

习题5.1

2.a.

```
findMinMax(A,low,high)
    if high == low:
        return A[low], A[low]
    elif high == low + 1:
        if A[low] <= A[high]:
            return A[low], A[high]
        else:
            return A[high], A[low]
    else:
        mid = (high + low) / 2
        leftMin, leftMax = findMinMax(A, low, mid)
        rightMin, rightMax = findMinMax(A, mid+1, high)
        min = min(leftMin, rightMin)
        max = max(leftMax, rightMax)
        return min, max
```

b. $T(2) = 1$

$$\begin{aligned} T(n) &= 2T\left(\frac{n}{2}\right) + 2 \\ &= 2(2T\left(\frac{n}{4}\right) + 2) + 2 \\ &= 4T\left(\frac{n}{4}\right) + 2 + 4 \\ &= 4(2T\left(\frac{n}{8}\right) + 2) + 2 + 4 \\ &= 8T\left(\frac{n}{8}\right) + 2 + 4 + 8 \\ &= \dots \\ \because n &= 2^k \\ &= 2^{k-1}T\left(\frac{n}{2^{k-1}}\right) + 2 + 4 + 8 + \dots + 2^{k-1} \\ &= 3 \cdot 2^{k-1} - 2 \\ &= \frac{3}{2}n - 2 \end{aligned}$$

c. 该算法效率为 $\frac{3}{2}n - 2$ ，
而蛮力算法效率为 $2(n-1)$

5. 已知 $f(n) \in \Theta(n^d)$, $d \geq 0$. 试:

$$T(n) \in \begin{cases} \Theta(n^d) & a < b^d \\ \Theta(n^d \log n) & a = b^d \\ \Theta(n^{\log_b a}) & a > b^d \end{cases} \quad T(n) = aT(n/b) + f(n)$$

a. $T(n) = 4T\left(\frac{n}{2}\right) + n$, $T(1) = 1$

$$a = 4, b = 2, d = 1$$

$$\therefore T(n) \in \Theta(n^2)$$

b. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$, $T(1) = 1$

$$a = 4, b = 2, d = 2$$

$$\therefore T(n) \in \Theta(n^2 \log 2)$$

c. $T(n) = 4T\left(\frac{n}{2}\right) + n^3$

$$a = 4, b = 2, d = 3$$

$$\therefore T(n) \in \Theta(n^3)$$

习题5.3

1. `floorCounter(T):`

`if T=∅ return 0`

`else return max(floorCounter(Tleft) + 1, floorCounter(Tright) + 1)`

效率类型: $O(n)$, 线性类型

2. 不正石角

`LeafCounter(T):`

`if T=∅ return 0`

`if Tleft=∅ and Tright=∅ return 1`

`return LeafCounter(Tleft) + LeafCounter(Tright)`

习题5.2

1. 快排 E,X,A,M,P,L,E

E X A M P L E
E E A M P L X
<u>A E E</u> M P L X
有序 M P L X
<u>A E E</u> L M P X
有序 有序

习题5.4

2. $2|0| \times 1|30$

$2|0|=2| \times |0+0|, a=2, b=0$

$1|30=1| \times |0^2+30, c=1, d=30$

$ac=2| \times 1|=23$

$bd=30$

$(a+b) \times (c+d)=22 \times 41=902$

$ac \times 10^4 + (a+b) \times (c+d) \times (d^2 + bd = 23) 4130$

7.

$$\begin{bmatrix} 10 & 2 & 1 \\ 4 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & 4 \\ 2 & 0 & 1 & 0 \\ 1 & 3 & 5 & 0 \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} 0 & 0 \\ 4 & 1 \end{bmatrix}, A_{12} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, B_{11} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix}, A_{22} = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}, B_{21} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}, B_{22} = \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$$M_1 = (A_{11} + A_{22}) (B_{11} + B_{22}) = \begin{bmatrix} 4 & 8 \\ 10 & 14 \end{bmatrix}$$

$$M_2 = (A_{21} + A_{12}) B_{11} = \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix}$$

$$M_3 = A_{11} (B_{12} - B_{22}) = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$M_4 = A_{22} (B_{21} - B_{11}) = \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix}$$

$$M_5 = (A_{11} + A_{12}) B_{22} = \begin{bmatrix} 8 & 3 \\ 10 & 6 \end{bmatrix}$$

$$M_6 = (A_{21} - A_{11}) (B_{11} + B_{12}) = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$$

$$M_7 = (A_{12} - A_{22}) (B_{21} + B_{22}) = \begin{bmatrix} 3 & 2 \\ -9 & -4 \end{bmatrix}$$

$$C_{11} = M_1 + M_4 - M_5 + M_7 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$C_{12} = M_3 + M_5 = \begin{bmatrix} 1 & 3 \\ 1 & 9 \end{bmatrix}$$

$$C_{21} = M_2 + M_6 = \begin{bmatrix} 8 & 1 \\ 5 & 8 \end{bmatrix}$$

$$C_{22} = M_1 - M_6 + M_3 + M_7 = \begin{bmatrix} 3 & 7 \\ 7 & 7 \end{bmatrix}$$

$$\therefore C = \begin{bmatrix} 5 & 4 & 7 & 3 \\ 4 & 5 & 1 & 9 \\ 8 & 1 & 3 & 7 \\ 5 & 8 & 7 & 7 \end{bmatrix}$$

习题5.5

7. 快包如何求 P_{max}

首先找到最左侧和最右侧的两个点，连成一条线，把所有点分成两部分。

再找离这条线最远的点，即 P_{max} ，然后以这个点为顶点，和原来的线形成两个三角形。

再递归处理每个三角形中的点，直到无 P_{max} ，剩下的即为凸包。

8. 假如有 n 个点，分层: $O(n)$, 层内处理: $O(\log n)$

$O(n \log n)$