

联立方程 (假设结论对称)

可以背的结论及验证方式

$$\begin{cases} y = kx + m \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \end{cases}$$

$$\therefore b^2 x^2 + a^2 (k^2 x^2 + 2kmx + m^2) - a^2 b^2 = 0$$

$$\therefore (a^2 k^2 + b^2) x^2 + 2kma^2 x + (a^2 m^2 - a^2 b^2) = 0$$

Checkpoint: 一次项为单项式 [1]

$$\Delta = 4k^2 m^2 a^4 - 4a^2 (m^2 - b^2) (a^2 k^2 + b^2)$$

$$= 4k^2 m^2 a^4 + 4a^2 (a^2 b^2 k^2 + b^4 - a^2 m^2 k^2 - b^2 m^2)$$

$$= 4a^4 b^2 k^2 + 4a^2 b^4 - 4a^2 b^2 m^2$$

$$= 4a^2 b^2 (a^2 k^2 + b^2 - m^2)$$

Checkpoint: 没有  $k^2 m^2$  四次项 [2]

椭圆相关 二次项系数 截距相关

$$\Delta > 0 \Leftrightarrow m^2 < a^2 k^2 + b^2$$

$$x_1 + x_2 = -\frac{2kma^2}{a^2 k^2 + b^2}$$

$$x_1 \cdot x_2 = \frac{a^2 (m^2 - b^2)}{a^2 k^2 + b^2}$$

$$|x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1 x_2} = \frac{2ab \sqrt{a^2 k^2 + b^2 - m^2}}{a^2 k^2 + b^2}$$

$$y_1 + y_2 = k(x_1 + x_2) + 2m = -\frac{2k^2 m a^2}{a^2 k^2 + b^2} + \frac{2k^2 m a^2 + 2mb^2}{a^2 k^2 + b^2} = \frac{2mb^2}{a^2 k^2 + b^2}$$

$$y_1 \cdot y_2 = (kx_1 + m)(kx_2 + m) = k^2 x_1 x_2 + km(x_1 + x_2) + m^2 = \frac{k^2 a^2 m^2 - k^2 a^2 b^2 - 2k^2 m^2 a^2 + k^2 a^2 m^2 + m^2 b^2}{a^2 k^2 + b^2} = \frac{b^2 (m^2 - k^2 a^2)}{a^2 k^2 + b^2}$$

Checkpoint: 消得很爽 [3]

$$|y_1 - y_2| = k|x_1 - x_2| = \frac{2abk \sqrt{a^2 k^2 + b^2 - m^2}}{a^2 k^2 + b^2}$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \frac{2ab}{a^2 k^2 + b^2} \sqrt{(k^2 + 1)(a^2 k^2 + b^2 - m^2)}$$

