UNIVERSITY OF SUSSEX
Scientific Computing
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Assignment 2
Deadline: 12pm on Monday, November 21st, 2016.
Penalties will be imposed for submissions beyond this date.
Final submission date: Tuesday, November 22nd, 2016
No submissions will be accepted beyond this date.

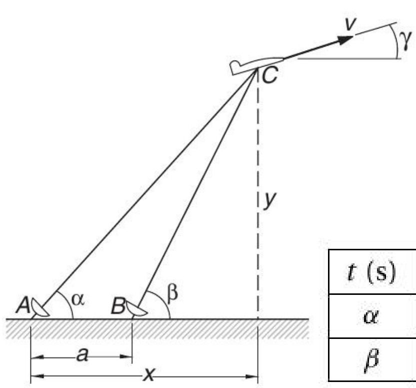
1. **Systems of equations:** The trajectory of a satellite orbiting the Earth is:

$$R = \frac{C}{1 + e\sin(\theta + \alpha)} \tag{1}$$

where (R, θ) are the polar coordinates of the satellite, and C, e, and α are constants (e is known as the eccentricity of the orbit, $C = a(1-e^2)$, where a is the orbit's semi-major axis, and α is the phase). If the satellite was observed at the three positions listed in the table, determine the smallest R of the

θ	-30°	0°	30°
R (km)	6870	6728	6615

trajectory (this is called pericentre of the orbit) and the corresponding value of θ. To solve the equations use both the provided code `newtonRaphson2.py` and the in-build routine `scipy.optimize.fsolve`. [30]



t (s)	9	10	11
α	54.80°	54.06°	53.34°
β	65.59°	64.59°	63.62°

2. The radar stations A and B, separated by the distance $a = 500\ m$, track the plane C by recording the angles α and β at 1-second intervals, as shown in the

figure. If three successive readings are shown in the table, calculate the speed v of the plane and the climb angle γ at $t = 10\ s$ as accurate as you can. The coordinates of the plane can be shown to be:

$$x = a\frac{\tan\beta}{\tan\beta - \tan\alpha}, \qquad y = a\frac{\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

[20]

3. (a) Derive the central difference approximation for $f''(x)$ accurate to $O(h^4)$ by applying Richardson extrapolation to the central difference approximation of $O(h^2)$.

   (b) Use the approximations in (a) to estimate the second derivative of $f(x) = x^3 + e^{-x}$ at $x = 1$ using $h = 0.1$ and $h = 0.5$. What are the errors for each of the two approximations? Do they behave as expected? [20]

4. **The Secant Function Integral:** The integral of the secant function:

$$\int_0^\theta \sec\phi d\phi$$

(remember that $\sec\phi = 1/\cos\phi$) for $-\pi/2 < \theta < \pi/2$ is important in navigation and the theory of map projections.

   a) Show analytically that it can be expressed in closed form as the inverse of the Gudermanian function

$$gd^{-1}(\theta) = \ln|\sec\theta + \tan\theta|$$

   b) Use `scipy.integrate.quad` to calculate the values for the integral at 1000 points across the relevant range above and compare graphically to the exact solution by plotting the decimal log of the absolute errors with respect to the exact solution. Discuss the results.

[30]