Efficient TFHE Bootstrapping in the Multiparty Setting

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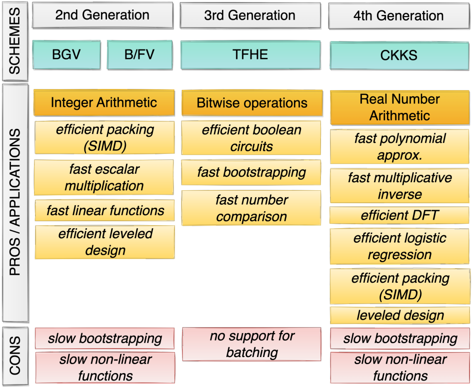
Motivation

- Handling multiple users securely and efficiently for privacy preserving protocol is important for real world applications.
- Two main approaches for multiple users: Multikey (MKHE) and Multiparty (MPHE)
- State of the art FHE schemes such as BGV, B/FV and CKKS are already efficiently extended to their MPHE version, but there is no concrete such extension of TFHE.

Scheme	**MKHE**	**MPHE**
BGV	[CZW17]	[Par21]
B/FV	[CDKS19, KLSW21]	[AJLA+12, MTPBH21, Par21, KLSW21]
TFHE	[CCS19, LP19]	This work, [LMK+23]
CKKS	[CDKS19]	[Par21]

Table: Main MKHE and MPHE extensions of the most well-known FHE schemes

Motivation



Thank you to Chiara Marcolla for providing the figure. Extracted from [MSM+23].

Contents

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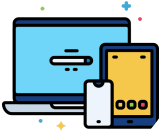
PC

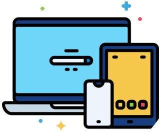


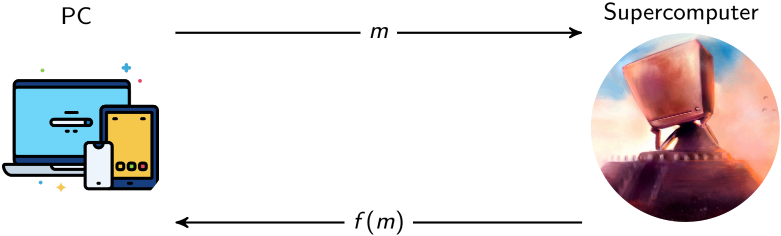
Supercomputer

PC

m

Supercomputer

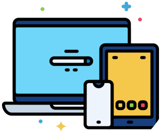



PC
m
Supercomputer
f(m)

PC

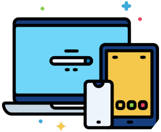
Supercomputer

(pk, sk)



PC

(pk, sk)



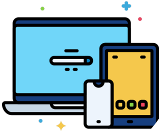
$Enc_{pk}(m)$ →

Supercomputer

Homomorphic Encryption: Delegation of computation

PC

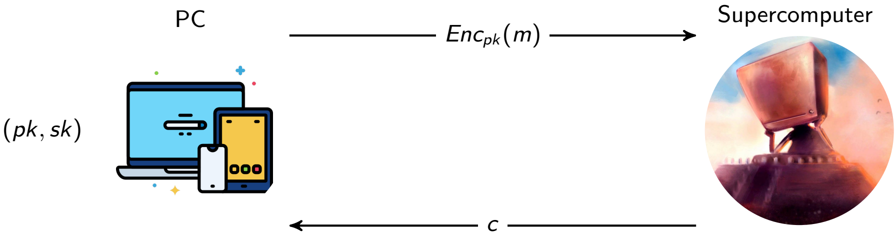
(pk, sk)



$\xrightarrow{Enc_{pk}(m)}$

Supercomputer

$\xleftarrow{c}$

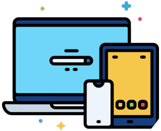


Properties:

Correctness: $Dec_{sk}(c) = f(m)$

PC

(pk, sk)



$\longrightarrow Enc_{pk}(m) \longrightarrow$

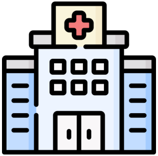
$\longleftarrow c \longleftarrow$

Supercomputer

Properties:

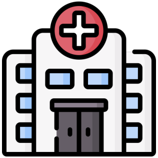
Correctness: $Dec_{sk}(c) = f(m)$
Compactness: The size of c is independent of f

