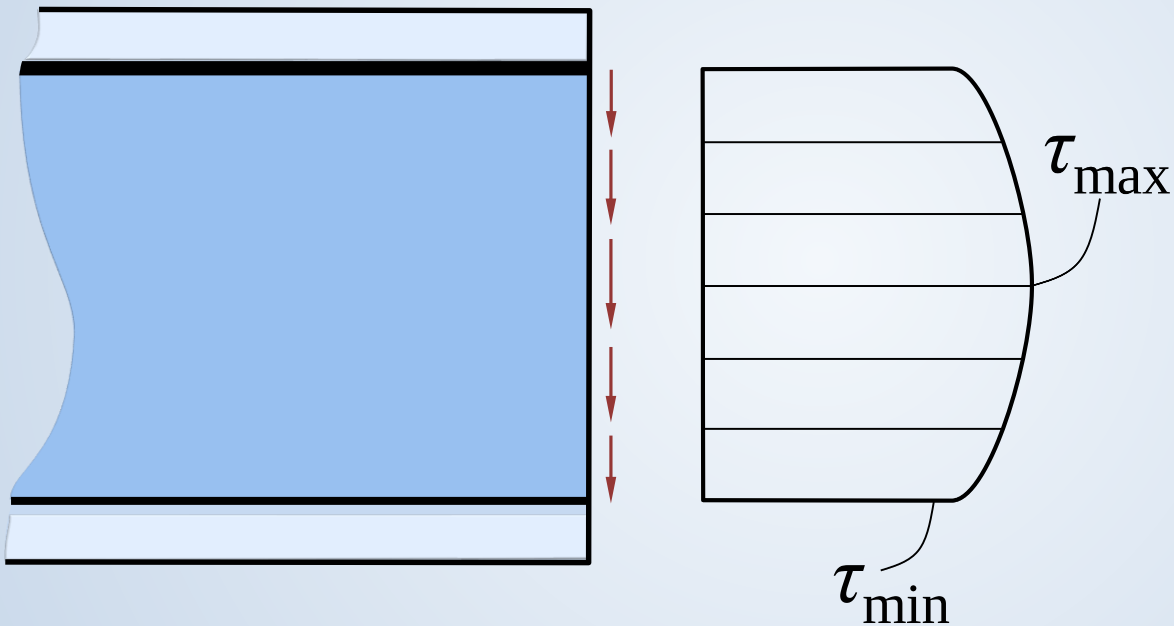
二. 工字形截面



$$\tau = \frac{F_Q S^*}{I_z b}$$

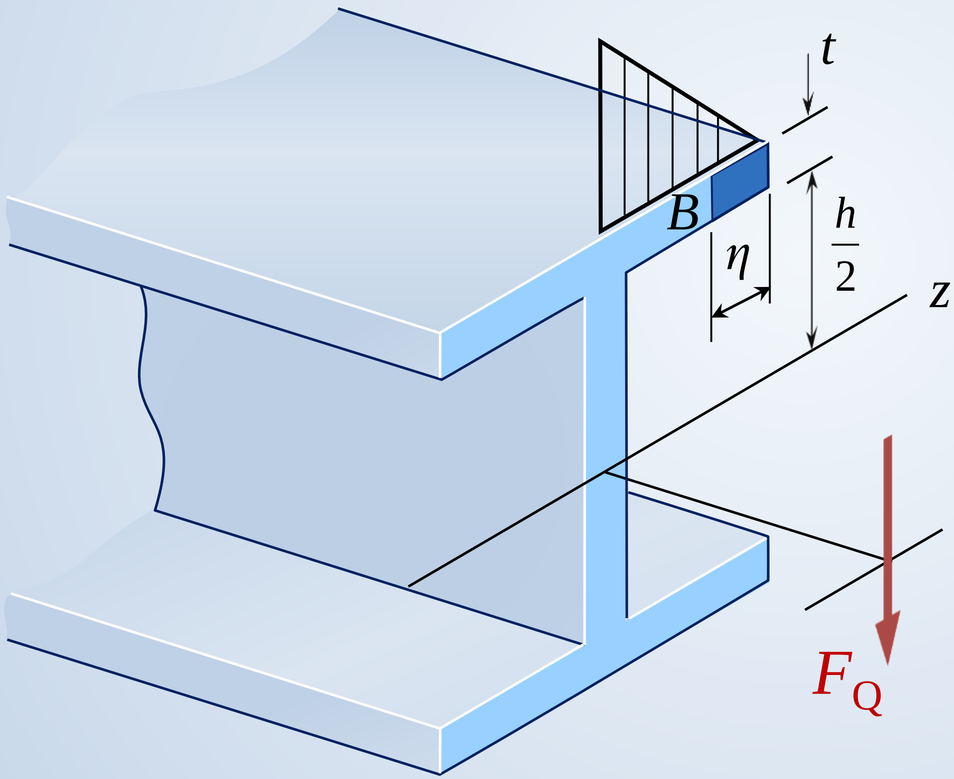
9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

