2.5 Poles and Zeros

Characteristic equation

Transfer function is $G(s)=\dfrac{p(s)}{q(s)}$ .The denominator polynomial $q(s)$, $q(s)=0$ is called the characteristic equation.

characteristic equation：

$$\boldsymbol{q(s)=0}$$

Poles: The roots of the characteristic equation $q(s)=0$.

Zeros: The roots of the numerator polynomic $p(s)=0$.

2.5 Poles and Zeros

Note:

- **The poles** determine **the character of the time response** and **the stability of the system**.
- **The zeros** do not affect the free modes.
- The weight of the modes may be affected by the zeros, especially when the zeros are located near the poles.

2.6 Basic terms

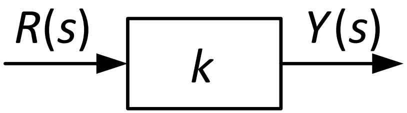
1.Proportional element

Model: $y(t)=k\cdot r(t)$, where k is a constant.

Transfer Function:

$$G(s)=\frac{Y(s)}{R(s)}=k$$

Character:

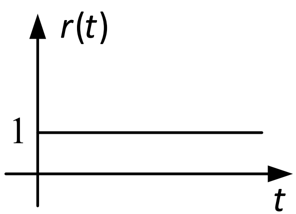


Note:

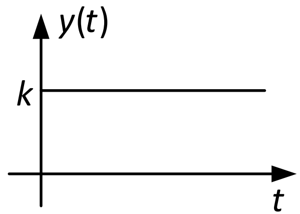
① The output is proportional to the input.

② fast response.

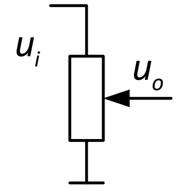
Input:

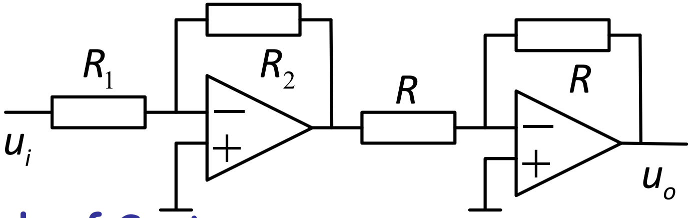


Output:



Physical devices





2.6 Basic terms

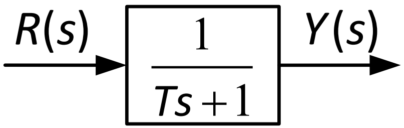
2.Inertial element

Model: $T\dfrac{dy(t)}{dt}+y(t)=r(t)$

Transfer Function:

$$G(s)=\frac{Y(s)}{R(s)}=\frac{1}{Ts+1}$$

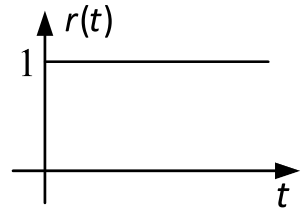
Character:



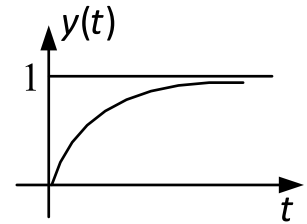
Note:

T: time constant.

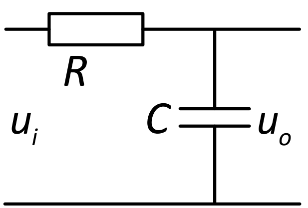
Input:

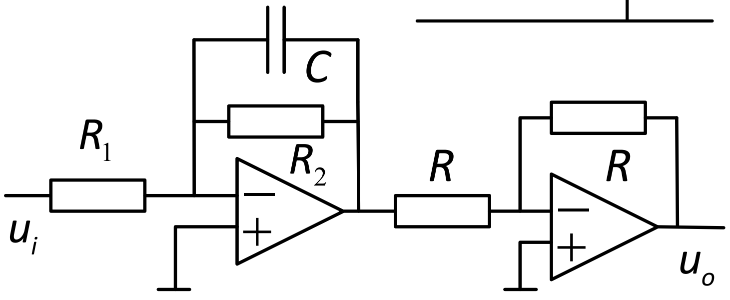


Output:



Physical devices





2.6 Basic terms

3.Integral element

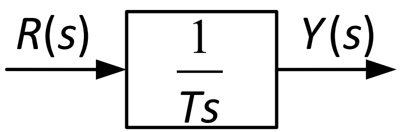
Model: $$T\frac{dy(t)}{dt}=r(t)$$

$$y(t)=\frac{1}{T}\int r(t)dt$$

Transfer Function:

$$G(s)=\frac{Y(s)}{R(s)}=\frac{1}{Ts}$$

Character:

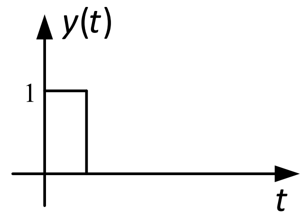


Note:

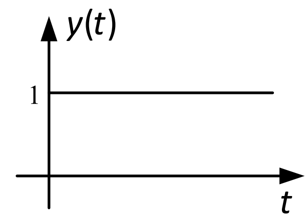
①T: time constant

②Memory facility

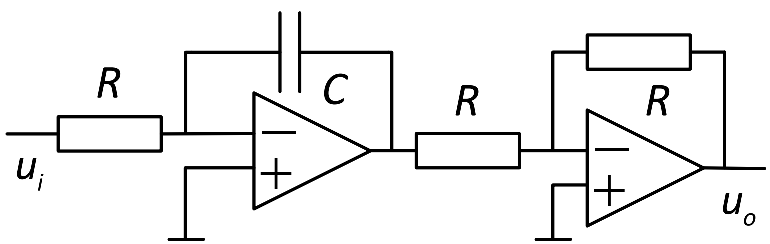
Input:



Output:



Physical devices



2.6 Basic terms

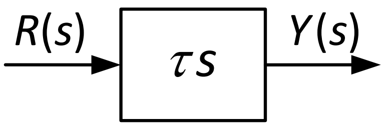
4.Differential element

Model: $y(t)=\tau\dfrac{dr(t)}{dt}$

Transfer Function:

$$G(s)=\frac{Y(s)}{R(s)}=\tau s$$

Character:

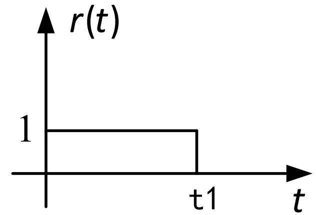


Note:

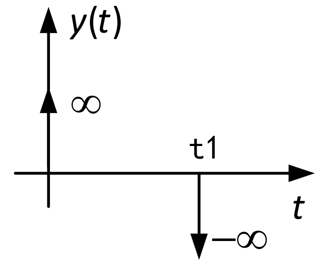
① τ : time constant

②y(t) shows the trend of r(t)

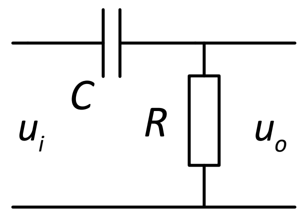
Input:

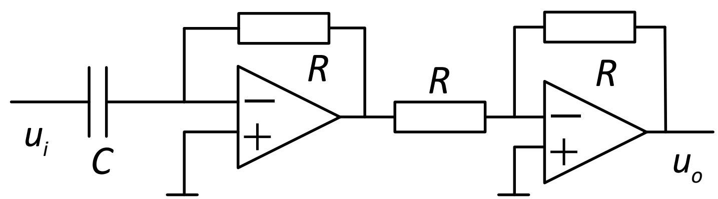


Output:



Physical devices





2.6 Basic terms

5.Oscillating element

Model:

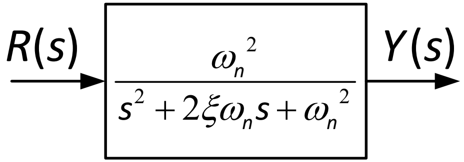
$$T^2\frac{d^2y(t)}{dt^2}+2\xi T\frac{dy(t)}{dt}+y(t)=r(t)$$

Transfer Function:

$$G(s)=\frac{Y(s)}{R(s)}=\frac{1}{T^2s^2+2\xi Ts+1}$$

$$=\frac{\omega_n^{\ 2}}{s^2+2\xi\omega_n s+\omega_n^{\ 2}}$$

Character:

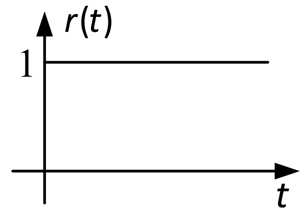


Note: $\omega_n=\frac{1}{T}$,Natural frequency

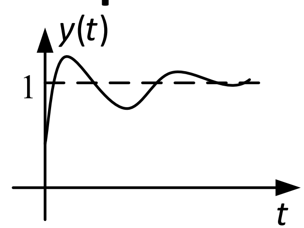
ξ ,Damping ratio

$$s_{1,2}=\begin{cases}-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1} & \xi\geq 1\\ -\xi\omega_n\pm j\omega_n\sqrt{\xi^2-1} & 0<\xi<1\end{cases}$$

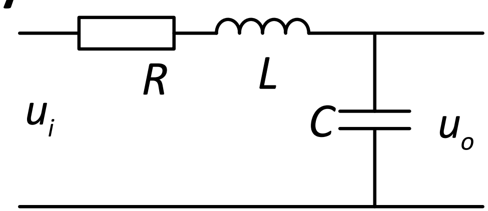
Input:



Output:



Physical devices



2.6 Basic terms

6.Dalay element

Model:

$$r(t) = f(t)$$

$$y(t) = f(t-\tau)$$

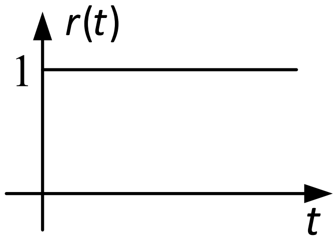
Transfer Function:

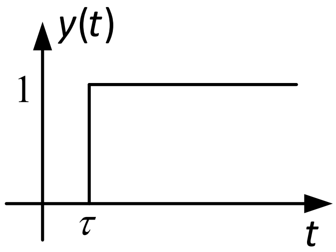
$$G(s) = \frac{Y(s)}{R(s)} = e^{-\tau s}$$

Character:

$$R(s) \longrightarrow \boxed{e^{-\tau s}} \longrightarrow Y(s)$$

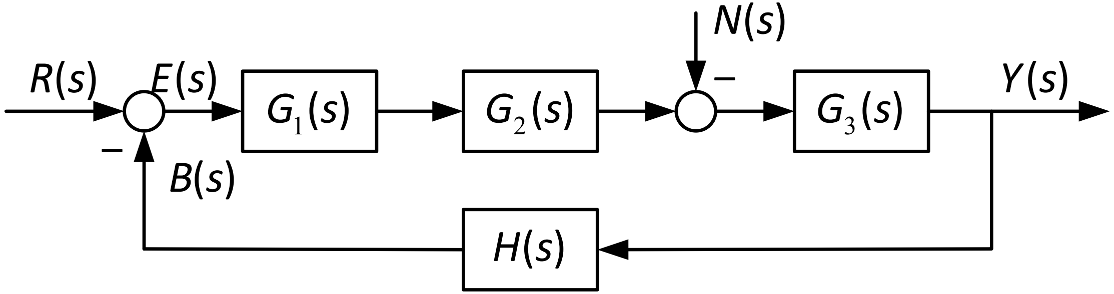
Output: (unit step Input)





2.7 Block diagram

Function: Representing the relationship of system variables by diagrammatic means.



