Note Title **VYŠETRITE PRIEBEH FUNKCIE** 10/24/2022

NA SKÚŠKE 1 A 6

$$f(x) = \frac{x}{x^2-4}$$

1) $D(f)$: $x^2-4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

$D(f) = R - \{-2, 2\}$

2) $f(x) = \frac{x}{x^2-4}$

$f(-x) = \frac{-x}{(-x)^2-4} = \frac{-x}{x^2-4} = -\frac{x}{x^2-4}$ } $f(x) = -f(-x)$ FUNKCIA JE NEPÁRNA

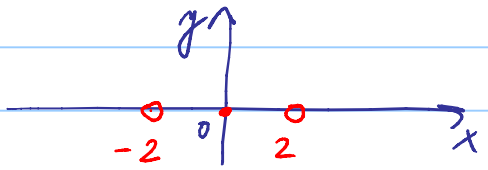
NIE JE PERIODICKÁ

JE SPOJITÁ

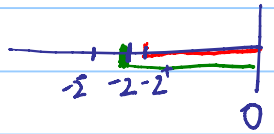
PRIESEČNÍKY S OSAMI:

$x = 0 \Rightarrow y = 0$

NB [0;0]

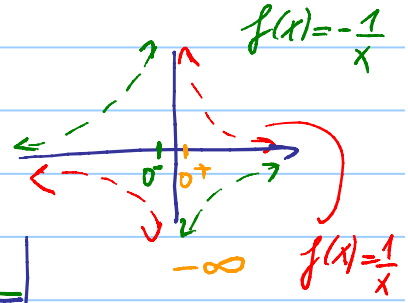


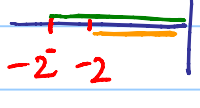
3) ABS: $x = -2$



$\lim_{x \to -2^+} \frac{x}{x^2-4} = \frac{-2}{0^-} = \infty$

$\lim_{x \to -2^-} \frac{x}{x^2-4} = \frac{-2}{0^+} = -\infty$

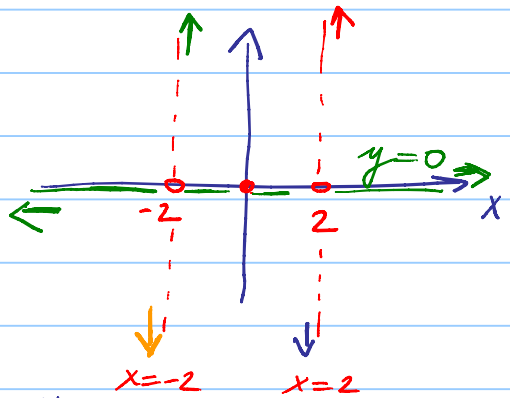




$x = 2$

$\lim_{x \to 2^+} \frac{x}{x^2-4} = \frac{2}{0^+} = \infty$

$\lim_{x \to 2^-} \frac{x}{x^2-4} = \frac{2}{0^-} = -\infty$



ASS $y = kx + q$

$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{\frac{x}{x^2-4}}{\frac{x}{1}} = \lim_{x \to \pm\infty} \frac{x \cdot 1}{x(x^2-4)} = 0$

$$q = \lim_{x\to\pm\infty}\left(f(x) - kx\right) = \lim_{x\to\pm\infty}\frac{x}{x^2-4} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\pm\infty}\frac{1}{2x} = 0$$

$$\boxed{y = 0}$$

4) $f'(x) = \left[\frac{x}{x^2-4}\right]' = \frac{1\cdot(x^2-4) - x\cdot 2x}{(x^2-4)^2} = \frac{x^2-4-2x^2}{(x^2-4)^2} = \frac{-4-x^2}{(x^2-4)^2} < 0$

VŽDY ⊖ (čitatel) VŽDY ⊕ (jmenovatel)

FUNKCIA JE RÝDZO KLESAJÚCA NA $(-\infty,-2) \sim (-2,2) \sim (2,\infty)$

5) $f''(x) = \left[\frac{-4-x^2}{(x^2-4)^2}\right]' = \frac{-2x(x^2-4)^2 - (-4-x^2)\,2(x^2-4)\cdot 2x}{(x^2-4)^4} =$

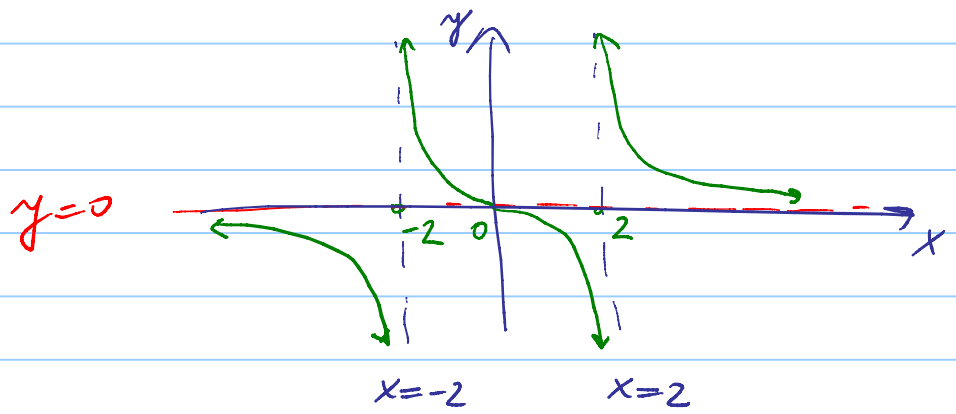
$= \frac{-2x^3+8x+(4+x^2)4x}{(x^2-4)^3} = \frac{-2x^3+8x+16x+4x^3}{(x^2-4)^3} = \frac{2x^3+24x}{(x^2-4)^3} =$