腹板



$$\tau_{腹} \approx \frac{F_Q}{A_{腹}}$$

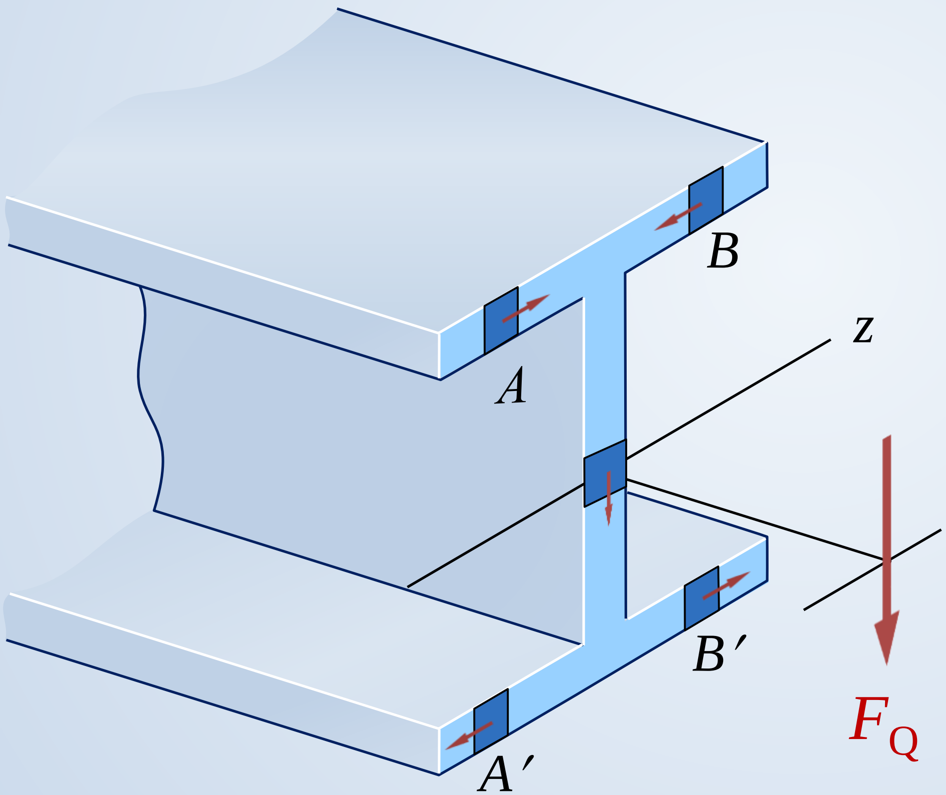
翼缘

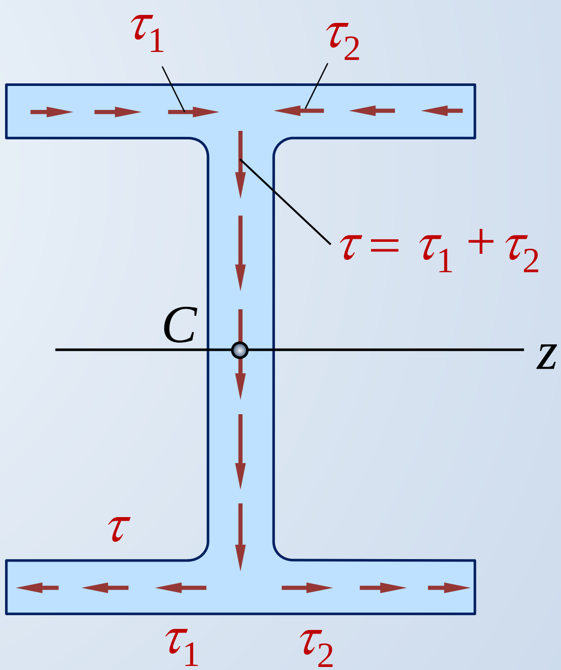


$$\tau_{翼} = \frac{F_Q S^*}{I_z t}$$

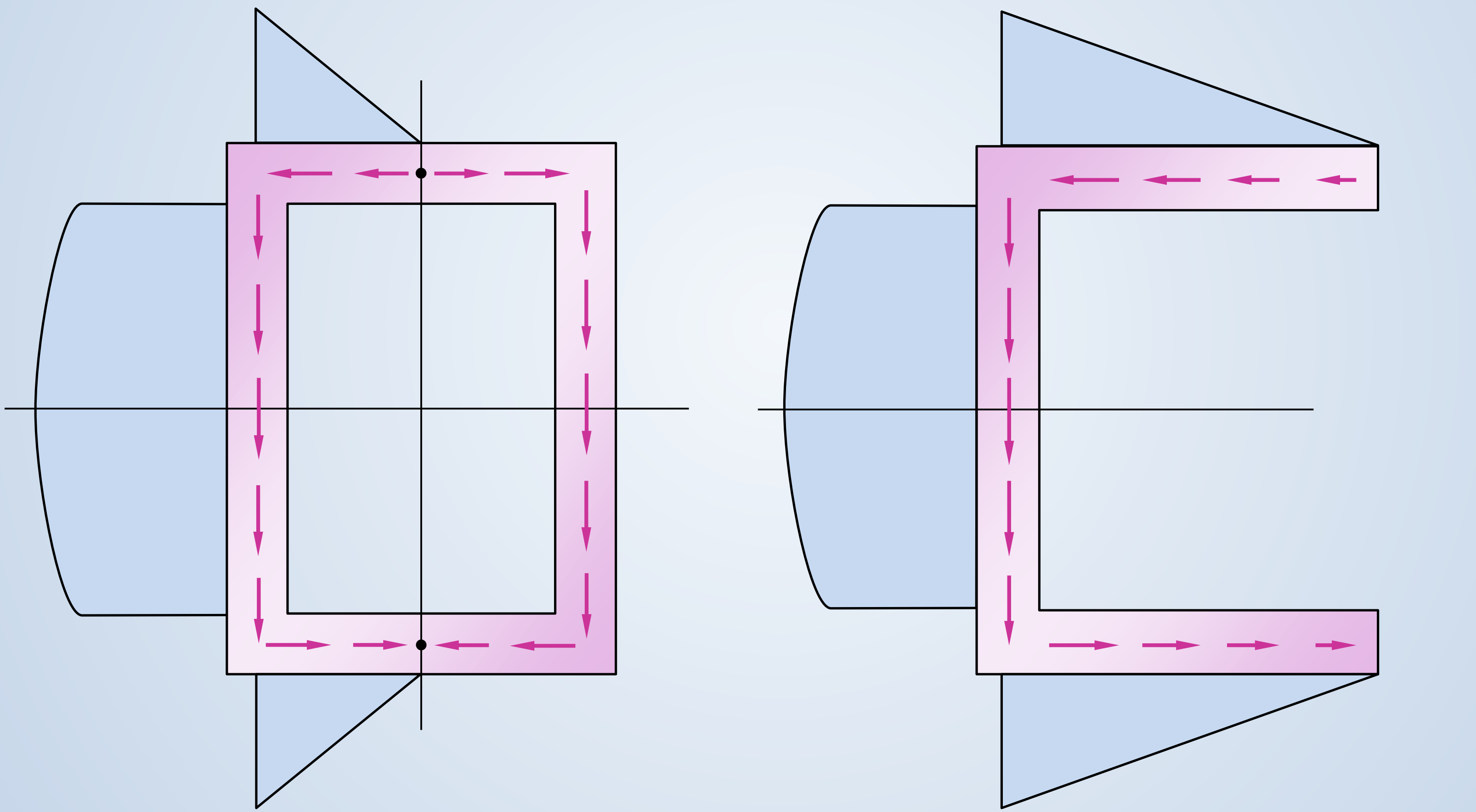
$$S^* = \eta t\left(\frac{h}{2} + \frac{t}{2}\right)$$

剪力流





9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件



三. 弯曲切应力强度条件

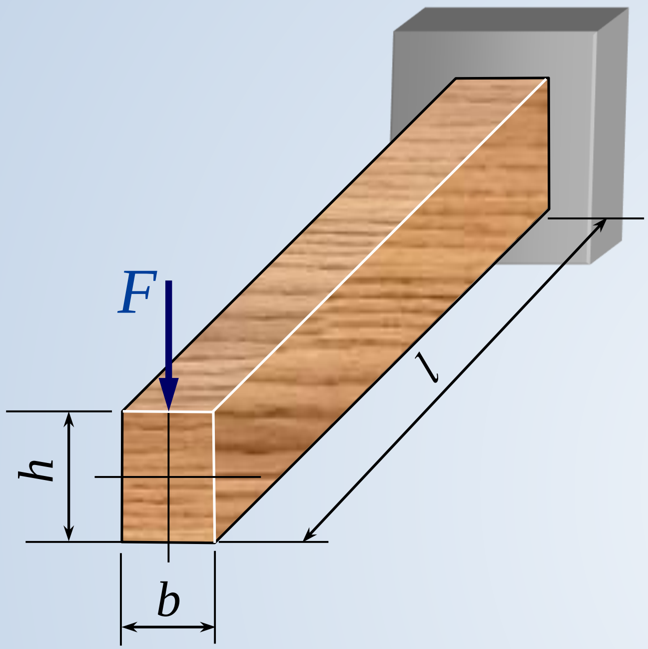
对于等直梁 $\tau_{\max} \leq [\tau]$

$$\tau_{\max} = \frac{F_{\mathrm{Q}\max} S^{*}_{z\max}}{I_z b} \leq [\tau]$$

四. 需要对切应力进行强度校核的情况

1. 短梁和集中力靠近支座
2. 木梁
3. 焊, 铆或胶合而成的梁
4. 薄壁截面梁

9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

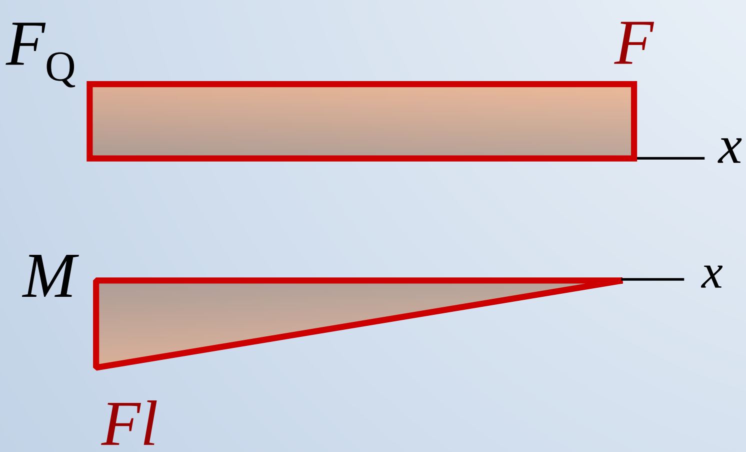


例5 已知 F, b, h, l. 求 $\dfrac{\sigma_{\max}}{\tau_{\max}}$

解:作F_Q， M 图

$\sigma_{\max}$发生在固定端上边缘

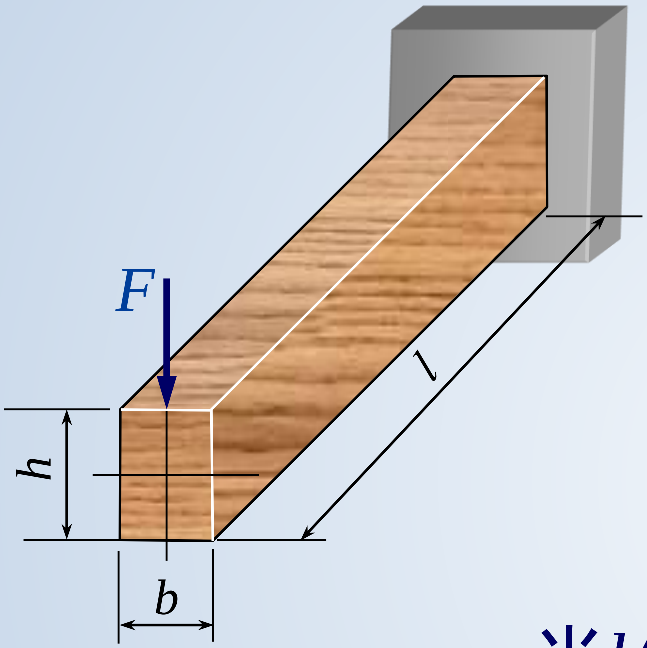
$$\sigma_{\max}=\frac{M_{\max}}{W_z}=\frac{Fl}{bh^2/6}=\frac{6Fl}{bh^2}$$



