New Shades of the Vehicle Routing Problem: Emerging Problem Formulations and Computational Intelligence Solution Methods

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***Abstract*—This paper presents an overview of recent advances in the field of the vehicle routing problem (VRP), based on papers published in high-quality journals during the period from January 2015 to July 2017. A distinctive feature of the presented survey is its focus on new versions of the VRP, which have recently emerged or gained momentum, and the corresponding new solution methods, with particular emphasis on computational intelligence (CI) approaches. The list of newly proposed or currently popular VRP formulations include last mile and same day delivery, crowdshipping, bike sharing systems, post-disaster response plans, local routing in large production or cargo plants, customer-centric VRP, autonomous delivery, unnamed aerial vehicle delivery, green VRP, waste collection VRP, rich VRP, or VRP with backhauls. Simultaneously, an adequate increase of interest in the application of traditional CI methods (e.g., genetic, memetic, ant colony or particle swarm optimization, simulated annealing, or their various hybrid versions) can be observed in the VRP domain. At the same time, approaches proven efficient in other optimization areas (e.g., hyperheuristics, methods based on Monte Carlo simulations, algorithms rooted in game theory and bi-level optimization—Stackelberg games,or cognitively motivated methods) have lately entered the VRP field and become a viable alternative to more traditional techniques. Since VRP is one of the fastest growing fields in the operations research area, we believe that an analysis of the recently published VRP papers from the perspective of their novelty in problem formulation and/or applied solution method can provide a true value for the CI community, especially young researchers entering the field and seeking challenges in this interesting and fast developing research area.**

***Index Terms*—Computational intelligence, vehicle routing, combinatorial optimization, metaheuristics.**

I. Introduction

THE Vehicle Routing Problem (VRP), was introduced in the literature as the *truck dispatching problem* by Dantzig and Ramser [1] in 1959 and proven to be NP-hard by Lenstra and Kan [2] in 1981. The classical version of the VRP consists in serving a set of customers by a fleet of homogeneous vehicles with routes beginning and ending at a specified depot. The optimization goal is to minimize the cost of delivery, i.e., the total length of routes of all vehicles. In the majority of cases, a capacity limit is imposed on the vehicles, leading to the Capacitated VRP (CVRP) formulation.

The VRP (CVRP) belongs to the most widely researched problems in the domain of Operations Research, mainly due to its practical relevance and combinatorial complexity. The importance of the VRP stems from its direct application to everyday business routines of distribution/service-providing companies. Due to a huge variety of practical implementations of the problem, the VRP literature covers a broad range of possible extensions to the classical problem formulation.

On a general note, there are two main factors which determine both the intrinsic difficulty of a given VRP formulation and the potential applicability of certain solution methods. These two dimensions along which the majority of VRP variants can be grouped are *static* vs. *dynamic* and *deterministic* vs. *stochastic* formulations. Following [3], in *static* problems partial vehicle routes constructed during a solution process do not change afterwards, while in *dynamic* formulations they may, and usually do, change (sometimes significantly) due to dynamic factors which are unknown beforehand and are only revealed later, when partial vehicle routes are already designed and frozen. In *deterministic* variants of the VRP all variables are deterministic, although in the dynamic case some of them are unknown *a priori*. In contrast, in *stochastic* versions some problem parameters are stochastic (e.g. customer demands, travel times, service times or service probabilities). The easiest variants are those which are static and deterministic. Dynamic and stochastic formulations are generally located on the other end of the difficulty spectrum.

From the application point of view, besides dynamism and stochasticity, there are several other features which differentiate the VRP formulations. The list of most popular problem aspects includes: *time windows (hard or soft)*, *pickups and deliveries* or *backhauls*, a *mixed (heterogeneous) vehicle fleet*, *split deliveries*, *loading/capacity constraints*, *multiple depots*, *open routes* (with no requirement to return to the depot at the end of a route [4]). Clearly the above list refers to the most popular aspects of the VRP only, and by no means should be treated as exhaustive. A recently published book [5] provides a comprehensive description of the most popular problem variants. For a detailed VRP taxonomy the reader can refer to [6].

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A. Main Contribution

The majority of the recently published VRP books or review papers can be assigned to one of the following two categories:

- surveys describing major types of the VRP from a general perspective of the entire VRP field, e.g. [5]–[7];
- surveys referring to particular types of the VRP, e.g. Dynamic VRP [3], Green VRP [8], Multi-attribute VRP [9], Waste Collection VRP [10], City VRP [11] or Periodic VRP [12], or considering particular aspects of the problem, e.g. the VRP taxonomy [6], [13].

A distinctive feature of this survey is its interest in emerging problem formulations and new CI-related solution methods. In terms of problem types the paper surveys:

- new VRP formulations which have emerged only recently, e.g. last mile and same day delivery, crowdshipping, bike sharing systems, post-disaster response plans, local routing in large production or cargo plants, customer-centric VRP, autonomous delivery, or UAV delivery,
- problem formulations which have been around for some time, but gained momentum over the last several years, e.g. Green VRP, Waste Collection VRP, Rich VRP or VRP with Backhauls.

With respect to solution methods the focus is on:

- recently developed approaches, previously not utilized in the VRP domain, that belong to the broadly understood CI field, e.g. methods based on Monte Carlo simulations, algorithms rooted in game theory and bi-level optimization (Stackelberg games), hyperheuristics, or cognitively motivated methods, as well as innovative applications of hybrid CI methods.

The remainder of the paper is organized as follows: the next section describes the applied methodology of paper selection and presents some statistics related to journals and papers chosen in this review. Section III discusses new, interesting problem formulations and is divided into subsections grouping particular types of the VRP. Section IV presents innovative and promising Computational Intelligence (CI) methods that have recently emerged in the field. The last section contains conclusions.

II. Scope of the Survey and Its Methodology

The main goal of this survey is the detection of new trends in the field of the VRP, which have emerged or become popular only recently. In order to accomplish this goal 400 research papers returned for the query *Vehicle Routing Problem* for the period January 2015–July 2017 were initially collected: 200 top papers from the *Scopus* database and 200 top papers from the *IEEE Xplore* digital library. After preliminary screening, around 200 papers (out of the 400) were promoted to the second stage of the selection process, in which all of them were looked through with the aim of tracing **new problem formulations, innovative solution methods and new benchmark repositories.**

The set of chosen papers was finally extended by the inclusion of some number of seminal papers in the field, several recent survey papers related to particular aspects of the VRP, and some number of relevant VRP and CI background papers, which are mentioned for the sake of completeness of the presentation of the new trends and accomplishments.

