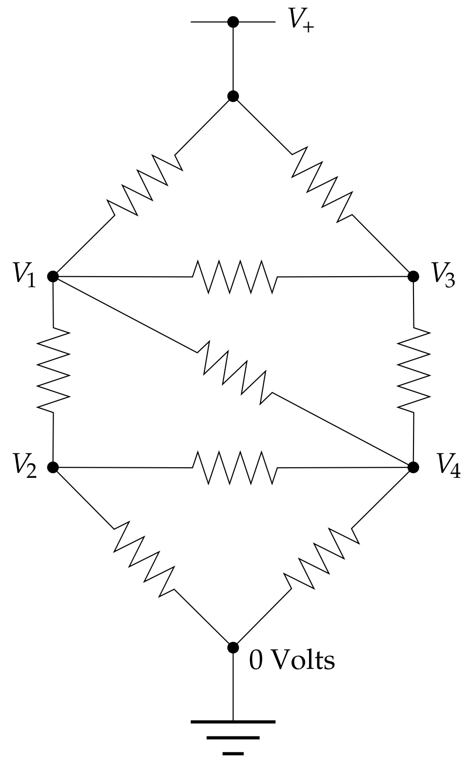
UNIVERSITY OF SUSSEX
Scientific Computing
Tutor: Dr. Ilian Iliev, Office: Pev III 4C5

Assignment 1
Deadline: 12pm on Thursday, October 30th, 2014.
Penalties will be imposed for submissions beyond this date.
Final submission date: Friday, October 31st, 2014.
No submissions will be accepted beyond this date.

1. Consider the following circuit of resistors:



All the resistors have the same resistance R. The power rail at the top is at voltage $V_+ = 5\,\text{V}$. What are the other four voltages, V_1 to V_4?

To answer this question we use Ohm's law and the Kirchhoff current law, which says that the total net current flow out of (or into) any junction in a circuit must be zero. Thus for the junction at voltage V_1, for instance, we have

$$\frac{V_1 - V_2}{R} + \frac{V_1 - V_3}{R} + \frac{V_1 - V_4}{R} + \frac{V_1 - V_+}{R} = 0,$$

or equivalently

$$4V_1 - V_2 - V_3 - V_4 = V_+.$$

a) Write similar equations for the other three junctions with unknown voltages.
b) Write a program to set up the four resulting equations in the standard way as a matrix A and a vector b, solve the system using Gaussian elimination and hence find the four voltages. For this you can use either the `solve` code from the `scipy.linalg` package or the provided `gaussElimin` code. In either case, check the help to understand how each code works in terms of input/output. E.g. `gaussElimin` *modifies* the input.
c) Use `lu` code from the `scipy.linalg` package to do the LU decomposition of the matrix you created in b). Again, please check carefully what the inputs are and how the results should be interpreted (i.e. what the L and the U matrices actually are).
d) Solve the system $Ax = b$ using this LU decomposition of the matrix A and right-hand sides given by b as derived in a) and b where the bottom voltage is not zero (ground), but is $V_0 = 1$ V, instead. Check the solutions you have found.

[40]

2. Solve the equations

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 2 & -1 \\ -1 & 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

by Gauss elimination with scaled row pivoting (`gaussPivot.py`). What happens if you try to do the same with no pivoting (using `gaussElimin.py`)?

[30]

3. **Fitting data points:**

The relative density ρ of air was measured at various altitudes h. The results were:

(a) Determine the relative density of air at 2 and 5 km using polynomial interpolation. Which order polynomial you need to use?

(b) Determine the relative density of air at 2 and 5 km using cubic spline interpolation.

(c) Use a quadratic least-squares fit to determine the relative air density at 2 and 5 km.

(d) Plot the data and the points you obtained in (a), (b) and (c). Which do you think might be most accurate here?

h (km)	0	1.525	3.050	4.575	6.10	7.625	9.150
ρ	1	0.8617	0.7385	0.6292	0.5328	0.4481	0.3741

Figure 1: Data for Problem 2.

[30]