$\tau_{\max}$发生在任意截面的中性轴上

$$\tau_{\max}=\frac{3F_Q}{2A}=\frac{3F}{2bh}$$

9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

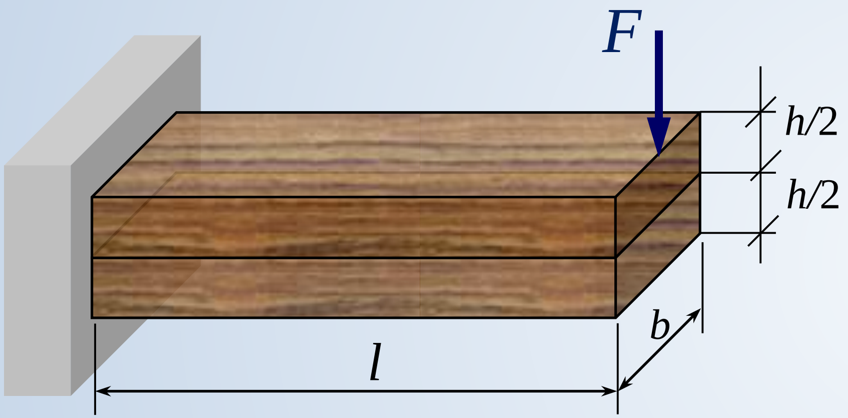


$$\frac{\sigma_{\max}}{\tau_{\max}}=\frac{\dfrac{6Fl}{bh^2}}{\dfrac{3F}{2bh}}=4\frac{l}{h}$$

当$l/h\geq 5$时， $\sigma_{\max}/\tau_{\max}\geq 20$

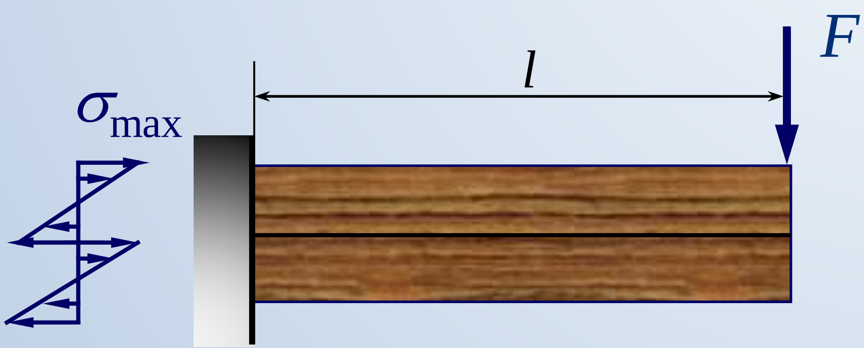
此情况下，弯曲切应力是次要的。

例6: (1) 两个相同材料的矩形截面叠梁. 设两梁间无摩擦, 求 $\sigma_{\max}$



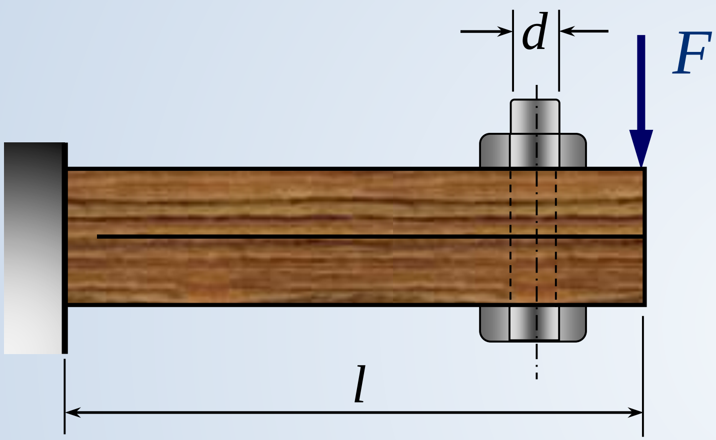
解: 每梁的变形相同, 各梁在自由端处所受外力均为 $F/2$,

$$M_{\max} = Fl/2$$



$$\sigma_{\max}^{(1)} = \frac{M_{\max}}{W_z} = \frac{\frac{1}{2}Fl}{\frac{1}{6}b\left(h/2\right)^2} = \frac{12Fl}{bh^2}$$

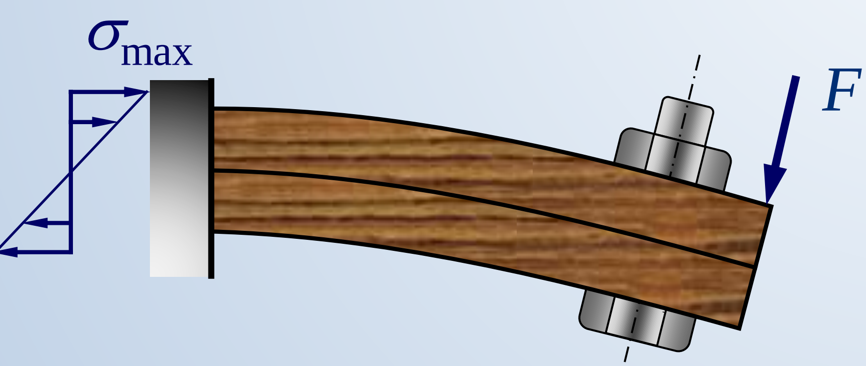
(2)在自由端有一直径为d 的螺栓,求$\sigma_{\max}$及螺栓截面的F_{Q1}



分析 两梁作为一整体, 故

$$M_{\max}=Fl$$

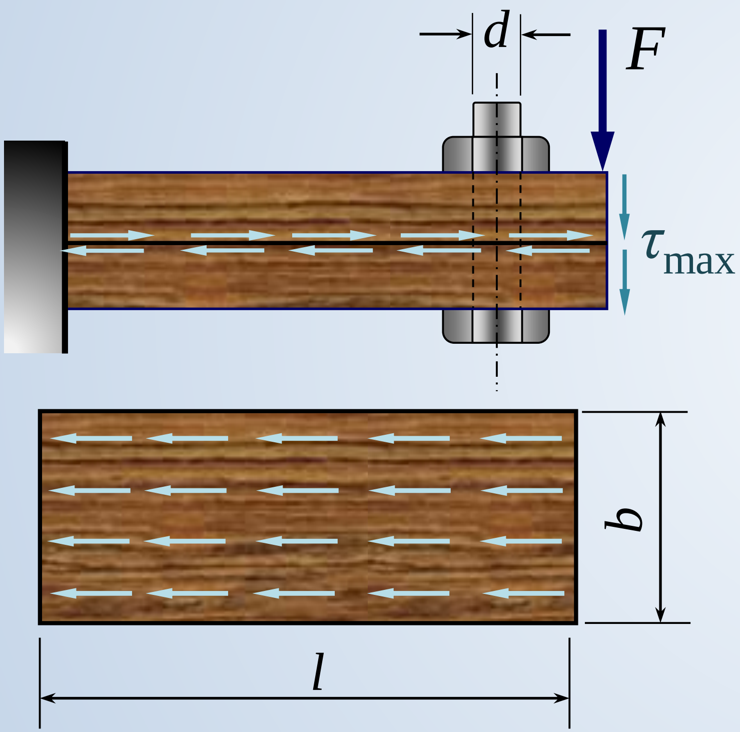
$$\sigma_{\max}^{(2)}=\frac{M_{\max}}{W_z}=\frac{Fl}{\frac{1}{6}bh^2}=\frac{6Fl}{bh^2}$$



$$\frac{\sigma_{\max}^{(1)}}{\sigma_{\max}^{(2)}}=2$$

加螺栓后，强度提高。

(3) 求螺栓截面的剪力F_{Q1}



在中性轴处有垂直中性轴τ_{max}

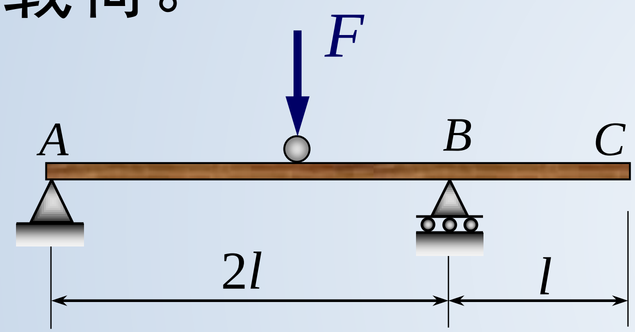
$$\tau_{max}=\frac{3F_Q}{2A}=\frac{3F}{2bh}$$

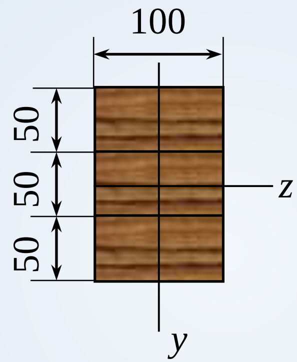
由切应力互等定理知，中性层面有均匀分布的τ_{max}

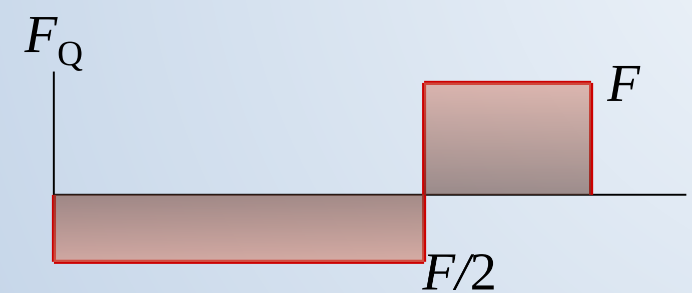
其合力与F_{Q1}平衡，即

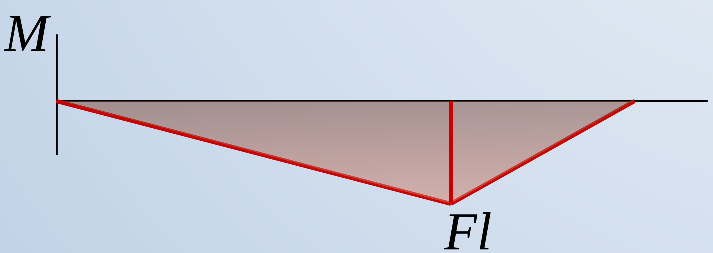
$$F_{Q1}=\tau_{max}bl=\frac{3Fl}{2h}$$

例7 三根材料相同的木板胶合而成的梁，l=1m, b=100mm, h=50mm. $[\tau]_{胶}$=0.34MPa,$[\sigma]_{木}$=10MPa，$[\tau]_{木}$=1MPa。试求许可载荷。









