Digital Control System

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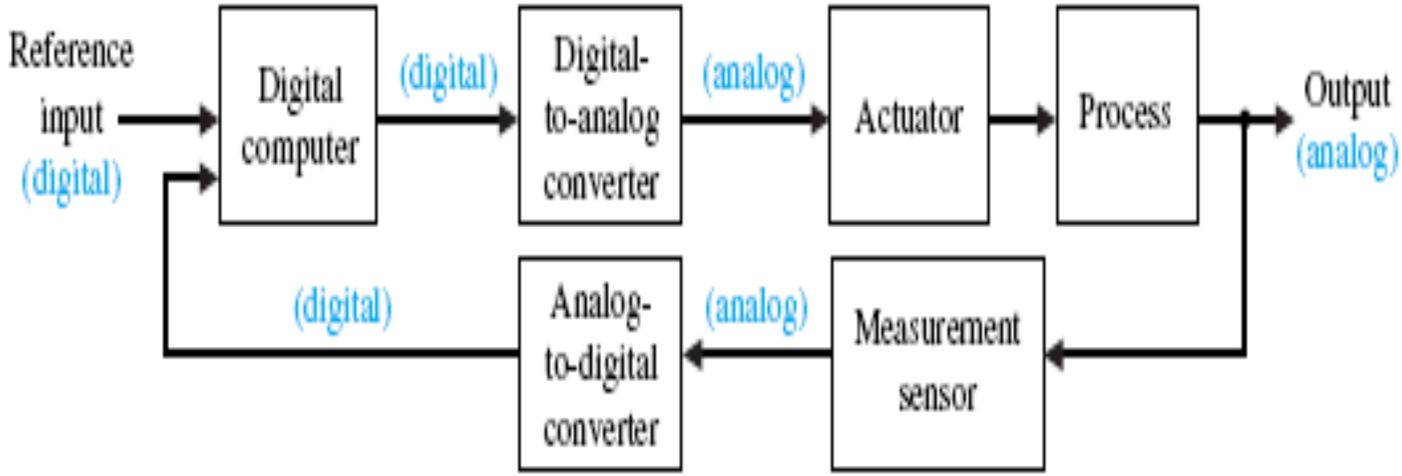
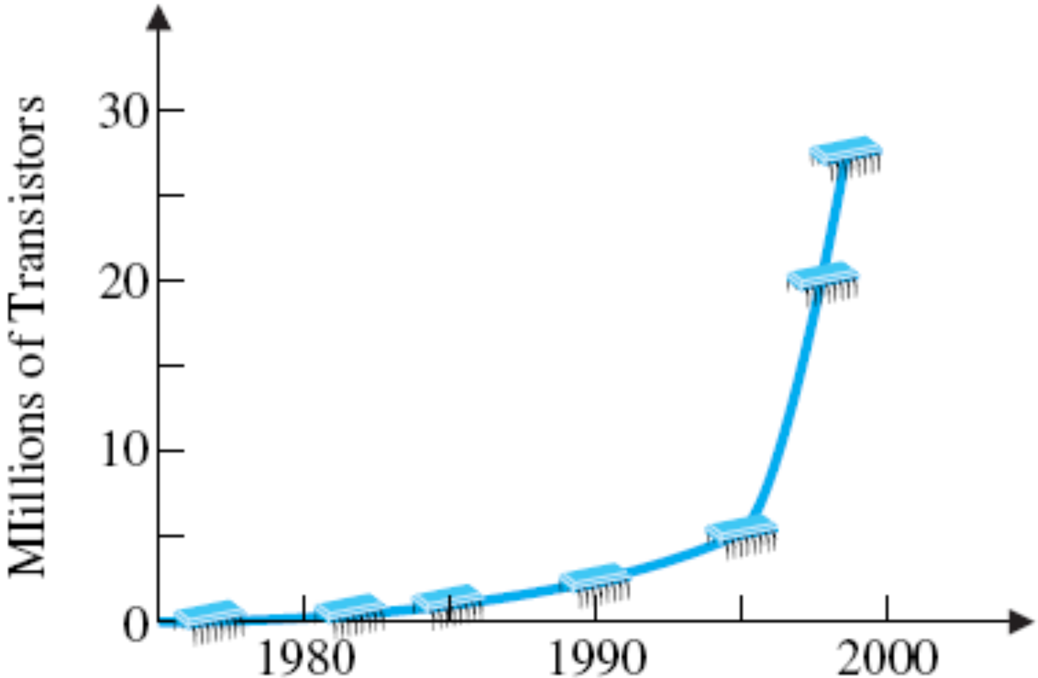
- Sampling, discretization, real time
  - AD/DA conversion, ZOH
- Mathematics
  - Impuls sampling
  - Nyquist frequency (Shannon)
  - z-transform transform
- Euler, Tustin, real z-transform

Configuration



Black diagram of a digital control systems

Development



INTEL microprocessors measured in millions of transistor（晶体管）

Development

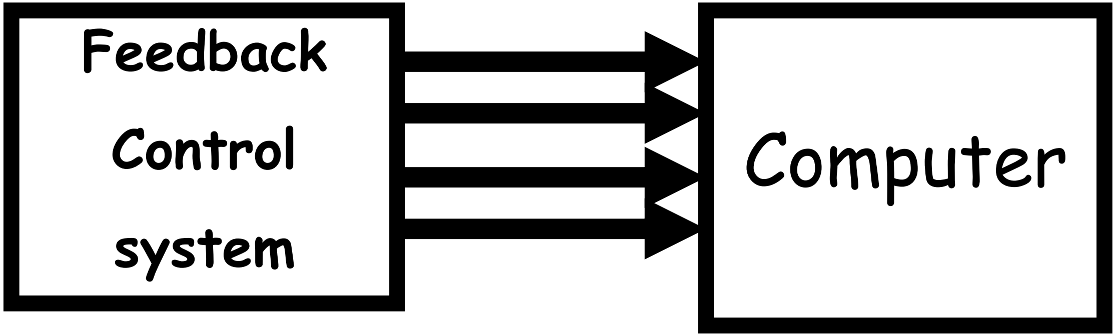
Boeing 757 and 767 features digital electronic

Why computers

- Advantages
  - General purpose hardware
  - Flexibility
  - More functionality

- Disadvantages
  - Sampled data:
  - Stability
  - High frequencies in control signal

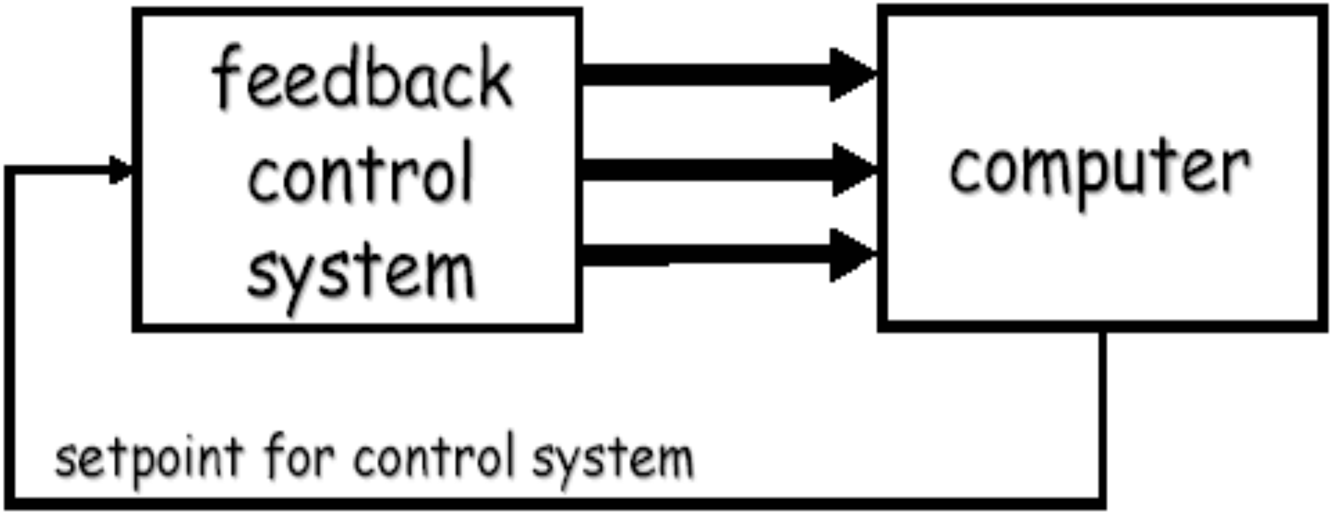
Configurations



Data logging(数据录入)

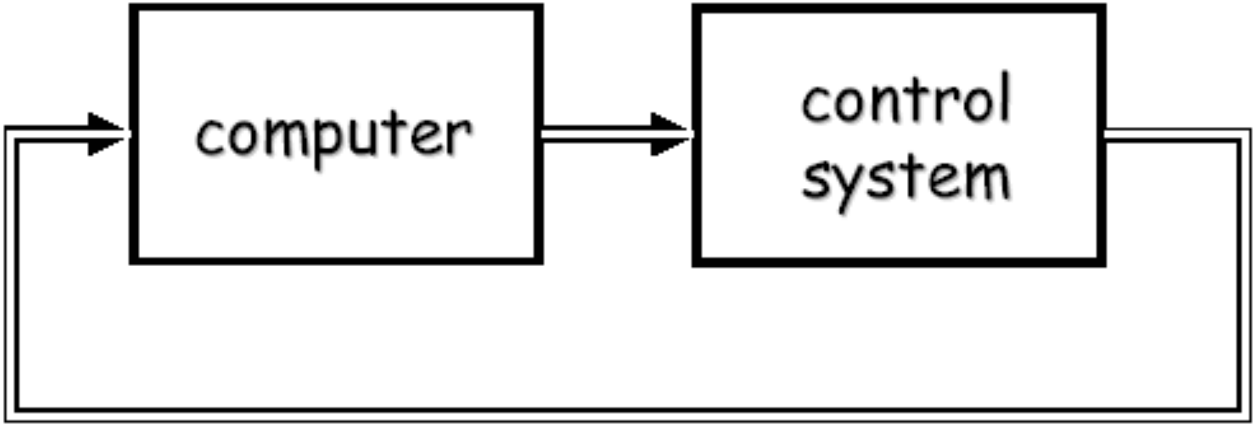
Computer = add on

Configurations



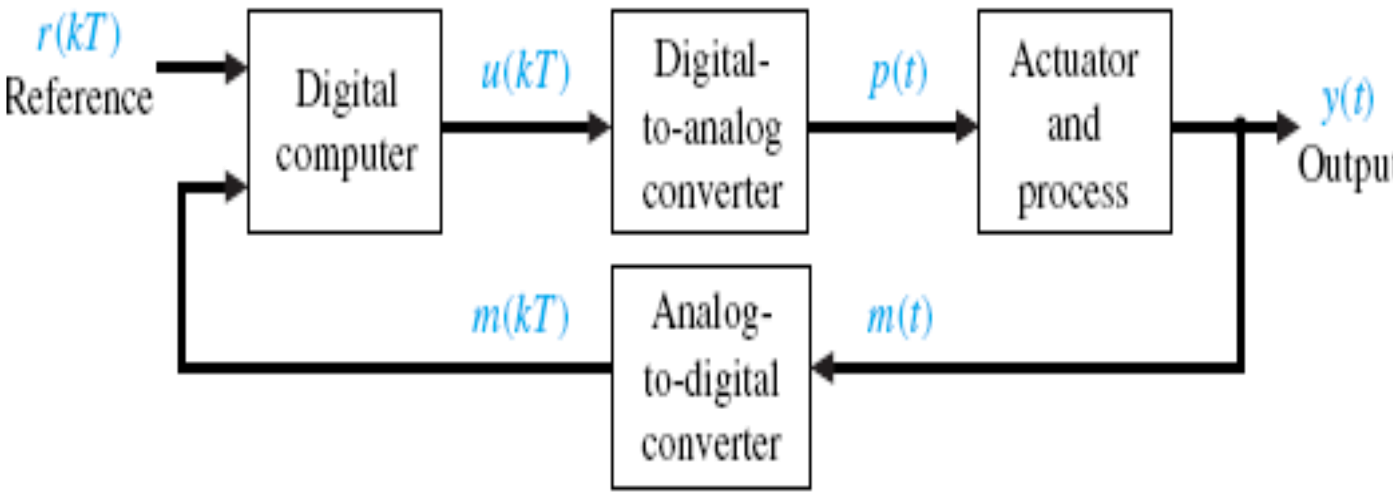
optimization

Configurations



Direct Digital Control

computer part of the control loop



A digital control system

Analogue versus digital

- Process: analogue:
  - a continuous time process
- Computer: data:
  - Data are measured
  - Computations take some time
  - Data are send to the outside
- Computer is digital
  - Means also 'limited accuracy'

Computer views

Seen from the computer:

- The real world is a discrete, digital world
- Requires other process descriptions
- Design methods that take the digital nature into account

Real time

- Computer should react fast enough
- Computer should react in time

- Soft real time:

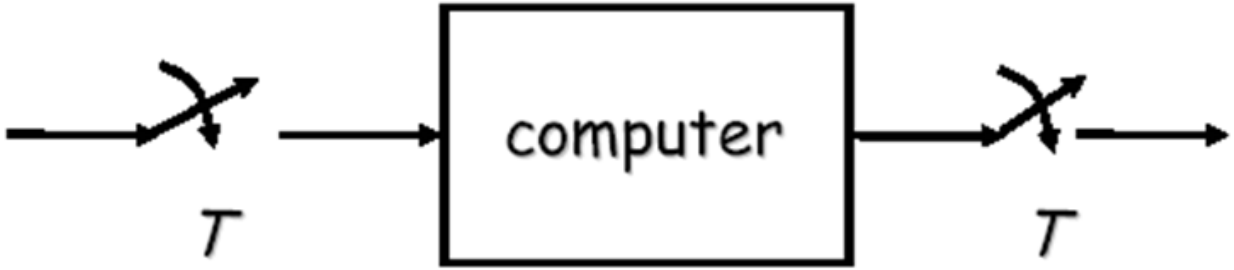
  e.g. automatic teller machine（自动提款机）
- Hard real time:

  all actions take place at accurately fixed time intervals

Sampled- Data system

- Fundamentals and Sampling process
- Z-transform and inverse z-transform analysis methods

