吉林大学
JILIN UNIVERSITY

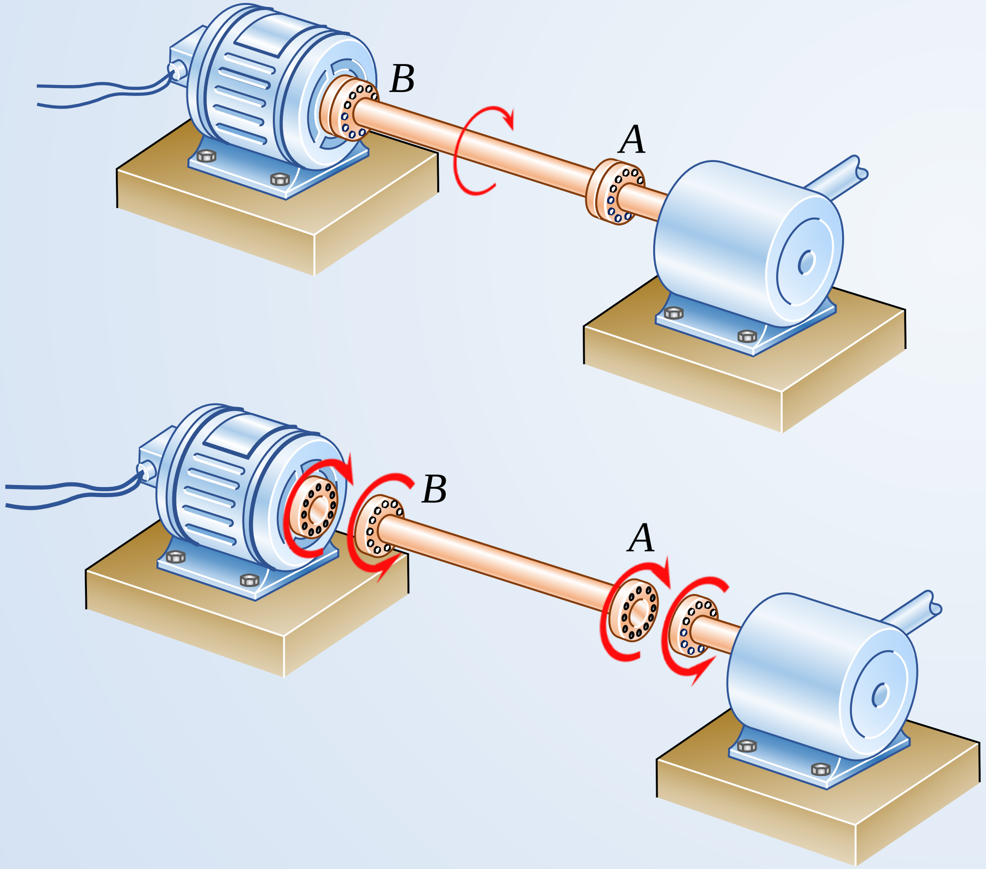
工程力学

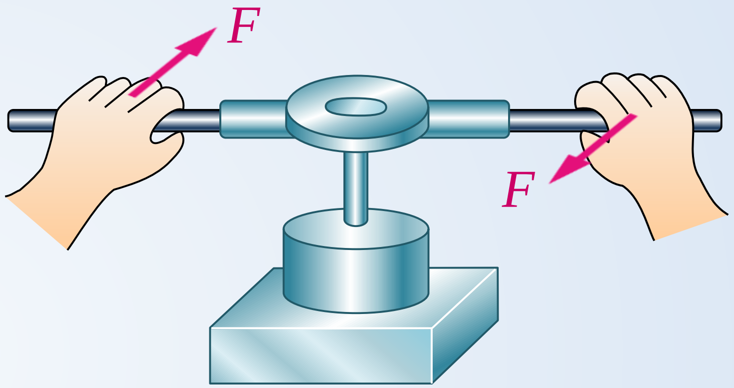
第7章 扭转与剪切

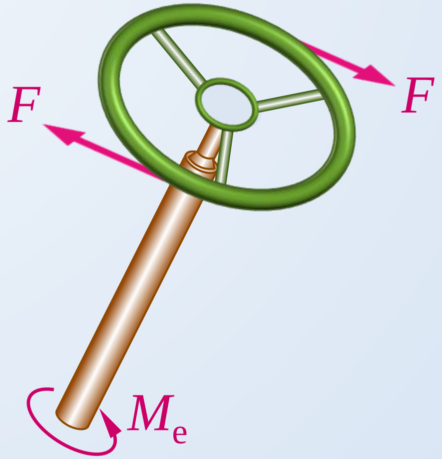
第7章 扭转与剪切

7.1 扭转的概念和实例

一. 工程实例







7.1 扭转的概念和实例

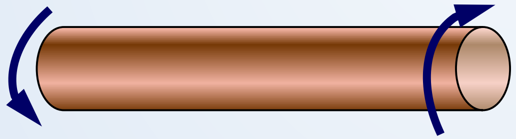
二. 受力特点:

力偶矩作用面垂直轴线, 即作用在横截面内

三. 变形特点

任意两横截面产生相对转动

四. 受力简图



五. 主要研究对象

以圆轴为主(等直轴, 阶梯轴, 空心轴)

7.2 外力偶矩的计算 扭矩和扭矩图

一. 外力偶矩的计算

1. 直接给出 M_e (N•m)

2. 给出功率, 转速

$$M_e = 9549 \frac{P}{n}$$

(kw)

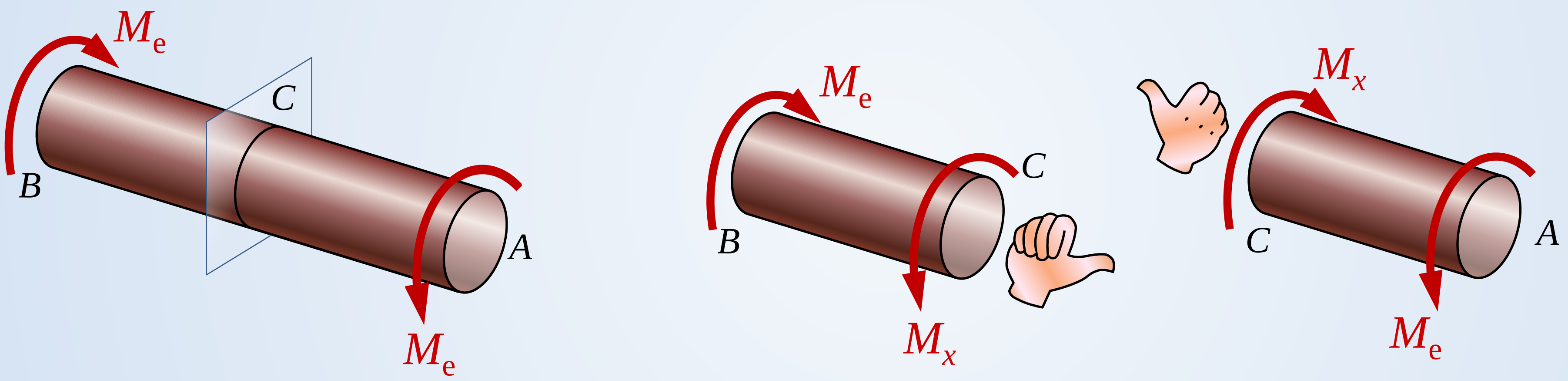
(N· m)

(r/min)

7.2 外力偶矩的计算 扭矩和扭矩图

二.横截面上的内力

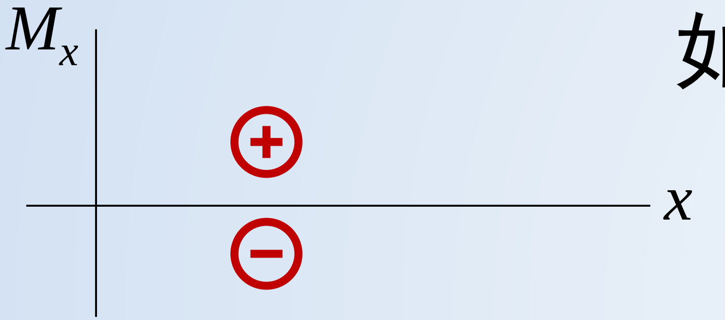
截面法求内力：截，取，代，平



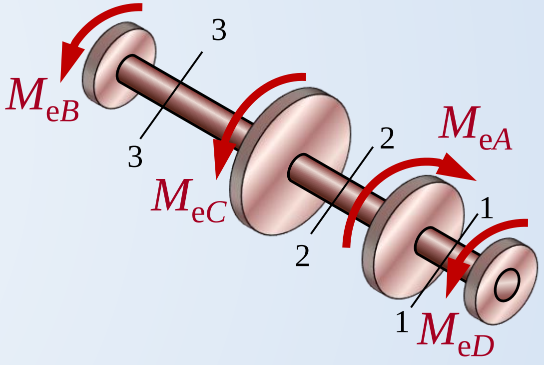
M_x 称为截面上的扭矩 $\sum M_x = 0$ $M_x - M_e = 0$ 即 $M_x = M_e$

按右手螺旋法：M_x 指离截面为正，指向截面为负。

三. 内力图(扭矩图)