解: 1. F 移到AB中点时

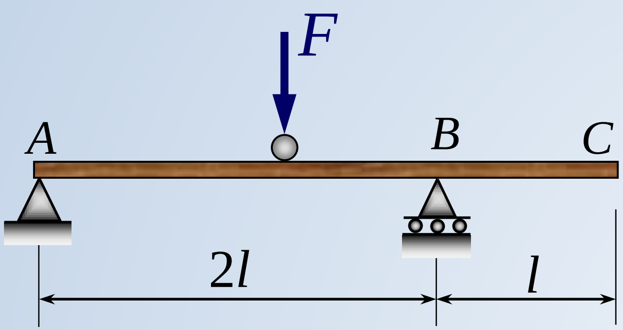
$$M_{\max} = \frac{Fl}{2} \qquad F_{Q\max} = F$$

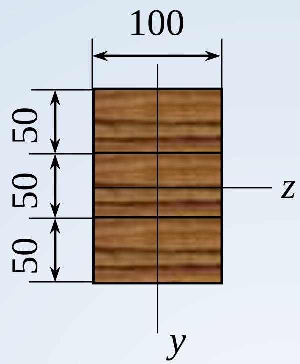
F 移到C点时

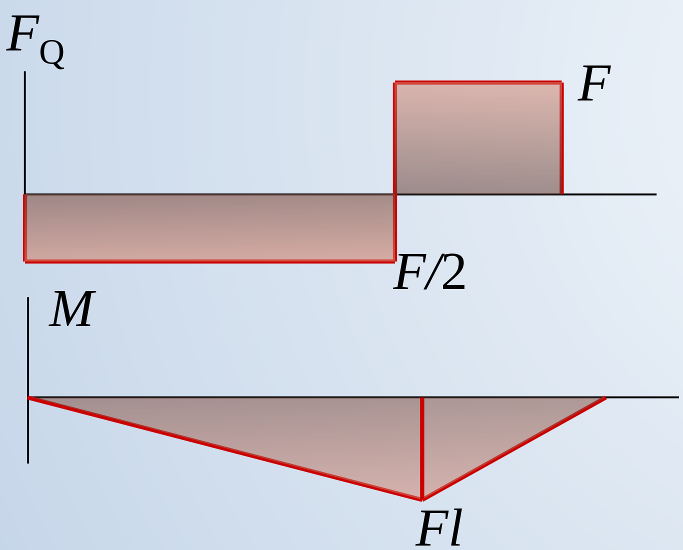
$$M_{\max} = Fl \qquad F_{Q\max} = F$$

比较得

$$M_{\max} = Fl \qquad F_{Q\max} = F$$







2. 胶合面剪切强度条件

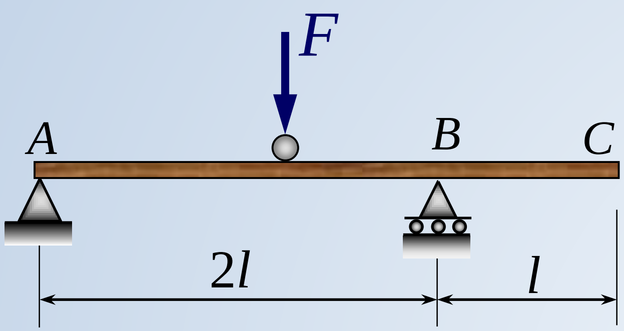
$$\tau_{胶}=\frac{F_{Q\max}S_z^*}{bI_z}=\frac{FS_z^*}{bI_z}\le[\tau]_{胶}$$

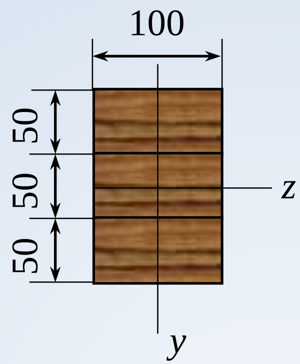
$$[F_1]\le\frac{[\tau]_{胶}bI_z}{S_z^*}$$

$$=\frac{0.34\times10^6\times0.1}{0.1\times0.05\times0.05}\cdot\frac{0.1\times0.15^3}{12}$$

$$=3.83\text{kN}$$

9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件

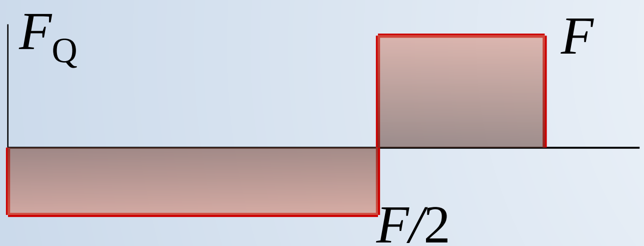


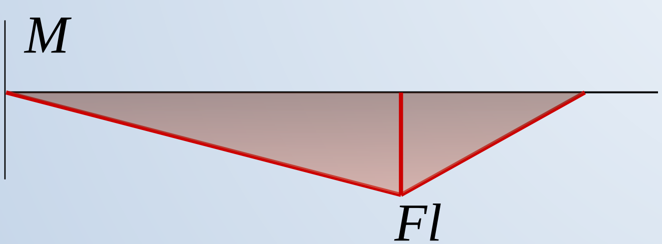


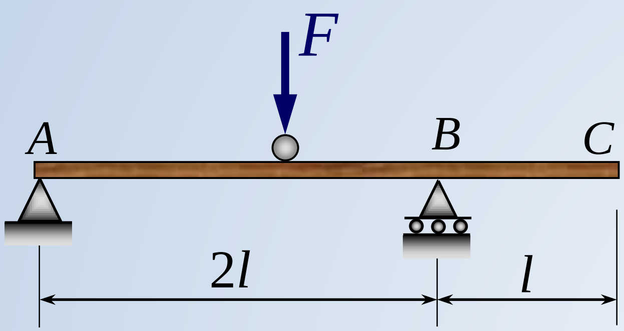
3. 梁的正应力强度条件

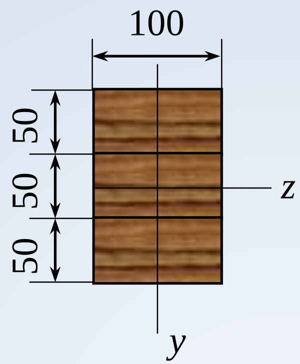
$$\sigma_{\max} = \frac{M_{\max}}{W_z} = \frac{6Fl}{bh^2} \le [\sigma]$$

$$[F_2] \le \frac{[\sigma]bh^2}{6l} = 3.75\text{kN}$$







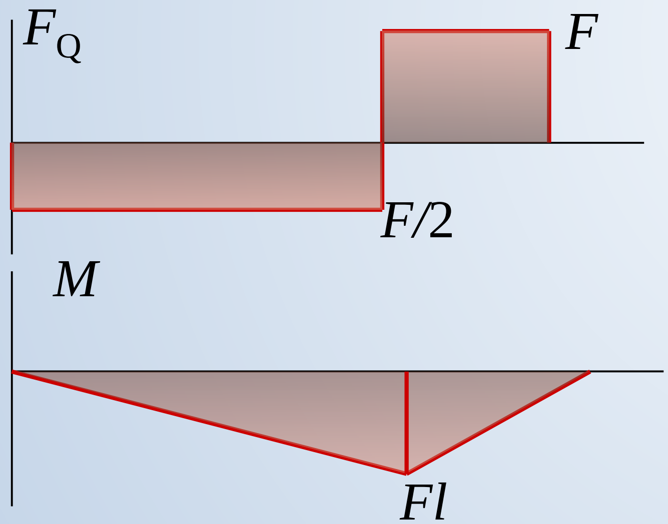


4. 梁的切应力强度条件

$$\tau_{\max} = \frac{3F_{\mathrm{Q}\max}}{2A} \le [\tau]$$

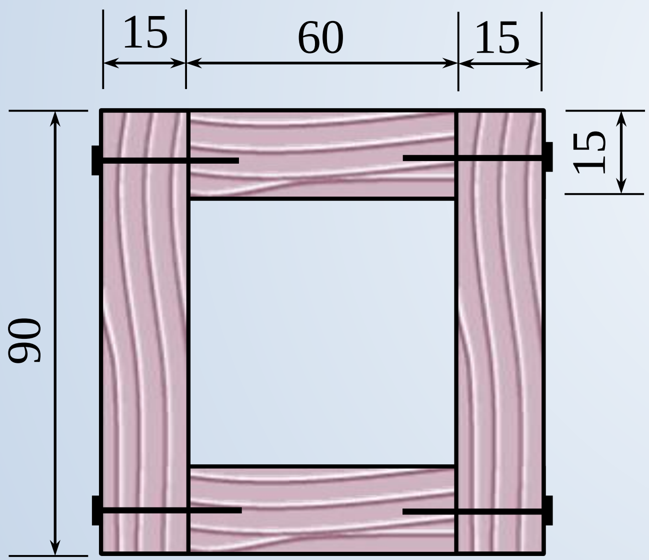
$$[F_3] \le \frac{[\tau]\cdot 2A}{3} = 10\,\mathrm{kN}$$

