Technical Report

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Derivation of the Markov Decision Process Matrices of the VMEC-in-a-Box Framework

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This Technical Report aims at deriving the transition probability matrix and the short-term reward matrix of the Markov Decision Process Σ, and the battery model of the VMEC-in-a-Box needed to calculate the service outage probability of a MEC Server.

I. TRANSITION PROBABILITY MATRIX OF THE MARKOV DECISION PROCESS Σ

In order to evaluate the transition probability of the Markov Decision Process Σ, let us apply the total probability theorem to the number of possible arrivals from both the areas, i.e. λ_1 and λ_2. We have:

$$P^{(Q1,Q2|a)}_{[(s'_{Q1},s'_{Q2}),(s''_{Q1},s''_{Q2})]}(s''_{N1},s''_{N2}) = \sum_{\substack{\forall\lambda_1\in\Im^{(\Lambda1)}\\ \forall\lambda_2\in\Im^{(\Lambda2)}}} B^{(\Lambda)}_{[(s''_{N1},s''_{N2}),\lambda_1,\lambda_2]} \cdot \Pr\left\{ \begin{matrix} S^{(Q1)}(n)=s''_{Q1}, \\ S^{(Q2)}(n)=s''_{Q2}, \end{matrix} \middle| \begin{matrix} S^{(Q1)}(n-1)=s'_{Q1}, \\ S^{(Q2)}(n-1)=s'_{Q2}, A(n)=a \\ \Lambda_1(n)=\lambda_1, \Lambda_2(n)=\lambda_2 \end{matrix} \right\} \tag{1}$$

The probability term of the previous equation can be evaluated by considering that, according to the choice made for the slot duration Δ, kept equal to the mean job service time on a MEC CE, b_1 jobs, if present, will be served in the queue Q_1, and b_2 jobs, if present, in the queue Q_2. The probability term in (1) can be written as follows:

$$P^{(Q1,Q2|a)}_{[(s'_{Q1},s'_{Q2}),(s''_{Q1},s''_{Q2})]}(s''_{N1},s''_{N2}) = f^{(Q1)}\begin{pmatrix} s'_{Q1},s'_{Q2},s''_{Q1},\sigma, \\ b_1,b_2,\lambda_1,\lambda_2 \end{pmatrix} \cdot f^{(Q2)}\begin{pmatrix} s'_{Q1},s'_{Q2},s''_{Q2},\sigma, \\ b_1,b_2,\lambda_1,\lambda_2 \end{pmatrix} \tag{2}$$

where $f^{(Q1)}(\cdot)$ and $f^{(Q2)}(\cdot)$ are functions providing us the one-slot evolution probabilities of the two MEC queues. Let us define three Boolean variables to decide the MEC server that can perform offload, according to the comparison between the queue lengths:

$$I_{\text{No-OL}} = \left[s'_{Q2} = s'_{Q1}\right] \qquad I_{Q1\to Q2} = \left[s'_{Q2} < s'_{Q1}\right] \qquad I_{Q2\to Q1} = \left[s'_{Q2} > s'_{Q1}\right] \tag{3}$$

They are used as follows: if $I_{\text{No-OL}}$ is true, none of the two MEC servers performs offload; otherwise, offload can be done by the MEC Server 1 if $I_{Q1\to Q2}$ is true, or by the MEC Server 2 if $I_{Q2\to Q1}$ is true.

The first function in (2) can be calculated taking into account that, according to the event sequence illustrated so far, the queue state at the beginning of the slot, s'_{Q1}, is decreased by the number of served jobs, b_1, and increased by the non-offloaded jobs, $\min\{\sigma, \lambda_1\}$. In case of queue overflow, it is truncated to the value K. So, we have:

$$f^{(Q1)}\begin{pmatrix} s'_{Q1}, s'_{Q2}, s''_{Q1}, \sigma, \\ b_1, b_2, \lambda_1, \lambda_2 \end{pmatrix} = \begin{cases} 1 & \text{if } \left(I_{\text{No-OL}} \text{ and } H^{(Q1)}_{\text{No-OL}}\right) \text{ or } \left(I_{Q1\to Q2} \text{ and } H^{(Q1)}_{Q1\to Q2}\right) \text{ or } \left(I_{Q2\to Q1} \text{ and } H^{(Q1)}_{Q2\to Q1}\right) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The Boolean conditions $H^{(Q1)}_{\text{No-OL}}$, $H^{(Q1)}_{Q1\to Q2}$ and $H^{(Q1)}_{Q2\to Q1}$ consider the transitions of the MEC Server 1 queue when no offloads are done, when there are possible offloads from the MEC Server 1 to the MEC Server 2 and when there are possible offloads from the MEC Server 2 to the MEC Server 1. They can be obtained as:

$$H^{(Q1)}_{\text{No-OL}} = \left[s''_{Q1} = \min\left\{\left(\max\left\{s'_{Q1} - b_1, 0\right\} + \lambda_1\right), K\right\}\right] \quad H^{(Q1)}_{Q1\to Q2} = \left[s''_{Q1} = \min\left\{\max\left\{s'_{Q1} - b_1, 0\right\} + \min\{\sigma, \lambda_1\}, K\right\}\right]$$
$$H^{(Q1)}_{Q2\to Q1} = \left[s''_{Q1} = \min\left\{ \begin{pmatrix} \max\left\{s'_{Q1} - b_1, 0\right\} + \\ + \lambda_1 + \left(\lambda_2 - \min\{\sigma, \lambda_2\}\right) \end{pmatrix}, K \right\} \right] \tag{5}$$

Dually, we can define $f^{(Q2)}\left(s'_{Q2}, s''_{Q2}, \sigma, b_2, \lambda_1, \lambda_2\right)$ for the MEC Server 2.

II. Short-term reward matrix of the Markov Decision Process Σ

In order to calculate the short-term reward matrix of the Markov Decision Process Σ, let us define the expected value of the immediate reward for a given transition from the state $\underline{s}'_{\Sigma}$ to the state $\underline{s}''_{\Sigma}$, and for a given action a for the state $\underline{s}'_{\Sigma}$. It is a weighed sum of the four key performance parameters characterizing the MEC Domain behavior, that is, the mean power consumption, the mean net revenue, the mean delay for a job processed by the MEC Domain and its loss probability. More in deep, we define the immediate reward associated to that transition as follows:

$$\Psi^{(\Sigma|a)}_{[\underline{s}'_{\Sigma}, \underline{s}''_{\Sigma}]} = -c_1 \overline{\xi}(a) - c_2 \overline{\vartheta}\left(\underline{s}''_{\Lambda}, a\right) - c_3 \overline{\phi}_M\left(\underline{s}''_{\Sigma}, a\right) - c_4 \overline{\psi}_M\left(\underline{s}''_{\Sigma}, a\right) \tag{6}$$

A. *The first term of (6): power consumption penalty*

The first term regards the penalty (it becomes a reward thanks to the minus sign) received for power consumption due to the active CEs, when the action $a = (b_1, b_2, \sigma)$ is performed according to the starting state $\underline{s}'_\Sigma$. It is given by:

$$\overline{\xi}(a) = (b_1 + b_2) \cdot \wp_{CE} \tag{7}$$

where $\wp_{CE}$ is the power consumption of each active CE. The terms b_1 and b_2 are the numbers of CEs that have been decided to be active, as part of the action *a*, in the current slot.

B. *The second term of (6): net revenue due to offload*

The net revenue due to offload is constituted by two parts:

a) a revenue that is proportional to the amount of received jobs that have been offloaded by the Vehicular Domain, i.e. $E\{\Lambda_{V\to M}\}$, with a constant of proportionality, $\Theta^{(OL)}_{V\to M}$, representing the price applied by the MPS to process one job in the MEC Domain;

b) a penalty that is proportional to the mean number of offloaded jobs to another MEC server, $E\{\Phi_{M\to M}|a\}$, with a constant of proportionality, $\Theta^{(OL)}_{M\to M}$, representing the per-job offload cost from the MEC Server 1 to the MEC Server 2.