TABLE I
List of Journals Considered in This Survey With at Least Two Cited VRP-Related Papers

Journal	#papers
European Journal of Operational Research	20
Transportation Research	8
Computers & Operations Research	7
Information Sciences	5
Applied Soft Computing	5
Networks	5
Computers and Industrial Engineering	5
Transportation Science	5
Expert Systems with Applications	5
Journal of the Operational Research Society	5
IEEE Transactions on Cybernetics	3
IEEE Transactions on Intelligent Transportation Systems	3
Operations Research	3
IEEE Transactions On Systems Man and Cybernetics: Systems	2
IEEE Systems Journal	2
IEEE Transactions on Evolutionary Computation	2

TABLE II
Breakdown of 117 Cited VRP-Related Papers by Publication Year/Time Periods. The Remaining 30 Papers From the Bibliography List (Not Considered Here) are Background Papers Presenting CI and Non-CI Methods With No Direct VRP Context, as Well as URLs Pointing to New Benchmark Repositories

Publication year / period	#papers
2017	30
2016	22
2015	15
2014	8
2013	8
2010 – 2012	7
2000 – 2009	12
1959 – 1999	15

The vast majority of the finally selected papers are journal publications from the most renowned venues. Table I presents the list of the most frequently cited journals.

A breakdown of the cited VRP papers by publication year/period is presented in Table II. The majority of the early published works (years 1959–1999 and 2000–2009) consists of seminal papers in the field, which are included in the survey for the sake of providing a comprehensive background for discussion of newer accomplishments.

We believe that the analysis of the recently published VRP papers from the perspective of their novelty in problem formulation and/or applied solution method can be of real value for the CI community, especially young researchers entering the field and seeking emerging challenges in this broad and fast developing area.

Certainly, classification of VRP variants or solution methods as *novel*, *emergent* or *gaining momentum* must be, to some extent, subjective and biased by the author's knowledge and vision of the field. One the other hand, this intuitive selection could be partly supported by the examination of the number of papers related to the topics of interest, published in recent years. To this end, some number of queries have been executed in the *Scopus* database among journal articles and reviews. The

TABLE III
NUMBER OF JOURNAL PAPERS WITH RESPECTIVE KEYWORDS FOUND IN A GIVEN YEAR/TIME PERIOD

Keywords	First paper	2000 - 2009	2010 - 2012	2013	2014	2015	2016	2017
last mile delivery	2008	3	3	2	2	0	6	9
crowdshipping	2016	—	—	—	—	—	2	2
green vehicle routing	2008	2	4	6	7	10	19	19
autonomous delivery vehicle	2004	4	2	2	1	4	1	7
bike sharing	2012	—	4	2	8	14	15	23
UAV delivery	2010	—	4	1	1	1	8	11

results, presented in Table III, support the claim about raising interest in certain VRP formulations in the last few years.

In order to avoid any potential misunderstandings, it is worth to underline that the focus on emerging trends in the field should in no way be interpreted as the lack of respect toward many other solid VRP accomplishments published in the period of interest, related to more "traditional" formulations of the problem and applying well-established solution methods.

III. NEW PROBLEM FORMULATIONS

The main criterion of paper selection in this survey was the presence of an innovative problem formulation or solution method. While each of the baseline VRP aspects mentioned in the introductory section (time frame, loading constraints, open routes, etc.) single-handedly adds a new layer of complexity to the plain VRP/CVRP formulation, such *one-dimensional* VRP extensions are, nevertheless, well-researched in the literature and gradually give ground to more complex problem variants. On a general note, the mainstream VRP research is developing in the three following directions:

1) new challenging problem extensions reflecting contemporary technical and societal challenges, demands and opportunities, e.g. *Green VRP*, *ad-hoc delivery*, or *last-mile delivery*,
2) synergetic combinations of several VRP aspects which lead to essentially new problem formulations, usually practically-motivated, e.g. *VRP with backhauls* or various formulations of *Rich VRP*,
3) new problem formulations addressing specific business or industry settings, e.g. *VRP for transportation of hazardous materials*, *humanitarian transportation in post-disaster settings*, or *local VRP in large industry or cargo plants*.

In the remainder of this section recent advances in the above-mentioned VRP subfields are discussed.

A. *New Formulations of VRP With Backhauls*

One of the fast developing variants of the VRP in recent years is *VRP with Backhauls* (VRPB), initially proposed in [14], which includes, as one of its main subcases, *VRP with Simultaneous Pick-up and Delivery* (VRPSPD). Simultaneous pick-up and delivery assumes that both *linehauls* (delivering goods) and *backhauls* (pick-up tasks) are requested simultaneously at a client's site [15]–[17]. While this situation is quite frequent in real-life scenarios, there are also two other VRPB formulations which gained momentum in recent years [18]. In the first one, known as *VRP with Delivery Before Pick-up* (VRPDBP) [19], [20], linehauls must be served before backhauls. In the other one, referred to as *VRP with Backhauls and Mixed-load* (VRPBM) or *VRP with Mixed Pickup and Delivery* (VRPMPD) [21], linehauls and backhauls can be served in any order.

The three above-mentioned variants of VRPB were initially introduced as stand-alone extensions of the VRP. Lately, they started to serve as a basis for new VRPB versions that address specific business requirements, most notably, loading constraints described below.

Two-dimensional loading VRP with clustered backhauls (2L-VRPB), which assumes that both types of demands (pickups and deliveries) are composed of non-stackable items, was proposed in [22]. Quite surprisingly, despite relatively high commonness in everyday transportation logistics, the problem has not been formally considered in the literature until recently [22]. The problem is approached with the Large Neighborhood Search (LNS) metaheuristic, with dedicated routing and packing local search heuristics.

VRPSPD is extended by adding constraints related to the transportation of non-stackable rectangular items was also proposed in [23] under the name *VRP with Simultaneous Pick-ups and Deliveries and Two-Dimensional Loading Constraints* (2L-SPD). A combination of 2L-CVRP [24] and VRPSPD makes the resulting problem truly demanding as 2D loading feasibility must be verified for each arc of the traveled routes. The problem can be further extended by imposing LIFO (Last In, First Out) constraints which prohibit the rearrangement of items on the route. Both versions (with and without LIFO) are solved with a two step approach: first a heuristic initial solution is constructed, which is then optimized with the help of three local exchange operators, previously used and evaluated by the authors in [25]. Also a new set of benchmarks [26] corresponding to 2L-SPD formulation is proposed in [23].

An extension along the lines of the classical VRPSPD formulation consisting in adding three-dimensional loading constraints, denoted as 3L-VRP, was proposed in [27]. The specification of the problem assumes that each demand is in the form of a set of 3D rectangular items (boxes) which must be carried by vehicles with a given 3D rectangular loading space. Furthermore, it may additionally be assumed that boxes, once loaded, must not be moved before the final unloading, i.e., no reloading effort is required while serving the customers (which is a variant of the LIFO constraint). This problem version is abbreviated as 3L-PDP. The solution method combines a routing procedure and a packing heuristic. The former is an extension of the LNS method used for the 1D-PDP (i.e. VRPSPD). The latter relies on a specific tree search problem representation. The authors propose a new set of 54 benchmark problems which are available at a dedicated website [28].

