静电场

总结

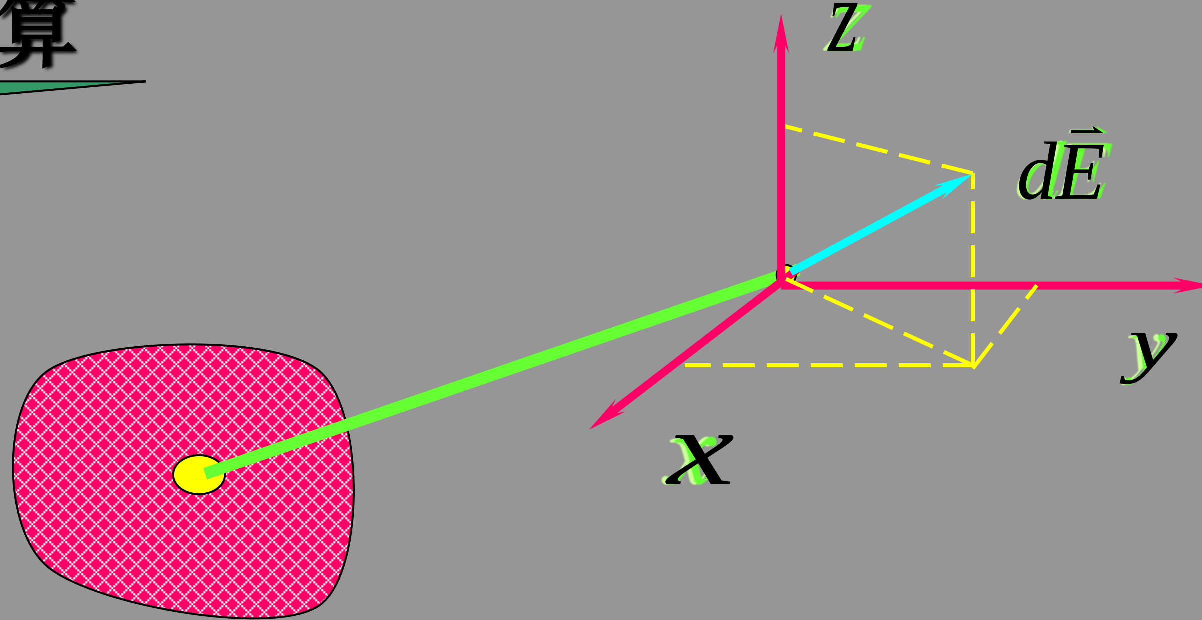
一：电场（定义法、高斯法）

$$
\begin{cases}
dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r^2}\text{(求出分量}E_x = \int dE_x\text{)} \\[2ex]
\oint \vec{D} \bullet d\vec{S} = \sum q_i
\end{cases}
$$

