Circuit Privacy for FHEW/TFHE-Style Fully Homomorphic Encryption in Practice

Kamil Kluczniak

Fully Homomorphic Encryption - FHE

sk

$$c = \mathsf{Enc}(sk, m)$$

$$\xrightarrow{\quad c \quad}$$

$$\xleftarrow{\quad y \quad}$$

$$F(m) = \mathsf{Dec}(sk, y)$$

$$\mathsf{Eval}(c, F) = y$$

Applications

Formally

$$\mathsf{Setup}(\lambda) : sk, pk$$

$$\mathsf{Enc}(pk, m) : c$$

$$\mathsf{Dec}(sk, c) : m$$

$$\mathsf{Eval}(pk, f, \{c_1, \ldots, c_n\}) : c$$

Correctness

$$\Pr\left[\mathsf{Dec}(sk, c) \neq f(m_1, \ldots, m_n) : c = \mathsf{Eval}(pk, f, \{c_1, \ldots, c_n\}), m_i = \mathsf{Dec}(sk, c_i)\right] \leq \mathsf{negl}(\lambda)$$

Formally

$$\mathsf{Setup}(\lambda) : sk, pk$$

$$\mathsf{Enc}(pk, m) : c$$

$$\mathsf{Dec}(sk, c) : m$$

$$\mathsf{Eval}(pk, f, \{c_1, \ldots, c_n\}) : c$$

Compactness

$$\left|\mathsf{Eval}(pk, f, \{c_1, \ldots, c_n\})\right| = \mathsf{poly}(\lambda, |m|)$$

Formally

$$\mathsf{Setup}(\lambda) : sk, pk$$

$$\mathsf{Enc}(pk, m) : c$$

$$\mathsf{Dec}(sk, c) : m$$

$$\mathsf{Eval}(pk, f, \{c_1, \ldots, c_n\}) : c$$

IND-CPA

$b \leftarrow \{0,1\}$

$\mathsf{Enc}(sk, m_b) = c$

m_0, m_1 ←

c →

$\hat{b} \stackrel{?}{=} b$

Something is Missing

sk

$c = \mathsf{Enc}(sk, m)$

$F(m) = \mathsf{Dec}(sk, c_{out})$

c

c_{out}

$\mathsf{Eval}(pk, F, c) = c_{out}$

Something is Missing

sk

$c = \mathsf{Enc}(sk, m)$

c

$F(m) = \mathsf{Dec}(sk, c_{out})$

c_{out}

$\mathsf{Eval}(pk, F, c) = c_{out}$

Trivial Attack:

1. Guess the function F
2. Run Eval locally
3. Check if your output matches the servers output

DETERMINISTIC!!!
Leaks information on F!

Something is Missing

sk

$c = \mathsf{Enc}(sk, m)$

$F(m) = \mathsf{Dec}(sk, c_{out})$

c →

← c_{out}

$\mathsf{Eval}(pk, F, c) = c_{out}$

Trivial Attack:

1. Guess the function F
2. Run Eval locally
3. Check if your output matches the servers output

DETERMINISTIC!!!
Leaks information on F!

