VYPOČÍTAJTE SÚČET RADU $\sum_{n=1}^{\infty} a_n$ POMOCOU POSTUPNOSTI ČIASTOČNÝCH SÚČTOV

ⓐ $$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+4)} = \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+4}$$

$$\frac{A}{n+1} + \frac{B}{n+4} \quad \dots \quad A = \frac{1}{3}\,;\ B = -\frac{1}{3}$$

$$= \frac{1}{3}\Big[\Big(\underbrace{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} + \cancel{\frac{1}{5}} + \cancel{\frac{1}{6}} + \cancel{\frac{1}{7}} + \dots + \cancel{\frac{1}{n+1}}\Big) -$$

$$- \Big(\cancel{\frac{1}{5}} + \cancel{\frac{1}{6}} + \cancel{\frac{1}{7}} + \dots + \cancel{\frac{1}{n+1}} + \underbrace{\frac{1}{n+2} + \frac{1}{n+3} + \frac{1}{n+4}}\Big)\Big]$$

$$s = \lim_{n\to\infty} s_n = \frac{1}{3}\lim_{n\to\infty}\Big(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} \underbrace{- \frac{1}{n+2} - \frac{1}{n+3} - \frac{1}{n+4}}_{=0}\Big)$$

$$= \frac{1}{3}\lim_{n\to\infty} \frac{6+4+3}{12} = \underline{\underline{\frac{13}{36}}}$$

ⓑ $$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = \sum_{n=1}^{\infty} \frac{2}{(n+3)(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+3} =$$

$$\frac{A}{n+3} + \frac{B}{n+1} \quad \dots \quad A = -1, \quad B = 1$$

$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty}\Big[\Big(\underbrace{\frac{1}{2} + \frac{1}{3}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{5}} + \dots + \cancel{\frac{1}{n+1}}\Big) -$$

$$- \Big(\cancel{\frac{1}{4}} + \cancel{\frac{1}{5}} + \dots + \cancel{\frac{1}{n+1}} + \underbrace{\frac{1}{n+2} + \frac{1}{n+3}}\Big)\Big]$$

$$= \lim_{n\to\infty}\Big(\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}\Big) = \underline{\underline{\frac{5}{6}}}$$

(c) $\sum_{n=1}^{\infty}\left(e^{\frac{1}{n}}-e^{\frac{1}{n+2}}\right)$

$$\lim_{n\to\infty}\left[\left(\underbrace{e^{1}+e^{\frac{1}{2}}}+\cancel{e^{\frac{1}{3}}}+\cancel{e^{\frac{1}{4}}}+\dots+\cancel{e^{\frac{1}{n}}}\right)-\left(\cancel{e^{\frac{1}{3}}}+\cancel{e^{\frac{1}{4}}}+\dots\right.\right.$$

$$\left.\left.\dots+\cancel{e^{\frac{1}{n}}}+\underbrace{e^{\frac{1}{n+1}}+e^{\frac{1}{n+2}}}\right)\right]=$$

$$=\lim_{n\to\infty}\left(e+\sqrt{e}\underbrace{-e^{\frac{1}{n+1}}-e^{\frac{1}{n+2}}}_{-2}\right)=\underline{\underline{e+\sqrt{e}-2}}$$

(a) $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots=\sum_{n=1}^{\infty}\frac{1}{(3n-2)(3n+1)}=$

$$\frac{A}{3n-2}+\frac{B}{3n+1}=\frac{3An+A+3Bn-2B}{(3n-2)(3n+1)}=\frac{n(3A+3B)+A-2B}{(3n-2)(3n+1)}$$

$A=\frac{1}{3}$; $B=-\frac{1}{3}$

$(n+1)$

$$=\underline{\frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{3n-2}-\frac{1}{3n+1}}$$

$$=\frac{1}{3}\left[\left(\frac{1}{1}+\cancel{\frac{1}{4}}+\cancel{\frac{1}{7}}+\dots+\cancel{\frac{1}{\underbrace{3(n-1)-2}_{3n-5}}}+\cancel{\frac{1}{3n-2}}\right)-\right.$$

$$\left.-\left(\cancel{\frac{1}{4}}+\cancel{\frac{1}{7}}+\dots+\cancel{\frac{1}{\underbrace{3(n-2)+1}_{3n-5}}}+\cancel{\frac{1}{\underbrace{3(n-1)+1}_{3n-2}}}+\frac{1}{3n+1}\right)\right]=$$

$$\Rightarrow\frac{1}{3}\lim_{n\to\infty}\left(1-\frac{1}{3n+1}\right)=\underline{\underline{\frac{1}{3}}}$$