电场强度计算

1.任意带电体

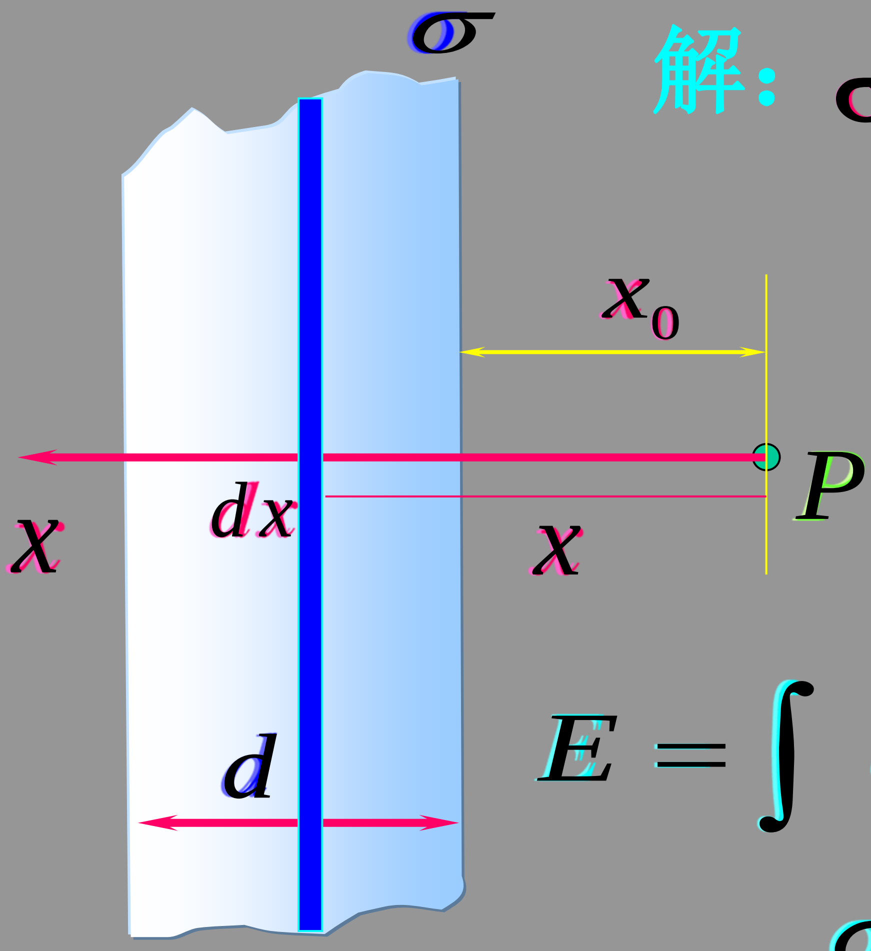
$$dE = \frac{dq}{4\pi\varepsilon r^2}$$



$$dq \begin{cases} \lambda dl \\ \sigma ds \\ \rho dV \end{cases}$$

$$\vec{E} \begin{cases} E_x = \int dE_x = \int dE\cos\alpha \\ E_y = \int dE_y = \int dE\cos\beta \\ E_z = \int dE_z = \int dE\cos\gamma \end{cases}$$

例1 计算面密度为σ、长、宽分别为l（很长）、d均匀带电平板侧面外附近对称处P点的场强。



解：$\sigma = \dfrac{dq}{l\,dx} \Rightarrow \lambda = \dfrac{dq}{l} = \sigma dx$

$$dE = \frac{\lambda}{2\pi\varepsilon x}$$

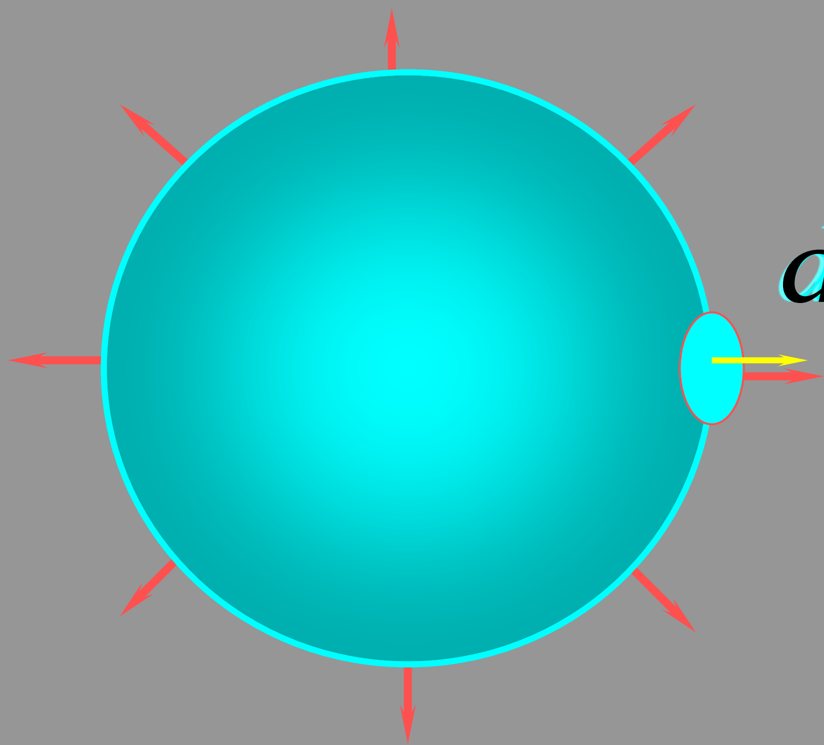
$$= \frac{\sigma dx}{2\pi\varepsilon x}$$

