

习题 5.1

2.a.

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findMinMax(A, low, high)
  if high = low:
    return A[low], A[low]
  elif high = low + 1:
    if A[low] <= A[high]:
      return A[low], A[high]
    else:
      return A[high], A[low]
  else:
    mid = (high + low) / 2
    leftMin, leftMax = findMinMax(A, low, mid)
    rightMin, rightMax = findMinMax(A, mid+1, high)
    min = min(leftMin, rightMin)
    max = max(leftMax, rightMax)
    return min, max

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b. $T(n) = 1$

$$\begin{aligned}
 T(n) &= 2T\left(\frac{n}{2}\right) + 2 \\
 &= 2(2T\left(\frac{n}{4}\right) + 2) + 2 \\
 &= 4T\left(\frac{n}{4}\right) + 2 + 4 \\
 &= 4(2T\left(\frac{n}{8}\right) + 2) + 2 + 4 \\
 &= 8T\left(\frac{n}{8}\right) + 2 + 4 + 8 \\
 &= \dots \\
 \because n &= 2^k \\
 \therefore &= 2^{k-1}T\left(\frac{n}{2^{k-1}}\right) + 2 + 4 + 8 + \dots + 2^{k-1} \\
 &= 3 \cdot 2^{k-1} - 2 \\
 &= \frac{3}{2}n - 2
 \end{aligned}$$

c. 该算法效率为 $\frac{3}{2}n - 2$,
而暴力算法效率为 $2(n-1)$

5. 已知 $f(n) \in \Theta(n^d)$, $d \geq 0, \mathbb{R}$:

$$T(n) \in \begin{cases} \Theta(n^d) & a < b^d \\ \Theta(n^d \log n) & a = b^d \\ \Theta(n^{a \log_b a}) & a > b^d \end{cases} \quad T(n) = aT(n/b) + f(n)$$

a. $T(n) = 4T\left(\frac{n}{2}\right) + n, T(1) = 1$

$a = 4, b = 2, d = 1$

$\therefore T(n) \in \Theta(n^2)$

b. $T(n) = 4T\left(\frac{n}{2}\right) + n^2, T(1) = 1$

$a = 4, b = 2, d = 2$

$\therefore T(n) \in \Theta(n^2 \log 2)$

c. $T(n) = 4T\left(\frac{n}{2}\right) + n^3$

$a = 4, b = 2, d = 3$

$\therefore T(n) \in \Theta(n^3)$

习题 5.3

1. floorCounter(T):

if $T = \emptyset$ return 0

else return max(floorCounter(T_{left}) + 1, floorCounter(T_{right}) + 1)

效率类型: $O(n)$, 线性类型

2. 不正确

LeafCounter(T):

if $T = \emptyset$ return 0

if $T_{left} = \emptyset$ and $T_{right} = \emptyset$ return 1

return LeafCounter(T_{left}) + LeafCounter(T_{right})

习题 5.2

1. 快排 E, X, A, M, P, L, E

E X A M P L E

E E A M P L X

A E E M P L X

有序 M L P X

A E E L M P X

有序 有序

习题 5.4

2. 2101×1130

$$2101 = 21 \times 10^3 + 01, a=21, b=01$$

$$1130 = 11 \times 10^3 + 30, c=11, d=30$$

$$ac = 21 \times 11 = 231$$

$$bd = 30$$

$$(a+b) \times (c+d) = 22 \times 41 = 902$$

$$ac \times 10^4 + (a+b) \times (c+d) \times 10^3 + bd = 231 \times 4130$$

7.

$$\begin{bmatrix} 10 & 2 & 1 \\ 4 & 1 & 0 \\ 0 & 1 & 3 \\ 5 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 4 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & 5 & 0 \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \quad B_{11} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \quad B_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix} \quad A_{22} = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \quad B_{21} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \quad B_{22} = \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$$M_1 = (A_{11} + A_{22})(B_{11} + B_{22}) = \begin{bmatrix} 4 & 8 \\ 20 & 14 \end{bmatrix}$$

$$M_2 = (A_{21} + A_{22})B_{11} = \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix}$$

$$M_3 = A_{11}(B_{12} - B_{22}) = \begin{bmatrix} -1 & 0 \\ -4 & 4 \end{bmatrix}$$

$$M_4 = A_{22}(B_{21} - B_{11}) = \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix}$$

$$M_5 = (A_{11} + A_{12})B_{22} = \begin{bmatrix} 8 & 8 \\ -10 & 3 \end{bmatrix}$$

$$M_6 = (A_{21} - A_{11})(B_{11} + B_{12}) = \begin{bmatrix} -2 & -3 \\ -2 & -3 \end{bmatrix}$$

$$M_7 = (A_{22} - A_{21})(B_{21} + B_{22}) = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$C_{11} = M_1 + M_4 - M_5 + M_7 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$C_{12} = M_3 + M_5 = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$$

$$C_{21} = M_2 + M_4 = \begin{bmatrix} 8 & 1 \\ 5 & 8 \end{bmatrix}$$

$$C_{22} = M_1 - M_2 + M_3 + M_6 = \begin{bmatrix} 3 & 7 \\ 7 & 1 \end{bmatrix}$$

$$\therefore C = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 5 & 1 \\ 8 & 1 & 3 \\ 5 & 8 & 7 \end{bmatrix}$$

习题 5.5

7. 快速如何求 p_{max}

首先找到最左和最右的两个点，连成一条线，把所有点分成两部分。

再找离这条线最远的点，即 p_{max} ，然后以这个点为顶点，和原来的线形成两个三角形。

再递归处理每个三角形中的点，直到无 p_{max} ，剩下的即为凸包。

8. 假设有 n 个点，分展： $O(n)$ ，展内处理： $O(\log n)$

$O(n \log n)$