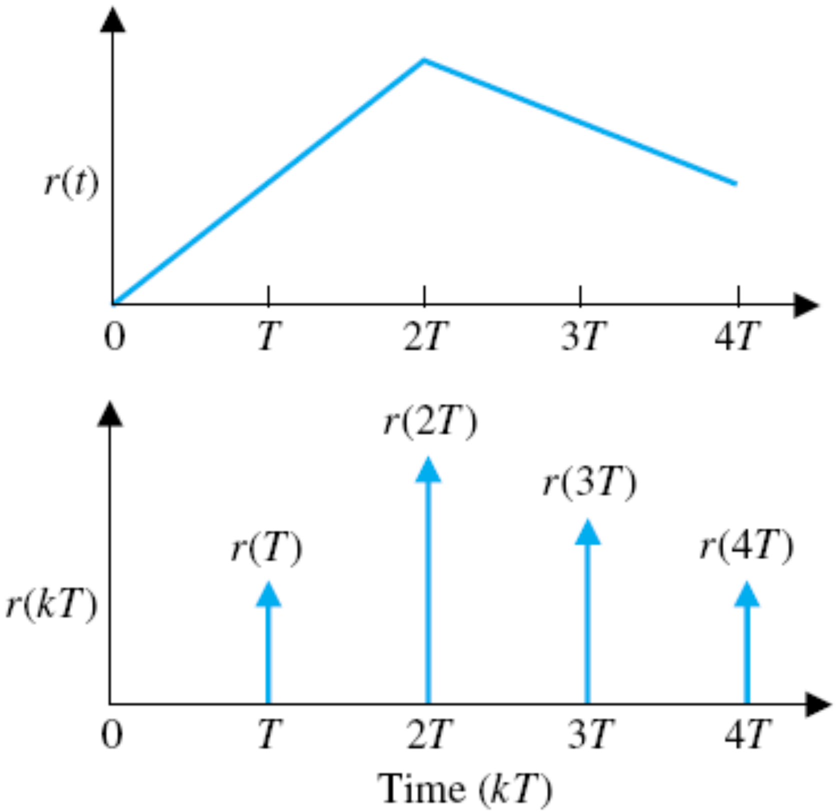
Ideal sampling

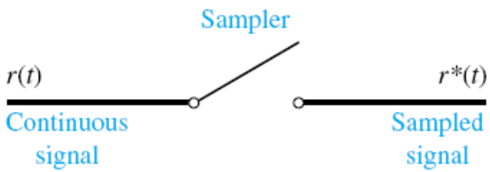


Switches close simultaneously at
$t = T, 2T, \ldots, kT, (k+1)T$

Discretization in time

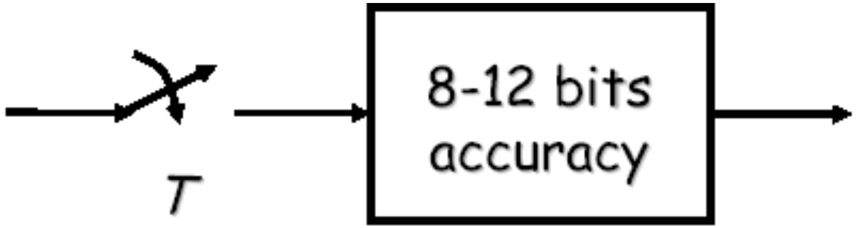
Ideal sampling

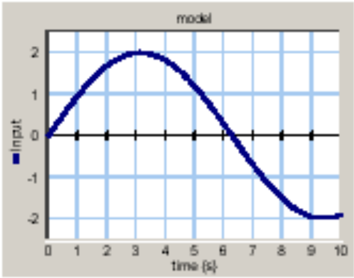




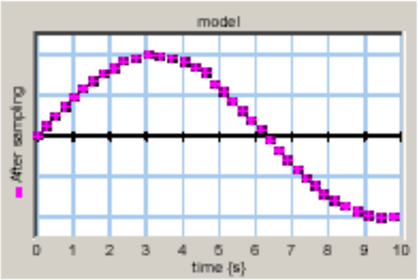
- limited accuracy
  - AD and
  - DA-converters
  - encoders

AD- Conversion



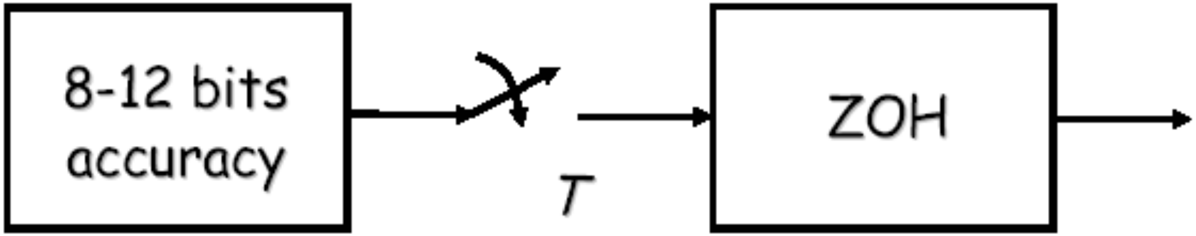


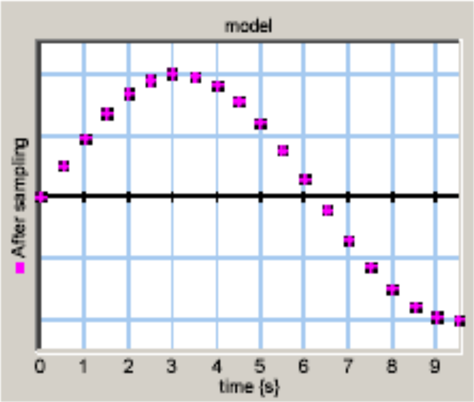
analogue
continous time

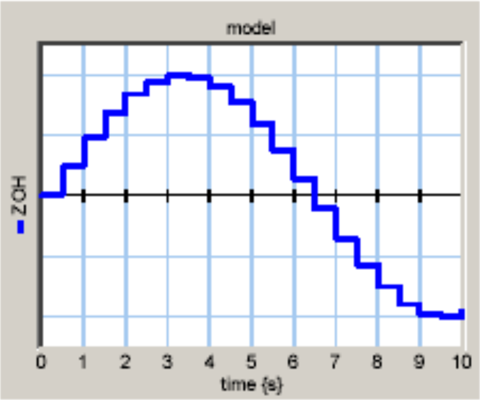


sampled and
discretised

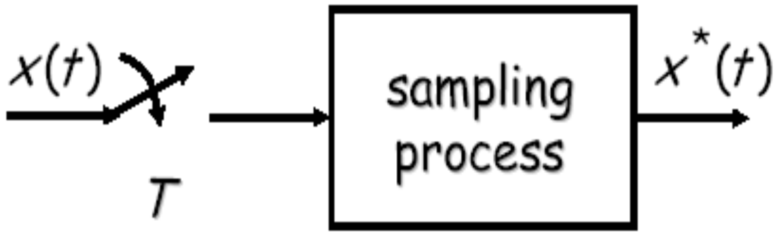
DA- Conversion

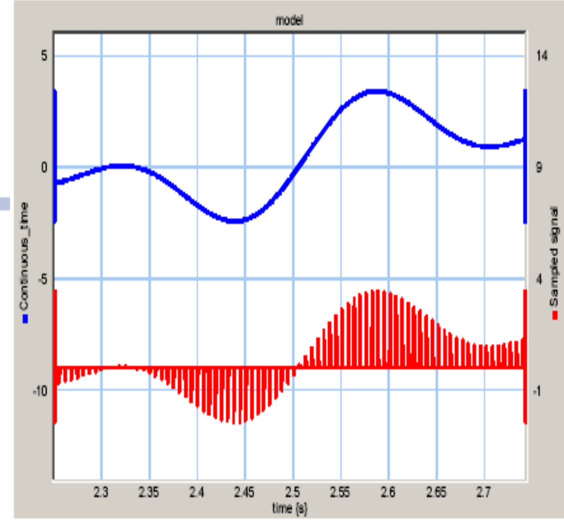


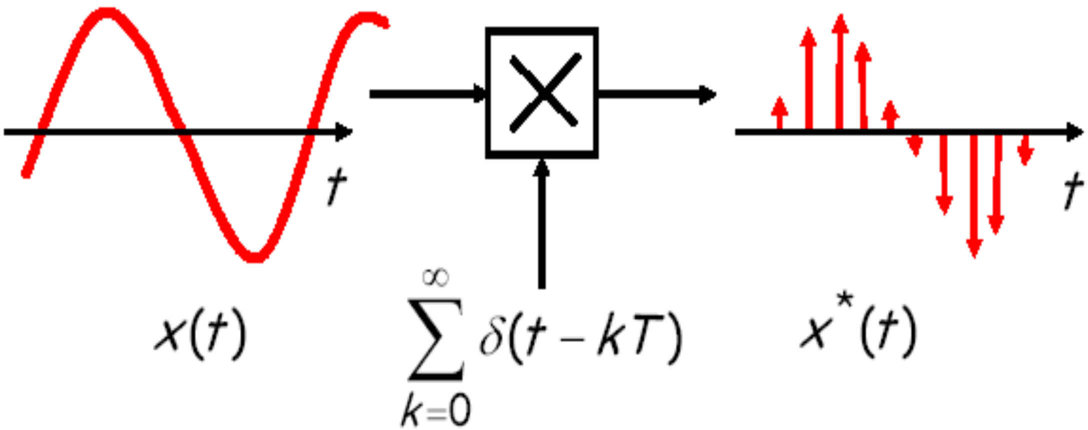




Sampling







•In a discrete system the signals have only values at the sample instants t = T, 2T, …

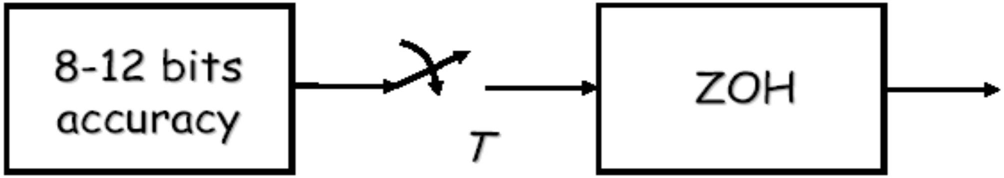
Sampling

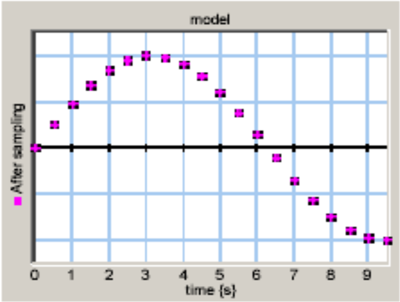
$$x^*(t) = \sum_{k=0}^{\infty} x(kT)\delta(t-kT)$$

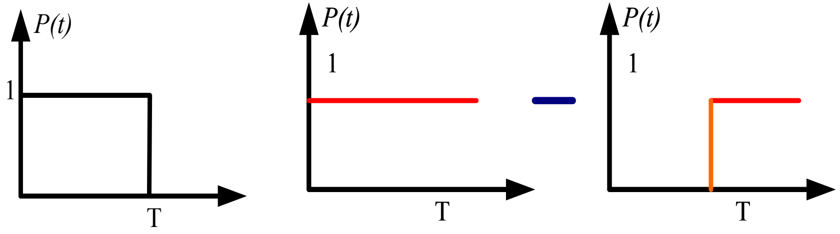
After Laplace transformation

$$\ell\{x(t)\} = x^*(s) = \sum_{k=0}^{\infty} x(kT)e^{-kTs}$$

zero-order holder (ZOH)







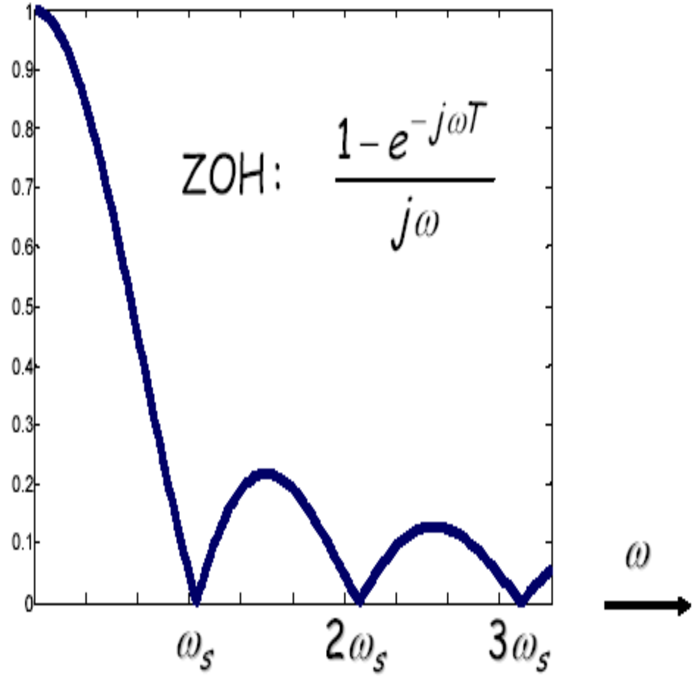
$$G_0(s) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1}{s}(1 - e^{-sT})$$

Reconstruction: ZOH

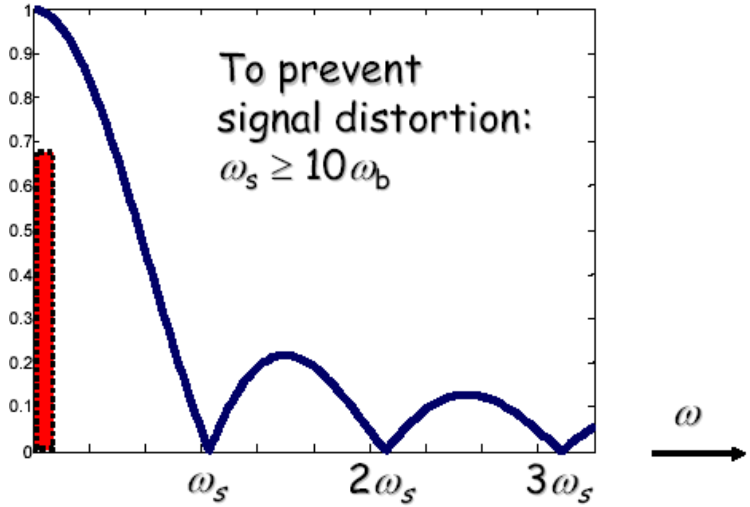
ZOH: $\dfrac{1-e^{-j\omega T}}{j\omega}$

MATLAB:

```
for k = 1:1:200,
w(k)=k/10
x(k)=abs((1-exp(-j*w(k)))/w(k))
end
plot (w,x)
```



Reconstruction: ZOH



Use a anti aliasing(low-pass) filter at the input

Take care $w_s \gg 2w_b$, eg. $w_s = 10w_b$

To eliminate high frequency at output

Use a low pass filter

Time domain $\Rightarrow$ **z-domain**

Z-transform Definition

When f(t)=0 for t<0, the impulse sequence is

$$\boxed{f^*(t) = f(t)\delta_T(t) = \sum_{k=0}^{\infty} f(t)\delta_T(t-kT)}$$

Using Laplace transform:

$$F^*(s) = \ell\left\{f^*(t)\right\} = \sum_{k=0}^{\infty} f(kT)e^{-kTS}$$

Let $\boxed{z = e^{TS}}$

So z-transform of function *f(t)*:

$$\boxed{Z\{f(t)\} = Z\left\{f^*(t)\right\} = \sum_{k=0}^{\infty} f(kT)z^{-k}}$$

Z-transform Example

e.g1. f(t)=1, determine F(z)?

$$Z\{f(t)\}=\sum_{k=0}^{\infty}f(kT)z^{-k}=\sum_{k=0}^{\infty}1\cdot z^{-k}=1+z^{-1}+z^{-2}+\cdots$$

$$=\frac{1}{1-z^{-1}}=\frac{z}{z-1}$$

e.g2. known $f(t)=e^{-at}$,determine f(z)?

$$Z\{f(t)\}=\sum_{k=0}^{\infty}f(kT)z^{-k}=\sum_{k=0}^{\infty}e^{-akT}\cdot z^{-k}=\sum_{k=0}^{\infty}(e^{aT}\cdot z)^{-k}$$

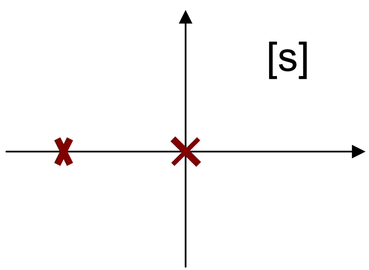
$$=\frac{1}{1-(e^{aT}z)^{-1}}=\frac{z}{z-e^{-aT}}$$

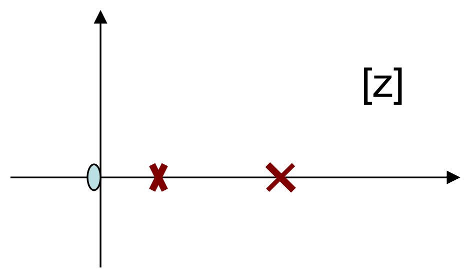
TF-Z-transform Example

e.g. $G(s)=\dfrac{a}{s(s+a)}$, **determine G(z)?**

$$G(s)=\frac{1}{s}-\frac{1}{s+a} \Rightarrow y(t)\Big|_{impuls} = \ell^{-1}\{G(s)\}=1-e^{-at}$$

$$G(z)=\frac{z}{z-1}-\frac{z}{z-e^{-aT}}=\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$$



