High-precision RNS-CKKS on small word-size architecture

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FHE.org Meetup

Jan 11th, 2024

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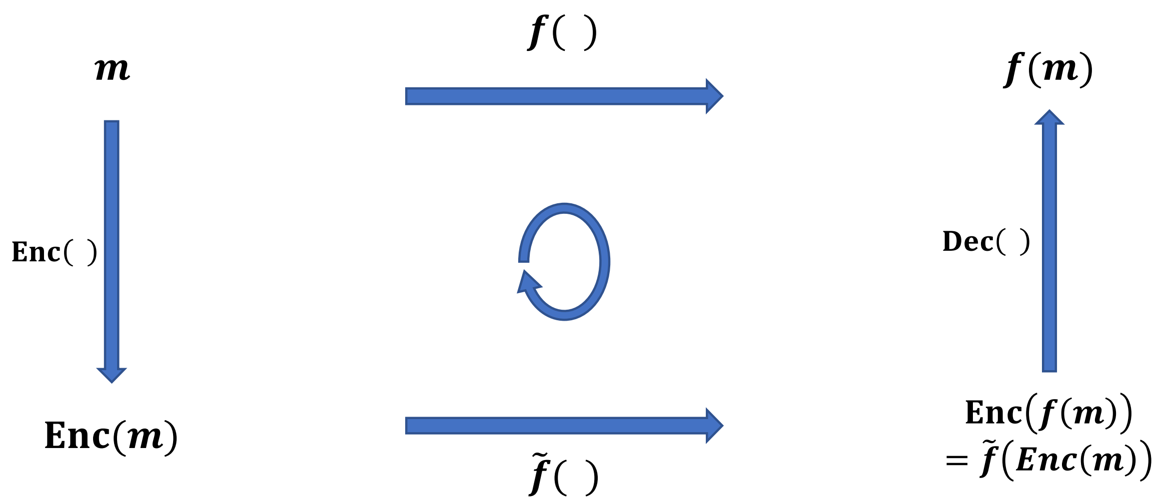
* This research was, in part, funded by the Defense Advanced Research Projects Agency (DARPA) through contract HR0011-21-3-0003. The views, opinions, and findings expressed are those of the authors and should not be interpreted as representing the official views or policies of the Department of Defense or the U.S. Government. Distribution Statement ‘A’ (Approved for Public Release, Distribution Unlimited).

Overview

- Enable high-precision RNS-CKKS on fixed but smaller word-size architectures
  - Single scaling → **Composite scaling**
- Enable functionally correct CKKS composite scaling in two open-source libraries
  - OpenFHE: C++, enabled by Intel labs
  - Lattigo: Go, enabled by Seoul National University (SNU)
- Demonstrate with secure parameters the **equivalence** between single and composite scaling
  - **7-layer CNN Inference** with longitudinal packing in OpenFHE-CKKS with composite scaling
  - **7-layer CNN Inference** with multiplexed packing in Lattigo-CKKS with composite scaling
  - **Logistic Regression Training** in OpenFHE-CKKS with composite scaling

Fully Homomorphic Encryption (FHE)

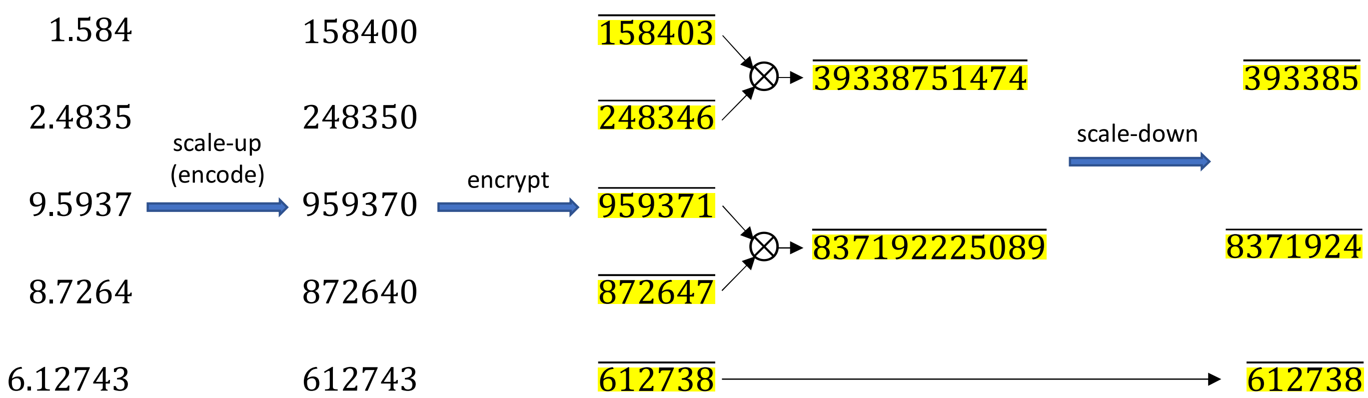
- Any computation on encrypted data “without decryption process”



CKKS: FHE for real-number arithmetic

How can we think of the "approximate" computation in CKKS?

