Control Laboratory

# Non-linear control system


## Ma Yan

Control Science and Engineering Department

Jilin University

mayan_maria@163.com

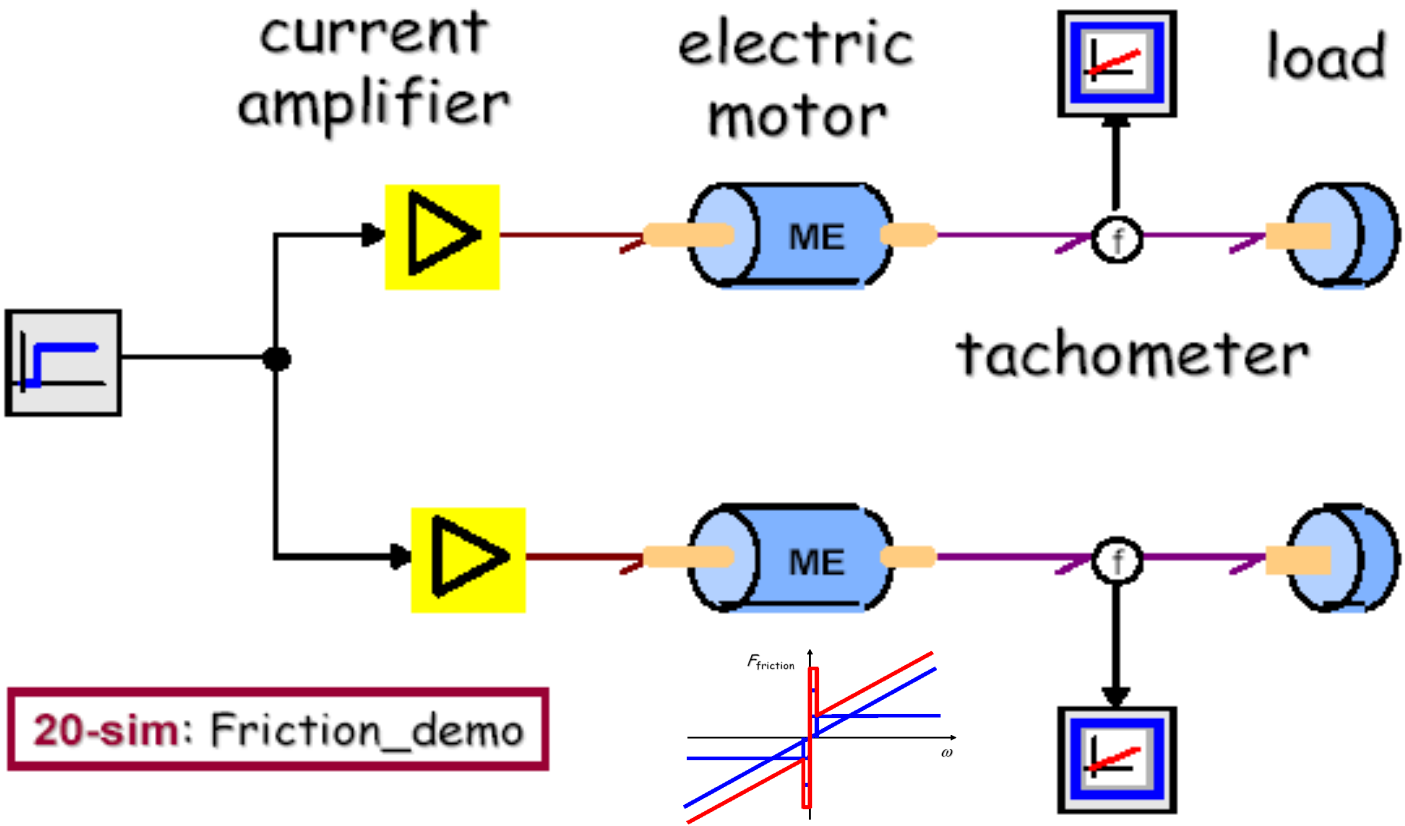
# Contents

- Types of non-linearity
- Reasons for the presence of non-linear elements
- Analysis in the phase plane
- Analysis with describing function