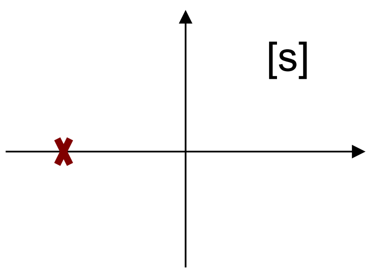


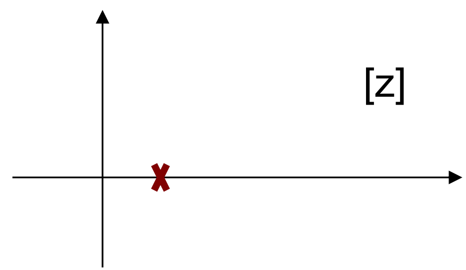
TF-Z-transform Example

e.g. $G(s) = \dfrac{1-e^{-Ts}}{s} \cdot \dfrac{1}{s+1}$, **determine G(z)?**

$$G(s) = (1-e^{-Ts})\frac{1}{s(s+1)} = (1-e^{-Ts})(\frac{1}{s} - \frac{1}{s+1})$$

$$G(z) = (1-z^{-1})(\frac{z}{z-1} - \frac{z}{z-e^{-T}}) = \frac{1-e^{-T}}{z-e^{-T}}$$





Z-transformTable

序号	拉氏变换 $E(s)$	时间函数 $e(t)$	Z变换 $E(z)$
1	1	$\delta(t)$	1
2	e^{-nsT}	$\delta(t-nT)$	z^{-n}
3	$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$
4	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
5	$\frac{1}{s^3}$	$\frac{t^2}{2!}$	$\frac{T^2z(z+1)}{2(z-1)^3}$
6	$\frac{1}{s^4}$	$\frac{t^3}{3!}$	$\frac{T^3z(z^2+4z+1)}{6(z-1)^4}$
7	$\frac{1}{s-(1/T)\ln a}$	$a^{t/T}$	$\frac{z}{z-a}$
8	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
9	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
10	$\frac{1}{(s+a)^3}$	$\frac{1}{2}t^2e^{-at}$	$\frac{T^2ze^{-aT}}{2(z-e^{-aT})^2}+\frac{T^2ze^{-2aT}}{(z-e^{-aT})^3}$
10	$\frac{1}{(s+a)^3}$	$\frac{1}{2}t^2e^{-at}$	$\frac{T^2ze^{-aT}}{2(z-e^{-aT})^2}+\frac{T^2ze^{-2aT}}{(z-e^{-aT})^3}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
12	$\frac{a}{s^2(s+a)}$	$t-\frac{1}{a}(1-e^{-aT})$	$\frac{Tz}{(z-1)^2}-\frac{(1-e^{-aT})z}{a(z-1)(z-e^{-aT})}$
13	$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(a-b)(c-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$	$\frac{z}{(b-a)(c-a)(z-e^{-aT})}+\frac{z}{(a-b)(c-b)(z-e^{-bT})}+\frac{z}{(a-c)(b-c)(z-e^{-cT})}$
14	$\frac{s+d}{(s+a)(s+b)(s+c)}$	$\frac{(d-a)}{(b-a)(c-a)}e^{-at}+\frac{(d-b)}{(a-b)(c-b)}e^{-bt}+\frac{(d-c)}{(a-c)(b-c)}e^{-ct}$	$\frac{(d-a)z}{(b-a)(c-a)(z-e^{-aT})}+\frac{(d-b)z}{(a-b)(c-b)(z-e^{-bT})}+\frac{(d-c)z}{(a-c)(b-c)(z-e^{-cT})}$
15	$\frac{abc}{s(s+a)(s+b)(s+c)}$	$1-\frac{bc}{(b-a)(c-a)}e^{-at}-\frac{ca}{(c-b)(a-b)}e^{-bt}-\frac{ab}{(a-c)(b-c)}e^{-ct}$	$\frac{z}{z-1}-\frac{bcz}{(b-a)(c-a)(z-e^{-aT})}-\frac{caz}{(c-b)(a-b)(z-e^{-bT})}-\frac{abz}{(a-c)(b-c)(z-e^{-cT})}$
16	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$
17	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$
18	$\frac{\omega}{s^2-\omega^2}$	$\sinh\omega t$	$\frac{z\sinh\omega T}{z^2-2z\cosh\omega T+1}$
19	$\frac{\omega}{s^2-\omega^2}$	$\cosh\omega t$	$\frac{z(z-\cosh\omega T)}{z^2-2z\cosh\omega T+1}$
20	$\frac{\omega^2}{s(s^2-\omega^2)}$	$1-\cos\omega t$	$\frac{z}{z-1}-\frac{z(z-\cosh\omega T)}{z^2-2z\cosh\omega T+1}$

Z-transform Theorems

1.Linear.Let

$$E_1(z) = Z[e_1(t)] \qquad E_2(z) = Z[e_2(t)]$$

$$Z[e_1(t) + e_2(t)] = E_1(z) + E_2(z)$$

$$Z[Ae_1(z)] = AE_1(z)$$

Z-transform linearity

Eg.

$$f(k) = 2 \cdot 1(k) + 4\delta(k), \qquad k = 0, 1, 2, \cdots \quad T = 1$$

$$F(z) = Z\{2 \cdot 1(k) + 4\delta(k)\}$$

$$= 2Z\{1(k)\} + 4Z\{\delta(k)\}$$

$$= \frac{2z}{z-1} + 4$$

$$= \frac{6z-4}{z-1}$$

Z-transform Theorems

2.Translation in time (time shift)

a. shifting to the right

$$\boxed{Z[e^*(t-pT)] = z^{-p}E(z)}$$

$$Let \qquad t = nT$$

$$Z[e^*(t-pT)] = Z[e^*(nT-pT)] = \sum_{n=0}^{\infty} e[(n-p)T]z^{-n}$$

$$= z^{-p}\sum_{n=0}^{\infty} e[(n-p)T]z^{-(n-p)}$$

$$= z^{-p}\sum_{m=-p}^{\infty} e(mT)z^{-m} = z^{-p}\sum_{m=0}^{\infty} e(mT)z^{-m}$$

$$= z^{-p}E(z)$$

Time Delay

Eg. $Z\{f(k-n)\} = z^{-n}F(z)$

$$f(k) = 4, \qquad k = 2,3,4,\cdots$$

$$F(z) = Z\{4\cdot 1(k-2)\} = 4z^{-2}Z\{1(k)\}$$

$$= z^{-2}\frac{4z}{z-1}$$

$$= \frac{4}{z(z-1)}$$

Z-transform Theorems

2. Translation in time (time shift)

b. shifting to the left

$$\boxed{Z[e^{*}(t+pT)] = z^{p}E(z) - \sum_{i=0}^{p-1} e(iT)z^{p-i}}$$

$$Let \qquad t = nT$$

$$Z[e^{*}(t+pT)] = Z[e^{*}(nT+pT)] = \sum_{n=0}^{\infty} e[(n+p)T]z^{-n}$$

$$= z^{p}\sum_{n=0}^{\infty} e[(n+p)T]z^{-(n+p)}$$

$$= z^{p}\sum_{m=p}^{\infty} e(mT)z^{-m} = z^{p}\sum_{m=0}^{\infty} e(mT)z^{-m} - z^{p}\sum_{i=0}^{p-1} e(iT)z^{-i}$$

$$= z^{p}E(z) - \sum_{i=0}^{p-1} e(iT)z^{p-i}$$

Time Advance Example Table13.1

$$\{f(k)\} = \{4, 8, 16, \cdots\}$$

Solve:

$$f(k) = 2^{k+2} = g(k+2) \qquad k = 0, 1, 2, \cdots$$

$$g(k) = 2^k,$$

$$F(z) = z^2 G(z) - z^2 g(0) - zg(1)$$

$$= z^2 \frac{z}{z-2} - z^2 - 2z$$

$$= \frac{4z}{z-2}$$

Z-transform Theorems

3. Final Theorem

if F(z) converges for |z|>1, and all poles of (1-z)F(z) are inside the unit circle, then

$$\boxed{f(\infty) = \lim_{k\to\infty} f(kT) = \lim_{z\to 1}[(1-z^{-1})F(z)]}$$

if $\lim_{z\to\infty} F(z)$ **exists, then the initial value of** $f(kT)$

$$\boxed{\lim_{k\to 0} f(kT) = \lim_{z\to\infty} F(z)}$$

final value does not apply to:

(*i*) An unbounded sequence.

(*ii*) An oscillatory sequence.

Multiplication by Exponential

4. $$\boxed{Z\left\{a^{-k}f(k)\right\}=F(az)}$$

Proof: $$Z\left\{a^{-k}f(k)\right\}=\sum_{k=0}^{\infty}a^{-k}f(k)z^{-k}$$

$$=\sum_{k=0}^{\infty}f(k)(az)^{-k}$$

$$=F(az)$$

Theory of Z-transform

Z 变换的性质定理

名称	时域序列关系	Z 域象函数关系
线性	$c_1 f_1(k) + c_2 f_2(k)$	$c_1 F_1(z) + c_2 F_2(z)$
移位性	（1） $f(k \pm m)$ （2） $f(k-m)U(k)^*$ $f(k-m)U(k-m)^*$ $f(k+m)U(k)^*$	$z^{\pm m} F(z)$ $z^{-m}[F(z) + \sum_{k=-m}^{-1} f(k) z^{-k}]$ $z^{-m} F(z)$ $z^{m}[F(z) - \sum_{k=0}^{m-1} f(k) z^{-k}]$
部分和	$f_1(k)$ $f(k) = \sum_{i=0}^{k} f_1(i)$	$F_1(z)$ $F(z) = \frac{z}{z-1} F_1(z)$
折叠性	$f(-k)$	$F(z^{-1})$
Z 域尺度变换性	$a^k f(k)$	$F(\frac{z}{a})$
Z 域微分性	$k^m f(k)$	$(-z\frac{d}{dz})^m F(z)$
Z 域积分性	$\frac{f(k)}{k+m}$ $k+m>0$ $\frac{f(k)}{k}$ $k>0$	$z^m \int_z^{\infty} \frac{F(x)}{x^{m+1}} dx$ $\int_z^{\infty} \frac{F(x)}{x} dx$
时域卷积定理	$f_1(k) * f_2(k)$	$F_1(k) F_2(k)$
初值定理	$f(0) = \lim_{z \to \infty} F(z)$	$f(m) = \lim z^m [F(z) - \sum_{k=0}^{m-1} f(k) z^{-k}]$
终值定理	$f(\infty) = \lim_{z \to 1} \frac{z-1}{z} F(z)$	

Inverse Z-transform

1 Long division or synthetic division

$$E(z)=\frac{1}{1-e^{-aT}z^{-1}} \Rightarrow e^{*}(t)=?$$

Let $e^{-aT}=0.5$

$$1+0.5z^{-1}+0.25z^{-2}+\cdots$$

$$1-0.5z^{-1}\overline{\big)\;1\qquad\qquad}$$

$$1-0.5z^{-1}$$

$$\overline{\big)\quad 0.5z^{-1}}$$

$$0.5z^{-1}-0.25z^{-2}$$

$$\overline{\big)\quad 0 \qquad 0.25z^{-2}}$$

Example

Inverse z-transform Solution: $F(z) = \dfrac{z+1}{z^2+0.2z+0.1}$

1. Long Division

$$
\begin{array}{r@{}l}
 & z^{-1} + 0.8z^{-2} - 0.26z^{-3} + \cdots \\
z^2+0.2z+0.1 \big) & z+1 \\
 & \underline{z + 0.2 + 0.1z^{-1}} \\
 & 0.8 - 0.1z^{-1} \\
 & \underline{0.8 + 0.16z^{-1} + 0.08z^{-2}} \\
 & -0.26z^{1} - \cdots
\end{array}
$$

2. Inverse Transformation

$$F(z) = 0 + z^{-1} + 0.8z^{-2} - 0.26z^{-3} + \cdots$$

$$\{f_k\} = \{0, 1, 0.8, -0.26, \cdots\}$$

$$f^*(t) = 0 + \delta(t-T) + 0.8\delta(t-2T) - 0.26\delta(t-3T) + \cdots$$

Inverse of Z-transform

1. Long division or synthetic division: gives as many terms of series as desired.

2. Partial fraction expansion and table look-up: similar to Laplace transform inversion.

e.g

$$E(z)=\frac{1}{1-e^{-aT}z^{-1}}$$

$$=1+0.5z^{-1}+0.25z^{-2}+\cdots$$

$$e^{*}(t)=\delta(t)+0.5\delta(t-T)+0.25\delta(t-2T)+\cdots$$

Partial Fraction Expansion

(i) Find the partial fraction expansion of $F(z)/z$.
(ii) Obtain the inverse transform $f(k)$ using
z-transform tables.

Three types of z-domain functions *F*(*z*):
1. ***F*(*z*)** with simple (non-repeated) real poles.
2. ***F*(*z*)** with complex conjugate & real poles.
3. ***F*(*z*)** with repeated poles.

Inverse Z-transform

2 Partial traction

$$E(z)=\frac{10z}{(z-1)(z-2)}$$

$$\frac{E(z)}{z}=\frac{10}{(z-1)(z-2)}=\frac{10}{z-2}-\frac{10}{z-1}$$

$$E(z)=\frac{10z}{z-2}-\frac{10z}{z-1}$$

$$e(nT)=10\cdot 2^{n}-10$$

$$e^{*}(t)=10\sum_{n=0}^{\infty}(-1+2^{n})\delta(t-nT)$$

I Simple Real Root

Residue of a complex function $F(z)$ at a simple pole z_i

$$A_i = (z - z_i)F(z)\Big|_{z \to z_j}$$

Residue = partial fraction coefficient of the ith term of the expansion

$$F(z) = \sum_{i=1}^{n} \frac{A_i}{z - z_i}$$

Example

Obtain the inverse z-transform of the function

$$F(z)=\frac{z+1}{z^2+0.3z+0.02}$$

Solution: Solve using two different methods.

