Matematická analýza cvičenie 3 ZS 2022-2023
utorok 4.9. 10:00 Live

Note Title 04/10/2022

ASYMPTOTY KU GRAFU FUNKCIE

ASYMPTOTA BEZ SMERNICE (ABS)

$$\lim_{x \to a^-} f(x) = \pm\infty \Rightarrow \text{ABS}: \; x = a$$

$$\lim_{x \to a^+} f(x) = \pm\infty$$

ASYMPTOTA SO SMERNICOU (ASS)

$$k_1 = \lim_{x \to \infty} \frac{f(x)}{x} \qquad k_2 = \lim_{x \to -\infty} \frac{f(x)}{x}$$

$$q_1 = \lim_{x \to \infty} \left(f(x) - k_1 \cdot x\right) \qquad q_2 = \lim_{x \to -\infty} \left(f(x) - k_2 \cdot x\right)$$

$$y = k_1 x + q_1 \qquad y = k_2 \cdot x + q_2$$

Pr: $y = \dfrac{x}{x-1}$

ABS: $D(f) = \{\forall x \in R;\; x - 1 \neq 0\} = R - \{1\}$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x}{\underbrace{x-1}_{\to 0^-}} = -\infty$$

$$\lim_{x \to 1^+} \frac{x}{\underbrace{x-1}_{\to 0^+}} = \infty$$

$\Rightarrow x = 1$

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ASS:

$$k_1 = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{x}{x-1}}{x} = \lim_{x\to\infty} \frac{x}{x^2-x}$$

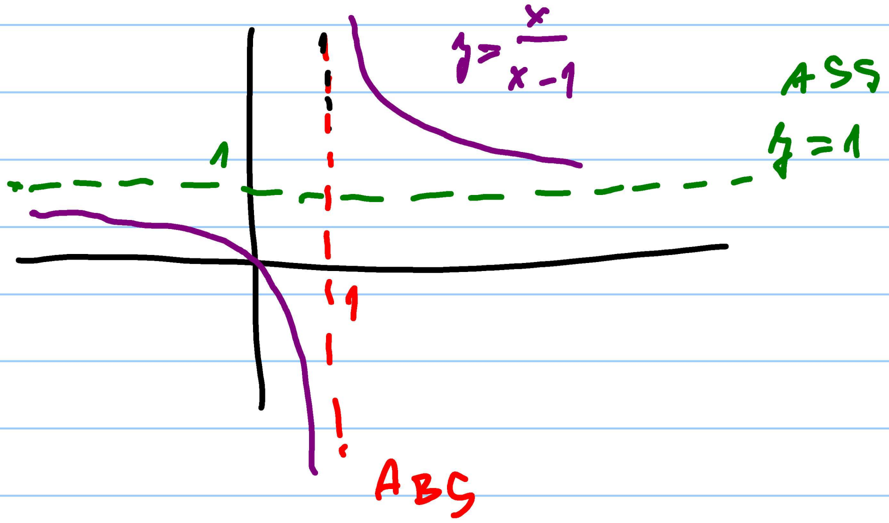
$$k_2 = \lim_{x\to -\infty} \frac{x}{x^2-1} = 0$$

$$q_1 = \lim_{x\to\infty} \left(\frac{x}{x-1} - 0\cdot x\right) = 1 = q_2$$

→ 1

$$y = 0\cdot x + 1 \qquad y = 1$$

$$f(x) = \frac{x-1+1}{x-1} = 1 + \frac{1}{x-1}$$



PR: $y = \sqrt{x^2-1}$ $\quad D(f)\quad x^2-1 \geq 0$

$$R \setminus (-1,1) = (-\infty, -1\rangle \cup \langle 1, \infty)$$

ABS

$$\lim_{x\to -1^-} \sqrt{x^2-1} = 0 \qquad \lim_{x\to 1^+} \sqrt{x^2-1} = 0$$

$\Rightarrow \nexists$ ABS

ASS:

$$k_1 = \lim_{x\to\infty} \frac{\sqrt{x^2-1}}{x} = \lim_{x\to\infty} \sqrt{\frac{x^2-1}{x^2}} = 1 = k_2$$

$\infty - \infty$ $$q_1 = \lim_{x\to\infty} \left(\sqrt{x^2-1} - 1\cdot x\right) \cdot \left(\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}+x}\right) =$$

$$\lim_{x\to\infty} \frac{\cancel{x^2} - 1 - \cancel{x^2}}{\underbrace{\sqrt{x^2-1} + x}_{\to\infty}} = 0 = q_2$$