$$E = \int dE = \int_{x_0}^{x_0+d} \frac{\sigma dx}{2\pi\varepsilon x}$$

$$= \frac{\sigma}{2\pi\varepsilon} \ln \frac{x_0 + d}{x_0}$$

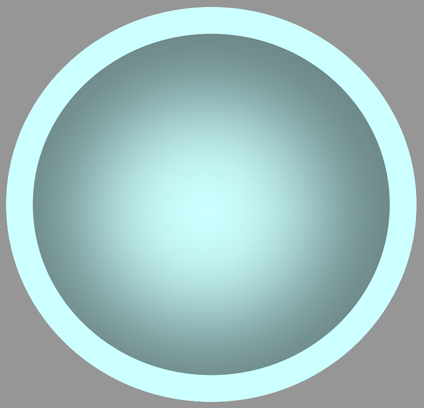
2.场分布对称的带电体

★球对称



$$d\vec{s} = ds\vec{n}$$

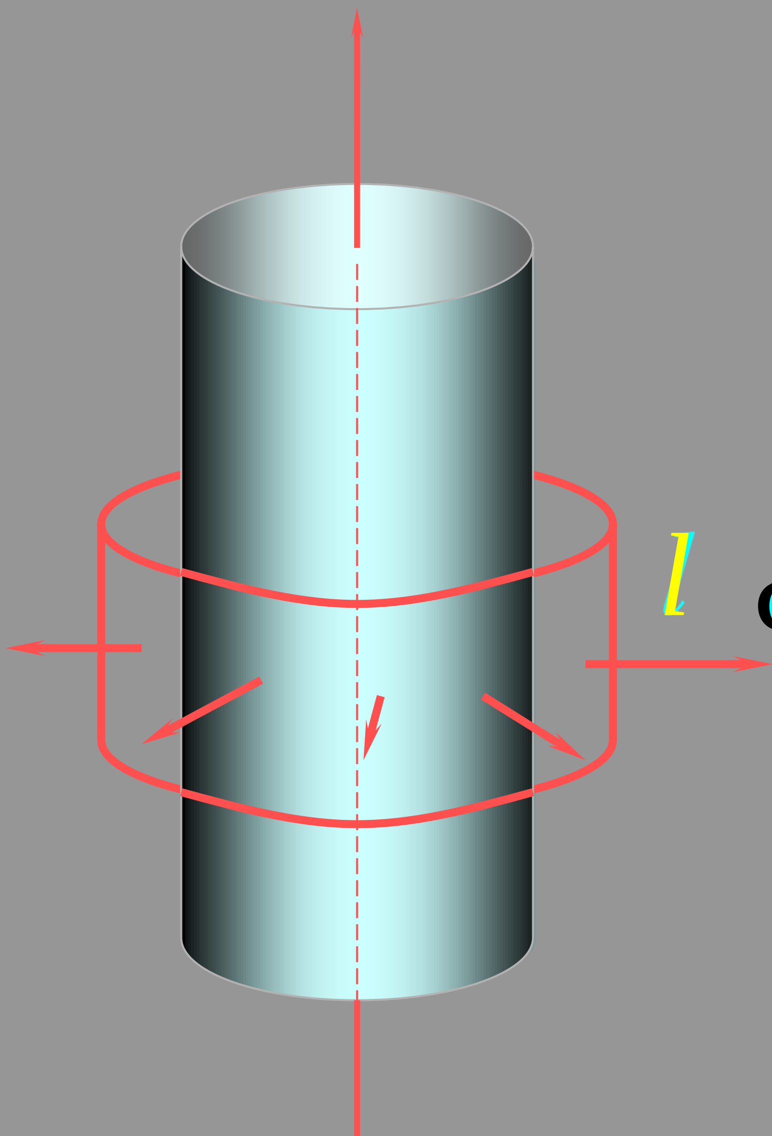
$$\vec{D} \,//\, d\vec{s}$$

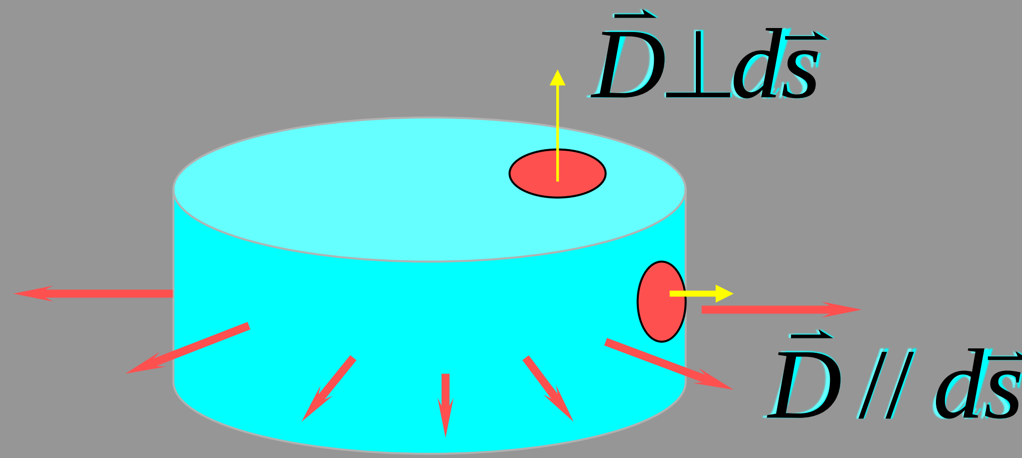