$$[F] = \{F_1,\ F_2,\ F_3\}_{\min} = 3.75\,\mathrm{kN}$$

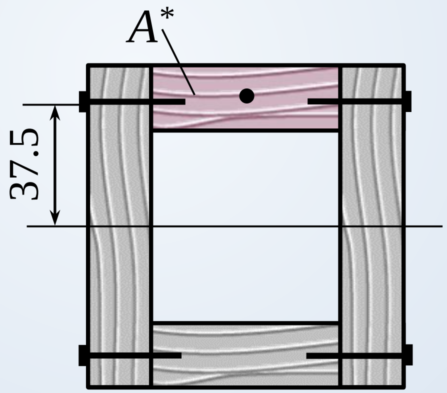


$$[F_1] = 3.83\,\mathrm{kN} \quad [F_2] = 3.75\,\mathrm{kN}$$

例8 由四块木板制成一个方形箱梁，已知钉子沿梁长间距为3.5cm。梁截面受到横向2400N剪力的作用，试求每个钉子的剪切力。



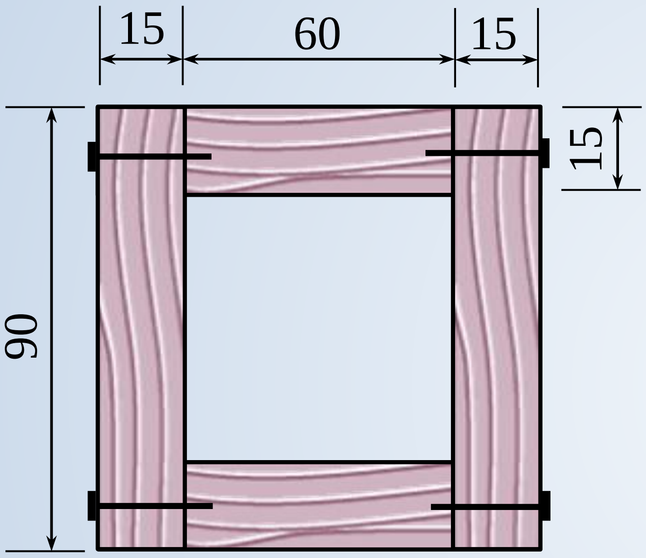
解：• 求上木板边缘的切应力.



$$S^* = A^* y$$
$$= (15 \times 60) \times 37.5 \times 10^{-9}$$
$$= 33.75 \times 10^{-6}\,\mathrm{m}^3$$

$$I = \tfrac{1}{12}(90)^4 - \tfrac{1}{12}(60)^4$$
$$= 4.39 \times 10^{-6}\,\mathrm{m}^4$$

9.4 横力弯曲时梁横截面上的切应力 弯曲切应力强度条件



钉子间距为35mm

$$\tau = \frac{F_Q S^*}{I\,2t}$$

$$= \frac{2400 \times 33.75 \times 10^{-6}}{4.388 \times 10^{-6} \times (2 \times 15 \times 10^{-3})}$$

$$= 0.615\mathrm{MPa}$$

- 求每个钉子的剪切力

$$F = \tau \cdot A$$

$$= \left(0.615 \times 10^{6}\right)\left(35 \times 10^{-3} \times 15 \times 10^{-3}\right)$$

$$= 323\mathrm{N}$$

$$\boxed{F = 323\mathrm{N}}$$

9.6 弯曲中心

一. 什么叫弯曲中心

截面上切应力合力的作用点叫弯曲中心, 也称弯心或剪心

注意

弯曲中心只与截面的形状和尺寸有关, 是一个几何点, 是截面的几何性质.

二. 只弯不扭的条件

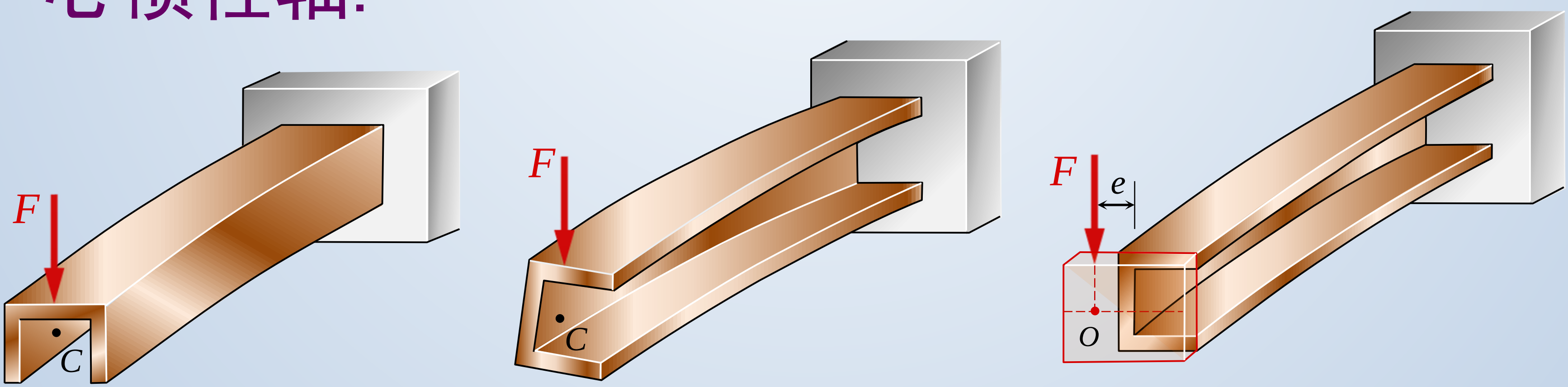
当横向力F通过弯心时, 则梁只弯而不扭.

9.6 弯曲中心

三. 产生平面弯曲的条件

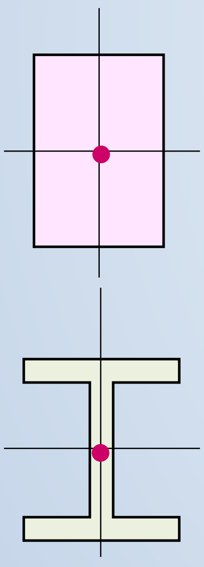
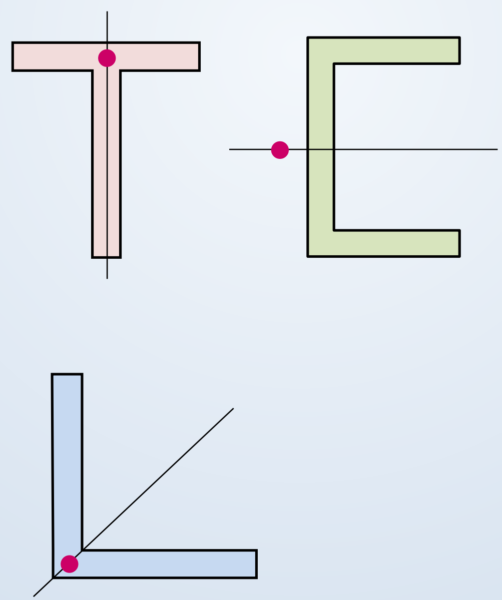
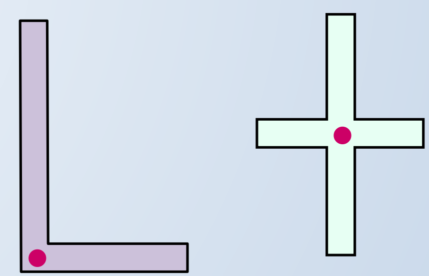
充分条件:梁截面有纵向对称轴, 所有载荷包括支反力都作用在纵向对称面内, 则梁一定产生平面弯曲.

充分必要条件:横向力过弯曲中心且平行主形心惯性轴.



