Note Title 28/10/2022

MA I 14.12.2017

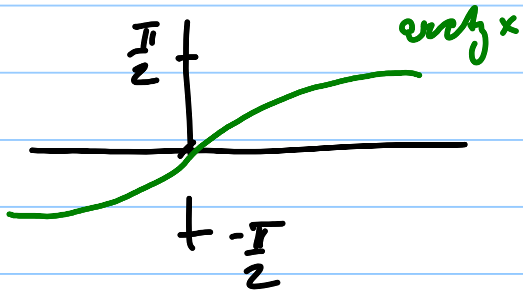
PZ: $f(x) = \operatorname{arctg}\left(\frac{1}{x+5}\right)$ NÁJDI $D(f)$ A f^{-1} AK EXISTUJE

$$D(f) = \{\forall x \in R;\ x+5 \neq 0\} = R - \{-5\}$$

$$\lim_{x\to\infty} \operatorname{arctg}\Big(\underbrace{\frac{1}{x+5}}_{\to 0}\Big) = 0 \qquad \lim_{x\to-\infty} \operatorname{arctg}\Big(\underbrace{\frac{1}{x+5}}_{\to 0}\Big) = 0$$

$$\lim_{x\to-5^-} \operatorname{arctg}\Big(\underbrace{\frac{1}{\underbrace{x+5}_{\to 0^-}}}_{\to -\infty}\Big) = -\frac{\pi}{2}$$

$$\lim_{x\to-5^+} \operatorname{arctg}\Big(\underbrace{\frac{1}{\underbrace{x+5}_{\to 0^+}}}_{\to \infty^+}\Big) = \frac{\pi}{2}$$



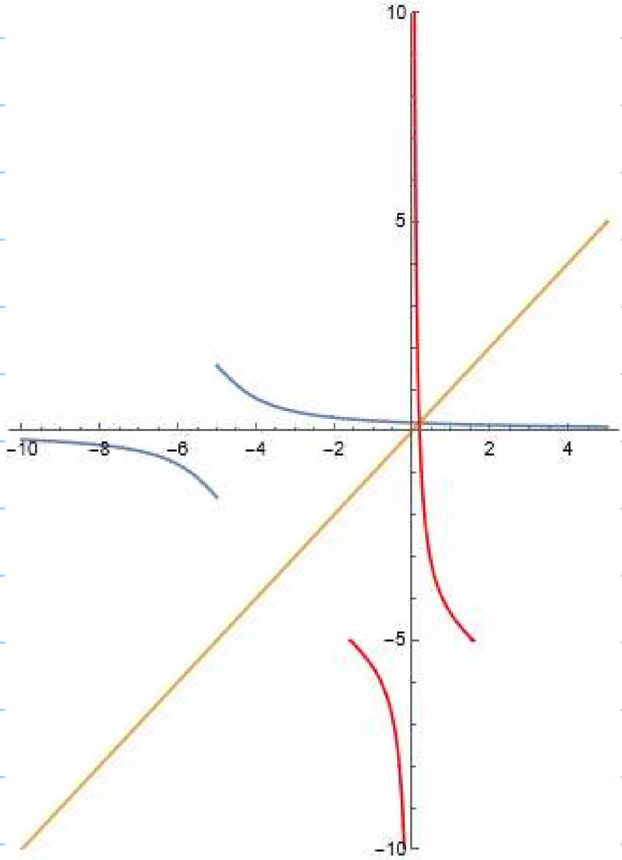
f^{-1}: $x = \operatorname{arctg} \frac{1}{y+5}$ $\quad / \operatorname{tg}$

$\operatorname{tg} x = \frac{1}{y+5}$ $\quad / \cdot (y+5)$

$(y+5)(\operatorname{tg} x) = 1$ $\quad / : \operatorname{tg} x$

$y+5 = \frac{1}{\operatorname{tg}(x)}$ $\quad / -5$

f^{-1}: $y = \frac{1}{\operatorname{tg} x} - 5$ $\qquad \forall x \in R;\ x \neq 0,\ x \neq -5$



