

$$E13.3 \quad y(kT) = kT$$

$$y(t) = t$$

$$Y(z) = \frac{Tz}{(z-1)^2}$$

$$E13.4 \quad Y(s) = \frac{s}{s(s+2)(s+10)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+10}$$

$$A = \frac{1}{4} \quad B = -\frac{5}{16} \quad C = \frac{1}{16}$$

$$Y(z) = \frac{1}{4} \cdot \frac{z}{(z-1)} - \frac{5}{16} \cdot \frac{z}{z-e^{-2T}} + \frac{1}{16} \cdot \frac{z}{z-e^{-10T}}$$

$$Y(z)|_{T=1} = \frac{1}{4} \frac{z}{z-1} - \frac{5}{16} \frac{z}{z-e^{-2}} + \frac{1}{16} \frac{z}{z-e^{-1}}$$

$$E13.6 \quad KG_p(s) = \frac{1-e^{-Ts}}{s} \cdot \frac{40}{s(s+2)}$$

$$G_c(j\omega) G_H(j\omega) = \frac{20(6.66j\omega+1)}{j\omega(0.5j\omega+1)(66.6j\omega+1)} \quad \text{p631 Sin.8 p840 E13.26}$$

$$G_c(s) = \frac{(6.66s+1)}{(66.6s+1)} = 0.1 \frac{s+0.15}{s+0.015}$$

$$= 0.1 + \frac{0.0135}{s+0.015}$$

$$D(z) = 0.1 + \frac{0.0135z}{z-e^{-0.015T}}$$

$$T=0.001 \quad = 0.1 + \frac{0.000135z}{z-0.999985}$$

$$= 0.1 \cdot \frac{z-0.99865}{z-0.999985}$$

E13.7

$$Y(z) = \frac{z^3 + 2z^2 + 1}{z^3 - 1.5z^2 + 0.5z}$$

$$= 1 + 3.5z^{-1} + 4.75z^{-2} + 6.375z^{-3} + \dots$$

$$1 + 3.5z^{-1} + 4.75z^{-2} + 6.375z^{-3}$$

$$z^3 - 1.5z^2 + 0.5z \overline{) z^3 + 2z^2 + 0z + 1}$$

$$z^3 - 1.5z^2 + 0.5z$$

$$3.5z^2 - 0.5z + 1$$

$$3.5z^2 - 5.25z + 1.75$$

$$4.75z - 0.75$$

$$4.75z - 7.125 + 2.375z^{-1}$$

$$6.375 - 2.375z^{-1}$$

$$Y(z) = \sum_{k=0}^{\infty} y(kT) z^{-k}$$

$$y(0) = 1 \quad y(1) = 3.5 \quad y(2) = 4.75 \quad y(3) = 6.375$$

$$E13.9 \quad (a) \quad Y(z) = \frac{z+1}{z^2-1} = z^{-1} + z^{-2} + z^{-3} + \dots$$

$$z^{-1} + z^{-2} + z^{-3}$$

$$z^2 - 1 \overline{) z + 1}$$

$$z - z^{-1}$$

$$1 + z^{-1}$$

$$1 - z^{-2}$$

$$z^{-1} + z^{-2}$$

$$Y(z) = \sum_{k=0}^{\infty} y(kT) z^{-k}$$

$$y(0) = 0, \quad y(1T) = 1, \quad y(2T) = 1, \quad y(3T) = 1$$

$$(b) \quad y(kT) = 1 - \delta(k)$$

$$k=0 \text{ off, } \delta(k) = 1 ;$$

$$k \neq 0 \text{ off, } \delta(k) = 0$$

$$E 13.12 \quad X(s) = \frac{s+1}{s^2+5s+6} = \frac{(s+1)}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = -1 \quad B = 2$$

$$X(z) = 2 \frac{z}{z-e^{-3T}} - \frac{z}{z-e^{-2T}}$$

$$\underline{T=1} \quad \frac{2z}{z-e^{-3}} - \frac{z}{z-e^{-2}}$$

$$E 13.13 \quad z^2 + (k-1.5)z + 0.5 = 0$$

$$z = \frac{w+1}{w-1}$$

$$\frac{(w+1)^2}{(w-1)^2} + (k-1.5) \frac{w+1}{w-1} + 0.5 = 0$$

$$kw^2 + w + (3-k) = 0$$

$$w^2 \quad k \quad 3-k$$

$$w^1 \quad 1$$

$$w^0 \quad 3-k$$

$$\begin{cases} 3-k > 0 \\ k > 0 \end{cases} \Rightarrow 0 < k < 3$$

$$P13.5 \quad G_p(s) = \frac{2}{s+2} \quad T=1$$

$$G_0(s) = \frac{1-e^{-Ts}}{s}$$

$$G_0 G_p(z) = (1-z^{-1}) \mathcal{Z} \left\{ \frac{2}{s(s+2)} \right\}$$

$$= \frac{z-1}{z} \cdot \frac{2 \cdot z(1-e^{-2T})}{(z-1)(z-e^{-2T})}$$

$$= \frac{2(1-e^{-2T})}{z-e^{-2T}}$$

$$G_a(z) = \frac{G_0 G_p(z)}{1 + G_0 G_p(z)} = \frac{2(1-e^{-2T})}{z - ze^{-2T} + 2} \stackrel{T=1}{=} \frac{2(1-e^{-2})}{z + (2 - 3e^{-2})}$$