PR5 ZISTITE, ČI RAD $\sum_{n=1}^{\infty} \frac{1}{n}$ KONVERGUJE

ALEBO DIVERGUJE

$$\sum_{n=1}^{\infty} 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots$$

UROBME POSTUPNOSŤ ČIASTOČNÝCH SÚČTOV

$$s_1 = 1$$

$$s_2 = 1+\frac{1}{2}$$

$$s_4 = 1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right) > 1+\frac{1}{2}+\underbrace{\left(\frac{1}{4}+\frac{1}{4}\right)}_{\frac{1}{2}} = 1+2\frac{1}{2}$$

$$s_8 = 1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right) > \underbrace{1+\frac{1}{2}+\underbrace{\left(\frac{1}{4}+\frac{1}{4}\right)}_{\frac{1}{2}}+\underbrace{\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)}_{\frac{1}{2}}}_{1+3.\frac{1}{2}}$$

$\Downarrow$

MAJORANTNÝ RAD

K

$$\underbrace{\lim_{m\to\infty}\left(1+m.\frac{1}{2}\right)}_{\infty} \Rightarrow \text{DIVERGUJE} \Rightarrow$$

MINORANTNÝ K RADU $\sum_{n=1}^{\infty}\frac{1}{n}$

RAD $\sum_{n=1}^{\infty}\frac{1}{n}$ SA NAZÝVA HARMONICKÝ RAD A

DIVERGUJE !

PR6 DOKÁŽTE, ŽE RAD $\sum_{n=1}^{\infty} a_n$ JE DIVERGENTNÝ

(OVERÍME NUTNÚ PODMIENKU KONVERGENCIE RADU,

T.J. $\lim_{n\to\infty} a_n = 0$)

MUSÍ PLATIŤ

ⓐ $\lim\limits_{n\to\infty} \frac{n-1}{n+1} = 1 \neq 0 \Rightarrow$ RAD $\sum\limits_{n=1}^{\infty} \frac{n-1}{n+1}$ DIVERGUJE

ⓑ $\sum\limits_{n=1}^{\infty} \operatorname{arctg} n \Rightarrow \lim\limits_{n\to\infty} \operatorname{arctg} n = \frac{\pi}{2} \neq 0$

$\Rightarrow$ RAD DIVERGUJE

ⓒ $\sum\limits_{n=1}^{\infty} 2n \Rightarrow \lim\limits_{n\to\infty} 2n = \infty \neq 0$ RAD $\sum\limits_{n=1}^{\infty} 2n$ DIVERGUJE

ⓓ $\sum\limits_{n=1}^{\infty} \frac{3n}{n+1} \Rightarrow \lim\limits_{n\to\infty} \frac{3n}{n+1} = 3 \neq 0 \Rightarrow$ RAD $\sum\limits_{n=1}^{\infty} \frac{3n}{n+1}$ DIVERGUJE

GEOMETRICKÍ RAD

PR9 a) $24+12+6+3+\ldots = 24\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right) =$

$\Rightarrow q = \left|\frac{1}{2}\right| < 1 \Rightarrow$ GR KONVERGUJE

$$\lim_{n\to\infty} s_n = \frac{a_1}{1-q} = \frac{24}{1-\frac{1}{2}} = \frac{24}{\frac{1}{2}} = \underline{\underline{48}}$$

b) $\sqrt{125} + \sqrt{25} + \sqrt{5} + \ldots = \sqrt{125}\left(1 + \sqrt{\frac{25}{125}} + \right.$

$\left. + \sqrt{\frac{5}{125}} + \ldots \right) = \sqrt{125}\left(1 + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}\sqrt{5}} + \ldots\right)$

$q = \left|\frac{1}{\sqrt{5}}\right| < 1 \ldots \Rightarrow$ GR KONVERGUJE

$$\lim_{n\to\infty} s_n = \frac{\sqrt{125}}{1-\frac{1}{\sqrt{5}}} = \frac{\sqrt{125}}{\frac{\sqrt{5}-1}{\sqrt{5}}} =$$

$$= \frac{\sqrt{125}\cdot\sqrt{5}}{\sqrt{5}-1} = \frac{5\sqrt{5}\cdot\sqrt{5}}{\sqrt{5}-1} = \frac{25}{\sqrt{5}-1}$$

(c) $\frac{e}{3}+\frac{e^2}{9}+\frac{e^3}{27}+\dots = \frac{e}{3}\left(1+\frac{e}{3}+\frac{e^2}{9}+\dots\right)$

$q=\frac{e}{3}<1 \Rightarrow$ RAD KONV. $\quad S=\frac{\frac{e}{3}}{1-\frac{e}{3}}=\frac{\frac{e}{3}}{\frac{3-e}{3}}=\underline{\underline{\frac{e}{3-e}}}$

(d) $\sum_{n=1}^{\infty}\frac{2^n}{3^{2n-1}}=\sum_{n=1}^{\infty}\frac{2^n}{3^{2n}\cdot 3^{-1}}=3\sum_{n=1}^{\infty}\frac{2^n}{9^n}=3\sum_{n=1}^{\infty}\left(\frac{2}{9}\right)^n$

$\Downarrow$

$q=\frac{2}{9}<1$

KONV.

$\lim_{n\to\infty}S_n=\frac{a_1}{1-q}=\underline{\underline{\frac{6}{7}}}$

PR12

(a) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+1}$

$\lim_{n\to\infty}\frac{1}{n+1}=0 \Rightarrow$ RAD $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+1}$ KONVERGIRA