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SCIENTIFIC COMPUTING
Assessment 2

3a) Using the central finite difference approximation to $f'(x)$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$$

We can rewrite as:

$$f'(x) = D(h) + O(h^2)$$

$$D(h) = \frac{f(x+h) - f(x-h)}{2h}$$

$$O(h^2) = e_2 h^2 + e_4 h^4 + \dots$$

(errors)

Eliminate $e_2 h^2$ error:

$$\cancel{f'(x) = D(2h) + e_2(2h)^2 +}$$

$$L(\cancel{f'(x)} = D(2h) + e_2(2h)^2 + e_4(2h)^4 + \dots$$

$$= D(2h) + 4e_2(h)^2 + 16e_4(h)^4$$

$$4f'(x) - L = 4D(h) + 4e_2(h)^2 + 4e_4(h)^4 - D(2h) - 4e_2h^2 - 16e_4(h)^4$$

$$= 4D(h) - D(2h) - 12e_4h^4$$

We therefore can find the fourth order approximation of the derivative as:

$$f'(x) = \frac{1}{3}\left(4\left(\frac{1}{2h}\left(f(x+h) - f(x-h)\right)\right) - \frac{1}{4h}\left(f(x+2h) - f(x-2h)\right)\right) + o(h^4)$$

$$= \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} + o(h^4)$$

4a) $\int_0^\theta \sec\phi \, d\phi = \int_0^\theta \sec\phi \frac{\sec(\phi)+\tan(\phi)}{\sec(\phi)+\tan(\phi)} \, d\phi$

$= \int_0^\theta \frac{\sec^2\phi + \sec\phi\tan\phi}{\sec\phi + \tan\phi} \, d\phi$

$$\boxed{u = \sec\phi + \tan\phi \qquad \frac{du}{d\phi} = \sec^2\phi + \sec\phi\tan\phi}$$

$\int_a^b \frac{du}{u} = \ln(u)\Big|_a^b = \ln\left|\sec(\phi)+\tan(\phi)\right|\Big|_0^\theta$

$\therefore \int_0^\theta \sec\phi \, d\phi = \ln\left|\sec(\theta) + \tan(\theta)\right|$