Candidate Number

THE UNIVERSITY OF SUSSEX

F3212

BSc, MPhys END OF YEAR TWO EXAMINATIONS, 2014

SCIENTIFIC COMPUTING

Friday, 10 January 2014 9.30 am. - 11.00 a.m.

DO NOT TURN OVER UNTIL INSTRUCTED TO BY THE CHIEF INVIGILATOR

Credit will be given for the best **TWO** *answers only.*
Total time allowed: **ONE and a HALF** *hours.*

Each question carries 20 marks. The approximate allocation of marks is shown in brackets by the questions.

A list of **physical constants** *is provided.*

Notes: *You are allowed to use your notes, as well as the course slides and codes available on Study Direct, but you must not contact other people (by email, chat or any other means). Please always explain your choice of method and code to use. Use a program that we/you wrote during the course whenever possible, and submit all programs as part of the solutions, along with a 'master sheet' write-up which summarises what you did, how to use your codes and the results that you obtained, any plots, etc. The master sheet and all programs have to be submitted (ideally in a single zip-ed file) via the drop box on the course site on Study Direct (please ensure that it worked before leaving the exam room!). The best two of the three problems count for the marks.*

1. **Differential equations:** A skydiver of mass m in a vertical free fall experiences an aerodynamic drag force $F_D = c_D \dot{y}^2$, where y is measured downward from the start of the fall. The differential equation describing the fall is

$$\ddot{y} = g - \frac{c_D}{m}\dot{y}^2$$

The overdots signify a derivative with respect to time. The skydiver starts from rest.

   (a) Convert the above system of equations to 2 first-order equations. [5]

   (b) Determine the time of a 5000-m fall. Use $g = 9.80665 m/s^2$, $c_D = 0.2028 kg/m$, and $m = 80$ kg. You can use a method of your choice, e.g. `scipy.integrate.odepack` with its default tolerance. [10]

   (c) Plot the skydiver's velocity vs. time. What is the terminal velocity of the diver (i.e. the velocity when he stops accelerating)? [5]

2. **Interpolation:** The table shows how the relative density ρ of air with respect to the one at sea level varies with altitude h.

$h\,(km)$	0	1.525	3.050	4.575	6.10	7.625	9.150
ρ	1	0.8617	0.7385	0.6292	0.5328	0.4481	0.3741

   a) Use the interpolating polynomial of maximum degree to estimate the value of ρ at $h = 2, 4$, and 8 km. What degree polynomial did you use? [8]

   b) Use a cubic spline to estimate the value of ρ at $h = 2, 4$, and 8 km. Are your results different from a)? Explain. [8]

   c) If the actual value at $h = 4$ km were 0.67, what are the relative and absolute errors of your estimates for $\rho(h = 4\,km)$ in (a) and (b)? [4]

3. **Numerical integration:** A power spike in an electric circuit results in the current

$$i(t) = i_0 e^{-t/t_0} \sin(2t/t_0)$$

across a resistor. The energy E dissipated by the resistor is

$$E = \int_0^\infty R[i(t)]^2 dt$$

(a) Using the data $i_0 = 100$ A, $R = 0.5\,\Omega$, and $t_0 = 0.01$ s, plot $[i(t)]^2$ for $t = 0$ to 1 s using linear x-axis and logarithmic y-axis. [14]

(b) Find E. [6]

End of Paper