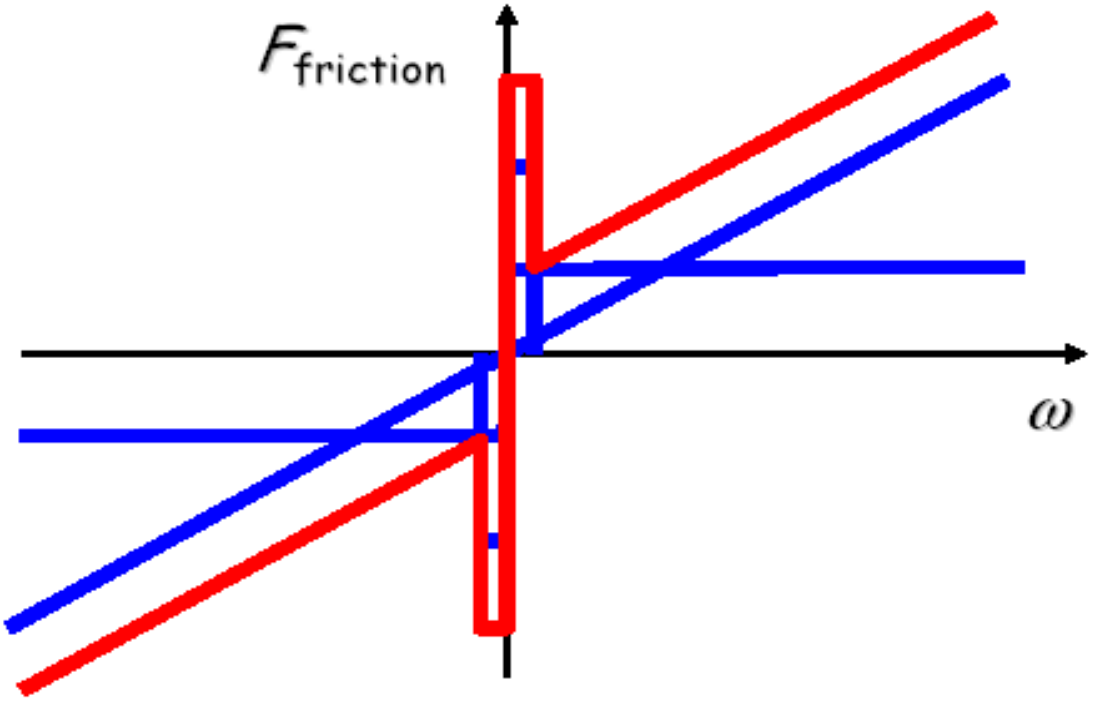
# Examples of non-linearity

- Friction in a mechanical system
- Saturation in an amplifier
- Switching elements, e.g. in thermostats.
- Operating-point-dependent parameters