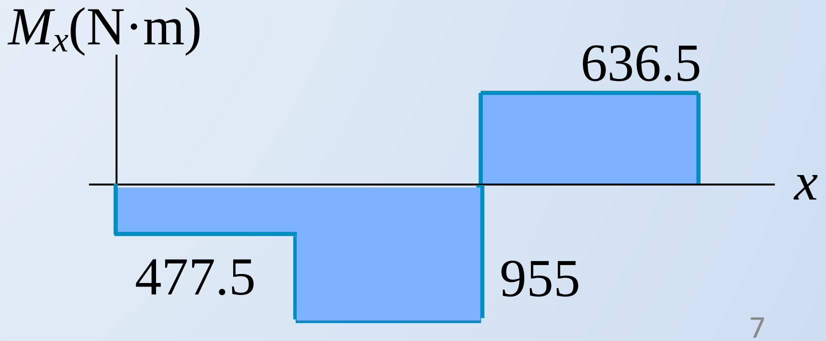
如同轴力图一样,将扭矩用图形表示称扭矩图



M_x图特点：

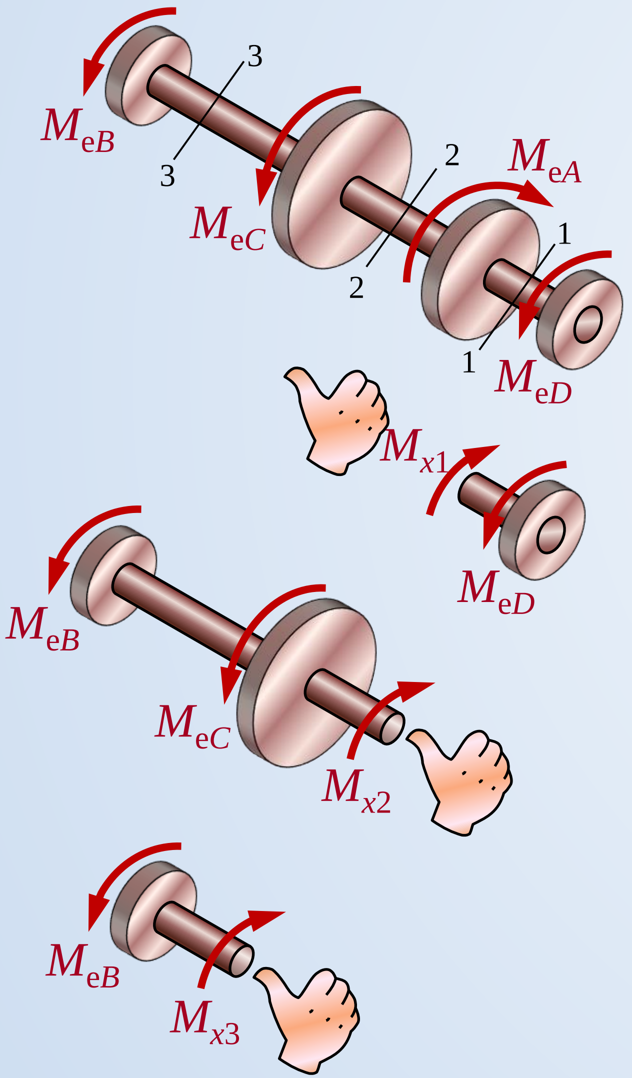
1.有M作用处, M_x图有突变,突变值=M

2.无力偶作用段, M_x图为水平线



7.2 外力偶矩的计算 扭矩和扭矩图

例1. 已知：n=300r/min , P_A=50kW, P_B = P_C =15kW, P_D =20kW



求：画扭矩图，判断危险截面。

解：1.求力偶矩

M_{eA}= 9549 ·P_A/n =9549×50/300=1591.5N·m

M_{eC}=M_{eB}=477.5N·m M_{eD} = 636.5N·m

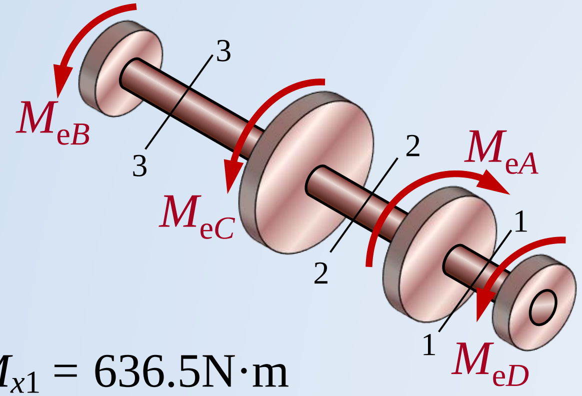
2.求扭矩

$$M_{x1} = M_{eD} = 636.5\text{N}\cdot\text{m}$$

$$M_{x2} = -(M_{eB}+M_{eC}) = -955\text{N}\cdot\text{m}$$

$$M_{x3} = -M_{eB} = -477.5\text{N}\cdot\text{m}$$

7.2 外力偶矩的计算 扭矩和扭矩图

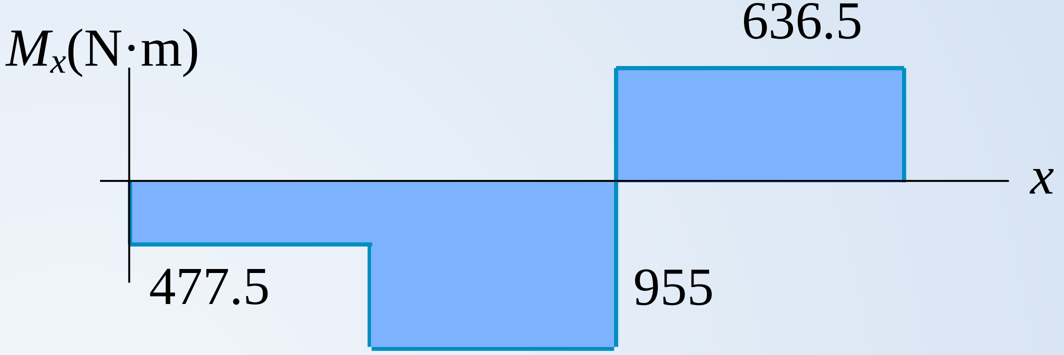


$M_{x1} = 636.5\text{N·m}$

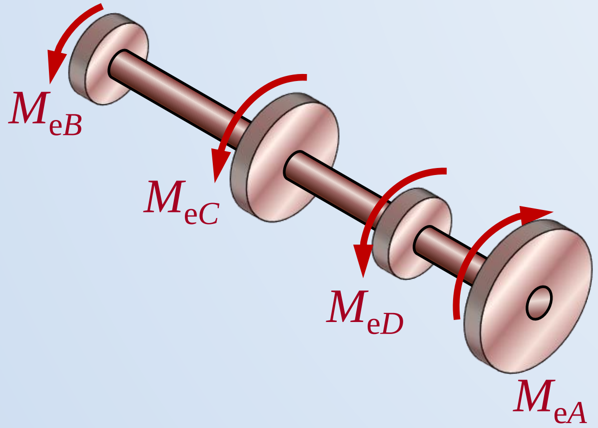
$M_{x2} = -955\text{N·m}$

$M_{x3} = -477.5\text{N·m}$

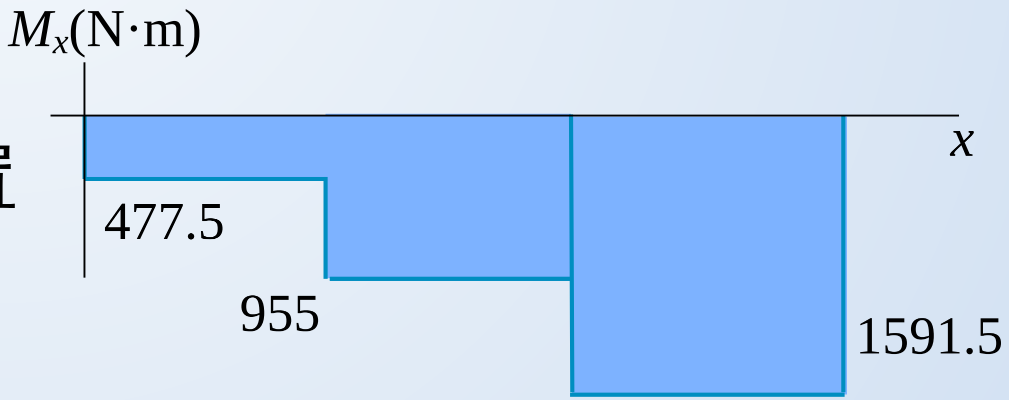
3.画扭矩图



危险截面： AC 段 $|M_x|_{max} = 955\,\text{N·m}$



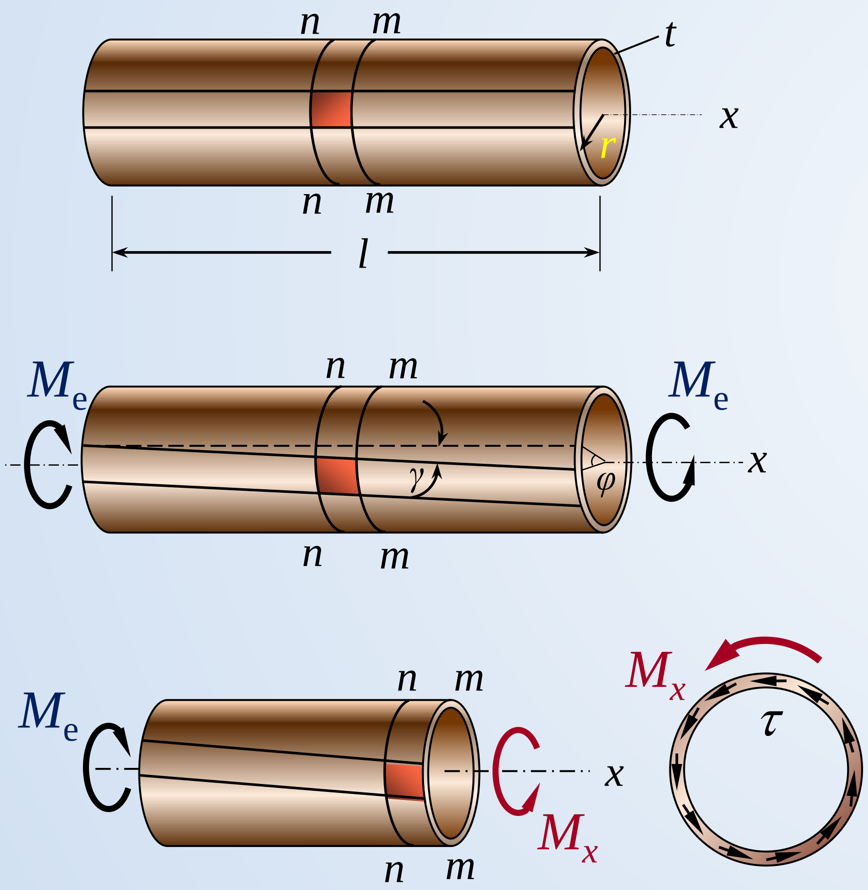
改变M_A的位置



$|M_x|_{max} = 1591.5\,\text{N·m}$ 不合理

7.3 薄壁圆筒的扭转 纯剪切

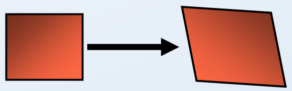
一. 薄壁筒扭转实验



实验观察　分析变形

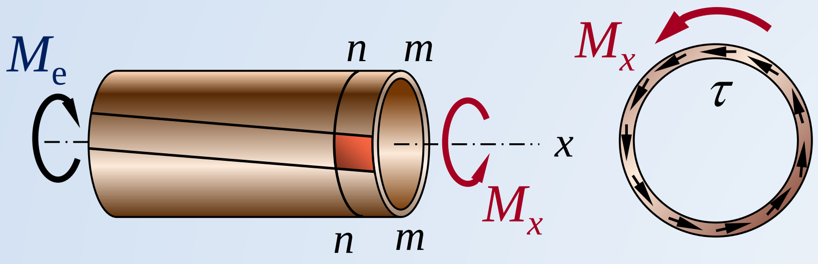
mn没变 $\varepsilon_x = 0$ $\Rightarrow$ $\sigma_x = 0$

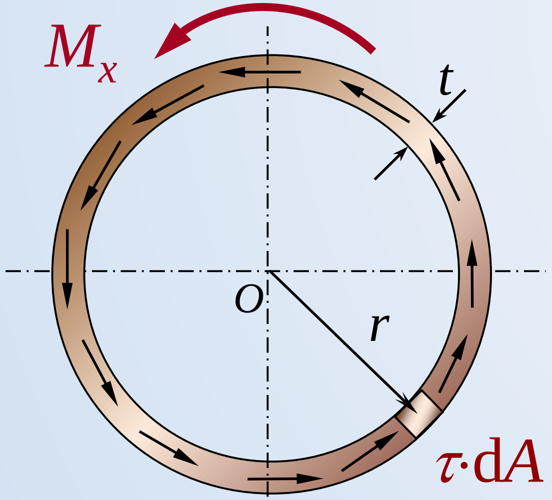
r没变 $\varepsilon_\theta = 0$ $\Rightarrow$ $\sigma_\theta = 0$



γ $\Rightarrow$ τ

7.3 薄壁圆筒的扭转 纯剪切





由于轴为薄壁,所以认为τ

沿t 均布. 即$\tau = C$

列平衡方程:

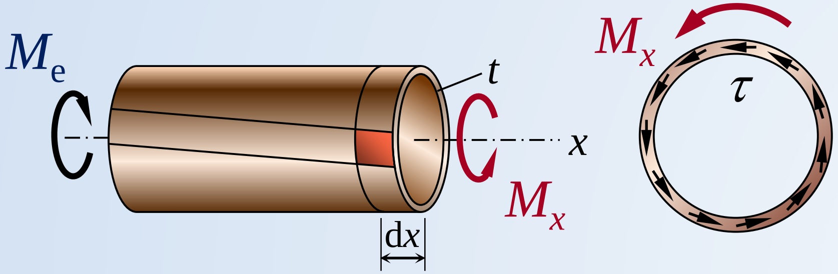
$$M_x = \int_A \tau \cdot \mathrm{d}A \cdot r = \tau \cdot 2\pi r t \cdot r$$

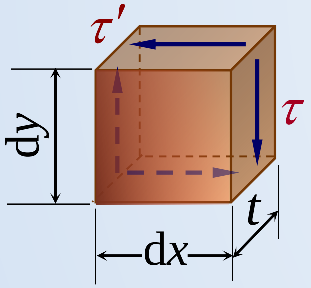
解得

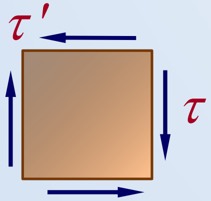
$$\boxed{\tau = \frac{M_x}{2\pi r^2 t}}$$

7.3 薄壁圆筒的扭转 纯剪切

二. 切应力互等定理







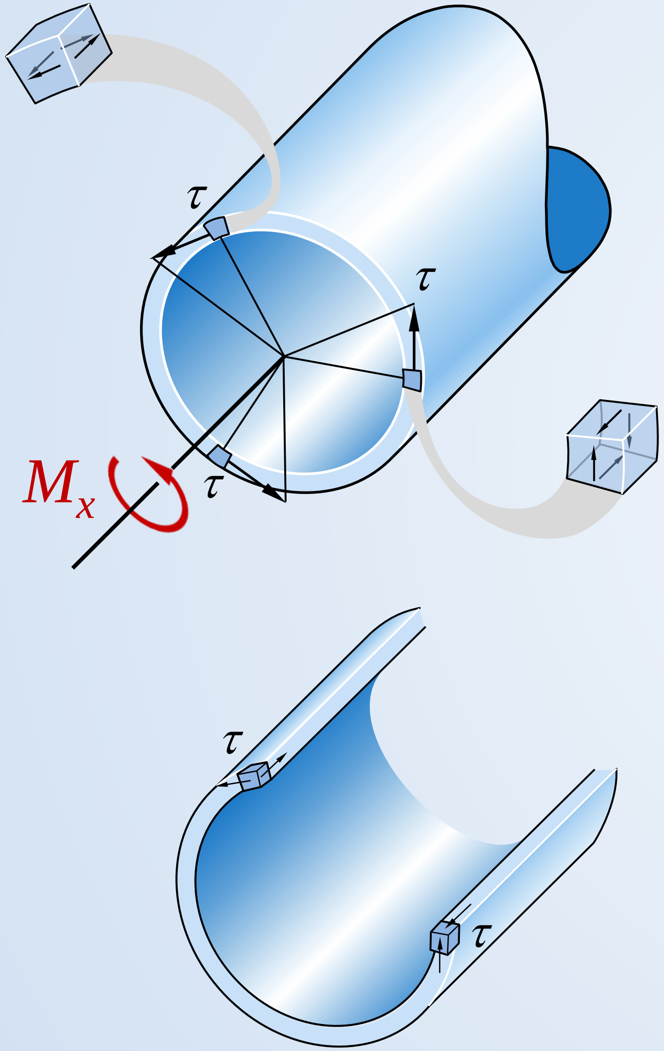
由微块的平衡条件可知:

$$\tau(\mathrm{d}y\cdot t)\mathrm{d}x = \tau'(\mathrm{d}x\cdot t)\mathrm{d}y$$

$$\tau = \tau'$$

互相垂直的两个平面上, 切应力必成对存在, 且大小相等,方向同时指向或背离两个面的交线。

7.3 薄壁圆筒的扭转 纯剪切



切应力互等定理口诀

相互垂直两平面，

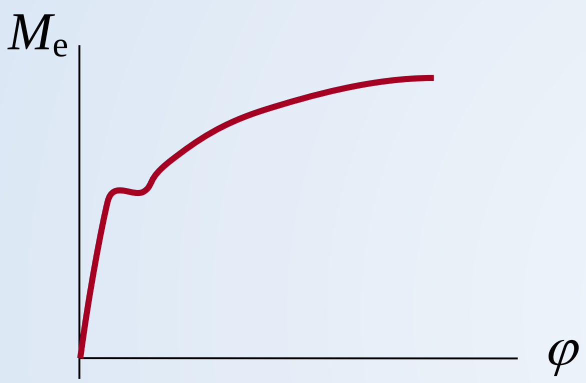
有切应力必成对，

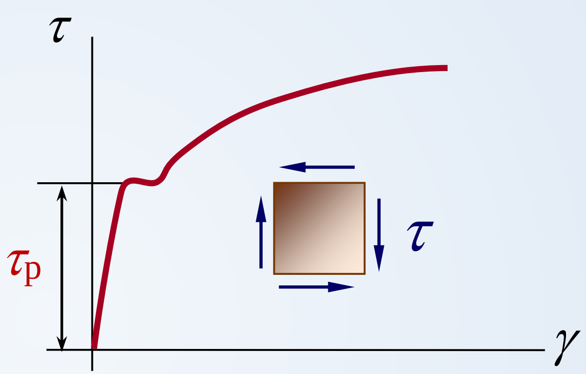
方向垂直于交线，

头对头或尾对尾。

7.3 薄壁圆筒的扭转 纯剪切

三. 剪切胡克定律





实验表明：$\tau \le \tau_p$ 时 $\tau \propto \gamma$

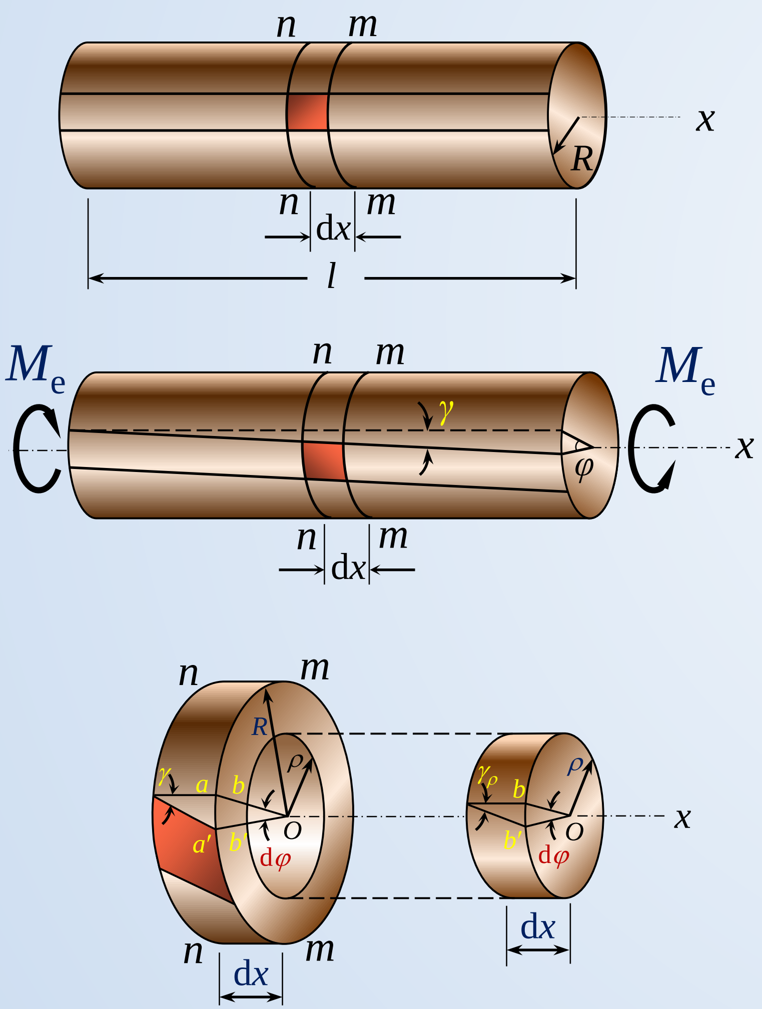
剪切胡克定律 $\boxed{\tau = G\gamma}$ G — 剪切弹性模量

