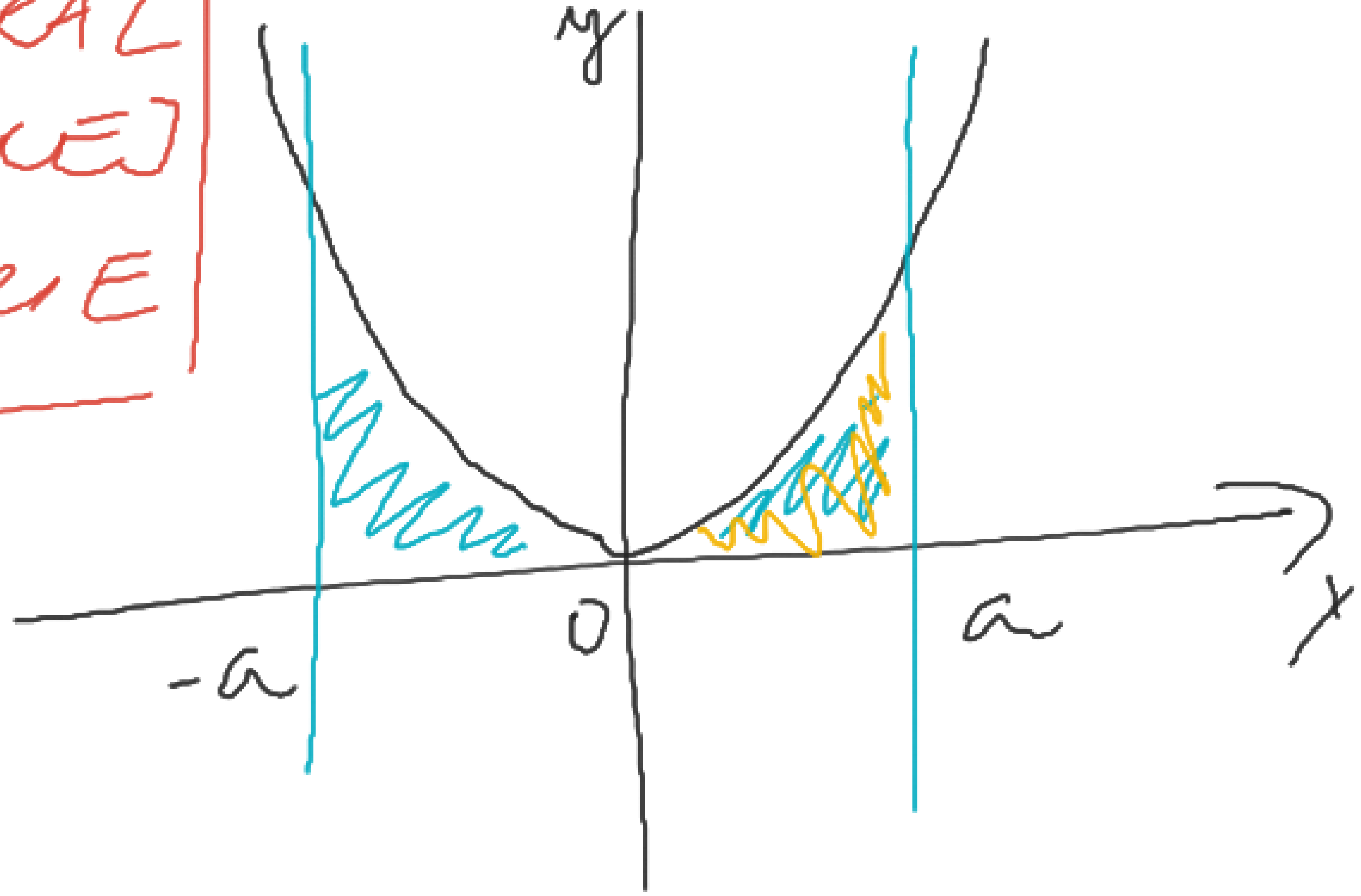
INTEGRÁL Z PÁRNEJ FUNKCIE

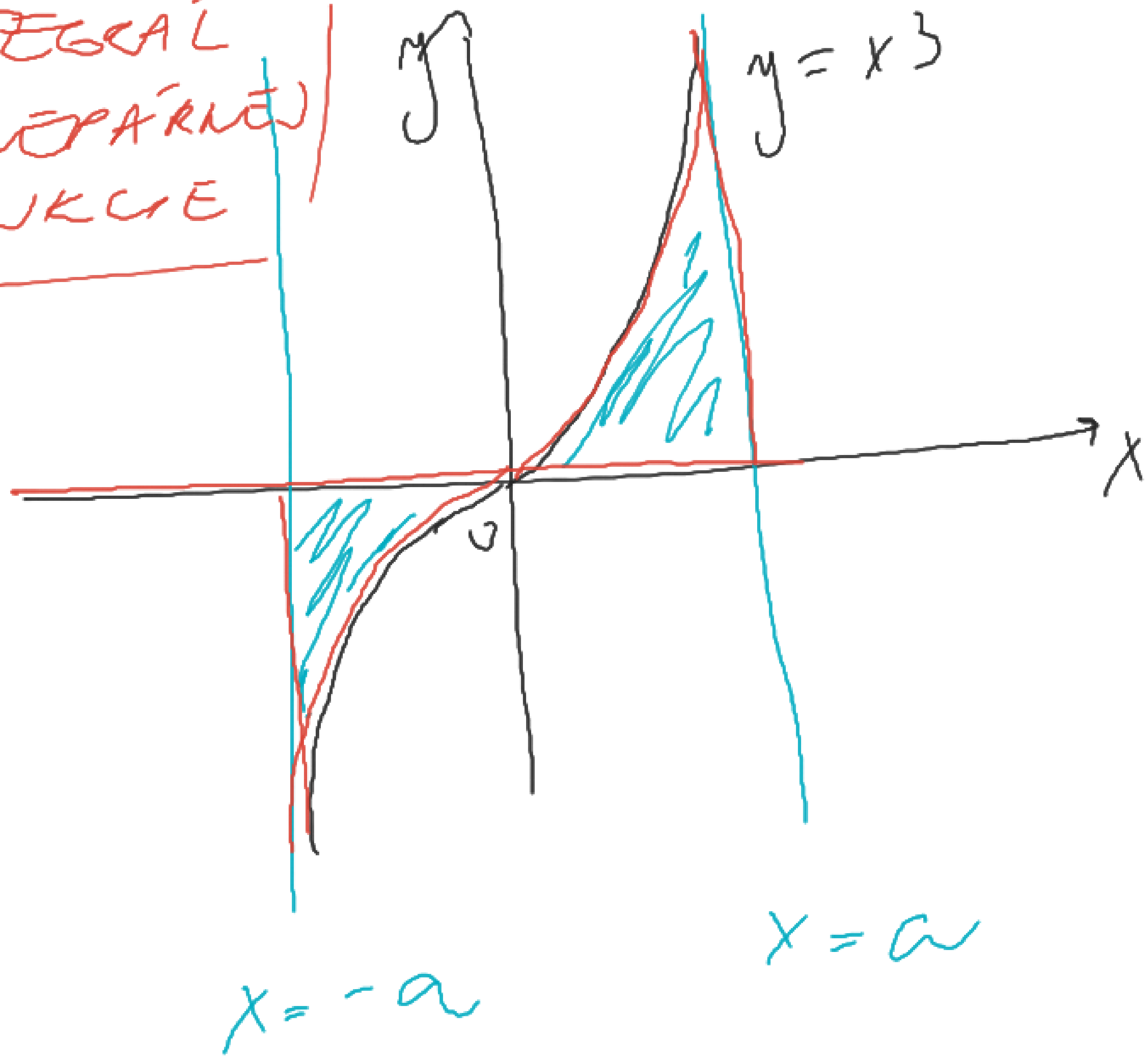


$y = x^2$

$$\boxed{\int_{-a}^{a} f(x)\,dx =}$$

$$= 2\int_{0}^{a} f(x)\,dx$$

INTEGRÁL
Z NEPÁRNEJ
FUNKCIE



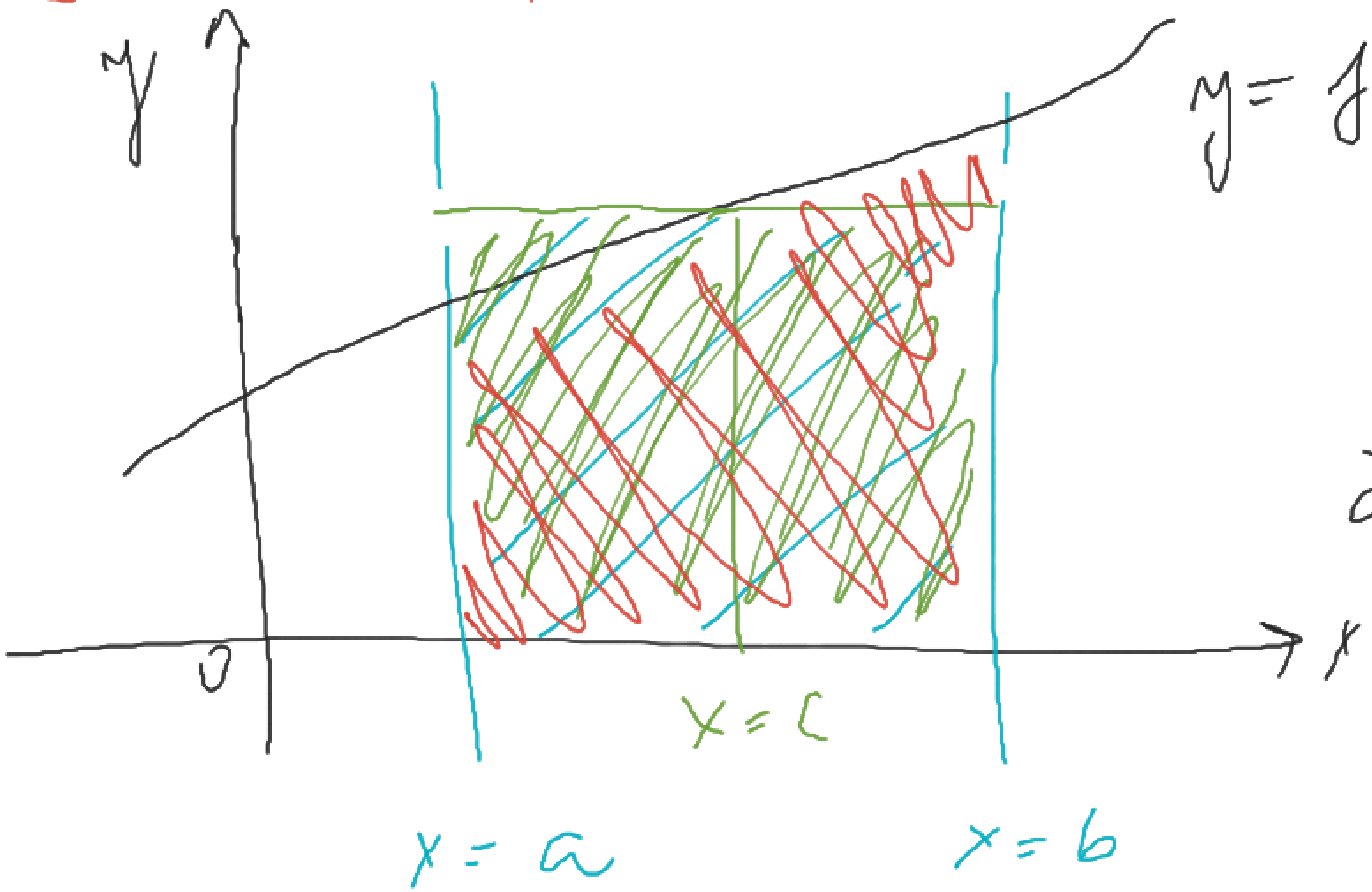
$$\int_{-a}^{a} f(x)\,dx = 0$$

PLOCHA MEDZI

f(x) a y = 0

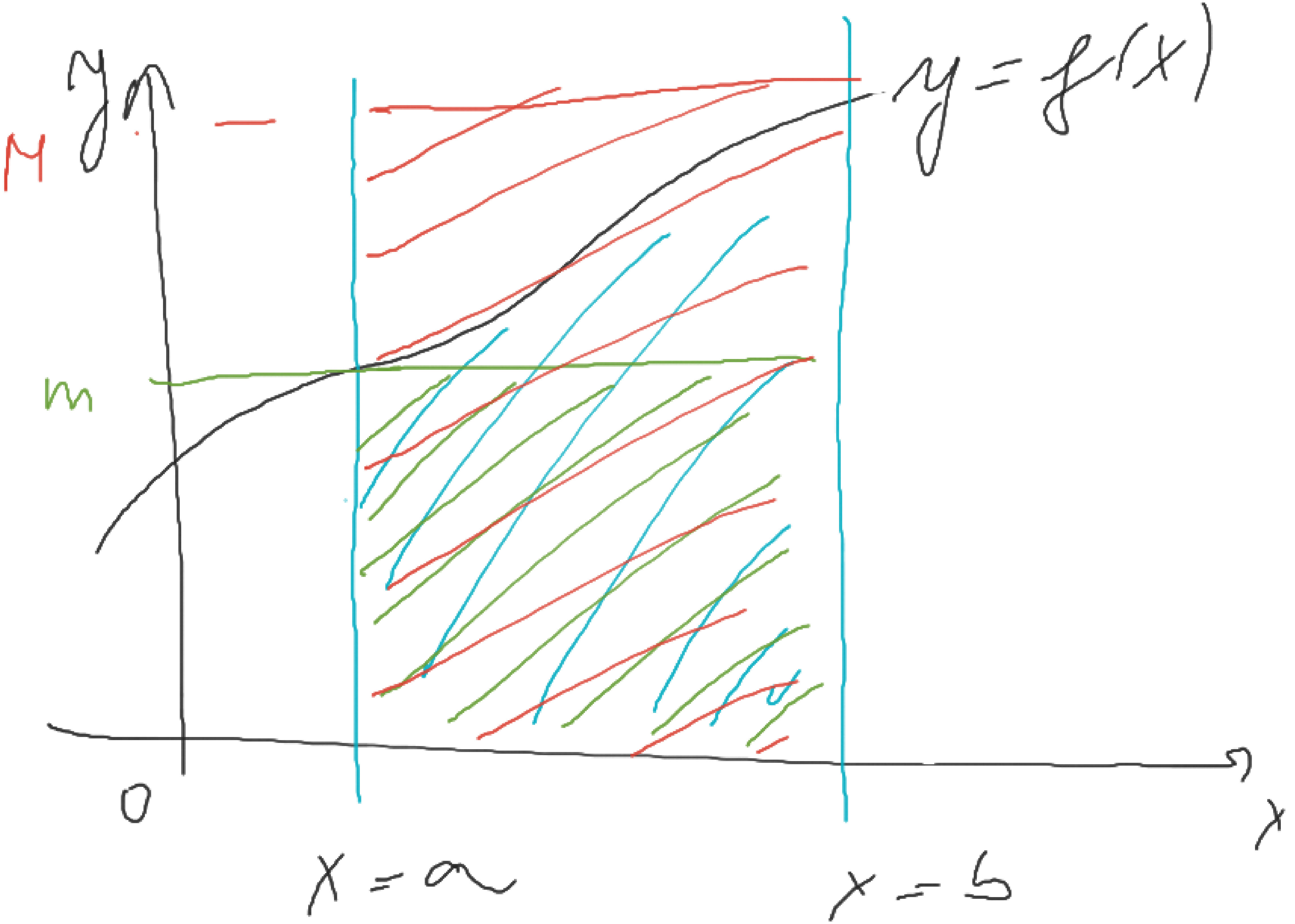
NEBUDE NULOVÁ

VETA O STREDNEJ HODNOTE PRE URČITÝ INTEGRÁL



$$\int_a^b f(x)\,dx = \underbrace{f(c)(b-a)}_{\text{PLOCHA OBDĹŽNIKA}}$$