(*i*) Partial Fraction Expansion (dividing by z)

$$\frac{F(z)}{z}=\frac{z+1}{z(z^2+0.3z+0.02)}$$

$$=\frac{A}{z}+\frac{B}{z+0.1}+\frac{C}{z+0.2}$$

Example

$$A = z\frac{F(z)}{z}\bigg|_{z=0} = F(0) = \frac{1}{0.02} = 50$$

$$B = (z+0.1)\frac{F(z)}{z}\bigg|_{z=-0.1} = \frac{1-0.1}{(0.1)(-0.1)} = -90$$

$$C = (z+0.2)\frac{F(z)}{z}\bigg|_{z=-0.2} = \frac{1-0.2}{(-0.2)(-0.1)} = 40$$

Partial fraction expansion

$$F(z) = \frac{50z}{z} - \frac{90z}{z+0.1} + \frac{40z}{z+0.2}$$

Example

(*ii*) Table Lookup

$$f(k)=\begin{cases}50\delta(k)-90(-0.1)^k+40(-0.2)^k, & k\geq 0\\ 0 & ,k<0\end{cases}$$

Note

f(0) = 0, so the time sequence can be rewritten as

$$f(k)=\begin{cases}-90(-0.1)^k+40(-0.2)^k, & k\geq 1\\ 0 & ,k<1\end{cases}$$

II Complex Conjugate & simple root

Use the following z-transforms

$$Z\left\{e^{-ak}\sin(kw_d)\right\} = \frac{e^{-a}\sin(w_d)z}{z^2 - 2e^{-a}\cos(w_d)z + e^{-2a}}$$

$$Z\left\{e^{-ak}\cos(kw_d)\right\} = \frac{z[z - e^{-a}\cos(w_d)]}{z^2 - 2e^{-a}\cos(w_d)z + e^{-2a}}$$

Example

Find the inverse z-transform of

$$F(z) = \frac{z^3 + 2z + 1}{(z-0.1)(z^2+z+0.5)}$$

Solution (i) Partial Fraction Expansion

Dividing by *z* gives

$$\frac{F(z)}{z} = \frac{A_1}{z} + \frac{A_2}{z-0.1} + \frac{Az+B}{z^2+z+0.5}$$

$$A_1 = F(0) = -20, \qquad A_1 = (z-0.1)\frac{F(z)}{z} = 19.689$$

Matlab

$G(z)$ numerator $5(z+3)$, denominator $z3+0.1z2+0.4z$

num = 5*[1, 3]

den = [1, 0.1, 0.4, 0]

% Multiplication of Polynomials

denp =conv(den1, den 2)

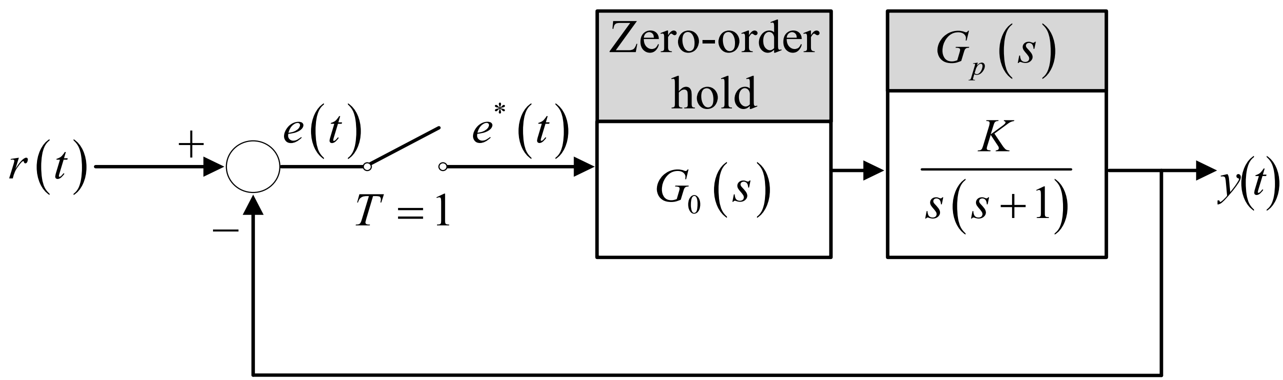
% Partial Fraction Coefficients

[r, p, k] = residue(num, den)

p = poles, **r** = residues , **k** = coefficients of the polynomial resulting from dividing the numerator by the denominator.

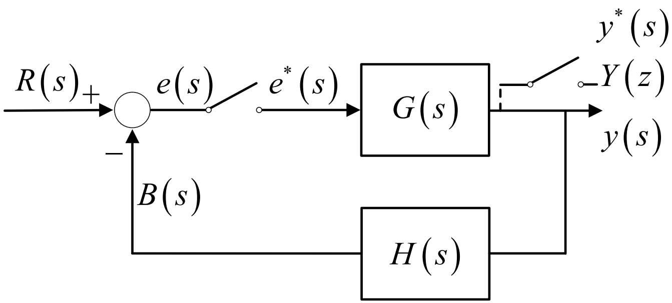
- **Mathematatic model of digital system**

13.3 Modeling of digital system



A close-loop digital system

Discrete-time system



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$$\boxed{G(z)H(z) \neq GH(z)}$$

a Discrete-time system

$$Y(s) = G(s)e^*(s)$$

$$e(s) = R(s) - B(s) = R(s) - H(s)Y(s)$$

$$e(s) = R(s) - B(s) = R(s) - H(s)G(s)e^*(s)$$

$$e^*(s) = R^*(s) - Z[H(s)G(s)]e^*(s)$$

$$e^*(s) = \frac{R^*(s)}{1 + Z[H(s)G(s)]}$$

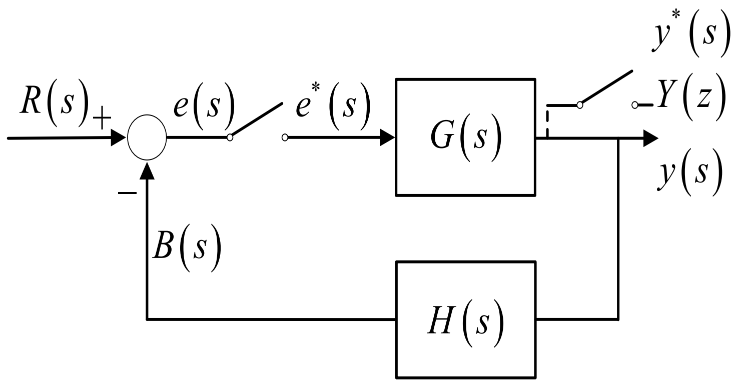
Let $Z[H(s)G(s)] = GH(z)$

$$Y(z) = \frac{G(z)R(z)}{1 + HG(z)}$$

Discrete-time system

Close-loop impulse TF:

$$\frac{Y(z)}{R(z)} = \frac{G(z)}{1+HG(z)}$$



Open-loop impulse TF:

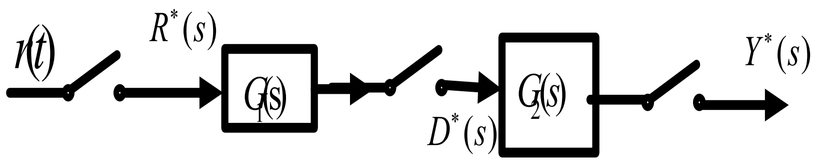
$$\frac{B(z)}{E(z)} = HG(z)$$

Error TF:

$$\frac{E(z)}{R(z)} = \frac{1}{1+HG(z)}$$

Cascade elements

Block diagram: sampler in the middle of two elements



$$D(z) = G_1(z)R(z)$$

$$Y(z) = G_2(z)D(z) = G_2(z)G_1(z)R(z)$$

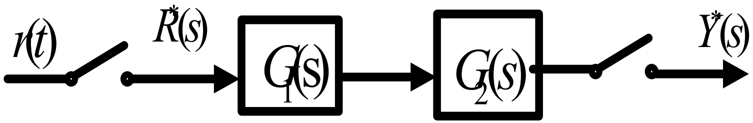
$$\frac{Y(z)}{R(z)} = G_2(z)G_1(z) = G(z)$$

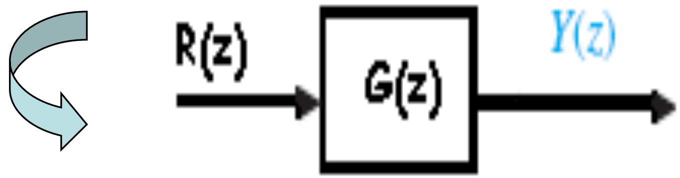
$$G(z) = G_2(z)G_1(z)$$

$$G(z) = \prod_{i=1}^{n} G_i(z)$$

Cascade elements

Block diagram: no-sampler in the middle of two elements





$$Y(z) = Z[G_2(s)G_1(s)]R(z)$$

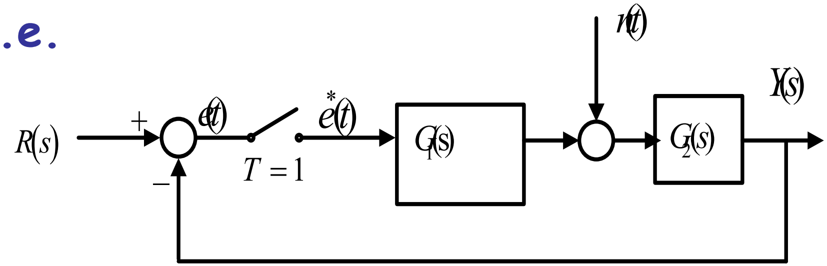
$$\frac{Y(z)}{R(z)} = G_2G_1(z) = G(z)$$

$$G(z) = G_2G_1(z)$$

$$\boxed{G(z) = Z\{\prod_{i=1}^{n} G_i(s)\}}$$

Example

i.e.

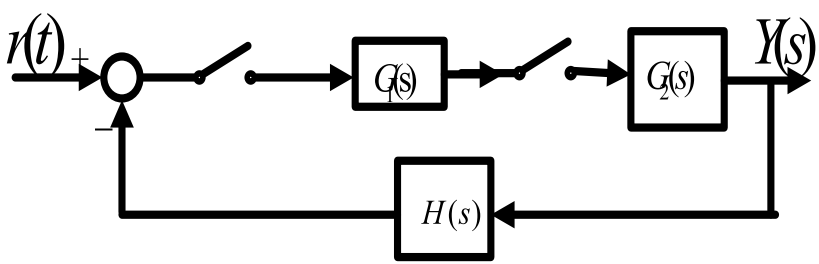


回路的定义

$$\frac{Y(z)}{R(z)} = \frac{G_2G_1(z)}{1+G_2G_1(z)}$$

$$Y(z) = \frac{G_2N(z)}{1+G_2G_1(z)}$$

i.e.

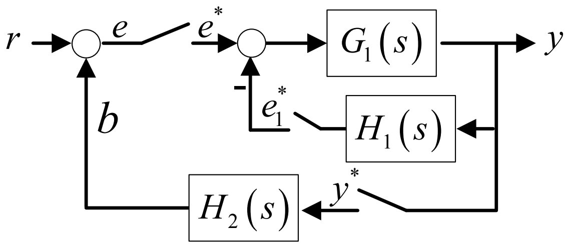


$$\frac{Y(z)}{R(z)} = \frac{G_1(z)G_2(z)}{1+G_1(z)HG_2(z)}$$

Example（Table 3）

Eg

The structure diagram of the discrete system is shown on the right, calculate the system pulse transfer function



answer

$$E_1(z)=G_1H_1(z)\cdot\left[E(z)-E_1(z)\right]$$

$$=\frac{G_1H_1(z)}{1+G_1H_1(z)}\cdot E(z)$$

$$G_{cl}(z)=\frac{Y(z)}{R(z)}$$

$$E(z)=R(z)-B(z)=R(z)-H_2(z)\cdot Y(z)$$

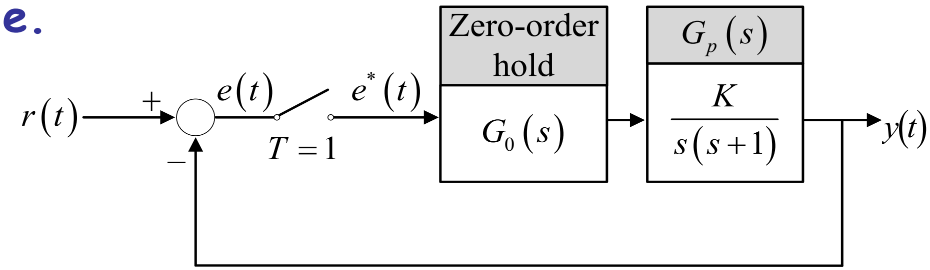
$$Y(z)=G_1(z)\cdot\left[E(z)-E_1(z)\right]$$

$$=\frac{G_1(z)\cdot\left[R(z)-H_2(z)\cdot Y(z)\right]}{1+G_1H_1(z)}$$

$$G_{cl}(z)=\frac{G_1(z)}{1+G_1H_1(z)+G_1(z)H_2(z)}$$

Example

i.e.



$$G_0(s) = \frac{1-e^{-Ts}}{s}$$

$$K = 1$$

$$open: G(z) = Z\{G_0 G_p\} = Z\{G_0 \frac{1}{s(s+1)}\}$$

$$G(z) = (1-z^{-1})Z\{\frac{1}{s^2(s+1)}\} = (1-z^{-1})Z\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\}$$

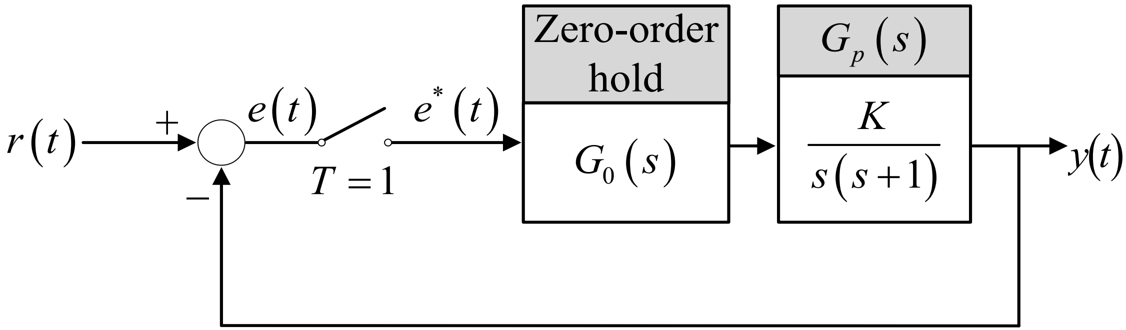
$$G(z) = (1-z^{-1})\{\frac{Tz^{-1}}{(1-z^{-1})^2} - \frac{1}{1-z^{-1}} + \frac{1}{1-z^{-1}e^{-T}}\}$$

$$G(z) = \frac{0.368z + 0.264}{z^2 - 1.368z + 0.368}$$

零阶保持器不改变系统的阶数

Example

i.e.



Close-loop impulse TF:

$$G_{cl}(z)=\frac{G(z)}{1+G(z)}=\frac{0.368z+0.264}{z^2-z+0.632}$$

先写公式再代数

Time-Analysis Method

Time response

Dynamic performance

1) Calculate impulse transfer function of the system

$$G_{cl}(z) = \frac{G(z)}{1 + GH(z)}$$

2) Calculate the $Y(z)$

$$Y(z) = G_{cl}(z)R(z) = y(0) + y(T)z^{-1} + y(2T)z^{-2} + \dots$$

3) $y^*(t) = y(0)\delta(t) + y(T)\delta(t-T) + y(2T)\delta(t-2T) + \dots$

4) Determine dynamic Index $P.O, t_s, t_r, t_p$

Time response

Example 1

The structure diagram of the discrete system is in Fig.1, where K=1, T=1

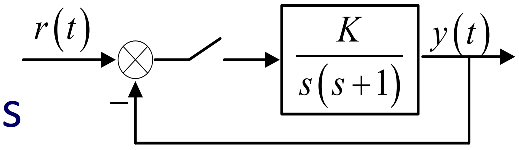


Fig.1

Determine dynamic indicators $\sigma\%, t_s$ when $r(t)=1(t)$

answer

Open-loop impulse TF

$$G(z)=Z\left[\frac{K}{s(s+1)}\right]=\frac{K\left(1-e^{-T}\right)z}{(z-1)\left(z-e^{-T}\right)}$$

$$=\frac{0.632z}{(z-1)(z-0.368)}$$

Close-loop impulse TF

$$G_{cl}(z)=\frac{G(z)}{1+G(z)}=\frac{0.632z}{z^2-0.736z+0.368}$$

Time response

$$R(z)=\frac{z}{z-1} \quad \Rightarrow \quad y(\infty)=\lim_{z\to 1}(z-1)\cdot G_{cl}(z)\cdot R(z)=1$$

$$Y(z)=G_{cl}(z)R(z)=\frac{0.632z^2}{z^3-1.736z^2+1.104z-0.368}$$

Calculate $y(t)$ by **long division**

$$Y(z)=0.632z^{-1}+1.097z^{-2}+1.207z^{-3}+1.117z^{-4}+1.014z^{-5}+0.964z^{-6}+\ldots$$

$$y^*(t)=0.632\delta(t-T)+1.097\delta(t-2T)+1.207\delta(t-3T)+1.117\delta(t-4T)$$
$$+1.014\delta(t-5T)+0.964\delta(t-6T)+\ldots$$

$$t_p=3T$$
$$\sigma\%=20.7\%$$

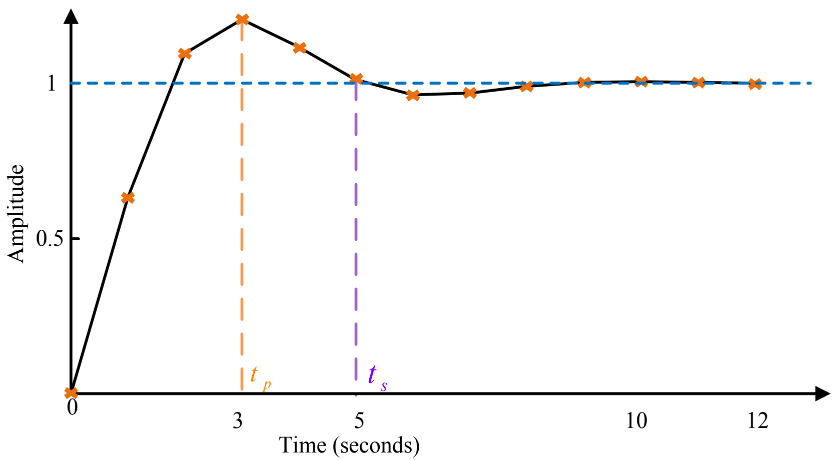
$$t_s=5T$$

Time response

$$R(z)=\frac{z}{z-1} \quad \Rightarrow \quad y(\infty)=\lim_{z\to 1}(z-1)\cdot G_{cl}(z)\cdot R(z)=1$$

$$Y(z)=G_{cl}(z)R(z)=\frac{0.632z^2}{z^3-1.736z^2+1.104z-0.368}$$

$$Y(z)=0.632z^{-1}+1.097z^{-2}+1.207z^{-3}+1.117z^{-4}+1.014z^{-5}+0.964z^{-6}+\ldots$$



$$t_s=5T$$

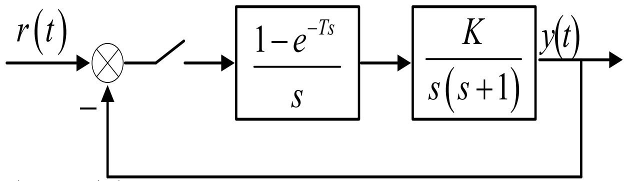
$$t_p=3T$$

$$\sigma\%=20.7\%$$

Time response

Example 2

The structure diagram of the discrete system is inFig.1 , where K=1, T=1



Determine dynamic indicators $\sigma\%, t_s$ when $r(t)=1(t)$

answer

$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}\cdot\frac{K}{s(s+1)}\right]$$