Variables

$R(s)$: Input signal

$Y(s)$: Output signal

$N(s)$: Disturbance signal

$B(s)$: Feedback signal

$E(s)$: Error signal

path

forward path: $R(s) \rightarrow Y(s)$

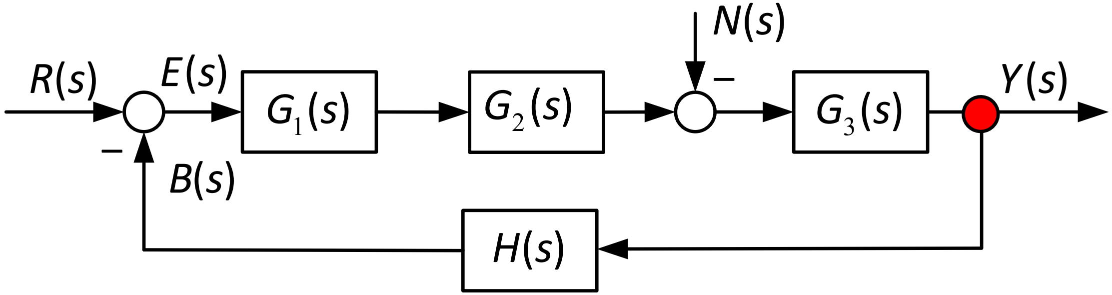
feedback path: $Y(s) \rightarrow B(s)$

Arrow

show the direction of the flow of the signal.

2.7 Block diagram

Function: Representing the relationship of system by diagrammatic means.

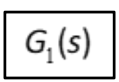


Block

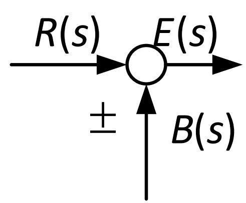
Point

Block

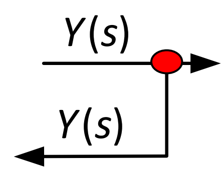
Transfer function of each component



Summing point

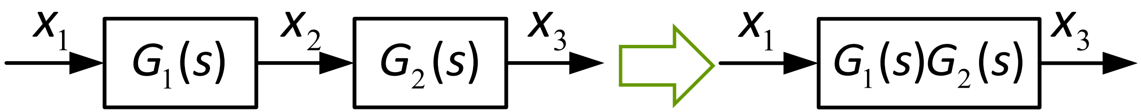


Pickoff point



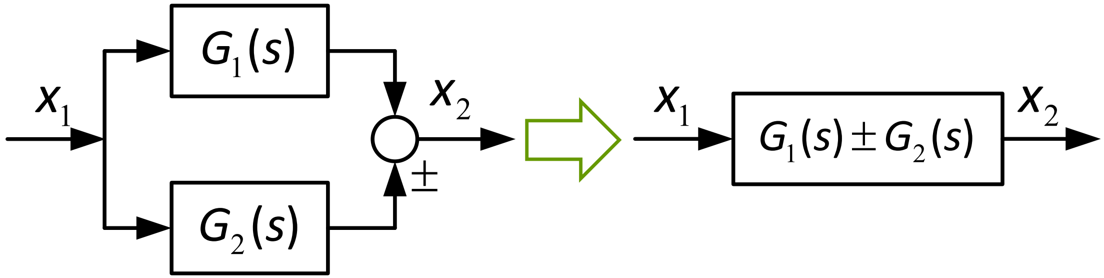
2.7 Block diagram reduction

- Combining blocks in cascade



$$x_3 = G_1(s)G_2(s)x_1$$

- Combining blocks in parallel

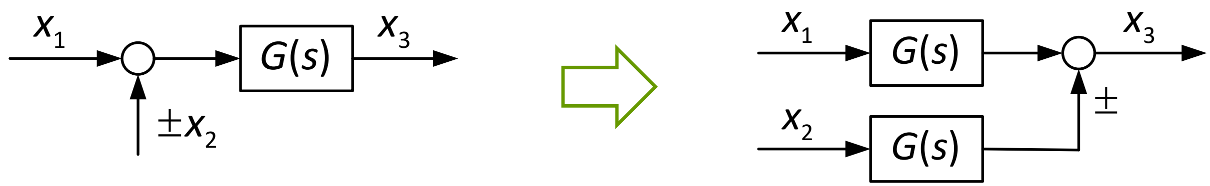


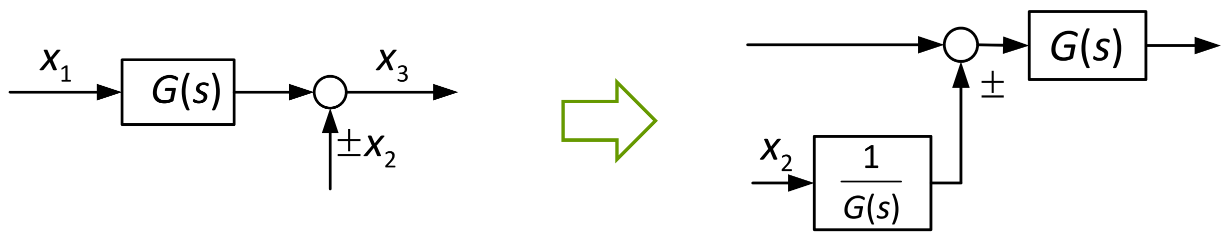
$$x_2 = [G_1(s) \pm G_2(s)]x_1$$

2.7 Block diagram reduction

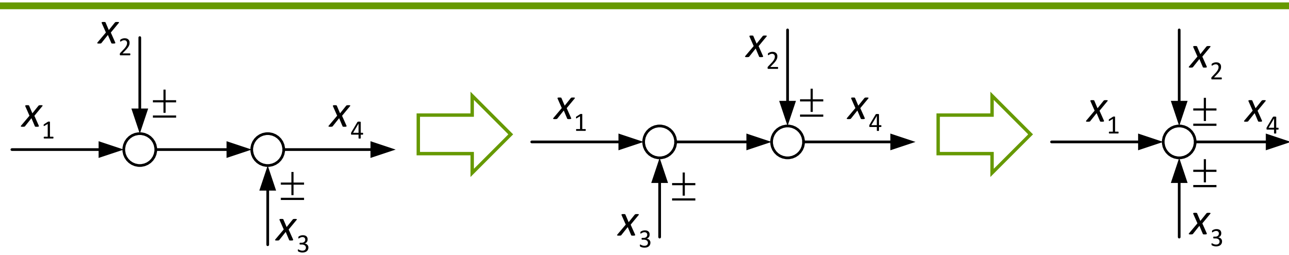
Moving a summing point

A.





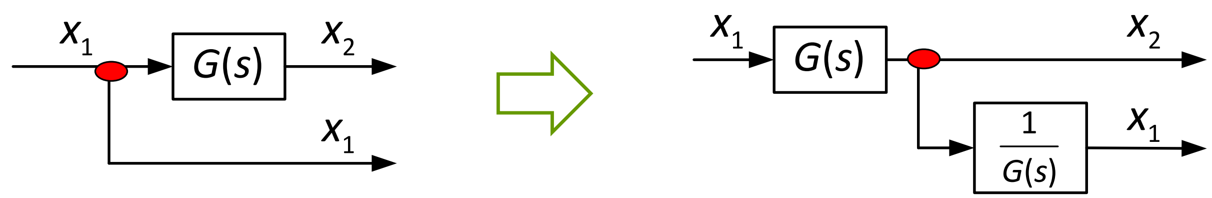
C.



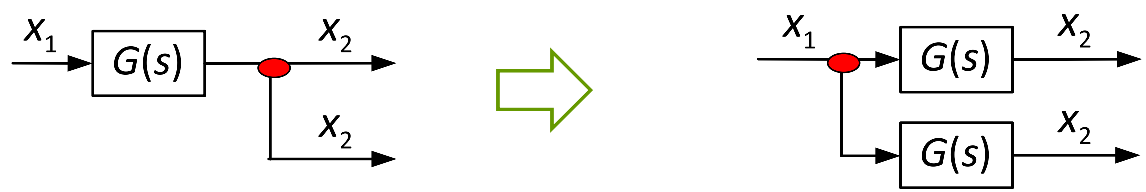
2.7 Block diagram reduction

Moving a pickoff point

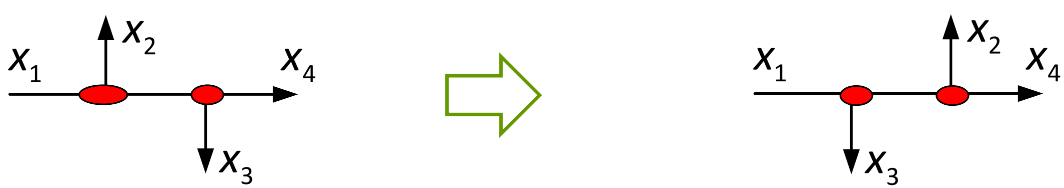
A.



B.

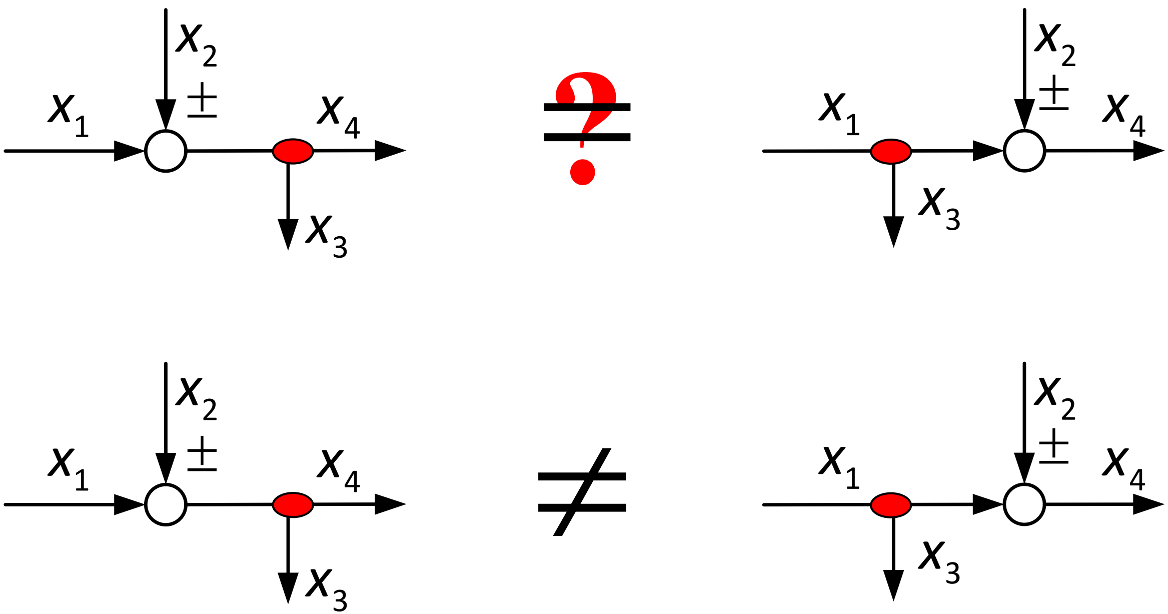


C.