ODHAD URČITÉHO INTEGRÁLU



$$\int \frac{\sin x}{x}\,dx$$

$$\int e^{-x^2}\,dx$$

PR 7

a

$$\int_{0}^{1} \frac{dx}{(2x+1)^3} = \left| \begin{array}{l} \boxed{t = 2x+1} \\ dt = 2\,dx \Rightarrow dx = \frac{dt}{2} \\ x_1 = 0 \Rightarrow t_1 = 2 \cdot 0 + 1 = 1 \\ x_2 = 1 \Rightarrow t_2 = 2 \cdot 1 + 1 = 3 \end{array} \right.$$

x_1, x_2

$$= \int_{1}^{3} \frac{dt}{2 \cdot t^3} = \left[\frac{1}{2} \cdot \frac{t^{-2}}{-2}\right]_1^3 = -\frac{1}{4}\left[\frac{1}{t^2}\right]_1^3 =$$

$$= -\frac{1}{4}\left[\frac{1}{9} - \frac{1}{1}\right] = -\frac{1}{4}\left(-\frac{8}{9}\right) = \underline{\underline{\frac{2}{9}}}$$

(b) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\overset{dt}{\cos x\, dx}}{\sin^2 x} = \left| \begin{array}{l} t = \sin x \\ dt = \cos x\, dx \\ x_1 = \frac{\pi}{4} \Rightarrow t_1 = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \\ x_2 = \frac{\pi}{2} \Rightarrow t_2 = \sin\frac{\pi}{2} = 1 \end{array} \right.$