Open-loop impulse TF

$$=\left(1-z^{-1}\right)K\cdot Z\left[\frac{1}{s^2(s+1)}\right]$$

$$=\frac{(z-1)K}{z}\cdot\left[\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right]$$

$$=\frac{0.368z+0.264}{(z-1)(z-0.368)}$$

Close-loop impulse TF

$$G_{cl}(z)=\frac{G(z)}{1+G(z)}=\frac{0.368z}{z^2-z+0.632}$$

Time response

$$R(z)=\frac{z}{z-1} \qquad y(\infty)=\lim_{z\to 1}(z-1)\cdot\phi(z)\cdot R(z)=1$$

$$Y(z)=G_{cl}(z)R(z)=\frac{0.368z^2+0.264z}{z^3-2z^2+1.632z-0.632}$$

Calculate y(t) by long division

$$Y(z)=0.368z^{-1}+z^{-2}+1.4z^{-3}+1.4z^{-4}+1.147z^{-5}+0.894z^{-6}+0.802z^{-7}$$
$$+0.868z^{-8}+0.994z^{-9}+1.077z^{-10}+1.081z^{-11}+1.032z^{-12}+0.981z^{-13}+\ldots$$

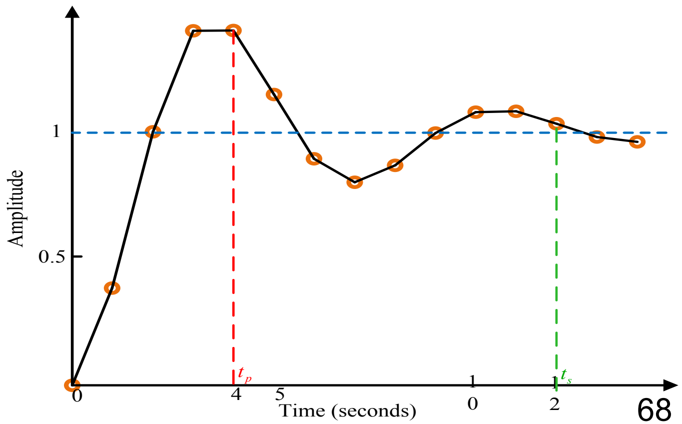
$$y^*(t)=0.368\delta(t-T)+\delta(t-2T)+1.4\delta(t-3T)+1.4\delta(t-4T)$$
$$+1.147\delta(t-5T)+0.894\delta(t-6T)+0.802\delta(t-7T)+0.868\delta(t-8T)$$
$$+0.994\delta(t-9T)+1.077\delta(t-10T)+1.081\delta(t-11T) \qquad t_p=3T\sim 4T$$
$$+1.032\delta(t-12T)+0.981\delta(t-13T)+\ldots \qquad t_s=12T \qquad \sigma\%=40\%$$

Time response

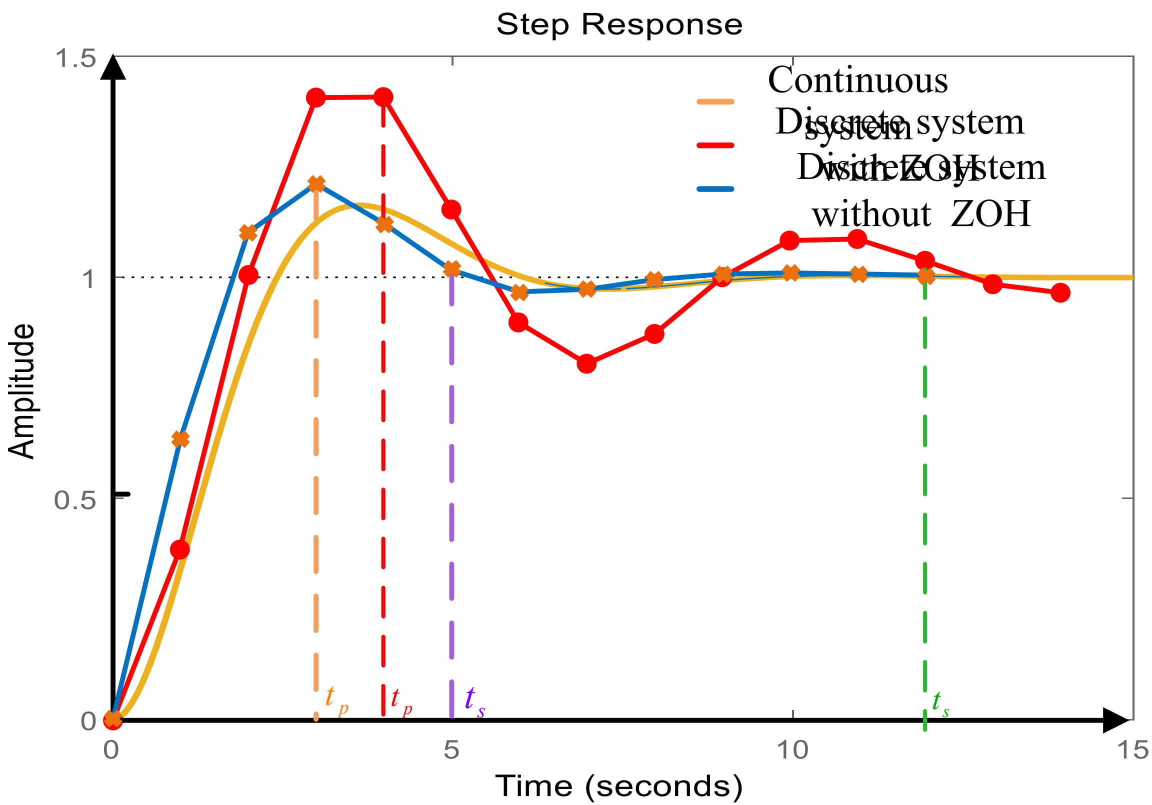
$$Y(z) = 0.368z^{-1} + z^{-2} + 1.4z^{-3} + 1.4z^{-4} + 1.147z^{-5} + 0.894z^{-6} + 0.802z^{-7}$$
$$+0.868z^{-8} + 0.994z^{-9} + 1.077z^{-10} + 1.081z^{-11} + 1.032z^{-12} + 0.981z^{-13} + \ldots$$

$$y^*(t) = 0.368\delta(t-T) + \delta(t-2T) + 1.4\delta(t-3T) + 1.4\delta(t-4T)$$
$$+1.147\delta(t-5T) + 0.894\delta(t-6T) + 0.802\delta(t-7T) + 0.868\delta(t-8T)$$
$$+0.994\delta(t-9T) + 1.077\delta(t-10T) + 1.081\delta(t-11T)$$
$$+1.032\delta(t-12T) + 0.981\delta(t-13T) + \ldots$$

$$t_p = 3T\text{~}4T$$
$$t_s = 12T$$
$$\sigma\% = 40\%$$
$$t_r = 2T$$

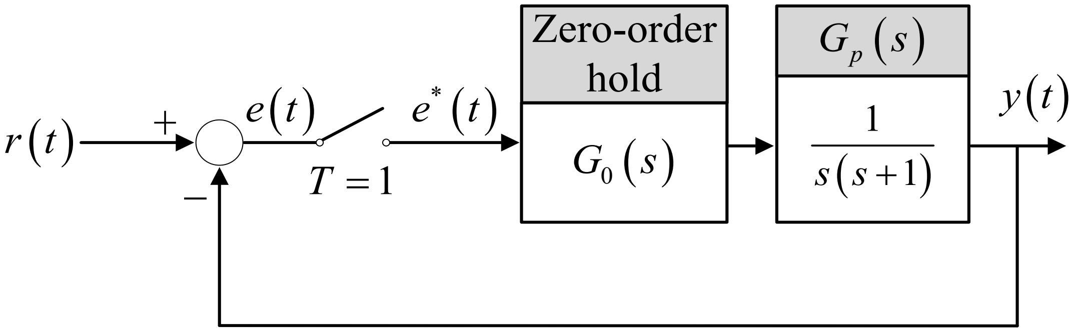


Time response



Time response

•Step response of digital system



$$open: G(z) = Z\{G_0 \frac{1}{s(s+1)}\} \Rightarrow G(z) = \frac{0.368z + 0.264}{z^2 - 1.368z + 0.368}$$

Close-loop impulse TF $$G_{cl}(z) = \frac{G(z)}{1+G(z)} = \frac{0.368z + 0.264}{z^2 - z + 0.632}$$

Time response

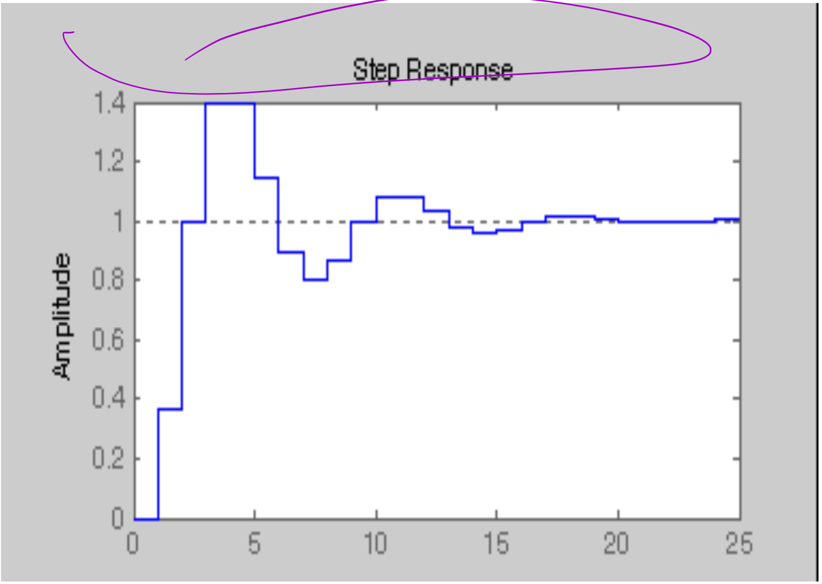
•Step response of digital system

$$
\begin{array}{r|l}
 & 0.368z^{-1} + z^{-2} + 1.4z^{-3} + 1.4z^{-4} + 1.14z^{-5} + \cdots \\
\hline
z^3 - 2z^2 + 1.632z - 0.632 & 0.368z^2 + 0.264z + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0
\end{array}
$$

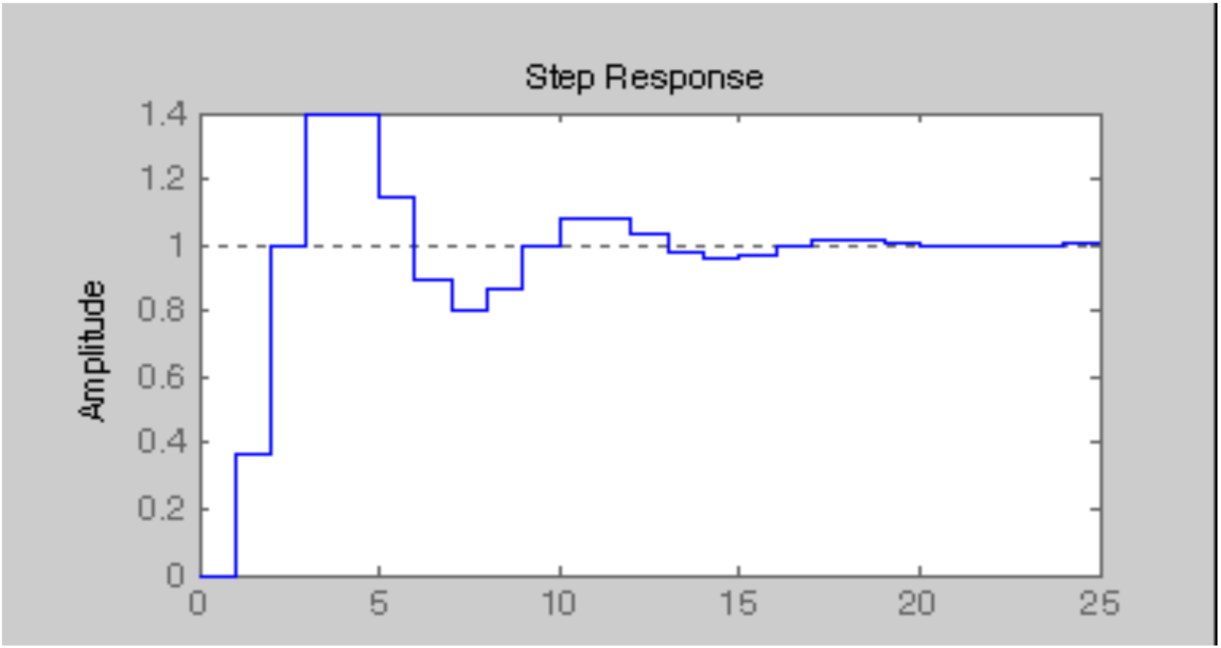
$$y^*(t) = 0.368\delta(t-T) + \delta(t-2T) + 1.4\delta(t-3T) + 1.4\delta(t-4T) + 1.14\delta(t-5T) + \cdots$$

$$G_{cl}(z) = \frac{G(z)}{1+G(z)} = \frac{0.368z + 0.264}{z^2 - z + 0.632}$$

```
num=[0.368 0.264];
den=[1 -1 0.632];
dstep(num,den)
```

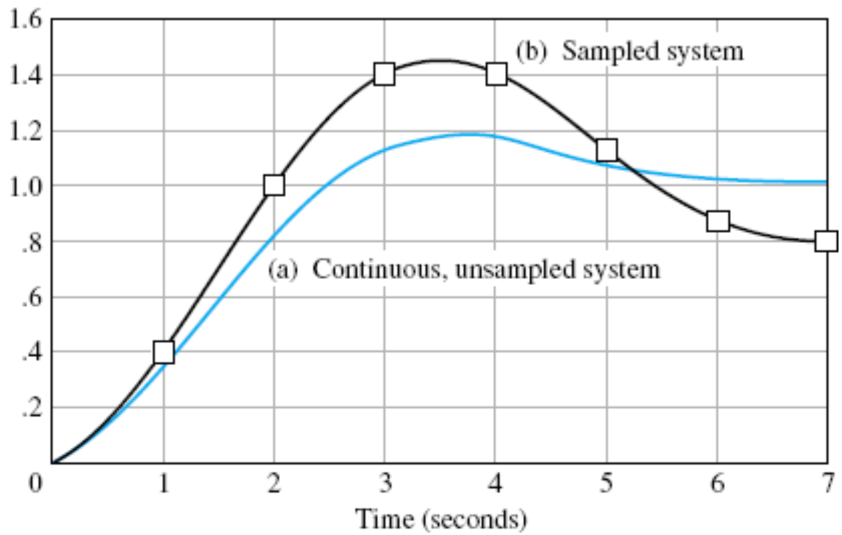


Time response



$$t_r = 2T \qquad t_p = (3 \sim 4)T \qquad t_s = 12T \qquad o.p = 40\%$$

Time response



The response of a second-order system: (a) continuous ($T = 0$), not sampled; (b) sampled system, $T = 1$ second.