Hospital A



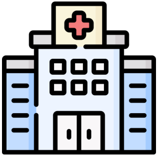
Supercomputer

Hospital B



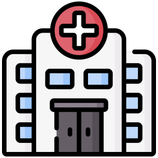
Hospital A

(pk, sk)

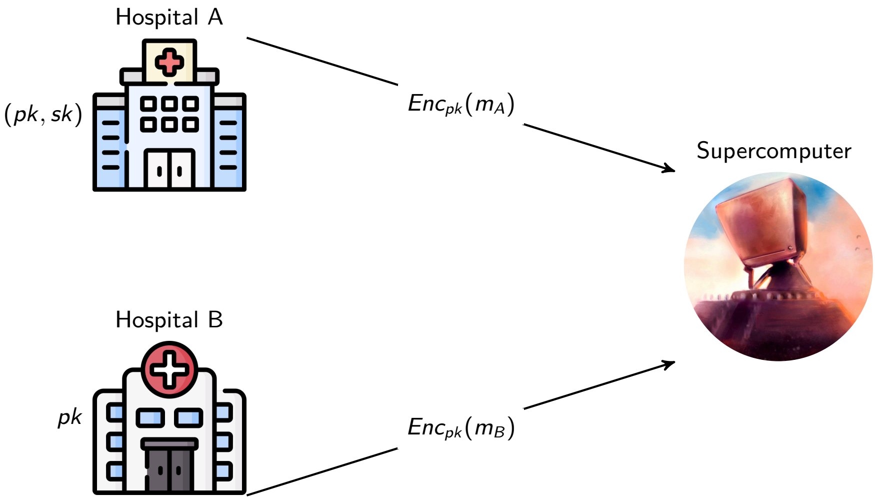


Hospital B

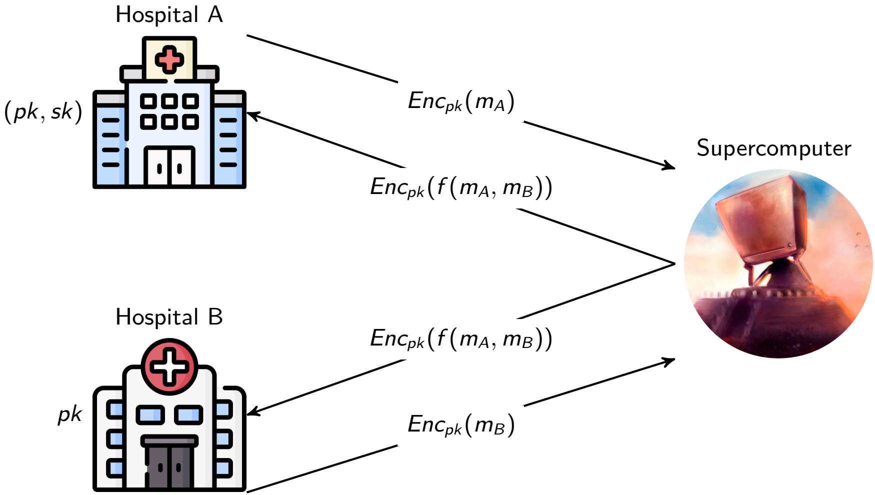
pk



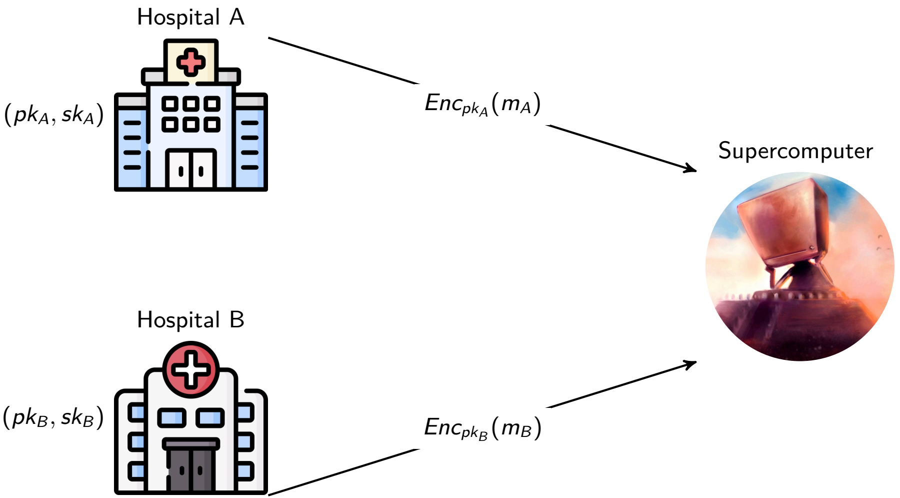
Supercomputer


Hospital A
(pk, sk)
$Enc_{pk}(m_A)$
Supercomputer
Hospital B
pk
$Enc_{pk}(m_B)$

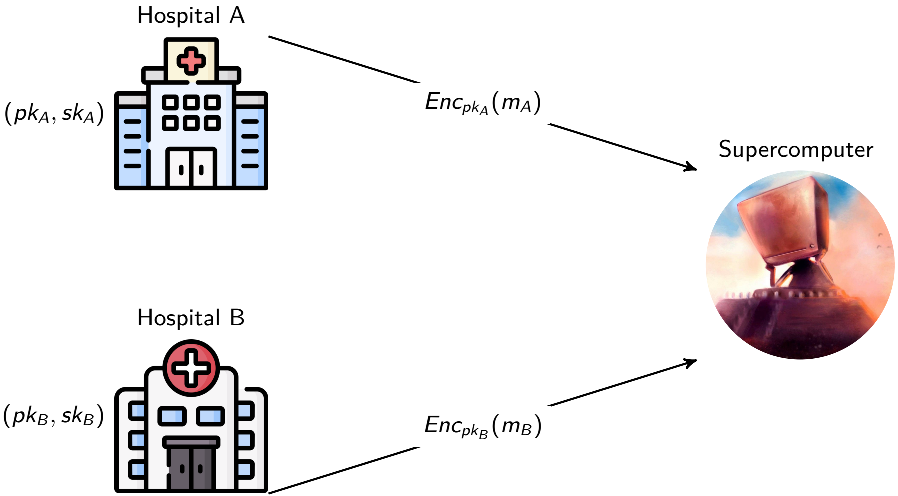