Circuit Privacy

$\mathsf{Eval}(pk, F, c)$

Has to be correct

c ←

→ c_{out}

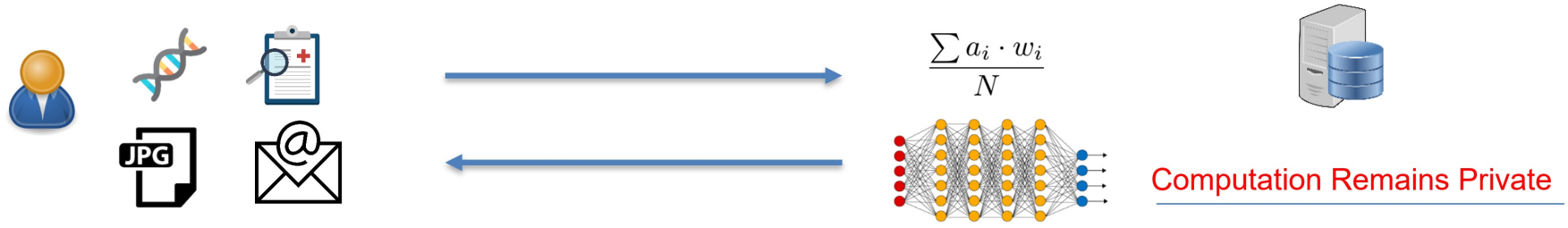
$\approx$

c →

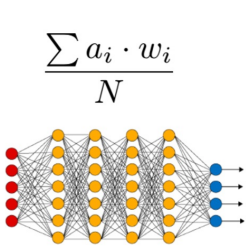
← c_{out}

$\mathsf{Sim}(pk, F(m))$

FHE With Circuit Privacy: Applications



FHE With Circuit Privacy: Applications



Computation Remains Private

Oblivious Pseudorandom Functions

sk

$c = \mathsf{Enc}(sk, m)$

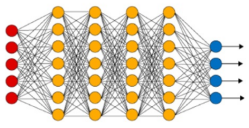
c

$c_{out} = \mathsf{Eval}(pk, \mathsf{PRF}(s, .), c)$

c_{out}

$\mathsf{PRF}(s, m) = \mathsf{Dec}(sk, c_{out})$

FHE With Circuit Privacy: Applications



Computation Remains Private

Blind Signatures

sk

$c = \mathsf{Enc}(sk, m)$

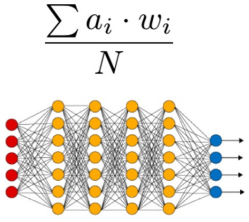
c

c_{out}

$\mathsf{SIGN}(s, m) = \mathsf{Dec}(sk, c_{out})$

$c_{out} = \mathsf{Eval}\big(pk, \mathsf{SIGN}(s, .), c\big)$

FHE With Circuit Privacy: Applications



Computation Remains Private

Secure Multi-Party Computation

sk

$c = \mathsf{Enc}(sk, m)$

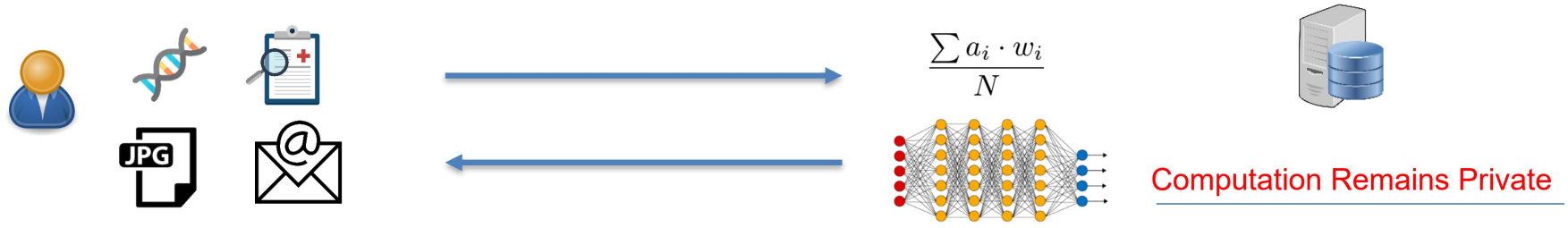
c

c_{out}

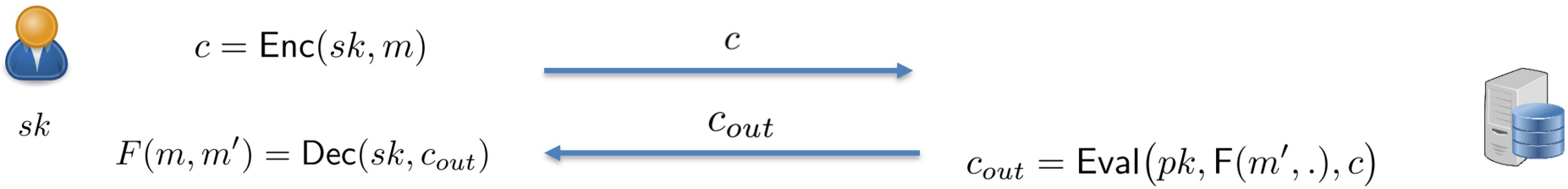
$F(m, m') = \mathsf{Dec}(sk, c_{out})$

$c_{out} = \mathsf{Eval}\big(pk, \mathsf{F}(m', .), c\big)$

FHE With Circuit Privacy: Applications



Secure Multi-Party Computation



Relaxed Notion of IND-CCA2 Security

[AGHV22]: Adi Akavia, Craig Gentry, Shai Halevi, and Margarita Vald. Achievable CCA2 relaxation for homomorphic encryption. TCC 2022

Frequently Asked Questions

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Q1: Does Computing a Universal Circuit Give Us Circuit Privacy?

Answer: No!

Q2: Does Adding a Freshly Sampled Ciphertext of Zero Give us Circuit Privacy?

Answer: Depends. But for LWE-based schemes the noise leaks information on the computation too.

Q3: Does Multikey FHE Give us Circuit Privacy?

Answer: Depends. You should be very careful...

See: Kamil Kluczniak, Giacomo Santato. On Circuit Private, Multikey and Threshold Approximate Homomorphic Encryption. IACR EPRINT 2023/301

Q4: If I homomorphically evaluate a neural network without circuit privacy does my entire model leak?

Answer: Doesn't seem to be so easy. But…. Who knows?

Contributions

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1) New Ciphertext Sanitization Algorithm

- Provable Secure (No Heuristics!!!)
- Ciphertexts are Sanitized with only one (randomized) bootstrapping invocation without increasing the parameters too much
- Generalized and Tight Analysis of the Gaussian Leftover hash Lemma

2) First Practical Implementation of a FHEW/TFHE-Style Circuit-Private FHE Scheme

- Parameters and Implementation for floating point arithmetic (FFT) and finite field arithmetic (NTT)
- Different arithmetic = different modulus = different type of Gaussian sampling: all implemented and compared
- Optimizations that result in deterministic computation as fast as non-circuit private
- Python Scripts to Estimate and help choose circuit private parameters for the scheme

3) New Parameters and Estimation for State-of-The-Art Previous Work

- Parameters and Tests for Ducas-Stehle Washing Machine

4) New Library: FHE-Deck

- FHEW/TFHE-style functions/programmable bootstrapping with Circuit Privacy by Default

How to Get a Circuit Private FHE

Idea 1: Let's Add a Fresh Ciphertext of Zero

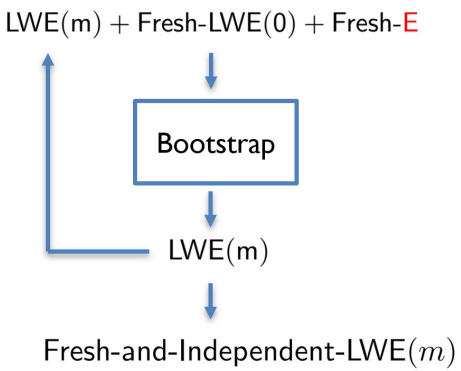
$$(\vec{a}, b = \langle \vec{a}, \vec{s} \rangle + m + e) + (\vec{a}_{\text{fresh}}, b_{\text{fresh}} = \langle \vec{a}_{\text{fresh}}, \vec{s} \rangle + e_{\text{fresh}})$$

$$= (\vec{a}_{\text{out}} = \vec{a} + \vec{a}_{\text{fresh}}, b_{\text{out}} = \langle \vec{a}_{\text{out}}, \vec{s} \rangle + m + e + e_{\text{fresh}})$$