有界输入有界输出

劳斯稳定判据

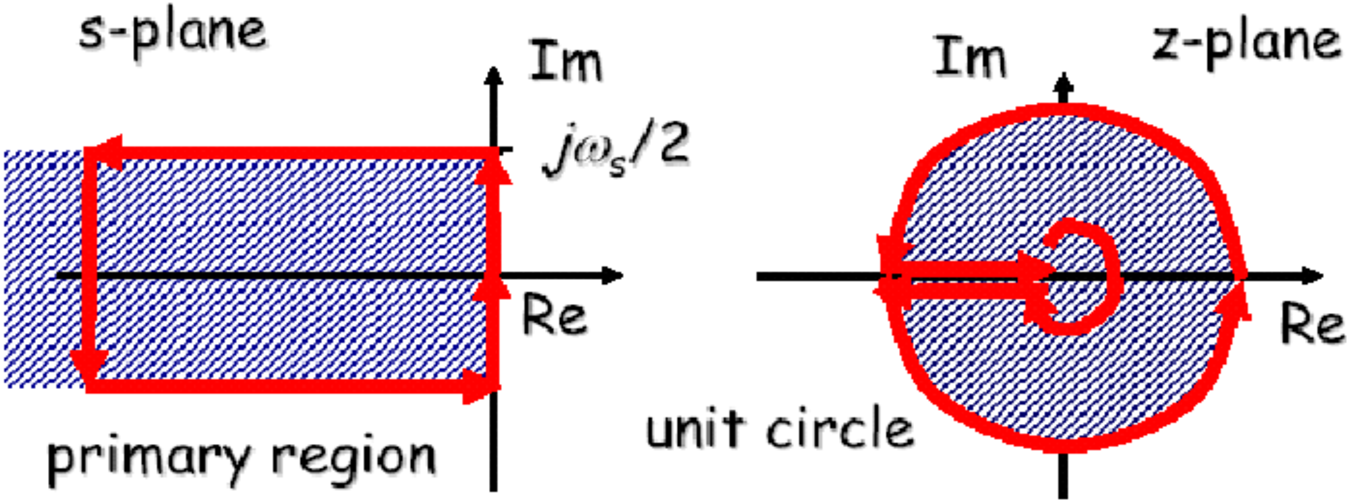
根轨迹

Stability Analysis

奈氏稳定判据，包围(-1, j0)点……

Time-Analysis - System stability

S-plane ➡ Z-plane

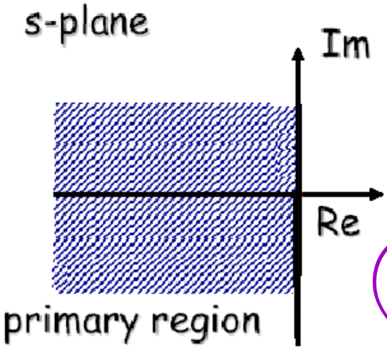


$$z = e^{sT}, \quad let \quad s = \sigma + jw$$

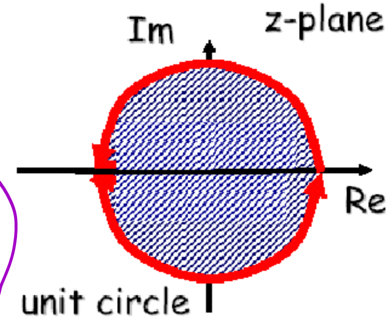
$$z = e^{(\sigma + jw)T} = e^{\sigma T} e^{jwT}$$

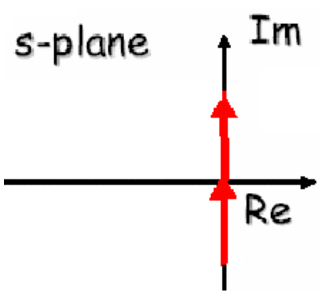
$\sigma < 0$ 稳定
$\sigma = 0$ 临界 $e^{\sigma T} = 1$
$\sigma > 0$

Time-Analysis -System stability



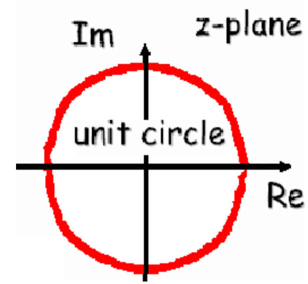
$\sigma < 0 \qquad |z| = e^{\sigma T} < 1$

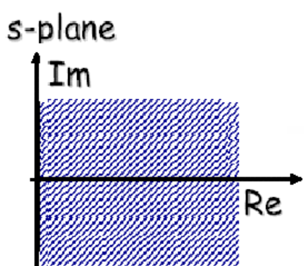




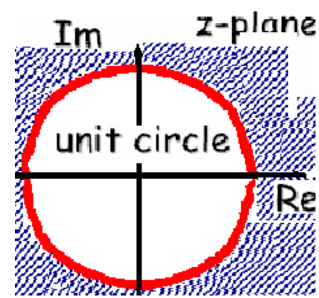
critical

$\sigma = 0 \qquad |z| = e^{\sigma T} = 1$



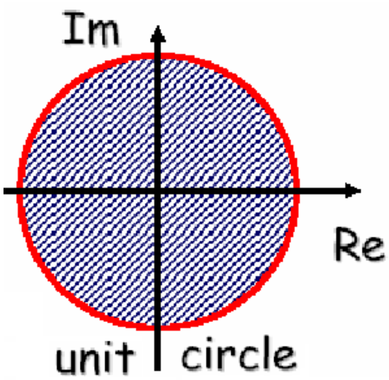


$\sigma > 0 \qquad |z| = e^{\sigma T} > 1$



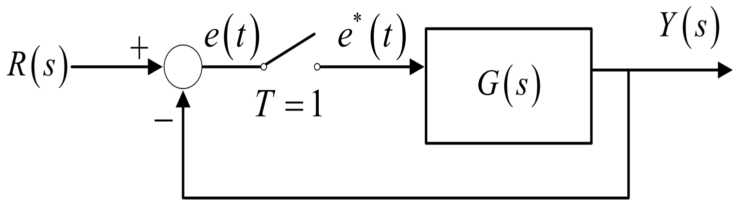
Stability Criterion

- A digital control system is stable if and only if all poles of close-loop digital system are in the unit circle, i.e. $|z|<1$.
- When all poles are on the circle, system is called marginal stable.



Example

i.e. we have $G(s)=\dfrac{k}{s(s+1)}$,determine stability.



$$G(z)=Z\{\frac{k}{s(s+1)}\}$$

$$=\frac{kz(1-e^{-T})}{(z-1)(z-e^{-T})}$$

$$CE: \quad D(z)=1+G(z)=z^2-[1-k+(1+k)e^{-T}]z+e^{-T}=0$$

1. k=1, T=1

$$\Rightarrow \quad D(z)=z^2-0.763z+0.368=0$$

$$z_{1,2}=-0.368\pm j0.482$$

$$|z_{1,2}|=\sqrt{0.368^2+0.482^2}=0.6064<1$$

The system is stable.

Example

1. k=5, T=1 $\Rightarrow$ $D(z)=z^2-1.792z+0.368=0$

$$z_1=-0.237 \qquad z_2=-1.555$$

$$|z_2|=1.555>1$$

The system is unstable.

2. k=5, T=0.1 $\Rightarrow$ $D(z)=0$

$$z_{1,2}=0.715\pm j0.608$$

$$|z_{1,2}|=0.94<1$$

The system is stable.

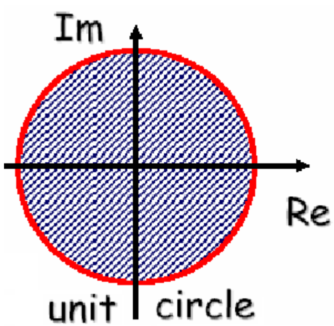
Notes: 1 |z| determine stability
2. stability relates with parameter k and T.

Time-Analysis Method-System stability

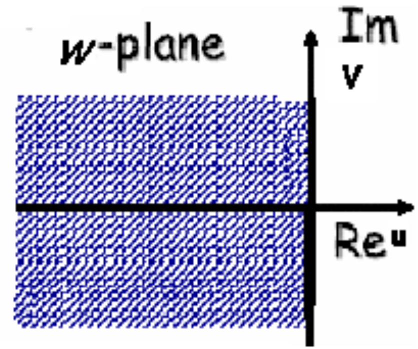
1. Routh criterion in [Z-plane],

let $\boxed{z = \dfrac{w+1}{w-1}}$ $\quad z = x + jy \qquad w = u + jv$

$$w = u + jv = \frac{z+1}{z-1} = \frac{x^2 + y^2 - 1}{(x-1)^2 + y^2} - j\frac{2y}{(x-1)^2 + y^2}$$

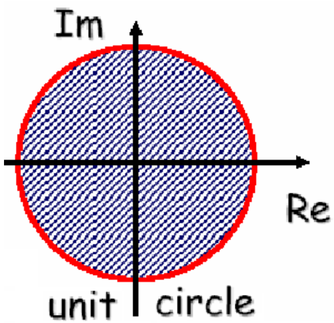


Z-plane

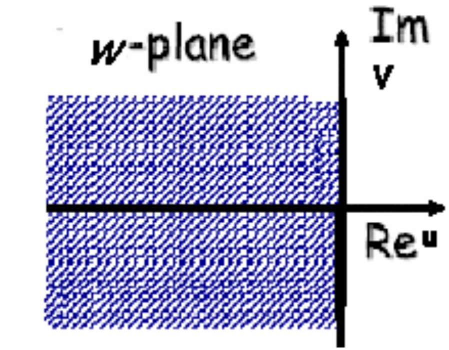


W-plane

Time-Analysis Method-System stability



Z-plane



W-plane

$x^2 + y^2 > 1$	Outside of circle	$u > 0$	Right in w-plane
$x^2 + y^2 = 1$	On the circle	$u = 0$	On Imaginary
$x^2 + y^2 < 1$	In the circle	$u < 0$	Left in w-plane

Similar Routh stability in s-plane

Time-Analysis -System stability

Routh's Stability Criterion in w field → $z = \dfrac{w+1}{w-1}$

Example 1 $D(z) = 45z^3 - 117z^2 + 119z - 39 = 0$

Judging the stability of the system

answer

$$D(w) = 45\left(\frac{w+1}{w-1}\right)^3 - 117\left(\frac{w+1}{w-1}\right)^2 + 119\left(\frac{w+1}{w-1}\right) - 39 = 0$$

$$D(w) = w^3 + 2w^2 + 2w + 40 = 0$$

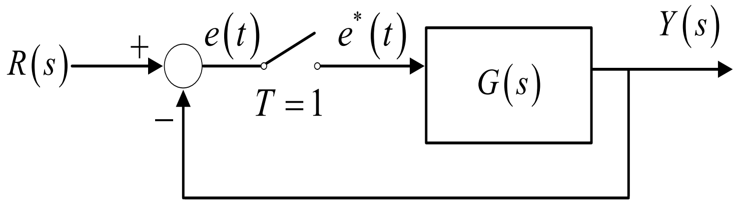
Routh:

w^3	1	2
w^2	2	40
w^1	-18	
w^0	40	

System is unstable

Time-Analysis -System stability

i.e. we have $G(s)=\dfrac{k}{s(0.1s+1)}$, $T=1s$, determine stability.



$$G(z)=Z\{\frac{k}{s(0.1s+1)}\}$$

$$=\frac{k0.632z}{z^2-1.368z+0.368}$$

$$CE: \quad D(z)=1+G(z)=z^2+[0.632k-1.368]z+0.368=0$$

Let $\boxed{z=\dfrac{w+1}{w-1}}$ $\Rightarrow$

$$D(w)=0.632kw^2+1.264w+(2.736-0.632k)=0$$

$$\begin{array}{lll} w^2 & 0.632k & 2.736-0.632k \\ w & 1.264 & \\ w^0 & 2.736-0.632k & \end{array}$$

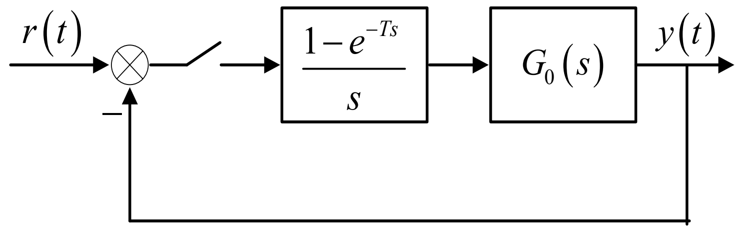
When $0<k<4.33$, the system is stable.