- **Imitation of "fixed-point" arithmetic in cleartext version**
- Example: computation of $1.584 \times 2.4835 \times 9.5937 \times 8.7264 \times 6.12743$ (≈ 2017.9897)



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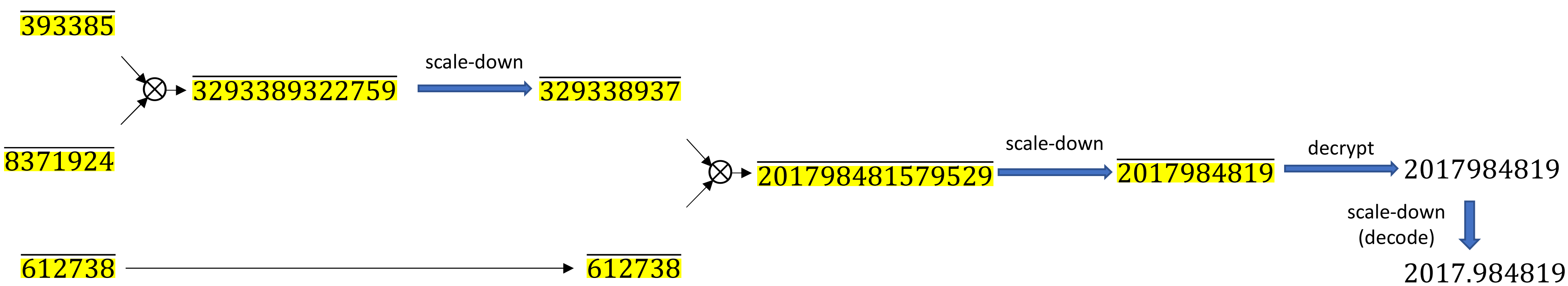

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Scaling Factor in CKKS

- Determine the **"initial precision bits"** under the decimal point
- CKKS Encoding/Encryption results in

$$m \mapsto \underbrace{\boldsymbol{\Delta}}_{\text{scaling factor}} \cdot m + \underbrace{e}_{\substack{\text{encoding/encryption error} \\ \text{(size determined by FHE params)}}}$$

- Larger $\boldsymbol{\Delta}$, start with higher precision
- Smaller $\boldsymbol{\Delta}$, start with lower precision

Scaling Factor in CKKS

- Exponential growth of Scaling Factor
  - $(\boldsymbol{\Delta} \cdot m) \cdot (\boldsymbol{\Delta} \cdot m') = \boldsymbol{\Delta}^{\mathbf{2}} \cdot mm'$
  - $\left(\boldsymbol{\Delta}^{\mathbf{2}^{k}} \cdot m\right) \cdot \left(\boldsymbol{\Delta}^{\mathbf{2}^{k}} \cdot m'\right) = \boldsymbol{\Delta}^{\mathbf{2}^{k+1}} \cdot mm'$
- How to control the growth of scaling factor?

“rescale”

- Rescale(ct): $ct \bmod \Delta^{\ell} \mapsto \left\lfloor \frac{ct}{\Delta} \right\rceil \bmod \Delta^{\ell-1}$ (from the context of “original” CKKS)
  - $(\boldsymbol{\Delta} \cdot m) \cdot (\boldsymbol{\Delta} \cdot m') = \boxed{\boldsymbol{\Delta}^{\mathbf{2}} \cdot mm' \xrightarrow{\text{Rescale } (1/\Delta)} \boldsymbol{\Delta} \cdot mm'}$

Rescale in RNS-CKKS

- RNS-CKKS
  - An efficient way to implement CKKS w/o big-number arithmetic
  - Ctxt moduli $Q_\ell = q_0 q_1 \cdots q_\ell$ for level ℓ (instead of modulo Δ^ℓ)

$$\mathbf{RNS}_{Q_\ell}(\boldsymbol{x}) \coloneqq (\boldsymbol{x} \ \mathbf{mod}\ \boldsymbol{q_0}, \boldsymbol{x} \ \mathbf{mod}\ \boldsymbol{q_1}, \dots, \boldsymbol{x} \ \mathbf{mod}\ \boldsymbol{q_\ell})$$

