Note Title 21/10/2022

PRIEBEH FUNKCIE

1) D(f) NULOVÉ BODY
2) PÁRNOSŤ NEPÁR
3) SPOJITOSŤ ABS
4) ASS
5) MONOTÓNNOSŤ A SB
6) EXTRÉMY
7) KONVEXNOSŤ A KONK.
KAND. NA INFLEXNÉ BODY
8) INFL. BODY
9) GRAF

$$f(x) = \frac{x^3}{2(x+1)^2}$$

1) $D(f) = \{\forall x \in R \,;\, 2(x+1)^2 \neq 0\} = R - \{-1\}$

$x+1 \neq 0 \iff x \neq -1$

NULOVÉ BODY: $f(x) = 0$

$$\frac{x^3}{2(x+1)^2} = 0 \iff x^3 = 0 \iff x = 0 \qquad N[0,0]$$

2) PÁR, NEPÁR: $f(-x) = f(x) \Rightarrow$ PÁRNA

$f(-x) = -f(x) \Rightarrow$ NEPÁRNA

$$f(-x) = \frac{(-x)^3}{2(-x+1)^2} = \frac{-x^3}{2(-x+1)^2} \begin{matrix} \neq -f(x) \\ \neq f(x) \end{matrix} \Bigg\} \begin{matrix} \text{ANI PÁR.} \\ \text{ANI. NEP.} \end{matrix}$$

POZN: FUNKCIA NIE JE ANI PÁRNA ANI NEPÁRNA, PRETOŽE D(f) NIE JE SYMETRICKÝ

3) SPOJITOST ABS:

$$\left.\begin{array}{l}\lim\limits_{x\to a^-} f(x)=\pm\infty \\ \lim\limits_{x\to a^+} f(x)=\pm\infty\end{array}\right\} x=a$$

TOČKA NESPOJITOSTI $a=-1$

$$\left.\begin{array}{l}\lim\limits_{x\to -1^-}\frac{x^3}{2(x+1)^2}=\frac{-1}{0^+}=-\infty \\ \lim\limits_{x\to -1^+}\frac{x^3}{2(x+1)^2}=\frac{-1}{0^+}=-\infty\end{array}\right\}$$

ABS: $x=-1$

$x^3\to -1$, $x+1\to 0^-$, $2(x+1)^2\to 0^+$

4) ABS

$$k_1=\lim_{x\to\infty}\frac{f(x)}{x} \qquad k_2=\lim_{x\to-\infty}\frac{f(x)}{x}$$

$$q_1=\lim_{x\to\infty}(f(x)-k_1x) \qquad q_2=\lim_{x\to-\infty}(f(x)-k_2\cdot x)$$

$$y_1=k_1x+q_1 \qquad y_2=k_2x+q_2$$

$$k_1=\lim_{x\to\infty}\frac{\frac{x^3}{2(x+1)^2}}{x}=\lim_{x\to\infty}\frac{x^3}{2x^3+4x^2+2x}=\frac{1}{2}$$

$4x^2+2x\to 0$

$$q_1=\lim_{x\to\infty}\left(\frac{x^3}{2(x+1)^2}-\frac{1}{2}\cdot x\right)=\lim_{x\to\infty}\frac{x^3-(x+1)^2\cdot x}{2(x+1)^2}=$$

$$=\lim_{x\to\infty}\frac{\cancel{x^3}-\cancel{x^3}-2x^2-x}{2(x+1)^2}=\lim_{x\to\infty}\frac{-2x^2-x}{2x^2+4x+2}=-1$$

$x\to 0$, $4x+2\to 0$

$$\boxed{y_1=\frac{1}{2}x-1}$$

$$k_2 = \lim_{x\to-\infty} \frac{\frac{x^3}{2(x+1)^2}}{x} = \frac{1}{2} = k_1$$

$$q_2 = \lim_{x\to-\infty} \left(\frac{x^3}{2(x+1)^2} - \frac{1}{2}x\right) = -1 = q_2 \qquad y_2 = y_1$$

(*)

$$\lim_{x\to\infty} \frac{x^3}{2x^3+4x^2+2x} \quad \frac{\frac{1}{x^3}}{\frac{1}{x^3}} \boxed{=} \lim_{x\to\infty} \frac{\underbrace{\frac{x^3}{x^3}}_{\to 1}}{\underbrace{\frac{2x^3}{x^3}}_{\to 2}+\underbrace{\frac{4x^2}{x^3}}_{\to 0}+\underbrace{\frac{2x}{x^3}}_{\to 0}} = \frac{1}{2+0+0}$$

$$\boxed{=} = \lim_{x\to\infty} \frac{3x^2}{6x^2+8x+2} \overset{L'}{=} \lim_{x\to\infty} \frac{6x}{12x+8} \overset{L'}{=} \lim_{x\to\infty} \frac{6}{12} = \frac{1}{2}$$

5) MONOTÓNNOSŤ

$f'(x) > 0$ RASTÚCA

$f'(x) < 0$ KLESAJÚCA

$f'(x) = 0$ STACIONÁRNE BODY

$$f'(x) = \left(\frac{x^3}{2(x+1)^2}\right)' = \frac{3x^2\cdot 2(x+1)^{\cancel{2}\,1} - x^3\cdot 2\cdot 2\cancel{(x+1)}^{2-1}\cdot 1}{\left(2(x+1)^2\right)^{\cancel{2}\,3}} =$$

$$= \frac{3x^2\cdot 2(x+1) - 4x^3}{2^2(x+1)^3} = \frac{6x^3+6x^2-4x^3}{4(x+1)^3} = \frac{2x^3+6x^2}{4(x+1)^3} =$$

$$= \frac{x^3+3x^2}{2(x+1)^3} = \frac{x^2(x+3)}{2(x+1)^3}$$

$SB: \; f'(x) = 0 \Leftrightarrow \quad x^2(x+3) = 0 \Leftrightarrow x = 0 \lor x = -3$