Multikey HE: Machine Learning based diagnosis



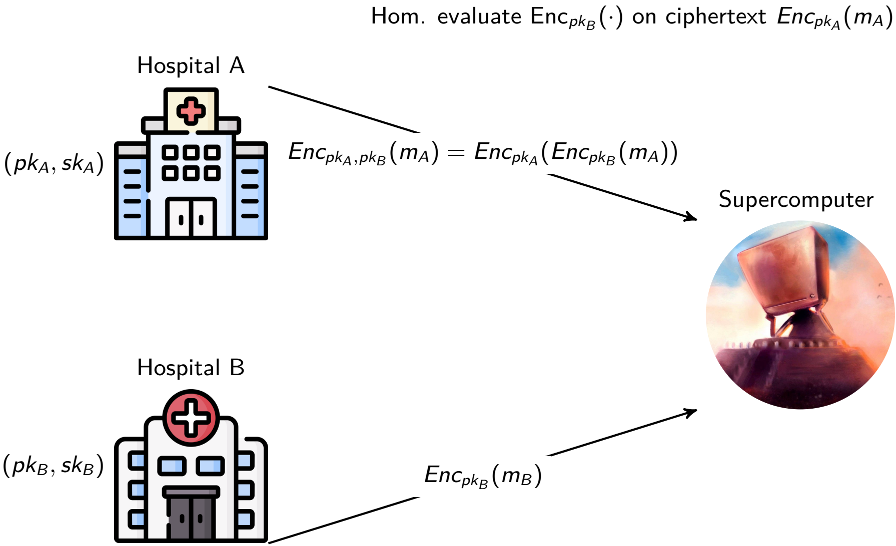
Multikey HE: Machine Learning based diagnosis



Hom. evaluate $\mathrm{Enc}_{pk_B}(\cdot)$ on ciphertext $Enc_{pk_A}(m_A)$



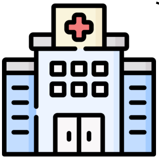
Multikey HE: Machine Learning based diagnosis



Hom. evaluate $\mathrm{Enc}_{pk_B}(\cdot)$ on ciphertext $Enc_{pk_A}(m_A)$

Hospital A

(pk_A, sk_A)