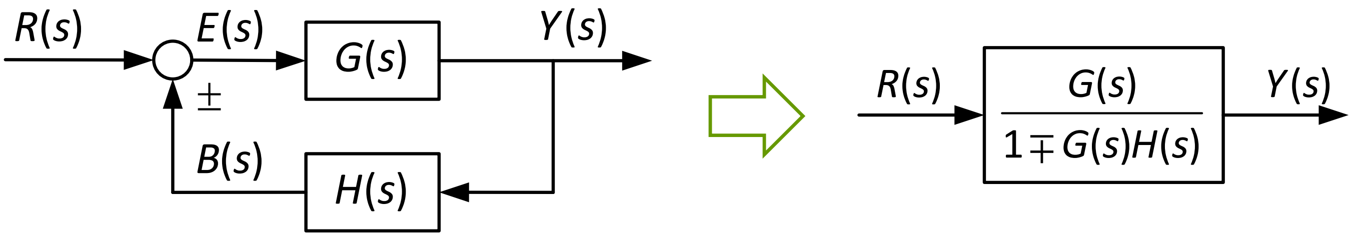
2.7 Block diagram reduction

- Moving a summing point



2.7 Block diagram reduction

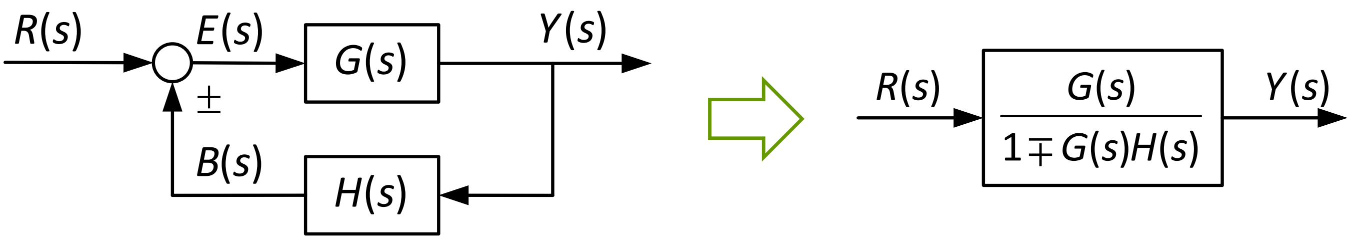
- Eliminating a feedback loop



$$G_{close}(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$

2.7 Block diagram reduction

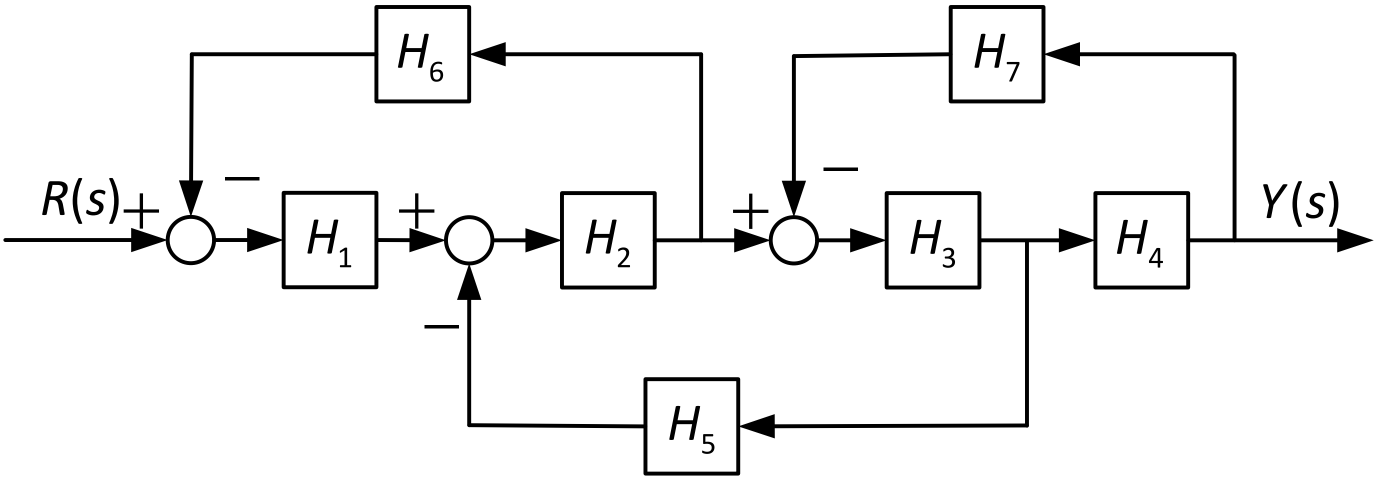
- Definitions



- The closed-loop transfer function $$G_{close}(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$
- Characteristic equation $$1 \mp G(s)H(s) = 0$$
- The open-loop transfer function $$G_{open}(s) = \frac{B(s)}{E(s)} = G(s)H(s)$$
- Error transfer function $$G_{error}(s) = \frac{E(s)}{R(s)} = \frac{1}{1 \mp G(s)H(s)}$$
- Forward transfer function $$\frac{Y(s)}{E(s)} = G(s)$$