Another interesting extension of VRPB was proposed in [29], where the problem is combined with *Multiple-Trip VRP* (MT-VRP) [30] (which assumes that a vehicle may perform several trips within a given time period), leading to the MT-VRPB variant. A MILP formulation is proposed and solved with the IBM ILOG CPLEX 12.5 optimizer for small and mid-size MT-VRPB instances. For large-size problems the Two-level VNS algorithm is applied which alternately switches between the two stages: outer and inner. The former is responsible for the construction of the transitory solution and the later aims at its local search-based improvement. The algorithm is tested on a new set of 168 MT-VRPB benchmark problems [31] proposed by the authors.

B. *Last-Mile and Same-Day Delivery*

One of the critical aspects of a delivery business nowadays is the *speed of delivery*. In some cases the expectation is that delivery be performed even within one day (*same-day delivery*). While transporting goods by means of fast trains or modern trucks running on highways is an efficient and cost-effective manner of delivery to the main depot or freight station, optimal organization of a depot-to-customer leg of the route seems to be a real challenge. This part of delivery process, known as the *last mile problem* (LMP), is currently extensively researched by practically all major retailers and delivery companies. Not surprisingly, the recent VRP literature also addresses this issue with growing interest.

An interesting approach to LMP is *VRP with roaming delivery locations* (VRPRDL) [32] which aims to find the least expensive set of delivery routes for a fleet of vehicles assuming that each customer's order is to be delivered to the trunk of their car, which is parked in one of predefined locations of an *a priori* known customer's itinerary. The task is formulated as a set-covering problem and solved by a branch-and-price algorithm. VRPRDL models real service processes in retail delivery segment. For instance, Amazon in collaboration with Audi and DHL has recently started to offer the so-called *trunk delivery service* in selected areas [33] (see also press releases from NBC News [34] and Audi [35]).

A similar VRPRDL formulation, which assumes that the customer's car follows a fixed itinerary and there is a number of spots on this route identified as possible delivery points, was proposed in [36]. The authors introduce a heuristic local improvement scheme for optimizing the sequence of deliveries (customers to be served) and the corresponding delivery locations. Experimental evaluation of the proposed heuristic reveals that substantial savings are possible compared with traditional home delivery.

One of the innovative approaches to last-mile delivery is the use of unnamed aerial vehicles (UAVs) [37]. There are two main advantages of using UAVs (drones) compared with traditional vehicle-based delivery. Firstly, drones are less expensive to maintain and their "labor cost" is lower due to the high degree of work automation. Secondly, drone-based delivery is often faster as drones do not require specific road infrastructure and are generally not vulnerable to the obstacle avoidance problem, which is frequently the case for road-based deliveries-due to traffic jams, accidents, road construction, maintenance work, etc. Furthermore, the utilization of drones is often the first option in the case of life-threatening, emergency situation deliveries, for instance, in hazardous or contaminated terrains.

Drone VRP settings are specific in the two following aspects. First of all, multi-trip routing is more natural and more feasible compared with vehicle-based delivery, and second, the effect of battery and payload weight is a critical factor in route planning [37]. In this respect, the authors propose two formulations of *VRP for Drone Delivery* (VRPDD) which address these two aspects: one with cost minimization under a given time limit and the other one with delivery time minimization within a constrained budget. A MILP formulation of the problem is proposed to solve instances with a moderate number of locations and the Simulated Annealing (SA) approach is used for finding heuristic sub-optimal solutions in large-scale scenarios.

Another paper devoted to the utilization of UAVs in last-mile delivery [38] considers the so-called *Fuel Constrained UAV Routing Problem* (FCURP). In this setting, a number of depots with refueling stations may be used by UAVs and the goal is to find the least-cost path (in terms of fuel usage) that visits each target at least once and does not violate the fuel constraint. The problem is essentially formulated as an extension of the Traveling Salesman Problem (TSP) with possible multiple visits and auxiliary refueling points that may (but do not have to) be visited.

C. *Crowdshipping, Ad-Hoc Carriers and Cooperative Logistics*

One of the viable and promising options in addressing LMP is *crowdshipping*, i.e., getting ordinary people (ad-hoc carriers) or drivers employed by the retail company to support professional delivery companies (such as UPS or FedEx), by dropping off small packages en-route to their destinations. Such an approach is considered by Walmart [39] and Amazon [40], among others. The following three papers [41]–[43] discuss the VRP setting that implements the idea of crowdshipping.

In the first scenario [41], a transportation company with professional drivers is supported by occasional drivers who are willing to make ad-hoc deliveries using their own vehicles, in return for some compensation, if a delivery point is not located too far from their destination. A typical example are in-store customers who are willing to deliver on-line ordered products to remote customers. The objective of the company (apart from a high level of customer satisfaction) is the minimization of the total delivery cost, i.e., the one associated with their own deliveries and that related to ad-hoc "private" deliveries (compensation for occasional drivers). The option of involving ordinary people in the realization of a delivery process leads to the *VRP with Occasional Drivers* (VRPOD) variant of the problem. A new multi-start heuristic approach is proposed, which yields close-to-optimal solutions as verified by a direct comparison with the MILP formulation of VRPOD, solved by commercial solvers.

In the related paper [42], a situation when the internal fleet of a retail/business company is supported by that of a professional external carrier, referred to as *VRP with Private fleet and common Carrier* (VRPPC), is studied. The main objective is to determine which deliveries should be covered by the company's own fleet and which by the professional carrier's. The decision is made in the context of the minimization of the total routing cost by means of a hybrid metaheuristic method developed by the author, which combines Tabu Search (TS) with the local injection of neighborhood client structures. The results show that the utilization of an external carrier for a small but well-chosen fraction of delivery tasks often leads to a significant reduction of the required number of routes made by the company's own fleet.

The VRPPC model presented in [42] is further extended in [43] by considering the following three aspects: two options of car rental for subcontractors (external carriers), a modified cost function which takes into account both the distance and the volume of delivery, and a discount policy related to the volume of the payload, offered to common carriers. A MILP formulation and three VNS-based heuristics are proposed for solving the task.

Another form of joint efforts aimed at streamlining the delivery process is the idea of *cooperative logistics* - a new and rapidly developing trend. In this framework, multiple carriers form coalitions so as to better serve the customers' demands and consequently gain more profit as a team [44], [45]. Cooperative logistics is becoming more and more popular, especially in the segment of small shippers, who are generally more flexible and ready to combine their resources to serve the overall pool of customers more effectively. The underlying idea is to avoid the under- and over-utilization of the fleets of each of the individual shippers by means of a temporal sharing of vehicles. The main question is how to allocate the combined pool of vehicles to serve all customers (satisfying their demands and service hours) at minimum cost, part of which depends on the mechanism of cost calculation of vehicle rental from one shipper to another.

