UNIVERSITY OF SUSSEX
Scientific Computing
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Problem Sheet 1 (Problem 2 will be assessed)
Deadline: 12pm on Monday, October 31st, 2016.
Penalties will be imposed for submissions beyond this date.
Final submission date: Tuesday, November 1st, 2016.
No submissions will be accepted beyond this date.

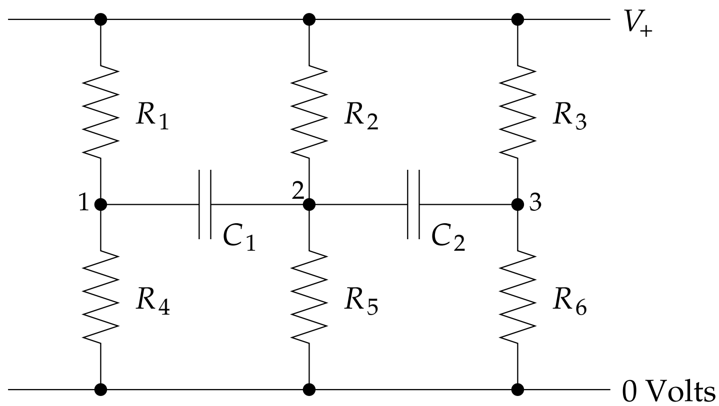
1. Use the built-in NumPy function `random.random` to create the $n \times n$ random matrix A, in which all elements have values between 0 and 10.

   (a) Use this to create an $n \times n$ matrix A for n=50, 100, 200, 300, and 400 using a `for` loop.

   (b) For this matrix A construct a vector b so that the solution to $Ax = b$ is $x = [1, 2, \ldots n]$.

   (c) Test your construction by applying Gauss elimination to compute x. What is the largest element-by-element error for each n? Plot the decimal log of the max element-by-element error vs. n. Comment on the results.

2. **(Assessed)** Consider the following RC circuit:



   The voltage V_+ is time-varying and sinusoidal of the form $V_+ = x_+ e^{i\omega t}$ with x_+ a constant. The resistors in the circuit can be treated using Ohm's law as usual. For the capacitors the charge Q and voltage V across them are related by the capacitor law $Q = CV$, where C is the capacitance. Differentiating both sides of this expression gives the current I flowing in on one side of the capacitor and out on the other:

$$I = \frac{dQ}{dt} = C\frac{dV}{dt}.$$

   (a) Assuming the voltages at the points labeled 1, 2, and 3 are of the form $V_1 = x_1 e^{i\omega t}$, $V_2 = x_2 e^{i\omega t}$, and $V_3 = x_3 e^{i\omega t}$, apply Kirchhoff's law at each of the three points, along with Ohm's law and the capacitor law, to show that the constants x_1, x_2, and x_3 satisfy the equations

$$\begin{aligned}
\left(\frac{1}{R_1}+\frac{1}{R_4}+\mathrm{i}\omega C_1\right)x_1 - \mathrm{i}\omega C_1 x_2 &= \frac{x_+}{R_1},\\
-\mathrm{i}\omega C_1 x_1 + \left(\frac{1}{R_2}+\frac{1}{R_5}+\mathrm{i}\omega C_1+\mathrm{i}\omega C_2\right)x_2 - \mathrm{i}\omega C_2 x_3 &= \frac{x_+}{R_2},\\
-\mathrm{i}\omega C_2 x_2 + \left(\frac{1}{R_3}+\frac{1}{R_6}+\mathrm{i}\omega C_2\right)x_3 &= \frac{x_+}{R_3}.
\end{aligned}$$

(b) Write a program to solve for x_1, x_2, and x_3 when

$$\begin{aligned}
&R_1 = R_3 = R_5 = 1\,\mathrm{k}\Omega,\\
&R_2 = R_4 = R_6 = 2\,\mathrm{k}\Omega,\\
&C_1 = 1\,\mu\mathrm{F}, \qquad C_2 = 0.5\,\mu\mathrm{F},\\
&x_+ = 3\,\mathrm{V}, \qquad \omega = 1000\,\mathrm{s}^{-1}.
\end{aligned}$$

Notice that the matrix for this problem has complex elements. You will need to define a complex array to hold it, but you can still use the `solve` function just as before to solve the equations—it works with either real or complex arguments. Using your solution have your program calculate and print the amplitudes of the three voltages V_1, V_2, and V_3 and their phases in degrees. (Hint: You may find the functions `polar` or `phase` in the `cmath` package useful. If `z` is a complex number then "`r,theta = polar(z)`" will return the modulus and phase (in radians) of `z` and "`theta = phase(z)`" will return the phase alone.) [40]