Idea 2: Let's Flood the Noise Term

$$(\vec{a}_{\text{out}}, b_{\text{out}} = \langle \vec{a}_{\text{out}}, \vec{s} \rangle + m + e + e_{\text{fresh}} + E)$$

Idea 3: Let's Add a Smaller Noise Term and Bootstrap Repeatedly

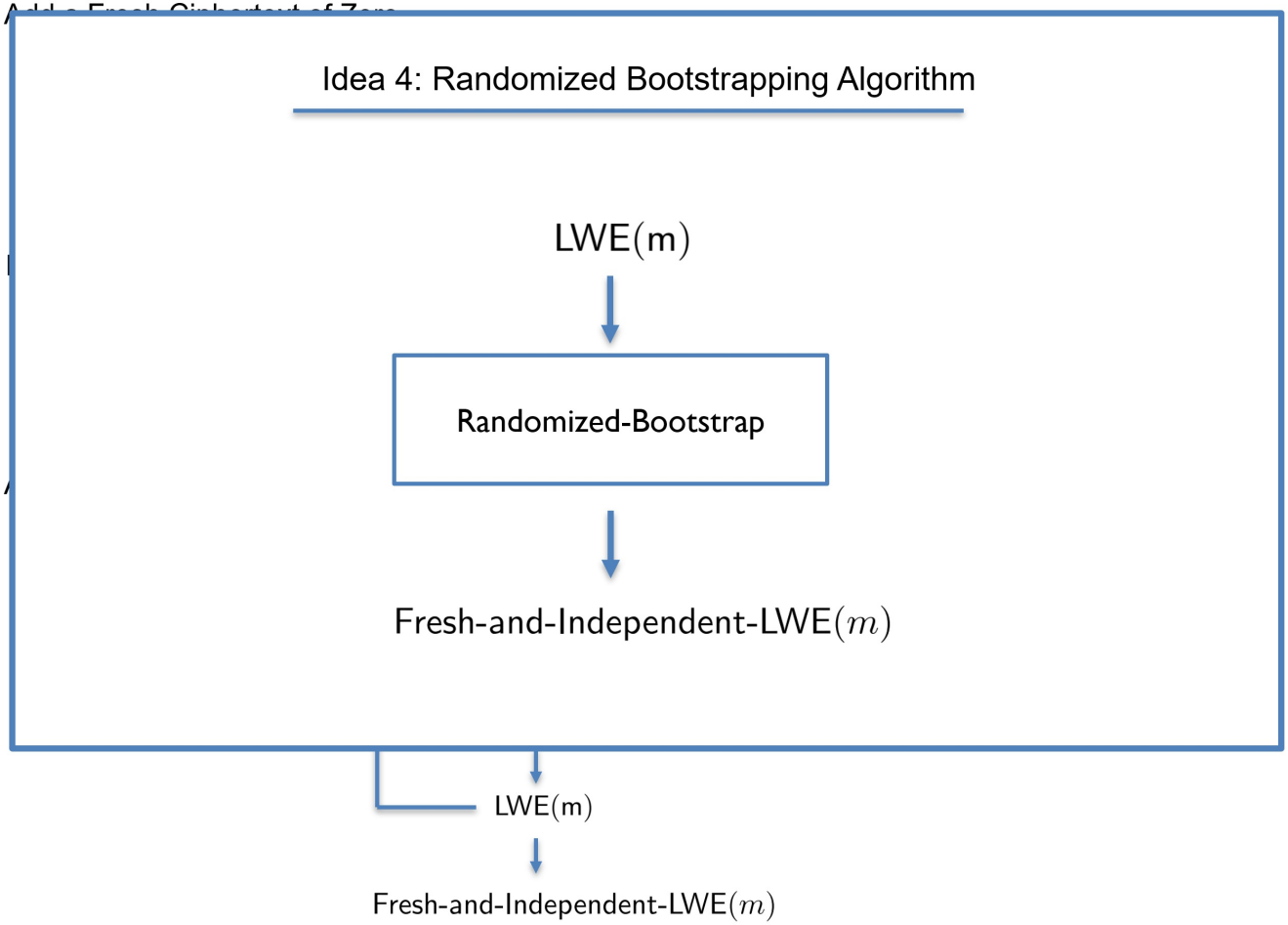


How to Get a Circuit Private FHE

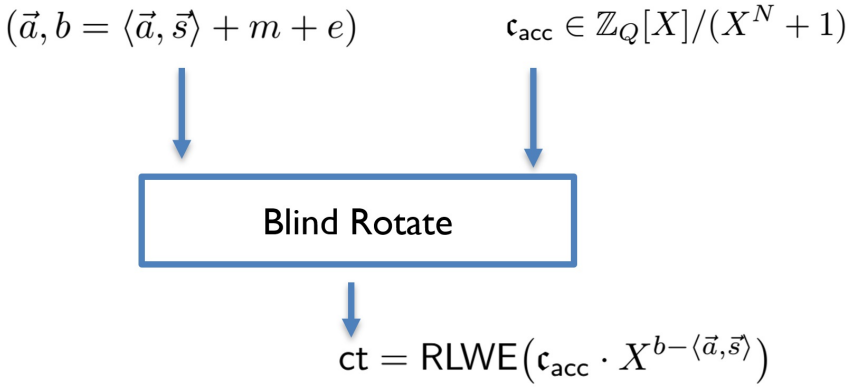
Idea 1: Let's Add a Fresh Ciphertext of Zero

