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SCIENTIFIC COMPUTING
Assessment 1(a)

1a) Kirchoff's Law for current $\Rightarrow \sum I_{in} = \sum I_{out}$

Ohm's law $\Rightarrow I_{R_j} = V_n / R_j = x_n e^{i\omega t} / R_j$

$$I_{C_k} = \frac{dQ}{dt} = C_k \frac{d}{dt}(V_n) = C_k \cdot \frac{d}{dt}\left(x_n e^{i\omega t}\right) = i\omega C_k x_n e^{i\omega t}$$

- Where n, j & k are variables dependent on the point in question

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(1) For point 1 : $(n = 1,2,+)(j = 1,4)$ & $(k = 1)$

$$I_{in} = I_{R_1} + I_{C_1} = x_+ e^{i\omega t}/R_1 + i\omega C_1 x_2 e^{i\omega t}$$

$$I_{out} = I_{R_1} + I_{R_4} + I_{C_4} = x_1 e^{i\omega t}/R_1 + x_1 e^{i\omega t}/R_4 + i\omega C_1 x_1 e^{i\omega t}$$

$$\therefore x_+/R_1 + i\omega C_1 x_2 = x_1/R_1 + x_1/R_4 + i\omega C_1 x_1$$

$$\Rightarrow \frac{x_+}{R_1} = x_1\left(\frac{1}{R_1} + \frac{1}{R_4} + i\omega C_1\right) - i\omega C_1 x_2$$

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(2) For point 2 : $(n = 1,2,3,+)$, $(j = 2,5)$ & $(k = 1,2)$

$$I_{in} = I_{R_2} + I_{C_1} + I_{C_2} = x_+ e^{i\omega t}/R_2 + i\omega C_1 x_1 e^{i\omega t} + i\omega C_2 x_3 e^{i\omega t}$$

$$I_{out} = \frac{x_2 e^{i\omega t}}{R_2} + \frac{x_2 e^{i\omega t}}{R_5} + i\omega C_1 x_2 e^{i\omega t} + i\omega C_2 x_2 e^{i\omega t} = I_{R_2} + I_{R_5} + I_{C_1} + I_{C_2}$$

$$\therefore \frac{x_+}{R_2} + i\omega C_1 x_1 + i\omega C_2 x_3 = \frac{x_2}{R_2} + \frac{x_2}{R_5} + i\omega C_1 x_2 + i\omega C_2 x_2$$

$$\Rightarrow \frac{x_+}{R_2} = -i\omega C_1 x_1 + x_2\left(\frac{1}{R_2} + \frac{1}{R_5} + i\omega C_1 + i\omega C_2\right) - i\omega C_2 x_3$$

(3) For point 3 : $(n = 2, 3, +)$ $(j = 3, 6)$ & $(k = 2)$

$$I_{in} = I_{R_3} + I_{C_2} = x_+ e^{i\omega t}/R_3 + i\omega C_2 x_2 e^{i\omega t}$$

$$I_{out} = I_{R_3} + I_{R_6} + I_{C_2} = x_3 e^{i\omega t}/R_3 + x_3 e^{i\omega t}/R_6 + i\omega C_2 x_3 e^{i\omega t}$$

$$\therefore \frac{x_+}{R_3} + i\omega C_2 x_2 = \frac{x_3}{R_3} + \frac{x_3}{R_6} + i\omega C_2 x_3$$

$$\Rightarrow \frac{x_+}{R_3} = x_3\left(\frac{1}{R_3} + \frac{1}{R_6} + i\omega C_2\right) - i\omega C_2 x_2$$

Therefore :

$$\left(\frac{1}{R_1} + \frac{1}{R_4} + i\omega C_1\right) x_1 - i\omega C_1 x_2 = \frac{x_+}{R_1}$$

$$-i\omega C_1 x_1 + \left(\frac{1}{R_2} + \frac{1}{R_5} + i\omega C_2\right) x_2 - i\omega C_2 x_3 = \frac{x_+}{R_2}$$

$$-i\omega C_2 x_2 + \left(\frac{1}{R_3} + \frac{1}{R_6} + i\omega C_2\right) x_3 = \frac{x_+}{R_3}$$

as required.