剪变模量

E,G,μ 三者关系：$G = \dfrac{E}{2(1+\mu)}$

7.4 圆轴扭转时的应力与强度条件

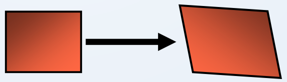
一. 圆轴扭转时横截面上的应力



实验观察:

mn没变 $\varepsilon_x = 0$ ⇒ $\sigma_x = 0$

R没变 $\varepsilon_\theta = 0$ ⇒ $\sigma_\theta = 0$

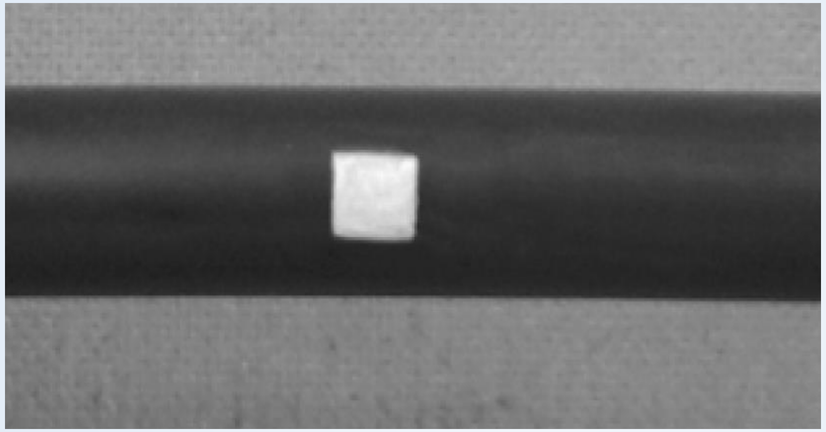
 γ ⇒ τ

假设： 刚性平面

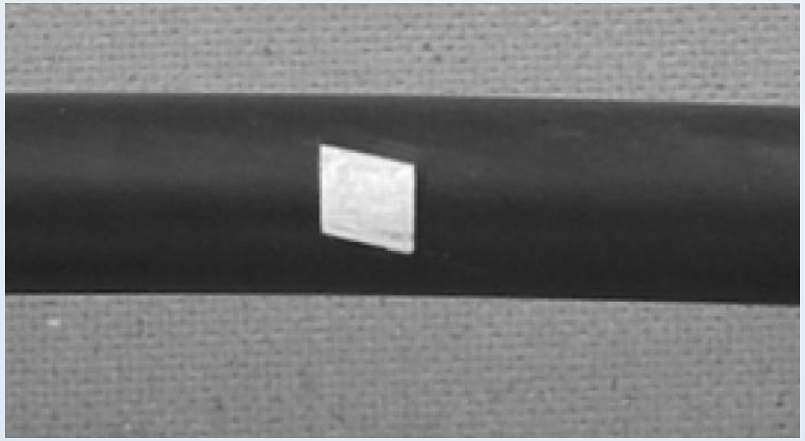
推理： 外 ⇒ 里

7.4 圆轴扭转时的应力与强度条件

实验结果

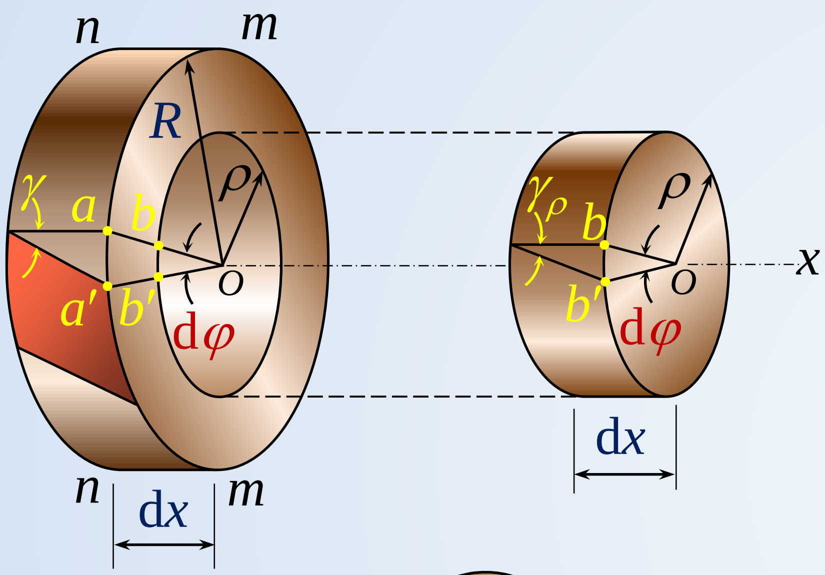


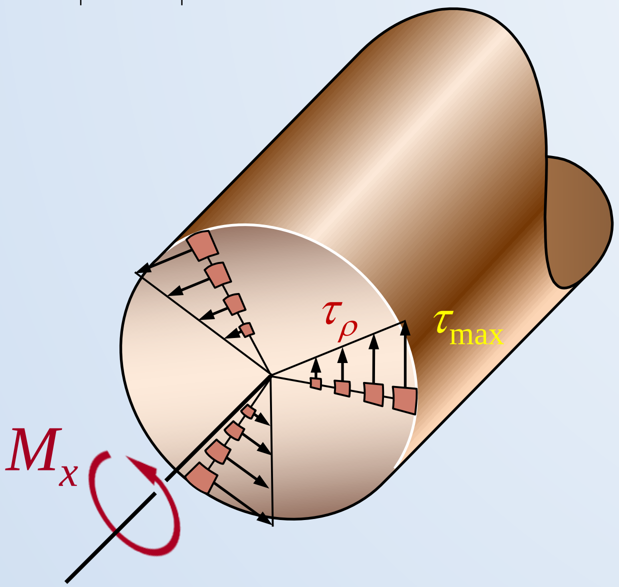
Before



After

7.4 圆轴扭转时的应力与强度条件





1. 几何方程

表面处 $aa' = \gamma \cdot \mathrm{d}x = \dfrac{d}{2} \cdot \mathrm{d}\varphi$

ρ处 $bb' = \gamma_\rho \cdot \mathrm{d}x = \rho \cdot \mathrm{d}\varphi$

得： $\gamma_\rho = \rho \dfrac{\mathrm{d}\varphi}{\mathrm{d}x}$

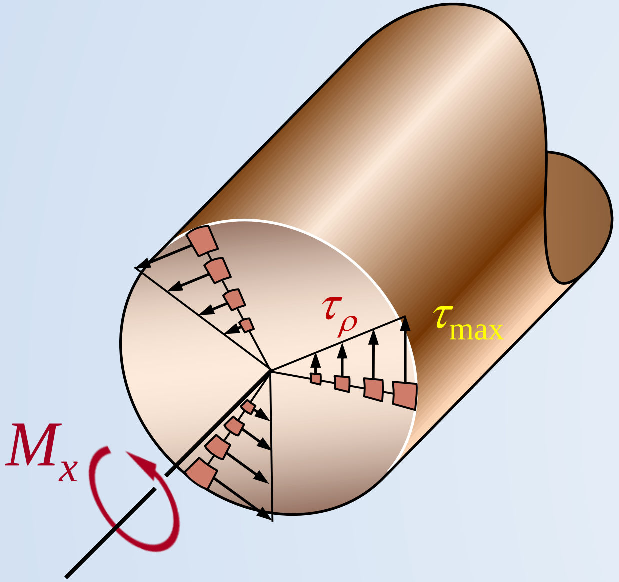
2. 物理方程

$\tau \le \tau_{\mathrm{p}}$ $\qquad \tau_\rho = G\gamma_\rho$

$$\tau_\rho = G\rho \frac{\mathrm{d}\varphi}{\mathrm{d}x}$$

7.4 圆轴扭转时的应力与强度条件

3. 静力方程



$$M_x = \int_A \tau_\rho \mathrm{d}A \cdot \rho = \int_A G\frac{\mathrm{d}\varphi}{\mathrm{d}x}\rho^2 \mathrm{d}A$$

$$\tau_\rho = G\rho\frac{\mathrm{d}\varphi}{\mathrm{d}x}$$

$$M_x = G\frac{\mathrm{d}\varphi}{\mathrm{d}x}\int_A \rho^2 \mathrm{d}A \qquad I_\mathrm{P} = \int_A \rho^2 \mathrm{d}A$$

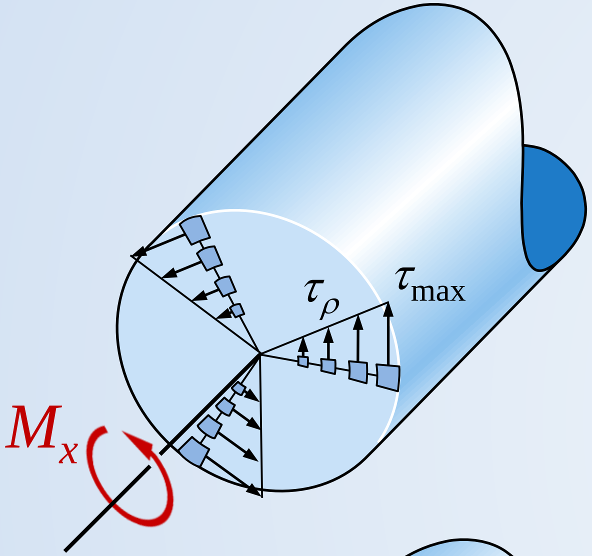
I_P 极惯性矩

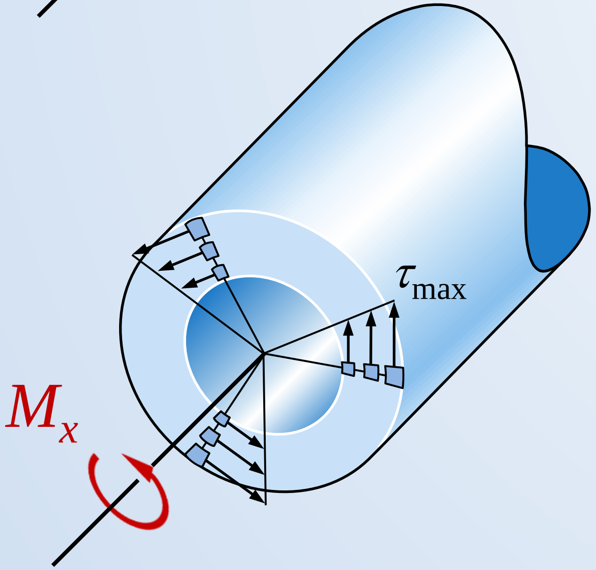
故 $M_x = G\dfrac{\mathrm{d}\varphi}{\mathrm{d}x}I_\mathrm{P}$

得 $\dfrac{\mathrm{d}\varphi}{\mathrm{d}x} = \dfrac{M_x}{GI_\mathrm{P}}$

GI_P 抗扭刚度

7.4 圆轴扭转时的应力与强度条件





$$\frac{d\varphi}{dx}=\frac{M_x}{GI_P}$$ 代入 $$\tau_\rho=G\rho\frac{d\varphi}{dx}$$

得

$$\boxed{\tau_\rho=\frac{M_x\cdot\rho}{I_P}}$$

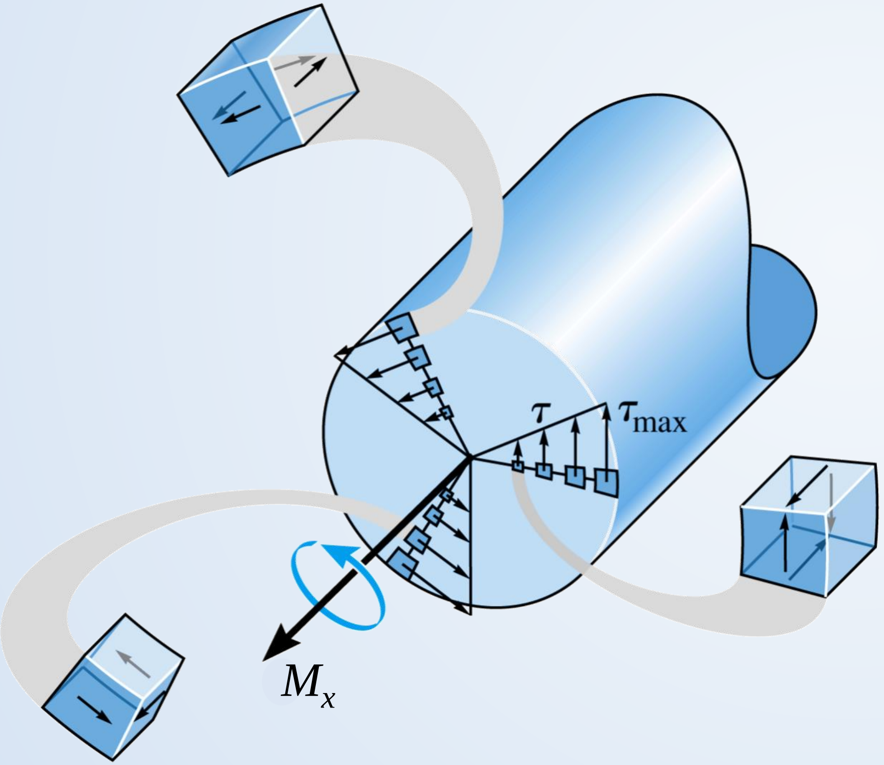
$$\rho_{max}=\frac{D}{2}$$ 令 $$W_P=\frac{I_P}{D/2}$$