$$= \int_{\frac{\sqrt{2}}{2}}^{1} \frac{dt}{t^2} = -\left[\frac{1}{t}\right]_{\frac{\sqrt{2}}{2}}^{1} = -\left(1 - \frac{2}{\sqrt{2}}\right) =$$

$$= -1 + \frac{\cancel{2}}{\cancel{\sqrt{2}}} \cdot \frac{\sqrt{2}}{\cancel{\sqrt{2}}} = \underline{-1 + \sqrt{2}}$$

c) $\displaystyle\int_0^{\ln 2} x e^x \, dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = e^x & v = e^x \end{matrix} \right| =$

$$= \left[x e^x \right]_0^{\ln 2} - \int_0^{\ln 2} e^x \, dx = \ln 2 \, e^{\ln 2} - 0 -$$

$$- \left[e^x \right]_0^{\ln 2} = 2 \ln 2 - e^{\ln 2} + e^0 =$$

$$= 2 \ln 2 - 2 + 1 = \underline{\underline{2 \ln 2 - 1}}$$

a) $\int_9^4 \frac{1-\sqrt{x}}{\sqrt{x}}\,dx = \left| \begin{array}{l} t=\sqrt{x} \\ t^2 = x \\ 2t\,dt = dx \end{array} \right.$ $\begin{array}{l} x_1 = 9 \Rightarrow t_1 = 3 \\ x_2 = 4 \Rightarrow t_2 = 2 \end{array}$

$= \int_3^2 \frac{1-t}{\cancel{t}} \cdot 2\cancel{t}\,dt = \left[2t - t^2\right]_3^2 = \cancel{4} - \cancel{4} -$