	$(-\infty, -3)$	$(-3, -1)$	$(-1, 0)$	$(0, \infty)$
x^2	+	+	+	+
$x+3$	−	+	+	+
$2(x+1)^3$	−	−	+	+
f'	+	−	+	+
	↗	↘	↗	↗

6) EXTRÉMY $\quad f''(SB) > 0 \quad$ LOK MIN

$f''(SB) < 0 \quad$ LOK MAX

$f''(SB) = 0 \Rightarrow$ KANDIDÁT NA IB

$$f'' = \left(\frac{x^3 + 3x^2}{2(x+1)^3}\right)' = \frac{(3x^2+6x)\,2(x+1)^{\cancel{3}\,1} - (x^3+3x^2)\,2\cdot 3\cdot \cancel{(x+1)^2}\cdot 1}{2^2\cdot\left((x+1)^{\cancel{3}}\right)^{\cancel{2}\,4}} =$$

$$= \frac{\cancel{6x^3} + \cancel{6x^2} + \cancel{12x^2} + 12x - \cancel{6x^3} - \cancel{18x^2}}{4(x+1)^4} = \frac{\cancel{12}^{3}x}{\cancel{4}(x+1)^4} =$$

$$= \frac{3x}{(x+1)^4}$$

$$f''(-3) = \frac{3\cdot(-3)}{(-3+1)^4} = \frac{-9}{16} < 0 \Rightarrow \text{LOK MAX}$$

$$f(-3) = \frac{(-3)^3}{2(-3+1)^2} = \frac{-27}{8} \qquad \text{L. MAX } \left[-3, -\frac{27}{8}\right]$$

$$f''(0) = \frac{3\cdot 0}{(0+1)^4} = 0 \Rightarrow \text{KANDIDÁT NA IB.}$$

7) KONVEXNOSŤ KONKÁVNOSŤ

$f''(x) > 0$ KONVEXNÁ

$f''(x) < 0$ KONKÁVNE

$f''(x) = 0$ KANDIDÁT NA IB.

$f'' = 0 \iff x = 0$

	$(-\infty, -1)$	$(-1, 0)$	$(0, \infty)$
$3x$	$-$	$-$	$+$
$(x+1)^4$	$+$	$+$	$+$
	$-$	$-$	$+$
	$\cap$	$\cap$	$\cup$

8) INFLEXNÉ BODY

$$f^{(n)}(IB) \neq 0$$

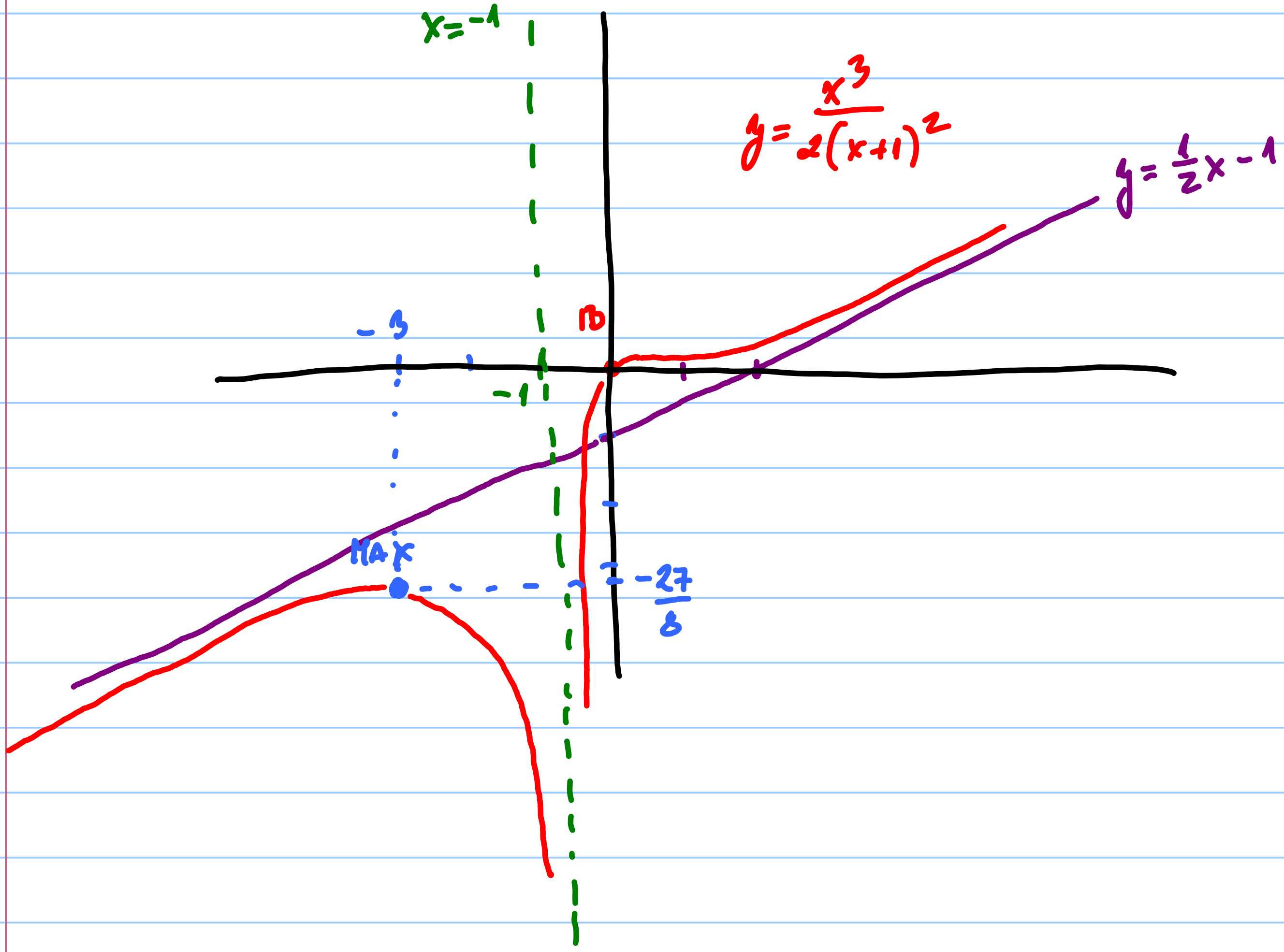
$n > 2$,
n - NEPÁRNE $\Rightarrow$ IB
n - PÁRNE $\Rightarrow$ EXTRÉM