2) $$\lim_{x\to 0}\left(\frac{\sin 5x}{\sqrt{x+2}+2}+4^{\sin x}\right)=$$

$$=\lim_{x\to 0}\frac{\overbrace{\sin 5x}^{\to 0}}{\underbrace{\sqrt{x+2}}_{\sqrt{2}}+2}+\lim_{x\to 0}4^{\overbrace{\sin x}^{\to 0}}=\frac{0}{\sqrt{2}+2}+1=1$$

$$\lim_{x\to 0}4^{\sin x}=4^0$$

$$\lim_{x\to 0}\left(\frac{\sin 5x}{\sqrt{x+2}-2}+4^{\sin x}\right)=$$

$$\lim_{x\to 0}\frac{\sin 5x}{\sqrt{x+2}-2}+\lim_{x\to 0}\underbrace{4^{\sin x}}_{\to 1}=\frac{0}{\sqrt{2}-2}+1=1$$

3) a) $f(x) = 5e^{\sin^2 x}$

$$f' = 5 \cdot e^{\sin^2 x} \cdot 2 \cdot \sin x \cdot \cos x$$

b) $f(x) = (2x)^{\operatorname{arctg} x}$

$$(f^g)' = f^g (g \ln f)' = f^g \left(g' \ln f + g \cdot \frac{1}{f} \cdot f' \right)$$

$$f'(x) = (2x)^{\operatorname{arctg} x} \cdot \left(2 \cdot \ln \operatorname{arctg} x + (2x) \cdot \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{1+x^2} \right.$$

4) NÁJDI t a n KU $f(x) = 2x^2 + 2x + 1$

$t \parallel p: \; y = -2x + 4$

$$k_t = k_p = -2$$

t: $y - f(x_0) = f'(x_0)(x - x_0)$

n: $y - f(x_0) = \frac{-1}{f'(x_0)}(x - x_0)$

$$f' = 4x + 2$$

$$f' = -2 \Rightarrow \quad 4x + 2 = -2 \quad /-2$$

$$4x = -4 \Rightarrow x = -1$$

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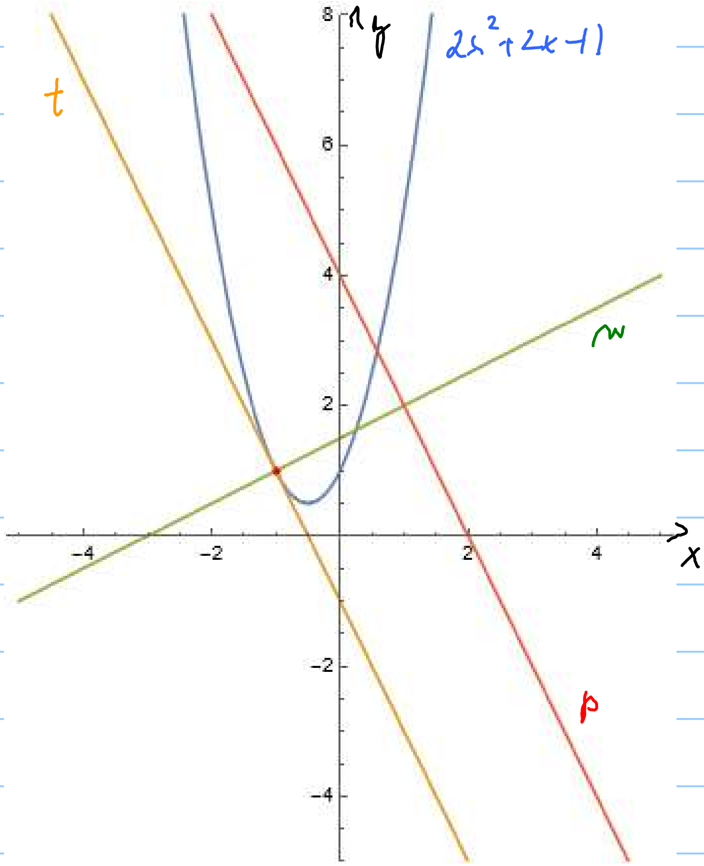
$$f(-1) = 2(-1)^2 + 2\cdot(-1) + 1 = 1$$

t: $y - 1 = -2(x+1) \Rightarrow y = -2x - 1$

n: $y - 1 = \frac{-1}{-2}(x+1) \Rightarrow y = \frac{1}{2}x + \frac{3}{2}$



y
$2x^2+2x+1$
t
m
p
x
8
6
4
2
-2
-4
-4
-2
2
4