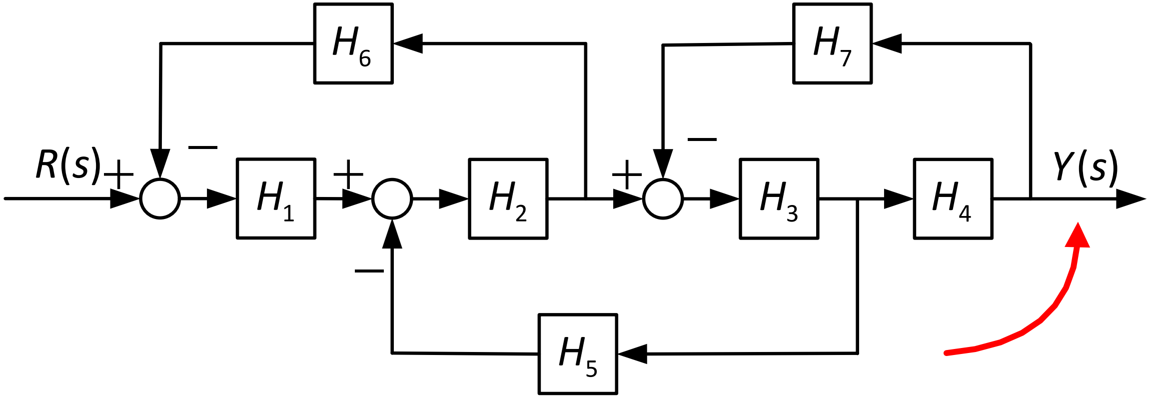
2.7 Block diagram reduction

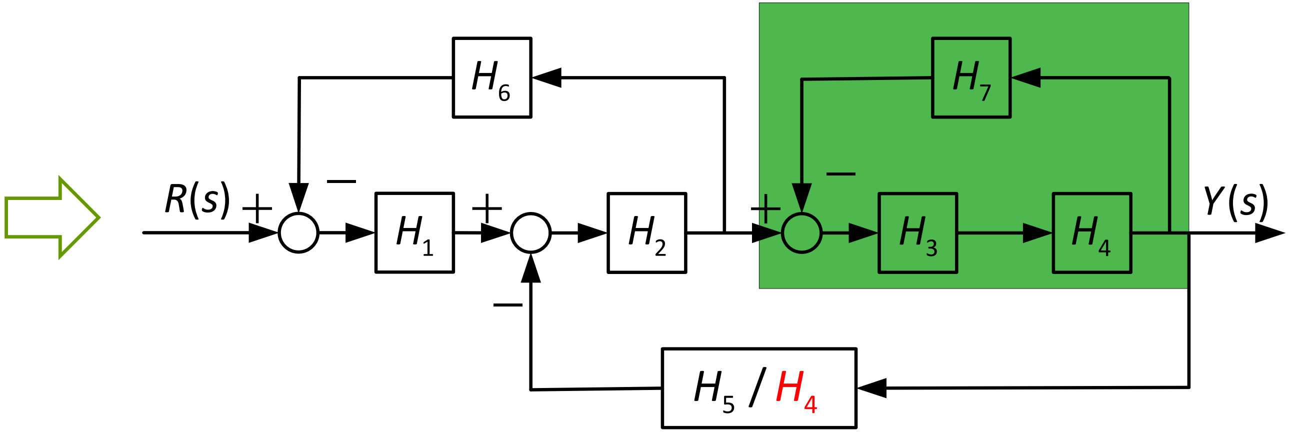
- Example



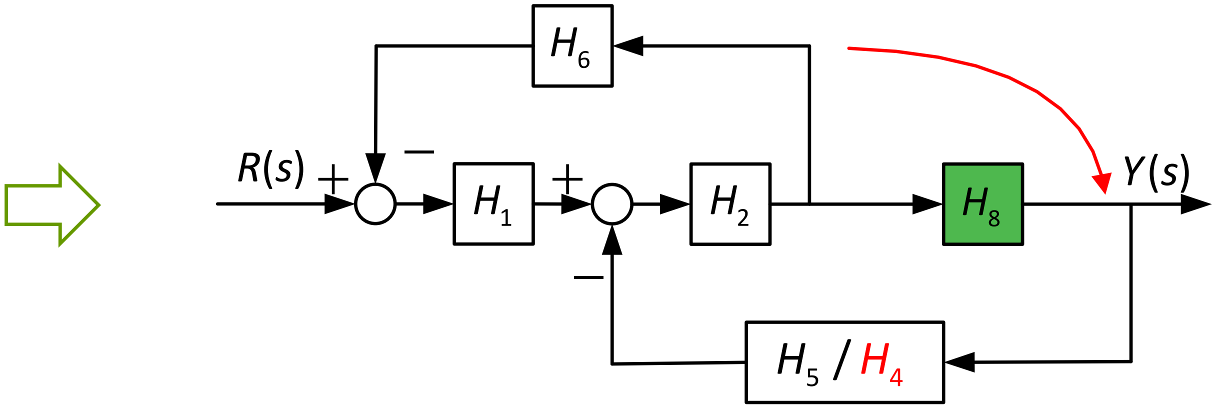
Exercise: Do it by yourself

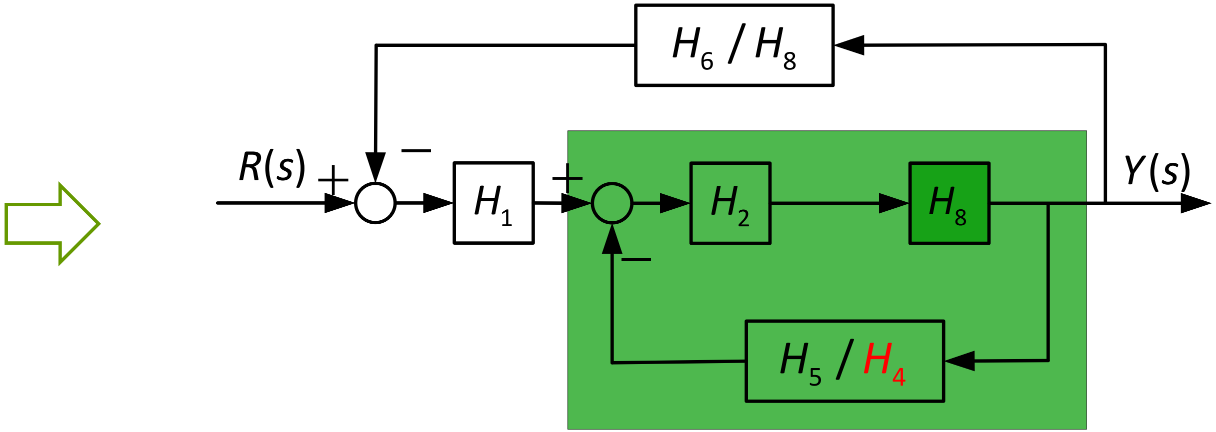
2.7 Block diagram reduction



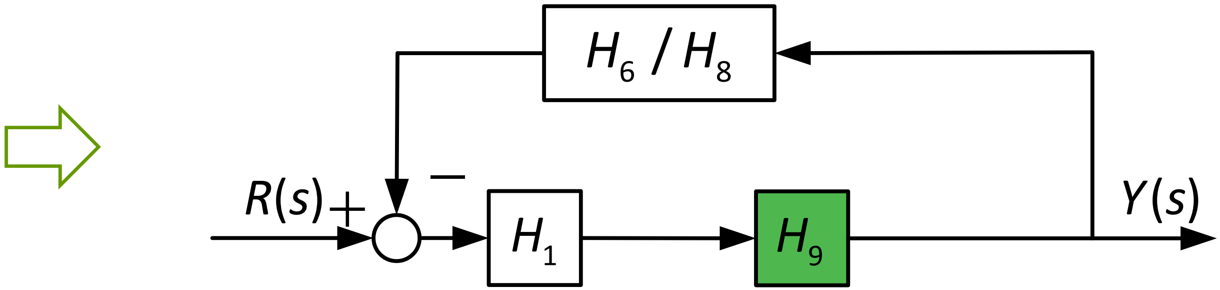


2.7 Block diagram reduction



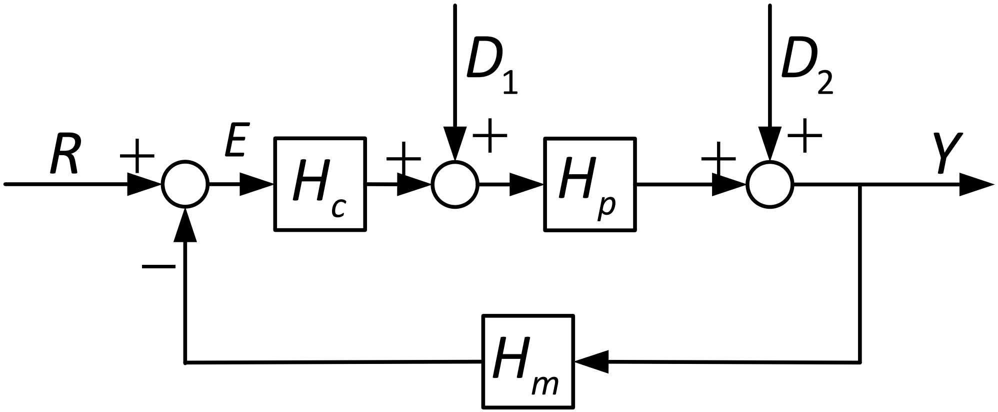


2.7 Block diagram reduction



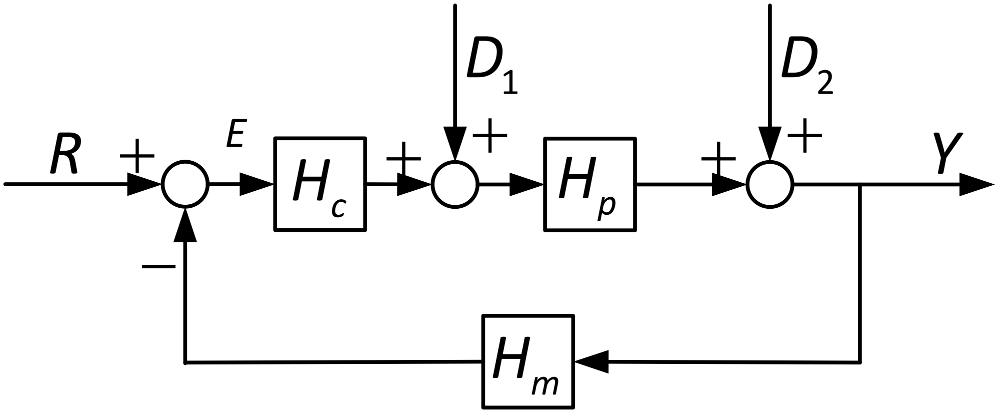
$$G_{close}(s)=\frac{Y(s)}{U(s)}=\frac{H_1H_9}{1+H_1H_9H_6/H_8}$$

2.7 Block diagram reduction



$$\frac{Y}{R} \quad \frac{E}{R} \quad \frac{Y}{D_1} \quad \frac{Y}{D_2}$$

2.7 Block diagram reduction

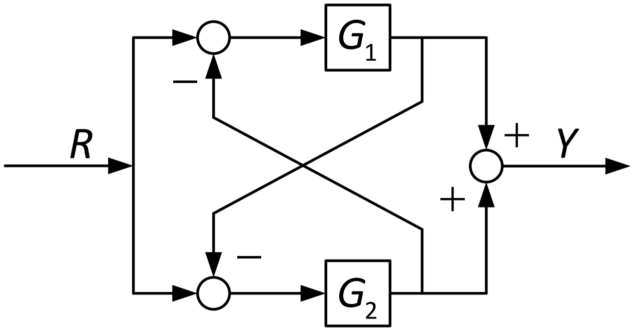


$$\frac{Y}{R} = \frac{H_c H_p}{1 + H_c H_p H_m} \qquad \frac{Y}{D_1} = \frac{H_p}{1 + H_c H_p H_m}$$

$$\frac{E}{R} = \frac{1}{1 + H_c H_p H_m} \qquad \frac{Y}{D_2} = \frac{1}{1 + H_c H_p H_m}$$

2.7 Black diagram reduction

- **Example**

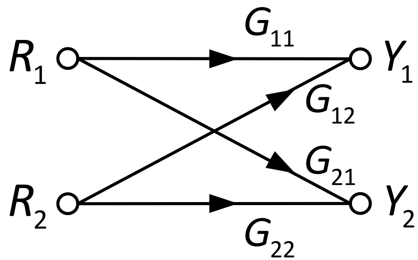


$$\frac{Y(s)}{R(s)} = \frac{G_1 + G_2 - 2G_1G_2}{1 - G_1G_2}$$

2.8 Signal Flow Graph

☐ Concept:

The signal-flow graph is the graph representation of the simultaneous equations, and is a diagram consisting of nodes that are connected by several directed branches and is a graphical representation of a set of linear relations.



The basic element:

① **Node:** input and output point or junctions→represent variable.

② **Branch:** a unidirectional path segment relates the input and the output.

③ **Arrow:** show the direction of the flow of the signal.

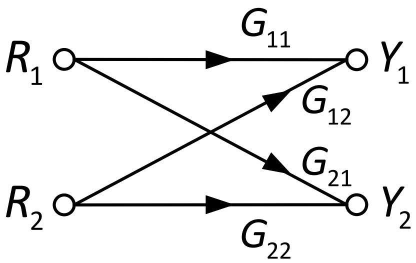
2.8 Signal Flow Graph

④ **Path:** a branch or a continuous sequence of branches that can be traversed from one signal (node) to another signal (node).

A closed path that originate and terminates on the path on the same node and along the path no node is met twice.

⑤ **Loop:** a closed path that originates and terminates on the same node, with no node being met twice along the path.

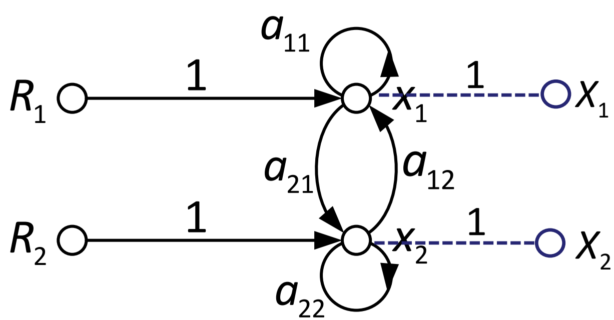
- **non-touching loop:** do not have a common node.
- **touching loop:** share one or more common node.



$$Y_1 = G_{11}R_1 + G_{12}R_2 \qquad \frac{Y_1}{R_1} = ? \quad \frac{Y_2}{R_2} = ?$$

$$Y_2 = G_{21}R_1 + G_{22}R_2$$

2.8 Signal Flow Graph



$$a_{11}x_1 + a_{12}x_2 + r_1 = x_1$$
$$a_{21}x_1 + a_{22}x_2 + r_2 = x_2$$

$\Rightarrow$

$$x_1(1-a_{11}) + x_2(-a_{12}) = r_1$$
$$x_1(-a_{21}) + x_2(1-a_{22}) = r_2$$

$\Rightarrow$

$$x_1 = \frac{(1-a_{22})r_1 + a_{12}r_2}{(1-a_{11})(1-a_{22}) - a_{12}a_{21}} = \frac{(1-a_{22})}{\Delta}r_1 + \frac{a_{12}}{\Delta}r_2$$

$$x_2 = \frac{(1-a_{11})r_2 + a_{21}r_1}{(1-a_{11})(1-a_{22}) - a_{12}a_{21}} = \frac{(1-a_{11})}{\Delta}r_1 + \frac{a_{21}}{\Delta}r_2$$

$$\Delta = (1-a_{11})(1-a_{22}) - a_{12}a_{21} = 1 - a_{11} - a_{22} + a_{11}a_{22} - a_{12}a_{21}$$

2.8 Signal Flow Graph

□ Mason's signal-flow gain formula

$$\frac{Y(s)}{R(s)} = T_{ij} = \frac{\Sigma_k P_{ijk}\Delta_{ijk}}{\Delta} = \frac{\Sigma_k P_k \Delta_k}{\Delta}$$

增益

P_{ijk} = gain of *k*th path from variable x_i to variable x_j.

Δ = determinant of the graph. 特征式

Δ_{ijk} = cofactor of the path P_{ijk}.

$$\Delta = 1 - \sum L_n + \sum L_m L_q - \sum L_r L_s L_t + \cdots$$

$\sum L_n$: Sum of all different loop gains.