Idea 2: Let's

Idea 3: Let's

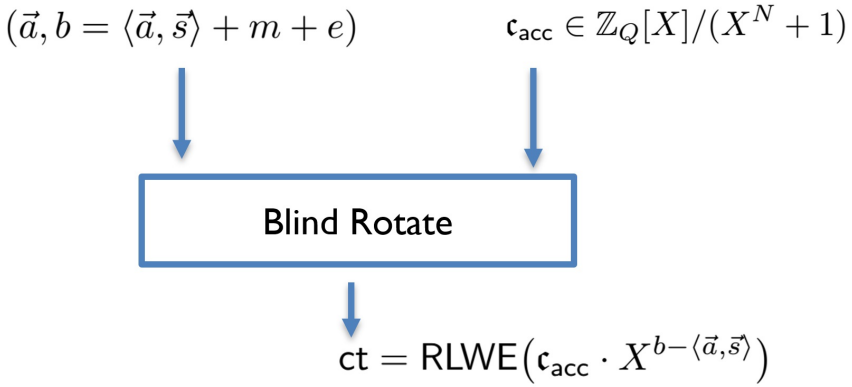


(Randomized) FHEW/TFHE-Style Bootstrapping



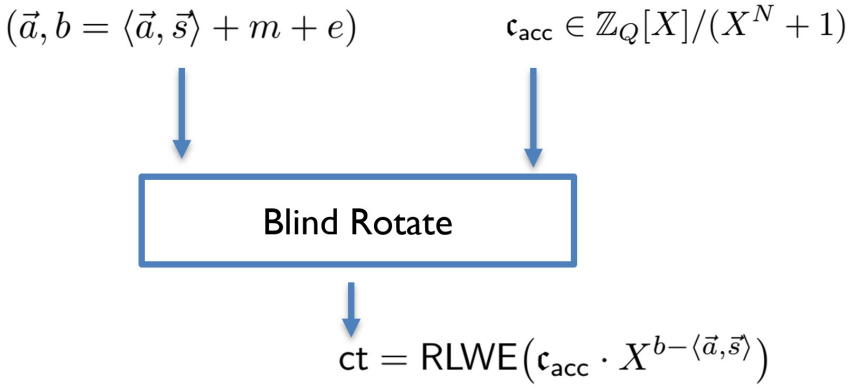
$\mathsf{Extract}(\mathsf{ct}) \to \mathsf{LWE}\big(\mathfrak{c}_{\mathsf{acc}} \cdot X^{b - \langle \vec{a}, \vec{s} \rangle}[1]\big)$

(Randomized) FHEW/TFHE-Style Bootstrapping



$\text{Extract}(\text{ct}) \to \text{LWE}\big(F(m)\big)$

(Randomized) FHEW/TFHE-Style Bootstrapping

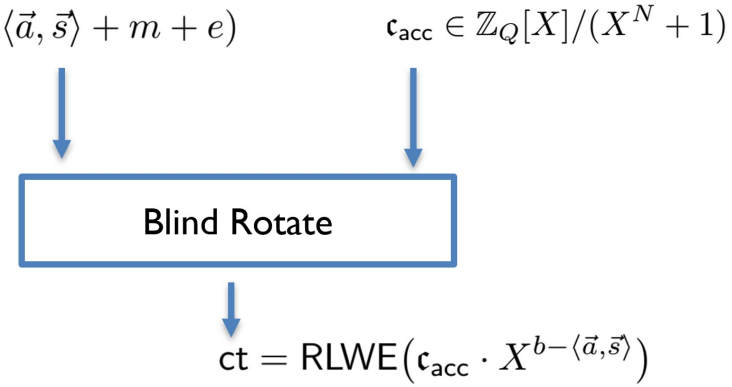


$\text{Extract}(\text{ct}) \to (\vec{a}_{\text{ext}}, b_{\text{ext}})$

where

$$b_{\text{ext}} = \langle \vec{a}_{\text{ext}}, \vec{s} \rangle + F(m) + e_{\text{ext}}$$

(Randomized) FHEW/TFHE-Style Bootstrapping



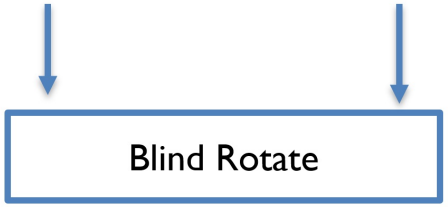
$$\text{Extract}(\text{ct}) \rightarrow (\vec{a}_{\text{ext}}, b_{\text{ext}}) + (\vec{a}_{\text{fresh}}, b_{\text{fresh}}) = (\vec{a}_{\text{out}}, b_{\text{out}})$$

where

$$b_{\text{out}} = \underbrace{\langle \vec{a}_{\text{out}}, \vec{s} \rangle}_{\text{Uniform}} + F(m) + e_{\text{fresh}} + \underbrace{e_{\text{ext}}}_{\text{Depends on the input ciphertext}}$$

(Randomized) FHEW/TFHE-Style Bootstrapping

$(\vec{a}, b = \langle \vec{a}, \vec{s} \rangle + m + e)$ $\quad \mathfrak{c}_{\text{acc}} \in \mathbb{Z}_Q[X]/(X^N + 1)$



$\text{ct} = \text{RLWE}\left(\mathfrak{c}_{\text{acc}} \cdot X^{b - \langle \vec{a}, \vec{s} \rangle}\right)$