The VRPPC setting can be regarded as a cooperative game in which multiple players are able to form coalitions and members of each coalition cooperate with each other [44], [45]. The question of how to allocate the payoff and the cost assigned to the whole coalition to its individual participants (carriers) is addressed in [44] by the concept of *cross-evaluation value*, and in [45] by the concept of *Myerson value* [46] from cooperative game theory. Both methods are further described in Section IV-B.

D. Rich VRP Formulations

In recent years, new application-oriented versions of the VRP, which combine various types of constraints with complex objective functions, have attracted the attention of researchers due to societal expectations and demands, as well as substantial technological and computational progress. Such VRP variants with complex implementation-oriented attributes are commonly regarded as *Rich VRPs* (RVRPs) [47], [48] or *Multi-attribute VRPs* [9], [49]. These new problem formulations are, most often, either *richer* versions of the existing variants (e.g. by considering additional constraints or problem aspects) or specific formulations tailored towards particular implementations of complex industrial/logistic processes.

One of the RVRP formulations - *Min-max VRP with Mixed Fleet and Mixed Service Demand* (MFMDVRP) has been recently proposed in [48]. The problem combines multiple demand types, multiple platform types and a constrained route structure. It is assumed in the paper that vehicles differ by service and transfer speed. Furthermore, they operate in demand points which are clustered into regions whose boundaries cannot be traversed, i.e., the operational regions are separated. The solution method is based on the ACO metaheuristic in which, unlike in typical ACO approaches to solving constrained optimization problems, both node and edge pheromone trails are applied. The former correspond to service times in nodes and the latter to transition times between nodes. Both of them contribute to the final vehicle-to-route assignment.

Another Rich VRP formulation was proposed in [50] where heterogeneous fleet, time windows, and backhauls mixed-loads are simultaneously considered. The objective is to optimize the total service cost by means of selecting an optimal number of vehicles of appropriate types and designing the optimal routes for this fleet. As is often the case with RVRP, the problem formulation originated in a real-life business scenario of mail delivery and pick-up service. The problem is solved with a Label-based Ant Colony System (LACS) which is a multi-attribute extension of the ACS method - see Section IV-A for further description.

Another paper regarding RVRP considers *VRP with Simultaneous Delivery and Pickup and Time Windows in a multi-objective context* (MO-VRPSDPTW). The following five minimization goals are simultaneously taken into account [51]: the number of vehicles, total travel distance, travel time on the longest route, total waiting time due to early arrivals and total delay time due to late arrivals. Two methods for solving such a generally defined MO-VRPSDPTW are proposed: multi-objective local search (previously introduced in [52]) and multi-objective MA. Both approaches are tested on two sets of MO-VRPSDPTW instances [53], [54] generated based on real-life data. The first one [53] is composed of 45 instances with different combinations of three sizes of customers, three types of vehicle capacities and five profiles of time windows. The other one [54] includes 56 revised Solomon instances [55], each with 100 customers.

Yet another practical realization of RVRP, integrating the routing and loading problem - the *Heterogeneous Fleet VRP with Three-dimensional loading constraints* (3L-HFVRP) - in which a cargo is composed of 3D rectangular-shaped weighted items, was proposed in [56]. The fleet is heterogeneous and each vehicle has a certain loading space, capacity, fixed cost and unit travel cost. The objective is to minimize the overall transportation cost (the sum of fixed costs of selected vehicles and the cost of their travel) while fulfilling the requested delivery demands. The problem is approached by the Adaptive Variable Neighborhood Search (AVNS) method and tested on a new specifically designed set of benchmark problems available on the project-related website [57]. The results for 3L-HFVRP are compared with those for the two underlying "component" problems: 3L-CVRP and HFVRP.

E. VRP Formulations for Specific Applications

In this section several contemporary VRP formulations which address particular application settings are summarized and discussed. **While these settings do not belong to mainstream VRP research, all of them seem to have strong research and application potential.**

1) Bike Sharing Systems: An interesting usage of the VRP framework in a large-scale inventory rebalancing problem is proposed in [58] in the context of a bike sharing system (BSS). Such systems are rapidly developing all over the world. In 2015 the estimated number of bike networks in major cites exceeded 1 000 and in another few hundred BSSs were under construction or seriously planned [59].

Solving the bike inventory problem requires addressing two major aspects: determining the optimal level of service at each bike sharing station and designing close-to-optimal routes for vehicles which will assure the requested inventory levels at these stations. The former task is usually addressed based on the analysis of historical data. The latter is most often stated and solved as a specific variant of the routing problem. More precisely, there are two major approaches to the task of rebalancing station inventory levels: *static* and *dynamic*. The first one, which is actually considered in the paper, assumes that the level of inventory at each station is frozen at some point at night for the time required for re-calculation of the routes and physical re-distribution of bicycles to be used during the next 24 h. The other approach, requires dynamic adaptation of bike re-balancing system, what makes the problem truly demanding during the daytime, particularly in rush hours. The authors propose a heuristic solution for the *static* case which consists in the application of a bike station clusterization procedure combined with service level feasibility and efficient routing costs. The method is tested on two large-scale BSSs located in Boston, MA and Washington, DC, respectively.

2) Transportation of Hazardous Materials: A specific implementation of the VRP in the area of transportation of hazardous materials is discussed in [60]. Due to the raising demand for this type of transportation, the problem gained special interest in recent years. In such expeditions, a potential threat to public (by means of leakage, explosion, poisoning or other accidents with serious consequences) is considered a major issue. Hence, the key factor is the minimization of the probability of an accident and the minimization of its potential consequences. In [60] a multi-depot formulation of the problem is proposed and solved by bi-level programming methods, discussed in more detail in Section IV-B.

Another approach to transportation of hazardous materials was recently proposed in [61] in the context of *Heterogeneous Fleet VRP* (HFVRP). Similarly to [60], the key objective is safety, which is understood as the minimization of the potential risk of an accident (and a subsequent ecological disaster), which non-linearly depends on the vehicle load and the size of population exposed to potential threats. The problem is approached with a VNS algorithm enhanced by a set partitioning procedure applied to the pool of candidate routes considered in the local search method embedded in the VNS.

3) Waste Collection VRP: The process of waste collection in urban areas belongs to the most important municipal services and due to specific environmental and recycling requirements needs to be planned and executed with extreme care and efficiency. One particularly relevant facet of Waste Collection VRP (WC-VRP) is e-waste which consists in collection of waste electrical and electronic equipment, and relies on efficient planning of container loading and route optimization [62].

A new WC-VRP formulation with intermediate depots has been recently proposed in [63], formulated as MILP and solved by CPLEX solver. The approach was further improved in [64] in which MILP formulation and CPLEX solver are utilized for small-size instances, and an improved Max-Min Ant System [65] is applied for problems beyond solver's capabilities. A MILP-based formulation (combined with BILP - *Binary Integer Linear Programming*) was also proposed in [66] for the case of solid waste collection. Yet another recent formulation of WC-VRP considers multiple trip case with working hours of the drivers and crews being one of the main concerns [67]. The problem is approached with the Simulate Annealing algorithm with a suitable cooling scheme.