Therefore, we have:

$$\overline{\vartheta}(\underline{s}''_\Lambda, a) = -\Theta^{(OL)}_{V\to M} \cdot E\{\Lambda_{V\to M}\} + \Theta^{(OL)}_{M\to M} \cdot E\{\Phi_{M\to M}|a)\} \tag{8}$$

The term $E\{\Lambda_{V\to M}\}$ can be derived by applying the total probability theorem to all the possible numbers of arrivals from the Areas 1 and 2 when the state of the underlying Markov chain of the bi-dimensional Markov modulated process is $\underline{s}''_\Lambda$:

$$E\{\Lambda_{V\to M}\} = \sum_{\forall \lambda_1 \in \Im^{(\Lambda 1)}} \sum_{\forall \lambda_2 \in \Im^{(\Lambda 2)}} (\lambda_1 + \lambda_2) \cdot B^{(\Lambda)}_{[\underline{s}''_\Lambda, \lambda_1, \lambda_2]} \tag{9}$$

Likewise, the term $E\{\Phi_{M\to M}|a\}$ can be derived considering the MEC server that can perform some offload when the system state is $\underline{s}''_\Sigma$:

$$E\{\Phi_{M\to M}|a\} = \sum_{\forall\lambda_1\in\Im^{(\Lambda 1)}}\sum_{\forall\lambda_2\in\Im^{(\Lambda 2)}} B^{(\Lambda)}_{[\underline{s}''_{\Lambda},\lambda_1,\lambda_2]}\cdot \\ \cdot\left[(\lambda_1-\min\{\sigma,\lambda_1\})\cdot I_{Q1\to Q2}+(\lambda_2-\min\{\sigma,\lambda_2\})\cdot I_{Q2\to Q1}\right] \tag{10}$$

C. The third term of (6): mean delay in the MEC Domain

The third term in (6) regards the delays suffered in the MEC Server 1 and MEC Server 2 queues. To this purpose, we indicate the number of jobs arriving from the Areas 1 and 2 and not lost, as $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, respectively.

In order to calculate $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, we have to account that each queue state goes through two different steps before the possible arrival of the offloaded jobs coming from the other MEC server. For example, for the MEC Server 1, starting from the state s'_{Q1} and after the departures, it reaches the state $s''_{Q1,INT1}=\max\{s'_{Q1}-b_1,0\}$; then, after the arrivals from the Area 1, we have two cases:

1. if offload from the MEC Server 1 is not allowed, all the λ_1 jobs arriving from the Area 1 are sent to Q_1, and therefore the queue state becomes $s''_{Q1,INT2}=\min\{s''_{Q1,INT1}+\lambda_1,K\}$;
2. if the MEC Server 1 is enabled to offload jobs to the other server, its queue state becomes $s''_{Q1,INT2}=\min\{s''_{Q1,INT1}+\min\{\sigma,\lambda_1\},K\}$.

Therefore, the number of job arrivals from the Area 1 that are accommodated in Q_1 are $\tilde{\lambda}_1=s''_{Q1,INT2}-s''_{Q1,INT1}$. Likewise, the number of job arrivals from the Area 2 that are accommodated in Q_2 are $\tilde{\lambda}_2=s''_{Q2,INT2}-s''_{Q2,INT1}$. They will suffer a delay due to the queues they find in the MEC Servers 1 and 2, respectively. More specifically, the mean queue lengths they find are:

$$\bar{q}_{a1\to Q1}=\left[s''_{Q1,INT2}+\left(s''_{Q1,INT1}+1\right)\right]/2 \tag{11}$$

$$\bar{q}_{a2\to Q2}=\left[s''_{Q2,INT2}+\left(s''_{Q2,INT1}+1\right)\right]/2 \tag{12}$$

Instead, the number of offloaded jobs from the MEC server with the shortest queue to the other one is:

- $\tilde{\lambda}_{OL}=\min\{\lambda_{OL},K-s''_{Q2,INT}\}$, where $\lambda_{OL}=\lambda_1-\min\{\sigma,\lambda_1\}$, if the offloading MEC server is the MEC Server 1;

- $\tilde{\lambda}_{OL} = \min\{\lambda_{OL}, K - s''_{Q1,INT}\}$, where $\lambda_{OL} = \lambda_2 - \min\{\sigma, \lambda_2\}$, if the offloading MEC server is the MEC Server 2.