$$R(s) = \frac{1}{s} \quad R(z) = \frac{z}{z-1}$$

$$Y(z) = R(z) \cdot G_a(z)$$

$$= \frac{z}{z-1} \cdot \frac{2(1-e^{-2})}{z + (2 - 3e^{-2})}$$

$$= \frac{1.73z}{z^2 + 0.59z - 1.59}$$

$$= 1.73z^{-1} - 1.0207z^{-2} + 3.3529z^{-3} + \dots$$

$$z^2 + 0.59z - 1.59 \overline{) \begin{array}{r} 1.73z^{-1} - 1.0207z^{-2} + 3.3529z^{-3} \\ 1.73z + 1.0207 - 2.7507z^{-1} \\ \hline -1.0207 + 2.7507z^{-1} \\ -1.0207 - 0.6022z^{-1} + 1.623 \\ \hline 3.3529z^{-1} \end{array}}$$

$$Y(z) = \sum_{k \rightarrow \infty} y(kT) z^{-k}$$

$$y(0) = 0 \quad y(T) = 1.73, \quad y(2T) = -1.0207$$

$$y(3T) = 2.3529$$

P13.6 $\lim_{k \rightarrow \infty} y(kT) = \lim_{z \rightarrow \infty} Y(z) = 0$

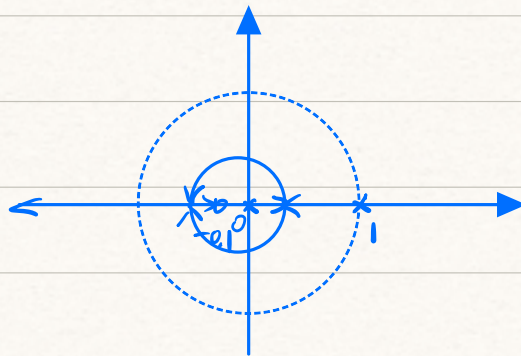
$$\lim_{z \rightarrow 1} (1-z) Y(z) = \lim_{z \rightarrow 1} \left(\frac{z-1}{z} \cdot \frac{z}{z-1} \cdot \frac{1.73}{z+1.59} \right)$$

$$= 0.67$$

P13.12 $G(z) = \frac{K(z+0.1)}{z(z-1)}$

① $n=2 \quad p_1=0 \quad p_2=1$

$m=1 \quad z_1 = -0.1$



② $\sigma = \frac{0+1+0.1}{2-1} = 1.1$

$\varphi = \frac{(2k+1)\pi}{2} = \pi, k \rightarrow \infty$

③ $\frac{1}{d} + \frac{1}{d-1} = \frac{1}{d+a_1}$

$$d^2 + a_2 d - a_1 = 0$$

$$d_1 = 0.23 \quad d_2 = -0.43$$

$$\frac{(2k+1)\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$④ \quad 1 + G(s) = 0$$

$$z^2 + (k-1)z + 0.1k = 0$$

$$z = \frac{w+1}{w-1}$$

$$\left(\frac{w+1}{w-1}\right)^2 + (k-1)\frac{w+1}{w-1} + 0.1k = 0$$

$$1.1k w^2 + (2 - 0.2k)w + (k - 0.9k) = 0$$

$$w^2 \quad 1.1k \quad 2 - 0.9k$$

$$w^1 \quad 2 - 0.2k$$

$$w^0 \quad 2 - 0.9k$$

$$\begin{cases} 1.1k > 0 \\ 2 - 0.2k > 0 \\ 2 - 0.9k > 0 \end{cases} \Rightarrow 0 < k < 2.22$$