$$f''' = \frac{3 \cdot (x+1)^4 - 3x \cdot 4(x+1)^3 \cdot 1}{((x+1)^4)^2}$$

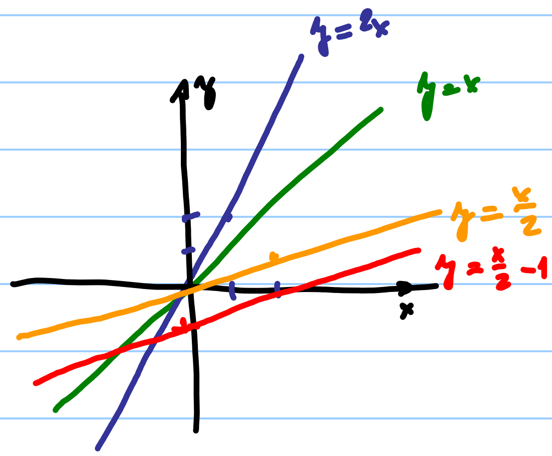
$$f'''(0) = \frac{3 \cdot (0+1)^4 - 3 \cdot 0 \cdot 4(0+1)^3 \cdot 1}{(0+1)^8} = \frac{3+0}{1} \neq 0$$

$\Rightarrow x = 0$ JE INFL. BOD.

$$f(0) = \frac{0^3}{2(0+1)^2} = 0 \qquad \text{IB } [0,0]$$



POMÔCKA:



PR: $y = \dfrac{1}{x^3+3}$

1) $D_f = \{\forall x \in R \;;\; x^3+3 \neq 0\} = R - \{-\sqrt[3]{3}\}$

2) $f(-x) = \dfrac{1}{(-x)^3+3} = \dfrac{1}{-x^3+3} = \dfrac{-1}{x^3-3} \neq -f(x)$
$\neq f(x)$

ANI. ANI.

3) ASS

$\lim\limits_{x \to -\sqrt[3]{3}^-} \dfrac{1}{x^3+3} = -\infty$ $\quad (x^3+3 \searrow 0^-)$

$\lim\limits_{x \to -\sqrt[3]{3}^+} \dfrac{1}{x^3+3} = \infty$ $\quad (x^3+3 \searrow 0^+)$

$x = -\sqrt[3]{3}$

4) ASS

$k_1 \lim\limits_{x \to \infty} \dfrac{\frac{1}{x^3+3}}{x} = \lim\limits_{x \to \infty} \dfrac{1}{x^4+x} = 0 = \lim\limits_{x \to -\infty} \dfrac{\frac{1}{x^3+3}}{x} = k_2$

$q_1 = \lim\limits_{x \to \infty} \dfrac{1}{x^3+3} - 0 \cdot x = 0 = q_2$

$y = 0 \cdot x + 0 = 0$

5) $f'(x) = -1(x^3+3)^{-2} \cdot 3x^2 = \dfrac{-3x^2}{(x^3+3)^2}$

$f(x) = (x^3+3)^{-1}$

$f' = \dfrac{0 \cdot \cancel{(x^3+3)} - 1 \cdot 3x^2}{(x^3+3)^2}$

SB: $f' = 0 \iff -3x^2 = 0 \iff x = 0$

	$(-\infty, -\sqrt[3]{3})$	$(-\sqrt[3]{3}, 0)$	$(0, \infty)$
$-3x^2$	$-$	$-$	$-$
$(x^3+2)^2$	$+$	$+$	$+$
	$(-)$ $\searrow$	$(-)$ $\searrow$	$(-)$ $\searrow$

6) $$f'' = \frac{-6x(x^3+3)^{\cancel{2}\,1} + 3x^2\cdot 2\cancel{(x^3+3)}\cdot 3x^2}{\left((x^3+3)^{\cancel{2}}\right)^{2}\ 3} =$$

$$= \frac{-6x^4 - 18x + 18x^4}{(x^3+3)^3} = \frac{12x^4 - 18x}{(x^3+3)^3} = \frac{6x(2x^3-3)}{(x^3+3)^3}$$

$$f''(0) = \frac{6\cdot 0(2\cdot 0^3 - 3)}{(0^3+3)^3} = 0$$

7) $$f''(x) = 0 \Leftrightarrow 6x(2x^3-3) = 0 \Leftrightarrow x = 0 \lor x = \sqrt[3]{\tfrac{3}{2}}$$