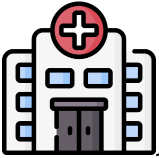


$Enc_{pk_A,pk_B}(m_A) = Enc_{pk_A}(Enc_{pk_B}(m_A))$

Supercomputer

Hospital B

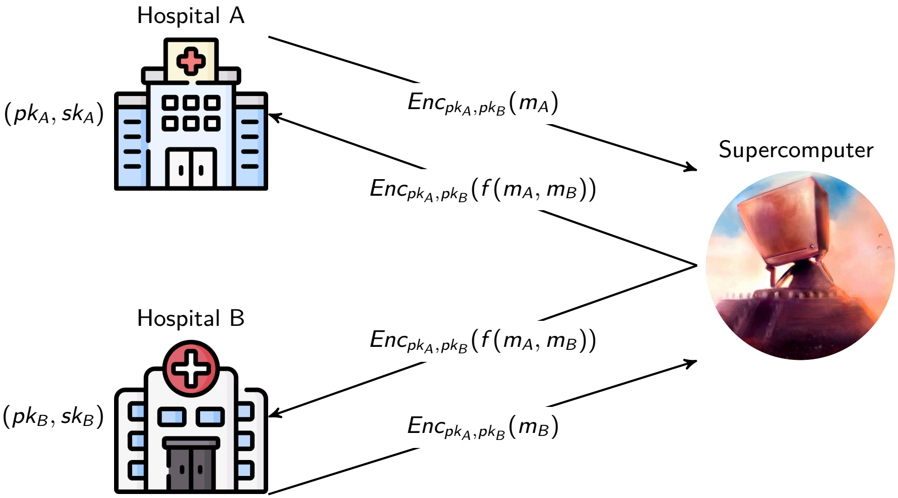
(pk_B, sk_B)



$Enc_{pk_A,pk_B}(m_B) = Enc_{pk_A}(Enc_{pk_B}(m_B))$

Multikey HE: Machine Learning based diagnosis

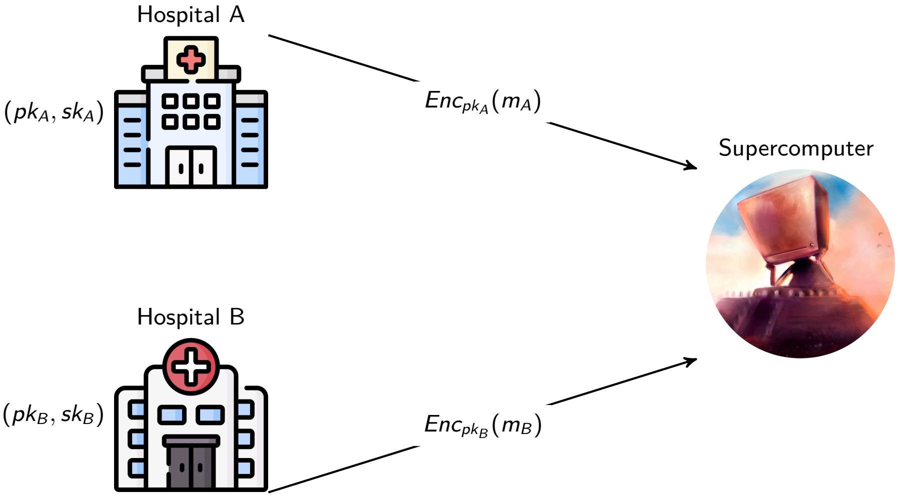
Hom. evaluate $\text{Enc}_{pk_B}(\cdot)$ on ciphertext $Enc_{pk_A}(m_A)$



Main problems with the MKHE approach

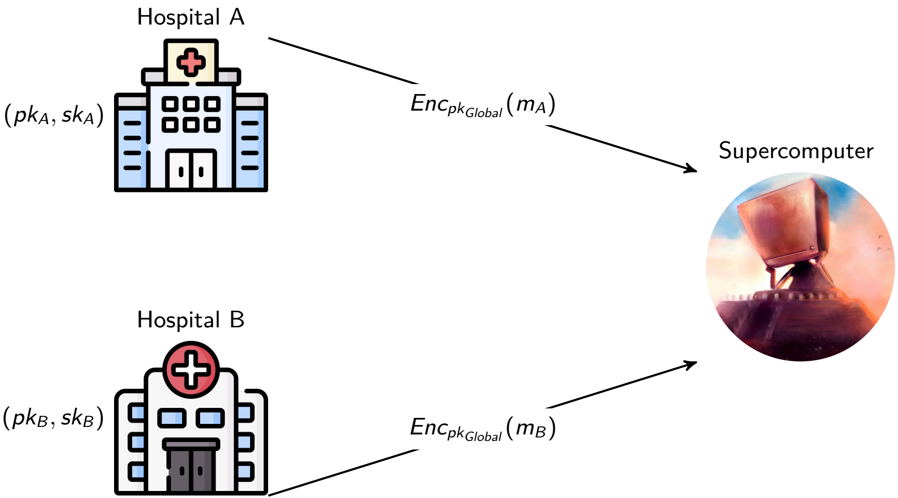
- **Problem 1**: Most constructions require an expensive ciphertext expansion mechanism.
- **Problem 2**: The size of the expanded ciphertexts grows linearly or quadratically on the number of parties.