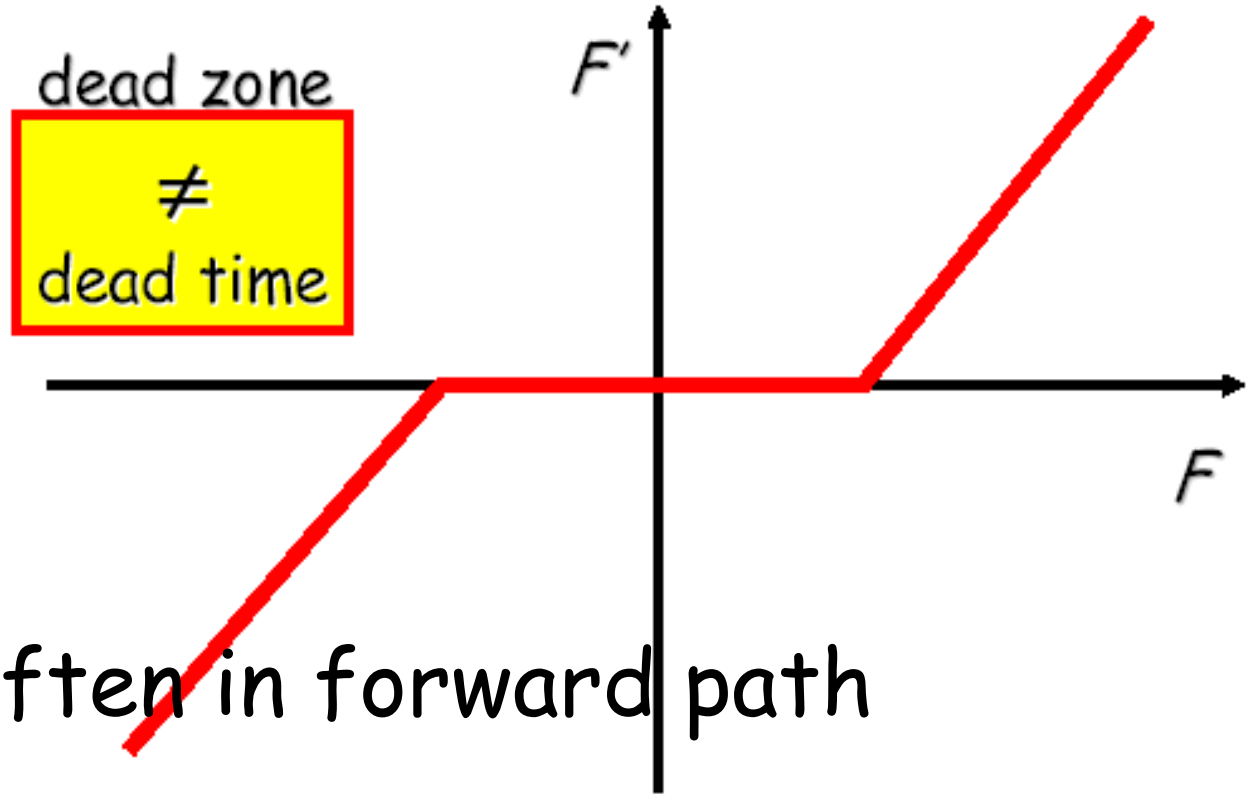
# Mechanical friction



# Friction characteristic



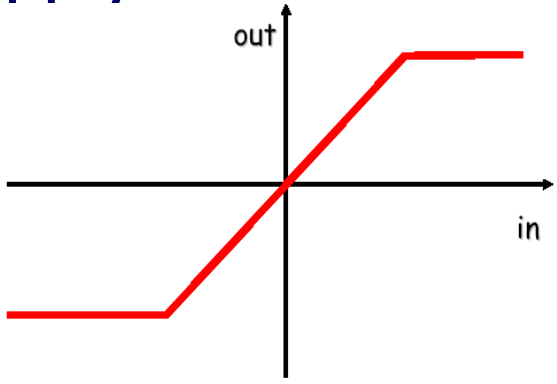
# Dead zone



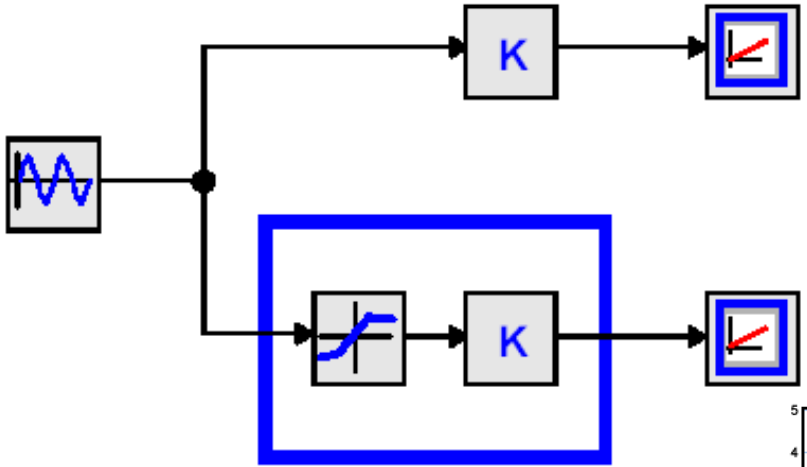

- Often in forward path

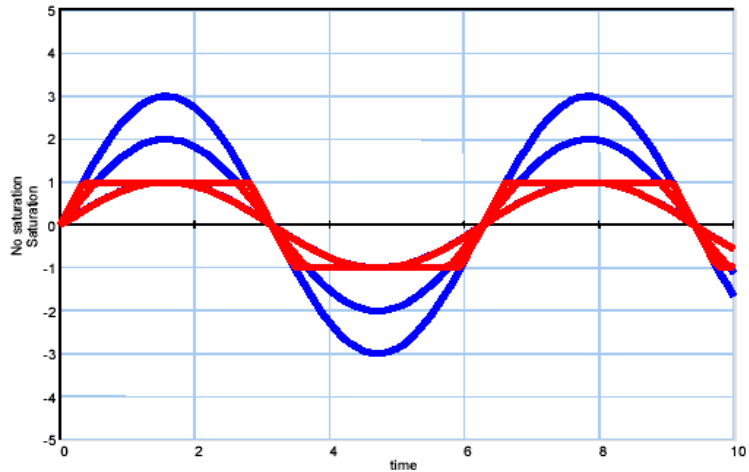
# Saturation

- Due to
  - maximum valve opening reached
  - output voltage of an amplifier limited by voltage of power supply
  - end stop

