UNIVERSITY OF SUSSEX
Scientific Computing
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Problem Sheet 3 (Problem 2 will be assessed)
Deadline: 12pm on Monday, October 31st, 2016.
Penalties will be imposed for submissions beyond this date.
Final submission date: Tuesday, November 1st, 2016.
No submissions will be accepted beyond this date.

1. **Interpolation and fitting:**

(a) Fit a straight line and a quadratic to the data in the Table below. Which fits better? (Hint: Compare the standard deviations.)

x	1.0	2.5	3.5	4.0	1.1	1.8	2.2	3.7
y	6.008	15.722	27.130	33.772	5.257	9.549	11.098	28.828

Figure 1: Data for straight line and quadratic fitting.

(b) Determine a and b so that $f(x) = axe^{bx}$ fits the following data in the least-squares sense. Compute the standard deviation.

x	0.5	1.0	1.5	2.0	2.5
y	0.541	0.398	0.232	0.106	0.052

Figure 2: Data for exponential function fitting.

(c) The drag coefficient c_D for an object moving through a fluid is defined as:

$$c_D = \frac{2F_{\mathrm{d}}}{\rho u^2 A}$$

where F_{d}, is the drag force, which is by definition the force component in the direction of the flow velocity, ρ is the mass density of the fluid, u is the flow speed of the object relative to the fluid and A is the reference area. The Reynolds number (Re) is a dimensionless quantity that is used to help predict the flow patterns in different fluid flow situations. Specifically, high Reynolds numbers correspond to turbulent flows, while low ones to laminar flows. The type of flow is related to the resistance experienced by an object moving through a fluid - high for turbulent flows, lower for laminar ones (thus e.g. airplanes and fast cars are constructed to minimize the former, so less air drag is experienced).

The table below shows the drag coefficient c_D of a sphere as a function of the Reynolds number Re.

Re	0.2	2	20	200	2000	20 000
c_D	103	13.9	2.72	0.800	0.401	0.433

Figure 3: Drag coefficient data.

Use the natural cubic spline to find c_D at Re $= 5$, 50, 500, and 5000, then do the same, but using a polynomial interpolant. Hint: use log-log scale in both cases (what happens if you don't?).

2. **(Assessed)** Interpolate the following function:

$$f(x) \quad = \quad \begin{cases} -1 & \text{if } x < 0 \\ +1 & \text{if } x \geq 0 \end{cases}$$

with 11 equally spaced points in the interval $[-\pi, \pi]$, as follows:

(a) Set up function values using the `linspace` command. Calculate (using the in-build `scipy.interpolate.barycentric_interpolate`) the values of the interpolation polynomial of maximal order to interpolate $f(x)$ (e.g. the Lagrangian interpolation polynomial) at the points $x = -1.5, 1$ and 3. Compare with the true values of $f(x)$ at the same points. What is the order of the interpolating polynomial?

(b) Now use the in-build Python function `interp1d` to interpolate the function $f(x)$ again, but now using cubic spline. Give again the values at $x = -1.5, 1$ and 3 for the interpolation and compare them with the true values of $f(x)$ at those points.

(c) Make a figure for $f(x)$ and its interpolations. On the figure plot the data points as circles and plot the exact function and interpolating functions from (b) and (c) as lines with different colours on a finer spacing (with 1000 points in the interval $[-\pi, \pi]$).

(d) Fit the same data from (a) above with the function $a * tanh(b * x + c)$. Add some random noise to the data (as real data from experimental measurements would have), as follows:

$$y_{noisy} = y + 0.1 * np.random.random(len(x))$$

Repeat the fit with the above function using the noisy data and plot both fits against the noisy data. How much did the parameters a, b and c change? What do you conclude?

Note: This last fit is related to the results of Planck and WMAP satellites on the timing and duration of the Epoch of Reionization, see e.g. `http://adsabs.harvard.edu/abs/2016A%26A...594A..13P`. [30]