- Rescale modulo Q_ℓ in RNS?
  - **No efficient** way to compute $\mathbf{x} \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{\Delta}} \cdot \boldsymbol{x} \right\rceil$
  - Instead, we can **efficiently** compute $\mathbf{x} \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q_\ell}} \cdot \boldsymbol{x} \right\rceil$
    - $\left\lfloor \frac{1}{q_\ell} \cdot x \right\rceil = q_\ell^{-1} \cdot (x - x \bmod q_\ell)$
    - Easy to obtain the RNS representation of $x \bmod q_\ell$
      - $\mathrm{RNS}_{Q_{\ell-1}}(x \bmod q_\ell) = (x \bmod q_\ell, x \bmod q_\ell, \dots, x \bmod q_\ell)$

Rescale in RNS-CKKS

- **Case 1: $\log \Delta$ < word-size**
  - Set each prime q_ℓ to be $\log \Delta$ bits
  - Perform the **"single scaling"**

$$\mathbf{x} \bmod Q_\ell \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q}_\ell} \cdot \boldsymbol{x} \right\rceil \bmod Q_{\ell-1}$$

- **Case 2: $\log \Delta$ > word-size**
  - Set each product of q_ℓ's to be $\log \Delta$ bits
  - Perform the **"composite scaling"**

Rescale in RNS-CKKS

- **Case 1: $\log \Delta$ < word-size**
  - Set each prime q_ℓ to be $\log \Delta$ bits
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$$\mathbf{x} \bmod Q_\ell \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q}_\ell} \cdot \boldsymbol{x} \right\rceil \bmod Q_{\ell-1}$$

- **Case 2: $\log \Delta$ > word-size**
  - Set each product of q_ℓ's to be $\log \Delta$ bits
  - Perform the **"composite scaling"** (degree = 2)

$$\mathbf{x} \bmod Q_\ell \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q}_\ell \boldsymbol{q}_{\ell-1}} \cdot \boldsymbol{x} \right\rceil \bmod Q_{\ell-2}$$

Rescale in RNS-CKKS

- **Case 1: $\log \Delta$ < word-size**
  - Set each prime q_ℓ to be $\log \Delta$ bits
  - Perform the **"single scaling"**

$$\mathbf{x} \bmod Q_\ell \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q}_\ell} \cdot \boldsymbol{x} \right\rceil \bmod Q_{\ell-1}$$

- **Case 2: $\log \Delta$ > word-size**
  - Set each product of q_ℓ's to be $\log \Delta$ bits
  - Perform the **"composite scaling"** (degree = 3)

$$\mathbf{x} \bmod Q_\ell \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q}_\ell \boldsymbol{q}_{\ell-1} \boldsymbol{q}_{\ell-2}} \cdot \boldsymbol{x} \right\rceil \bmod Q_{\ell-3}$$

Rescale in RNS-CKKS

- **Case 1: $\log \Delta$ < word-size**
  - Set each prime q_ℓ to be $\log \Delta$ bits
  - Perform the **"single scaling"**

$$\mathbf{x} \bmod Q_\ell \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q}_\ell} \cdot \boldsymbol{x} \right\rceil \bmod Q_{\ell-1}$$

- **Case 2: $\log \Delta$ > word-size**
  - Set each product of q_ℓ's to be $\log \Delta$ bits
  - Perform the **"composite scaling"** (degree = t)

$$\mathbf{x} \bmod Q_\ell \mapsto \left\lfloor \frac{\mathbf{1}}{\boldsymbol{q}_\ell \cdots \boldsymbol{q}_{\ell-t+1}} \cdot \boldsymbol{x} \right\rceil \bmod Q_{\ell-t}$$

Rescale in RNS-CKKS

- **Examples**
  - $\log\Delta = \mathbf{30}$, word-size $= \mathbf{64}$: single scaling
  - $\log\Delta = \mathbf{30}$, word-size $= \mathbf{32}$: single scaling
  - $\log\Delta = \mathbf{50}$, word-size $= \mathbf{64}$: single scaling
  - $\log\Delta = \mathbf{50}$, word-size $= \mathbf{32}$: composite scaling (double-prime)
  - $\log\Delta = \mathbf{70}$, word-size $= \mathbf{64}$: composite scaling (double-prime)
  - $\log\Delta = \mathbf{70}$, word-size $= \mathbf{32}$: composite scaling (triple-prime)

