Control Laboratory

Chapter 4 Feedback Control System Characteristics

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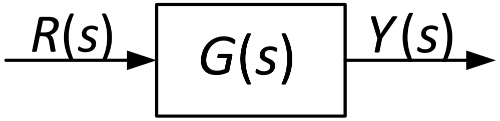
Contents

4.1 Open- & Closed-Loop

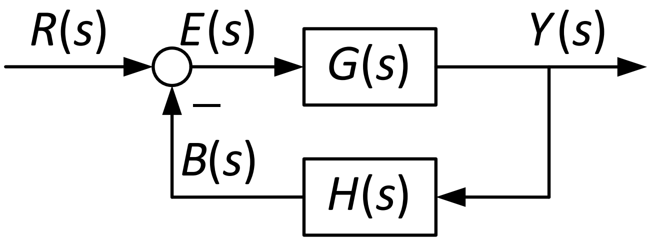
Open-loop system

- An open-loop (direct) system operates without feedback and directly generates the output in response to an input signal.



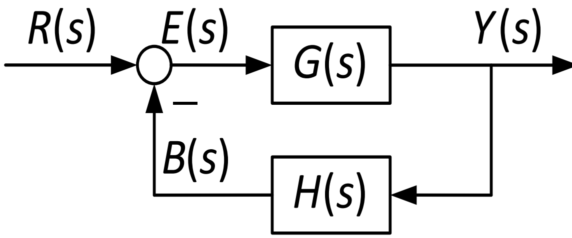
Closed-loop system

- A closed-loop system uses a measurement of the output signal and a comparison with the desired output to generate an error signal that is used by the controller to adjust the actuator.



4.2 Sensitivity of System

The closed-loop system $Y(s) = \dfrac{G(s)}{1+G(s)H(s)} R(s)$



If $G(s)H(s) >> 1 \Rightarrow Y(s) \cong \dfrac{1}{H(s)} R(s)$

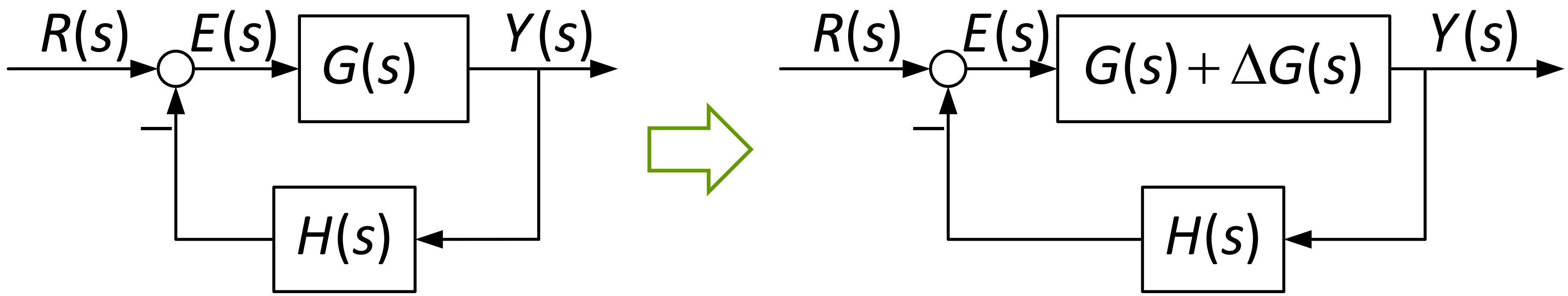
$$\big(G(s)H(s) >> 1\big) \,\&\, \big(H(s) = 1\big) \Rightarrow Y(s) \cong R(s)$$

- The condition $G(s)H(s)$>>1 may cause the system response to be highly oscillatory and even unstable.
- Increasing the magnitude of the loop gain reduces the effect of $G(s)$ on the output is an exceedingly useful result.
- **The first advantage of a feedback system** is that the effect of the variation of the parameters of the process, $G(s)$, is reduced.