Multiparty HE to the rescue



Multiparty HE to the rescue

$$pk_{Global} = pk_A + pk_B$$



Multikey vs Multiparty HE

- MKHE
  - Pros
    - Parties can join the protocol at any time
    - Faster key generation than MPHE
  - Cons
    - Requires ciphertext expansion
    - The size of ciphertexts grows with the number of parties involved
- MPHE
  - Pros
    - Similar performance to single-key HE schemes
    - No ciphertext expansion
  - Cons
    - Fix set of users during setup phase
    - No other parties can join the protocol afterwards

Contents

- The core operation of TFHE bootstrapping is *blind rotation*.
  - By *rotation*, we mean:

$$P(X) = a_0 + a_1 X + \cdots + a_{\mu-1} X^{\mu-1} + a_\mu X^\mu + \cdots + a_{N-1} X^{N-1} \in \mathcal{R}_q$$

$$P(X) \cdot X^{-\mu} = a_\mu + a_{\mu+1} X + \cdots + a_{N-1} X^{N-\mu-1} - a_0 X^{N-\mu} - \cdots - a_{\mu-1} X^{N-1} \in \mathcal{R}_q$$

  - By *blind*, we mean that we convert a $\mathrm{LWE}_s(\mu) = (a_1, \ldots, a_n, b)$ into a RLWE encryption of $X^{-\mu} \cdot v$, where v is a test polynomial and

$$-\mu \approx -b + \sum_{j=1}^{n} s_j a_j, \text{ with } s = (s_1, \ldots, s_n) \in \{0,1\}^n$$

  - In the binary case, we require n bootstrapping keys, computed as $bsk[j] \leftarrow RGSW(s_j)$.

Algorithm 1 Blind rotation in the binary case

1: $\mathrm{acc} \leftarrow (0, \ldots, 0, X^{-b} \cdot v)$
2: **for** $j = 1$ to n **do**
3: $\quad \mathrm{acc} \leftarrow \mathrm{acc} + \mathrm{bsk}[j] \boxdot ((X^{a_j} - 1) \cdot \mathrm{acc})$
4: **end for**
5: **return** acc

- Assume that we have a secret key space $\mathcal{S} = \{0, \ldots, k\}$.
- We can write

$$X^{s_j a_j} = \sum_{i=0}^{k} \mathbb{1}\{i = s_j\} X^{i \cdot a_j} = \sum_{i=1}^{k} \mathbb{1}\{i = s_j\}(X^{i \cdot a_j} - 1)$$

- Therefore, we can compute acc by setting $bsk[k(j-1)+i] \leftarrow RGSW(\mathbb{1}\{i = s_j\})$ and iterating

$$acc \leftarrow acc + \left(\sum_{i=1}^{k} (X^{i \cdot a_j} - 1) bsk[k(j-1)+i] \right) \boxdot acc$$

Blind rotation

- Let us consider $\mathcal{S} = \{0, 1, 2, 3, 4\}$ the secret key $s = (1, 2, 3, 4)$.
- Recall that $bsk[k(j-1)+i] \leftarrow RGSW(\mathbb{1}\{i = s_j\})$
- Define $RGSW(m) := \bar{m}$, then:

Blind rotation

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$$(j = 1, i = 1) \quad bsk[1] = \bar{1}$$

Blind rotation

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- Define $RGSW(m) := \bar{m}$, then:

$$(j = 1, i = 1) \quad bsk[1] = \bar{1}$$
$$(j = 1, i = 2) \quad bsk[2] = \bar{0}$$

Blind rotation

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- Recall that $bsk[k(j-1)+i] \leftarrow RGSW(\mathbb{1}\{i = s_j\})$
- Define $RGSW(m) := \bar{m}$, then:

$$
\begin{aligned}
(j = 1, i = 1) \quad & bsk[1] = \bar{1} \\
(j = 1, i = 2) \quad & bsk[2] = \bar{0} \\
(j = 1, i = 3) \quad & bsk[3] = \bar{0}
\end{aligned}
$$

