INTEGROVANIE RACIONÁLNYCH FUNKCIÍ

Note Title 11/7/2022

PR1 ROZLOŽTE NA PARCIÁLNE ZLOMKY:

a) $\frac{1}{x^2-4} = \boxed{\frac{1}{(x-2)(x+2)}} \; \ldots \; \frac{A}{(x-2)} + \frac{B}{(x+2)} = \boxed{\frac{Ax+2A+Bx-2B}{(x-2)(x+2)}}$

$* \quad 1 = Ax + Bx + 2A - 2B$

$\boxed{0} \cdot x^1 \boxed{+1} = x^1 \boxed{A+B} \boxed{+2A-2B}$

$0 = A + B \Rightarrow A = -B$ $\quad A = \frac{1}{4}$

$1 = 2A - 2B$

$1 = -2B - 2B \Rightarrow -4B = 1$

$B = -\frac{1}{4}$

$= \frac{\frac{1}{4}}{(x-2)} + \frac{-\frac{1}{4}}{x+2}$

b) $\frac{5x^2-17x+12}{x^3-4x^2+4x} = \frac{5x^2-17x+12}{x\underbrace{(x^2-4x+4)}_{(a-b)^2}} = \frac{5x^2-17x+12}{x(x-2)^2} \ldots \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$\frac{5x^2-17x+12}{x(x-2)^2} = \frac{A\overbrace{(x^2-4x+4)}^{(x-2)^2} + Bx(x-2) + Cx}{x(x-2)^2}$

$5x^2 - 17x + 12 = Ax^2 - 4Ax + 4A + Bx^2 - 2Bx + Cx$

$5 = A + B$ $\quad B = 5 - A = 5 - 3 = 2$

$-17 = -4A - 2B + C$ $\quad C = -17 + 4A + 2B =$

$12 = 4A \Rightarrow A = 3$ $\quad = -17 + 12 + 4 = -1$

$= \frac{3}{x} + \frac{2}{x-2} - \frac{1}{(x-2)^2}$

c) $\frac{2x-3}{x^3+2x^2-x-2} = \frac{2x-3}{x^2(x+2)-1(x+2)} = \frac{2x-3}{(x+2)(x^2-1)} = \frac{2x-3}{(x+2)(x-1)(x+1)}$

$\frac{2x-3}{(x+2)(x-1)(x+1)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x+1}$

$$2x-3 = A(x^2-1) + B(x+2)(x+1) + C(x+2)(x-1)$$

$$2x-3 = Ax^2-A + B(x^2+3x+2) + C(x^2-x+2x-2)$$

$$0x^2+2x-3 = Ax^2-A + Bx^2+3Bx+2B + Cx^2+Cx-2C$$

$$\left.\begin{aligned} 0 &= A+B+C \\ 2 &= 3B+C \\ -3 &= -A+2B-2C \end{aligned}\right\} \quad A=-\frac{1}{6} \quad B=\frac{5}{2} \quad C=-\frac{7}{3}$$

d) $$\frac{x^3-3x^2-3x-10}{(x-1)^2(x^2+4)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+4} = \frac{\quad}{(x-1)^2(x^2+4)}$$

$$x^3-3x^2-3x-10 = A(x-1)(x^2+4) + B(x^2+4) + (Cx+D)\underbrace{(x-1)^2}_{(x^2-2x+1)}$$

$$1x^3-3x^2-3x-10 = A(x^3+4x-x^2-4) + Bx^2+4B + Cx^3-2Cx^2+Cx+Dx^2-2Dx+D$$

$$1 = A+C \Rightarrow A = 1-C$$

$$-3 = -A+B-2C+D \Rightarrow -3 = -1+C+B-2C+D$$

$$-2 = B-C+D$$

$$-3 = 4A+C-2D \rightarrow -3 = 4-4C+C-2D$$

$$-7 = -3C-2D$$

$$-10 = -4A+4B+D \rightarrow -10 = -4+4C+4B+D$$

$$-6 = 4B+4C+D$$

$$\left.\begin{aligned} A&=0 \\ B&=-3 \\ C&=1 \\ D&=2 \end{aligned}\right\} \quad \frac{-3}{(x-1)^2} + \frac{x+2}{x^2+4}$$

PR2 VYPOČÍTAJTE INTEGRÁLY:

a) $$\int \frac{x^3-2x^2+9}{x^2-x-2}\,dx = \int x-1+\frac{x+7}{x^2-x-2}\,dx = \frac{x^2}{2} - x + \int \frac{x+7}{(x-2)(x+1)}\,dx$$

(*)

$$(x^3-2x^2+9):(x^2-x-2) = x-1+\frac{x+7}{x^2-x-2}$$

$$\begin{aligned} &-(x^3-x^2-2x) \\ &\quad -x^2+2x+9 \\ &-(-x^2+x+2) \\ &\quad x+7 \end{aligned}$$