4) Transportation of Valuable Materials or Goods: A fairly new VRP formulation devoted to vital or valuable shipments, which are exposed to potential robbery or terrorist attacks (e.g. fuel distribution, money transports, transfer of prisoners, etc.), has been recently proposed in [68]. In this setting the underlying threat is the possibility of the interdiction of the arcs of the planned route by an *attacker* whose goal is to disrupt parts of the network so as to minimize the flow of transported commodities or maximize the expected tour length. The problem is formulated in the attacker-evaded framework of Stackelberg Security Games [69], [70] (see Section IV-B for a further discussion).

5) Post-Disaster Response Plans: In post-disaster response plans, emergency transportation plays one of the major roles. The respective VRP formulation in this scenario - the *Cumulative Multidepot VRP* (cum-MDVRP) [71] - combines two popular VRP variants: *cumulative VRP* and *multidepot VRP*. The authors propose a novel metaheuristic approach to solving the problem which integrates ACO with Clarke and Write's Savings Algorithm [72] and 2-opt local optimization. The resulting method allows ants to return to the depot multiple times. Whenever an ant leaves the depot, it is fully loaded. Following the ACO setting proposed in [73], the final solution is constructed from partial solutions developed by different ants.

6) Local Routing in Large Production or Cargo Plants: Another contemporary VRP formulation refers to the problem of efficient routing in a large plant, which becomes a more and more relevant task in the era of global, transnational markets. An example of such a setting, which refers to a large air cargo plant, is proposed in [74]. The automated freight handling system considered in the paper uses multi-capacity rail-guided vehicles working on linear trucks. The problem is formulated as MILP and solved either as a static instance or as a dynamic multi-step decision problem with a *rolling horizon*. In the latter case, Whenever a new request arrives or there are yet-non-assigned requests in the system, the current solution is recomputed within a given horizon. Observe that the rolling horizon is a *smoother* realization of the *time slots* approach, widely-applied in dynamic versions of the VRP, in which the working time is divided into time slots and new requests which arrive during the current slot wait to be handled in the immediately following slot (see [75]–[77] for some recent examples of the utilization of the time slots technique in solving VRP with stochastic demands/dynamic requests).

7) Optimal Local Routing of People: Yet another new VRP variant addressing contemporary demands is considered in [78]. The *Demand Weighted VRP* (DWVRP) refers to carrying people on certain routes with the goal of minimizing their journey time. Such a scenario is encountered, for instance, in a two-way shuttle service between an airport and a set of hotels located in a certain area, or a two-way bus service between a car pool lot and a terminal. The objective is to minimize the traveled distance weighted by the number of passengers. A particular setting considered in the paper is motivated by a bus shuttle service for students of the University of Cincinnati, USA. The authors propose a MILP formulation of the problem and apply a branch-price-and-cut algorithm to solve its large-scale instances.

A recently published paper focusing on an effective commuter transit system [79] considers the setting of a highly urbanized area and a fleet of highly-efficient Electric Vehicles (EVs). The aim is to choose the optimal route for each EV from a given origin to a given destination that would satisfy the welfare of passengers in terms of the minimization of travel time and distance, while maximizing the energy efficiency, assuming a given geographical distribution of charging stations. The problem is formulated as a Mixed Integer Quadratically Constrained Problem and essentially extends the *VRP with Pickup and Delivery* (VRPPD) formulation by adding the aspect of battery charging and taking into account human traveler satisfaction. The paper concludes that in densely populated areas a fleet of shared EVs offers a highly competitive commuting solution. This observation is in line with the outcomes of [80] where similar operational settings of an autonomous EV fleet are discussed.

8) Customer-Centric VRP: Customer satisfaction plays a central role in the competitive delivery market and for this reason some VRP formulations consider the minimization of the customers' waiting time in the delivery process as one of the key objectives. Such problems are known as *Cumulative VRPs* [81] or *Customer-Centric VRPs* [82].

The maximization of customer satisfaction in the *Customer-centric, Multi-commodity VRP with Split Delivery* (CMVRPSD) version of VRP is the focus of [83]. CMVRPSD aims at the minimization of the total waiting time of the customers, assuming multiple types of commodities and the possibility of fulfilling a customer's request by any number of available vehicles. Such a formulation involves two kinds of operational decisions: the construction of vehicle routes with respect to individual customers with possibly multiple visits, and the quantification of delivered goods to be loaded (in a depot) and partly unloaded (at each customer's site). The problem is formulated in two ways, i.e., as two independent MILP models and solved with a heuristic approach combining SA and VNS. A suitable temperature schedule allows to maintain the global exploration capabilities of SA while the heuristic local search of VNS is responsible for the exploitation aspect.

9) Robust VRP: A separate segment of the VRP literature refers to the *Robust VRP* formulation, which assumes that customer demands are uncertain - usually sampled from some unknown but fixed distribution - and hence there is a strong need for robust solutions, i.e., ones which are effective for a broad class of possible problem realizations. Clearly, a robust solution is not necessarily optimal for the particular future realization of the distribution of demands, but generally works well for a wide range of potential scenarios.

A recent paper in this area [84] provides a theoretical asymptotic analysis of the *worst-case scenario*, i.e., a situation in which induced loads are as undesired as possible, for two popular VRP models: CVRP and VRPTW (*VRP with Time Windows*).

F. Green VRP

One of the fastest-growing research directions within the VRP area is *Green VRP* or *Green Logistics* concentrated on the reduction of air pollution. To this end Green VRP explores the pros and cons of wide utilization of environmentally friendly vehicles, most commonly Electric Vehicles (EVs) or Hybrid Vehicles (HVs), in particular in last-mile delivery and urban logistics. Each year the number of papers related to Green VRP is increasing (cf. Table III) due to the raising industrial and environmental demand and adequate technological advancement currently observed in this area. In the Green Logistics framework EVs and HVs have a clear advantage over petrol-based vehicles, but their usage on a massive scale is hindered by the limited operational range, as a consequence of scarce battery charging infrastructure. A detailed presentation of the Green VRP research area can be found in a dedicated survey paper [8]. The following review considers a selection of recent Green VRP publications, dated after the publication of [8].

The battery swap station location-routing problem with capacitated electric vehicles (BSS-EV-LPR) has been recently introduced in [85], in the context of planning the infrastructure for battery swapping in EVs. Subsequent work [86] considers BSS-EV-LPR within the classical VRP formulation, i.e., with a set of customers and their demands that need to be served by a fleet of homogeneous EVs with a certain capacity and limited driving range, stationed at a single depot. In order to stay operational, the EVs need to plan their visits at battery swap stations where depleted batteries are swapped for fully charged ones. Consequently, the BSS-EV-LPR simultaneously considers the two following aspects: the optimization of the locations of battery swapping stations (from a set of candidate locations), each with an associated construction cost, and the minimization of the sum of the total travel cost and the total swapping stations construction cost. The problem is approached with the AVNS algorithm, whose quality is tested on a new set of problem-related benchmarks (not published), that allow to test the impact of the swapping stations construction costs on the selection of the station locations.