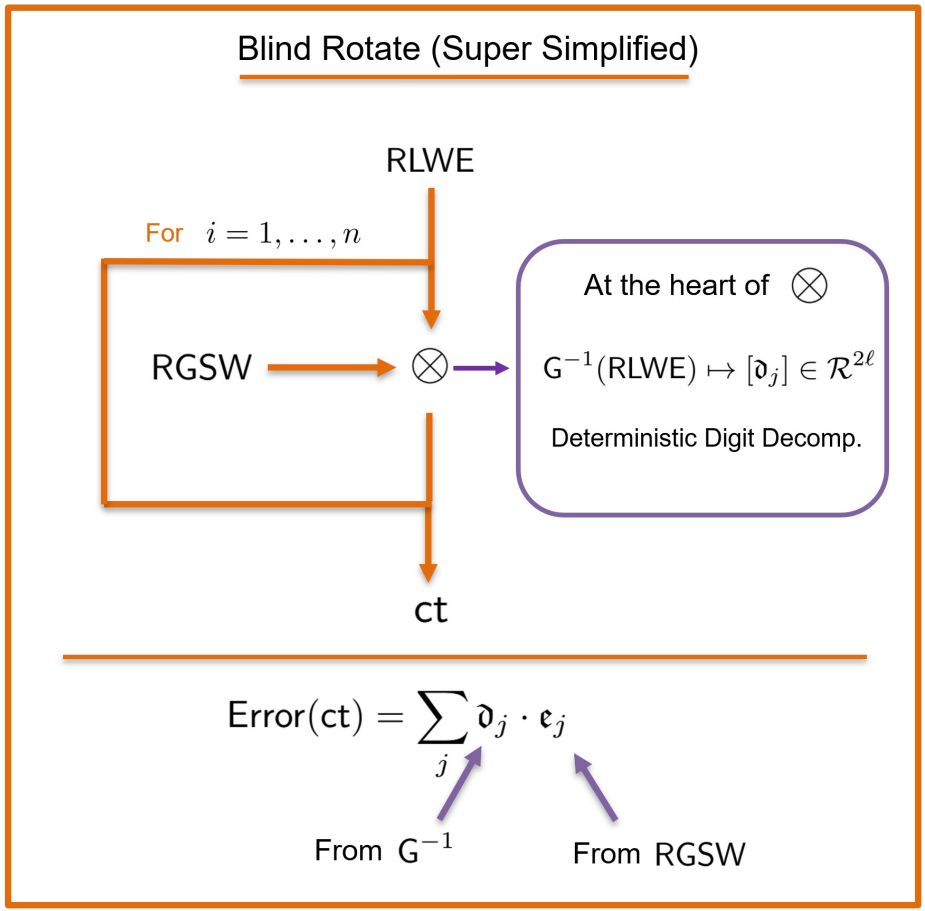
$\text{Extract}(\text{ct}) \to (\vec{a}_{\text{ext}}, b_{\text{ext}}) + (\vec{a}_{\text{fresh}}, b_{\text{fresh}}) = (\vec{a}_{\text{out}}, b_{\text{out}})$

where

$$b_{\text{out}} = \langle \vec{a}_{\text{out}}, \vec{s} \rangle + F(m) + e_{\text{fresh}} + e_{\text{ext}}$$

Uniform

Depends on the input ciphertext



(Randomized) FHEW/TFHE-Style Bootstrapping

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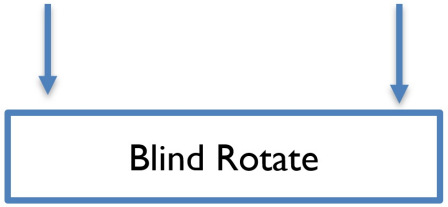
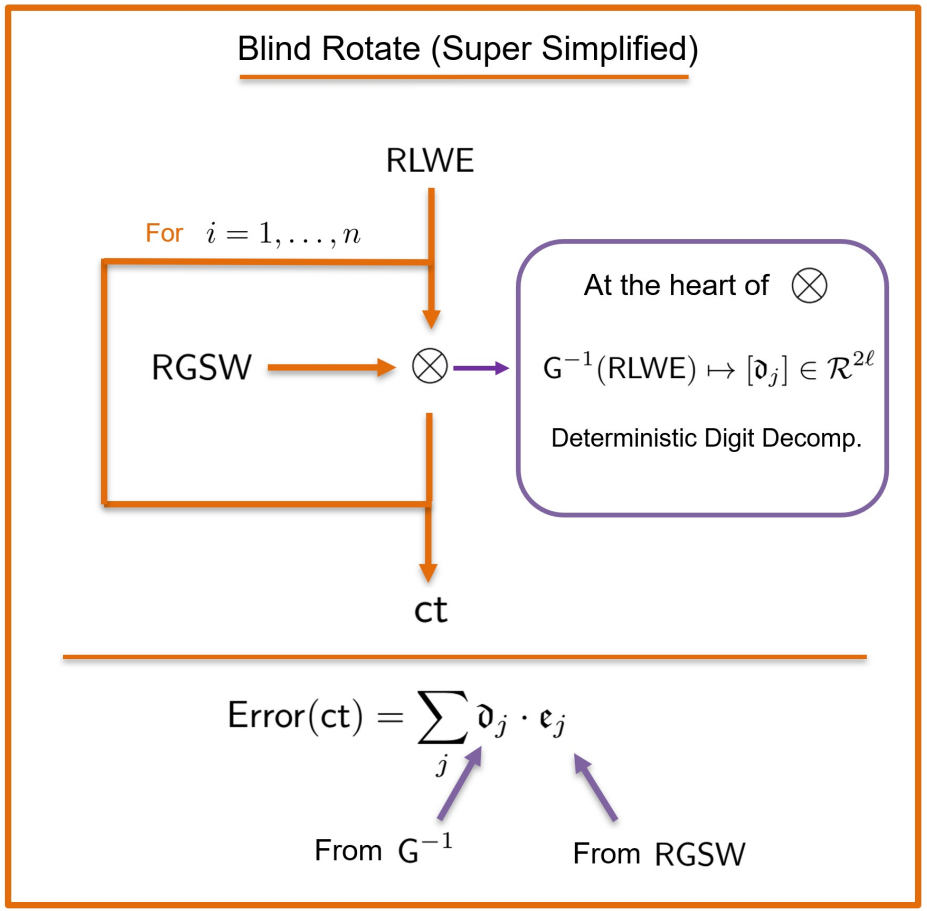


$\text{ct} = \text{RLWE}\left(\mathfrak{c}_{\text{acc}} \cdot X^{b - \langle \vec{a}, \vec{s} \rangle}\right)$