The mean queues the offloaded jobs find on the other server where they are offloaded are:

$$\bar{q}_{a1\to Q2} = I_{Q1\to Q2} \cdot \frac{s''_{Q2} + \left(s''_{Q2,INT2} + 1\right)}{2} \tag{13}$$

$$\bar{q}_{a2\to Q1} = I_{Q2\to Q1} \cdot \frac{s''_{Q1} + \left(s''_{Q1,INT2} + 1\right)}{2} \tag{14}$$

where the terms $I_{Q1\to Q2}$ and $I_{Q2\to Q1}$ indicate whether there is some offload from the MEC Server 1 to the MEC Server 2 or vice versa.

Therefore, applying the total probability theorem to the number of jobs arriving from both the areas, the mean delay that the MPS estimates when the system is in the state $\underline{s}'_{\Sigma}$ and the action a is performed, is:

$$\begin{aligned} \bar{\phi}_M\left(\underline{s}''_{\Sigma}, a\right) = \sum_{\forall \lambda_1 \in \Im^{(\Lambda 1)}} \sum_{\forall \lambda_2 \in \Im^{(\Lambda 2)}} B^{(\Lambda)}_{[\underline{s}''_{\Lambda}, \lambda_1, \lambda_2]} \cdot \frac{1}{\tilde{\lambda}_1 + \tilde{\lambda}_2 + \tilde{\lambda}_{OL}} \cdot \\ \cdot \left[\frac{\tilde{\lambda}_1}{b_1} \bar{q}_{a1\to Q1} + \frac{\tilde{\lambda}_{OL}}{b_2} \bar{q}_{a1\to Q2} + \frac{\tilde{\lambda}_2}{b_2} \bar{q}_{a2\to Q2} + \frac{\tilde{\lambda}_{OL}}{b_1} \bar{q}_{a2\to Q1} \right] \end{aligned} \tag{15}$$

where we have divided the mean queue lengths experienced by the arrived jobs with the current queue service rates, and weighed them with the number of jobs that enter each queue.

D. *The fourth term of (6): loss probability in the MEC Domain*

The loss probability suffered by jobs in the MEC Domain can be calculated as the ratio between the number of jobs lost in the two MEC servers, $L_{M1} + L_{M2}$, and the number of jobs arrived to the MEC Domain in the same slot, $\lambda_1 + \lambda_2$:

$$\bar{\Psi}_M\left(\underline{s}''_{\Sigma}, a\right) = \sum_{\forall \lambda_1 \in \Im^{(\Lambda 1)}} \sum_{\forall \lambda_2 \in \Im^{(\Lambda 2)}} B^{(\Lambda)}_{[\underline{s}''_{\Lambda}, \lambda_1, \lambda_2]} \cdot \frac{L_{M1} + L_{M2}}{\lambda_1 + \lambda_2} \tag{16}$$

The number L_{M1} of lost jobs in the MEC Server 1 is the sum of:

- L_{Q1}, representing the number of jobs arrived from the Area 1, which have not been offloaded and have not found any room in Q_1;

- L_{OF1}, representing the number of jobs that should be offloaded (according to the MPS decision) to Q_2, but are discarded by the OFMEC block of the MEC Server 1 because they would not found any rooms in Q_2.

In order to calculate L_{Q1}, we consider that it only depends on the arrivals from the Area 1, since jobs from the Area 2 are offloaded by the OFMEC block of the MEC Server 2 only if they are able to be accommodated in Q_1. Therefore, let us consider the Q_1 starting state, s'_{Q1}. It is decreased by the departure of b_1 jobs, and then increased by the arrival of either λ_1 or $\min\{\sigma,\lambda_1\}$ jobs (whether the MEC Server 1 is not enabled or enabled for offloading, respectively). The term L_{Q1} is not null only if the resulting queue length is higher than *K* after departures and local arrivals. Therefore, its value is:

$$L_{Q1} = \max\left\{\left[\begin{matrix}\max\{s'_{Q1}-b_1,0\}+\\ (1-I_{Q1\to Q2})\cdot\lambda_1+I_{Q1\to Q2}\cdot\min\{\sigma,\lambda_1\}-K\end{matrix}\right],0\right\} \tag{17}$$