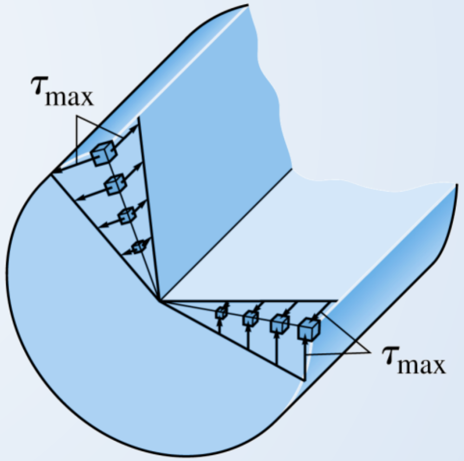
$$\boxed{\tau_{max}=\frac{M_x}{W_P}}$$

W_P 抗扭截面模量

7.4 圆轴扭转时的应力与强度条件

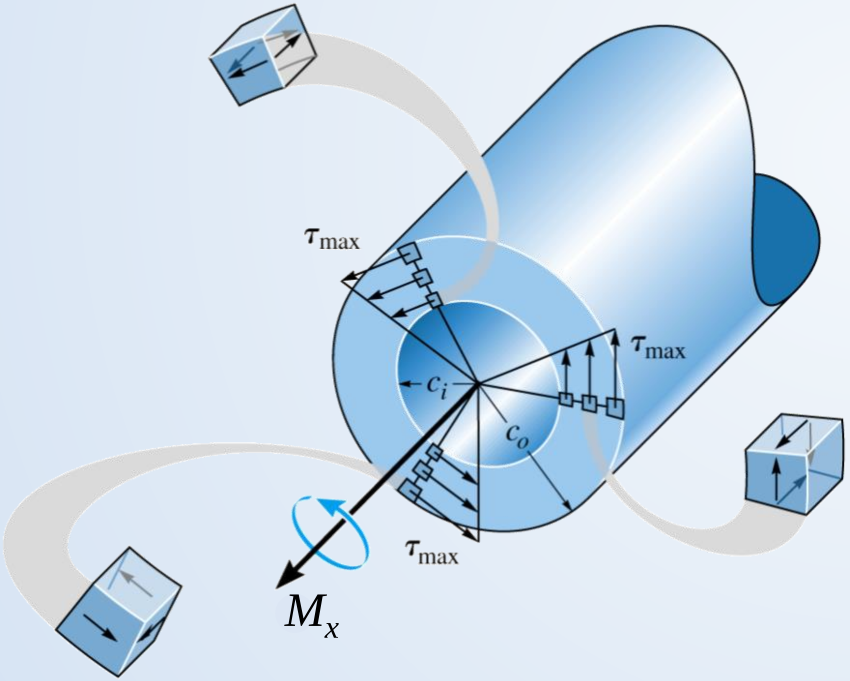
切应力互等定理

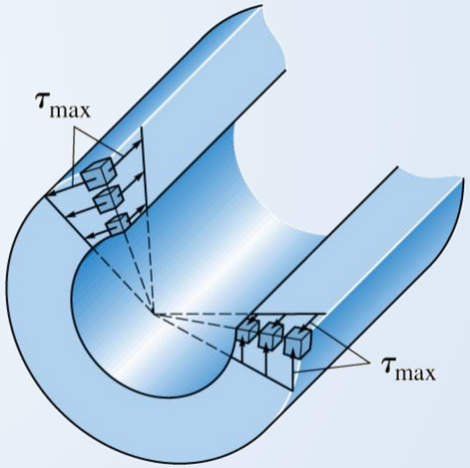




7.4 圆轴扭转时的应力与强度条件

切应力互等定理





7.4 圆轴扭转时的应力与强度条件

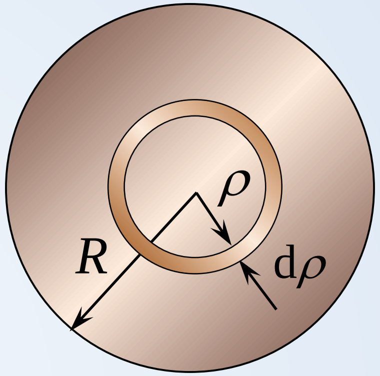
二. 计算 I_{P}, W_P

1. 实心圆截面

$$I_{\mathrm{P}} = \int_A \rho^2 \mathrm{d}A = \int_0^{\frac{D}{2}} \rho^2 2\pi\rho \mathrm{d}\rho$$

$$= 2\pi \int_0^{\frac{D}{2}} \rho^3 \mathrm{d}\rho = \frac{\pi D^4}{32}$$

$$W_{\mathrm{P}} = \frac{I_{\mathrm{P}}}{R} = \frac{I_{\mathrm{P}}}{D/2} = \frac{\pi D^3}{16}$$



$$\boxed{I_{\mathrm{P}} = \frac{\pi D^4}{32}}$$

$$\boxed{W_{\mathrm{P}} = \frac{\pi D^3}{16}}$$

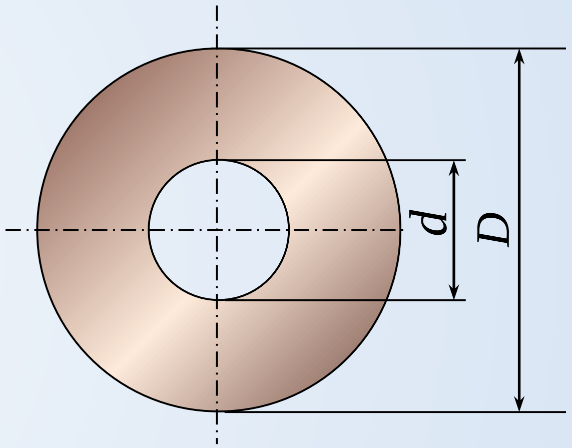
7.4 圆轴扭转时的应力与强度条件

2. 空心圆截面

$$I_{\mathrm{P}} = \frac{\pi D^4}{32} - \frac{\pi d^4}{32} = \frac{\pi (D^4 - d^4)}{32} = \frac{\pi D^4}{32}(1-\alpha^4)$$

$$W_{\mathrm{P}} = \frac{I_{\mathrm{P}}}{R} = \frac{I_{\mathrm{P}}}{D/2} = \frac{\pi D^3}{16}(1-\alpha^4)$$

$$\boxed{I_{\mathrm{P}} = \frac{\pi D^4}{32}(1-\alpha^4)} \qquad \boxed{W_{\mathrm{P}} = \frac{\pi D^3}{16}(1-\alpha^4)}$$



$$\frac{d}{D} = \alpha$$

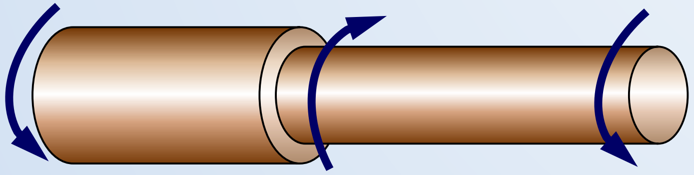
三. 圆轴扭转时的强度条件

对等直轴:

$$\tau_{\max} = \frac{M_{x\max}}{W_P} \le [\tau]$$

$M_{x\max}$为危险截面扭矩

对阶梯轴:



$$\tau_{\max} = \frac{M_{x\max}}{W_P} \le [\tau]$$

分段计算，求出 $\tau_{\max}$

7.4 圆轴扭转时的应力与强度条件

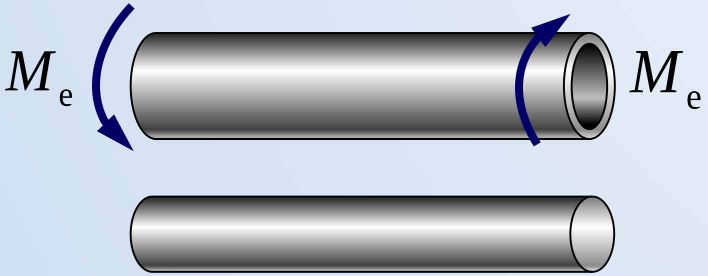
例2 已知：$D=76\text{mm}, t=2.5\text{mm}, [\tau]=100\text{MPa} \quad M_e=1.98\text{kN}\cdot\text{m}$

1.校核扭转强度 2.改为强度相同实心轴 求 $D', W_{空}/W_{实}$

解： 1。·求内力 $M_{x\max}=M_e=1.98\text{kN}\cdot\text{m}$

· 求 $\tau_{\max}$ 代入 $\boxed{\tau_{\max}=\dfrac{M_{x\max}}{W_p}\le[\tau]}$

$$\alpha=\frac{d}{D}=\frac{D-2t}{D}=\frac{76-2\times2.5}{76}=0.935$$

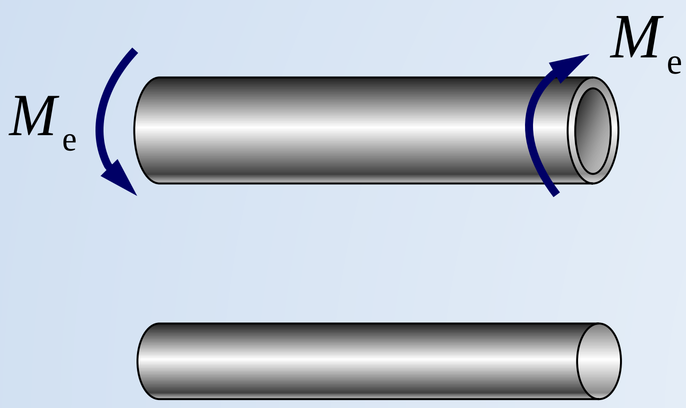


$$W_p=\frac{\pi D^3}{16}(1-\alpha^4)=20.3\times10^{-6}\text{m}^3$$

$$\tau_{\max}=\frac{M_{x\max}}{W_p}=\frac{1.98\times10^3}{20.3\times10^{-6}}=97.5\text{MPa}\le[\tau]$$

此轴安全

7.4 圆轴扭转时的应力与强度条件



2。 ·两轴强度相等，故:

$$\tau_{\max} = \frac{M_{x\max}}{W_p} = \frac{1.98\times10^3}{\pi D'^3/16} = 97.5\times10^6\ \text{Pa}$$

$$D' = \sqrt[3]{\frac{1.98\times10^3\times16}{\pi\cdot97.5\times10^6}} = 0.0469\,\text{m}$$

$$\boxed{D' = 46.9\,\text{mm}}$$

·比较重量:

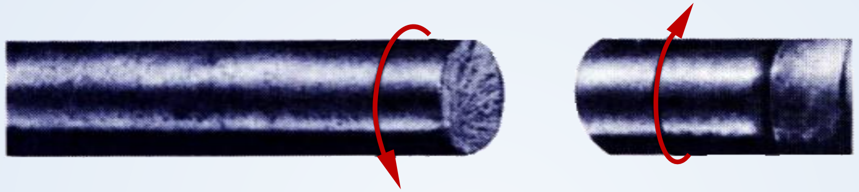
$$\frac{W_{空}}{W_{实}} = \frac{A_{空}}{A_{实}} = \frac{D^2-d^2}{D'^2} = \frac{76^2-(76-2\times2.5)^2}{46.9^2} = 0.334$$

$$\boxed{\frac{W_{空}}{W_{实}} = 0.334}$$

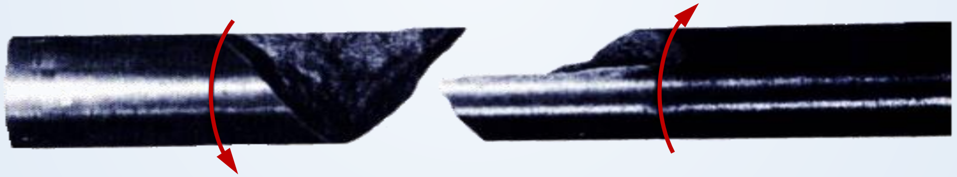
显然,空心轴比实心轴节省材料. 在扭转轴设计中,选用空心轴是一种合理的设计.

四. 圆轴扭转时斜截面的应力

低碳钢



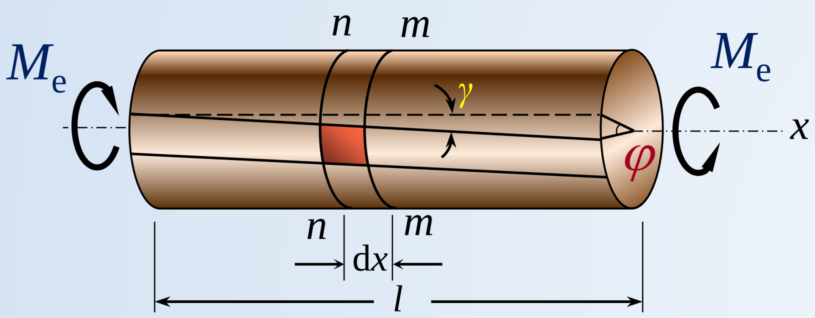
铸铁



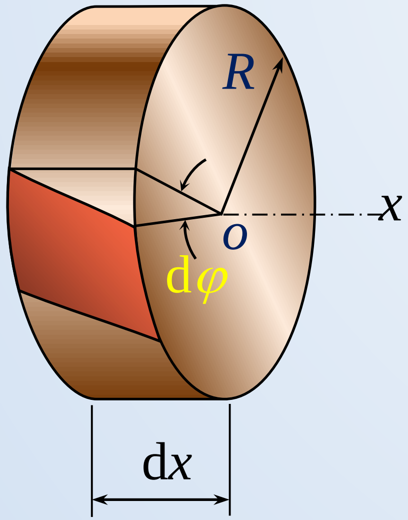
如何解释扭转破坏产生的原因呢？

7.5 圆轴扭转时的变形与刚度条件

一.两横截面间相对扭转角φ



由前节 $\boxed{\dfrac{d\varphi}{dx}=\dfrac{M_x}{GI_P}}$

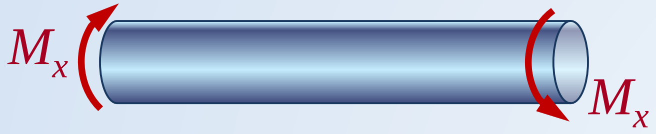


得 $$d\varphi=\frac{M_x}{GI_P}dx$$

积分得 $$\varphi=\int_l d\varphi=\int_0^l \frac{M_x}{GI_P}dx$$

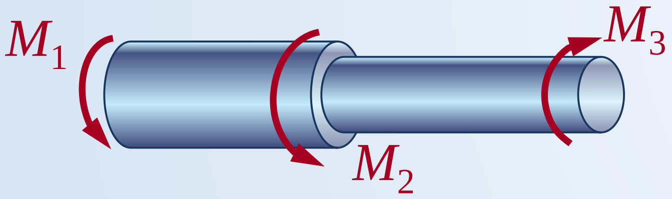
7.5 圆轴扭转时的变形与刚度条件

1. 当l段内M_x、GI_P为常数



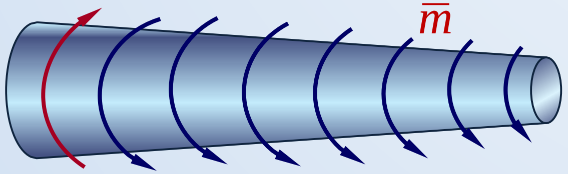
$$U = W = \frac{1}{2}M_x\varphi = \frac{M_x^2 l}{2GI_P}$$