(*) $\displaystyle\int \frac{x+7}{(x-2)(x+1)}\,dx \quad \dots \quad \frac{A}{x-2}+\frac{B}{x+1}=\frac{Ax+A+Bx-2B}{(x-2)(x+1)}$

$$x+7 = x(A+B)+A-2B$$

$$1 = A+B \quad (-1) \quad \Rightarrow \underline{A} = 1-B = 1-(-2) = \underline{3}$$

$$7 = A-2B$$

$$6 = -3B \Rightarrow \underline{\underline{B=-2}}$$

$$= \frac{x^2}{2} - x + \int \frac{3}{x-2} - \frac{2}{x+1}\,dx = \frac{x^2}{2} - x + 3\ln|x-2| - 2\ln|x+1| + c$$

$$= \left(\frac{x^2}{2} - x + \ln|x-2|^3 - \ln|x+1|^2 + c = \frac{x^2}{2} - x + \ln\frac{|x-2|^3}{|x+1|^2} + c \right)$$

b) $\displaystyle\int \frac{x}{x^3-3x+2}\,dx = (*)$

	1	0	-3	2
1	1	1	-2	0

$$(x^3-3x+2) = (x-1)(x^2+x-2) =$$

$$= (x-1)(x+2)(x-1) = (x+2)(x-1)^2$$

$$\frac{x}{x^3-3x+2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x-1)^2} = \frac{A(x-1)(x+2) + B(x-1)^2 + C(x+2)}{(x+2)(x-1)^2}$$

$$x = A(x^2+2x-x-2) + B(x^2-2x+1) + Cx + 2C$$

$$0\cdot x^2 + 0 + 1x^1 = Ax^2 + 2Ax - Ax - 2A + Bx^2 - 2Bx + B + Cx + 2C$$

$$0 = A+B$$

$$1 = 2A - A - 2B + C = A - 2B + C$$

$$0 = -2A + B + 2C$$

$$A = \frac{2}{9}, \quad C = \frac{1}{3}, \quad B = -\frac{2}{9}$$

$$= \int \frac{\frac{2}{9}}{x-1} + \frac{-\frac{2}{9}}{x+2} + \frac{\frac{1}{3}}{(x-1)^2}\,dx = \frac{2}{9}\int\frac{1}{x-1}\,dx - \frac{2}{9}\int\frac{1}{x+2}\,dx +$$

$$\frac{1}{3}\int\frac{1}{(x-1)^2}\,dx = \frac{2}{9}\ln|x-1| - \frac{2}{9}\ln|x+2| - \frac{1}{3(x-1)} + c$$

$$\int \frac{1}{(x-1)^2}\,dx = \left| \begin{matrix} t = x-1 \\ dt = dx \end{matrix} \right| = \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = \frac{t^{-1}}{-1} + c = -\frac{1}{t} + c$$

c) $\int \frac{x^2+x+12}{x^3+7x^2+11x+5}\,dx =$

	1	7	11	5
-5	1	2	1	0

$= \int \frac{x^2+x+12}{(x+5)(x^2+2x+1)}\,dx = \int \frac{x^2+x+12}{(x+5)(x+1)^2}\,dx =$

$\frac{x^2+x+12}{(x+5)(x+1)^2} = \frac{A}{x+5} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ $\quad \left.\begin{matrix} A = 2 \\ B = -1 \\ C = 3 \end{matrix}\right.$

$= \int \frac{2}{x+5} - \frac{1}{x+1} + \frac{3}{(x+1)^2}\,dx = 2\ln|x+5| - \ln|x+1| - \frac{3}{(x+1)} + C$

d) $\int \frac{7-x}{x^3-x^2+3x+5}\,dx =$

	1	-1	3	5
-1	1	-2	5	0

$= \int \frac{7-x}{(x+1)\underbrace{(x^2-2x+5)}}\,dx = \ldots \frac{A}{x+1} + \frac{Bx+C}{x^2-2x+5}$

$D = b^2-4ac = 4-4\cdot5 < 0$

$\ldots A=1;\ B=-1;\ C=2$

$= \int \frac{1}{x+1} + \frac{-x+2}{x^2-2x+5}\,dx = \ln|x+1| - \frac{1}{2}\int \frac{2(x-2)}{x^2-2x+5}\,dx =$

$= \ln|x+1| - \frac{1}{2}\int \frac{2x-\overbrace{4}^{-2-2}}{x^2-2x+5}\,dx = \ln|x+1| - \frac{1}{2}\int \frac{2x-2}{x^2-2x+5} - \frac{2}{x^2-2x+5}\,dx = \ln|x+1| - \frac{1}{2}\ln(x^2-2x+5) + \int \frac{1}{x^2-2x+5}\,dx$

$= \ln|x+1| - \frac{1}{2}\ln(x^2-2x+5) + \int \frac{1}{(x-1)^2\underbrace{-1+5}_{+4 = a^2}}\,dx =$

$= \ln|x+1| - \frac{1}{2}\ln(x^2-2x+5) + \frac{1}{2}\operatorname{arctg}\frac{x-1}{2} + C$