Note Title 11/28/2022

POSTUPNOSTI

PR1 $a_1 = \frac{2-3}{1+1} = -\frac{1}{2}$; $a_2 = \frac{1}{3}$; $a_3 = \frac{3}{4}$; ...

$a_n = \frac{2n-3}{n+1}$

$$\lim_{n\to\infty} \frac{2n-3}{n+1} = \lim_{n\to\infty} \frac{\cancel{n}(2-\frac{3}{n})}{\cancel{n}(1+\frac{1}{n})} = \underline{\underline{2}}$$

($\frac{3}{n} \to 0$, $\frac{1}{n} \to 0$)

$$\boxed{|a_n - L| < \varepsilon}$$

$$\left|\frac{2n_0-3}{n_0+1} - 2\right| < \frac{1}{1000}$$

$$\left|\frac{\cancel{2n_0}-3-\cancel{2n_0}-2}{n_0+1}\right| < \frac{1}{1000}$$

$$\frac{|-5|}{n_0+1} < \frac{1}{1000}$$

$$\frac{5}{n_0+1} < \frac{1}{1000} \Rightarrow 5000 < n_0+1$$

$$\underline{\underline{n_0 > 4999}}$$

PR2 $\{0{,}9;\ 0{,}99;\ 0{,}999, \dots\}_{n=1}^{\infty} = \left\{\frac{9}{10}, \frac{99}{100}, \frac{999}{1000}, \dots\right\}_{n=1}^{\infty} =$

$$= \left\{1-\frac{1}{10^n}\right\}_{n=1}^{\infty} \qquad \lim_{n\to\infty} a_n = \lim_{n\to\infty} 1-\frac{1}{10^n} = \underline{\underline{1}}$$

a_n

PR3

a) $\left\{\frac{1+2+3+\dots n}{n+2} - \frac{n}{2}\right\}_{n=1}^{\infty} = \left\{\frac{(1+n)\cdot n}{2(n+2)} - \frac{n}{2}\right\}_{n=1}^{\infty} = \left\{\frac{n+n^2-n^2-2n}{2(n+2)}\right\}_{n=1}^{\infty} =$

($1+2+3+\dots n = \frac{(1+n)\cdot n}{2}$)

$$= \left\{\frac{-n}{2(n+2)}\right\}_{n=1}^{\infty} \Rightarrow \lim_{n\to\infty} \frac{-n}{2(n+2)} = \underline{\underline{-\frac{1}{2}}}$$

b) $\left\{\left(3-\frac{1}{n}\right)\sqrt{\frac{n}{4n+1}}\right\}_{n=1}^{\infty} \Rightarrow \lim_{n\to\infty} \left(3-\frac{1}{n}\right)\sqrt{\frac{n}{4n+1}} =$

$$= \lim_{n\to\infty} \left(3-\frac{1}{n}\right)\sqrt{\frac{\cancel{n}\cdot(1)}{\cancel{n}(4+\frac{1}{n})}} = \underline{\underline{\frac{3}{2}}}$$

($\frac{1}{n} \to 0$)

(c) $a_n = \sqrt{1+n^2} - n$

$$\lim_{n\to\infty} \left(\sqrt{1+n^2} - n\right) \cdot \frac{\left(\sqrt{1+n^2}+n\right)}{\left(\sqrt{1+n^2}+n\right)} = \lim_{n\to\infty} \frac{1+\cancel{n^2}-\cancel{n^2}}{\left(\sqrt{1+n^2}+n\right)} = \underline{\underline{0}}$$

(d) $a_n = \left(1+\frac{1}{4n}\right)^{1-3n} \Rightarrow \lim_{n\to\infty} \left(1+\frac{1}{4n}\right)^{1-3n} =$

$$\boxed{\begin{array}{l} 4n = t \Rightarrow n = \frac{t}{4} \\ n\to\infty \Rightarrow t\to\infty \end{array}}$$

$$= \lim_{t\to\infty} \left(1+\frac{1}{t}\right)^{1-\frac{3t}{4}} =$$

$$= \lim_{t\to\infty} \left(1+\frac{1}{t}\right)^{1} \cdot \Big[\underbrace{\lim_{t\to\infty} \left(1+\frac{1}{t}\right)^{t}}_{e}\Big]^{-\frac{3}{4}} = 1 \cdot e^{-\frac{3}{4}} = \underline{\underline{e^{-\frac{3}{4}}}}$$