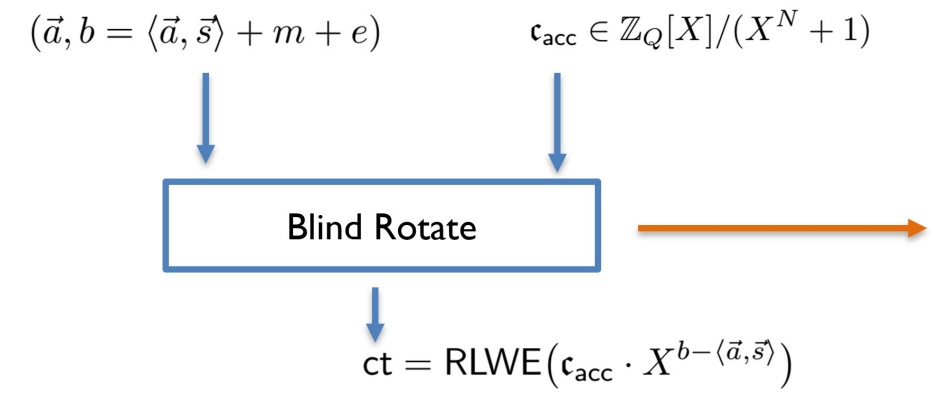
$\text{Extract}(\text{ct}) \rightarrow (\vec{a}_{\text{ext}}, b_{\text{ext}}) + (\vec{a}_{\text{fresh}}, b_{\text{fresh}}) = (\vec{a}_{\text{out}}, b_{\text{out}})$

where

$$b_{\text{out}} = \underbrace{\langle \vec{a}_{\text{out}}, \vec{s} \rangle}_{\text{Uniform}} + F(m) + e_{\text{fresh}} + \underbrace{\sum_k d_k \cdot e_k}_{\text{Depends on the input ciphertext}}$$



(Randomized) FHEW/TFHE-Style Bootstrapping



$$\text{Extract}(\text{ct}) \to (\vec{a}_{\text{ext}}, b_{\text{ext}}) + (\vec{a}_{\text{fresh}}, b_{\text{fresh}}) = (\vec{a}_{\text{out}}, b_{\text{out}})$$

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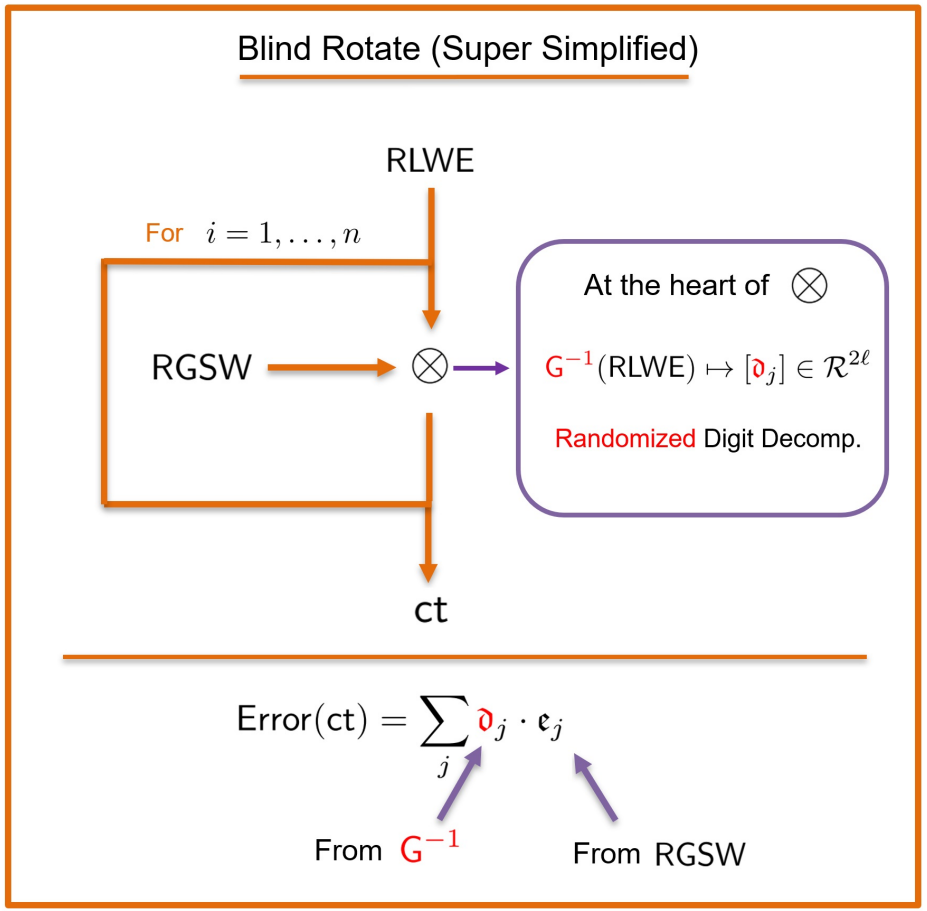
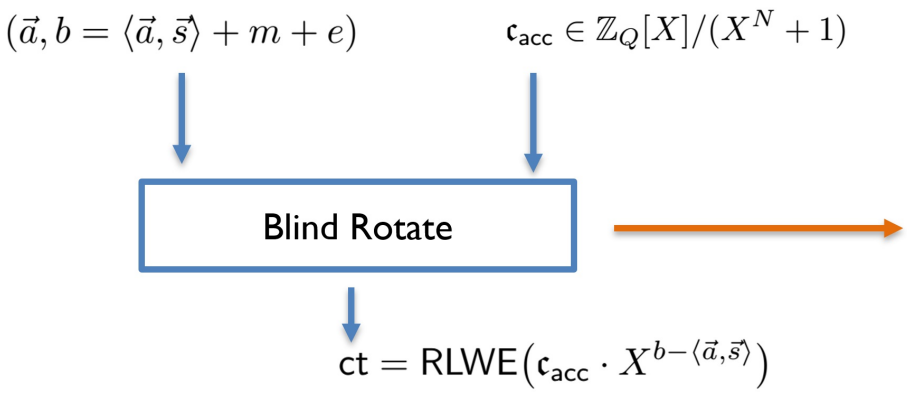
$$b_{\text{out}} = \underbrace{\langle \vec{a}_{\text{out}}, \vec{s} \rangle}_{\text{Uniform}} + F(m) + e_{\text{fresh}} + \underbrace{\sum_k d_k \cdot e_k}_{\text{Depends on the input ciphertext}}$$