	$-\infty, -\sqrt[3]{3}$	$(-\sqrt[3]{3}, 0)$	$(0, \sqrt[3]{\frac{3}{2}})$	$(\sqrt[3]{\frac{3}{2}}, \infty)$
$6x$	$-$	$-$	$+$	$+$
$2x^3-3$	$-$	$-$	$-$	$+$
$(x^3+3)^3$	$-$	$+$	$+$	$+$
	$-$	$+$	$-$	$+$
	$\cap$	$\cup$	$\cap$	$\cup$

8) $$f''' = \frac{(48x^3-18)(x^3+3)^{\cancel{3}\,1} - (12x^4-18x)\cdot 3\cancel{(x^3+3)}^2\cdot 3x^2}{\left((x^3+3)^{\cancel{3}}\right)^{2}\ 4} =$$

$$= \frac{(48x^3-18)(x^3+3) - (12x^4-18x)\cdot 9x^2}{(x^3+3)^4}$$

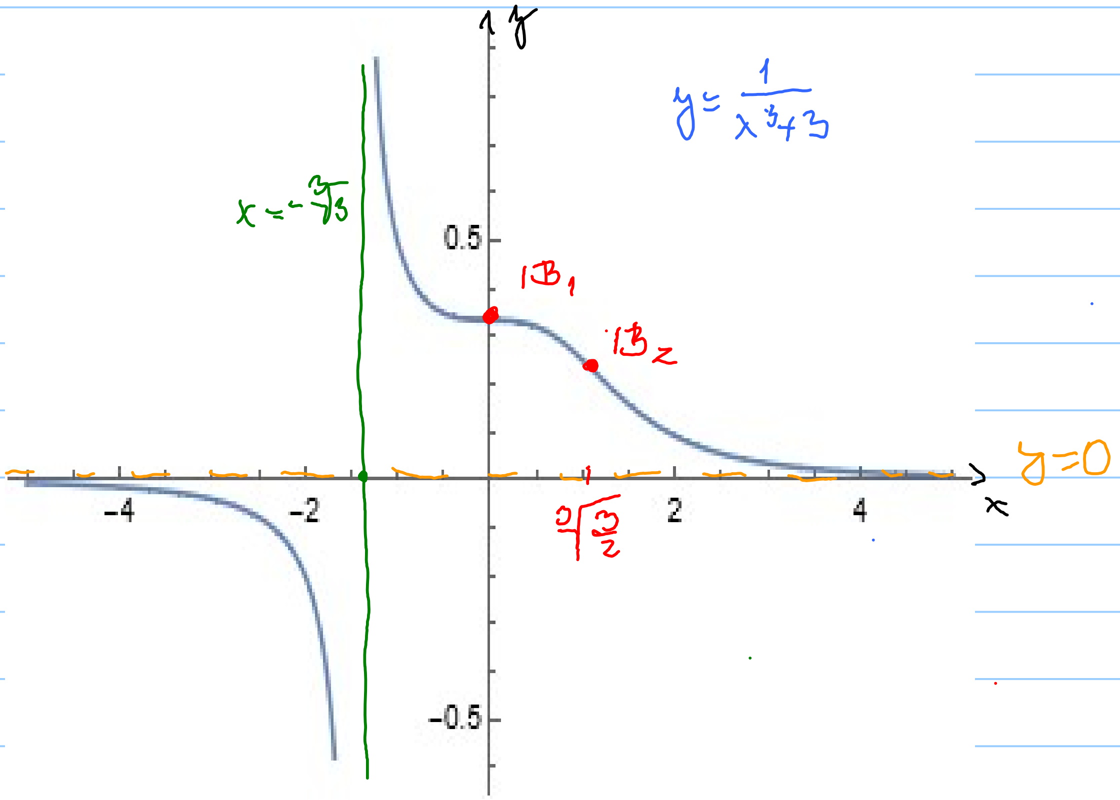
$$f'''(0) = \frac{(48\cdot 0^3 - 18)(0^3+3) - (12\cdot 0^4 - 18\cdot 0)\cdot 9\cdot 0^2}{(0^3+3)^4} = \frac{-54-0}{81} \neq 0$$

$$\Rightarrow \quad x = 0 \quad \text{JE} \quad \text{IB} \qquad f(0) = \frac{1}{0^3+3} = \frac{1}{3} \qquad \text{IB}_1\left[0, \tfrac{1}{3}\right]$$

$$f'''\left(\sqrt[3]{\tfrac{3}{2}}\right) = \frac{\left(48\cdot\left(\sqrt[3]{\frac{3}{2}}\right)^3 - 18\right)\left(\left(\sqrt[3]{\frac{3}{2}}\right)^3+3\right) - \left(12\left(\sqrt[3]{\frac{3}{2}}\right)^4 - 18\cdot\sqrt[3]{\frac{3}{2}}\right)\cdot 9\left(\sqrt[3]{\frac{3}{2}}\right)^2}{\left(\left(\sqrt[3]{\frac{3}{2}}\right)^3+3\right)^4} =$$

$$= \frac{\left(48\cdot\frac{3}{2} - 18\right)\left(\frac{3}{2}+3\right) - \underbrace{\left(12\cdot\frac{3}{2}\cdot\sqrt[3]{\frac{3}{2}} - 18\cdot\sqrt[3]{\frac{3}{2}}\right)}_{=0}\cdot 9\left(\sqrt[3]{\frac{3}{2}}\right)^2}{\left(\frac{3}{2}+3\right)^4} \neq 0$$

$$\Rightarrow \quad x = \sqrt[3]{\tfrac{3}{2}} \quad \text{JE} \quad \text{IB} \qquad f\left(\sqrt[3]{\tfrac{3}{2}}\right) = \frac{1}{\left(\sqrt[3]{\frac{3}{2}}\right)^3+3} = \frac{1}{\frac{3}{2}+3} = \frac{2}{9}$$



POZOR OPRAVA !!!