$\vdots \qquad \vdots \qquad \vdots$

Precision Issue due to Rescale

- Original CKKS: **NO** precision issue
  - ➢ Scaling factor is **ALWAYS** preserved as $\boldsymbol{\Delta}$
- RNS-CKKS: **YES** precision issue
  - ➢ Scaling factor is **NOT** be preserved as $\boldsymbol{\Delta}$
    - ○ Division by q_ℓ's, instead of Δ
    - ○ $\boldsymbol{\Delta^2/q_\ell \neq \Delta}$
  - ➢ Critical Impact to **Homomorphic Addition**
    - ○ $\mathrm{Enc}(\Delta \cdot m) + \mathrm{Enc}(\Delta' \cdot m') = \mathrm{Enc}(\Delta \cdot (m + \boldsymbol{\Delta'/\Delta} \cdot m'))$
      $\neq \mathrm{Enc}(\Delta \cdot (m + m'))$
    - ○ The ratio $\boldsymbol{\Delta'/\Delta}$ $(\neq 1)$ directly harms the precision

Precision Issue due to Rescale

- **Solution 1: Choose the primes properly**
  - To keep the scaling factors (not equal but) **very close to Δ**
  - Single Scaling
    - Requirement: $\boldsymbol{q_\ell \simeq \Delta}$ (proposed in original RNS-CKKS)
    - $\boldsymbol{\Delta^2 / q_\ell \simeq \Delta}$
  - Composite Scaling
    - Requirement: $\boldsymbol{q_\ell q_{\ell-1} \simeq \Delta}$
    - $\boldsymbol{\Delta^2 / q_\ell q_{\ell-1} \simeq \Delta}$
  - **Precision** (Single Scaling v.s. Composite Scaling)
    - **NO Difference** in Mult + Relin + Rescale
    - **Closeness** of q_ℓ (resp. $q_\ell q_{\ell-1}$) and Δ affects the **Add Precision**

Precision Issue due to Rescale

- **Solution 2: Exact Scaling**
  - Differences v.s. Solution 1
    - Scaling factor $\boldsymbol{\Delta_i}$ for each level $\boldsymbol{i}$
    - $\boldsymbol{\Delta_i}$'s are **NOT** required to be **very close to Δ**
    - Adjust the ciphertext scaling factors to $\boldsymbol{\Delta_i}$ before Add and Mult
    - As a result, we "always" add two ciphertexts with "same" scaling factors
  - **Precision** (Single Scaling v.s. Composite Scaling)
    - **NO Difference** in Mult + Relin + Rescale
    - **NO Difference** in Add
  - We implemented 32-bit RNS-CKKS in OpenFHE and Lattigo with **Solution 2**
    - "FLEXIBLEAUTO" mode in OpenFHE
    - Bootstrapping enabled in both libraries

Theoretical Analysis on Precision

$\mathrm{Rescale}(ct)$: $\quad ct \bmod Q_i \mapsto \left\lfloor \frac{1}{q_i} \cdot ct \right\rceil \bmod Q_{i-1}$ (single scaling)

$\mathrm{Rescale}^{(t)}(ct)$: $\quad ct \bmod Q_i \mapsto \left\lfloor \frac{1}{q_i q_{i-1} \cdots q_{i-t+1}} \cdot ct \right\rceil \bmod Q_{i-t}$ (composite scaling)

Theorem. *Let* B_{rs}, B_{comp-} *be the upper bounds of the error induced by* $\mathrm{Rescale}(\cdot)$ *and* $\mathrm{Rescale}^{(t)}(\cdot)$*, respectively. Then, it holds that*

$$B_{comp-rs} \leq \left(\frac{1}{q_i q_{i-1} \cdots q_{i-t+1}} + \frac{1}{q_i q_{i-1} \cdots q_{i-t+2}} + \cdots + \frac{1}{q_i} + 1 \right) B_{rs} \approx \left(\frac{1}{q_i} + 1 \right) B_{rs}$$

Hence, composite scaling results in less than $\log\left(\frac{1}{q_i} + 1\right) \approx \frac{3.322}{q_i}$ *bit* ***precision loss****, which is* ***negligible****, compared to single scaling.*

Experimental Results

7-layer CNN Inference (CIFAR-10)