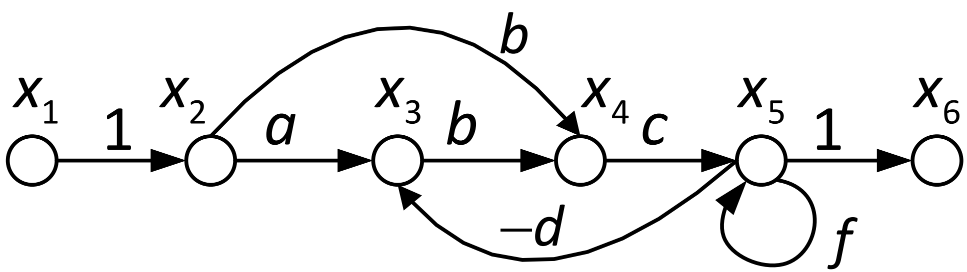
$\sum L_m L_q$: Sum of the gain products of all combinations of two non-touching loops.

$\sum L_r L_s L_t$: Sum of the gain products of all combinations of three – non-touching loops. 互不接触的3个

2.8 Signal Flow Graph

☐ Example:

A signal flow graph is a diagram consisting of nodes that are connected by several directed branches and is a graphical representation of a set of linear relations.

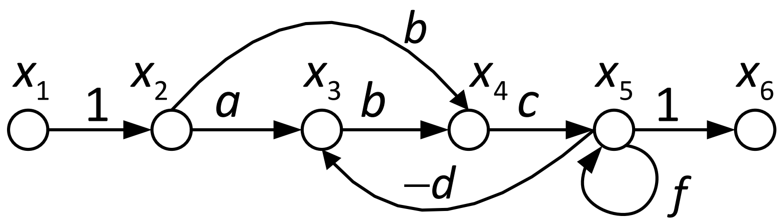


The basic element:

① **Nodes:** input and output point or junctions→represent variable.

② **Branch:** a unidirectional path segment relates the input and the output.

2.8 Signal Flow Graph



The signal-flow graph is the graph representation of the simultaneous equations.

$$x_2 = x_1 \qquad x_3 = ax_2 - dx_5 \qquad x_4 = bx_2 + bx_3$$

$$x_5 = cx_4 + fx_5 \qquad x_6 = x_5$$

③ **path:** a branch or a continuous sequence of branches that can be traversed from one signal (node) to another signal (node).

④ **Loop:** a closed path that originates and terminates on the same node, with no node being met twice along the path.

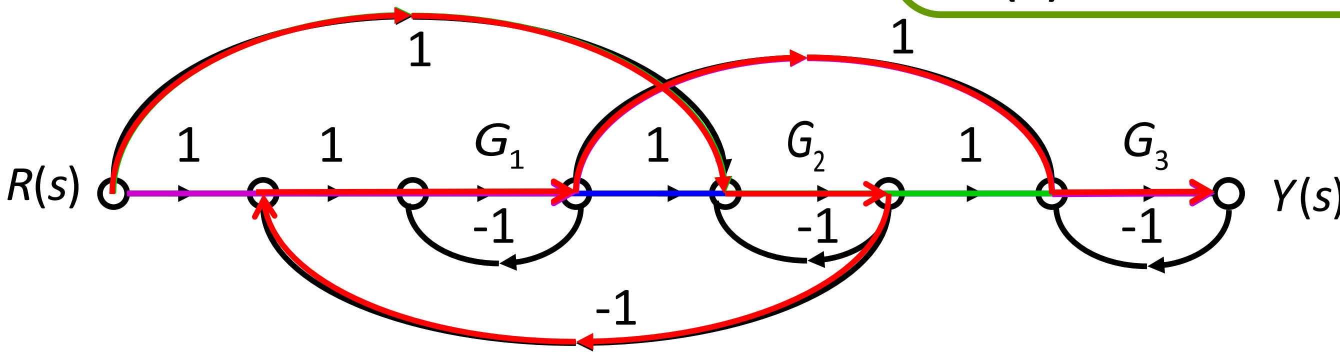
non-touching loop: do not have a common node.

touching loop: share one or more common node.

2.8 Signal Flow

$$\frac{Y(s)}{R(s)}=\frac{\sum_{k=1}^{n}P_k\Delta k}{\Delta}$$

□ Example:



$$P_1=G_1G_2G_3,\ P_2=G_2G_3,\ P_3=G_1G_3,\ P_4=-G_1G_2G_3$$

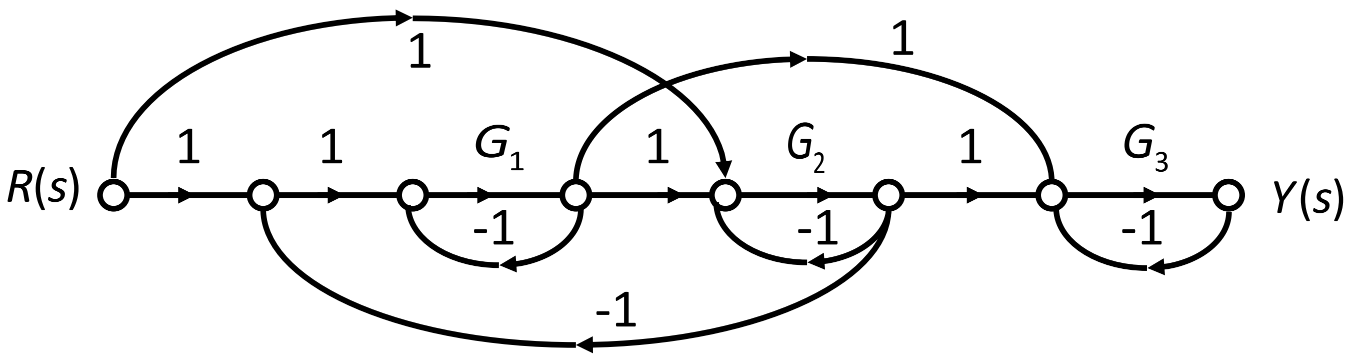
$$\sum L_n=-G_1-G_2-G_3-G_1G_2 \qquad \sum L_rL_sL_t=(-G_1)(-G_2)(-G_3)$$

$$\sum L_mL_q=(-G_1)(-G_2)+(-G_1)(-G_3)+(-G_2)(-G_3)+(-G_1G_2)(-G_3)$$

$$\Delta=1-\sum L_n+\sum L_mL_q-\sum L_rL_sL_t$$

2.8 Signal Flow Graph

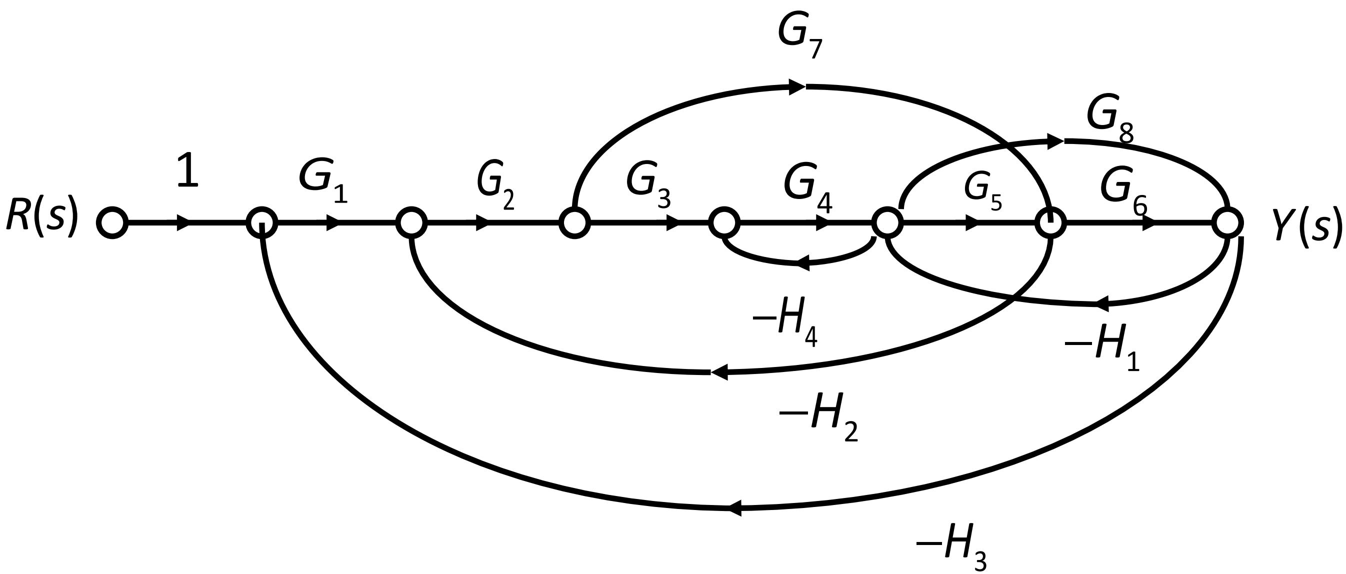
Answer



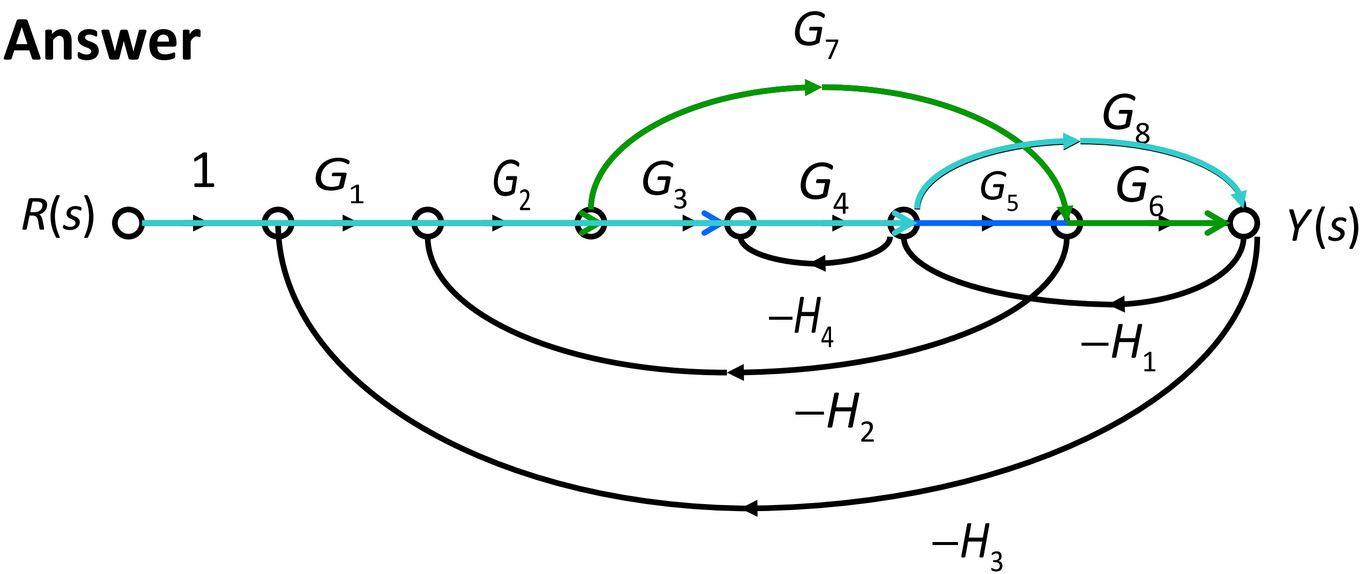
$$\frac{Y(s)}{R(s)} = \frac{\sum_{k=1}^{n} P_k \Delta_k}{\Delta} = \frac{2G_1G_2G_3 + G_2G_3 + G_1G_3}{1 + G_1 + G_2 + G_3 + 2G_1G_2 + G_1G_3 + G_2G_3 + 2G_1G_2G_3}$$

