实现一个手写数字识别的算法(使用神经网络算法)

MNIST数据集:

训练(train)	50,000
验证(validation)	10,000
测试(test)	10,000

```
#神经网络
class Network(object):
def __init__(self, sizes):
        self.num_layers = len(sizes)
        self.sizes = sizes
        self.biases = [np.random.randn(y, 1) for y in sizes[1:]]
        self.weights = [np.random.randn(y, x)
                        for x, y in zip(sizes[:-1], sizes[1:])]
```

sizes: 每层神经元的个数, 例如: 第一层2个神经元,第二层3个神经元:

```
net = Network([2, 3, 1]);
```

`np.random.rand(y, 1)`: 随机从正态分布(均值0, 方差1)中生成 `net.weights[1]`: 存储连接第二层和第三层的权重 (Python索引从0开始数)

$$\alpha' = \sigma(\omega * \alpha + b)$$

> 正向传播

```
    def feedforward(self, a):
        """Return the output of the network if "a" is input."""
        for b, w in zip(self.biases, self.weights): zip函数把两个矩阵（实际上是多个维度不同的向量组成）组合
            a = sigmoid(np.dot(w, a)+b)
        return a
```

> Backprogation:反向传播

随机梯度下降更新公式:

权重更新

$$w_k \to w'_k = w_k - \eta \partial C_x / \partial w_k$$

偏向更新

$$b_l \to b'_l = b_l - \eta \partial C_x / \partial b_l$$

```
#随机梯度下降算法
def SGD(self, training_data list形式, epochs, mini_batch_size 每次用于梯度下降的实例的大小, eta 学习率,
            test_data=None):
        """Train the neural network using mini-batch stochastic
        gradient descent.  The "training_data" is a list of tuples
        "(x 输入特征向量, y x数据所属label)" representing the training inputs and the desired
        outputs.  The other non-optional parameters are
        self-explanatory.  If "test_data" is provided then the
        network will be evaluated against the test data after each
        epoch, and partial progress printed out.  This is useful for
        tracking progress, but slows things down substantially."""
        if test_data: n_test = len(test_data)
        n = len(training_data)
        for j in xrange(epochs):
            random.shuffle(training_data)
            mini_batches = [
                training_data[k:k+mini_batch_size]
                for k in xrange(0, n, mini_batch_size)]
            for mini_batch in mini_batches:
                self.update_mini_batch(mini_batch, eta)
            if test_data:
                print "Epoch {0}: {1} / {2}".format(
                    j, self.evaluate(test_data), n_test)
            else:
                print "Epoch {0} complete".format(j)
```

```
# 反向传播代码
def update_mini_batch(self, mini_batch, eta):
    """Update the network's weights and biases by applying
    gradient descent using backpropagation to a single mini batch.
    The "mini_batch" is a list of tuples "(x, y)", and "eta"
    is the learning rate."""
    nabla_b = [np.zeros(b.shape) for b in self.biases]
    nabla_w = [np.zeros(w.shape) for w in self.weights]
    for x, y in mini_batch:
    # 调用backpropagation算法求出偏导数
        delta_nabla_b, delta_nabla_w = self.backprop(x, y)
        nabla_b = [nb+dnb for nb, dnb in zip(nabla_b, delta_nabla_b)]
        nabla_w = [nw+dnw for nw, dnw in zip(nabla_w, delta_nabla_w)]
```

```
        self.weights = [w-(eta/len(mini_batch))*nw
                        for w, nw in zip(self.weights, nabla_w)]
        self.biases = [b-(eta/len(mini_batch))*nb
                       for b, nb in zip(self.biases, nabla_b)]
```

```
        #这行代码在梯度下降最重要，backpropagation算法下次笔记详细说明
        delta_nabla_b, delta_nabla_w = self.backprop(x, y)
```