$$D \cdot 4\pi r^2 = Q$$ 电荷均匀分布

$$D \cdot 4\pi r^2 = \int_r \rho(r) 4\pi r^2 dr$$ 电荷非均匀分布

★轴对称



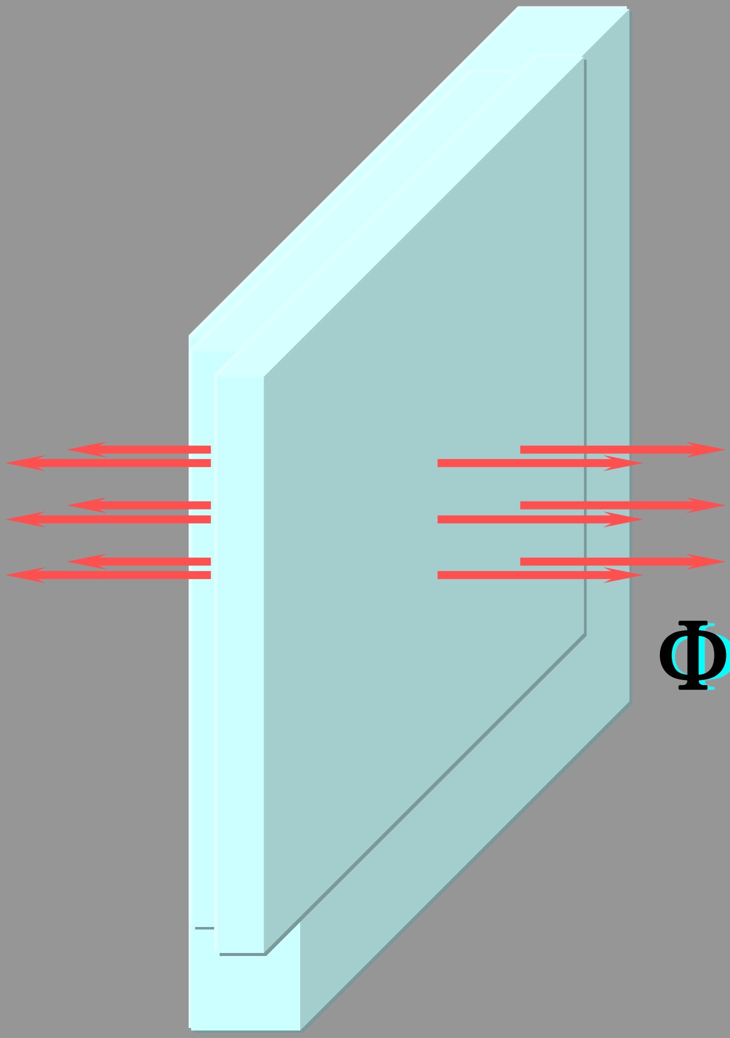


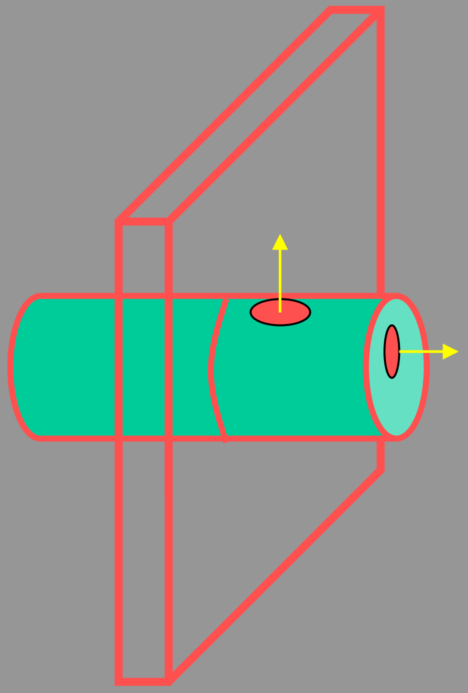
$$\Phi = \int_{上} \vec{D} \cdot d\vec{s} + \int_{下} \vec{D} \cdot d\vec{s} + \int_{侧} \vec{D} \cdot d\vec{s}$$