$- (6-9) = \underline{\underline{3}}$

PR 12

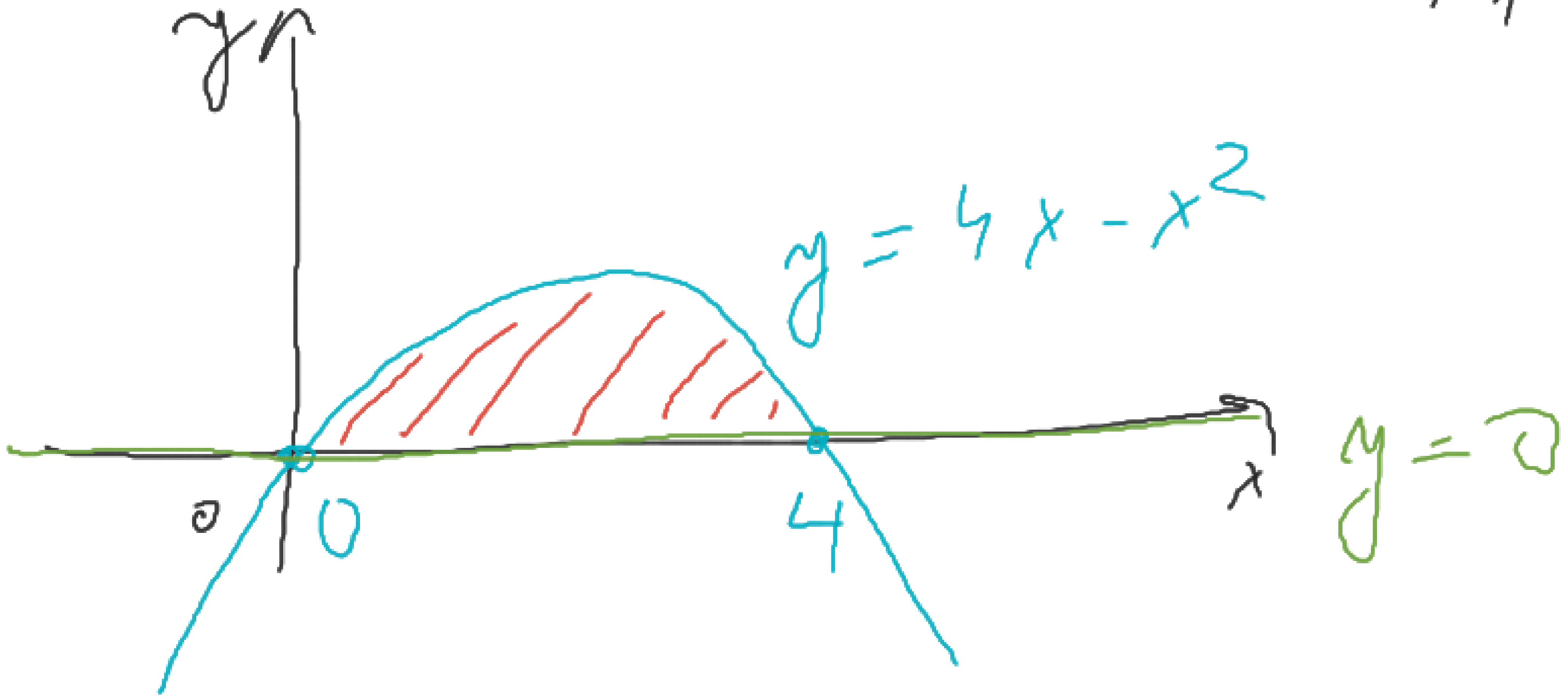
a) $\boxed{y = 4x - x^2},$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\} \Rightarrow 4x - x^2 = 0$

$Ox \Rightarrow \boxed{y = 0}$

$x(4 - x) = 0$

$\Downarrow \qquad \Downarrow$

$x_1 = 0 \qquad x_2 = 4$



$$P = \int_0^4 4x - x^2 - 0 \, dx =$$

$$= \left[\frac{2 \cancel{4} x^2}{\cancel{2}} - \frac{x^3}{3} \right]_0^4 = 2 \cdot 16 - \frac{4 \cdot 16}{3} = \frac{6 \cdot 16 - 4 \cdot 16}{3} =$$

$$= \frac{2 \cdot 16}{3} = \underline{\underline{\frac{32}{3}}} \; \mathrm{j}^2$$