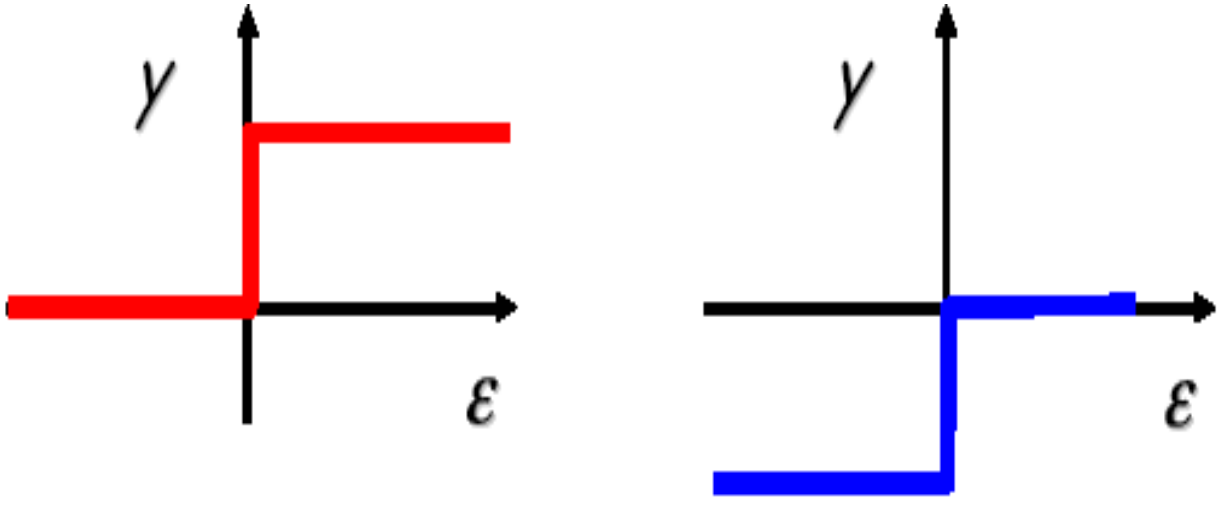


# Saturation in amplifier



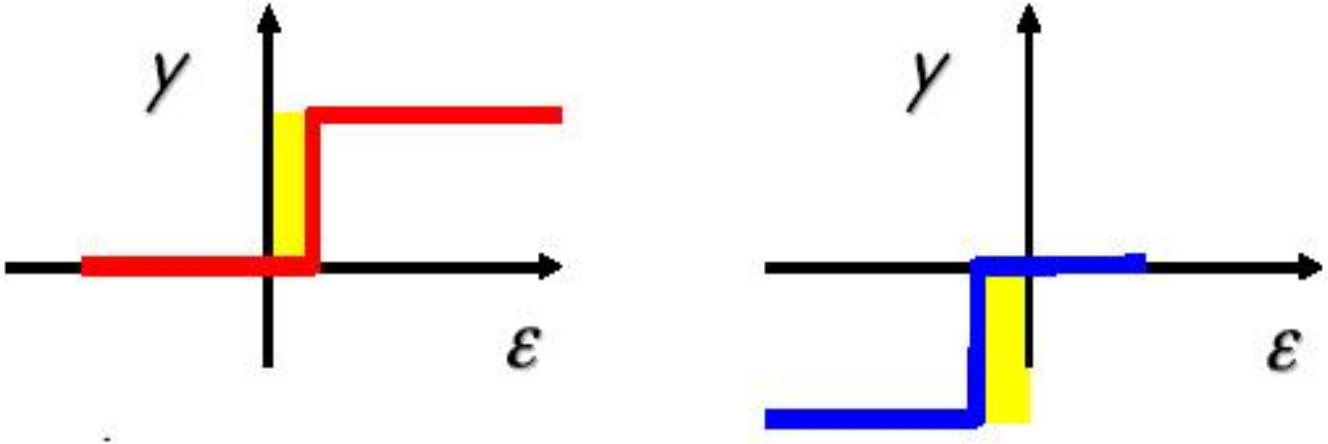
# Thermostat（恒温器）




relay:
if x> 0, y= $y_{max}$
else y=0 .