PR.: $y = \frac{\ln x}{x}$

1) $D(f) = \{\forall x \in R;\ x \neq 0;\ x > 0\} = (0, \infty)$

NULOVÉ BODY $f(x) = 0$

$$\frac{\ln x}{x} = 0 \Leftrightarrow \ln x = 0 \Leftrightarrow x = 1 \quad NB[1,0]$$

2) PÁR, NEPÁR

$f(-x) = f(x)$ PÁRNA

$f(-x) = -f(x)$ NEPÁR.

$$f(-x) = \frac{\ln(-x)}{-x} = -\frac{\ln(-x)}{x} \begin{matrix} \neq f(x) \\ \neq -f(x) \end{matrix} \Big\} \text{ ANI. PÁR. ANI NEPÁR}$$

(VYPLÝVA AJ Z NESYMETRIE $D(f)$)

3) SPOJITOSŤ A ABS

$-\frac{\infty}{0}$

$$\lim_{x \to 0^+} \frac{\ln x}{x} = \lim_{x \to 0^+} \frac{1}{x} \cdot \ln x = \infty \cdot (-\infty) = -\infty$$

$\boxed{x = 0}$

4) ASS

$k_1 = \lim_{x \to \infty} \frac{f(x)}{x}$ $\qquad$ $k_2 = \lim_{x \to -\infty} \frac{f(x)}{x}$

$q_1 = \lim_{x \to \infty} (f(x) - k_1 \cdot x)$ $\qquad$ $q_2 = \lim_{x \to -\infty} (f(x) - k_2 \cdot x)$

$y_1 = k_1 x + q_1$ $\qquad$ $y_2 = k_2 \cdot x + q_2$

$$k_1 = \lim_{x\to\infty} \frac{\frac{\ln x}{x}}{x} = \lim_{x\to\infty} \frac{\ln x}{x^2} \overset{\frac{\infty}{\infty}}{\underset{=}{L'}} \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$$

$$q_1 = \lim_{x\to\infty} \frac{\ln x}{x} - 0\cdot x \overset{L'}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$$

$$y = 0\cdot x + 0 \Rightarrow \boxed{y = 0}$$

5) MONOTÓNNOSŤ A SB

$f'(x) > 0$ RASTÚCA

$f'(x) < 0$ KLESAJÚCA

$f'(x) = 0 \Rightarrow$ STACIONÁRNY BOD

$$f'(x) = \left(\frac{\ln x}{x}\right)' = \frac{\frac{1}{x}\cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$$f'(x) = 0 \Leftrightarrow 1 - \ln x = 0 \Leftrightarrow \ln x = 1 \Leftrightarrow x = e$$

SB $x = e$

	$(0, e)$	(e, ∞)
$1 - \ln x$	+	−
x^2	+	+
	+	−
	↗	↘

6) EXTRÉMY

$f''(SB) > 0$ LOK MIN

$f''(SB) < 0$ LOK MAX

$f''(SB) = 0$ KANDIDÁT NA IB

$$f'' = \frac{\left(0 - \frac{1}{x}\right)x^{\cancel{2}} - (1 - \ln x)\cdot 2\cancel{x}}{\cancel{(x^2)^2}\ 3} = \frac{-1 - 2 + 2\ln x}{x^3} =$$

$$= \frac{-3 + 2\ln x}{x^3}$$

$$f''(e) = \frac{-3 + 2\cdot 1}{e^3} = \frac{-1}{e^3} < 0 \Rightarrow \text{LOK. MAX}$$

$$f(e) = \frac{\ln e}{e} = \frac{1}{e} \qquad \text{MAX}\left[e, \frac{1}{e}\right]$$

KONVEXNOSŤ A KONKÁVNOSŤ

$f'' > 0$ KONVEXNÁ $\cup$

$f'' < 0$ KONKÁVNA $\cap$

$f'' = 0$ KAND. NA IB.

$$f'' = 0 \Leftrightarrow -3 + 2\ln x = 0 \Leftrightarrow \ln x = \frac{3}{2} \Leftrightarrow$$

$$x = e^{\frac{3}{2}}$$

	$(0, e^{\frac{3}{2}})$	$(e^{\frac{3}{2}}, \infty)$
$-3+2\ln x$	$-$	$+$
x^3	$+$	$+$
	$-$	$+$
	$\cap$	$\cup$