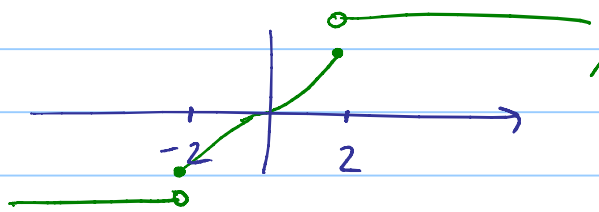
$= \frac{2x(x^2+12)}{(x^2-4)^3}$ (0, ≠)

$D(f): \mathbb{R}-\{-2,2\}$

	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
$(2x)$	⊖	⊖	⊕	⊕
$2x(x^2+12)$	⊖	⊖	⊕	⊕
$(x^2-4)^3$	⊕	⊖	⊖	⊕
$\frac{2x(x^2+12)}{(x^2-4)^3}$	⊖	⊕	⊖	⊕
	∩	∪	∩	∪

INFLEXNÝ BOD [0;0]

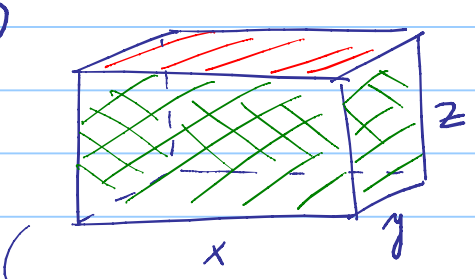




NESPOJITÁ

SLOVNÉ ÚLOHY

PR 1



BUDEME POČÍTAŤ BEZ PODLAHY

$V = 500\ m^3$

$V = xy.z = 500 \Rightarrow z = \frac{500}{xy}$

$x = 2y \longrightarrow z = \frac{500}{2y^2}$

$C(x,y,z) = \boxed{3xy} + \boxed{2(xz + yz)}$

$C(y,z) = 3.2y.y + 2(2yz + yz)$

$C(y) = 6y^2 + 2\left(2y.\frac{500}{2y^2} + y.\frac{500}{2y^2}\right)$

$= 6y^2 + \frac{1000}{y} + \frac{500}{y} = 6y^2 + 1500.y^{-1}$

$C' = 12y - 1500y^{-2} = 12y - \frac{1500}{y^2} = \frac{12y^3 - 1500}{y^2}$

$C' = 0 \iff 12y^3 - 1500 = 0$

$y^3 = \frac{1500}{12} = 125$

$\underline{y = 5\ m} \Rightarrow x = 2.y = \underline{10\ m}$

$z = \frac{500}{2.25} = \frac{500}{50} = \underline{10\ m}$

$C'' = 12 + 2.1500\,y^{-3} \Rightarrow C''(5) > 0 \Rightarrow$ NAŠLI SME MINIMUM

PR 2 $V(r,h) = \pi r^2 h = 1[\ell] \longrightarrow h = \frac{1}{\pi r^2}$

$S(r,h) = 2\pi r^2 + 2\pi r.h = 2\pi r(r+h)$

$S(r) = 2\pi r^2 + 2\pi r.\frac{1}{\pi r^2} = 2\pi r^2 + \frac{2}{r} = 2\pi r^2 + 2r^{-1}$

$S'(r) = 4\pi r - 2r^{-2} = 4\pi r - \frac{2}{r^2} = \frac{4\pi r^3 - 2}{r^2}$

$S'(r) = 0 \iff 4\pi r^3 - 2 = 0$

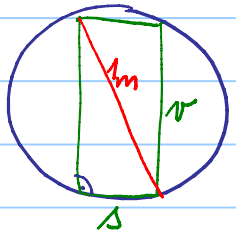
$\underline{r = \sqrt[3]{\frac{2}{4\pi}} = \sqrt[3]{\frac{1}{2\pi}}}$

$S''(r) = 4\pi + 4r^{-3} = 4\pi + \frac{4}{r^3} > 0$ NAŠLI SME MINIMUM

DOPOČÍTAME $h = \frac{1}{\pi\left(\frac{1}{2\pi}\right)^{\frac{2}{3}}} = \frac{1}{\pi\left(\frac{1}{4\pi^2}\right)^{\frac{1}{3}}} = \left(\frac{1}{\pi^3}\right)^{\frac{1}{3}} =$

$= \sqrt[3]{\frac{4}{\pi}}$

PR3



N = hmotnosť

$N = c.s.r^2$

⇓

konštanta = charakteristika dreva

$s^2 + r^2 = 1$

$r^2 = 1 - s^2$

$N(s) = c.s.(1-s^2) = cs - cs^3$

$N'(s) = c - c.3s^2$

$N'(s) = 0 \iff c - 3cs^2 = 0 \quad / : c$

$1 - 3s^2 = 0$

$3s^2 = 1$

$s = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3}$

$r^2 = 1 - \frac{1}{3} = \frac{2}{3}$

$r = \sqrt{\frac{2}{3}}$

$N''(s) = -3c.2s = -6cs$

$N'\left(\frac{\sqrt{3}}{3}\right) = -6.c.\frac{\sqrt{3}}{3} = -2c\sqrt{3} < 0$