Another paper [87] considers the VRP with a heterogeneous EV fleet, time windows, and recharging stations locations, introduced by the authors as *Electric Fleet Size and Mix VRP with Time Windows and Recharging Stations* (E-FSMFTW). The problem consists in selecting the optimal fleet of heterogeneous EVs, constructing their routes that include recharging schedules (times and locations), and fulfilling the customer time window constraints, so as to minimize the total cost of service, for a given set of customers and demands. A critical aspect of E-FSMFTW is the choice of EVs since acquisition costs and operational costs may vary significantly between particular EVs. For instance, while larger EVs are more expensive, they, at the same time, allow for planning longer route segments without recharging. The problem is solved by a branch-and-price method (if the calculation of the exact solution is feasible) or by an Adaptive LNS method with an embedded local search heuristic (otherwise). A new set of benchmarks based on modified Solomon instances [88], devoted specifically to E-FSMFTW, is also proposed in [87].

One of the potential solutions to the limited driving range of EVs (stemming from constrained battery capacity) is the provision of a public network of charging stations. Along this line, a simulation model of routing and charging points reservation is presented in [89] and its efficacy evaluated based on the public charging infrastructure deployed in Ireland in early 2016. Extensive Monte Carlo simulations based on population density and the estimated distribution of route directions and lengths show substantial advantages of using a routing and charging points reservation system on a nationwide (versus local) scale in terms of a much better utilization of charging points, and consequently, an effective increase of the number of EVs served.

An innovative approach to the problem of recharging EVs or swapping their batteries in the context of urban freight distribution is introduced and evaluated in [90], under the name *Modular electric VRP* (MeVRP). Its novelty stems from the proposed modularity of serving vehicles with a possibility of leaving a discharged module at a destination and picking it up later on (e.g. on the return trip to the depot), which may lead to electricity savings. MeVRP is modeled as MILP and solved by a CEPLEX solver.

An alternative to EVs is the use of HVs, working both electrically and on petrol [91]. The respective VRP model is known in the literature as *Hybrid VRP* (HVRP). The propulsion mode can be changed at any time during the HV operation, and is automatically switched to traditional fuel when the electric battery is depleted. The unitary cost of using the electric battery is smaller than that of traditional fuel, but - obviously - electric batteries have limited capacity and require recharging at dedicated stations, whose availability is limited. HVRP is formulated in [91] as MILP and solved using an LNS metaheuristic proposed by the authors.

Another HVRP paper [92] considers a similar framework of hybrid vehicles which can be switched to either one of the two power sources depending on available infrastructure. A solution method employs the SA technique with restart strategy (SA_RS) with two popular variants of acceptance probability: *Boltzmann function* and *Cauchy function*. Initial numerical tests on classical (non-hybrid) CVRP proved that the Cauchy acceptance probability function was preferable over the Boltzmann one, so the final experiments in HVRP settings were performed for SA_RS with the Cauchy function. The main results suggest that, taking into account the available refueling and recharging infrastructure, the use of HVs is overall more cost-effective than using purely electric or traditional petrol-based vehicles.

An interesting approach to eco-friendly city routing (with no direct reference to EVs or HVs) has been recently proposed in [93], where the authors formulate the *Green Multi-objective Multi-attribute VRP* (G-MoMaVRP). G-MoaMaVRP considers four real-world VRP aspects: time windows, simultaneous pick-up and delivery, heterogeneous fleet, and heterogeneity of traffic congestion (which is of particular importance in urban logistics), and combine them with the task of the minimization of the environmental impact caused by the delivery realization. The proposed multi-objective bi-level optimization approach (discussed further in Section IV-B) is tested on existing benchmarks, as well as on two real-life data sets describing Singapore traffic.

G. *Autonomous Ground/Air Delivery*

One of the very hot topics in the VRP domain is *autonomous ground/air delivery*. Large-scale research projects within this area have been recently announced by Ford teamed with Postmates, USPS in cooperation with FedEx and DHL, Natilus (drone delivery), and Toyota together with Pizza Hut. Despite its infancy development level and the lack of serious research papers, the area of autonomous delivery will most probably become one of the research targets in the coming years, posing inter-disciplinary problems and requiring adequate solutions that extend beyond constrained route planning.

IV. New Computational Intelligence Solution Methods

In the early stage of the VRP development, the most common approaches were **exact methods**, e.g. branch-and-bound [94], [95] or branch-and-cut [96], [97], as well as straightforward **heuristic approaches**. The latter included constructive methods which gradually build a feasible solution, e.g. the Savings Algorithm [72], and 2-phase methods, which divide the task of finding the optimal set of routes into two subproblems. In the first phase, the customers are assigned to vehicles and then, in the second phase, the routes for them are constructed (with possible repetitions of this cycle), e.g. the Fisher and Jaikumar method [98], the Sweep Algorithm [99] or Taillard's algorithm [100].

Later on, **metaheuristic methods relying on combinations of local improvement schemes or operators** became popular and widely applied to various new VRP specifications. The most notable examples include Adaptive Large Neighbourhood Search (ALNS) [101]–[103], Variable Neighbourhood Search (VNS) [61], [104], [105], the Greedy Randomized Adaptive

Search Procedure (GRASP) [106], [107], Simulated Annealing (SA) [92], [108] or Tabu Search (TS) [109], [110].

Since the 1990s, **population or agent-based metaheuristic approaches** have gained momentum [111], in particular Genetic Algorithms (GAs) [112], [113], Memetic Algorithms (MAs) [76], [114]–[116], Ant Colony Optimization (ACO) [117], [118], Particle Swarm Optimization (PSO) [77], [119] or Artificial Bee Colony optimization (ABC) [120], [121].

All the above-mentioned methods are well-known in the VRP community and thoroughly researched in various VRP contexts. Generally speaking, in the vast majority of cases, the application of a suitable metaheuristic method (either one of those relying on local search improvement, or one of population-based or agent-based CI methods), when adequately tuned and tailored towards specific problem constraints, usually leads to a strong solution, acceptable from a practical (business) perspective.

On the other hand, despite the efficacy of these well-established solution methods, several new ideas **transferred from other optimization domains** have recently entered the VRP field, offering a **viable alternative** to the existing CI and non-CI methods. In our opinion, these new solution concepts, which belong to broadly understood CI area, have potential for future development and more frequent usage in the field of the VRP. The list of these emerging methods includes *game-theoretic and bi-level optimization methods*, *Monte Carlo simulations*, *hyperheuristic approaches*, *cognitively motivated approaches*, as well as *particular hybridizations of baseline CI methods*. Some of them address new, specific problem variants (e.g. *Vehicle Routing Games* which refer to cooperative logistics, or *Capacitated VRP with Traffic Jams*), others are applied to more typical VRP formulations, offering however, innovative solution schemes.

