ZÁPOČTOVKA M4 1. OPRAVNÝ ZÁPOČET 2018

1) ZDERIVUJ

a) $$\left(\frac{\cot g^3 x}{2+5^{\arcsin x}}\right)' =$$

$$= \frac{3\cdot \cot g^2 x \cdot \frac{-1}{\sin^2 x}\cdot\left(2+5^{\arcsin x}\right) - \cot g^3 x \cdot \left(5^{\arcsin x}\cdot \ln 5 \cdot \frac{1}{\sqrt{1-x^2}}\right)}{\left(2+5^{\arcsin x}\right)^2}$$

$$(f^g)' = f^g (g \ln f)' = f^g\left(g'\ln f + g\frac{1}{f}\cdot f'\right)$$

b) $$\left(\left(\operatorname{arctg} x\right)^{2-x^2}\right)' =$$

$$= \left(\operatorname{arctg} x\right)^{2-x^2}\left(-2x\cdot \ln \operatorname{arctg} x + (2-x^2)\cdot\frac{1}{\operatorname{arctg} x}\cdot\frac{1}{1+x^2}\right)$$

2) $f(x) = \sqrt{1-\log_2(2-x)}$ URČI $D(f)$ f^{-1} AK EXISTUJE

1) ln $\quad 2-x>0 \Rightarrow x<2 \quad (-\infty, 2) = K_1$

2) $\sqrt{\ }$ $\quad 1-\log_2(2-x) \geq 0$

$\log_2(2-x) \leq 1$

$\log_2(2-x) \leq 1 \Leftrightarrow 2-x \leq 2 \Leftrightarrow$

$-x \leq 0$

$x \geq 0 \quad \langle 0, \infty) = K_2$

$$K_1 \cap K_2 = \langle 0; 2)$$ $$H(f) = \langle 0, \infty)$$

f^{-1}: $x = \sqrt{1 - \log_2(2-y)}$

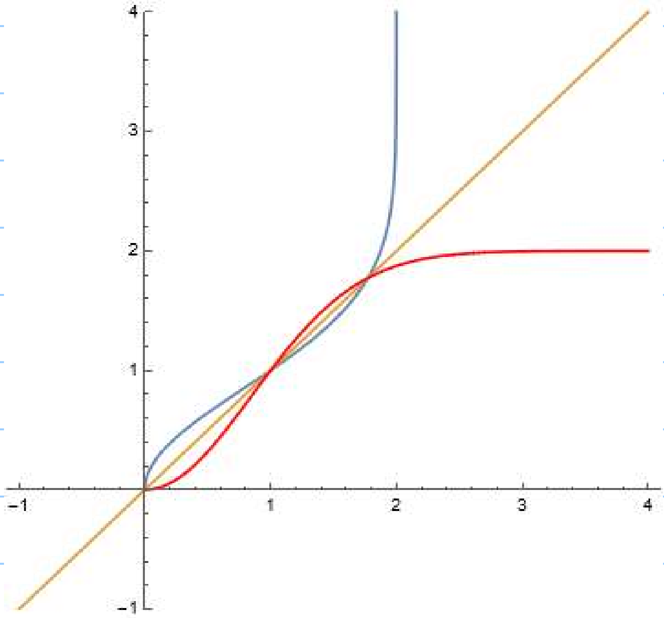
$$x^2 = 1 - \log_2(2-y) \quad / + \log_2(2-y) - x^2$$

$$\log_2(2-y) = 1 - x^2$$

$$2^{\log_2}(2-y) = 2^{1-x^2}$$

$$2 - y = 2^{1-x^2} \Rightarrow y = 2 - 2^{1-x^2} \quad x \in \langle 0, \infty)$$

$$H(f) = \langle 0, 2)$$



3) BEZ L'HOSPITALA

$$\lim_{x \to -\infty} \left(\left(\frac{3x+3}{3x-1} \right)^{3x+1} + 3^{2x} \right) =$$

$$= \underbrace{\lim_{x \to -\infty} \left(\frac{3x+3}{3x-1} \right)^{3x+1}}_{A} + \underbrace{\lim_{x \to -\infty} 3^{2x}}_{B}$$

B: $\lim\limits_{x\to-\infty} 3^{2x} = 3^{-2\cdot\infty} = \dfrac{1}{3^{2\cdot\infty}} = 0$

A: $\lim\limits_{x\to-\infty}\left(\dfrac{3x+3}{3x-1}\right)^{3x+1} = \lim\limits_{x\to-\infty}\left(\dfrac{3x-1+1+3}{3x-1}\right)^{3x+1} =$

$= \lim\limits_{x\to-\infty}\left(1+\dfrac{4}{3x-1}\right)^{3x+1} = \lim\limits_{x\to-\infty}\left(1+\dfrac{1}{\frac{3x-1}{4}}\right)^{3x+1} =$

$= \lim\limits_{x\to-\infty}\underbrace{\left(1+\dfrac{1}{\frac{3x-1}{4}}\right)}_{\to e}^{\overbrace{\frac{3x-1}{4}\cdot\frac{4}{3x-1}}^{1=}\cdot(3x+1)} =$

$= e^{\lim\limits_{x\to-\infty}\frac{4(3x+1)}{3x-1}} = e^{\lim\limits_{x\to-\infty}\frac{12x+4}{3x-1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}} =$

$= e^{\lim\limits_{x\to-\infty}\frac{\frac{12x}{x}+\frac{4}{x}}{\frac{3x}{x}-\frac{1}{x}}} = e^{\frac{12}{3}} = e^4$

$\frac{12x}{x}\to 12$, $\frac{4}{x}\to 0$, $\frac{3x}{x}\to 3$, $\frac{1}{x}\to 0$

$\lim\limits_{x\to-\infty} f(x) = e^4 + 0 = e^4$

4) Nájdi t a n k $f(x) = 2x^2 + 2x + 5$
$n \parallel p: 2y - x - 3 = 0$

$k_n = k_p$

$p: 2y - x - 3 = 0 \quad / + x + 3$

$2y = x + 3 \quad /: 2$

$y = \frac{1}{2}(x+3) \Rightarrow k_p = \frac{1}{2}$

$f' = 4x + 2 \Rightarrow -\frac{1}{f'} = \frac{1}{2} \Rightarrow f' = -2$

$4x + 2 = -2 \quad / -2$

$4x = -4 \quad /: 4$

$x = -1 \Rightarrow f(-1) = 2 \cdot (-1)^2 + 2 \cdot (-1) + 5 = 5$

$T[-1, 5]$

$t: \quad y - 5 = -2(x+1) \Rightarrow y = -2x + 3$

$n: \quad y - 5 = \frac{1}{2}(x+1) \Rightarrow y = \frac{x}{2} + \frac{11}{2}$