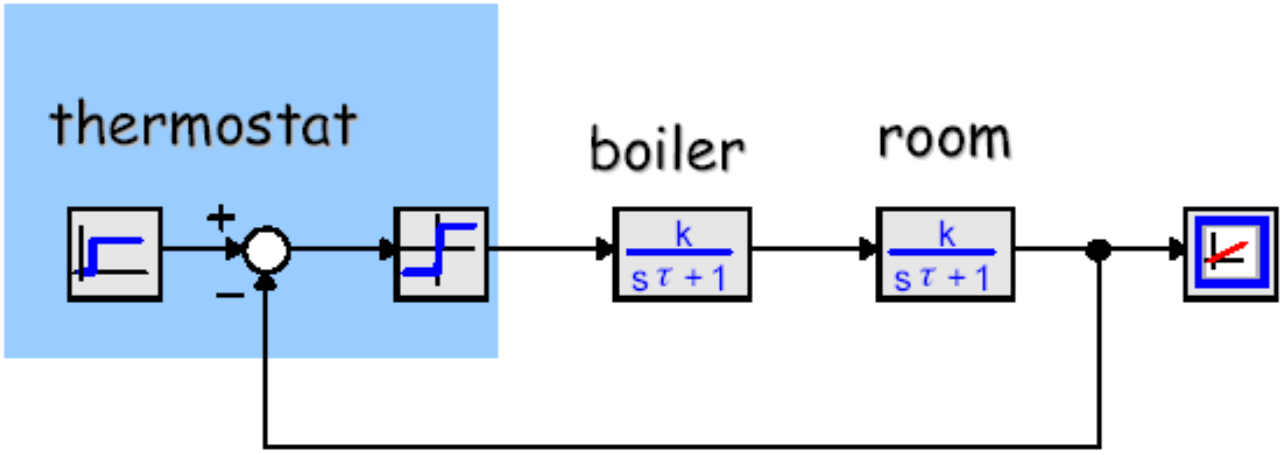
different control systems for heating and cooling

