UNIVERSITY OF SUSSEX
Scientific Computing
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Problem Sheet 7 (Problem 2 will be assessed)
Deadline: 12pm on Monday, November 21st, 2016.
Penalties will be imposed for submissions beyond this date.
Final submission date: Tuesday, November 22nd, 2016
No submissions will be accepted beyond this date.

1. **Diffraction of light:** In optics, you have learned that light bends around objects, i.e. exhibits diffraction. One of the simplest cases to study is the bending of light around a straight edge. In this case, we find that the intensity of light varies as we move away from the edge according to:

$$I = 0.5 I_0 \{[C(v) + 0.5]^2 + [S(v) + 0.5]^2\}$$

where I_0 is the intensity of the incident light, v is proportional to the distance travelled, and $C(v)$ and $S(v)$ are the Fresnel integrals:

$$C(v) = \int_0^v \cos(\pi w^2/2) dw$$

and

$$S(v) = \int_0^v \sin(\pi w^2/2) dw$$

Using `scipy.integrate.quad` or Romberg integration, numerically integrate the Fresnel integrals, and thus evaluate I/I_0 as a function of v for $0 \leq v \leq 10$. Plot your results for C, S and I/I_0. Do they agree with what you have learned about diffraction in Optics? As an extra twist, try to this *computationally efficiently.*

Challenge: Derive the formula for I.

2. **The Secant Function Integral:** The integral of the secant function:

$$\int_0^\theta \sec\phi d\phi$$

(remember that $\sec\phi = 1/\cos\phi$) for $-\pi/2 < \theta < \pi/2$ is important in navigation and the theory of map projections.

a) Show analytically that it can be expressed in closed form as the inverse of the Gudermanian function

$$gd^{-1}(\theta) = \ln|\sec\theta + \tan\theta|$$

b) Use `scipy.integrate.quad` to calculate the values for the integral at 1000 points across the relevant range above and compare graphically to the exact solution by plotting the decimal log of the absolute errors with respect to the exact solution. Discuss the results.