9.6 弯曲中心

四. 常见截面弯心的大致位置

有两个对称轴，形心即是	有一个对称轴，则一定在对称轴上	有几支组成，则在支的交点上
		

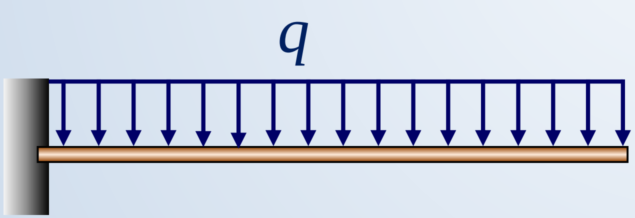
9.7 提高弯曲强度的主要措施

弯曲强度主要取决于$\sigma_{\max}$

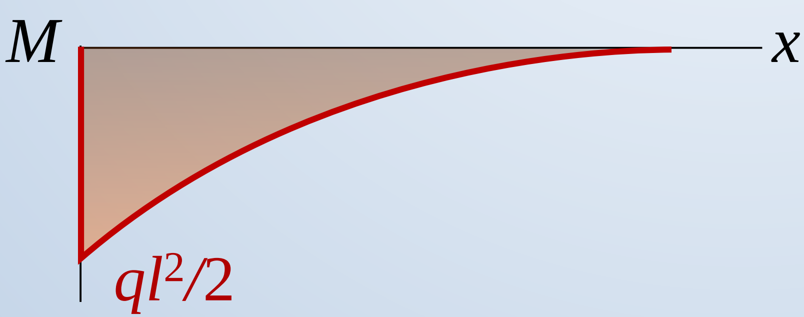
$$\sigma_{\max}=\frac{M_{\max}}{W_z}\le[\sigma]$$