4.2 Sensitivity of System

The effect of parameter variations

- The closed-loop system



$$Y(s)=\frac{G(s)}{1+G(s)H(s)}R(s), Y(s)+\Delta Y(s)=\frac{G(s)+\Delta G(s)}{1+(G(s)+\Delta G(s))H(s)}R(s)$$

$$\Delta Y(s)=\frac{[G(s)+\Delta G(s)]R(s)}{1+[G(s)+\Delta G(s)]H(s)}-\frac{G(s)R(s)}{[1+G(s)H(s)]}$$

$$=\frac{\Delta G(s)R(s)}{[1+G(s)H(s)+\Delta G(s)H(s)][1+G(s)H(s)]}$$

4.2 Sensitivity of System

When $GH(s) >> \Delta G(s)H(s)$

$$\boxed{\Delta Y(s) = \frac{\Delta G(s)}{(1+GH(s))^2}R(s)}$$

The change in the output of the closed-loop system is reduced by the factor $[1+GH(s)]$.

- **The open-loop system**

$$\boxed{\Delta Y(s) = \Delta G(s)R(s)}$$

4.2 Sensitivity of System

- Conception

System sensitivity is the ratio of the change in the system transfer function to the change of a process transfer function (or parameter) for a small incremental change.

If the system transfer function is $T(s)=\dfrac{Y(s)}{R(s)}$,and therefore the sensitivity is defined as $\boxed{S=\dfrac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)}}$

In the limit, for small incremental changes, Equation becomes

$$\boxed{S=\frac{\partial T/T}{\partial G/G}=\frac{\partial \ln T}{\partial \ln G}}$$

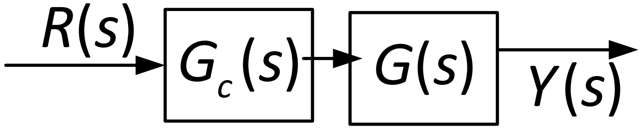
- The open-loop system: $S=\dfrac{\partial T/T}{\partial G/G}=1$
- The closed-loop system: $S=\dfrac{\partial T}{\partial G}\cdot\dfrac{G}{T}=\dfrac{1}{(1+GH)^2}\cdot\dfrac{G}{G/(1+GH)}=\dfrac{1}{1+GH}<1$

4.3 Control of the Transient Response of Control System

The transient response is the response of a system as a function of time.

To provide a satisfactory response:

- Open -loop control system:



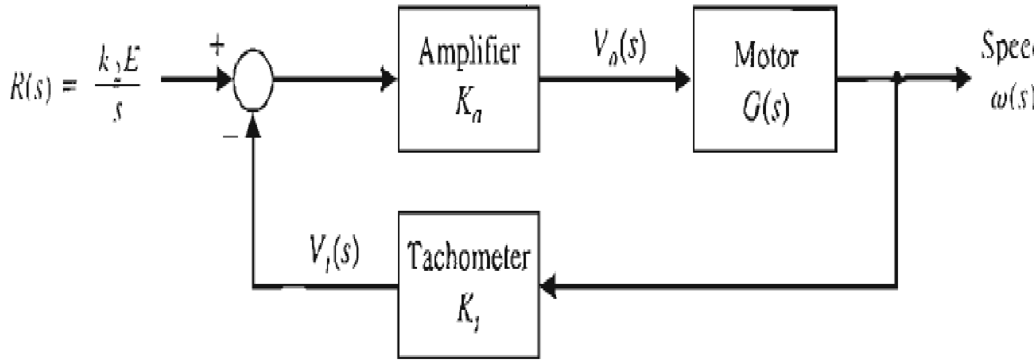
  - replace the process, $G(s)$, with a more suitable process.
  - insert a suitable cascade controller, $Gc(s)$, preceding the process, $G(s)$.

- Closed-loop system:
  - adjust the feedback loop parameters

4.3 Control of the Transient Response of Control System

Example

closed-loop system



$$\frac{\omega(s)}{V_a(s)}=G(s)=\frac{K_1}{(\tau_1 s+1)},$$

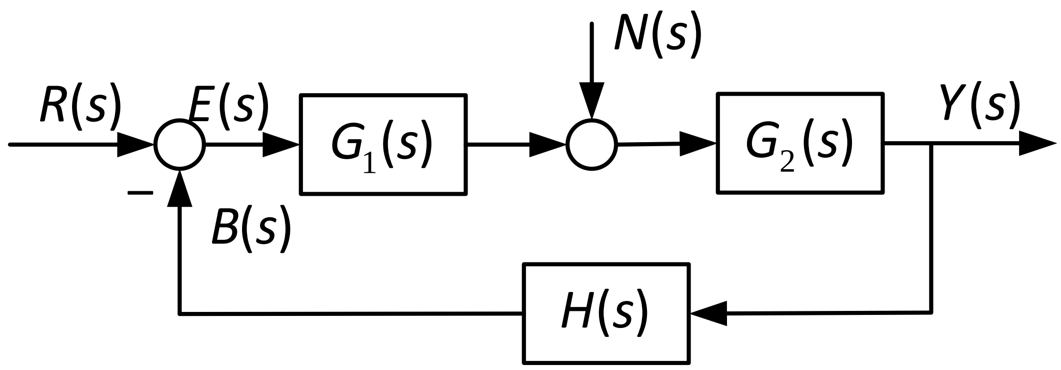
$$\frac{\omega(s)}{R(s)}=\frac{K_aG(s)}{1+K_aK_tG(s)}=\frac{K_aK_1}{\tau_1 s+1+K_aK_tK_1}=\frac{K_aK_1/\tau_1}{s+\left(1+K_aK_tK_1\right)/\tau_1}$$