$$= \int_{侧} \vec{D} \cdot d\vec{s} = D \cdot 2\pi r l$$

$$= \lambda l$$

★面对称



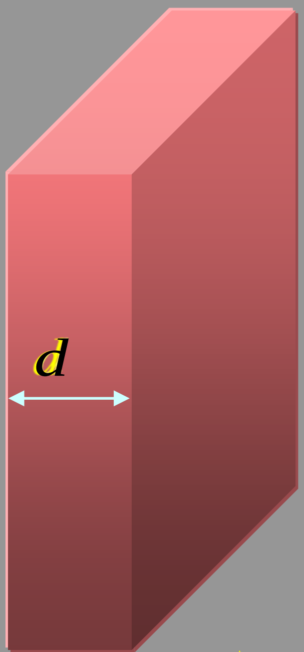


$$\Phi = \int_{左} \vec{D}\cdot d\vec{s} + \int_{右} \vec{D}\cdot d\vec{s} + \int_{侧} \vec{D}\cdot d\vec{s}$$

$$= \int_{左} \vec{D}\cdot d\vec{s} + \int_{右} \vec{D}\cdot d\vec{s}$$

$$= 2D\cdot \Delta S = \sigma \cdot \Delta S$$

例 2 ρ = 常数 计算场分布（此题目预留有bug一处）



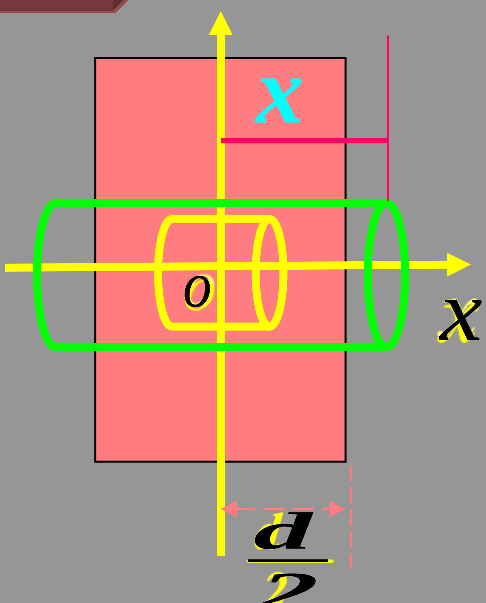
$x < \frac{d}{2}$

$$\Phi = \int_{左} \vec{D}\cdot d\vec{s} + \int_{右} \vec{D}\cdot d\vec{s} + \int_{侧} \vec{D}\cdot d\vec{s}$$

$$= \int_{左} \vec{D}\cdot d\vec{s} + \int_{右} \vec{D}\cdot d\vec{s}$$

$$= 2D\cdot \Delta S = \rho\cdot 2x\Delta S$$

$$D = \rho x \qquad E = \frac{\rho x}{\varepsilon}$$



$x > \frac{d}{2}$ $\quad 2D\cdot\Delta S = \rho\cdot d\Delta S$

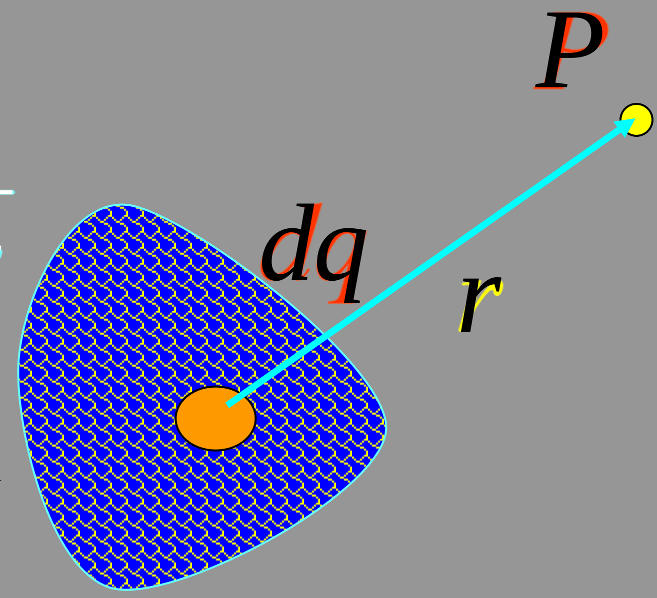
$$D = \frac{\rho d}{2} \qquad E = \frac{\rho d}{2\varepsilon}$$