一. 合理安排梁的受力情况

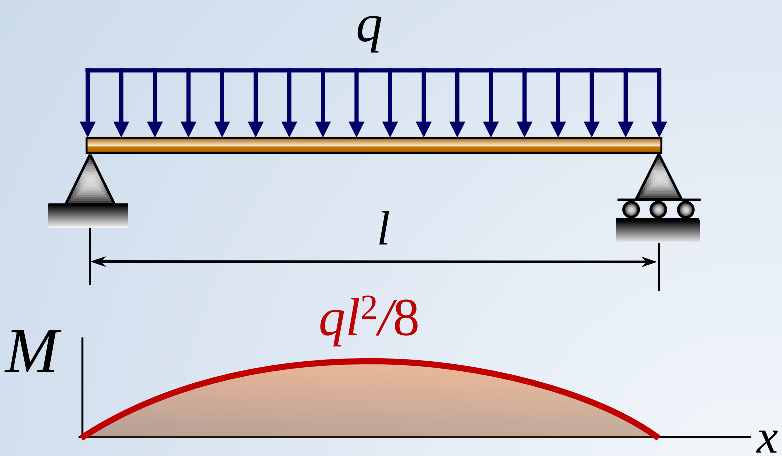
1. 合理设计和布置支座



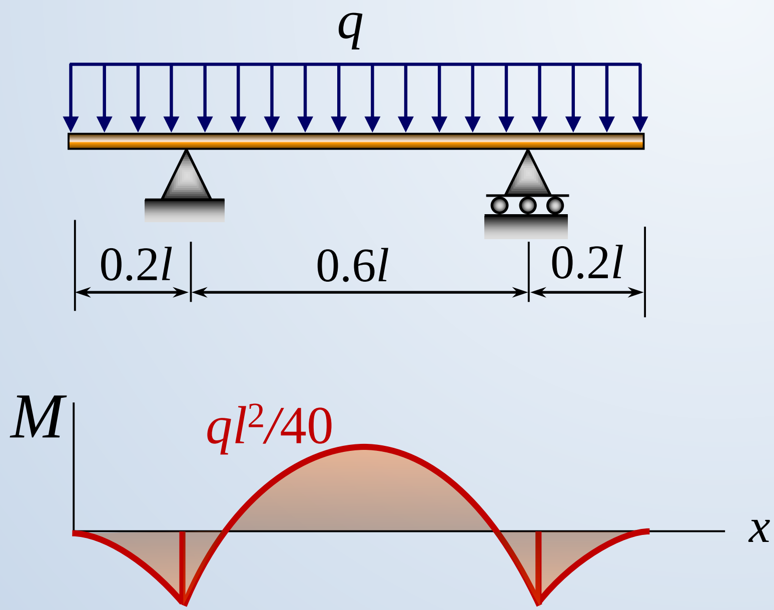
$$M_{\max}=\frac{ql^2}{2}=0.5ql^2$$



9.7 提高弯曲强度的主要措施



$$M_{\max} = \frac{ql^2}{8} = 0.125ql^2$$

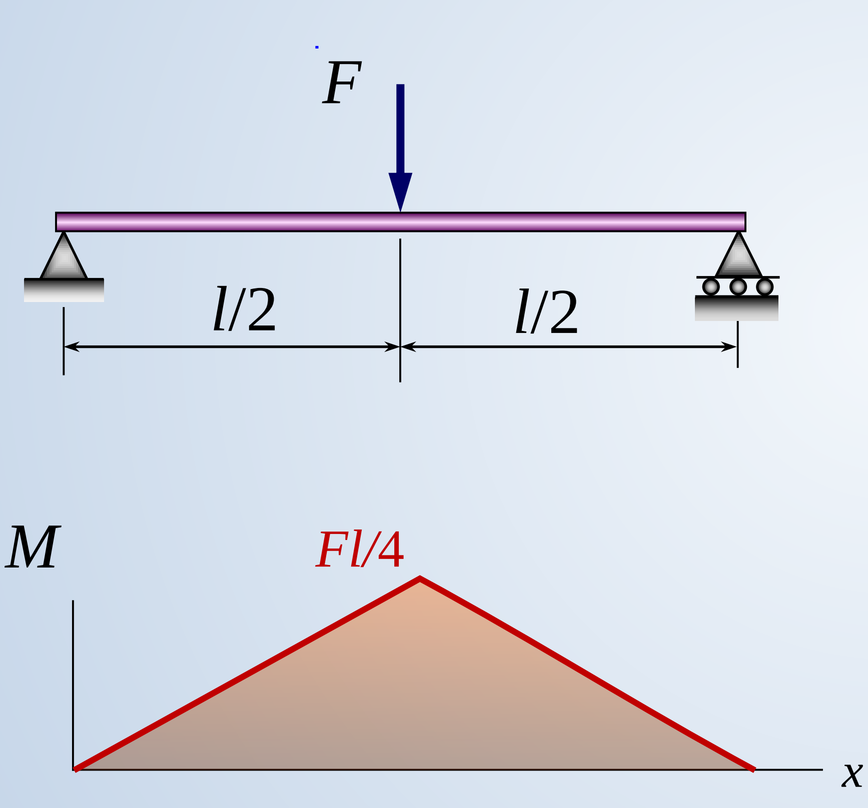


$$M_{\max} = \frac{ql^2}{40} = 0.025ql^2$$

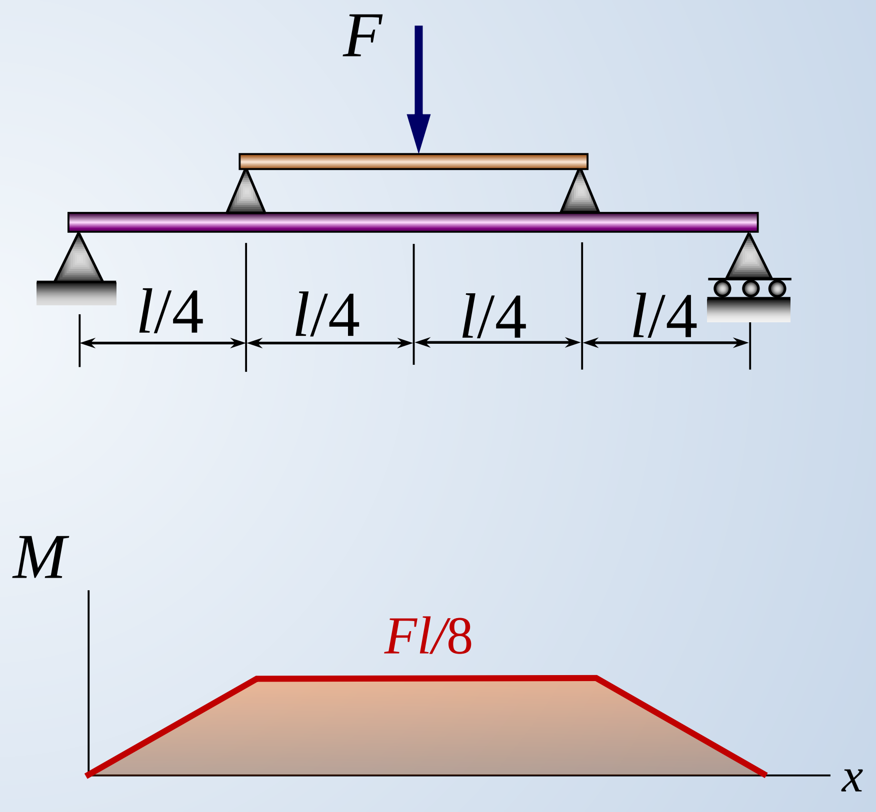
最优位置？

5.7 提高弯曲强度的主要措施

2. 将集中载荷适当分散



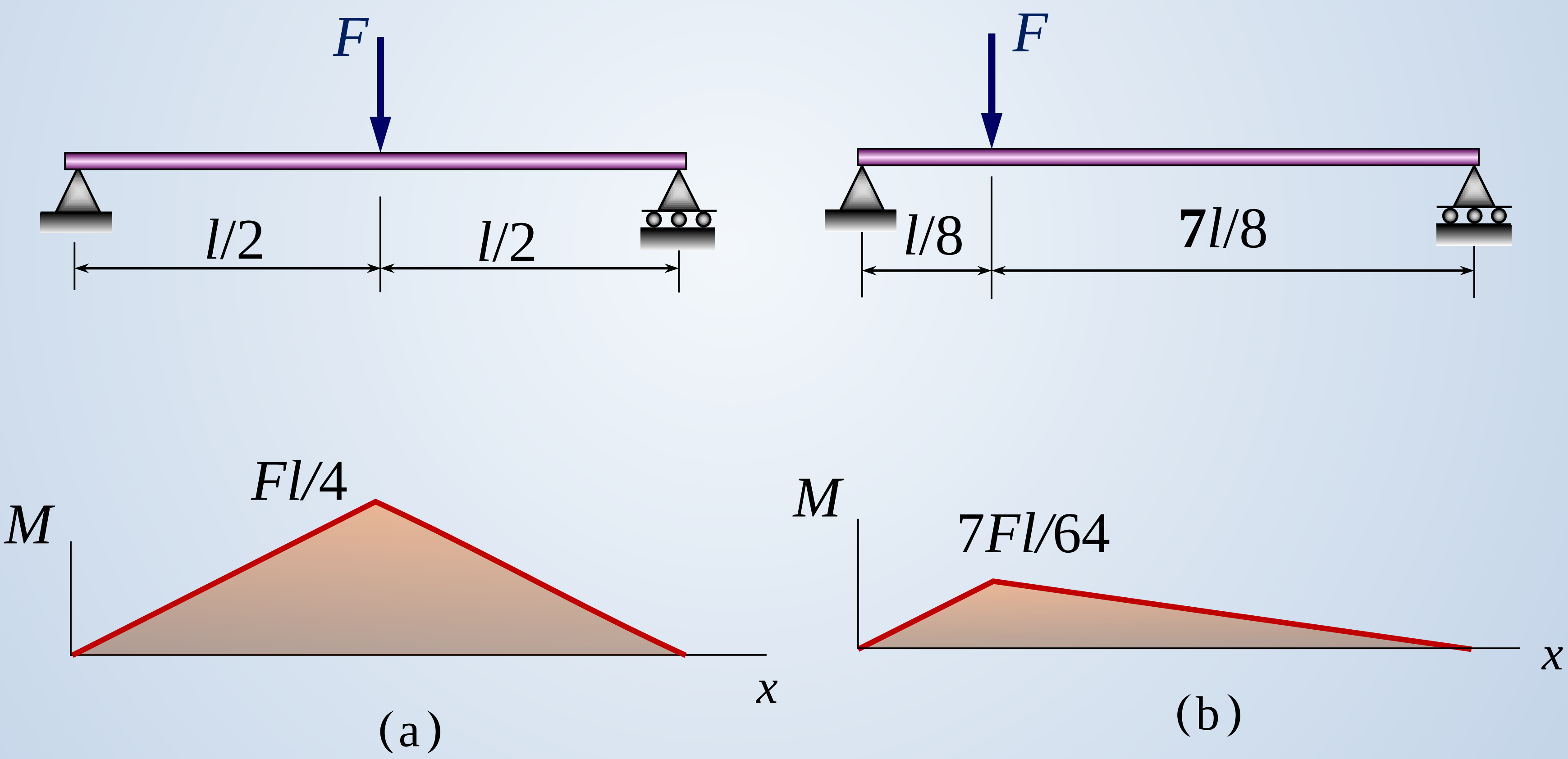
(a)



(b)

9.7 提高弯曲强度的主要措施

3. 集中载荷尽量靠近支座



9.7 提高弯曲强度的主要措施

二. 合理的截面设计

1.塑性材料 $[\sigma_t]=[\sigma_c]$，

应尽量制成对称截面，使面积分布远离中性轴，由

$$\sigma_{\max}=\frac{M_{\max}}{W_z}\le[\sigma]$$

知，在不增加面积的情况下，W_z越大越好，以矩形截面为例

$$\sigma_{\max}=\frac{M_{\max}}{W_z}=\frac{M_{\max}}{\frac{1}{6}bh^2}=\frac{6M_{\max}}{Ah}$$

显然，增大h可提高强度。

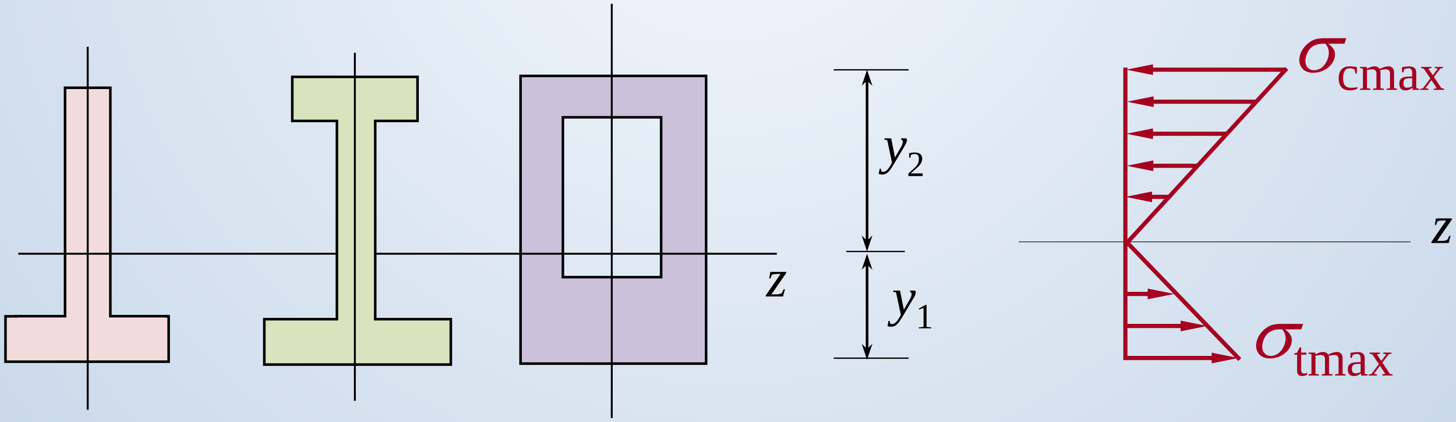
9.7 提高弯曲强度的主要措施

2. 脆性材料$[\sigma_t]<[\sigma_c]$，使

$$\frac{\sigma_{t\max}}{\sigma_{c\max}}=\frac{M_{\max}\cdot y_1}{I_z}\bigg/\frac{M_{\max}\cdot y_2}{I_z}=\frac{y_1}{y_2}=\frac{[\sigma_t]}{[\sigma_c]}$$

尽量制成截面对中性轴不对称

放置时使靠近中性轴边缘受拉

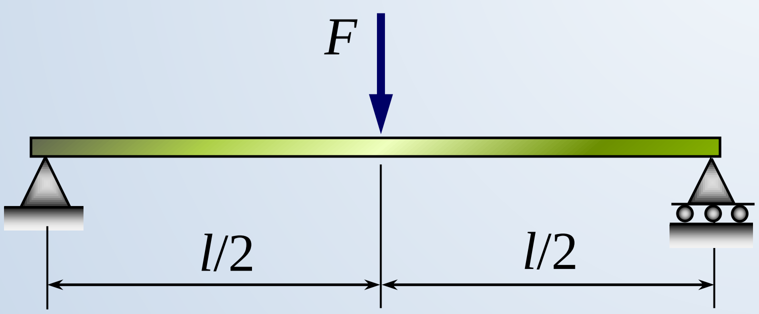


9.7 提高弯曲强度的主要措施

三.等强度梁的概念

如使各截面上危险点的应力都同时达到许用应力, 则称该梁为等强度梁.