The term L_{OF1}, on the other hand, is the difference between the number of jobs that the OFMEC block has to offload to the MEC Server 2 and the number ρ_2 of available rooms in Q_2 after departures and local arrivals, that is:

$$L_{OF1} = \max\{I_{Q1\to Q2}\cdot\min\{\sigma,\lambda_1\}-\rho_2,0\} \tag{18}$$

where ρ_2 is the difference between the maximum queue size *K* and the Q_2 queue size after departures and local arrivals from the Area 2:

$$\rho_2 = \max\left\{K-\left[\begin{matrix}\max\{s'_{Q2}-b_2,0\}+\\ (1-I_{Q2\to Q1})\cdot\lambda_2+I_{Q2\to Q1}\cdot\min\{\sigma,\lambda_2\}\end{matrix}\right],0\right\} \tag{19}$$

The terms needed to calculate the number L_{M2} of lost jobs in the MEC Server 2 can be calculated at the same way.

III. BATTERY MODEL TO CALCULATE THE SERVICE OUTAGE PROBABILITY OF A MEC SERVER

In this section, we derive the transition probability matrix of the battery behavior $\underline{S}^{(BT)}(n)$ of a MEC server, aimed at deriving its service outage probability. Its generic element can be written as the product of three terms:

$$P^{(BT)}_{[\underline{s}'_{BT},\underline{s}''_{BT}]} = P^{(RG)}_{[s'_{RG},s''_{RG}]} \cdot P^{(CF)}_{[b',b'']} \cdot P^{(QB)}_{[s'_{QB},s''_{QB}]}\left(s''_{RG},b''\right) \tag{20}$$

The matrix $P^{(RG)}$ is the transition probability matrix of the SBBP process $RG(n)$ describing the behavior of the microeolic power generator. It, together with the matrix $B^{(RG)}$ giving us the probability distribution of the amount of charge generated for each state of the wind, is an input of the problem. They are defined as follows:

$$P^{(RG)}_{[s'_{RG},s''_{RG}]} = \Pr\left\{S^{(RG)}(n) = s''_{RG} \middle| S^{(RG)}(n-1) = s'_{RG}\right\} \quad \text{and} \quad B^{(WG)}_{[s''_{RG},\beta_{RG}]} = \Pr\left\{RG(n) = \beta_{RG} \middle| S^{(RG)}(n) = s''_{RG}\right\} \tag{21}$$

In order to calculate the matrix $P^{(CF)}$, let us define the subset $\Im^{(\Sigma,1)}_{b}$ of $\Im^{(\Sigma)}$ whose states are the ones that are characterized by an action that activates b CEs in the MEC Server 1 (for the MEC Server 2, it can be calculated in the same way):

$$\Im^{(\Sigma,1)}_{b} = \left\{\underline{s}_{\Sigma} \in \Im^{(\Sigma)} \text{ such that } a^{*}_{\underline{s}_{\Sigma}} = (b,b_2,\sigma), \forall b_2, \forall \sigma\right\} \tag{22}$$

where $a^{*}_{\underline{s}_{\Sigma}}$ indicates the best action associated to the state $\underline{s}_{\Sigma}$.

Now, the probability of transition for the process $S^{(CF)}(n)$, representing the number of active CEs in the MEC Server 1, from the generic state b' of the process $S^{(CF)}(n)$ (i.e. b' active CEs) to b'' can be easily calculated as follows:

$$P^{(CF)}_{[b',b'']} = \Pr\left\{S^{(CF)}(n) = b'' \middle| S^{(CF)}(n-1) = b'\right\} = \sum_{\forall \underline{s}'_{\Sigma} \in \Im^{(\Sigma,1)}_{b'}} \sum_{\forall \underline{s}''_{\Sigma} \in \Im^{(\Sigma,1)}_{b''}} P^{(\Sigma)}_{[\underline{s}'_{\Sigma},\underline{s}''_{\Sigma}]} \cdot \frac{\pi^{(\Sigma)}_{[\underline{s}'_{\Sigma}]}}{\sum_{\forall \underline{\tilde{s}}_{\Sigma} \in \Im^{(\Sigma,1)}_{b'}} \pi^{(\Sigma)}_{[\underline{\tilde{s}}_{\Sigma}]}} \tag{23}$$

where $\Im^{(\Sigma,1)}_{b'}$ and $\Im^{(\Sigma,1)}_{b''}$ are the subsets of the state space $\Im^{(\Sigma)}$ containing the states $\underline{s}_{\Sigma}$ characterized by an action that activates, in the MEC Server 1, b' and b'' CEs, respectively.

