Note Title 25/10/2022

PRŮBĚH FUNKCE 2.

$f(x) = x^2 e^{\frac{1}{x}}$

1) $D(f) = \{ \forall x \in \mathbb{R};\ x \neq 0 \} = \mathbb{R} - \{0\}$

NB $f(x) = 0 \iff x = 0 \notin D(f) \Rightarrow \nexists$ /0B

2) PÁR. / NEPÁR.

$f(-x) = (-x)^2 e^{\frac{1}{-x}} = x^2 \cdot \frac{1}{e^{x}}$ $\begin{array}{l} \neq f(x) \\ \neq -f(x) \end{array} \Big\}$ ANI P, ANI N

3) SPOJ. ABS

$0 \cdot (\pm\infty)$ $\lim_{x \to 0^-} x^2 \cdot e^{\frac{1}{x}} = \lim_{x \to 0^-} \frac{e^{\frac{1}{x}}}{\frac{1}{x^2}} \overset{L'}{\underset{\frac{-\infty}{\infty}}{=}} \lim_{x \to 0^-} \frac{e^{\frac{1}{x}} \cdot (-x^{-2})}{2(-x^{-3})} =$

$= \lim_{x \to 0^-} \frac{e^{\frac{1}{x}}}{2x^{-1}} \overset{L'}{=} \lim_{x \to 0^-} \frac{e^{\frac{1}{x}} \cdot (-x^{-2})}{-2 \cdot 2x^{-2}} = \lim_{x \to 0^-} \frac{e^{\frac{1}{x}}}{4} = \frac{1}{4e^{\infty}} = 0$

$\lim_{x \to 0^+}$ — " — $= \lim_{x \to 0^+} \frac{e^{\frac{1}{x}}}{4} = \frac{e^{\infty}}{4} = \infty$

ABS: $x = 0$

ABS:

$k_1 = \lim_{x \to \infty} \frac{x^2 e^{\frac{1}{x}}}{x} = \lim_{x \to \infty} \underset{\to\infty}{x}\, e^{\overset{\to 0}{\frac{1}{x}}} = \infty \cdot 1 = \infty$

$k_2 = \lim_{x \to -\infty} \frac{x^2 e^{\frac{1}{x}}}{x} = \lim_{x \to -\infty} x \cdot e^{\frac{1}{x}} = -\infty \cdot 1 = -\infty$

$\Big\}$ $\nexists$ ABS

MONOTÓNNOSŤ

$$f'(x) = \left(x^2 \cdot e^{\frac{1}{x}}\right)' = 2x\, e^{\frac{1}{x}} + x^2 \cdot e^{\frac{1}{x}} \cdot \left(-x^{-2}\right) = 2x e^{\frac{1}{x}} - e^{\frac{1}{x}} =$$

$$= e^{\frac{1}{x}}(2x-1)$$

SB: $f'(x) = 0 \iff 2x-1 = 0 \iff x = \frac{1}{2}$

	$(-\infty, 0)$	$(0, \frac{1}{2})$	$(\frac{1}{2}, \infty)$
$e^{\frac{1}{x}}$	+	+	+
$2x-1$	−	−	+
	↘	↘	↗

EXTRÉMY

$$f'' = \left(e^{\frac{1}{x}}\left(\frac{-1}{x^2}\right)\right)(2x-1) + e^{\frac{1}{x}} \cdot 2 = e^{\frac{1}{x}}\left(\frac{-2}{x} + \frac{1}{x^2} + 2\right)$$

$$f''\left(\tfrac{1}{2}\right) = e^{\frac{1}{\frac{1}{2}}}\left(\frac{-2}{\frac{1}{2}} + \frac{1}{\left(\frac{1}{2}\right)^2} + 2\right) = e^2(-4+4+2) = 2e^2 > 0$$

$\Rightarrow$ LOK. MIN

$$f\left(\tfrac{1}{2}\right) = \left(\tfrac{1}{2}\right)^2 e^{\frac{1}{\frac{1}{2}}} = \tfrac{1}{4} e^2 \qquad \text{L. MIN}\left[\tfrac{1}{2}, \tfrac{e^2}{4}\right]$$

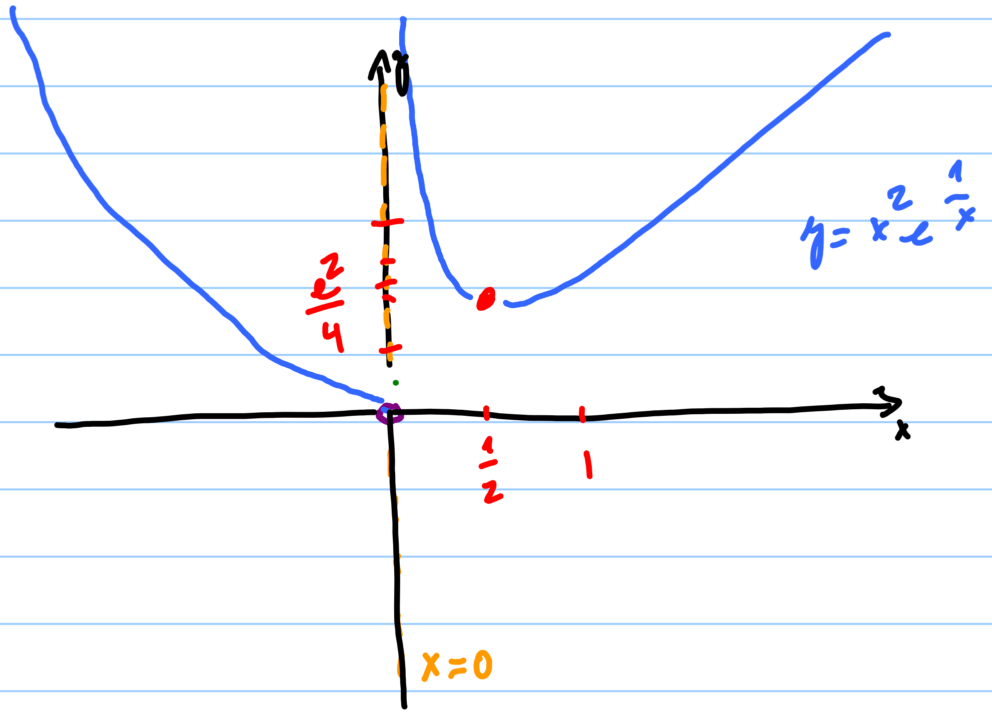
KONVEX. KONKÁVNOSŤ

KB $f'' = 0 \iff \left(\frac{-2}{x} + \frac{1}{x^2} + 2\right) = 0 \iff$

$$\frac{-2x + 1 + 2x^2}{x^2} = 0 \iff 2x^2 - 2x + 1 = 0$$

$D = 4 - 4 \cdot 2 \cdot 1 < 0 \Rightarrow \nexists$ RIEŠ. v $\mathbb{R} \Rightarrow \nexists$ IB

	$(-\infty, 0)$	$(0, \infty)$
$e^{\frac{1}{x}}$	+	+
$-\frac{2}{x} + \frac{1}{x^2} + 2$	+	+
	$\cup$	$\cup$



$$\lim_{x\to\infty} x^2 e^{\frac{1}{x}} = \lim_{x\to\infty} \frac{e^{\frac{1}{x}}}{\frac{1}{x^2}} = \infty$$

$$\lim_{x\to-\infty} x^2 e^{\frac{1}{x}} = -\infty \cdot e^0 = (-\infty)^2 \cdot 1 = \infty$$