Low Latency Evaluation of AES via (leveled) TFHE

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Advanced Encryption Scheme (AES)

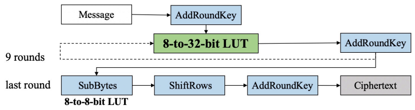
The Advanced Encryption Standard (AES) is a widely embraced block encryption standard by the United States federal government, known for its efficiency and prevalent use in securing sensitive information across diverse applications.

Motivation: AES stands as one of the top choices for application in the transciphering framework. How to achieve efficient AES evaluation has been a major challenge for the transciphering community.

Implementation methods of AES:

(1) Using four basic functions
SubBytes, RowShifts, MixColumns and AddRoundKey

(2) Using LUT-based implementation
Merge SubBytes, RowShifts and MixColumns three functions into 8-to-32-bit LUT, as follows. We present faster evaluation of AES using this implementation based on leveled TFHE.



TFHE

The TFHE scheme is based on the LWE and RLWE problems and it supports efficient gate bootstrapping, functional(programmable) bootstrapping and circuit bootstrapping.

(1) TFHE ciphertexts:
TLWE(m)、TRLWE(m(x))、TRGSW(m(x))

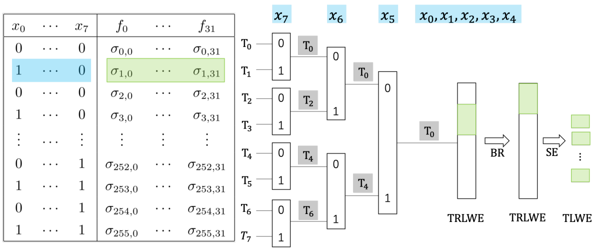
(2) Building Blocks:
- External Multiplication ⊡: TRGSW×TRLWE→TRLWE
- $\mathrm{CMUX}(c, d_1, d_0)$: $c ⊡ (d_1 - d_0) + d_0$
- KeySwitching: TLWE→T(R)LWE、TRLWE→TRLWE
- SampleExtraction(SE): TRLWE→TLWE
- BlindRotation(BR): Rotate the test polynomial blindly

(3) Bootstrapping types in TFHE:
- Identity bootstrapping
- Gate bootstrapping
- Functional (Programmable) bootstrapping
- Full domain functional bootstrapping
- Multi-value bootstrapping (PBSmanyLUT)
- Circuit bootstrapping: TLWE→TRGSW

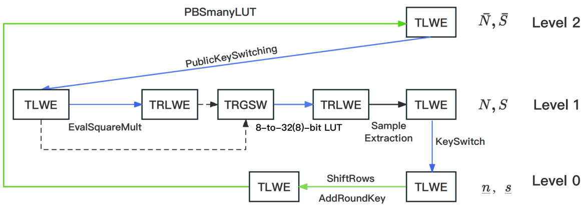
Evaluation Framework

Message Encoding: $\{0,1\} \to \{0, \frac{1}{2}\}$ over the Torus

1、Efficient 8-to-32-bit lookup table using CMUX and mixed packing



2、Efficient AES evaluation framework based on leveled TFHE



ShiftRows and AddRoundKey can be evaluated at Level 0 for free, while SubBytes can be performed in Level 1 efficiently.

Efficient Circuit Bootstrapping

New TLWE-to-TRGSW Conversion: Bridge the evaluation of AES

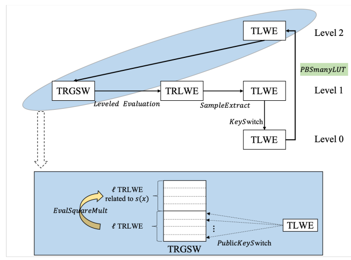
(1) **PBSmanyLUT:** $\mathrm{TLWE}(m) \to \mathrm{TLWE}\left(m \cdot \frac{1}{B^{-j}}\right), j = 1, \cdots, \ell$

(2) The second ℓ rows of TRGSW can be constructed by **PublicKeySwitch:**

$$\mathrm{TLWE}(m) \to \mathrm{TRLWE}\left(m \cdot \frac{1}{B^{-j}}\right), j = 1, \cdots, \ell$$

The first ℓ rows of TRGSW can be constructed by **EvalSquareMult:**

$$\mathrm{TRLWE}\left(m \cdot \frac{1}{B^{-j}}\right) \to \mathrm{TRLWE}\left(-s \cdot m \cdot \frac{1}{B^{-j}}\right), j = 1, \cdots, \ell$$



A test polynomial that satisfies the **negacyclic** property for PBSmanyLUT as follows:

$$P(X) = \sum_{i=0}^{\frac{N}{2^{\rho}\cdot 2}-1} \sum_{j=0}^{2^{\rho}-1} (-1) \cdot \frac{1}{2B_g^j} X^{2^{\rho}\cdot i + j} + \sum_{i=\frac{N}{2^{\rho}\cdot 2}}^{\frac{N}{2^{\rho}}-1} \sum_{j=0}^{2^{\rho}-1} \frac{1}{2B_g^j} X^{2^{\rho}\cdot i + j}, \text{ where } \rho = \lceil \log_2 \ell \rceil$$

Performance

Experimental environment

a single core of Intel(R) Core(TM) i5-11500 CPU @ 2.70GHz and 32 GB RAM, running the Ubuntu 20.04 operating system.

Implementation result

Table: Comparison of AES-128 evaluation latency based on different schemes

Scheme	Evaluation mode	Latency	Amortized
BGV	Leveled[GHS12]	4 mins	2 s
	Bootstrapping[GHS12]	18 mins	6 s
CKKS	Bootstrapping[ADE+23]	31mins	56.7 ms*
TFHE	Functional bootstrapping[SMK22]	4.2 mins	4.2 mins*
	Functional bootstrapping[TCBS23]	270 s	270 s
	Functional bootstrapping[BPR23]	211 s	211 s
	Ours(Leveled)	**46 s**	**46 s**

Our AES evaluation latency based on leveled TFHE is about 5x improvement over the state-of-the-art work in terms of latency.

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