2. 当M_x、GI_P为分段常数



$$U = \sum_{i=1}^{n}\frac{M_{xi}^2 l_i}{2GI_{Pi}}$$

3. 当M_x沿x为连续函数$M_x(x)$

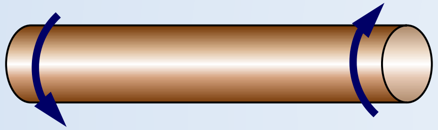


$$U = \int\frac{M_x^2(x)}{2GI_P}\mathrm{d}x$$

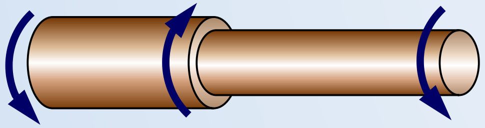
7.5 圆轴扭转时的变形与刚度条件

二. 刚度条件

对等直轴: $\theta = \dfrac{\mathrm{d}\varphi}{\mathrm{d}x} = \dfrac{M_x}{GI_\mathrm{P}}$ θ单位长度的扭转角



$$\theta_{\max} = \frac{M_{x\max}}{GI_\mathrm{P}} \cdot \frac{180}{\pi} \le [\theta](^\circ/\mathrm{m})$$



$$\theta_{\max} = \frac{M_{x\max}}{GI_\mathrm{P}} \cdot \frac{180}{\pi} \le [\theta](^\circ/\mathrm{m})$$

阶梯轴,分段校核

精密机床, $[\theta]=(0.25\sim0.5)^\circ/\mathrm{m}$;

一般传动轴, $[\theta]=(0.5\sim1)^\circ/\mathrm{m}$; $[\theta]=(2\sim4)^\circ/\mathrm{m}$;

7.5 圆轴扭转时的变形与刚度条件

三.计算

强度条件
刚度条件

解决三类问题

1.校核
2.设计
3.确载

步骤

1.求外力M_e

2.求内力（画M_x图 —$M_{x\max}$）

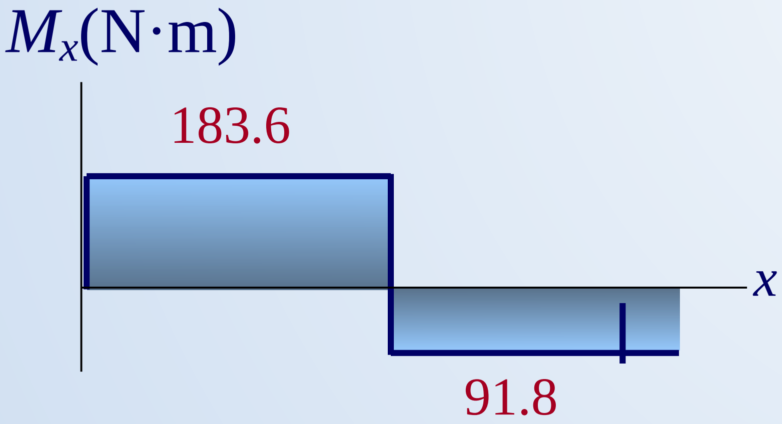
3.强度计算 $$\tau_{\max}=\frac{M_{x\max}}{W_p}\le[\tau]$$ （先计算I_P，W_P）

刚度计算 $$\theta_{\max}=\frac{M_{x\max}}{GI_P}\cdot\frac{180}{\pi}\le[\theta](^\circ/\mathrm{m})$$ 等直圆轴扭转

7.5 圆轴扭转时的变形与刚度条件

例3. 已知: P_A=6kW, P_B=4kW, P_C=2kW, $[\tau]$=30MPa, $[\theta]$=1°/m, G=80GPa,

n=208转/min, 求：$d=?$



解: ·计算外力矩:

$$M = 9549\frac{P}{n} = 9549\cdot\frac{6}{208} = 275.4\text{N·m}$$

$$M_B = 183.6\text{N·m} \qquad M_C = 91.8\text{N·m}$$

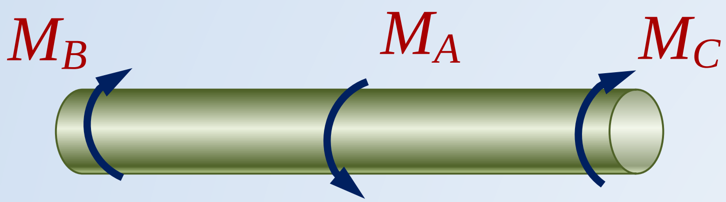
·求内力（扭矩图）

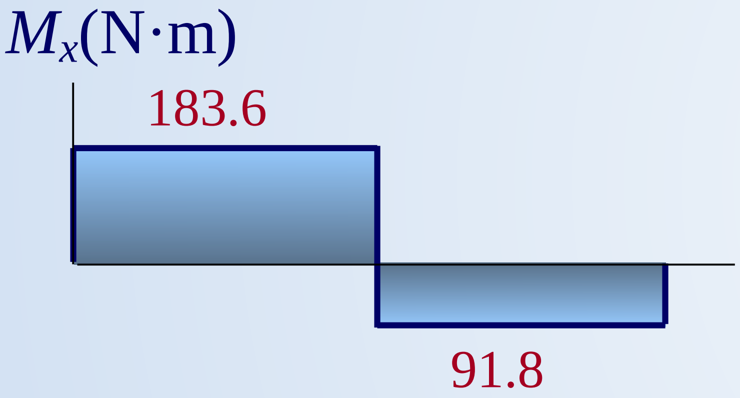
$$M_{x\max} = 183.6\text{N}\cdot\text{m}$$

·由强度条件

$$\tau_{\max} = \frac{M_{x\max}}{W_p} \le [\tau] \quad 其中\ W_p = \frac{\pi d^3}{16}$$

7.5 圆轴扭转时的变形与刚度条件





$$M_{x\max} = 183.6\,\mathrm{N\cdot m}$$

得 $d_1 \geq \sqrt[3]{\dfrac{16M_{x\max}}{\pi[\tau]}} = \sqrt[3]{\dfrac{16\times182.6}{\pi\times30\times10^6}} = 31.5\times10^{-3}\,\mathrm{m} = 31.5\mathrm{mm}$

·由刚度条件

$$\theta_{\max} = \frac{M_{x\max}}{GI_P}\cdot\frac{180}{\pi} \leq [\theta] \quad \text{其中}\ I_p = \frac{\pi d^4}{32}$$

得 $d_2 \geq \sqrt[4]{\dfrac{32M_{x\max}\cdot180}{G\pi^2[\tau]}} = \sqrt[4]{\dfrac{32\times183.6\times180}{80\times10^9\cdot\pi^2\cdot1}} = 34\times10^{-3}\,\mathrm{m}$

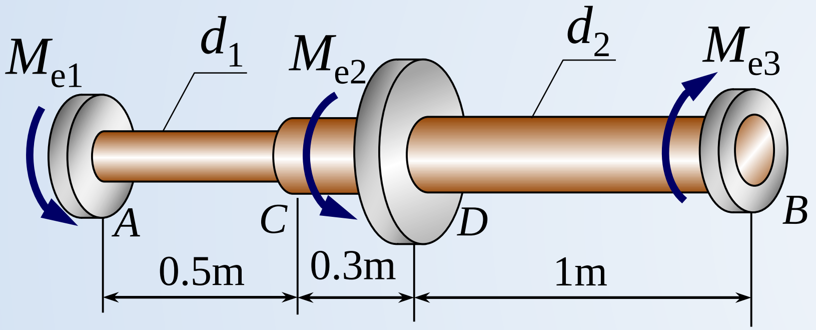
$= 34\mathrm{mm}$

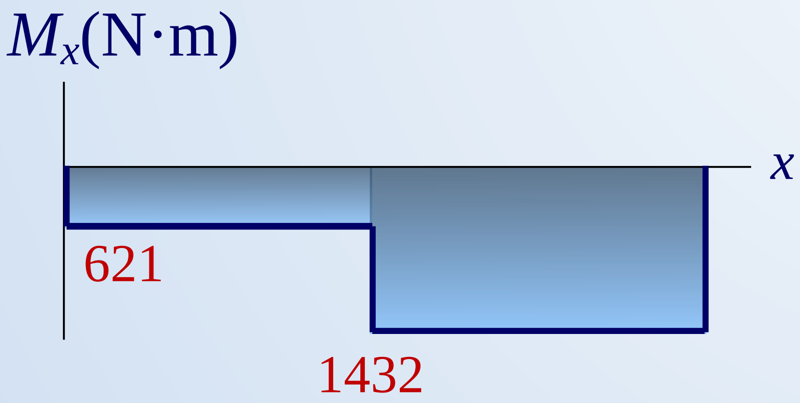
$$\boxed{\text{取直径}\ d = \{d_1, d_2\}_{\max} = 34\mathrm{mm}}$$

7.5 圆轴扭转时的变形与刚度条件

例4 阶梯轴d_1=4cm, d_2=7cm, P_3=30kW, P_1=13kW,n=200r/min, [τ]=60MPa

G=80GPa, [θ]=2°/m 试校核轴的强度和刚度





解：1.求外力偶矩

$$M_{e1}=9549\frac{P_1}{n}=621\text{N}\cdot\text{m}$$

$$M_{e3}=9549\frac{P_3}{n}=1432\text{N}\cdot\text{m}$$

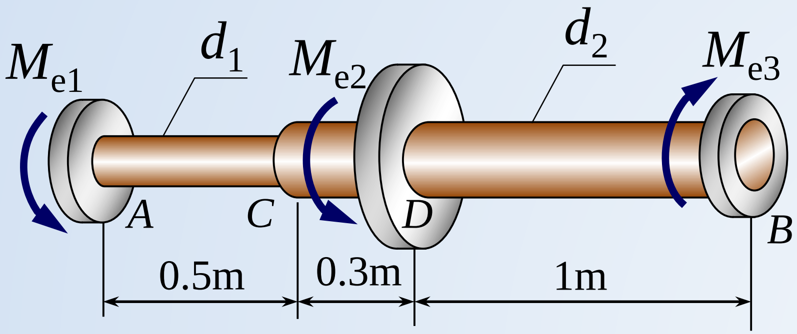
$$M_{e2}=M_{e3}-M_{e1}=811\text{N}\cdot\text{m}$$

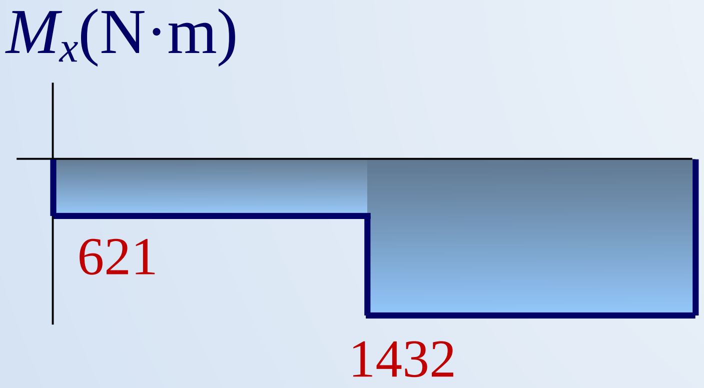
2.求内力(画M_x图,判断危险截面)

AC段 $M_{1\max}=621\text{N}\cdot\text{m}$

DB段 $M_{2\max}=1432\text{N}\cdot\text{m}$

7.5 圆轴扭转时的变形与刚度条件





3.分段作校核

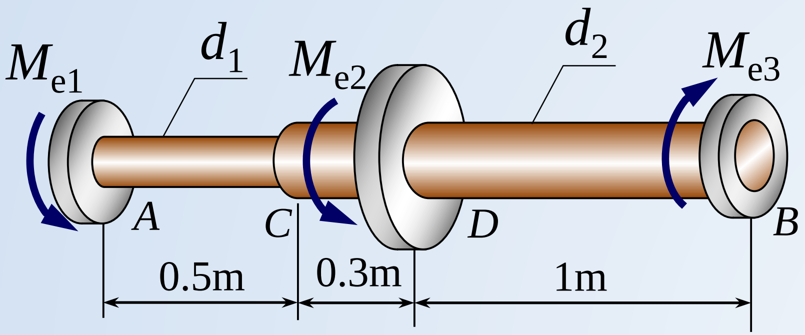
·校核*AC*段

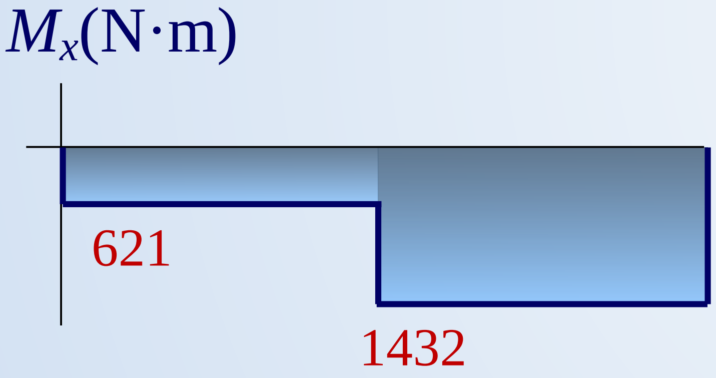
$$\tau_1=\frac{M_{1\max}}{W_{P1}}=\frac{M_{1\max}}{\frac{1}{16}\pi d_1^3}=\frac{16\times621}{\pi\cdot0.04^3}=49.4\,\text{MPa}<[\tau]$$

$$\theta_1=\frac{M_{1\max}}{GI_{P1}}\cdot\frac{180}{\pi}=\frac{M_{1\max}}{G\cdot\frac{1}{32}\pi d_1^4}\cdot\frac{180}{\pi}=\frac{32\times621\times180}{80\times10^9\cdot\pi^2\cdot0.04^4}$$

$$=1.77^\circ\,/\,\text{m}<[\theta]$$

7.5 圆轴扭转时的变形与刚度条件





·校核*DB*段

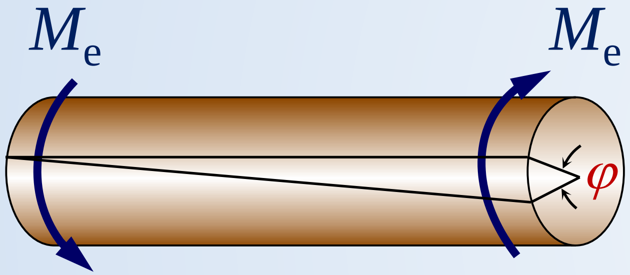
$$\tau_2 = \frac{M_{2\max}}{W_{P2}} = \frac{M_{2\max}}{\frac{1}{16}\pi d_2^3} = \frac{16\times1432}{\pi\cdot0.07^3} = 21.3\text{MPa} < [\tau]$$

