NETWORKDYNAMICS.JL

JACOBIAN VECTOR PRODUCT

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Explicit Solvers

$$x_{k+1} = F(x_k)$$

Implicit Solvers

$$G(x_k, x_{k+1}) = 0$$
$$g(x_{k+1}) = 0$$

Rootfinding problem

$$x_{k+1} = x_k - J(x_k)^{-1} g(x_k)$$

Newton Method

1. Solve $\boxed{Jv = g(x_k) =: w}$
2. Update $x_{k+1} = x_k - v$

$$Jv = g(x_k) =: w$$

1) Find Jacobian

$$v = J^{-1}w$$

memory problems

2) Use Jacobian Vector Product (JVP)

$$Jv = w$$

$$w_i = \sum_j^m J_{ij}\, v_j \quad \Leftrightarrow \quad w_i = \sum_j^m \frac{df_i}{dx_j} v_j = \nabla\, f_i(x)\, v$$

$$Jv = \nabla f\, v$$

$$Jv = \lim_{\epsilon \to 0} \frac{f(x + v\epsilon) - f(x)}{\epsilon}$$

$$Jv \approx \frac{f(x + v\epsilon) - f(x)}{\epsilon} \qquad \epsilon \neq 0$$