2.8 Signal Flow Graph

Example



- **Answer**



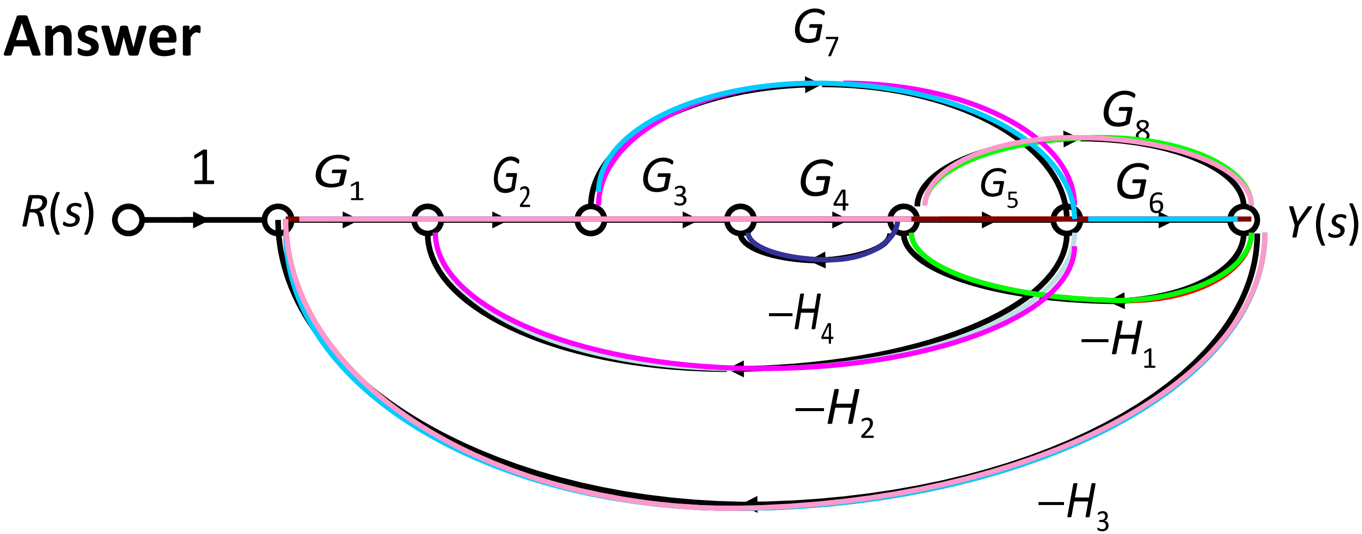
$$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$$

$$P_2 = G_1 G_2 G_7 G_6$$

$$P_3 = G_1 G_2 G_3 G_4 G_8$$

2.8 Signal Flow Graph

- **Answer**



$$L_1 = -G_2G_3G_4G_5H_2 \qquad L_5 = -G_4H_4$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4)$$

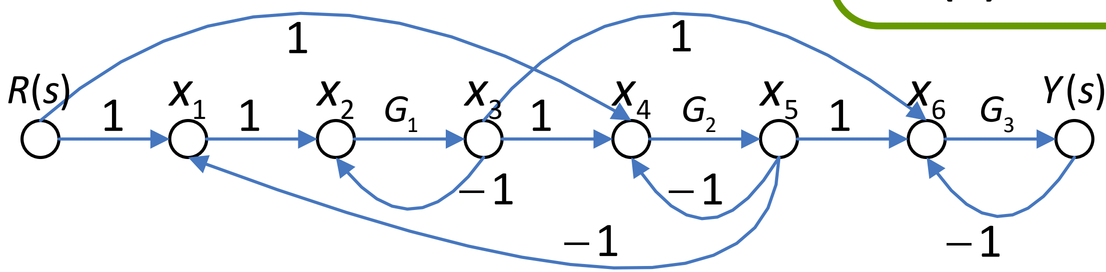
$$L_3 = -G_8H_1 \qquad L_7 = -G_1G_2G_7G_6H_3$$

$$L_4 = -G_7H_2G_2 \qquad L_8 = -G_1G_2G_3G_4G_8H_3$$

2.8 Signal Flow

$$\frac{Y(s)}{R(s)} = \frac{\sum_{k=1}^{n} P_k \Delta_k}{\Delta}$$

□ Mason's signal-flow gain formula



1. ***n***: The number of all possible paths from $R(s)$ to $Y(s)$ (no node encountered more than once).

$$n=4$$

2. $\boldsymbol{P_k}$: The k-th path gain

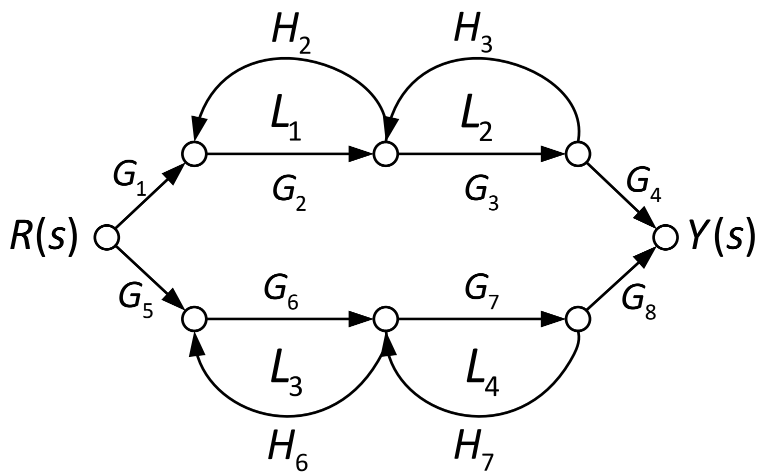
$$P_1 = G_1G_2G_3,\ P_2 = G_2G_3,\ P_3 = G_1G_3,\ P_4 = -G_1G_2G_3$$

3. Δ : Determinant of the graph

$$\Delta = 1 - \sum L_n + \sum L_m L_q - \sum L_r L_s L_t + \cdots$$

2.8 Signal Flow Graph

Example



Answer

$$k=2, P_1=G_1G_2G_3G_4, P_2=G_5G_6G_7G_8$$

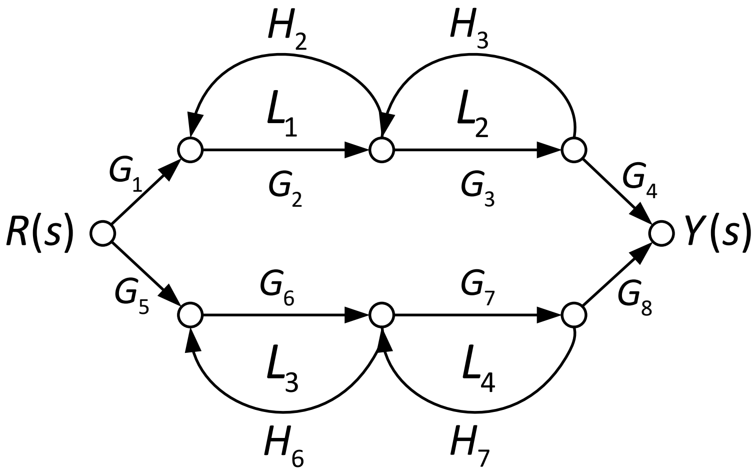
$$N=4, L_1=G_2H_1, L_2=G_3H_2, L_3=G_6H_3, L_4=G_7H_4$$

$$\Delta_1=1-(L_3+L_4), \Delta_2=1-(L_2+L_1)$$

$$\Delta=1-(L_1+L_2+L_3+L_4)+(L_1L_3+L_1L_4+L_2L_3+L_2L_4)$$

2.8 Signal Flow Graph

Example

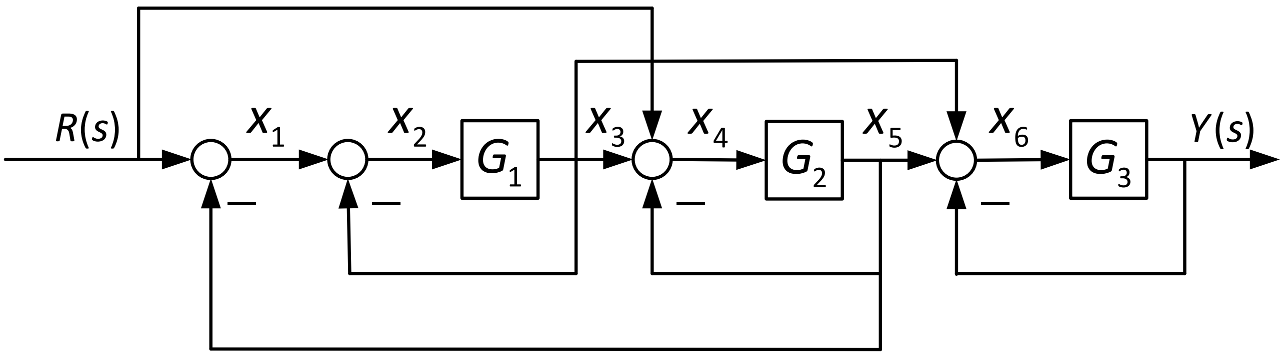


Answer

$$\frac{Y(s)}{R(s)} = T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

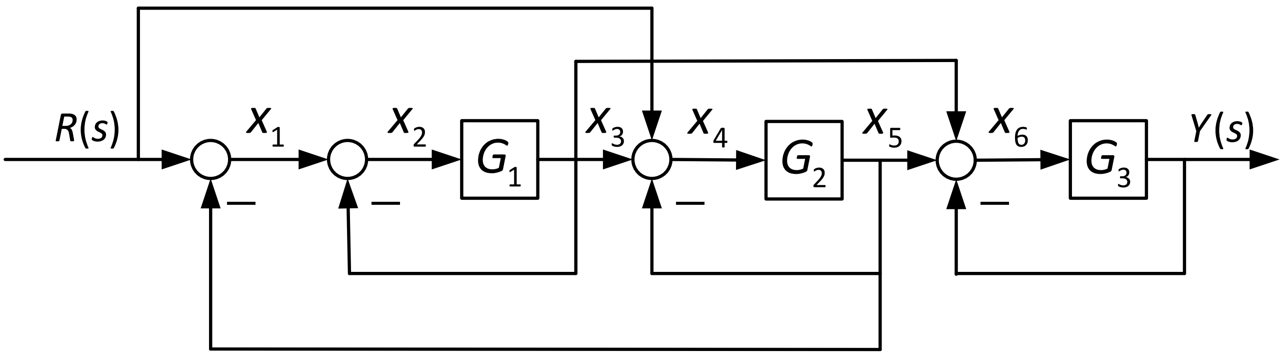
$$= \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$