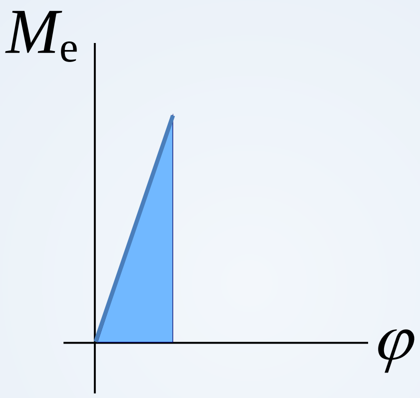
$$\theta_1 = \frac{M_{2\max}}{GI_{P2}}\cdot\frac{180}{\pi} = \frac{M_{2\max}}{G\cdot\frac{1}{16}\pi d_2^4}\cdot\frac{180}{\pi} = \frac{32\times1432\times180}{80\times10^9\cdot\pi^2\cdot0.07^4}$$

$$= 0.435^\circ/\text{m} < [\theta]$$

$\boxed{\text{此轴安全}}$

四. 圆轴扭转时弹性变形能

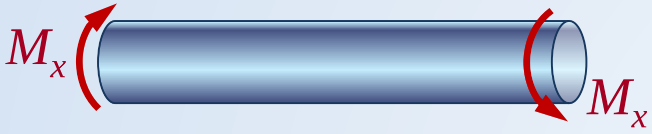




$$\tau \leq \sigma_P \qquad U = W = \frac{1}{2} M_e \varphi$$

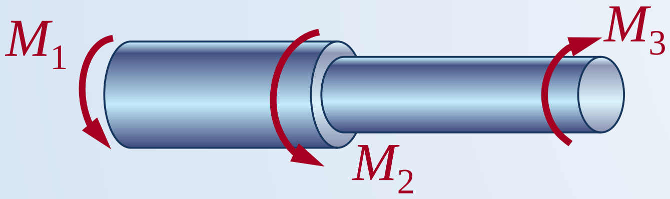
7.5 圆轴扭转时的变形与刚度条件

1. 当l段内M_x、GI_P为常数



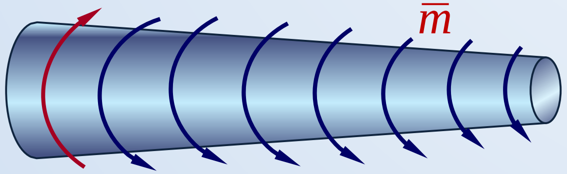
$$U = W = \frac{1}{2} M_x \varphi = \frac{M_x^2 l}{2GI_P}$$

2. 当M_x、GI_P为分段常数



$$U = \sum_{i=1}^{n} \frac{M_{xi}^2 l_i}{2GI_{Pi}}$$

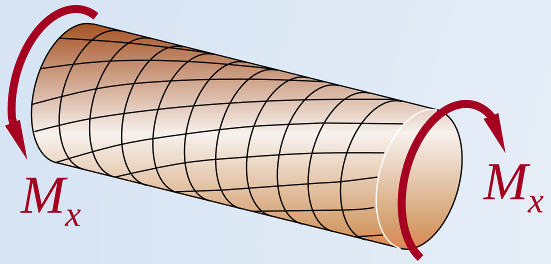
3. 当M_x沿x为连续函数$M_x(x)$

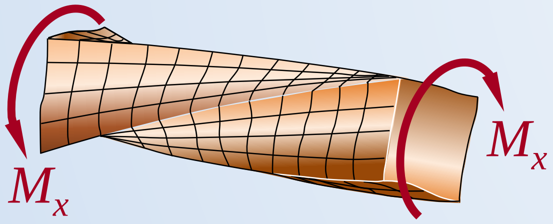


$$U = \int \frac{M_x^2(x)}{2GI_P} \mathrm{d}x$$

7.6 非圆截面杆扭转的概念

一.非圆截面杆和圆截面杆扭转时的区别





变形特点:

圆截面杆：刚性平面

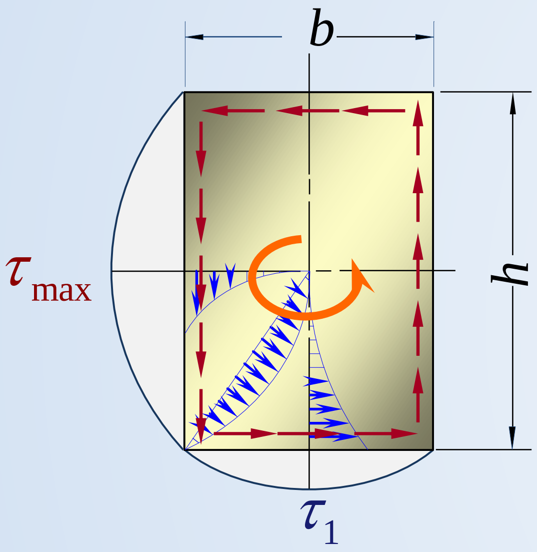
非圆截面杆：横截面产生翘曲.

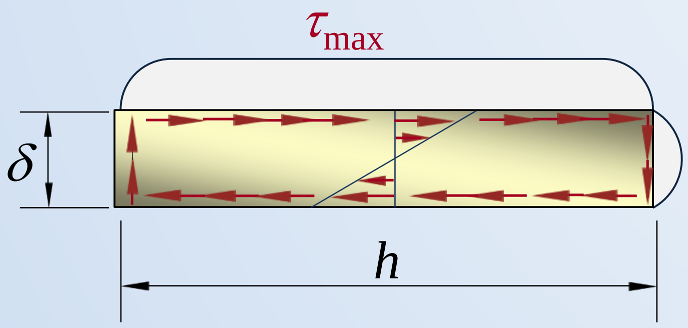
前面的公式均不适用

引用弹性理论的结论.

7.6 非圆截面杆扭转的概念

二. 矩形截面杆的扭转





横截面上切应力分布特点:

1.周边的τ必与周边相切

2.外尖角处 $\tau \equiv 0$

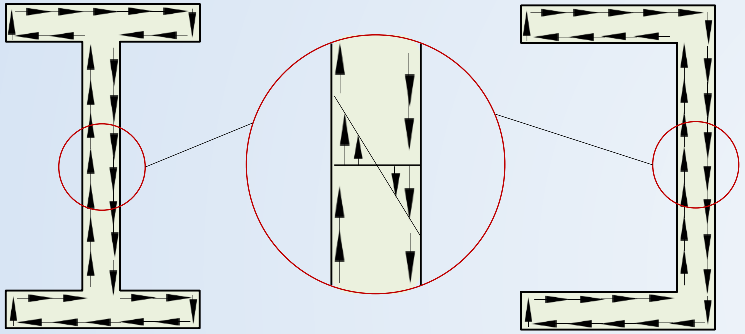
3. τ_{max}发生在长边中点

$$\tau_{max} = \frac{M_x}{\alpha h b^2}$$ α与h/b有关

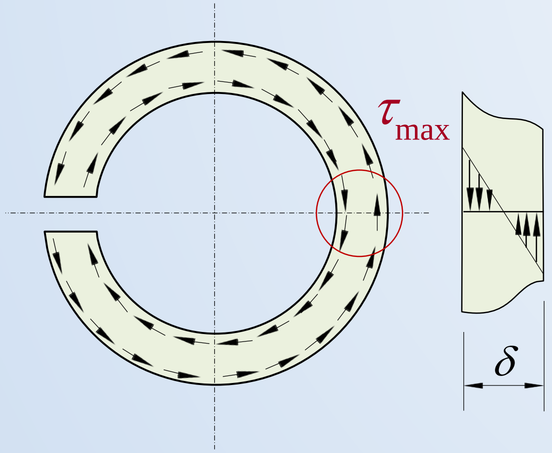
4. 当$h/b>10$时 $\tau_{max} = \dfrac{M_x}{\frac{1}{3} h\delta^2}$

7.7 薄壁杆件的自由扭转

一. 开口薄壁杆件的自由扭转



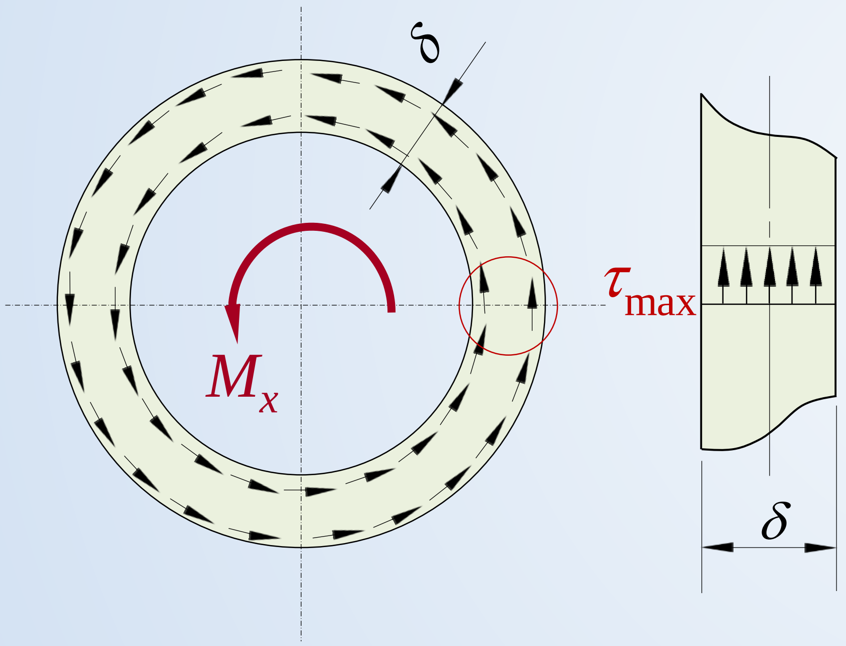
$$\tau_{\max} = \frac{M_x \delta_{\max}}{I_t} \qquad I_t = \eta \sum \frac{1}{3} h_i \delta_i^3$$



中线为曲线的开口薄壁杆件，计算时可将截面展开，作为狭长矩形截面处理。

7.7 薄壁杆件的自由扭转

二. 闭口薄壁杆件的自由扭转



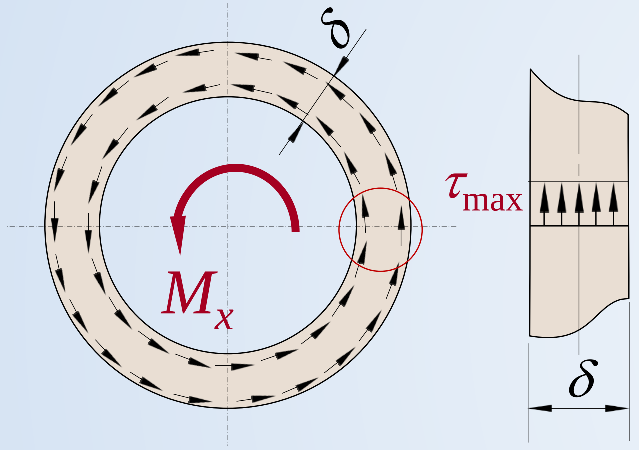
$$\tau = \frac{t}{\delta} = \frac{M_x}{2\omega\delta}$$

$$\varphi = \frac{M_x lS}{4G\omega^2\delta} = \frac{M_x l}{GI_t}$$

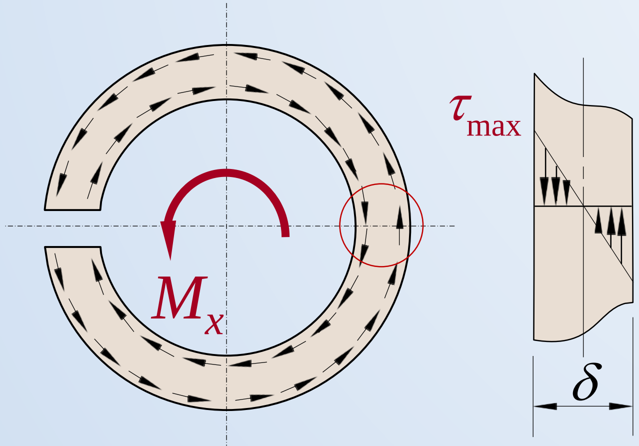
ω是截面中线所围面积

S是截面中线的长度

三.开口薄壁杆件与闭口薄壁杆件扭转的比较



$$\tau_{\max 闭口}=\frac{M_x}{2\omega\delta}=\frac{2M_x}{\pi d^2\delta}$$



$$\tau_{\max 开口}=\frac{M_x}{\frac{1}{3}h\delta^2}=\frac{3M_x}{\pi d\delta^2}$$

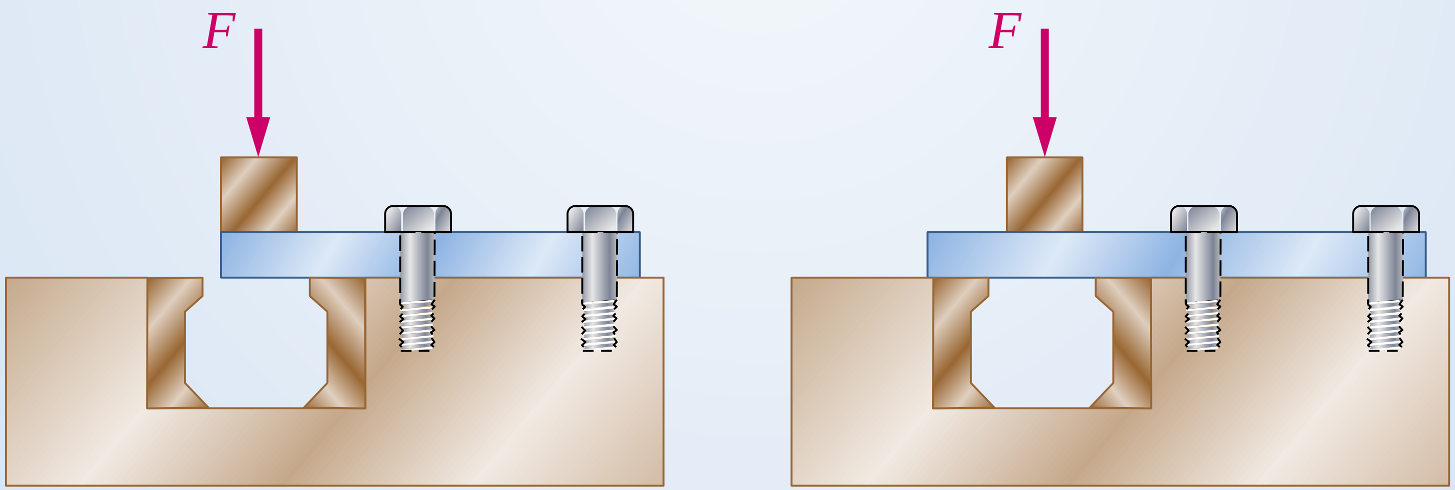
$d>>\delta$, 开口薄壁杆件的应力远大于闭口薄壁杆件的杆件, 所以工程上不采用开口薄壁杆件