8, INFLEXNÝ BOD $\qquad f^{(n)}(\text{KIB}) \neq 0 \quad n > 2,$

n - NEPÁRNE $\Rightarrow$ IB

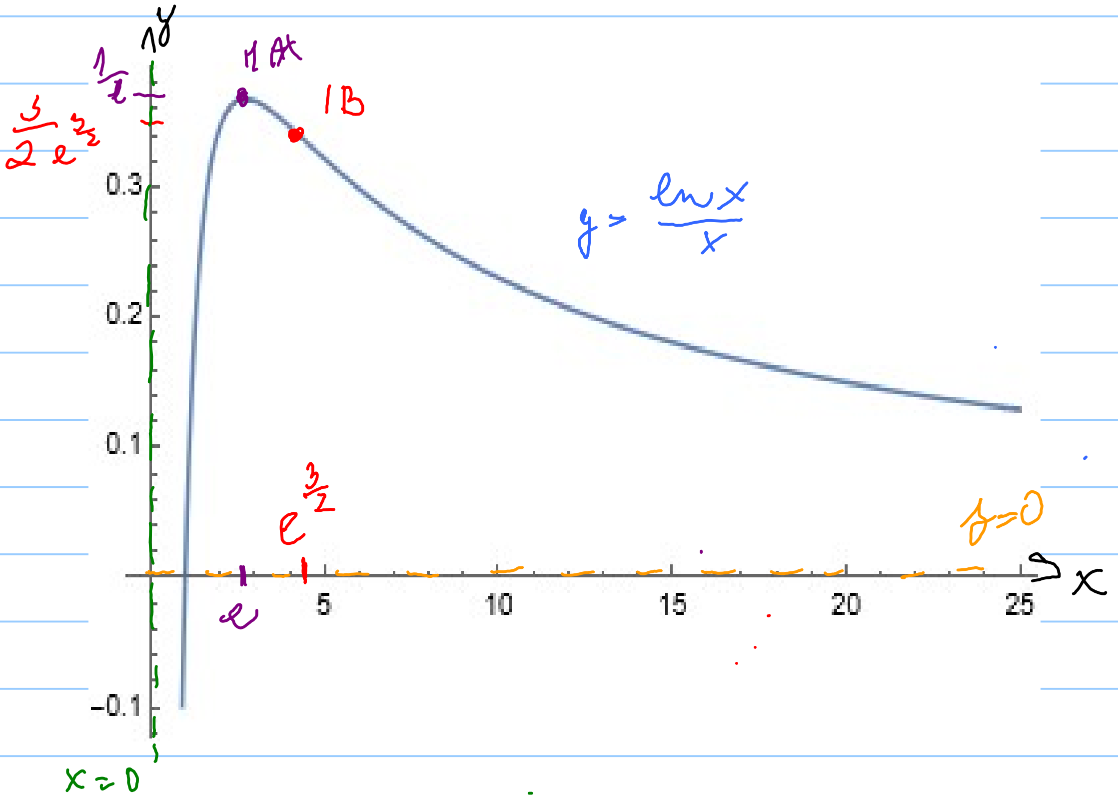
n - PÁRNE $\Rightarrow$ EXTR.

$$f'''(x) = \frac{2\cdot\frac{1}{x}\cdot \cancel{x^3} - (-3+2\ln x)\cdot 3\cancel{x}^2}{(\cancel{x^3})^2} = \frac{2+6-6\ln x}{x^4}$$

$$f'''(e^{\frac{3}{2}}) = \frac{8-6\cdot \ln e^{\frac{3}{2}}}{(e^{\frac{3}{2}})^4} = \frac{8-6\cdot\frac{3}{2}}{e^6} = \frac{-1}{e^6} \neq 0$$

$\Rightarrow x = e^{\frac{3}{2}}$ JE INFL. BOD $\quad f(e^{\frac{3}{2}}) = \frac{\ln e^{\frac{3}{2}}}{e^{\frac{3}{2}}} = \frac{\frac{3}{2}}{e^{\frac{3}{2}}} =$

$= \frac{3}{2e^{3/2}}$ $\qquad$ IB$\left[e^{\frac{3}{2}}, \frac{3}{2e^{\frac{3}{2}}}\right]$



PR : $y = x + \frac{1}{x}$

1) $D(f) = \{\forall x \in R;\ x \neq 0\} = R - \{0\}$

$NB = \quad f(x) = 0 \iff \frac{x^2+1}{x} = 0 \iff x^2+1 = 0$

NIKDY $\Rightarrow \nexists$ NB

2) $f(-x) = \frac{(-x)^2+1}{-x} = -\frac{x^2+1}{x} = -f(x)$ – NEPÁRNA

3) $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} x + \frac{1}{x} = -\infty$, $x \to 0^-$

$\lim\limits_{x \to 0^+} x + \frac{1}{x} = \infty$, $x \to 0^+$

ABS: $x = 0$

4) ASS

$k_1 = \lim\limits_{x \to \infty} \frac{\frac{x^2+1}{x}}{x} = \lim\limits_{x \to \infty} \frac{x^2+1}{x^2} = \frac{1}{1} = 1$

$q_1 = \lim\limits_{x \to \infty} \left(\frac{x^2+1}{x} - 1 \cdot x\right) = \lim\limits_{x \to \infty} \left(\frac{x^2+1-x^2}{x}\right) =$

$= \lim\limits_{x \to \infty} \frac{1}{x} = 0 \qquad y_1 = 1 \cdot x + 0 \Rightarrow \boxed{y = x}$

$k_2 = \lim\limits_{x \to -\infty} \frac{x^2+1}{x^2} = 1 = k_1$

$q_2 = \lim\limits_{x \to -\infty} \left(\frac{x^2+1}{x} - x\right) = \lim\limits_{x \to -\infty} \left(\frac{1}{x}\right) = 0 = q_1$

