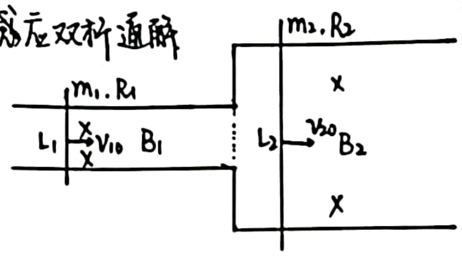
电磁感应双杆通解



[illegible]电流正方向

$E_1 = B_1L_1v_1$, $E_2 = -B_2L_2v_2$, $B_1L_1v_1 - B_2L_2v_2 = \bar{E}_{\sigma} = I(R_1+R_2)$. $\therefore I = \dfrac{B_1L_1v_1 - B_2L_2v_2}{R_1+R_2}$.

$F_1 = -IL_1B_1 = m_1\dfrac{dv_1}{dt}$, $F_2 = IL_2B_2 = m_2\dfrac{dv_2}{dt}$. $\therefore \dfrac{m_1}{L_1B_1}\Delta v_1 + \dfrac{m_2}{L_2B_2}\Delta v_2 = 0$,

$\lambda_1F_1 + \lambda_2F_2 = I(\lambda_2L_2B_2 - \lambda_1L_1B_1) = \dfrac{d(m_2v_2\lambda_2 + m_1v_1\lambda_1)}{dt} = \dfrac{(\lambda_2L_2B_2 - \lambda_1L_1B_1)(B_1L_1v_1 - B_2L_2v_2)}{R_1+R_2}$

令 $\lambda_1 = \dfrac{B_1L_1}{m_1}$, $\lambda_2 = -\dfrac{B_2L_2}{m_2}$. 记 $V = B_1L_1v_1 - B_2L_2v_2$.

$\dfrac{dV}{dt} = \left(\dfrac{B_1^2L_1^2}{m_1} + \dfrac{B_2^2L_2^2}{m_2}\right)\dfrac{-V}{R_1+R_2}$, $\ln V = -\dfrac{m_2B_1^2L_1^2 + m_1B_2^2L_2^2}{m_1m_2(R_1+R_2)}t + \ln C$, $C = B_1L_1v_{10} - B_2L_2v_{20}$

$\therefore B_1L_1v_1 - B_2L_2v_2 = (B_1L_1v_{10} - B_2L_2v_{20})e^{-\frac{m_2B_1^2L_1^2 + m_1B_2^2L_2^2}{m_1m_2(R_1+R_2)}t}$, 记 $\xi = \exp\left(-\dfrac{m_2B_1^2L_1^2 + m_1B_2^2L_2^2}{m_1m_2(R_1+R_2)}t\right)$

$\therefore B_1L_1(v_1 - \xi v_{10}) = B_2L_2(v_2 - \xi v_{20})$ 记 $-\dfrac{m_1}{L_1B_1}\Delta v_1 = \dfrac{m_2}{L_2B_2}\Delta v_2 = u$.

$\therefore v_1 = v_{10} + \Delta v_1 = v_{10} - \dfrac{L_1B_1}{m_1}u$, $v_2 = v_{20} + \Delta v_2 = v_{20} + \dfrac{L_2B_2}{m_2}u$,

$\therefore B_1L_1\left(v_{10} - \xi v_{10} - \dfrac{L_1B_1}{m_1}u\right) = B_2L_2\left(v_{20} - \xi v_{20} + \dfrac{L_2B_2}{m_2}u\right)$,

$\therefore u\left(\dfrac{B_1^2L_1^2}{m_1} + \dfrac{B_2^2L_2^2}{m_2}\right) = (1-\xi)(B_1L_1v_{10} - B_2L_2v_{20})$, $\therefore u = \dfrac{m_1m_2(B_1L_1v_{10} - B_2L_2v_{20})}{m_2B_1^2L_1^2 + m_1B_2^2L_2^2}(1-\xi)$.

记 $A_c = \dfrac{m_2B_1^2L_1^2 + m_1B_2^2L_2^2}{m_1m_2}$, $A_v = B_1L_1v_{10} - B_2L_2v_{20}$, $u = \dfrac{A_v}{A_c}\left(1 - e^{-\frac{A_c}{R_1+R_2}t}\right)$.