$$\omega(t)=\frac{K_aK_1}{1+K_aK_tK_1}\left(k_2E\right)(1-e^{-t\cdot(1+K_aK_tK_1)/\tau_1})$$

- Adjust the amplifier K_a and the tachometer gain constant K_t to meet the required transient response

4.4 Disturbance Signals

① Unwanted disturbance



Let $R(s)=0$

$$Y(s)=\frac{G_2}{1+G_1G_2H}N(s),$$

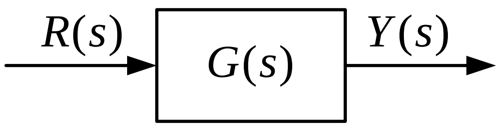
$$Y(s)\approx\frac{1}{G_1H}N(s) \quad \Leftrightarrow \quad close\text{-}loop$$

$$Y(s)=G_2N(s) \quad \Leftrightarrow \quad open\text{-}loop$$

If $G_1(s)H(s)$ is made sufficiently large, the effect of the disturbance can be decreased by closed-loop feedback.

4.5 Steady-state Error

Open-loop system

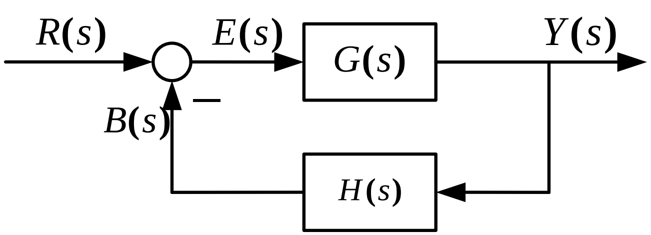


$$E_0(s) = R(s) - Y(s) = [1 - G(s)]R(s)$$

$$e_0(\infty) = \lim_{s \to 0} s[1 - G(s)]R(s)$$

$$\xrightarrow{R(s)=\frac{1}{s}} 1 - G(0)$$

Closed-loop system



$$E(s) = R(s) - Y(s)$$

$$\xrightarrow{H(s)=1} \frac{1}{1+G(s)} R(s)$$

$$e(\infty) = \lim_{s \to 0} s \frac{1}{1+G(s)} R(s)$$

$$\xrightarrow{R(s)=\frac{1}{s}} \frac{1}{1+G(0)}$$

$$\frac{1}{1+G(0)} < 1 - G(0)$$

4.6 Advantage And Disadvantage

Advantage

1. Decrease the sensitivity
   - The open-loop system: $S = \dfrac{\partial T / T}{\partial G / G} = 1$
   - The closed-loop system: $S = \dfrac{\partial T}{\partial G} \cdot \dfrac{G}{T} = \dfrac{1}{1+GH}$
2. Adjust the transient response

$$\omega_{\text{open-loop}}(t) = K_1 k_2 E \cdot (1 - e^{-t/\tau_1}) \Rightarrow \tau_1$$

$$\omega_{\text{closed-loop}}(t) = \frac{K_a K_1}{1 + K_a K_t K_1}\left(k_2 E\right)(1 - e^{-t \cdot (1+K_a K_t K_1)/\tau_1}) \Rightarrow K_a, K_t$$

3. Reject the disturbance or noise signals
4. Reduce the steady-state error

4.6 Advantage And Disadvantage

□ Disadvantage

1. increased number of components and complexity
   – sensor is often expensive and introduces noise and inaccuracies into the system
2. Loss of gain
   – single-loop system, the open-loop gain is $G(s)$, and is reduced to $G(s)/(1+ G(s))$ in a unity negative feedback system
3. Increase the possibility of instability.
   – Whereas the open-loop system is stable, the closed-loop system may not be always stable.