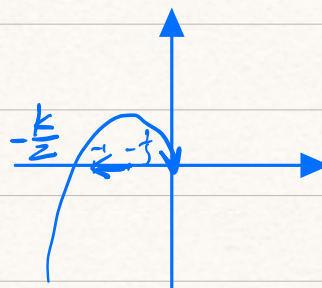
$$8-17 \quad (1) -\frac{1}{NA} = -\frac{A+2}{A+b}$$

$$G(s) = \frac{k}{s(s+1)^2}$$

$$G(j\omega) = \frac{k}{j\omega(j\omega+1)^2}$$

$$|G(j\omega)| = \frac{k}{\omega(\omega^2+1)}$$

$$\angle \theta = -\frac{\pi}{2} - 2 \arctan \omega$$



ω	0	1	5	10	$+\infty$
$ G(j\omega) $	$+\infty$	$8.5k$	$2.008k$	$0.00099k$	0

$$-0 \quad -90^\circ \quad -180^\circ \quad -247^\circ \quad -259^\circ \quad -270^\circ$$

$$-\frac{1}{N(s)} = -\frac{A+2}{A+b}$$

$$\frac{d}{dA} \left(-\frac{1}{N(s)} \right) = -\frac{A+b-A-2}{(A+b)^2} = -\frac{4}{(A+b)^2} < 0$$

$$-\frac{1}{N(s)} = \begin{cases} -\frac{1}{3}, & A=0 \\ -1, & A=\infty \end{cases}$$

$$-\frac{k}{2} > -\frac{1}{3}, \text{ 即 } k < \frac{2}{3}, \text{ 稳定,}$$

$$-\frac{k}{2} < -1 \text{ 即 } k > 2, \text{ 不稳定}$$

$$-1 < -\frac{k}{2} < -\frac{1}{3} \text{ 即 } \frac{2}{3} < k < 2, \text{ 产生周期运动}$$

(b) 稳定

$$\omega=1$$

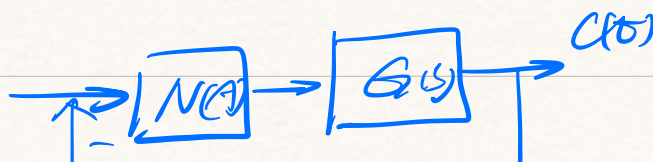
$$|G(j\omega)|_{\omega=1} = \frac{k}{2} = \frac{1}{N(s)} = \frac{A+2}{A+b}$$

$$A = \frac{2-3k}{\frac{k}{2}-1}$$

$$8-18 \quad N(s) = \frac{4M}{\pi A} \sqrt{1 - \left(\frac{k}{A}\right)^2} - j \frac{4M}{\pi A^2}$$

$$= \frac{4}{2A} \sqrt{1 - \left(\frac{0.2}{A}\right)^2} - j \frac{4}{\pi A^2}$$

$$G_c G(s) = \frac{10}{s(s+1)}$$

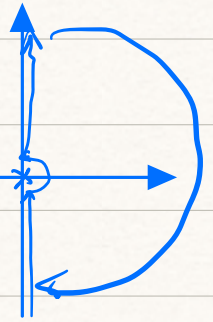
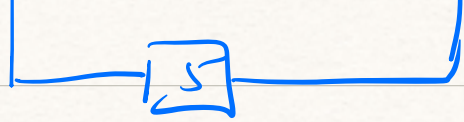


$$G_c G(\bar{w}) = \frac{10}{j\omega(j\omega+1)}$$

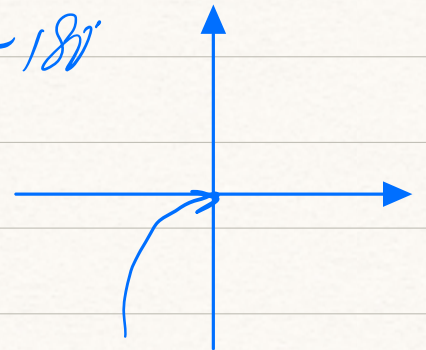
$$|G_c G(\bar{w})| = \frac{10}{\omega \sqrt{\omega^2+1}}$$

$$\angle \theta = -\frac{\pi}{2} - \arctan \omega$$

ω	0	1	5	100	$+\infty$
$ G_c G(\bar{w}) $	∞	7.1	0.4	0.001	0
$\angle \theta$	-90°	-135°	-169°	-174°	-180°



$$\begin{aligned} -\frac{1}{NA} &= -\frac{1}{\frac{4}{\pi A} \sqrt{1 - \left(\frac{0.2}{A}\right)^2} - j \frac{4}{\pi A}} \\ &= -\frac{\sqrt{A^2 - 0.04} + j}{\frac{4}{\pi} \left(1 - \frac{0.04}{A^2}\right) + \frac{4}{\pi A^2}} \\ &= -\frac{\pi A^2}{4} \frac{1}{\sqrt{A^2 - 0.04} - j} \\ &= -\frac{\pi A^2}{4} \frac{\sqrt{A^2 - 0.04} + j}{A^2 + 0.96} \end{aligned}$$