$y = 1\cdot x + 0$ $\boxed{y = x}$

PŘ! $y = \frac{x^5}{x^2-1}$ $D(f) = \{\forall x \in \mathbb{R};\ x^2 - 1 \neq 0\} = \mathbb{R} - \{-1; 1\}$

ABS $$\lim_{x\to -1^-} \frac{\overset{\to -1}{x}}{\underbrace{x^2-1}_{\to 0^+}} = -\infty \qquad \lim_{x\to -1^+} \frac{\overset{\to -1}{x}}{\underbrace{x^2-1}_{\to 0^-}} = \infty$$

$$\lim_{x\to 1^-} \frac{\overset{\to 1}{x}}{\underbrace{x^2-1}_{\to 0^-}} = -\infty \qquad \lim_{x\to 1^+} \frac{\overset{\to 1}{x}}{\underbrace{x^2-1}_{\to 0^+}} = \infty$$

$\boxed{x = -1}$ $\boxed{x = 1}$

Ass.: $$k_1 = \lim_{x\to\infty} \frac{\frac{x^5}{x^2-1}}{x} = \lim_{x\to\infty} \frac{x^5}{x^3 - x} = \infty \Rightarrow \nexists \text{ ASS}$$

$$k_2 = \lim_{x\to-\infty} \frac{x^5}{x^3-x} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to-\infty} \frac{\overbrace{\frac{x^5}{x^3}}^{\to x^2}}{\frac{x^3}{x^3} - \underbrace{\frac{x}{x^3}}_{\to 0}} = \infty$$

7) : $y = x^2 + 1$

$D(A) = \mathbb{R} \Rightarrow \nexists$ ASS

$k_1 = \lim_{x \to \infty} x^2 + 1 = \infty$ $k_2 = \lim_{x \to -\infty} x^2 + 1 = \infty$

$\Rightarrow \nexists$ ASS

DERIVÁCIA FUNKCIE

$$(x^n)' = n x^{n-1}$$

$$y = 2x^5 + 4x^2 - \frac{1}{x^3} + x - 1$$

$$y' = 5 \cdot 2 \cdot x^{5-1} + 2 \cdot 4 \cdot x^{2-1} - (-3) \cdot x^{-3-1} + 1 \cdot x^{1-1} - 0 =$$

$$= 10x^4 + 8x + \frac{3}{x^4} + 1$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$y = \underbrace{(2x^3 + 4)}_{f} \cdot \underbrace{\sin x}_{g}$$

$$y' = \underbrace{(6x^2 + 0)}_{f'} \cdot \underbrace{\sin x}_{g} + \underbrace{(2x^3 + 4)}_{f} \cdot \underbrace{\cos x}_{g'}$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$y = \frac{\cos x}{2x^3}$$

$$y' = \frac{(-\sin x) \cdot 2x^3 - \cos x \cdot (6x^2)}{(2x^3)^2}$$

$$(f(g))' = f'(g)\cdot g'$$

Př $y = \ln(2x^3+3)$

$$y' = \frac{1}{2x^3+3}\cdot(2x^3+3)' = \frac{1}{2x^3+3}\cdot(6x^2)$$

$$(f^g)' = f^g\cdot(g\ln f)'$$

$$\left(e^{\ln f^g}\right)' = \left(e^{g\ln f}\right)' = e^{g\cdot\ln f}\cdot(g\ln f)' =$$

$$= f^g\left(g'\cdot\ln f + g\cdot\frac{1}{f}\cdot f'\right)$$

$y = x^{2x}$

$$y' = x^{2x}\cdot(2x\ln x)' = x^{2x}\left(2\cdot\ln x + 2x\cdot\frac{1}{x}\cdot 1\right) =$$

$$= x^{2x}\cdot(2\ln x + 2) = 2x^{2x}(\ln x + 2)$$

DOTYČNICA A NORMÁLA KU GRAFU FUNKCIE V x_0

$$t: \quad y - f(x_0) = f'(x_0)(x - x_0)$$

$$n: \quad y - f(x_0) = \frac{-1}{f'(x_0)}(x - x_0)$$

PR: NÁJDI t a n: $f(x) = x \ln x$ v $x_0 = e$

$f(x_0) = e \cdot \ln e = e \cdot 1 = e \rightarrow A[e, e]$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$f'(x_0) = \ln e + 1 = 1 + 1 = 2$

$t: \quad y - e = 2(x - e)$

$n: \quad y - e = \frac{-1}{2}(x - e)$