11.5 Stability analysis

Example 2

The structure diagram of the discrete system is shown on the right, where

$$G_0(s)=\frac{2}{(0.1s+1)(0.05s+1)}$$



Judging the stability of the system, when T=0.1s

answer

$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}\cdot G_0(s)\right]=\frac{z\text{ - }1}{z}\cdot Z\left[\frac{G_0(s)}{s}\right]$$

$$\frac{G_0(s)}{s}=\frac{2}{s(0.1s+1)(0.05s+1)}=\frac{A_1}{s}+\frac{A_2}{0.1s+1}+\frac{A_3}{0.05s+1}$$

$$A_1=\lim_{s\to 0}\left[s\cdot\frac{2}{s(0.1s+1)(0.05s+1)}\right]=2$$

11.5 Stability analysis

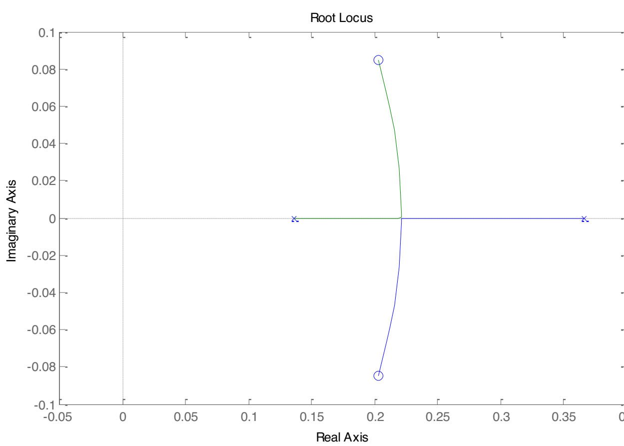
$$A_2 = \lim_{s \to -10} \left[(0.1s+1) \cdot \frac{2}{s(0.1s+1)(0.05s+1)} \right] = -0.4$$

$$A_3 = \lim_{s \to -20} \left[(0.05s+1) \cdot \frac{2}{s(0.1s+1)(0.05s+1)} \right] = 0.1$$

$$G(z) = \frac{z-1}{z} \cdot Z\left[\frac{2}{s} - \frac{0.4}{0.1s+1} + \frac{0.1}{0.05s+1} \right]$$

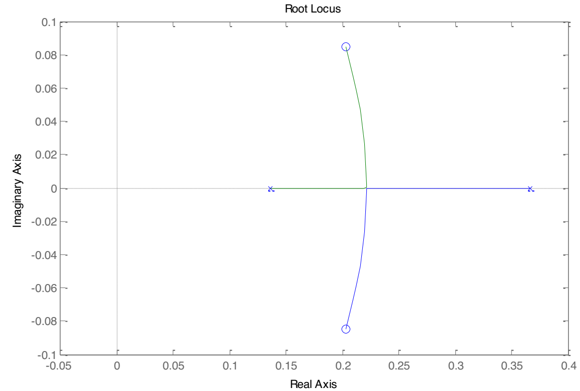
$$= \frac{z-1}{z} \left[\frac{2z}{z-1} - \frac{0.4z}{z-e^{-10T}} + \frac{0.1z}{z-e^{-20T}} \right]$$

$$= \frac{1.7z^2 - 0.689z + 0.082}{z^2 - 0.503z + 0.05}$$



11.5 Stability analysis

$$\boxed{1+G(z)=0} \quad D(z)=2.7z^2-1.192z+0.132=0$$



$$D(z)=z^2-0.441z+0.05=0$$

M1:

$$|z_{1,2}|=|-0.2207\pm0.0127j|=0.2211<1$$

M2:

$$\boxed{z=\frac{w+1}{w-1}}$$

$$D(w)=0.609w^2+1.9w+1.491=0$$

Routh:

$$\begin{array}{lll} w^2 & 0.609 & 1.491 \\ w^1 & 1.9 & \\ w^0 & 1.491 & \end{array}$$

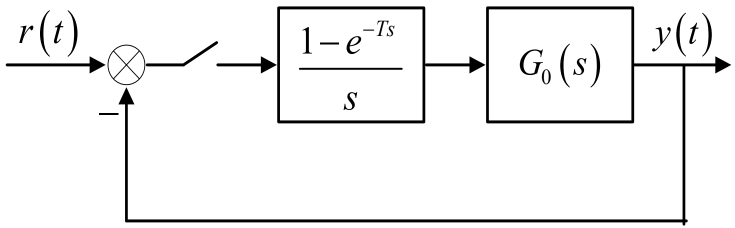
System is stable

Time-Analysis -System stability

Example 3

The structure diagram of the discrete system is shown on the right, where

$$G_0(s)=\frac{2}{s(0.1s+1)(0.05s+1)}$$



Judging the stability of the system, when T=0.1s

answer

$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}\cdot G_0(s)\right]=\frac{z\text{-}1}{z}\cdot Z\left[\frac{G_0(s)}{s}\right]$$

$$G_0(z)=\frac{z\text{-}1}{z}Z\left\{\frac{2}{s^2(0.1s+1)(0.05s+1)}\right\}=\frac{z\text{-}1}{z}Z\left\{\frac{A_1}{s^2}+\frac{A_2}{s}+\frac{A_3}{0.1s+1}+\frac{A_4}{0.05s+1}\right\}$$

$$A_1=\lim_{s\to 0}\left[s^2\cdot\frac{2}{s^2(0.1s+1)(0.05s+1)}\right]=2$$

$$A_2=\lim_{s\to 0}\frac{d}{ds}\left[s^2\cdot\frac{2}{s^2(0.1s+1)(0.05s+1)}\right]=-0.3$$

Time-Analysis -System stability

$$A_3 = \lim_{s \to -10} \left[(0.1s+1) \cdot \frac{2}{s^2(0.1s+1)(0.05s+1)} \right] = 0.04$$

$$A_4 = \lim_{s \to -20} \left[(0.05s+1) \cdot \frac{2}{s^2(0.1s+1)(0.05s+1)} \right] = -0.05$$

$$G(z) = \frac{z\text{-}1}{z} \cdot Z\left[\frac{2}{s^2} - \frac{0.3}{s} + \frac{0.04}{0.1s+1} - \frac{0.05}{0.05s+1} \right]$$

$$= \frac{z-1}{z}\left[\frac{2Tz}{(z-1)^2} - \frac{0.3z}{z-1} + \frac{0.04z}{z-e^{-10T}} - \frac{0.05z}{z-e^{-20T}} \right]$$

$$= \frac{-0.31z^3 + 0.684z^2 + 0.303z + 0.038}{z^3 - 1.503z^2 + 0.553z - 0.05}$$

Time-Analysis -System stability

$$\boxed{1+G(z)=0} \qquad D(z)=0.69z^3-0.819z^2+0.856z-0.012=0$$

$$D(z)=z^3-1.19z^2+1.24z-0.02=0$$

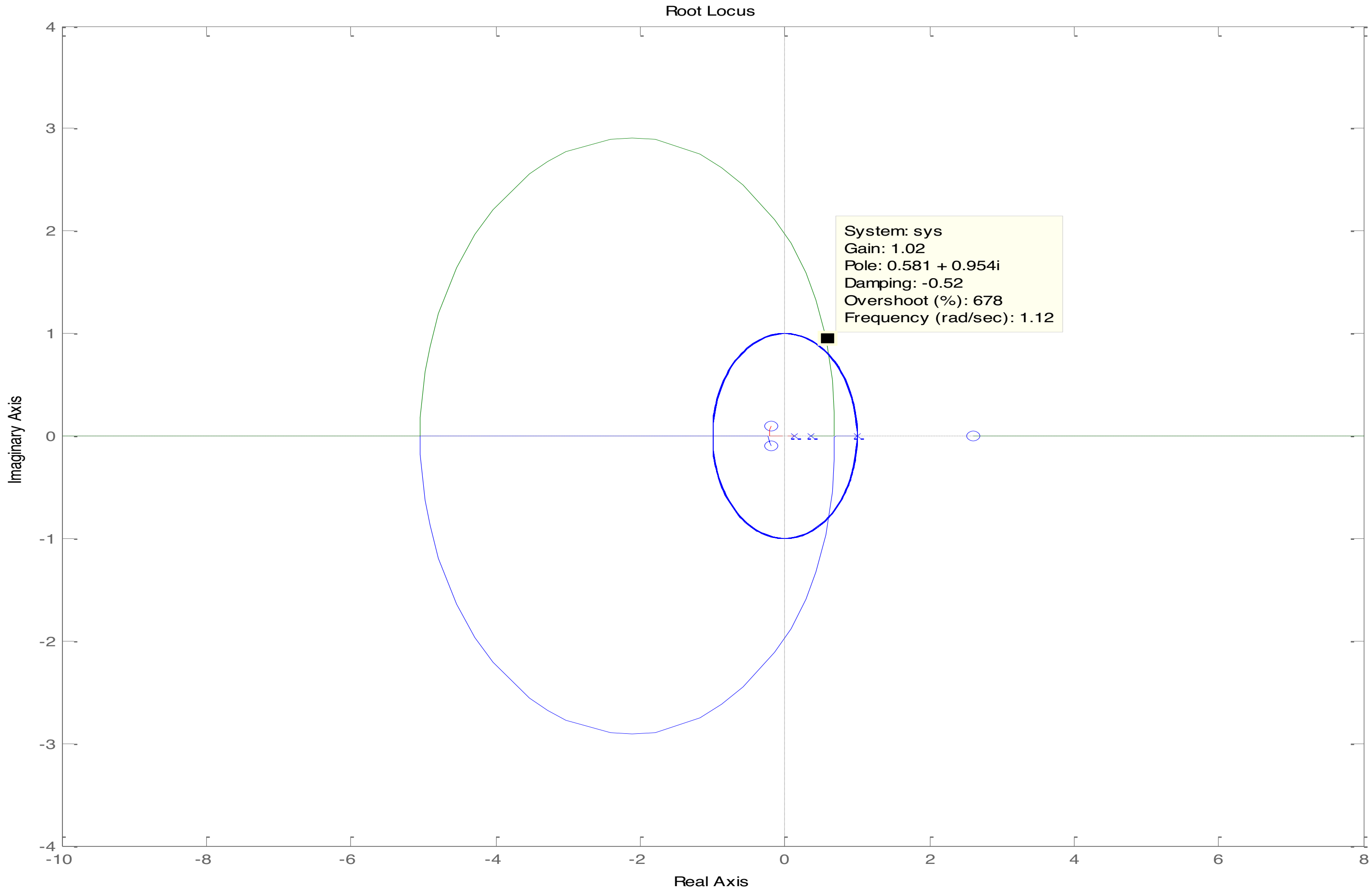
$$\boxed{z=\frac{w+1}{w-1}} \qquad D(w)=1.03w^3+0.63w^2+2.89w+3.45=0$$

Routh:

$$\begin{array}{lll} w^3 & 1.03 & 2.89 \\ w^2 & 0.63 & 3.45 \\ w^1 & -2.75 & \\ w^0 & 3.45 & \end{array}$$

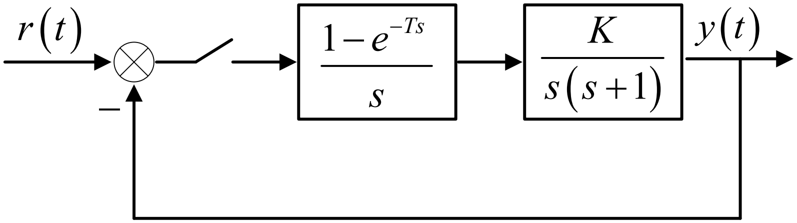
System is unstable

Time-Analysis -System stability



Time-Analysis -System stability

Example 3 The structure diagram of the discrete system is shown on the right



If T=1s, find the range of K to make the system stable

answer

$$G(z)=Z\left[\frac{1-e^{-Ts}}{s}\cdot\frac{K}{s(s+1)}\right]=\left(1-z^{-1}\right)K\cdot Z\left[\frac{1}{s^2(s+1)}\right]$$

$$=\frac{(z-1)K}{z}\cdot Z\left[\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s+1}\right]$$

$$=\frac{(z-1)K}{z}\cdot\left[\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right]\overset{T=1}{=}\frac{0.368K(z+0.718)}{(z-1)(z-0.368)}$$

Time-Analysis -System stability

Close-loop TF:

$$G_{cl}(z)=\frac{G(z)}{1+G(z)}=\frac{0.368K(z+0.718)}{z^2+(0.368K-1.368)z+(0.264K+0.368)}$$

CE: $D(z)=z^2+(0.368K-1.368)z+(0.264+0.368)=0$

$$D(w)=\left(\frac{w+1}{w-1}\right)^2+(0.368K-1.368)\left(\frac{w+1}{w-1}\right)+(0.264+0.368)$$

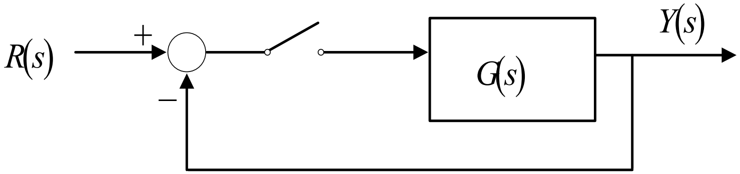
$$=0.632Kw^2+(1.264-0.528K)w+(2.736-0.104K)=0$$

$$\begin{cases}K>0\\1.264-0.528K>0\\2.736-0.104K>0\end{cases}\qquad\begin{cases}K>0\\K<2.394\\K<26.3\end{cases}\qquad 0<K<2.394$$

- Time-Analysis -Stable-state error

Time-Analysis -Stable-state error

i.e. the system is shown:



$$E(z) = R(z) - Y(z) = \frac{1}{1+G(z)} R(z)$$

From the final value Theorem, the stable-state error:

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{z \to 1} (1 - z^{-1}) E(z) = \lim_{z \to 1} (1 - z^{-1}) \frac{1}{1+G(z)} R(z)$$

$$G_{open-loop}(z) = G(z) = \frac{K(z-z_1)(z-z_2)\cdots(z-z_m)}{(z-1)^{\gamma}(z-p_1)\cdots(z-p_{n-v})}$$

Time-Analysis -Stable-state error

System type

i.e. the system is shown:

$\gamma = 0$ ⟹ "0" type system

$\gamma = 1$ ⟹ "I" type system

$\gamma = 2$ ⟹ "II" type system

Let

K_p ⟹ The step (position) error coefficient

K_v ⟹ The ramp (vecolation) error coefficient

K_a ⟹ The step (accelarition) error coefficient

Time-Analysis -Stable-state error

The number of poles at z=1 of open-loop impulse TF is called the type of system

1. The unit step

$$e(\infty)=\lim_{z\to 1}(1-z^{-1})\frac{1}{1+G(z)}\boxed{\frac{z}{z-1}}=\frac{1}{1+\lim_{z\to 1}G(z)}$$

Let $\boxed{K_p=\lim_{z\to 1}G(z)}$

So "0" type $K_p=\lim_{z\to 1}G(z)$ $e(\infty)=\dfrac{1}{1+K_p}$

"I" type $K_p=\infty$ $e(\infty)=0$

"II" typa $K_p=\infty$ $e(\infty)=0$

Time-Analysis -Stable-state error

2. The unit ramp input

$$e(\infty) = \lim_{z\to 1}(1-z^{-1})\frac{1}{1+G(z)}\boxed{\frac{Tz}{(z-1)^2}} = \frac{T}{(z-1)\lim\limits_{z\to 1} G(z)}$$

Let $\boxed{K_v = (z-1)\lim\limits_{z\to 1} G(z) = const}$

So "0" type $K_v = 0$ $e(\infty) = \infty$

"I" type $K_v = (z-1)\lim\limits_{z\to 1} G(z)$ $e(\infty) = \dfrac{T}{K_v}$

"II" typa $K_v = \infty$ $e(\infty) = 0$

Time-Analysis -Stable-state error

2. The unit ramp input

$$e(\infty) = \lim_{z\to 1}(1-z^{-1})\frac{1}{1+G(z)}\boxed{\frac{T^2(z+1)}{2(z-1)^3}} = \frac{T^2}{(z-1)^2\lim\limits_{z\to 1}G(z)}$$

Let $\boxed{K_a = (z-1)^2\lim\limits_{z\to 1}G(z) = const}$

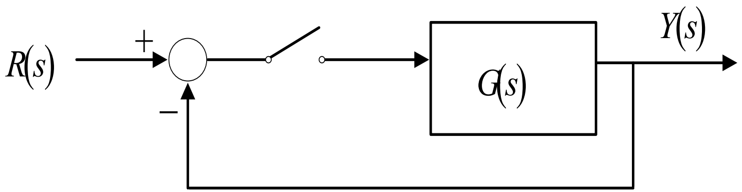
So "0" type $K_a = 0$ $\quad e(\infty) = \infty$

"I" type $K_a = 0$ $\quad e(\infty) = \infty$

"II" type $K_a = (z-1)^2\lim\limits_{z\to 1}G(z)$ $\quad e(\infty) = \dfrac{T^2}{K_a}$

