UNIVERSITY OF SUSSEX
Scientific Computing
Tutor: Dr. Ilian Iliev, Office: Pev 3 4C5

Problem Sheet 7 (Problem 2 will be assessed)
Deadline: 12pm on Monday, December 5th, 2016.
Penalties will be imposed for submissions beyond this date.
Final submission date: Tuesday, December 6th, 2016
No submissions will be accepted beyond this date.

1. **Conical float**

   a) Modify the provided Runge-Kutta code to use the Euler method, instead and integrate the system

   $$\begin{aligned} y' &= z \\ z' &= -y \end{aligned}$$

   with $y(0) = 0, z(0) = 1$ over the interval [0,20] with 200 steps.

   b) Do the same using the 4th order Runge-Kutta method with the provided code.

   c) Plot y from both methods and (on a separate plot) their absolute errors. What do you observe? What is the correct solution? What do you think is the reason for these results? Roughly how many points do we need for the Euler method to produce an acceptable (visually indistinguishable) solution?

2. A conical float (see figure) is free to slide on a vertical rod. When the float is disturbed from its equilibrium position, it undergoes oscillating motion described by the differential equation

   $$\ddot{y} = g(1 - ay^3)$$

   where $a = 1\,m^{-3}$ (determined by the density and dimensions of the float) and $g = 9.80665\,m/s^2$ is the acceleration of gravity.

   (a) If the float is raised to the position $y_0 = 0.1\ m$ and released (with zero initial velocity), solve the equation for $t = 0$ to $2.5\,s$ and estimate the period and the amplitude of the oscillations. Do the same for initial position $y_0 = 0.9\ m$. Plot the results. How does the initial displacement affect the results?

(b) Now consider the linear version of the same problem

$$\ddot{y} = g(1 - ay)$$

with same values of g and a and again estimate the period and amplitude of the oscillations for $y_0 = 0.1$ m and $y_0 = 0.9$ m.

(c) Modify your program so that instead of plotting x against t, it plots dx/dt against x, i.e., the velocity of the oscillators against its position. Such a plot is called a phase space plot. Make sure you say **pylab.axis('equal')** to ensure the plot axes have the same length for the phase space plots. Discuss.