(Randomized) FHEW/TFHE-Style Bootstrapping



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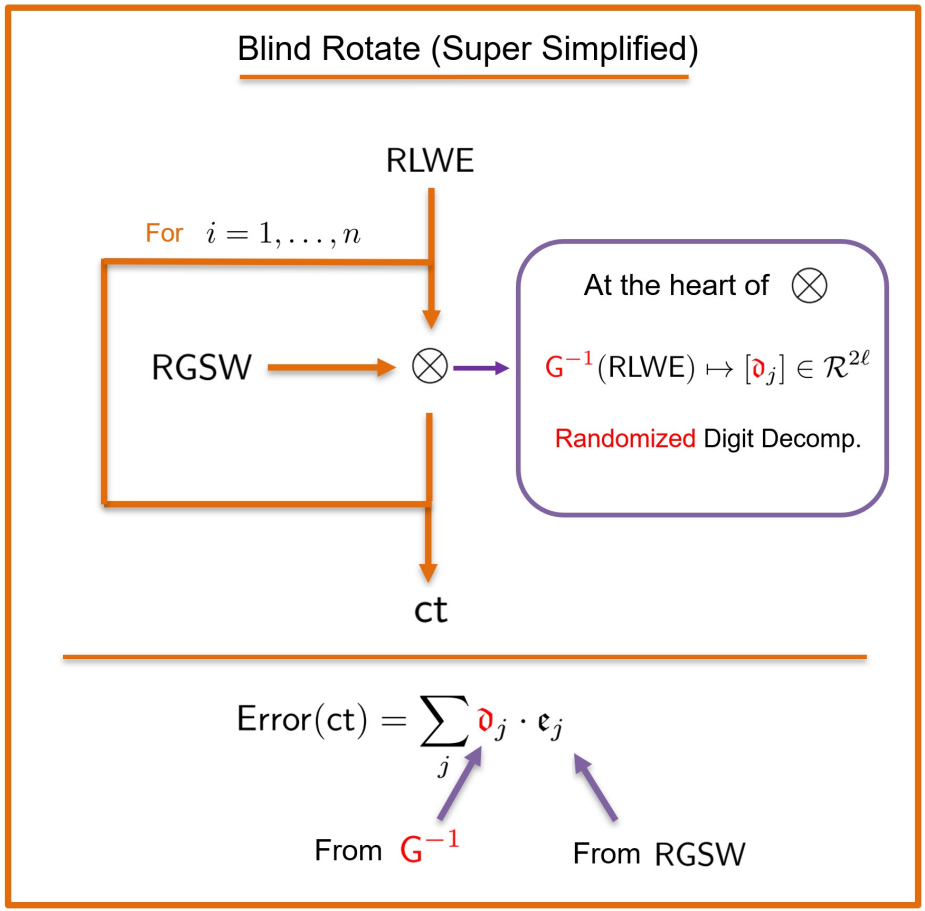
where

$$b_{\text{out}} = \underbrace{\langle \vec{a}_{\text{out}}, \vec{s} \rangle}_{\text{Uniform}} + F(m) + e_{\text{fresh}} + \underbrace{\sum_k d_k \cdot e_k}_{\overset{s}{\approx} e_{\text{new}}}$$

1) Mask by fresh LWE sample

2) Independent Gaussian via G^{-1}



The Proof

$$b_{\text{out}} = \langle \vec{a}_{\text{out}}, \vec{s} \rangle + \mathfrak{c}_{\text{acc}} \cdot X^{b - \langle \vec{a}, \vec{s} \rangle}[1] + e_{\text{fresh}} + d + e_{\text{ext}}$$

$$b_{\text{out}} = \langle \vec{a}_{\text{out}}, \vec{s} \rangle + \mathfrak{c}_{\text{acc}} \cdot X^{b - \langle \vec{a}, \vec{s} \rangle}[1] + e_{\text{fresh}} + d + \sum_k d_k \cdot e_k$$

$$b_{\text{out}} = \langle \vec{a}_{\text{out}}, \vec{s} \rangle + F(m) + e_{\text{fresh}} + d + \sum_k d_k \cdot e_k$$

uniform random

$$b_{\text{out}} = \langle \vec{a}_{\text{out}}, \vec{s} \rangle + F(m) + e_{\text{fresh}} + e_{\text{new}}$$

1) Show that $e_{\text{ext}} = \sum_k d_k \cdot d_k$

2) Assume that $\mathfrak{c}_{\text{acc}} \cdot X^{b - \langle \vec{a}, \vec{s} \rangle}[1] = F(m)$ from **Correctness**

3) $\vec{a}_{\text{out}} = \vec{a}_{\text{ext}} + \vec{a}_{\text{fresh}}$ uniform random, because $\vec{a}_{\text{fresh}}$ is close to uniform from Leftover Hash Lemma

4) $d + \sum_k d_k \cdot d_k \overset{s}{\approx} e_{\text{new}}$ from Gaussian Leftover Hash Lemma (Generalized Lemma 3.6 from [BDPMW16])

[BDPMW16]: *Florian Bourse, Rafaël Del Pino, Michele Minelli, and Hoeteck Wee. FHE Circuit Privacy Almost For Free. Crypto 2016*