✵二：电势（两种定义）

$$\begin{cases} dU = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r} \text{（注意电势是标量）} \\ U_P = \displaystyle\int_P^{\infty} \vec{E} \bullet d\vec{l} = \int_P^{Q} \vec{E} \bullet d\vec{l} + \int_Q^{\infty} \vec{E} \bullet d\vec{l} \\ U_{AB} = \displaystyle\int_A^{B} \vec{E} \bullet d\vec{l} = \int_A^{Q} \vec{E} \bullet d\vec{l} + \int_Q^{B} \vec{E} \bullet d\vec{l} \end{cases}$$

二、电势的计算

迭加法：$U=\int dU=\int\frac{dq}{4\pi\varepsilon r}$



适用于非对称电荷连续体

定义法：$U_P=\int_P^{参考点}dU=\int_P^{参考点}\vec{E}\cdot d\vec{l}$

由高斯定理求得

沿电场线方向

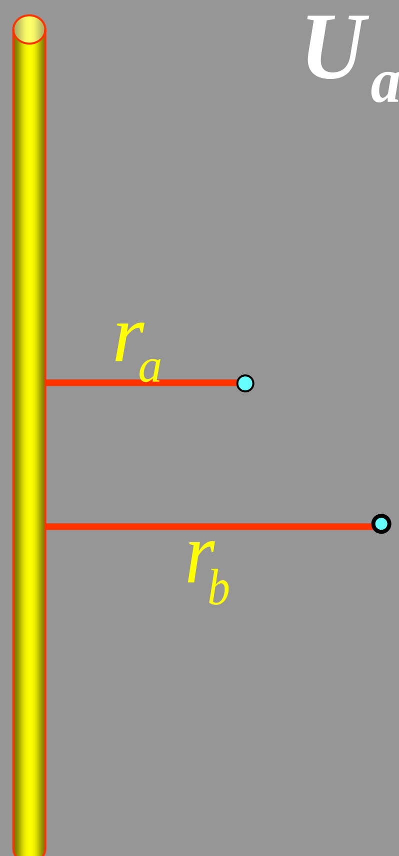
适用于电荷分布产生对称场的带电体

电压：$U_{PQ}=\int_P^{Q}dU=\int_P^{Q}\vec{E}\cdot d\vec{l}$

PQ之间的电场

例3 “无限长”均匀带电直线，电荷线密度为λ，求到直线的距离为r_a的一点a的电势

解：由高斯定理得 $E=\dfrac{\lambda}{2\pi\varepsilon r}$



$$U_{ab}=U_a-U_b=\int_{r_a}^{r_b}\vec{E}_{ab}\cdot d\vec{r}=\int_{r_a}^{r_b}\frac{\lambda}{2\pi\varepsilon r}dr$$

$$=-\frac{\lambda}{2\pi\varepsilon}(\ln r_a-\ln r_b)$$

令$U_b=\dfrac{\lambda}{2\pi\varepsilon}\ln r_b=0$

则$U_a=-\dfrac{\lambda}{2\pi\varepsilon}\ln r_a$

✵三： 电容（能量及能量密度）

$$\begin{cases} C = \dfrac{q}{U_{AB}} \\ W_e = \dfrac{1}{2} C U_{AB}^2 \\ w_e = \dfrac{1}{2} \varepsilon E^2 \end{cases}$$

三（1）、电容的计算

基本公式

$$C = \frac{Q}{U_{AB}}$$

一个极板所带的电量

两个极板之间的电势差

$$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l}$$

两极板间电场强度，由高斯定理求得

三（2）、电场能量的计算

基本公式

能量密度 $w = \dfrac{dW}{dV} = \dfrac{1}{2}DE = \dfrac{1}{2}\varepsilon E^2$

场能量 $W = \int w dV = \int \dfrac{1}{2}\varepsilon E^2 dV$

$$dV = \begin{cases} 4\pi r^2 dr \text{ (球分布的场空间)} \\ 2\pi r l dr \text{ (轴分布的场空间)} \end{cases}$$

- The end