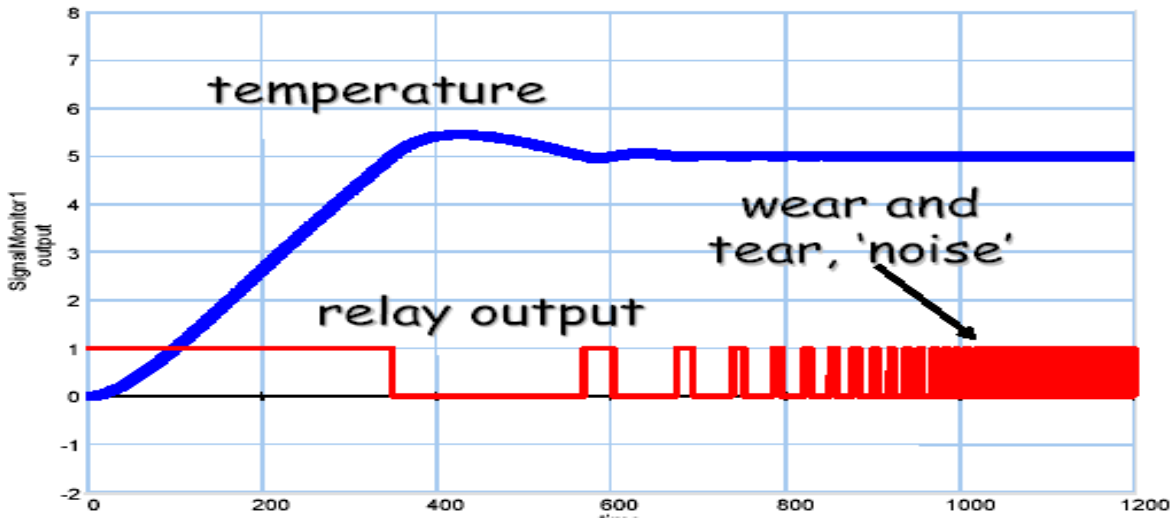
# Thermostat with dead zone



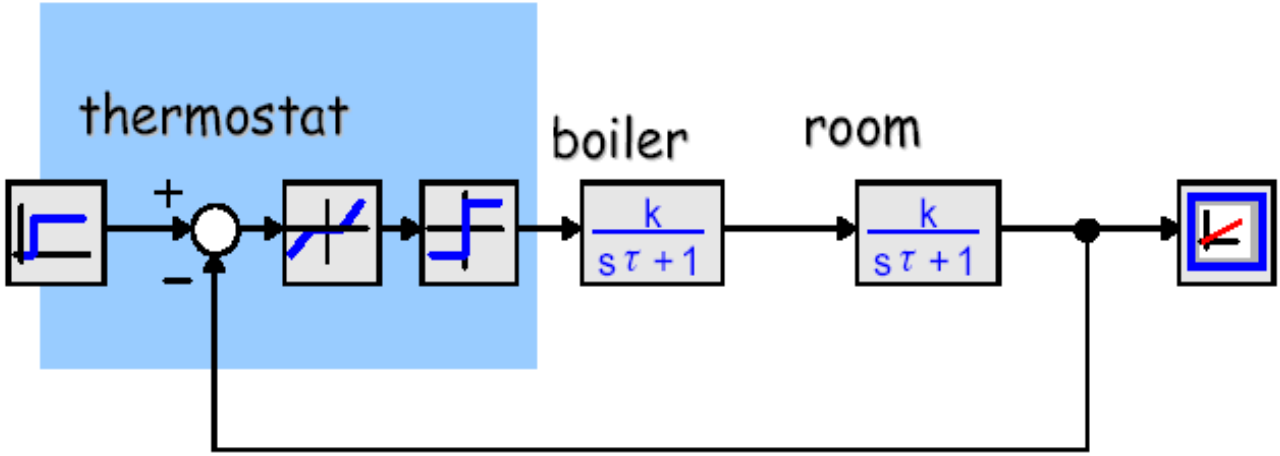

It makes no sense to damp the overshoot caused by the heating systems or by switching on the air conditioning