- Implementation in OpenFHE with longitudinal packing
  - **Unit tests** with **Same Precision**

Unit tests	Precision bits 64-bit single scaling	Precision bits 32-bit composite scaling
Fully connected	39	39
ReLU	40	40
Mean pool	41	41
Convolution	39	39
Bootstrapping	12	12

========== **Parameters** ==========

Ring dimension : **65536**

Scaling factor : $\mathbf{2^{58}}$

- **Same** for both cases

Primes

- **58-bit** primes for 64-bit case
- **(29, 29)-bit** primes for 32-bit case
  - Double-prime scaling
  - 58 = 29 + 29

Security

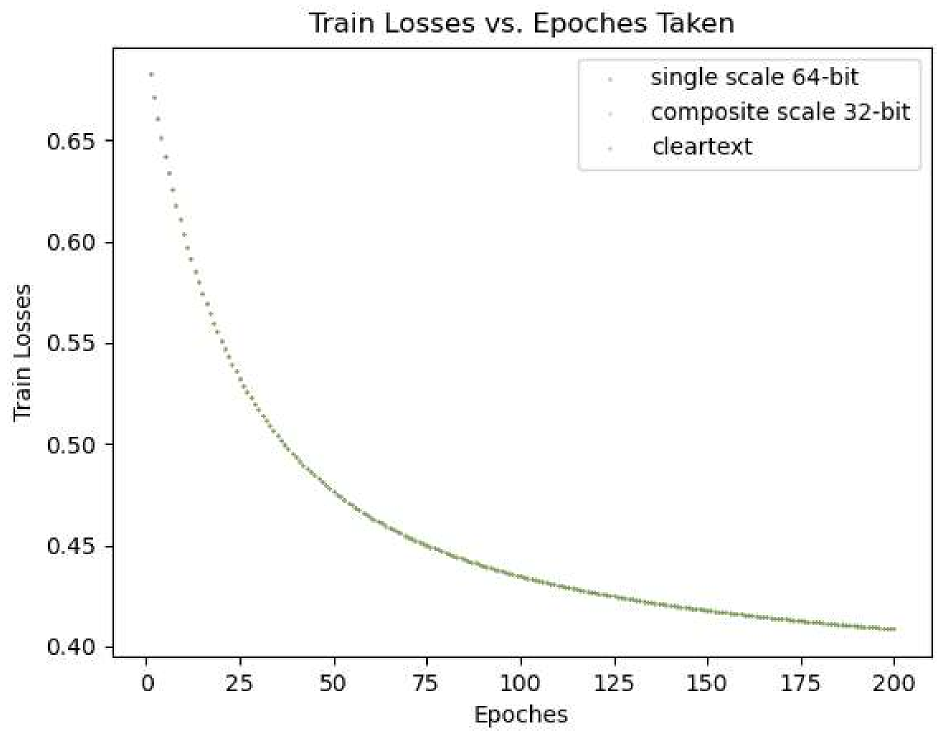
- **Same** for both cases

- Implementation in Lattigo with multiplexed packing
  - The **end-to-end CNN Inference results match** up to **5** digits after the decimal point
  - 14 consecutive bootstrapping (2 per layer, before and after ReLU)

Experimental Results

Logistic Regression Training

- Reference code: https://github.com/openfheorg/openfhe-logreg-training-examples
- 1 bootstrapping per epoch



========== Parameters ==========

Ring dimension : 32768

Scaling factor : 2^{58}

- **Same** for both cases

Primes

- **58-bit** primes for 64-bit case
- **(29, 29)-bit** primes for 32-bit case
  - Double-prime scaling
  - 58 = 29 + 29

Security

- **Same** for both cases

 # Bootstrapping

- **Same** for both cases

Wrap-up

- **Result:** Enable high-precision RNS-CKKS on small word-size architectures without multi-precision arithmetic
  - Use of small word-size: GPU, FPGA, Embedded devices, etc.
  - Arbitrary precision for bootstrapping combined with Meta-BTS
- **Limitation:** Choice of scaling factor
  - Lower bound exists on each prime (NTT condition)
  - E.g., $\Delta = 2^{40} \rightarrow$ two 20-bit primes for double-prime scaling
    - How many 20-bit "NTT-friendly" primes exist for the dimension $N = 2^{16}$?
  - Several small intervals that are not usable as scaling factor
- **Implementation:** Not public yet but planning for open-sourcing composite-scaling variant of OpenFHE-CKKS

https://eprint.iacr.org/2023/1462