7.8 剪切和挤压的实用计算

一. 剪切构件的受力和变形特点

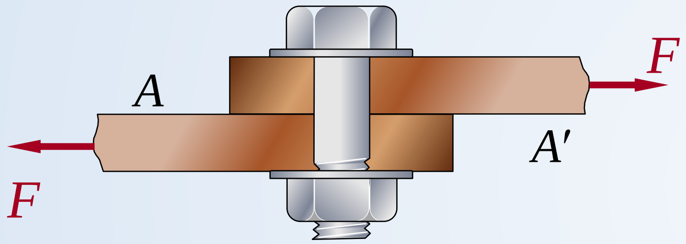
受力特点: 外力大小相等、方向相反,且作用线相距很近

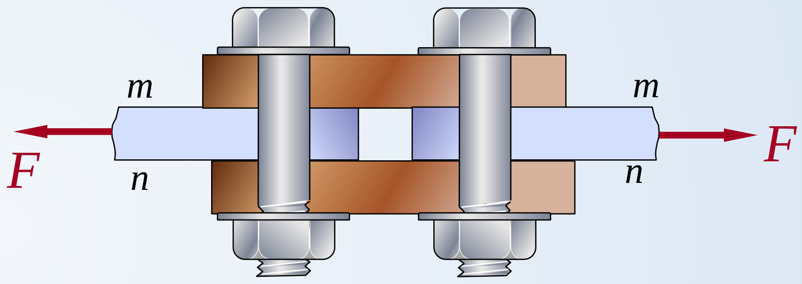
变形特点: 剪切面发生相对错动

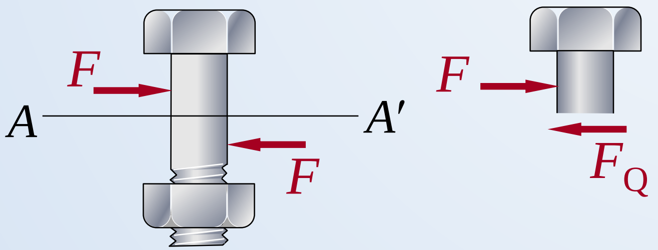


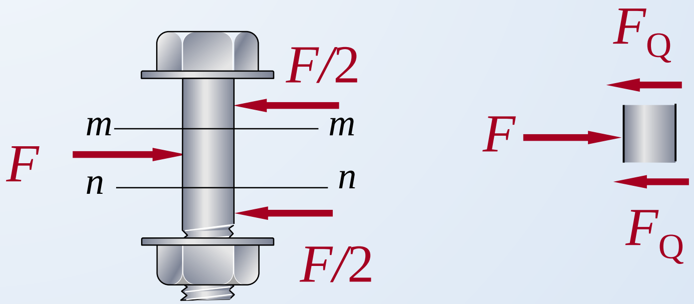
7.8 剪切和挤压的实用计算

工程实例: 螺栓



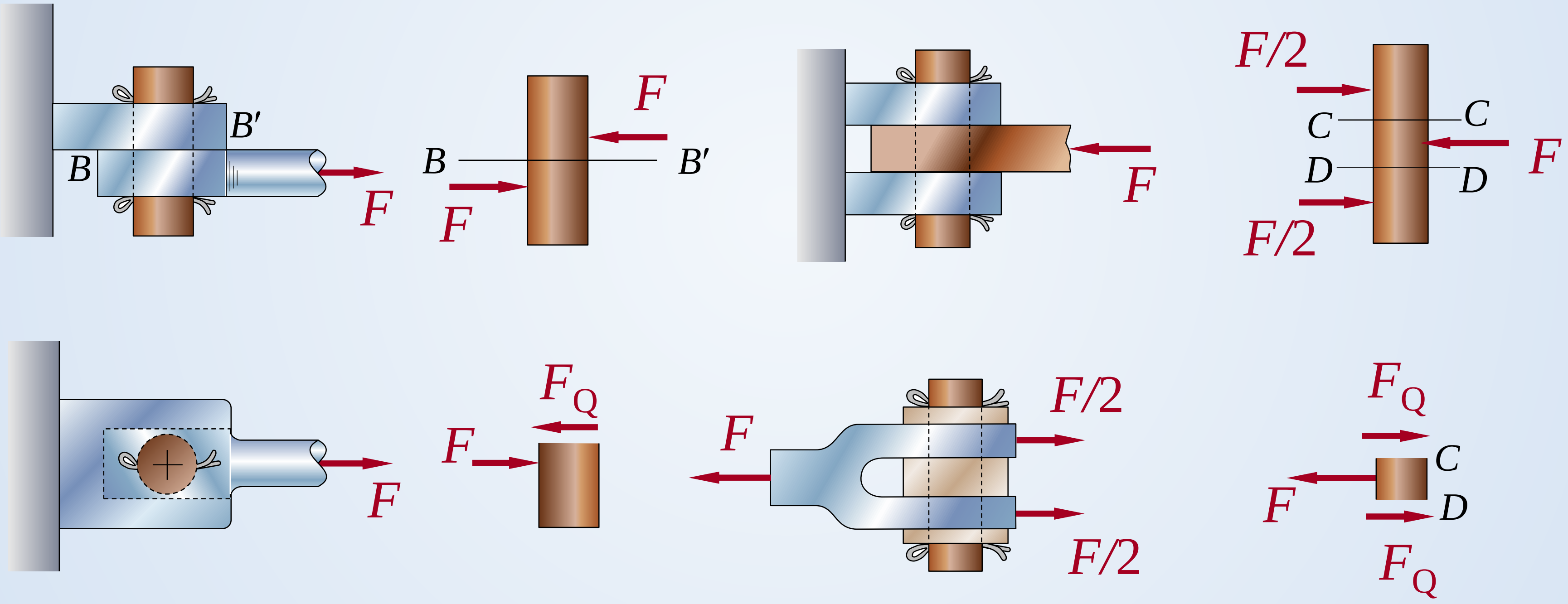




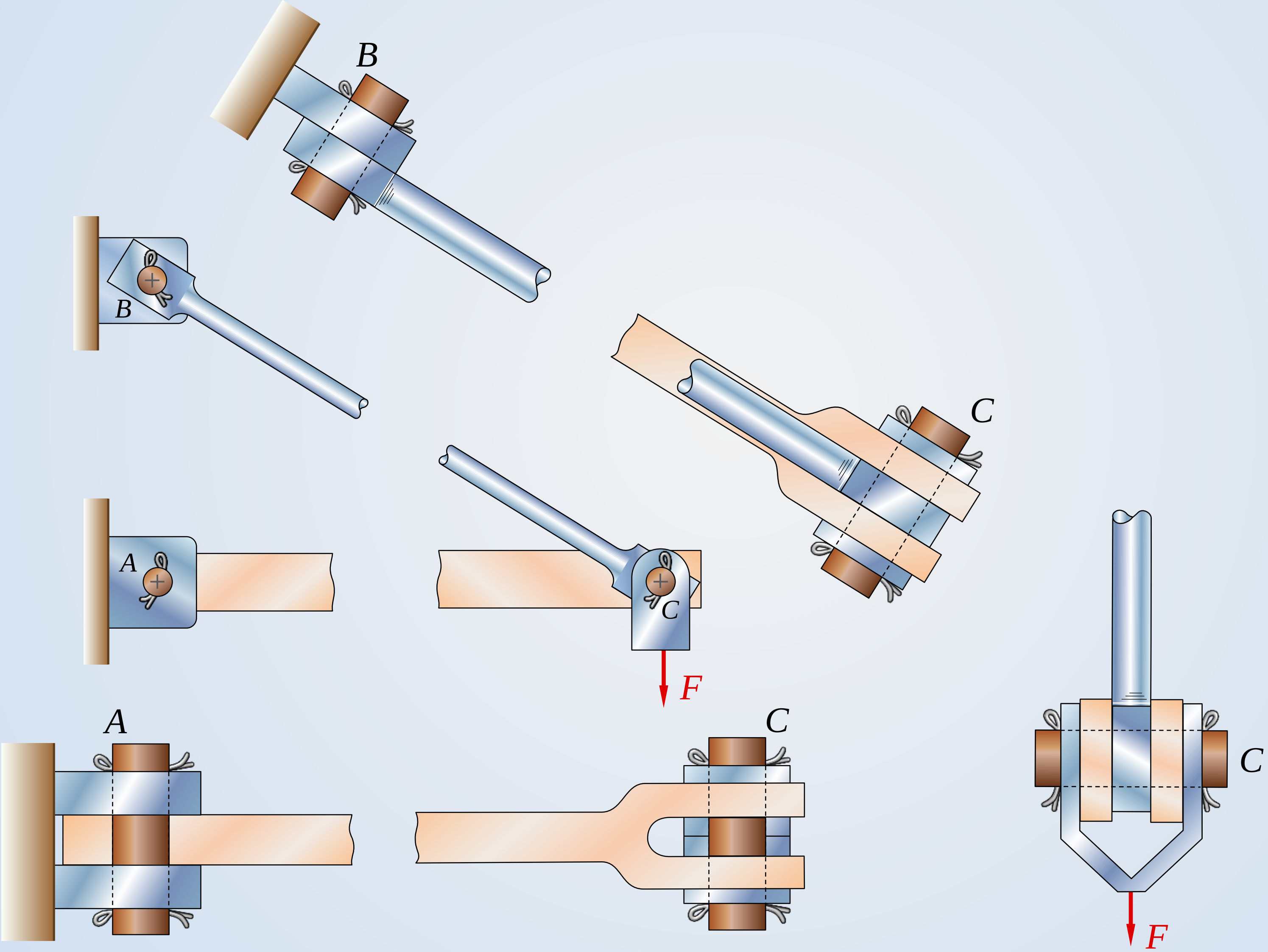


7.8 剪切和挤压的实用计算

销钉



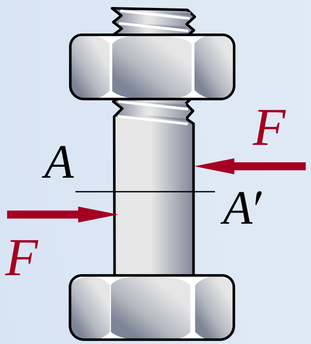
7.8 剪切和挤压的实用计算



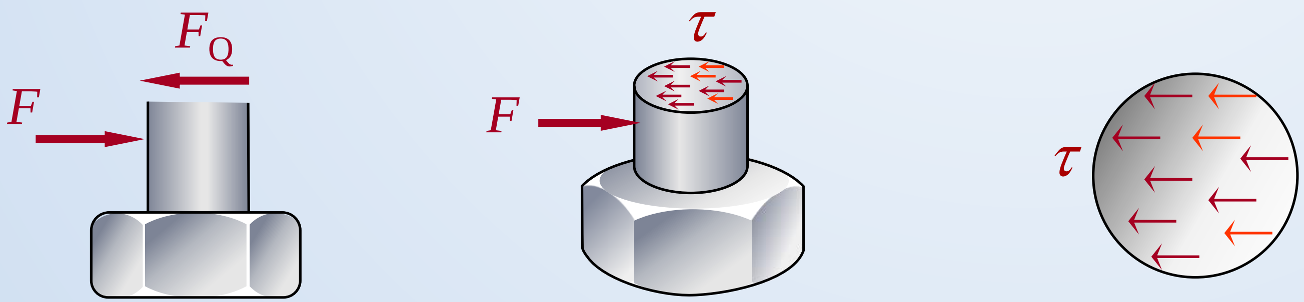
7.8 剪切和挤压的实用计算

二. 剪切的实用计算

实用计算方法: 内力分布复杂, τ不能推导,只能作出尽量反映实际的假设,简化计算。



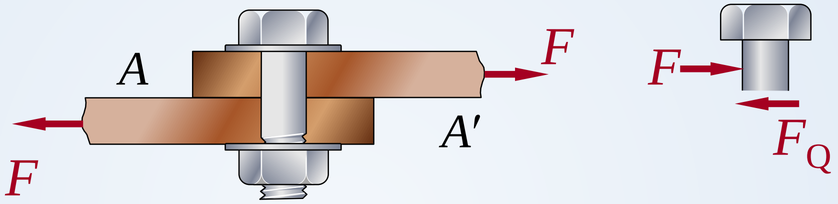
1. 认为受剪面上只有剪力F_Q
2. τ 平行F_Q, 方向同F_Q
3. 切应力在受剪面上均匀分布



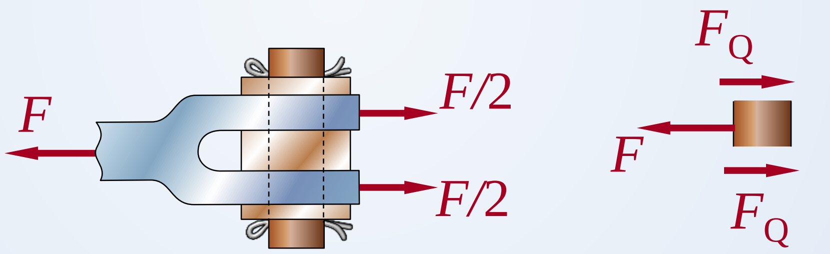
7.8 剪切和挤压的实用计算

剪切强度条件 $$\boxed{\tau=\frac{F_Q}{A}\le[\tau]}$$

单剪 $F_Q=F$



双剪 $F_Q=F/2$

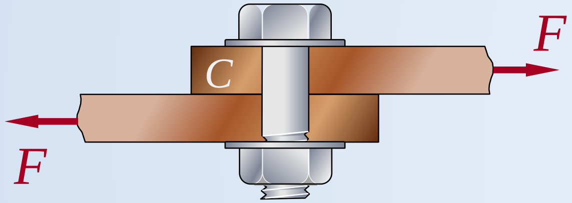


应用 $\left\{\begin{array}{l}\text{安全计算 (联接的钉, 键要满足剪切强度条件)} \\ \text{破坏计算（安全销、安全阀、冲剪板...)}\end{array}\right.$