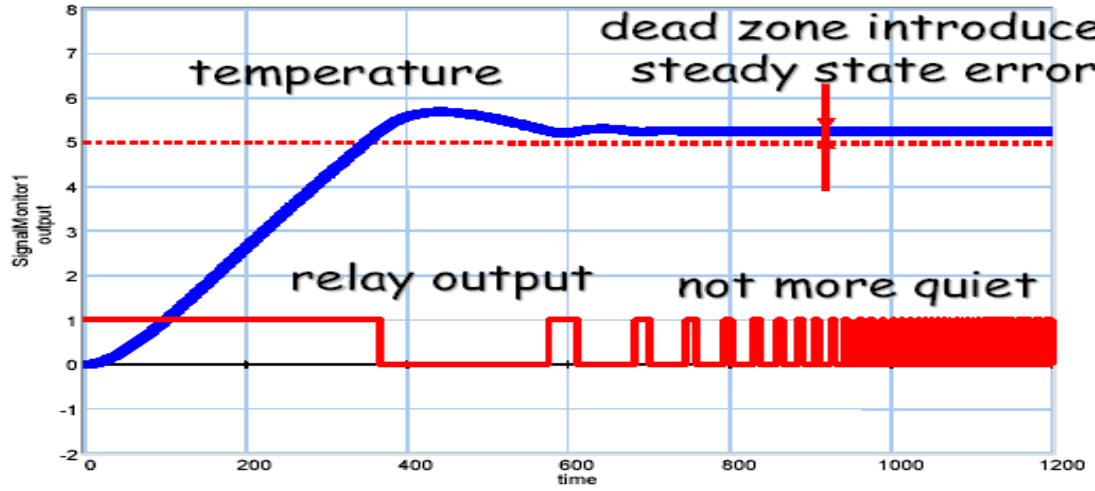
# Room control



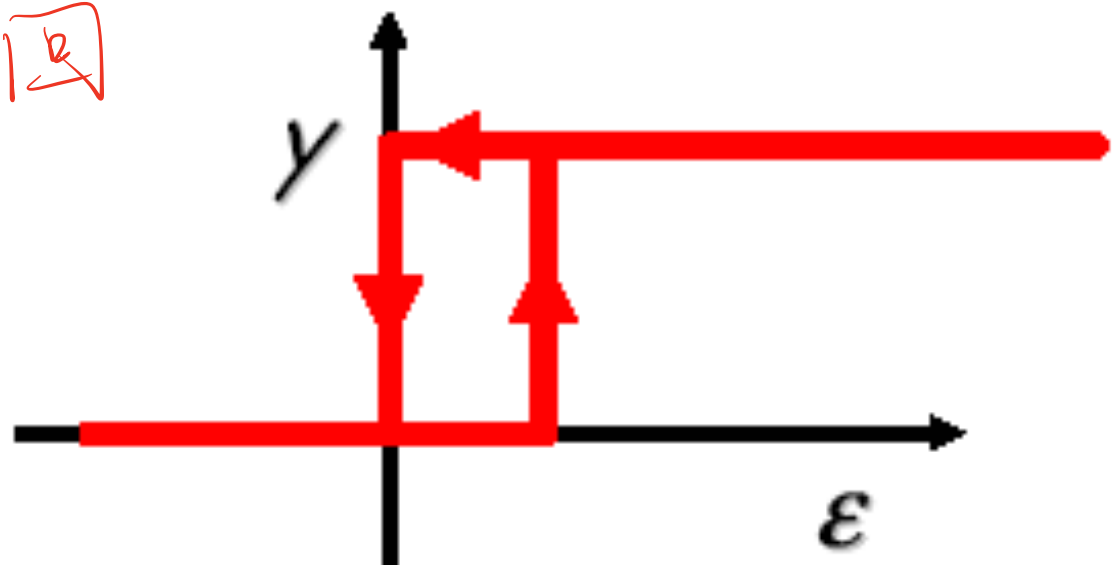
# Relay plus dead zone



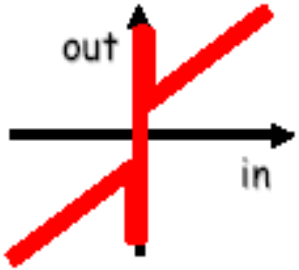



# Relay plus Hysteresis

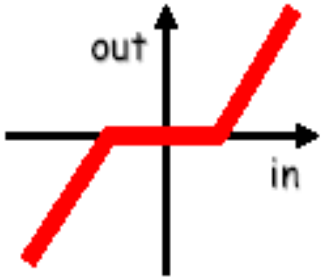
滞回



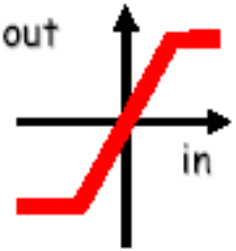
# Overview



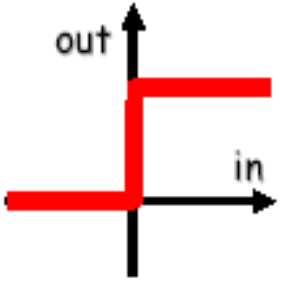

friction



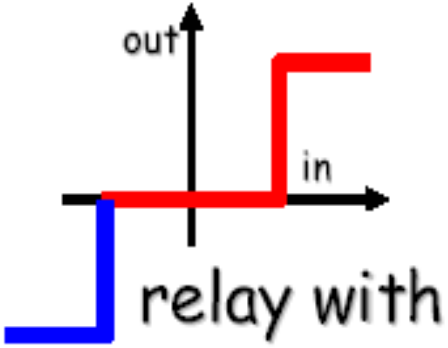

dead zone/band



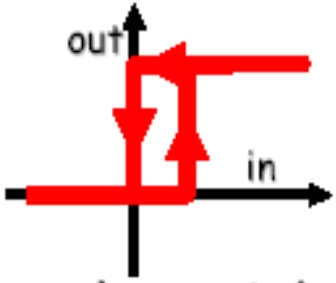

saturation




relay




relay with dead zone




relay with hysteresis

# Why non-linearities ?

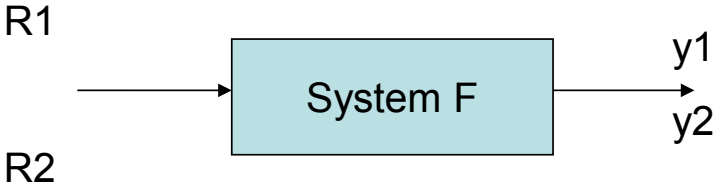
- Sometimes it is difficult or too expensive to make a linear system
  - friction, saturation
- sometimes wanted
  - safety (maximum relay)
- It is cheap
  - relay is a cheap power amplifier, e.g. thermostat, integrated with other functions: sensing, set-point

# Linear or non-linear

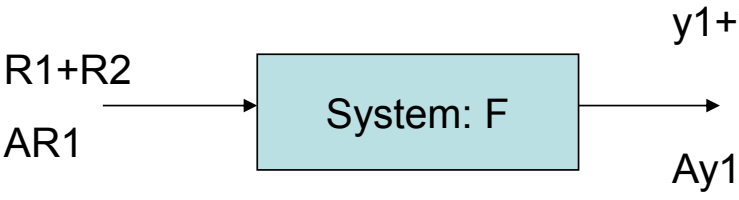
- Linear system: a linear system is a system to which the principle of superposition（叠加原理） applies.

- Non-linear system: A non-linear system is not linear; that is, a nonlinear system doesn't use superposition principle.

# Linearity Principle




$y1(t) = F(t)R1(t)$
$y2(t) = F(t)R2(t)$



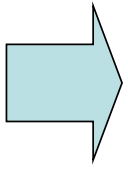

$y1+y2 = F(t)\,(R1+R2)$
$y1 = F(t)\,AR1 = Ay1$

Linear system

# Examples of non-Linearity

Examples:

1. $y(t) = \cos(at) \times r(t)$
2. $y(t) = kr(t)$
3. $y(t) = \cos[w \bullet r(t)]$



$$Y(s) = \frac{as}{s^2 + a^2} \times R(s)$$

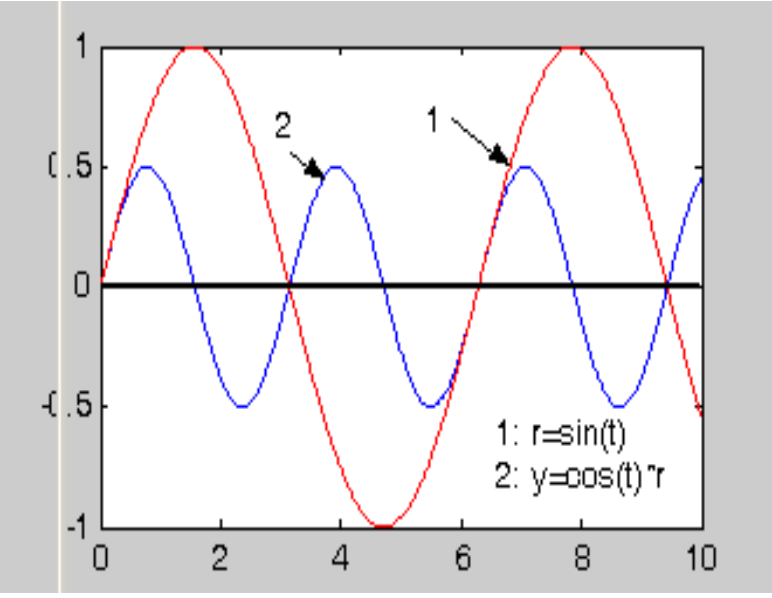
$$Y(s) = kR(s)$$

$$Y(s) = \ell\{\cos[w \bullet r(t)]\}$$

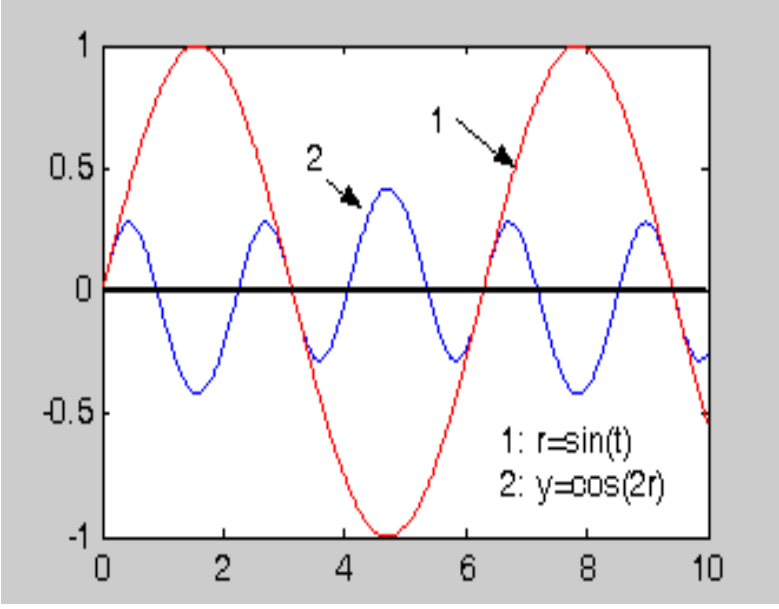
Notes: 1 and 2 are linear systems.
3 is a non-linear system

# Examples of non-Linearity

$$y(t) = \cos(at) \times r(t)$$




$$y(t) = \cos[w \bullet r(t)]$$




Notes: 1 are linear.

3 is a non-linear

# Examples

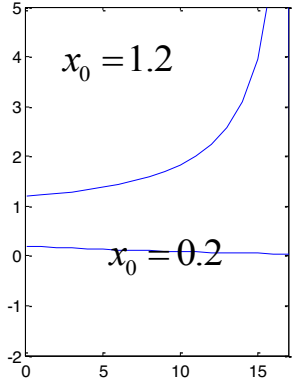
Examples: the equation of a non-linear system is:

$$\dot{x} = -x(1-x) \qquad \dot{x} = 0 \Rightarrow equilibrium \quad x = 0; \;\; x = 1$$

Solutions

$$\frac{x}{1-x} = ce^{-t}, \qquad c = \frac{x_0}{1-x_0}$$

$$so \quad x(t) = \frac{x_0 e^{-t}}{1 - x_0 + x_0 e^{-t}}$$



# Notes

Different initial value

$$\dot{x} = -x(1-x) \qquad \dot{x} = 0 \Rightarrow equilibrium \qquad x = 0;\ \ x = 1$$

- Notes:
  1. The response depends on the initial value.
  2. There is stable oscillation in the non-liner system.
  3. The output of nonlinear system due to sinusoid input is complex

# Unwanted non-linearity

- Behavior of a good feedback system is only determined by the elements in the feedback path.
- High gain feedback can compensate for unwanted non-linearity, such as friction.