$v_1 = v_{10} - \dfrac{L_1B_1A_v}{m_1A_c}\left(1 - e^{-\frac{A_c}{R_1+R_2}t}\right)$, $v_2 = v_{20} + \dfrac{L_2B_2A_v}{m_2A_c}\left(1 - e^{-\frac{A_c}{R_1+R_2}t}\right)$,

记初始距离 D_0. $D = D_0 - \int_0^t v_1dt + \int_0^t v_2dt = D_0 - \int_0^t (v_1 - v_2)dt$.

$v_1 - v_2 = (v_{10} - v_{20}) - \left(\dfrac{L_1B_1}{m_1} - \dfrac{L_2B_2}{m_2}\right)\cdot\dfrac{A_v}{A_c}\left(1 - e^{-\frac{A_c}{R_1+R_2}t}\right)$.

$\therefore D = D_0 - \left[(v_{10} - v_{20}) - \left(\dfrac{L_1B_1}{m_1} - \dfrac{L_2B_2}{m_2}\right)\dfrac{A_v}{A_c}\right]t + \int_{(0)}^{(t)}\left(\dfrac{L_1B_1}{m_1} - \dfrac{L_2B_2}{m_2}\right)\dfrac{A_v(R_1+R_2)}{A_c^2}e^{-\frac{A_c}{R_1+R_2}t}\,d\left(-\dfrac{A_c}{R_1+R_2}t\right)$

$= D_0 - (v_{10} - v_{20})t + \left(\dfrac{L_1B_1m_2 - L_2B_2m_1}{m_1m_2}\dfrac{A_v}{A_c}\right)t - \dfrac{(m_2L_1B_1 - m_1L_2B_2)A_v(R_1+R_2)}{m_1m_2A_c^2}\left(1 - e^{-\frac{A_c}{R_1+R_2}t}\right)$

$\because V = A_ve^{-\frac{A_c}{R_1+R_2}t}$, $I = \dfrac{V}{R_1+R_2}$.

$\therefore Q = \int_0^t I^2(R_1+R_2)dt = \int_0^t \dfrac{A_v^2e^{-\frac{2A_c}{R_1+R_2}t}}{R_1+R_2}dt = \dfrac{A_v^2}{2A_c} - \dfrac{A_v^2}{2A_c}e^{-\frac{2A_c}{R_1+R_2}t}$

$$\left.\begin{aligned} E_{k0} &= \tfrac{1}{2}m_1v_{10}^2 + \tfrac{1}{2}m_2v_{20}^2 \\ E_{kf} &= \tfrac{1}{2}m_1\left[v_{10} - \tfrac{L_1B_1A_v}{m_1A_c}\left(1 - e^{-\frac{A_c}{R_1+R_2}t}\right)\right]^2 + \tfrac{1}{2}m_2\left(v_{20} + \tfrac{L_2B_2A_v}{m_2A_c} - \tfrac{L_2B_2A_v}{m_2A_c}e^{-\frac{A_c}{R_1+R_2}t}\right)^2 \end{aligned}\right\} \Rightarrow E_{k0} - E_{kf} = Q$$

$q = \int_0^t Idt = \int_0^t \dfrac{A_v}{R_1+R_2}e^{-\frac{A_c}{R_1+R_2}t}dt = \dfrac{A_v}{A_c} - \dfrac{A_v}{A_c}e^{-\frac{A_c}{R_1+R_2}t}$

低速电磁场换系公式

原则：电磁场可以变，受力不能变，对任意运动成立

静系：$Oxyz$　　动系：$O'x'y'z'$，相对静系以V沿x正向运动

$$F_x = (E_x + v_y B_z - v_z B_y)q = F'_x = [E'_x + v_y B'_z - v_z B'_y]q$$

$$F_y = (E_y + v_z B_x - v_x B_z)q = F'_y = [E'_y + v_z B'_x - (v_x - V)B'_z]q$$

$$F_z = (E_z + v_x B_y - v_y B_x)q = F'_z = [E'_z + (v_x - V)B'_y - v_y B'_x]q$$

$$\Rightarrow E_x = E'_x \qquad E_y = E'_y + VB'_z \qquad E_z = E'_z - VB'_y$$

$$B_x = B'_x \qquad B_y = B'_y \qquad B_z = B'_z$$

$$\Rightarrow \vec{E} = \vec{E}' + \vec{B}'\times\vec{V} = \vec{E}' + \vec{B}\times\vec{V} \qquad \vec{B} = \vec{B}'$$

$$\vec{E}' = \vec{E} + \vec{V}\times\vec{B} = \vec{E} + \vec{V}\times\vec{B}' \qquad \vec{B}' = \vec{B}$$