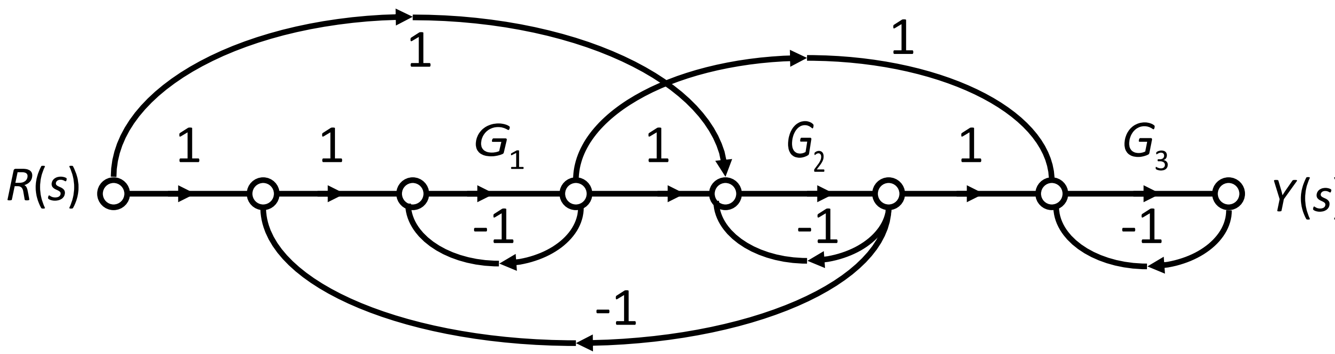
2.8 Signal Flow Graph



Exercise: Do it yourself

2.8 Signal Flow Graph





2.8 Signal Flow Graph

$\sum L_n$: Sum of all different loop gains

$$\sum L_n = -G_1 - G_2 - G_3 - G_1G_2$$

$\sum L_m L_q$: Sum of the gain products of all combinations of two non-touching loops.

$$\sum L_m L_q = (-G_1)(-G_2) + (-G_1)(-G_3) + (-G_2)(-G_3) + (-G_1G_2)(-G_3)$$

$\sum L_r L_s L_t$: Sum of the gain products of all combinations of three nontouching loops.

$$\sum L_r L_s L_t = (-G_1)(-G_2)(-G_3)$$

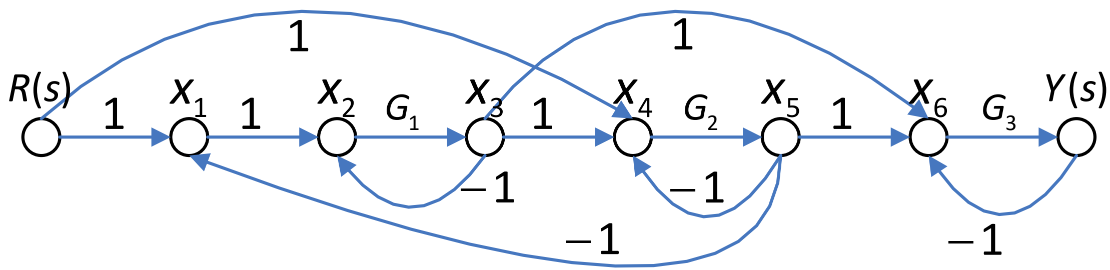
4. Δ_k : Cofactor of the paths P_k

The determinant with the loops touching the *k*th path removed.

$$\Delta_1 = 1,\ \Delta_2 = 1-(-G_1),\ \Delta_3 = 1-(-G_2),\ \Delta_4 = 1$$

2.8 Signal Flow Graph

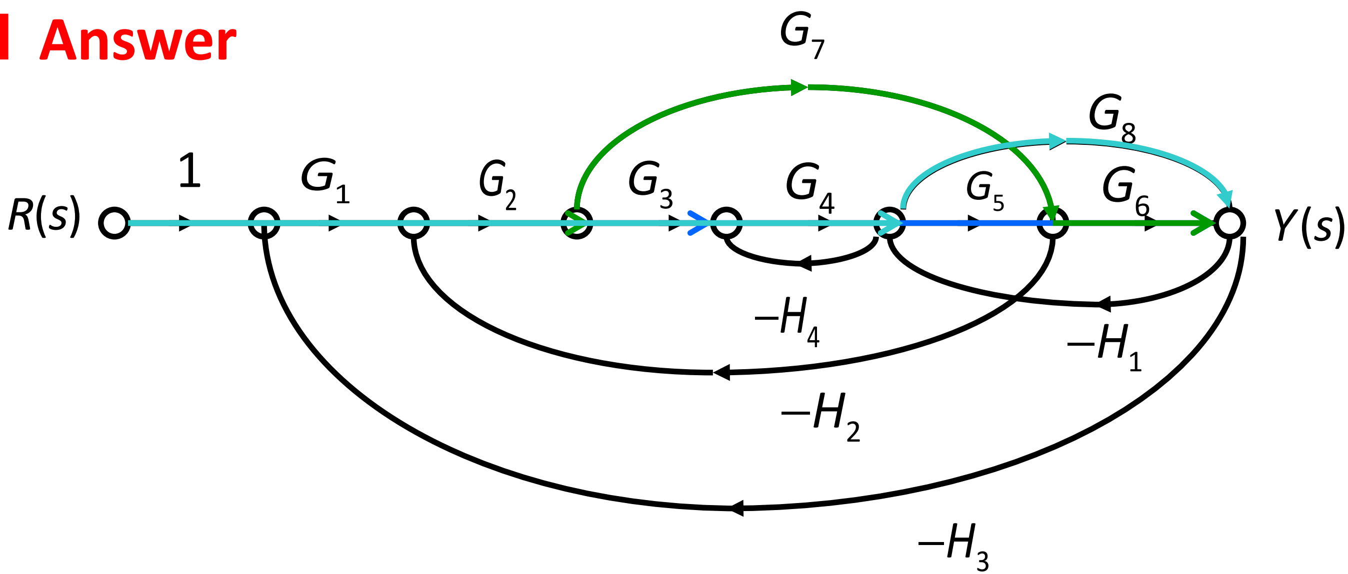
Mason's signal-flow gain formula



$$\frac{Y(s)}{R(s)} = \frac{\sum_{k=1}^{n} P_k \Delta_k}{\Delta} = \frac{2G_1G_2G_3 + G_2G_3 + G_1G_3}{1 + G_1 + G_2 + G_3 + 2G_1G_2 + G_1G_3 + G_2G_3 + 2G_1G_2G_3}$$

2.8 Signal Flow Graph

Answer



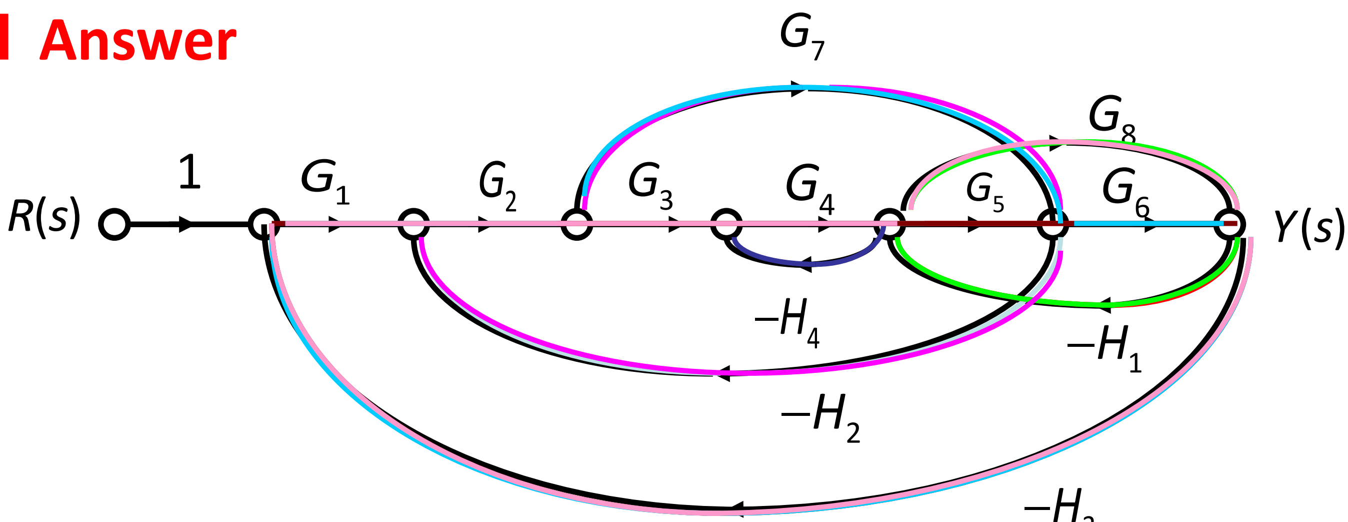
$$P_1 = G_1G_2G_3G_4G_5G_6$$

$$P_2 = G_1G_2G_7G_6$$

$$P_3 = G_1G_2G_3G_4G_8$$

2.8 Signal Flow Graph

Answer



$$L_1 = -G_2G_3G_4G_5H_2 \qquad L_5 = -G_4H_4$$

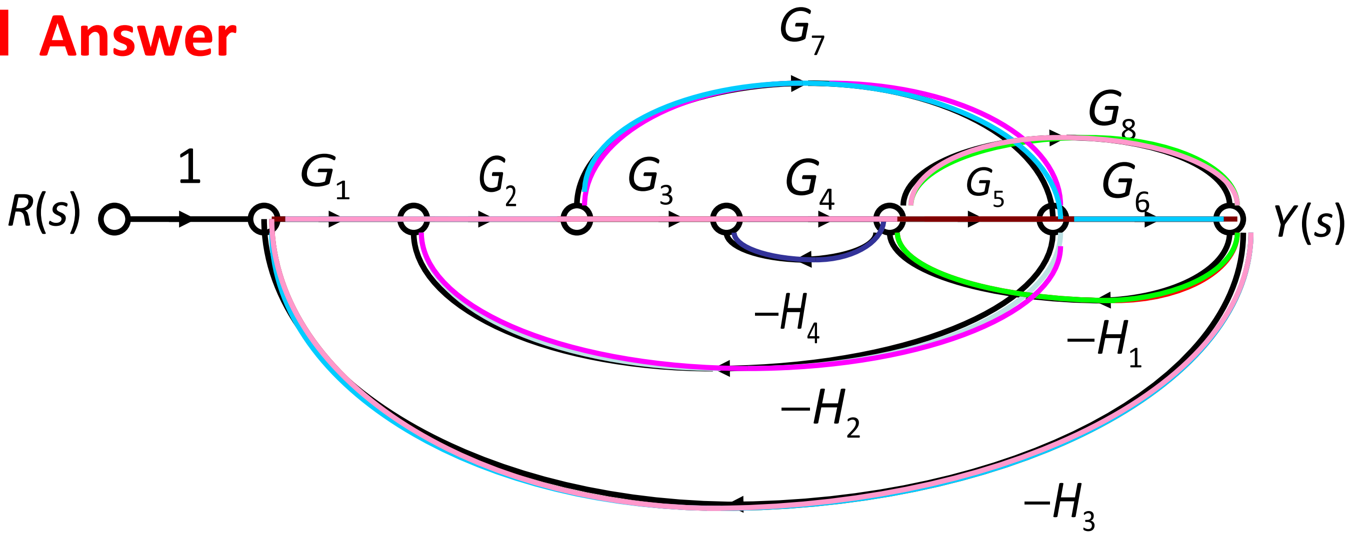
$$\Delta = 1-(L_1+L_2+L_3+L_4+L_5+L_6+L_7+L_8)+(L_5L_7+L_5L_4+L_3L_4)$$