完整的神经网络实现代码

代码原github地址

```
import random

# Third-party libraries
import numpy as np

class Network(object):

    def __init__(self, sizes):
        """The list ``sizes`` contains the number of neurons in the
        respective layers of the network.  For example, if the list
        was [2, 3, 1] then it would be a three-layer network, with the
        first layer containing 2 neurons, the second layer 3 neurons,
        and the third layer 1 neuron.  The biases and weights for the
        network are initialized randomly, using a Gaussian
        distribution with mean 0, and variance 1.  Note that the first
        layer is assumed to be an input layer, and by convention we
        won't set any biases for those neurons, since biases are only
        ever used in computing the outputs from later layers."""
        self.num_layers = len(sizes)
        self.sizes = sizes
        self.biases = [np.random.randn(y, 1) for y in sizes[1:]]
        self.weights = [np.random.randn(y, x)
                        for x, y in zip(sizes[:-1], sizes[1:])]

    def feedforward(self, a):
        """Return the output of the network if ``a`` is input."""
        for b, w in zip(self.biases, self.weights):
            a = sigmoid(np.dot(w, a)+b)
        return a

    def SGD(self, training_data, epochs, mini_batch_size, eta,
            test_data=None):
        """Train the neural network using mini-batch stochastic
        gradient descent.  The ``training_data`` is a list of tuples
        ``(x, y)`` representing the training inputs and the desired
        outputs.  The other non-optional parameters are
        self-explanatory.  If ``test_data`` is provided then the
```

```
        network will be evaluated against the test data after each
        epoch, and partial progress printed out.  This is useful for
        tracking progress, but slows things down substantially."""
        if test_data: n_test = len(test_data)
        n = len(training_data)
        for j in xrange(epochs):
            random.shuffle(training_data)
            mini_batches = [
                training_data[k:k+mini_batch_size]
                for k in xrange(0, n, mini_batch_size)]
            for mini_batch in mini_batches:
                self.update_mini_batch(mini_batch, eta)
            if test_data:
                print "Epoch {0}: {1} / {2}".format(
                    j, self.evaluate(test_data), n_test)
            else:
                print "Epoch {0} complete".format(j)

    def update_mini_batch(self, mini_batch, eta):
        """Update the network's weights and biases by applying
        gradient descent using backpropagation to a single mini batch.
        The ``mini_batch`` is a list of tuples ``(x, y)``, and ``eta``
        is the learning rate."""
        nabla_b = [np.zeros(b.shape) for b in self.biases]
        nabla_w = [np.zeros(w.shape) for w in self.weights]
        for x, y in mini_batch:
            delta_nabla_b, delta_nabla_w = self.backprop(x, y)
            nabla_b = [nb+dnb for nb, dnb in zip(nabla_b, delta_nabla_b)]
            nabla_w = [nw+dnw for nw, dnw in zip(nabla_w, delta_nabla_w)]
        self.weights = [w-(eta/len(mini_batch))*nw
                        for w, nw in zip(self.weights, nabla_w)]
        self.biases = [b-(eta/len(mini_batch))*nb
                       for b, nb in zip(self.biases, nabla_b)]

    def backprop(self, x, y):
        """Return a tuple ``(nabla_b, nabla_w)`` representing the
        gradient for the cost function C_x.  ``nabla_b`` and
        ``nabla_w`` are layer-by-layer lists of numpy arrays, similar
        to ``self.biases`` and ``self.weights``."""
        nabla_b = [np.zeros(b.shape) for b in self.biases]
        nabla_w = [np.zeros(w.shape) for w in self.weights]
        # feedforward
        activation = x
        activations = [x] # list to store all the activations, layer by layer
        zs = [] # list to store all the z vectors, layer by layer
        for b, w in zip(self.biases, self.weights):
            z = np.dot(w, activation)+b
            zs.append(z)
            activation = sigmoid(z)
            activations.append(activation)
        # backward pass
        delta = self.cost_derivative(activations[-1], y) * \
            sigmoid_prime(zs[-1])
        nabla_b[-1] = delta
```

```
        nabla_w[-1] = np.dot(delta, activations[-2].transpose())
        # Note that the variable l in the loop below is used a little
        # differently to the notation in Chapter 2 of the book.  Here,
        # l = 1 means the last layer of neurons, l = 2 is the
        # second-last layer, and so on.  It's a renumbering of the
        # scheme in the book, used here to take advantage of the fact
        # that Python can use negative indices in lists.
        for l in xrange(2, self.num_layers):
            z = zs[-l]
            sp = sigmoid_prime(z)
            delta = np.dot(self.weights[-l+1].transpose(), delta) * sp
            nabla_b[-l] = delta
            nabla_w[-l] = np.dot(delta, activations[-l-1].transpose())
        return (nabla_b, nabla_w)

    def evaluate(self, test_data):
        """Return the number of test inputs for which the neural
        network outputs the correct result. Note that the neural
        network's output is assumed to be the index of whichever
        neuron in the final layer has the highest activation."""
        test_results = [(np.argmax(self.feedforward(x)), y)
                        for (x, y) in test_data]
        return sum(int(x == y) for (x, y) in test_results)

    def cost_derivative(self, output_activations, y):
        """Return the vector of partial derivatives \partial C_x /
        \partial a for the output activations."""
        return (output_activations-y)

#### Miscellaneous functions
def sigmoid(z):
    """The sigmoid function."""
    return 1.0/(1.0+np.exp(-z))

def sigmoid_prime(z):
    """Derivative of the sigmoid function."""
    return sigmoid(z)*(1-sigmoid(z))
```

实现一个手写数字识别程序

> 手写识别的参考代码

> 1.加载mnist数据集 2.查看下返回的训练集、测试集和验证集的数据类型和长度（数据结构搞清楚） 3.创建神经网络 4.进行梯度下降，获得优化后的神经网络模型参数 5.对算法进行准确性评估

```
#coding=utf-8

import mnist_loader
from network import Network

trainDataset,validationDataset,testDataset = mnist_loader.load_data_wrapper()
```

```
# 训练集是一个50000长度的list，每个元素是一个元祖(x,y),x表示输入特征(768),y表示所属数字label
# print(len(trainDataset))
# print(len(trainDataset[0]))
# # x的结构
# print(trainDataset[0][0].shape)
# # y的结构
# print(trainDataset[0][1].shape)

# list里面是每层神经网络的神经数量
network = Network([784,50,10])
# 进行梯度下降，并测试
network.SGD(trainDataset,30,10,3.0,test_data=testDataset)
```