Implementations, Tests and Comparisons

- Computational Security for IND-CPA: 128-bit

- Statistical Security for Circuit Privacy: 80-bit

- Ring: $\mathbb{Z}_Q[X]/(X^N+1)$ where $N = 2^{11}$

- Schemes: Our and [DS16]

- Implementations: Number Theoretic Transform (Intel Hexl) vs. Fast Fourier Transform (FFTW)

Parameters, Schemes and Implementations

Impl.	BK [MB]	Scheme	Bootstr.	$\log_2$(mod)	Mul-Det	Mul-Sim
NTT (Intel Hexl)	134	Our	1	48 (prime)	3	7
		[DS16]	5			
FFT (FFTW, 64-bit)	168	Our	1	36 (power-of-2)	4	10
		[DS16]	10			

[DS16]: Léo Ducas and Damien Stehlé. Sanitization of FHE ciphertexts. Eurocrypt 2016

The (Randomized) Decomposition Algorithm

Special modulus of the form $Q = b^k$

Used in FFT-Based Implementation

$\textsc{SampleG}(q = b^k, s, u)$
for $i = 0, \ldots, k-1$:
  $x_i \leftarrow \mathcal{D}_{b\mathbb{Z}+u,s}$
  $u := (u - x_i)/b \in \mathbb{Z}.$
return $(x_0, \ldots, x_{k-1})$.

NTT vs. FFT (Brief) Summary

NTT	Larger Mod	Complex G^{-1}
FFT	Numerical Errors	Simple G^{-1}

Arbitrary Modulus

Used in NTT-Based Implementation

$\textsc{SampleG}(s, \mathbf{u} = [u]_b^k, \mathbf{q} = [q]_b^k)$
  $\sigma := s/(b+1)$
  $\mathbf{p} \leftarrow \textsc{Perturb}(\sigma)$
  for $i = 0, \ldots, k-1$:
    $c_i := (c_{i-1} + u_i - p_i)/b$
  $\mathbf{z} \leftarrow \textsc{SampleD}(\sigma, \mathbf{c})$
  for $i = 0, \ldots, k-2$:
    $t_i := b \cdot z_i - z_{i-1} + q_i \cdot z_{k-1} + u_i$
  $t_{k-1} := q_{k-1} \cdot z_{k-1} - z_{k-2} + u_{k-1}$
  return t

$\textsc{Perturb}(\sigma)$
  $\beta := 0$
  for $i = 0, \ldots, k-1$:
    $c_i := \beta/l_i$, and $\sigma_i := \sigma/l_i$
    $z_i \leftarrow \lfloor c_i \rfloor + \textsc{SampleZ}_t(\sigma_i, \lfloor c_i \rceil_{[0,1)}, s)$
    $\beta := -z_i h_i$
  $p_0 := (2b+1)z_0 + bz_1$
  for $i := 1, \ldots, k-1$:
    $p_i := b(z_{i-1} + 2z_i + z_{i+1})$
  return p

$\textsc{SampleD}(\sigma, \mathbf{c})$
  $z_{k-1} \leftarrow \lfloor -c_{k-1}/d_{k-1} \rfloor$
  $z_{k-1} \leftarrow z_{k-1} + \textsc{SampleZ}_t(\sigma/d_{k-1}, \lfloor -c_{k-1}/d_{k-1} \rceil_{[0,1)}, s)$
  $\mathbf{c} := \mathbf{c} + z_{k-1}\mathbf{d}$
  for $i \in \{0, \ldots, k-2\}$:
    $z_i \leftarrow \lfloor -c_i \rfloor + \textsc{SampleZ}_t(\sigma, \lfloor -c_i \rceil_{[0,1)}, s)$
  return z

[DS16]: Nicholas Genise and Daniele Micciancio. Faster Gaussian Sampling for Trapdoor Lattices with Arbitrary Modulus. Eurocrypt 2018

Implementations, Tests and Comparisons

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Parameters, Schemes and Implementations

Impl.	BK [MB]	Scheme	Bootstr.	$\log_2$(mod)	Mul-Det	Mul-Sim
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		[DS16]	10			

Tests and Comparison

Impl.	Scheme	Total [s] det	Total [s] simul	Bootstrap [s] det	Bootstrap [s] simul	G^{-1}
NTT	Ours	0.14	1.44	0.14	1.44	78%
	[DS16]		3.55		0.71	–
FFT	Ours	0.27	1.22	0.27	1.22	42%
	[DS16]		6.38		0.63	–

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FHE-Deck: Circuit Privacy by Default

https://github.com/fhe-deck

- Implementations of (different types of functional/PBS) variants of **FHEW/TFHE/NTRUnium/FINAL** schemes
- All parameters sets support **Circuit Privacy**
- User doesn't have to think much about it
- …. and estimator scripts to choose new parameter sets

Example

```
lwe_ct ct2 = context.encrypt(2);
lwe_ct mct3 = context.encrypt(-3);

auto fun_relu = [](long m) -> long {
    if(m >= 0){
        return m;
    }else{
        return 0;
    }
};

lwe_ct ct_relu = context.eval_lut(&ct2, fun_relu);
std::cout << "Decrypt(ct_relu): " << context.decrypt(&ct_relu) << std::endl;
ct_relu = context.eval_lut(&mct3, fun_relu);
std::cout << "Decrypt(ct_relu): " << context.decrypt(&ct_relu) << std::endl;

std::ofstream fout("filename.txt");
fout << ct_relu;
```

Decrypt(ct_relu): 2

Decrypt(ct_relu): 0

Sanitizes the Ciphertext

Thank You

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FHE-Deck: Circuit Privacy by Default

https://github.com/fhe-deck

(Newest Version will be Available Next Week)

Circuit Privacy for FHEW/TFHE-Style Fully Homomorphic Encryption in Practice

Kamil Kluczniak

https://eprint.iacr.org/2022/1459.pdf

On Circuit Private, Multikey and Threshold Approximate Homomorphic Encryption

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https://eprint.iacr.org/2023/301.pdf