$$L_3 = -G_8H_1 \qquad L_7 = -G_1G_2G_7G_6H_3$$

$$L_4 = -G_7H_2G_2 \qquad L_8 = -G_1G_2G_3G_4G_8H_3$$

2.8 Signal Flow Graph

Answer



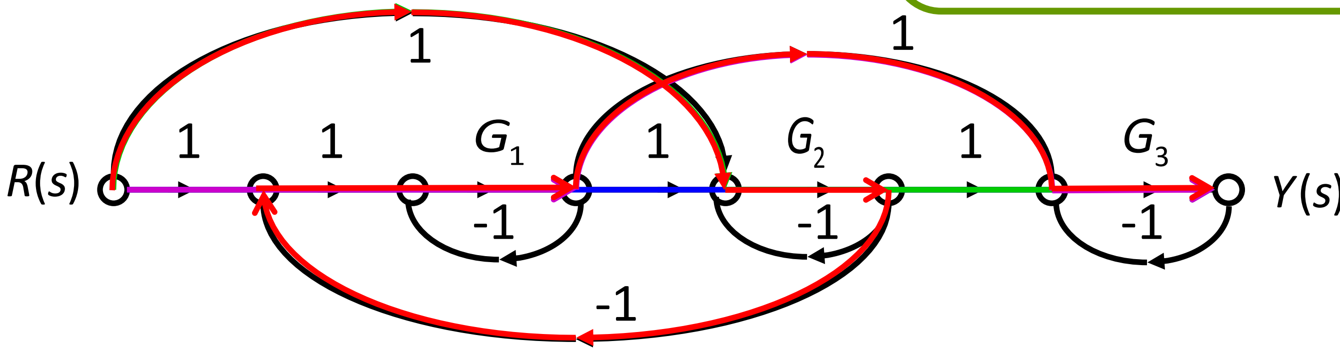
$$\Delta_1 = \Delta_3 = 1, \Delta_2 = 1 - L_5 = 1 + G_4 H_4$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}$$

2.8 Signal Flow

$$\frac{Y(s)}{R(s)} = \frac{\sum_{k=1}^{n} P_k \Delta k}{\Delta}$$

Answer



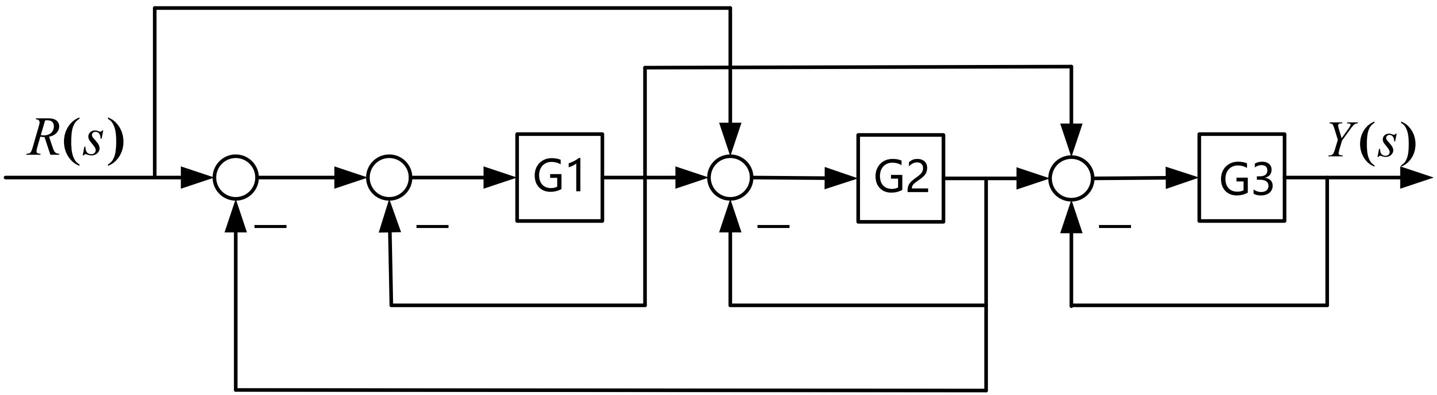
$$P_1 = G_1 G_2 G_3,\ P_2 = G_2 G_3,\ P_3 = G_1 G_3,\ P_4 = -G_1 G_2 G_3$$

$$\sum L_n = -G_1 - G_2 - G_3 - G_1 G_2 \qquad \sum L_r L_s L_t = (-G_1)(-G_2)(-G_3)$$

$$\sum L_m L_q = (-G_1)(-G_2) + (-G_1)(-G_3) + (-G_2)(-G_3) + (-G_1 G_2)(-G_3)$$

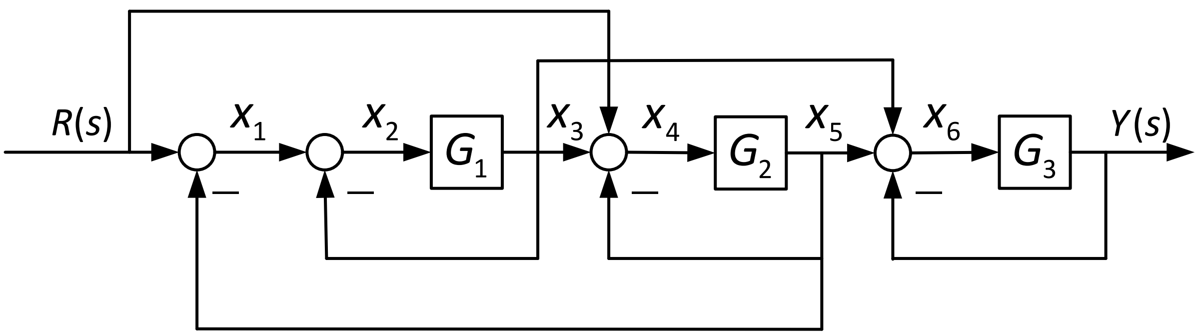
$$\Delta = 1 - \sum L_n + \sum L_m L_q - \sum L_r L_s L_t$$

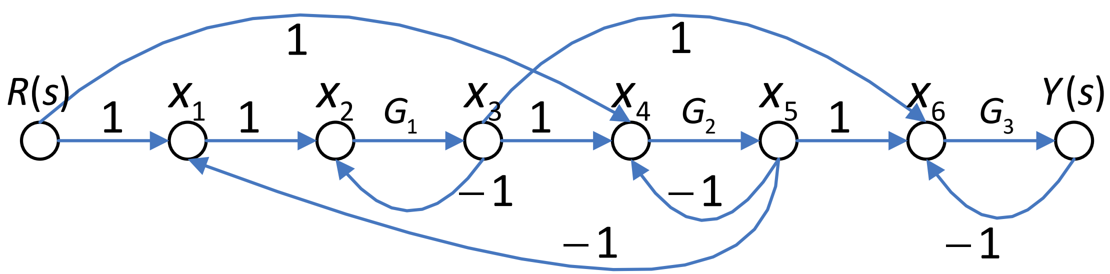
2.9 Block Diagram-Signal Flow Graph



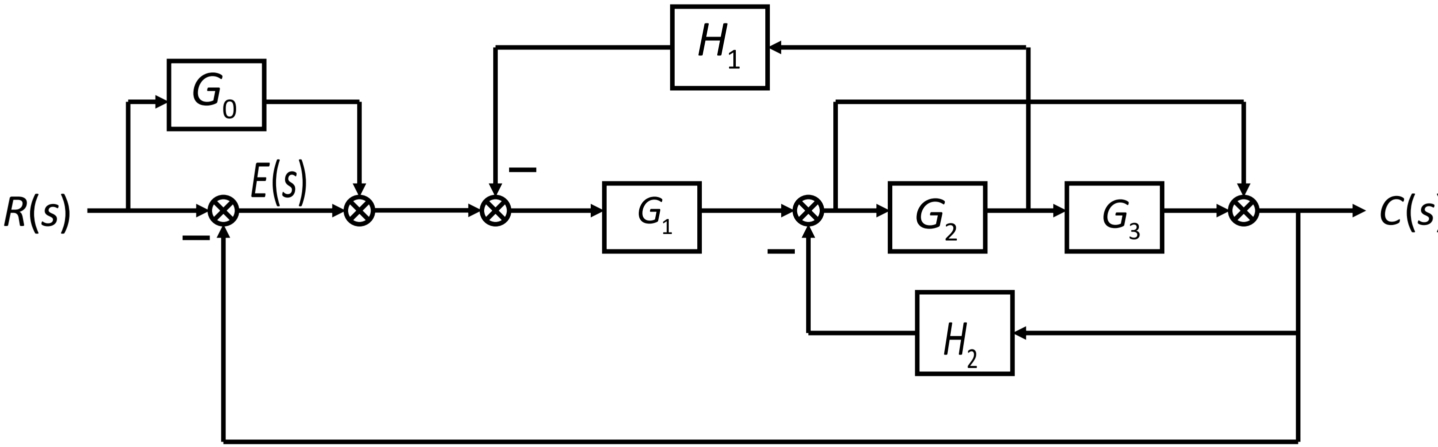
Exercise: Do it by ***yourself***

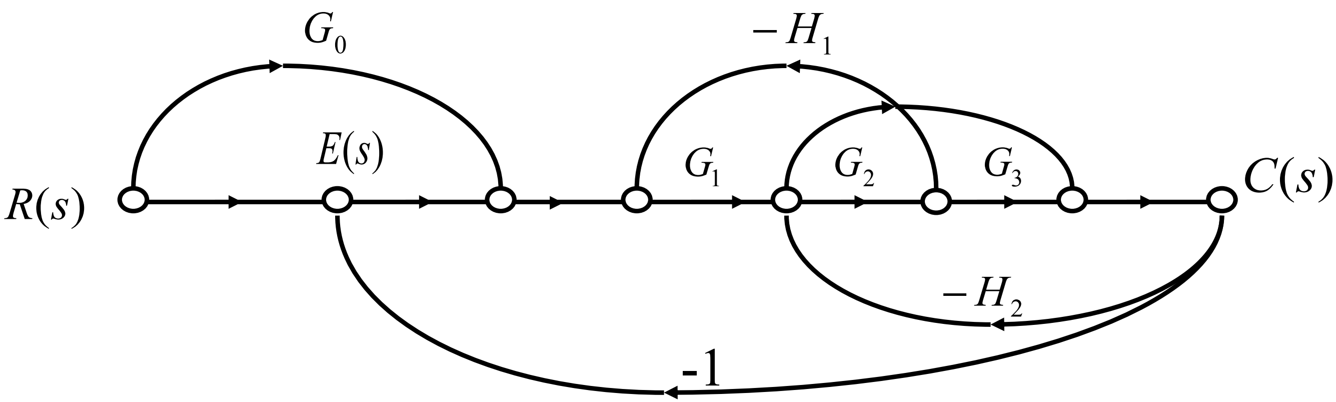
2.9 Block Diagram-Signal Flow Graph





2.9 Block Diagram-Signal Flow Graph





2.10 Summary α推

- **Establish differential equations**

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- **Transfer function**
- **Block diagram**
  - The closed-loop TF
  - The open-loop TF
  - Error TF
- **Draw signal-flow graph**
  - Mason's gain formula

小李老板