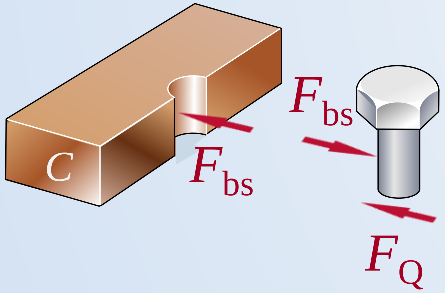
7.8 剪切和挤压的实用计算

三. 挤压的实用计算

联接件与被联接件之间接触--挤压力F_{bs}



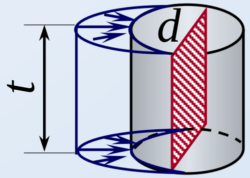
1.假定在挤压面上挤压应力是均匀分布的



2.当接触面为圆柱形,用直径平面作为挤压面

挤压强度条件

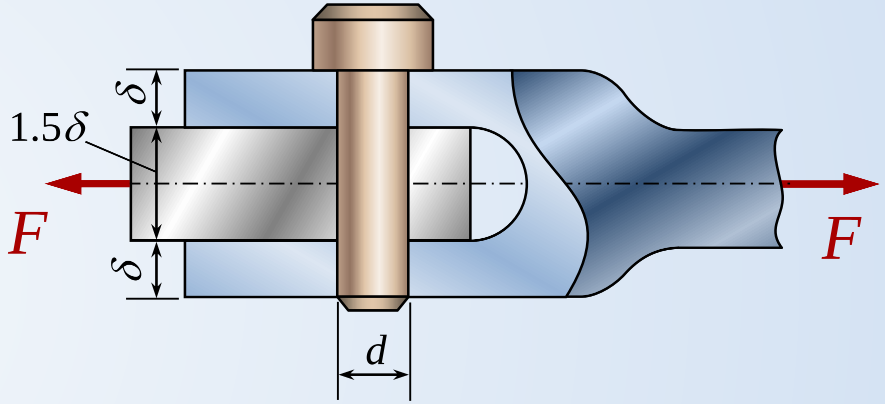
$$\sigma_{bs} = \frac{P_{bs}}{A_{bs}} \leq [\sigma_{bs}]$$



$$A_{bs} = td$$

7.8 剪切和挤压的实用计算

例5 电瓶车挂钩由插销连接，插销$[\tau]$=30MPa, $[\sigma_{bs}]$=60MPa, d=20mm, δ=8mm, 牵引力F=15kN. 试校核 插销的强度。



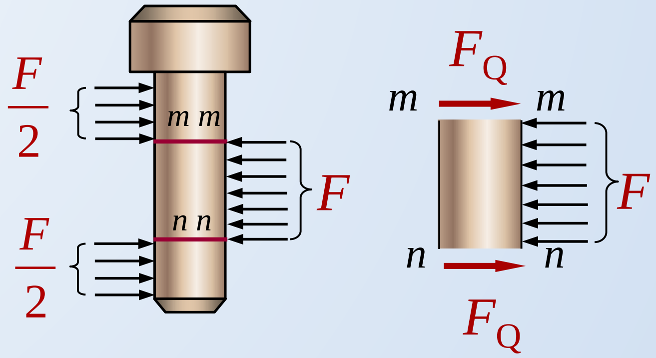
解： 1.校核插销剪切强度

两个剪切面： *mm* 和 *nn*

剪切面上的剪力： $F_Q = \dfrac{F}{2}$

剪切面面积： $A = \dfrac{\pi d^2}{4}$

校核：$\tau = \dfrac{F_Q}{A} = \dfrac{\frac{1}{2}F}{\frac{1}{4}\pi d^2} = \dfrac{2F}{\pi d^2} = \dfrac{2\times 15\times 10^3}{\pi\times 20^2\times 10^{-6}} = 23.9\text{MPa} < [\tau]$



插销满足剪切强度

7.8 剪切和挤压的实用计算

2.校核插销挤压强度

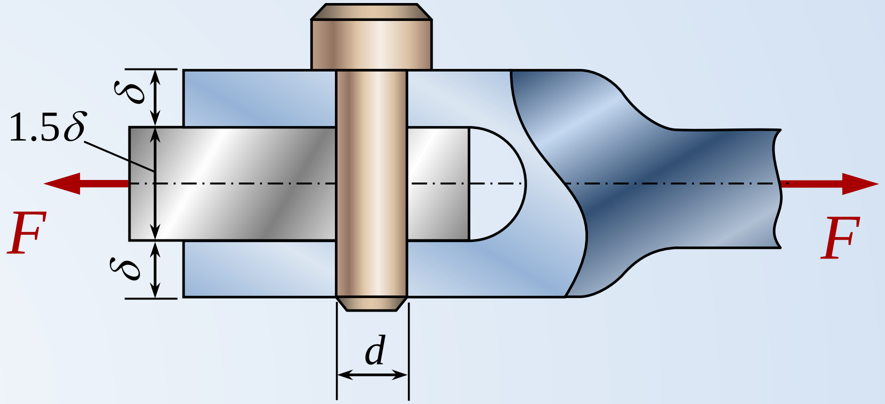
计算挤压力 $F_{bs1}=\dfrac{F}{2}\quad F_{bs2}=F$

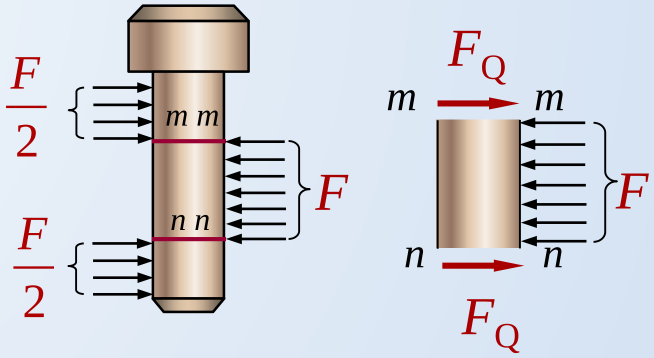
计算挤压面面积 $A_1=\delta\cdot d\quad A_2=1.5\delta\cdot d$

校核：$\sigma_{bs1}=\dfrac{F_{bs1}}{A_1}=\dfrac{F/2}{\delta\cdot d}=\dfrac{F}{2\delta d}$

$$\sigma_{bs2}=\frac{F_{bs2}}{A_2}=\frac{F}{1.5\delta\cdot d}=\frac{15\times10^3}{1.5\times8\times20\times10^{-6}}=62.5\,\mathrm{MPa}$$

$$\frac{\sigma_{bs2}-[\sigma]}{[\sigma]}=\frac{62.5-60}{60}=4.17\%<5\%$$

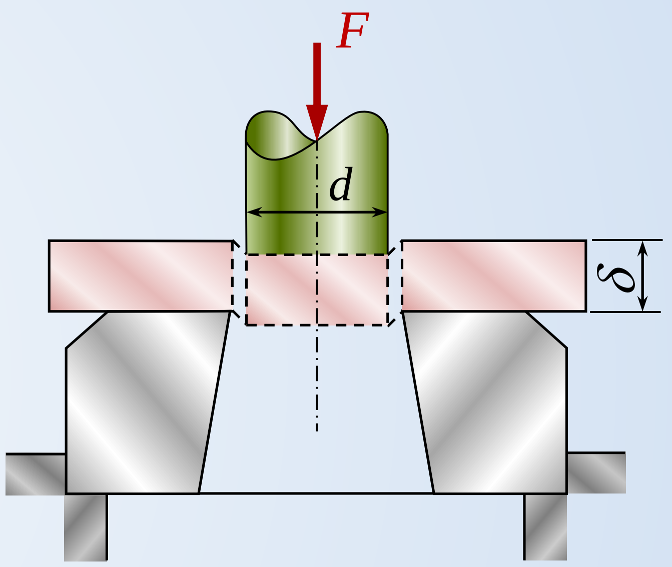




插销满足挤压强度条件

7.8 剪切和挤压的实用计算

例6 板厚δ=5mm, 剪切强度极限τ_b=320MPa，如用冲床冲出直径d=15mm的孔, 需要多大的力F?

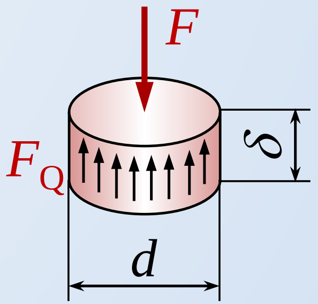


解：分析 冲孔就是发生剪切破坏条件 $\tau > \tau_b$

剪切面上的剪力 $F_Q = F$

计算剪切面积 $A = \pi d \delta$

由破坏条件 $\tau > \tau_b$ $\quad \dfrac{F_Q}{A} = \dfrac{F}{\pi d \delta} > \tau_b$



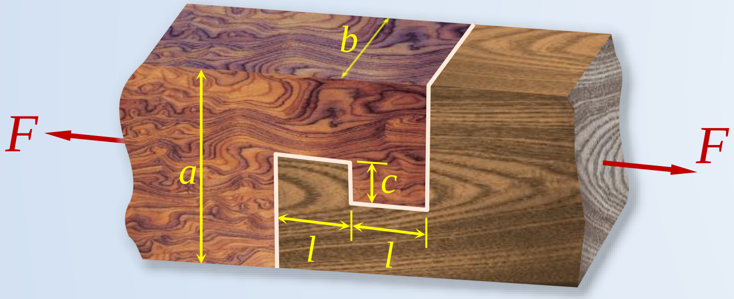
得 $F \geq \pi d \delta \tau_b = \pi \cdot 15 \times 5 \times 10^{-6} \times 320 \times 10^{6} = 75.5\text{kN}$

$\boxed{F\text{至少为}75.5\text{kN力}}$

7.8 剪切和挤压的实用计算

例7 木榫接头，当F 作用时，求：接头的剪切面积和挤压面积，

并求 $\tau,\sigma_{\mathrm{bs}}$



解：接头的剪切面积： $A=bl$

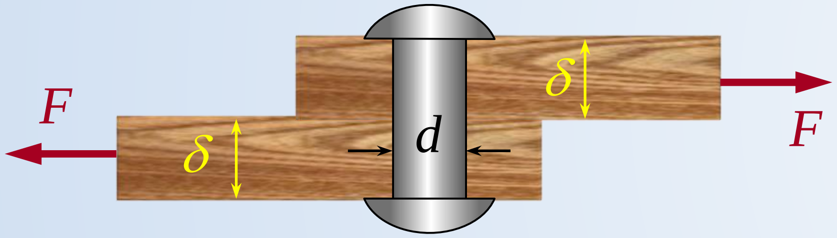
$$\tau=\frac{F_{\mathrm{Q}}}{A}=\frac{F}{lb}$$

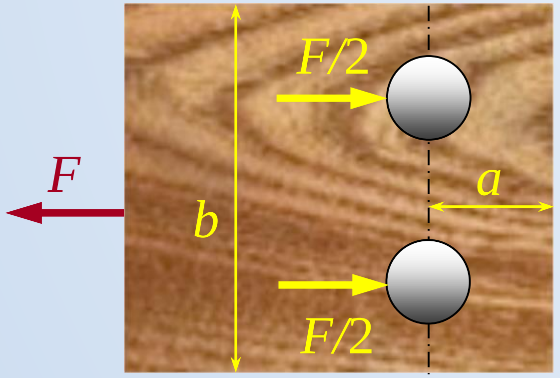
接头的挤压面积： $A_{\mathrm{bs}}=cb$

$$\sigma_{\mathrm{bs}}=\frac{F_{\mathrm{bs}}}{A_{\mathrm{bs}}}=\frac{F}{cb}$$

7.8 剪切和挤压的实用计算

例8 接头，受轴向力F 作用。已知F=50kN b=150mm，δ=10mm，d=17mm，a=80mm，[σ]=160MPa，[τ]=120MPa，[σ_{bs}]=320MPa，铆钉和板的材料相同，试校核其强度。





解：1.板的拉伸强度

$$\sigma=\frac{F_{\mathrm{N}}}{A}=\frac{F}{(b-2d)\delta}=\frac{50\times10^{3}}{(0.15-2\times0.017)\times0.01}=43.1\mathrm{MPa}<[\sigma]$$

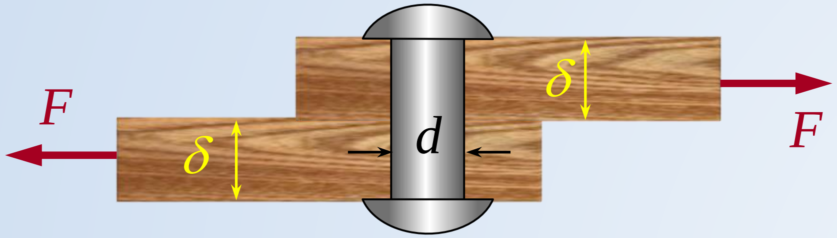
板的拉伸强度足够

2.板的剪切强度

$$\tau=\frac{F_{\mathrm{Q}}}{A}=\frac{F}{4a\delta}=\frac{50\times10^{3}}{4\times0.08\times0.01}=15.6\mathrm{MPa}<[\tau]$$

板的剪切强度足够

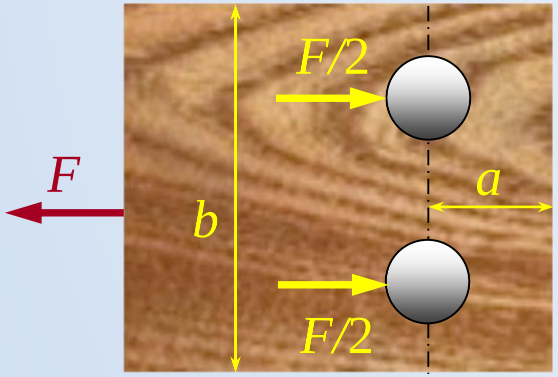
7.8 剪切和挤压的实用计算



3.铆钉的剪切强度

$$\tau=\frac{F_Q}{A}=\frac{F/2}{\pi d^2/4}=\frac{2F}{\pi d^2}=\frac{2\times 50\times 10^3}{\pi\times 0.017^2}=110\text{MPa}<[\tau]$$

$\boxed{\text{铆钉的剪切强度足够}}$



4.板和铆钉的挤压强度

$$\sigma_{bs}=\frac{F_{bs}}{A_{bs}}=\frac{F}{2d\delta}=\frac{50\times 10^3}{2\times 0.017\times 0.01}=147\text{MPa}<[\sigma_{bs}]$$

$\boxed{\text{铆钉的挤压强度足够}}$

Thank you!