A. *Hybridizations of Baseline Methods*

In recent years many synergetic CI approaches which combine two or more baseline methods have been developed in the field of the VRP, partly in response to the emergence of more complex problem formulations. One of the interesting examples is a combination of TS and ABC, proposed in [122] regarding *VRP with time windows and pallet loading constraints* (VRPTWP). In the combined TS-ABC approach, TS is used for fast generation of high quality solutions (food sources for bees), which are then explored and optimized by the ABC algorithm.

Another combination of two metaheuristics is proposed in [123] in the context of *Multidepot and Periodic VRP* (MDPVRP), where MA is supported by the SA method equipped with a set of random heuristic operators. Furthermore, the initial MA population is constructed using a mixture of stochastic and greedy learning. According to the authors, such a hybridization of the stochastic component and greediness is crucial in the avoidance of premature convergence, and allows to maintain the appropriate balance between exploration and exploitation.

Another interesting hybrid approach, combining ACO with the *multi-attribute labeling technique* [21] and leading to the *multi-attribute Label-based Ant Colony System* (LACS), has been recently proposed in [50], in the context of Rich VRP (cf. Section III-D), which simultaneously considers fleet heterogeneity, time windows and mixed backhauls. Each set of labels in the LACS is associated with a particular problem aspect: *demand labels* refer to delivery amount, service time and time window specification, *vehicle labels* address the vehicles' parameters and availability, and *route labels* concern the length, overall planned demand and already served demand on the respective route. The main advantage of the label-based approach is its generality (various constraints can be naturally combined into one coherent system) and flexibility (changes in the problem parametrization require only minor adjustment of the overall system).

B. *Game-Theoretic Approaches and Bi-Level Optimization Methods*

In this section several methods rooted in game theory and bi-level optimization, recently applied in the VRP domain, are summarized.

The first contribution considers the so-called *Vehicle Routing Games* (VRG) which model the behavior of delivery companies as players in a cooperative game [44]. One of the main issues in rendering collaborative delivery services is the fair division of the total profit and cost of service among coalition members. To this end the concept of *cross-reference value* is applied and its efficacy experimentally evaluated. For each player, the cross-reference value is calculated as an average of his or her self-evaluation and peer-evaluations made by all other coalition members. Even though the VRG formulation addresses the profit/cost allocation problem from a rather simplistic perspective, it can be generally concluded that the underlying ideas presented in [44] have application potential in more complex realizations of VRP shared delivery services, modeled as cooperative games, after an adequate adjustment of the assessment mechanism.

Cooperative game theory was also applied in [45] to solve the VRPTW. In the proposed solution method it is assumed that a pool of vehicles is created for the purpose of rendering cooperative delivery service. The main questions, apart from the optimality of the service cost, concern an efficient way of vehicle pool creation and a fair allocation of costs and benefits among shippers. The former is addressed by means of *Myerson value* [46] and the latter through an *overlapping coalition formation game* (OCFG). In OCFG each delivery company can simultaneously form several coalitions with other companies in various configurations and with various levels of their own vehicle contribution to the overall coalition fleet. These appropriate quotas of vehicle involvement, as well as a fair cost allocation mechanism among cooperating shippers follow the *Nash Equilibrium conditions*.

In [124] heterogeneous *VRP with Multiple Time Windows* (VRPMTW) is considered in a general context of multi-objective optimization. The underlying claim is that in order for the transportation industry to apply *any* type of VRP solution, the following three aspects need to be addressed in the objective function: minimization of the total travel cost, maximization of the minimal customer utility, and maximization of the minimal driver utility. The first objective is an obvious expectation from a transportation company viewpoint, the next one refers to the quality of service, and the last one is related to the drivers' contractual agreements. These three subjects (company, customers, drivers) are regarded as players in a multiple-agent noncooperative game with multiple objectives. The solution method is a hybrid VNS and TS approach which chooses a *Pareto nondominated set of solutions* that satisfy the *Nash Equilibrium conditions* for the considered game model.

An attacker-evader scenario in Stackelberg Games framework applied to a specific aspect of VRP, namely to the transportation of valuable goods, materials or people [68], discussed in Section III-E4, represents another interesting example of a game theoretic approach. The proposed bi-level problem formulation is solved by the *Benders decomposition algorithm* [125] (in the exact case) and two bi-level metaheuristic methods (with approximate solutions). Both approximate approaches employ *Bi-level version of the PSO* (Bi-PSO) with suitable problem representation and an adequate objective function on each of the two levels.

A bi-level optimization approach is also proposed in [60] to address the problem of transportation of hazardous materials (cf. Section III-E2). The solution method involves a fuzzy bi-level programming model, in which the upper level formulation is responsible for the allocation of customers to the depots (constrained by depot capacities and customer demands), and the lower level problem determines route optimality for a given depot and the associated set of customers. Four fuzzy, simulation-based heuristic algorithms are proposed for solving the problem.

One more example of utilization of the bi-level optimization paradigm in the VRP domain is presented in [93] in the context of multi-objective G-MoMaVRP, discussed in Section III-F. The method extends the bi-level formulation of the classical single-objective VRP proposed in [126] to its multi-objective and multi-attribute form. In short, G-MoMaVRP is decomposed into two levels. The upper level relies on an multi-objective evolutionary method and is responsible for defining the optimal sequence of visits for all customers. The lower lever is tackled by a multi-objective extension of Dijkstra's shortest path algorithm [127] and its role is to assign customers to particular routes.

C. Solution Methods Relying on Monte-Carlo Simulations

In [128] a new version of *Dynamic VRP* (DVRP) called *Capacitated VRP with Traffic Jams* (CVRPwTJ) was proposed and solved using the *Upper Confidence bounds applied to Trees* (UCT) method [129]. In CVRPwTJ traffic jams occur randomly with predefined intensity and length distributions. Consequently, static CVRP is transformed into a dynamic, nondeterministic scheduling problem. Except for a new problem formulation, the paper proposes a qualitatively new solution approach which relies on the UCT method (used for the first time in the VRP literature). UCT is a state-of-the-art approach in game domain (e.g. in Go [130], [131] or General Game Playing [132], [133]) and also a strong solution technique in the area of planning and scheduling [134], [135]. In short, UCT explores the state space provided in the form of a tree, by means of Monte Carlo (MC) simulations of possible problem solutions, in a way that assures an efficient exploration to exploitation balance. The most challenging issue in the proposed approach is finding a suitable mapping of the CVRPwTJ structure onto a tree-like problem representation required by the UCT search method.