Time-Analysis -Stable-state error

i.e. the system is shown:



$$G(s)=\frac{2}{s(0.1s+1)},T=0.1$$

$$G(z)=Z\{\frac{2}{s(0.1s+1)}\}=\frac{1.264z}{(z-1)(z-0.368)}$$

This is a "I" type system:, so when the input is

the unit step, $K_v=0$ $e(\infty)=\infty$

the unit ramp, $K_v=(z-1)\lim_{z\to 1}G(z)=\frac{1.264}{0.632}$ $e(\infty)=\frac{T}{K_v}=0.05$

the acceleration, $K_v=\infty$ $e(\infty)=0$

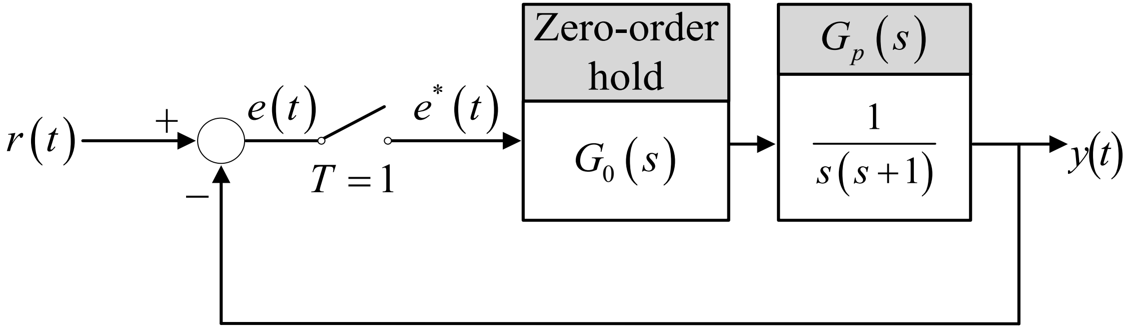
先写公式再代数

Analysis

- Z-plane not easy for design.
- All root locus rules remain valid.
- Pheudo frequency design

Time response

•Step response of digital system



$$open: G(z) = Z\{G_0 \frac{1}{s(s+1)}\} \Rightarrow G(z) = \frac{0.368z+0.264}{z^2-1.368z+0.368}$$

Close-loopimpulse TF

$$G_{cl}(z) = \frac{G(z)}{1+G(z)} = \frac{0.368z+0.264}{z^2-z+0.632}$$

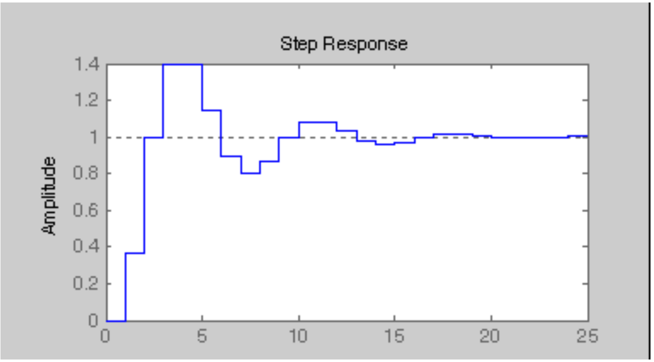
Time response

•Step response of digital system

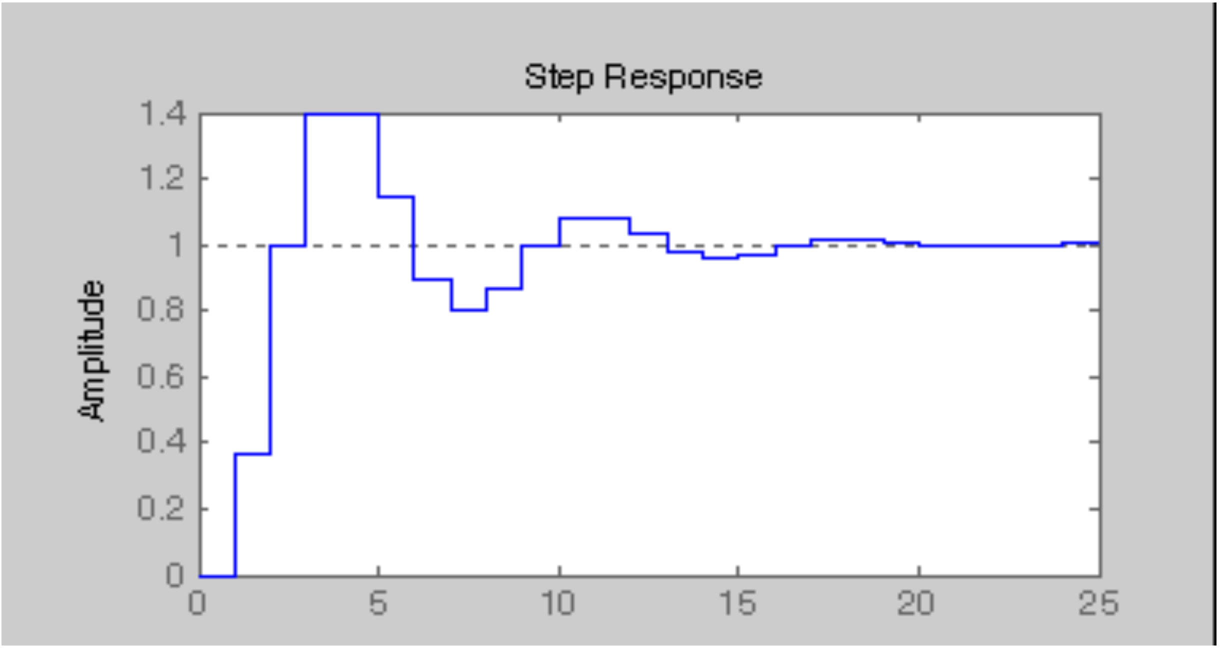
$$\begin{array}{r} 0.368z^{-1} + z^{-2} + 1.4z^{-3} + 1.4z^{-4} + 1.14z^{-5} + \cdots \\ z^3 - 2z^2 + 1.632z - 0.632 \overline{\big) 0.368z^2 + 0.264z + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0} \end{array}$$

$$y^*(t) = 0.368\delta(t-T) + \delta(t-2T) + 1.4\delta(t-3T) + 1.4\delta(t-4T) + 1.14\delta(t-5T) + \cdots$$

```
num=[0.368 0.264];
den=[1 -1 0.632];
dstep(num,den)
```

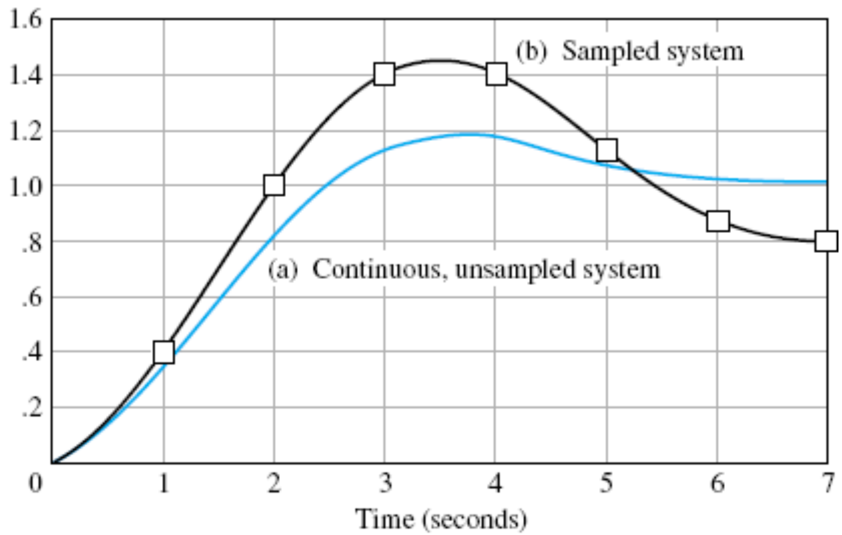


Time response



$$t_r = 2T \qquad t_p = (3 \sim 4)T \qquad t_s = 12T \qquad o.p = 40\%$$

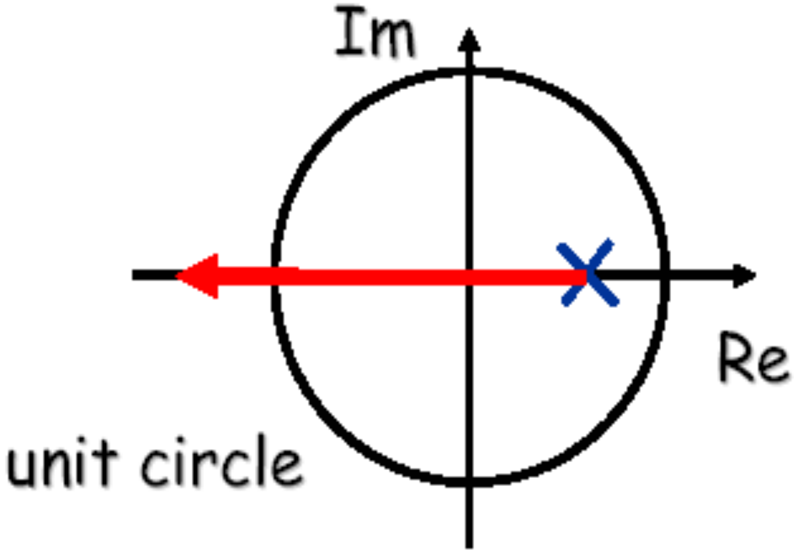
Time response



The response of a second-order system: (a) continuous ($T = 0$), not sampled; (b) sampled system, $T = 1$ second.

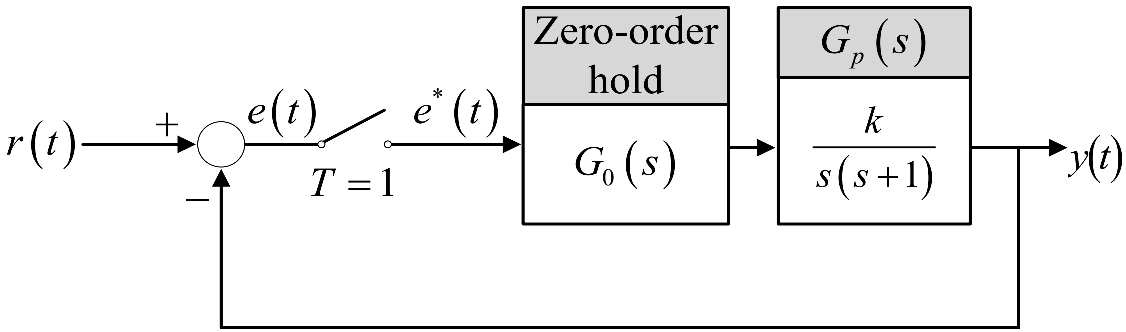
Root loci of First-order digital system

First-order system $$H_L = \frac{X(z)}{U(z)} = \frac{K[1-e^{-aT}]}{z-e^{-aT}}$$



Closed loop can become unstable !!

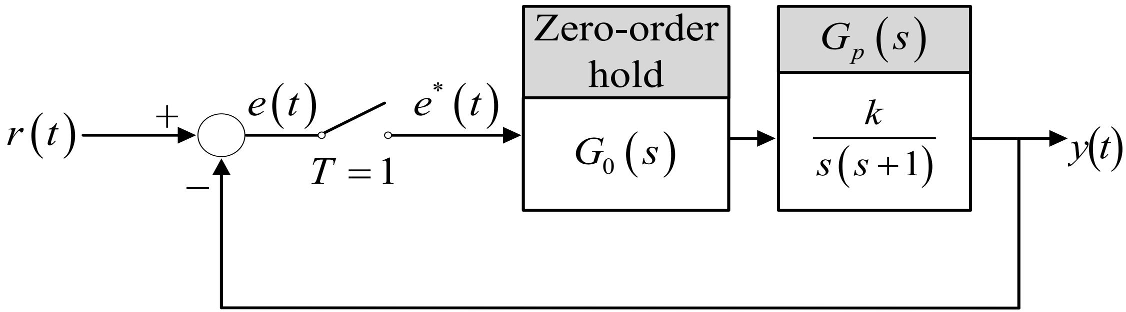
Root loci of the digital system



$$open: G(z) = Z\{G_0 \frac{k}{s(s+1)}\} \Rightarrow G(z) = \frac{k(0.368z+0.264)}{z^2-1.368z+0.368}$$

$$G(z) = \frac{0.368k(z+0.717)}{(z-1)(z-0.368)} = \frac{k^*(z+0.717)}{(z-1)(z-0.368)}$$

Root loci of the digital system



$$open: G(z)=\frac{0.368k(z+0.717)}{(z-1)(z-0.368)}=\frac{k^*(z+0.717)}{(z-1)(z-0.368)}$$

$$\text{poles}: z=1, z=0.368, Zeros: z=-0.717$$

n=2,m=1;2 branches;

Real axis loci: 0.368~1; -∞~-0.717;

Break-away point and break-in point:

$$z_1=0.638;\quad z_2=-2.08$$

CE:1+G(z)=0; ⇨ $\frac{d(-k)}{dz}=d\left\{\frac{z^2-1.368z+0.368}{0.368z+0.264}\right\}/dz=0$

Root loci of the digital system

$$D(z) = z^2 + (k^* - 1.368)z + 0.368 + 0.717k^* = 0$$

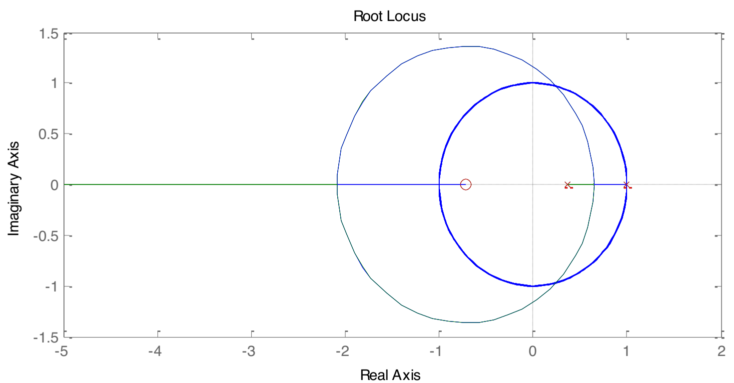
$$Let \qquad z = \frac{w+1}{w-1}$$

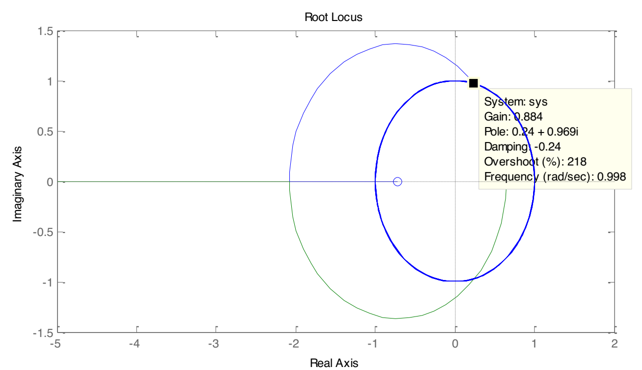
$$D(w) = 1.707k^* w^2 + (1.264 - 1.434k^*)w + 2.736 - 0.293k^* = 0$$

$$1.707k^* > 0, \qquad 1.264 - 1.434k^* > 0 \qquad 2.736 - 0.293k^* > 0$$

When $0 < k^* < 0.88$, System is stable

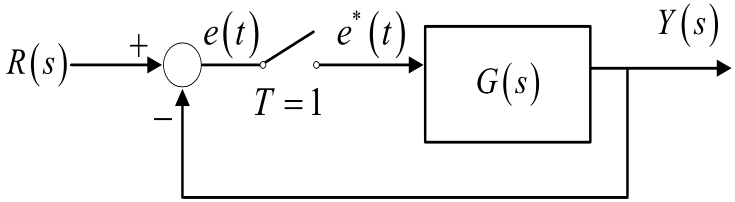
```
>>t=0:0.1:100;plot(sin(t),cos(t)),hold on, rlocus([1,0.717],[1 -1.368 0.368])
```





Pseudo Frequency Response of the digital system

i.e. we have $G(s) = \dfrac{k}{s(0.1s+1)}, T = 1s$, determine stability.



$$G(z) = Z\{\frac{k}{s(0.1s+1)}\}$$

$$= \frac{k0.632z}{z^2 - 1.368z + 0.368}$$

Let $\boxed{z = \dfrac{w+1}{w-1}}$ $\Rightarrow G(w) = \left.\dfrac{k0.632z}{z^2 - 1.368z + 0.368}\right|_{z=\frac{w+1}{w-1}}$

Let $w = jv$

$$G(jv) = -$$