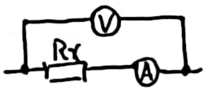
即定系换动系出$\vec{V}\times\vec{B}$涡旋电场，动系换定系出$\vec{B}\times\vec{V}$涡旋电场。

用途：1. 复合场代替配速法；

2. 动磁场电磁感应本质理解。

测量电阻大小的内外接 $\sqrt{R_A R_V}$ 判据推导

① 外接：



$R_{外} = R_x + R_A \quad \Rightarrow \quad \Delta_{外} = |R_{外} - R_x| = R_A$

② 内接

$R_{内} = \frac{R_V R_x}{R_x + R_V} \quad \Rightarrow \quad \Delta_{内} = |R_{内} - R_x| = \frac{R_x^2}{R_x + R_V}$

考虑到电阻实际值 R_x 不变，相对误差 $\frac{\Delta}{R_x}$ 与绝对误差 Δ 大小关系一样.

$\Delta_{外} \div \Delta_{内} = \frac{R_A(R_x + R_V)}{R_x^2}$,

当内接误差小时，$\frac{\Delta_{外}}{\Delta_{内}} > 1$，即 $R_x^2 - R_A R_x - R_A R_V < 0$，(且 $R_x > 0$).

$\therefore 0 < R_x < \frac{1}{2}(R_A + \sqrt{R_A^2 + 4R_A R_V})$，当 $R_A \ll R_V$ 时（一般情形），$0 < R_x < \sqrt{R_A R_V}$。

同理，当外接误差小时，$R_x > \sqrt{R_A R_V}$。

（注：由此可见，$R_x = \sqrt{R_A R_V}$ 时，应内接，但应用价值不大）

期末物理第8题

氢原子电子质量 m，电荷 $-e$，外加磁场 B，角速度变化 $\Delta\omega$（很小），半径 r 不变，

利用物理学中常用方法 —— 爆解 + 差不多得了 —— 判断 $\Delta\omega$：

外加磁场前，$mr\omega^2 = k\frac{e^2}{r^2}$ ①

外加磁场后，$mr\omega'^2 = k\frac{e^2}{r^2} \pm e\omega' rB$ ②

法1：

考虑到 $\Delta\omega = \omega' - \omega$ 很小，$(\Delta\omega)^2$ 舍弃，$\frac{\Delta\omega}{\omega}$ 舍弃，

$\omega'^2 = (\omega+\Delta\omega)^2 \approx \omega^2 + 2\omega\Delta\omega$，

则由②，$mr\omega^2 + 2mr\omega\Delta\omega = k\frac{e^2}{r^2} \pm (e\omega rB + e\Delta\omega rB)$，

与①比较，$2mr\omega\Delta\omega = \pm(e\omega rB + e\Delta\omega rB)$，

即 $\Delta\omega = \pm\left(\frac{eB}{2m} + \frac{eB\Delta\omega}{2m\omega}\right) \approx \pm\frac{eB}{2m}$。

法2：（贴近高中但相对严谨），

由①，$\omega = \sqrt{\frac{ke^2}{mr^3}}$ ③，

由②，$\omega' = \frac{\pm erB \pm \sqrt{e^2r^2B^2 + \frac{ke^2m}{r}}}{2mr}$

考虑到 $\Delta\omega$ 很小的前提是 B 很小，

且 ω' 产生变化来源于 B，

$\omega' \approx \pm\frac{eB}{2m} + \sqrt{\frac{ke^2}{mr^3}}$，

$\Delta\omega = \omega' - \omega = \pm\frac{eB}{2m}$

由于 $\Delta\omega$ 改变电子运动产生的磁场，且与外磁场方向相反，

大部分材料具微弱的抗磁性。