Blind rotation

- Let us consider $\mathcal{S} = \{0, 1, 2, 3, 4\}$ the secret key $s = (1, 2, 3, 4)$.
- Recall that $bsk[k(j-1)+i] \leftarrow RGSW(\mathbb{1}\{i = s_j\})$
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\begin{aligned}
(j = 1, i = 1) \quad & bsk[1] = \bar{1} \\
(j = 1, i = 2) \quad & bsk[2] = \bar{0} \\
(j = 1, i = 3) \quad & bsk[3] = \bar{0} \\
(j = 1, i = 4) \quad & bsk[4] = \bar{0}
\end{aligned}
$$

- Let us consider $\mathcal{S} = \{0, 1, 2, 3, 4\}$ the secret key $s = (1, 2, 3, 4)$.
- Recall that $bsk[k(j-1)+i] \leftarrow RGSW(\mathbb{1}\{i = s_j\})$
- Define $RGSW(m) := \bar{m}$, then:

$$
\begin{aligned}
(j=1, i=1) \quad & bsk[1] = \bar{1} & \qquad (j=2, i=1) \quad & bsk[5] = \bar{0} \\
(j=1, i=2) \quad & bsk[2] = \bar{0} & \qquad (j=2, i=2) \quad & bsk[6] = \bar{1} \\
(j=1, i=3) \quad & bsk[3] = \bar{0} & \qquad (j=2, i=3) \quad & bsk[7] = \bar{0} \\
(j=1, i=4) \quad & bsk[4] = \bar{0} & \qquad (j=2, i=4) \quad & bsk[8] = \bar{0}
\end{aligned}
$$

$$
\begin{aligned}
(j=3, i=1) \quad & bsk[9] = \bar{0} & \qquad (j=4, i=1) \quad & bsk[13] = \bar{0} \\
(j=3, i=2) \quad & bsk[10] = \bar{0} & \qquad (j=4, i=2) \quad & bsk[14] = \bar{0} \\
(j=3, i=3) \quad & bsk[11] = \bar{1} & \qquad (j=4, i=3) \quad & bsk[15] = \bar{0} \\
(j=3, i=4) \quad & bsk[12] = \bar{0} & \qquad (j=4, i=4) \quad & bsk[16] = \bar{1}
\end{aligned}
$$

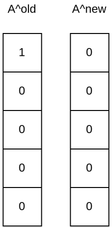
Blind rotation

- Consider a set of parties $\mathcal{P}_1, \ldots, \mathcal{P}_4$. Notation $RGSW(m) := \bar{m}$
- Each party i has a secret key $s_i \in \{0,1\}^n$.
- For example, assume that $s_{1,1} = 1, s_{2,1} = 1, s_{3,1} = 1, s_{4,1} = 0$.
- In our MPHE scheme, the global LWE secret key s_G will have $s_{G,1} = 3$.
- In this example, we also have $\mathcal{S} = \{0,1,2,3,4\}$.
- Therefore, the global bootstrapping key bsk_G will start with

$$
\begin{aligned}
(j=1, i=1) \quad & bsk_G[1] = \bar{0} \\
(j=1, i=2) \quad & bsk_G[2] = \bar{0} \\
(j=1, i=3) \quad & bsk_G[3] = \bar{1} \\
(j=1, i=4) \quad & bsk_G[4] = \bar{0}
\end{aligned}
$$

- Problem: the server only has encryptions of the secret keys
- We need a way to go from $(\bar{s}_{1,1}, \bar{s}_{2,1}, \bar{s}_{3,1}, \bar{s}_{4,1})$ to $(\bar{0}, \bar{0}, \bar{1}, \bar{0})$

Homomorphic Indicator (k = 4)



- For all $j = 0$ set

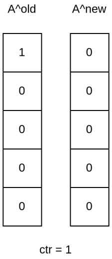
$$A^{new}[0] \leftarrow 0 \qquad \text{if ctr} = 1$$
$$A^{new}[0] \leftarrow A^{old}[0] \qquad \text{otherwise}$$

- For all $j \in \{1, \ldots, k\}$ set

$$A^{new}[j] \leftarrow A^{old}[j-1] \qquad \text{if ctr} = 1$$
$$A^{new}[j] \leftarrow A^{old}[j] \qquad \text{otherwise}$$

$$s_{1,1} = 1, s_{2,1} = 1, s_{3,1} = 1, s_{4,1} = 0$$

Homomorphic Indicator (k = 4)