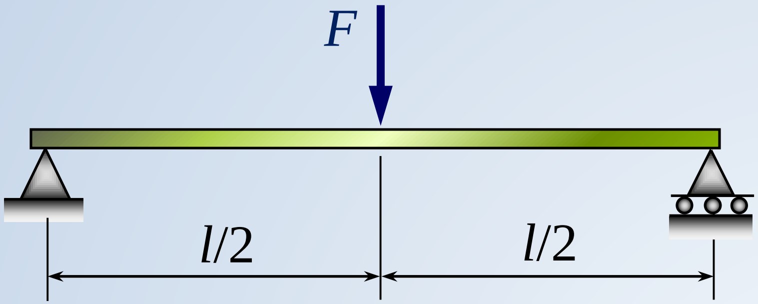
根据等强度梁的要求，应有:



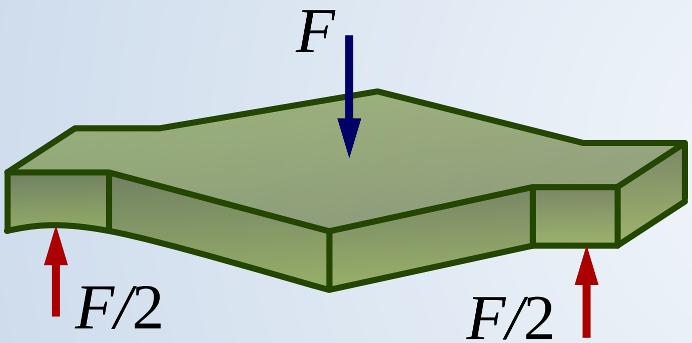
$$\sigma_{\max}=\frac{M(x)}{W_z(x)}=[\sigma]$$

即 $$W_z(x)=\frac{M(x)}{[\sigma]}$$

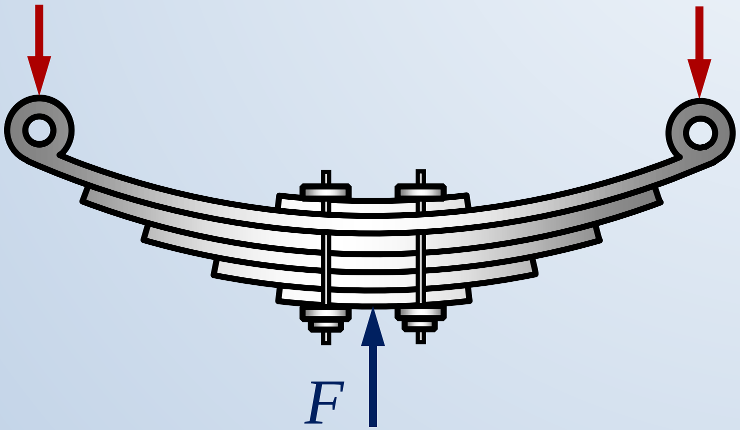
9.7 提高弯曲强度的主要措施



$$W_z(x)=\frac{M(x)}{[\sigma]}$$

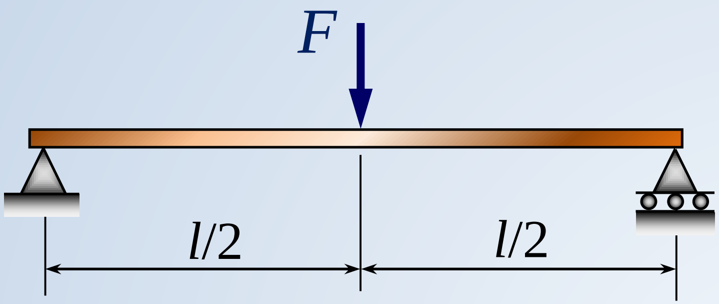


$$W_z(x)=\frac{b(x)h(x)^2}{6}=\frac{M(x)}{[\sigma]}=\frac{\frac{1}{2}Fx}{[\sigma]}$$

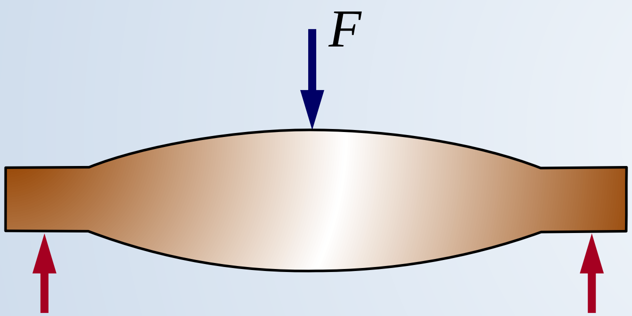


$$h=c \qquad b(x)=\frac{3F}{[\sigma]h^2}x$$

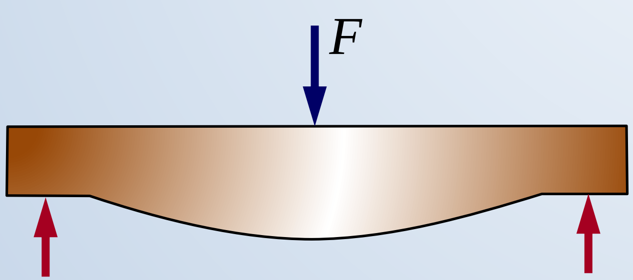
9.7 提高弯曲强度的主要措施

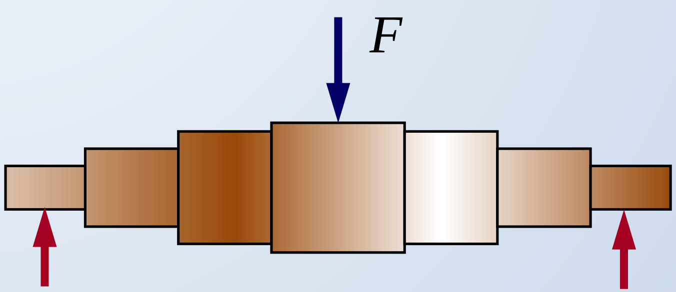


$$W_z(x)=\frac{b(x)h(x)^2}{6}=\frac{M(x)}{[\sigma]}=\frac{\frac{1}{2}Fx}{[\sigma]}$$



$$b=c \qquad h(x)=\sqrt{\frac{3Fx}{b[\sigma]}}$$



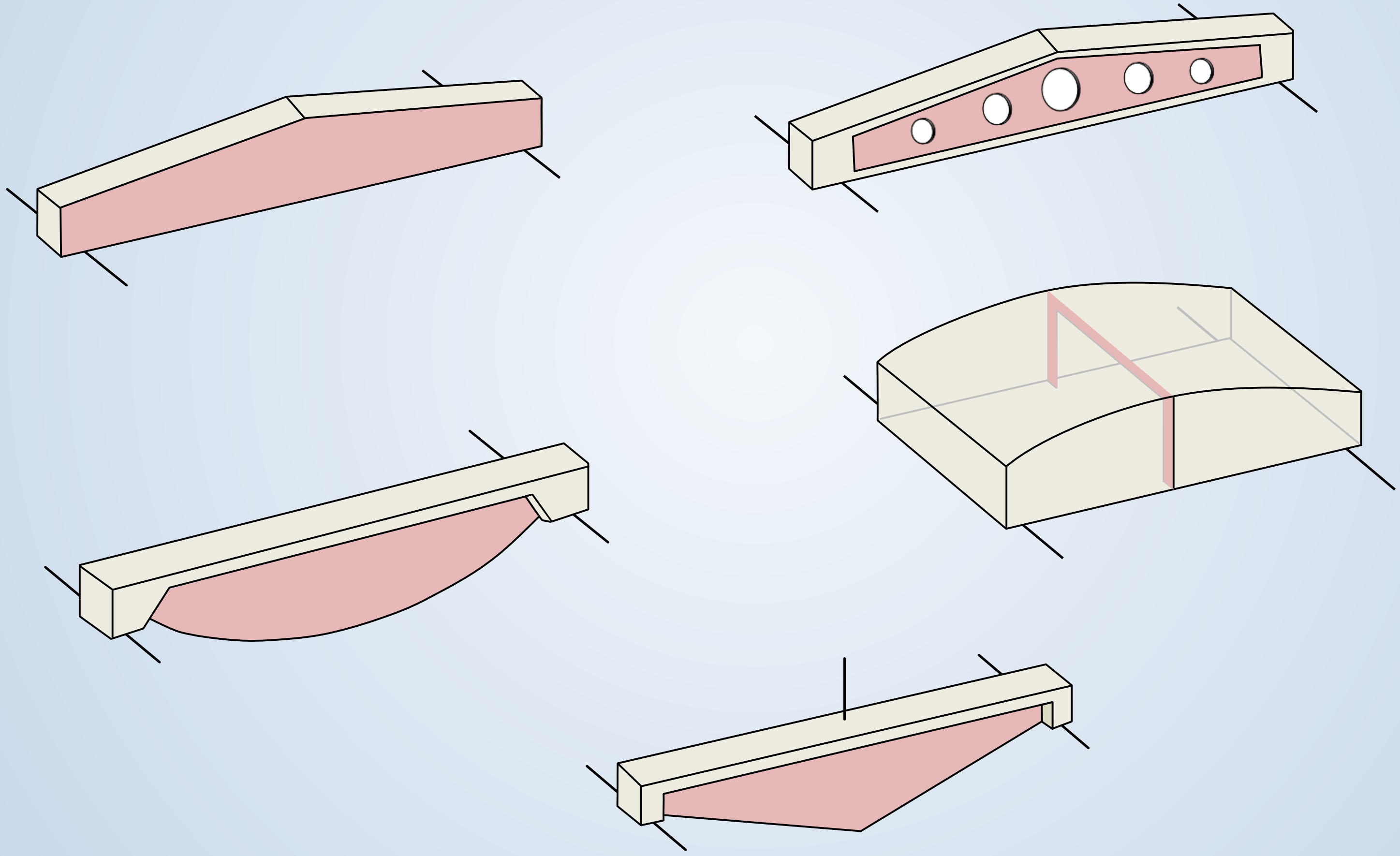


9.7 提高弯曲强度的主要措施

合理的截面形状

9.7 提高弯曲强度的主要措施

变截面梁



Thank you !