Finally, as far as the matrix $P^{(QB)}\left(s''_{RG},b''\right)$ in (20) is concerned, it contains the transition probabilities of the battery charge level from the slot $n-1$ to the slot *n*. It depends on the state of the wind generator and the load (constituted by the active CEs of the considered MEC Server) in the arrival slot *n*.

Let us indicate the number of QoCs that b'' active CEs drain from the battery as ξ_{CF}. Since it may be not an integer, we approximate it to the closest integers with probabilities depending on the distance from them.

More specifically, we will approximate ξ_{CF} to $\xi_{CF}^{(-)} = \lfloor \xi_{CF} \rfloor$ with probability $p_{CF}^{(-)}$, and to $\xi_{CF}^{(+)} = \lceil \xi_{CF} \rceil$ with probability $p_{CF}^{(+)}$, where $p_{CF}^{(-)} = \xi_{CF}^{(+)} - \xi_{CF}$ and $p_{CF}^{(+)} = \xi_{CF} - \xi_{CF}^{(-)}$. For example, if $\xi_{CF} = 3.7$, it will be rounded to $\phi_{CF}^{(+)} = 4.0$ with probability $p_{CF}^{(+)} = 0.7$, or to $\xi_{CF}^{(-)} = 3.0$ with probability $p_{CF}^{(-)} = 0.3$. Therefore, the generic element of $P^{(QB)}(s''_{WG}, b'')$ can be calculated as follows:

$$
\begin{aligned}
P^{(QB)}_{[s'_{QB}, s''_{QB}]}(s''_{RG}, b'') &= \sum_{\forall \beta_{RG} \in \Psi^{(RG)}} \cdot B^{(WG)}_{[s''_{RG}, \beta_{RG}]} \cdot \Pr\left\{ S^{(QB)}(n) = s''_{QB} \left| \begin{matrix} S^{(QB)}(n-1) = s'_{QB}, S^{(RG)}(n) = s''_{RG}, \\ S^{(CF)}(n) = b'' \end{matrix} \right. \right\} = \\
&= \sum_{\forall \beta_{RG} \in \Psi^{(RG)}} \cdot B^{(RG)}_{[s''_{RG}, \beta_{RG}]} \cdot \begin{cases} B^{(WG)}_{[s''_{WG}, \beta_{RG}]} \cdot p_{CF}^{(+)} & \text{if } C_{QB}^{(+)} \\ B^{(RG)}_{[s''_{RG}, \beta_{RG}]} \cdot p_{CF}^{(-)} & \text{if } C_{QB}^{(-)} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\tag{24}
$$

where $C_{QB}^{(+)}$ and $C_{QB}^{(-)}$ are two Boolean conditions representing the transition of the battery charge level (expressed in QoBs) from the state s'_{QB} to the state s''_{QB} when β_{RG} QoBs arrived to the battery from the wind generator during the slot *n*, and either $\xi_{CF}^{(+)}$ or $\xi_{CF}^{(-)}$ QoBs have been drained by the active CEs during the same slot:

$$
C_{QB}^{(+)} = \left[s''_{QB} = \min\left\{ \max\left\{ s'_{QB} + \beta_{RG} - \xi_{CF}^{(+)}, 0 \right\}, K_{QoB} \right\} \right] \tag{25}
$$

$$
C_{QB}^{(-)} = \left[s''_{QB} = \min\left\{ \max\left\{ s'_{QB} + \beta_{RG} - \xi_{CF}^{(-)}, 0 \right\}, K_{QoB} \right\} \right] \tag{26}
$$

The maximum with 0 and the minimum with K_{QoB} avoid that s''_{QB} assumes negative values or values greater than K_{QoB}.