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$$
\begin{aligned}
A^{new}[0] &\leftarrow 0 && \text{if ctr} = 1 \\
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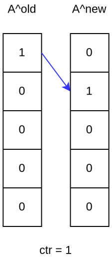

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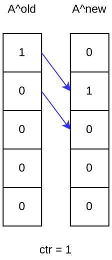

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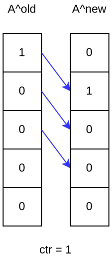
Homomorphic Indicator (k = 4)



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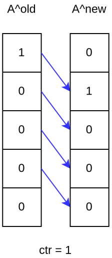
Homomorphic Indicator (k = 4)



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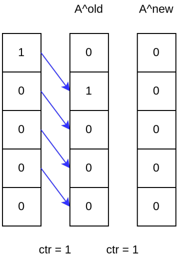

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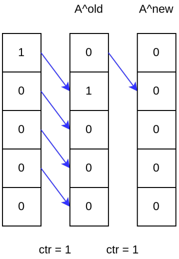

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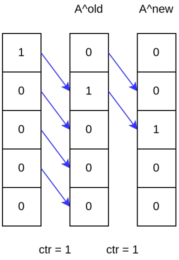
Homomorphic Indicator (k = 4)



- For all $j = 0$ set

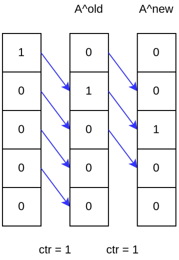
$$A^{new}[0] \leftarrow 0 \qquad \text{if ctr} = 1$$
$$A^{new}[0] \leftarrow A^{old}[0] \qquad \text{otherwise}$$

- For all $j \in \{1, \ldots, k\}$ set

$$A^{new}[j] \leftarrow A^{old}[j-1] \qquad \text{if ctr} = 1$$
$$A^{new}[j] \leftarrow A^{old}[j] \qquad \text{otherwise}$$

$$s_{1,1} = 1, \text{ctr} = 1, s_{3,1} = 1, s_{4,1} = 0$$

Homomorphic Indicator (k = 4)



- For all $j = 0$ set

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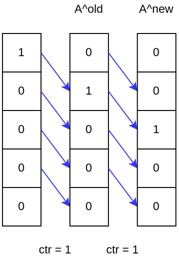
Homomorphic Indicator (k = 4)



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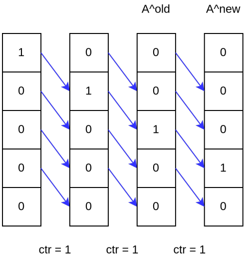
$$s_{1,1} = 1, \text{ctr} = 1, s_{3,1} = 1, s_{4,1} = 0$$

Homomorphic Indicator (k = 4)



- For all $j = 0$ set

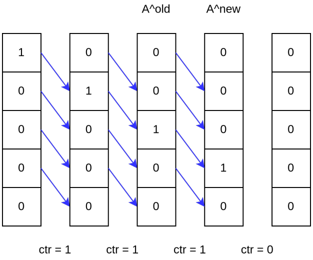
$$A^{new}[0] \leftarrow 0 \qquad \text{if ctr} = 1$$
$$A^{new}[0] \leftarrow A^{old}[0] \qquad \text{otherwise}$$

- For all $j \in \{1, \ldots, k\}$ set

$$A^{new}[j] \leftarrow A^{old}[j-1] \qquad \text{if ctr} = 1$$
$$A^{new}[j] \leftarrow A^{old}[j] \qquad \text{otherwise}$$

$$s_{1,1} = 1, s_{2,1} = 1, \text{ctr} = 1, s_{4,1} = 0$$

Homomorphic Indicator (k = 4)



- For all $j = 0$ set

$$A^{new}[0] \leftarrow 0 \quad \text{if ctr} = 1$$
$$A^{new}[0] \leftarrow A^{old}[0] \quad \text{otherwise}$$

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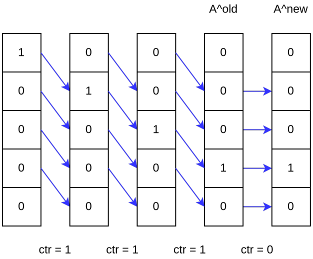
Homomorphic Indicator (k = 4)