5) $f'(x) = 1 + \frac{-1}{x^2} = \frac{x^2-1}{x^2}$

SB $f' = 0 \iff x^2 - 1 = 0 \iff x = \pm 1$

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$x^2 - 1$	+	−	−	+
x^2	+	+	+	+
	+	−	−	+
	↗	↘	↘	↗

6)

$$f'' = \left(1 - (x^{-2})\right)' = 0 - (-2) \cdot x^{-3} = \frac{2}{x^3}$$

$f''(-1) = \frac{2}{(-1)^3} = -2 < 0$ LOK MAX $[-1, -2]$

$f(-1) = -1 + \frac{1}{-1} = -2$

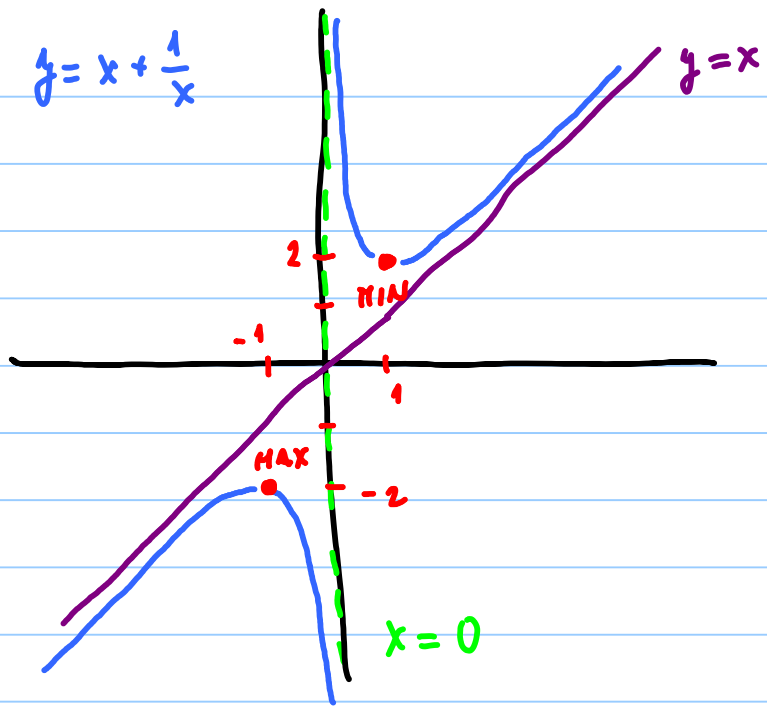
$f''(1) = \frac{2}{1^3} = 2 > 0$ LOK MIN $[1, 2]$

$f(1) = 1 + \frac{1}{1} = 2$

7)

$f'' = 0 \iff \frac{2}{x^3} = 0$ NIKDY $\Rightarrow \nexists$ KAND. NA SB

	$(-\infty, 0)$	$(0, \infty)$
$\frac{2}{x^3}$	−	+
	$\cap$	$\cup$



PŘ: $y = e^{-x^2}$

1) $D(f) = \{\forall x \in \mathbb{R}\} = \mathbb{R}$

2) $f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$ PARNÁ

3) SPOJITÁ NA $\mathbb{R} \Rightarrow \nexists$ ABS

4) $k_1 = \lim_{x\to\infty} \frac{e^{-x^2}}{x} = \lim_{x\to\infty} \frac{\frac{1}{e^{x^2}}}{x} = \lim_{x\to\infty} \frac{1}{x e^{x^2}} = 0 = k_2$

$q_1 = \lim_{x\to\infty} e^{-x^2} - 0\cdot x = \lim_{x\to\infty} \frac{1}{e^{x^2}} = 0 = q_2$

ASS: $\boxed{y=0}$

5) $f'(x) = e^{-x^2}\cdot(-2x) = \frac{-2x}{e^{x^2}}$

$f' = 0 \iff -2x = 0 \iff x = 0$

	$(-\infty, 0)$	$(0, \infty)$
$-2x$	$+$	$-$
e^{x^2}	$+$	$+$
	$+$	$-$
	$\nearrow$	$\searrow$

6)

$$f'' = \frac{-2\cdot \cancel{e^{x^2}} - (-2x)\cdot \cancel{e^{x^2}}\cdot 2x}{\left(e^{x^2}\right)^{\cancel{2}}} = \frac{-2 + 4x^2}{e^{x^2}}$$

$$f''(0) = \frac{-2+4\cdot 0^2}{e^{0^2}} = \frac{-2}{1} = -2 < 0 \quad \text{LOK. MAX}$$

$$f(0) = e^{-0^2} = e^0 = 1 \qquad \text{L. MAX } [0,1]$$

7)

$$f''=0 \Leftrightarrow 4x^2-2=0 \Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x = \pm\frac{1}{\sqrt{2}}$$

	$\left(-\infty, -\frac{1}{\sqrt{2}}\right)$	$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$	$\left(\frac{1}{\sqrt{2}}, \infty\right)$
$4x^2-2$	$+$	$-$	$+$
e^{x^2}	$+$	$+$	$+$
	$+$	$-$	$+$
	$\cup$	$\cap$	$\cup$