MC simulations were also utilized in the problem of modeling efficient EV routing using battery charging infrastructure recently deployed in Ireland [89], mentioned in Section III-F. Massive MC simulations taking into account the density of the Irish population, the specified number of EVs, and the estimated trip lengths distribution helped in assessing the capacity of the proposed routing model on a nationwide scale. Also, the impact of potential faults of particular elements of the charging infrastructure on the efficacy of the whole model was quantitatively assessed in the simulations allowing for the detection of critical parts of the charging system.

D. Hyperheuristic Approaches

On the operational level, the hyperheuristic approach to problem solving consists of two layers: the upper layer maintains a general problem solving strategy, and the lower layer contains a set of simple problem-based strategies (problem-dependent heuristics). The role of the upper level strategy is to choose the lower level heuristics (which are directly used to solve a given combinatorial optimization problem), analyze their performance and accept or reject their outcomes.

Two recent papers [136], [137] propose two hyperheuristic approaches applicable to a wide class of Constrained Optimization Problems (COPs), including various VRP formulations. In the first paper [136], the role of the upper level strategy is played by the Dynamic Multi-armed Bandit mechanism [138], which is a specific implementation of a general Multi-armed Bandit scheme [129] (applied also in [128], under the name *UCT method*, as discussed above in Section IV-C). The acceptance mechanism is provided by means of a gene expression programming algorithm described in the paper in detail. The method is applied to two different COPs: exam timetabling and DVRP (Kilby *et al.* instances [139]), leading to high quality solutions.

The other paper [137] implements the hyperheuristic methodology in a different way. Here, the higher level heuristic is generated automatically by means of a gene expression programming algorithm, as part of the solution process. The generality and efficacy of the proposed method is verified on a suite of six different COPs which includes the classical VRP formulation.

E. Cognitively Motivated Methods

An interesting cognitively-motivated approach to the classical CVRP - very different from the mainstream literature - has been recently introduced in [140]. The baseline idea is known as the *human-in-the-loop* paradigm and boils down to the cooperative solving of demanding optimization problems by human and machine. In most implementations of the human-in-the-loop scenarios, humans and computers *interact with each other* during the collaborative effort of finding the optimal solution. One of the possible realizations of such a paradigm is the human guidance of machine search for the optimal solution, based on his or her cognitive abilities [141], [142].

In [140] human performance in solving CVRP is assessed in two main scenarios. In the first one, live feedback on constraints violation from the system is accompanied by directive feedback concerning the planned route. In the other, more restrictive setting, only live explanatory feedback, without additional information regarding the route planning strategy, is provided. In both scenarios human performance is visibly better than in the situation when no interaction with the machine is possible. No significant performance difference between the two above-mentioned settings was noticed.

Another key aspect of cognitively-motivated human-like problem solving is the implicit transfer learning by means of efficient utilization of interdomain knowledge in a multi-task optimization paradigm. Two recent approaches of this kind (though with no explicit reference to human cognitive abilities) were proposed in [143] and [144]. In the former paper, *knowledge memes* explore two different, but related, problem domains: CVRP and CARP (*Capacitated Arc Routing Problem* [145]), combining knowledge learnt in both domains to boost the search process. In the latter approach, which refers to DVRP, the evolutionary search process aims to discover (learn) the baseline structure of the optimized solution at an early stage and uses this knowledge to streamline the search when the dynamics occurs. The system is tested on the standard set of 21 benchmarks of Kilby *et al.* [139].

F. The Lack of Neural Methods

It came as a surprise to the author that neural networks (NNs) have practically not been utilized in the VRP domain, even though they have been successfully applied to solving constrained optimization problems for years. Since 1985 and publication of the seminal paper by Hopfield and Tank [146] there has been a strong resurgence in the field lasting about two decades, which has led to "massive" application of NNs to a wide variety of constrained optimization tasks (e.g. the N-Queens Problem [147], [148], the Quadratic Knapsack Problem [149], and - most notably - the Traveling Salesman Problem [150], [151], to name just a few examples).

Despite this initial burst of interest in solving constrained optimization problems with the use of NNs, apparently, over the years they have not proven useful for solving optimization problems whose structure changes in time (in a dynamic, stochastic, or any other way). The main reason for that should, most probably, be attributed to the lack of efficient mechanisms for capturing the drifting form of the error function, which represents the estimated solution cost. Due to the changing structure of the problem, the corresponding goal function also evolves in time. Therefore, it seems that some kind of goal adaptation mechanism during NN training, when applied to solving dynamic or stochastic VRP formulations, is indispensable.

A recent paper related to constrained optimization with NNs and reinforcement learning [152] presents a novel approach to (static) TSP, which may potentially be extended to solving the VRP. The proposed method relies on learning a stochastic policy over potential solutions, using *the expected reward as an objective*. One of potential research directions is the modification of this method to the case of *reward mechanisms changing in time*. Another interesting application of NNs trained with Deep RL was proposed in [153] for the (static) CVRP with potential extension to dynamic VRP formulations. The solution relies on certain modifications of the *Pointer Network* introduced by Vinyals *et al.* [154]. The trained model possesses transfer learning properties and can be used for new problem instances, with no need for re-training, if only the new instances are generated from the same (training) distribution. Another paper worth mentioning in the context of NN applications to dynamic VRPs [155] presents an MA-based approach to VRPwSD guided by the Extreme Learning Machine (ELM) [156]. The ELM enhances the memetic algorithm by handling the process of learning the knowledge memes from previous VRP instances in an automated way.

V. Conclusions

The field of the Vehicle Routing Problem is developing at an enormous speed-each year hundreds of new papers are published in high-quality journals and presented at top conferences. VRP-related publications appear in various domain venues, from transportation, through constrained optimization, computational intelligence, to logistics and planning. For this reason new survey papers appear practically each year, usually addressing specific problem types (e.g. Green VRP [8], Dynamic VRP [3], Multi-attribute VRP [9], Waste collection VRP [10], City VRP [11], Periodic VRP [12] and other). Some surveys also aim to extend the taxonomy of the VRP so as to keep it up-to-date with emerging problem variants and new solution methods [6], [13].

In this review we focus on high-quality journal papers published between January 2015 and July 2017. In comparison with the above-cited surveys, a different approach has been adopted in this work, where the main criteria of paper selection are *originality* and *novelty* of the considered problem formulation and/or *innovativeness* of the proposed solution method. This way, as stated in the title, ***emerging problem formulations and new CI solution methods*** in the VRP domain are identified in this review, for possible further development by the interested readers.

New problem formulations discussed in this paper are often accompanied by a suitable set of benchmark problems, tailored towards specific requirements of the considered VRP variant. These new benchmark sets are listed alongside the descriptions of the respective problem specifications.

We believe that this survey will be helpful to researchers looking for challenging, not yet deeply explored problem variants, or emerging innovative solution methods in the broad and fast-growing Vehicle Routing Problem domain.

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