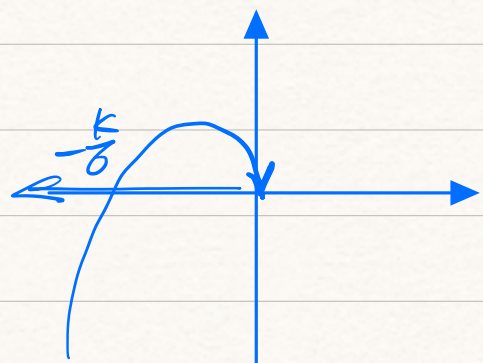
8-23

(1)

$$G(s) = \frac{k}{s(s+1)(s+2)}$$

$$G(\bar{w}) = \frac{k}{j\omega(j\omega+1)(j\omega+2)}$$

$$|G(\bar{w})| = \frac{k}{\omega \sqrt{\omega^2+1} \sqrt{\omega^2+4}}$$



$$\angle \theta = -\frac{\pi}{2} - \arctan \omega - \arctan \frac{\omega}{2}$$

ω	0	1	5	10	$+\infty$
$ G(j\omega) $	$+\infty$	0.3k	0.007k	0.001k	0
$\angle \theta$	-90°	-162°	-237°	-253°	-270°

$$\angle \theta = -180^\circ$$

$$\arctan \omega + \arctan \frac{\omega}{2} = 90^\circ$$

$$\arctan \frac{\omega + \frac{\omega}{2}}{1 - \frac{\omega^2}{2}} = 90^\circ$$

$$\omega = \sqrt{2}$$

$$|G(j\omega)|_{\omega=\sqrt{2}} = \frac{k}{\sqrt{2} \sqrt{3} \sqrt{6}} = \frac{k}{6}$$

$$N(A) = \frac{4A}{2A} = \frac{4}{2A}$$

$$-\frac{1}{N(A)} = -\frac{2A}{4} = \begin{cases} -0, & A=0 \\ -\infty, & A=+\infty \end{cases}$$

$$\frac{k}{6} = \frac{2A}{4} \Rightarrow A = \frac{2k}{3\pi}$$

$$\text{以 } \omega = \sqrt{2}, A = \frac{2k}{3\pi} \text{ 共振}$$

$$(2) \quad G(s) = \frac{ke^{-Ts}}{s(s+1)(s+2)}$$

$$G(j\omega) = \frac{ke^{-j\omega T}}{j\omega(j\omega+1)(j\omega+2)}$$

$$|G(j\omega)| = \frac{k}{\omega \sqrt{\omega^2+1} \sqrt{\omega^2+4}}$$

$$\angle \theta = -\tau \omega - \frac{\pi}{2} - \arctan \omega - \arctan \frac{\pi}{2}$$

$$-\frac{1}{NA} = -\frac{\pi}{2}$$

$$|G(j\omega)|_{\omega=1} = \frac{k}{\sqrt{2}\sqrt{5}} = \frac{\pi}{2}$$

$$k = \frac{\sqrt{10}}{2} \pi$$

$$\angle \theta|_{\omega=1} = -\tau - 90^\circ - 45^\circ - 26.6^\circ = -180^\circ$$

$$\tau = 18.4^\circ$$

$$8-24 \quad G_c \cdot G(s) = \frac{10K}{s(Ts+1)(0.1s+1)}$$

$$G_c G(j\omega) = \frac{10K}{j\omega(j\omega T+1)(0.1j\omega+1)}$$

$$|G_c G(j\omega)| = \frac{10K}{\omega \sqrt{\omega^2 T^2 + 1} \sqrt{0.01\omega^2 + 1}}$$

$$\angle \theta = -\frac{\pi}{2} - \arctan T\omega - \arctan 0.1\omega$$

$$NA = \frac{PM}{\pi A} = \frac{4\sqrt{2}}{\pi A}$$

$$-\frac{1}{NA} = -\frac{\pi A}{4\sqrt{2}}$$

$$|G_c G(j\omega)|_{\omega=10} = \frac{K}{\sqrt{100T^2+1} \sqrt{2}}$$

$$\angle \theta|_{\omega=10} = -90^\circ - \arctan 10T - 45^\circ = -180^\circ$$

$$\arctan 10T = 45^\circ$$

$$T = 0.1$$

$$\frac{K}{\sqrt{2}} = \frac{\pi \cdot 0.1}{4\sqrt{2}}, \quad \frac{10}{\sqrt{2}}$$

$$\sqrt{2} \sqrt{2} - \frac{4\sqrt{2}}{\sqrt{T^2+1}}$$

$$k = \frac{0.12}{2\sqrt{2}} \cdot \frac{10}{\sqrt{1.01}} = \frac{2}{2\sqrt{2.02}}$$