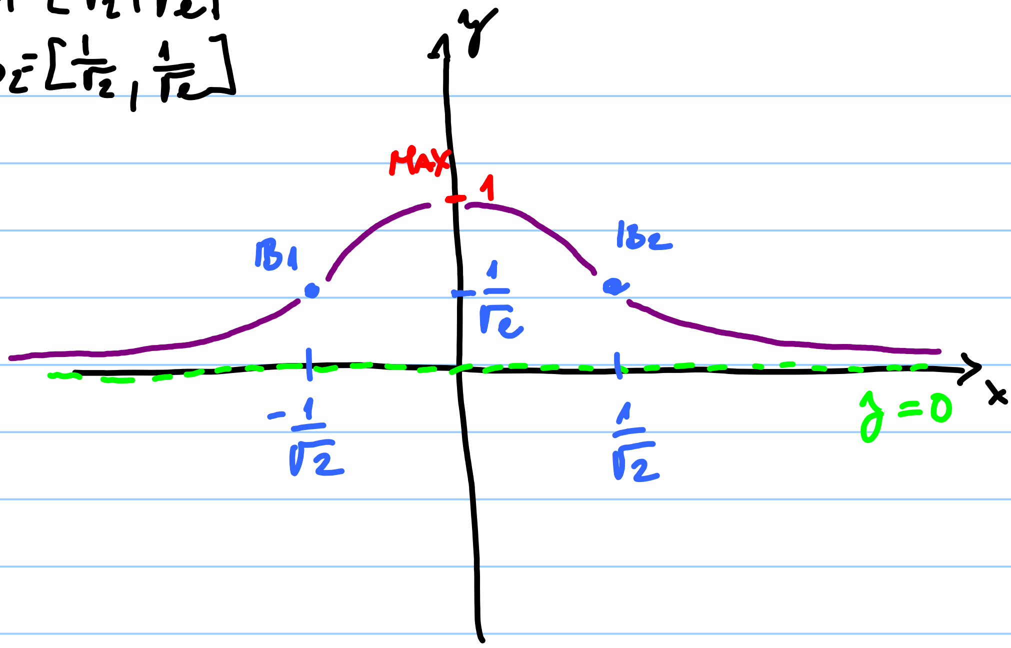
8) V BODE $x = -\frac{1}{\sqrt{2}}$ JE $f(x)$ SPOJITÁ ZĽAVA KONVEXNÁ SPRAVA KONKÁVNA $\Rightarrow$ JE V TOMTO BODE INFLEXNÝ BOD

ANALOGICKY PRE $x = \frac{1}{\sqrt{2}}$

$$f\left(-\frac{1}{\sqrt{2}}\right) = f\left(\frac{1}{\sqrt{2}}\right) = e^{-\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{e}}$$

$B_1 = \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right]$

$B_2 = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right]$



PR: $y = \frac{1}{x^2-4}$

1) $D(f) = \{\forall x \in \mathbb{R} ; \ x^2 - 4 \neq 0\} = \mathbb{R} - \{\pm 2\}$

2) $f(-x) = \frac{1}{(-x)^2-4} = \frac{1}{x^2-4} = f(x)$ PARNA

3)

$\lim\limits_{x \to -2^-} \frac{1}{x^2-4} = \infty$ $\to 0^+$

$\lim\limits_{x \to -2^+} \frac{1}{x^2-4} = -\infty$ $\to 0^-$

$\lim\limits_{x \to 2^-} \frac{1}{x^2-4} = -\infty$

$\lim\limits_{x \to 2^+} \frac{1}{x^2-4} = \infty$

ABS: $x = -2$ $x = 2$

4) ASS

$k_1 \ \lim\limits_{x \to \infty} \frac{\frac{1}{x^2-4}}{x} = \lim\limits_{x \to \infty} \frac{1}{x^3-x} = 0 = k_2$

$$q_1 = \lim_{x\to\infty} \frac{1}{x^2-4} - 0\cdot x = 0 = q_2$$

$$\boxed{y=0}$$

5) $$f'(x) = \left(\left(x^2-4\right)^{-1}\right)' = -1\left(x^2-4\right)^{-2}\cdot 2x = \frac{-2x}{\left(x^2-4\right)^2}$$

$$f'=0 \iff -2x=0 \iff x=0$$

	$(-\infty,-2)$	$(-2,0)$	$(0,2)$	$(2,\infty)$
$-2x$	$+$	$+$	$-$	$-$
$(x^2-4)^2$	$+$	$+$	$+$	$+$
	$+$	$+$	$-$	$-$
	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$

6) $$f''(x) = \frac{-2\cdot\left(x^2-4\right)^{\cancel{2}\,1} - (-2x)\,2\cdot\cancel{\left(x^2-4\right)}\cdot 2x}{\left(\left(x^2-4\right)^{\cancel{2}}\right)^{\cancel{2}\,3}}$$

$$= \frac{-2x^2+8+8x^2}{\left(x^2-4\right)^3} = \frac{6x^2+8}{\left(x^2-4\right)^3}$$

$$f''(0) = \frac{6\cdot 0^2+8}{\left(0^2-4\right)^3} = \frac{-8}{64} = -\frac{1}{8} < 0 \quad \text{LOK MAX}$$

$$f(0) = \frac{1}{0^2-4} = -\frac{1}{4} \qquad \text{MAX } \left[0, -\frac{1}{4}\right]$$

7) $$f''=0 \iff 6x^2+8=0 \quad \text{NIKDY} \Rightarrow \nexists \text{ K.B.}$$

	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
$6x+8$	$+$	$+$	$+$
$(x^2-4)^3$	$+$	$-$	$+$
	$+$	$-$	$+$
	$\cup$	$\cap$	$\cup$

8) $\nexists$ IB