- For all $j = 0$ set

$$A^{new}[0] \leftarrow 0 \qquad \text{if ctr} = 1$$
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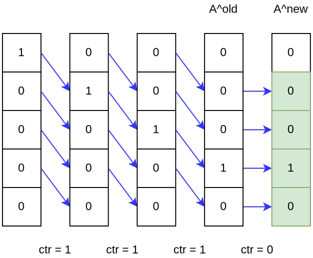

- For all $j = 0$ set

$$A^{new}[0] \leftarrow 0 \qquad \text{if ctr} = 1$$
$$A^{new}[0] \leftarrow A^{old}[0] \qquad \text{otherwise}$$

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$$A^{new}[j] \leftarrow A^{old}[j-1] \qquad \text{if ctr} = 1$$
$$A^{new}[j] \leftarrow A^{old}[j] \qquad \text{otherwise}$$

$$s_{1,1} = 1, s_{2,1} = 1, s_{3,1} = 1, \text{ctr} = 0$$

Homomorphic Indicator

Algorithm 2 Homomorphic Indicator (Hom.Indicator)

Require: $\{\mathbf{C}_i\}_{i \in [m]}, \mathsf{A}^{new}$ and A^{old}.
Ensure: A^{old}.

```
for i ← 1 to k do
    for j ← 1 to k do
        A^new[j] := CMUX_⊠(C_i, A^old[j], A^old[j − 1])
    end for
    A^new[0] := A^old[0] ⊠ (1 − C_i)
    for j ← 0 to k do
        A^old[j] := A^new[j]
    end for
end for
```

Bootstrapping Keys Generation

Algorithm 3 Global bootstrapping key generation

Require: $\{\mathsf{bsk}_i\}_{i \in [k]}, \mathsf{A}^{new}$ and A^{old}.

Ensure: $\widehat{\mathsf{bsk}}$.

```
for t ← 0 to n − 1 do
    for i ← 1 to k do
        Parse C_{i,t} := bsk_i[t]
    end for
    A := Hom.Indicator({C_{i,t}}_{i∈[k]}, A^new, A^old)
    bsk^[t] := [A[1], . . . , A[k]]
    Refresh A^new and A^old
end for
```

$$
\begin{aligned}
&\textbf{for } t \leftarrow 0 \text{ to } n-1 \textbf{ do}\\
&\quad \textbf{for } i \leftarrow 1 \text{ to } k \textbf{ do}\\
&\quad\quad \text{Parse } \mathbf{C}_{i,t} := \mathsf{bsk}_i[t]\\
&\quad \textbf{end for}\\
&\quad \mathsf{A} := \mathsf{Hom.Indicator}(\{\mathbf{C}_{i,t}\}_{i \in [k]}, \mathsf{A}^{new}, \mathsf{A}^{old})\\
&\quad \widehat{\mathsf{bsk}}[t] := [\mathsf{A}[1], \ldots, \mathsf{A}[k]]\\
&\quad \text{Refresh } \mathsf{A}^{new} \text{ and } \mathsf{A}^{old}\\
&\textbf{end for}
\end{aligned}
$$

Contents

k	N	n	log q	log Q	$\sigma_{\text{rlwe}}(=\theta)$	σ_{lwe}	B	l	Time (in seconds)	Bootstrapping noise	Bsk noise
2	2048	530	32	64	$1.85 \cdot 2^{4.2}$	2^{17}	12	3	**0.20**	56.2 (24.2)	35.91
							6	8	0.48	**45.6 (13.6)**	30.37
4	2048	495	32	64	$1.85 \cdot 2^{4.2}$	2^{17}	11	4	**0.33**	56.12 (24.12)	36.95
							7	7	0.59	**48.97 (16.97)**	32.98
8	2048	495	32	64	$1.85 \cdot 2^{4.2}$	2^{17}	8	4	**0.46**	57.51 (57.51)	40.29
							7	6	0.70	**50.65 (18.65)**	33.85
16	2048	495	32	64	$1.85 \cdot 2^{4.2}$	2^{17}	10	5	**0.90**	58.37 (26.37)	38.02
							7	6	1.06	**52.79 (20.79)**	35.81

Table: Parameter sets recommended achieving at least 110-bit security based on LWE estimator for different number parties k.The last three columns correspond to the average of 500 NAND operations, each performed with a freshly encrypted LWE ciphertext.

Thank you!

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