PRIEBEH FUNKCIE

$$\boxed{f(x) = x \ln(x^2)}$$

① $D(f)$

$x^2 > 0$

$|x| > 0$

$$\boxed{D(f) = \mathbb{R} - \{0\}}$$

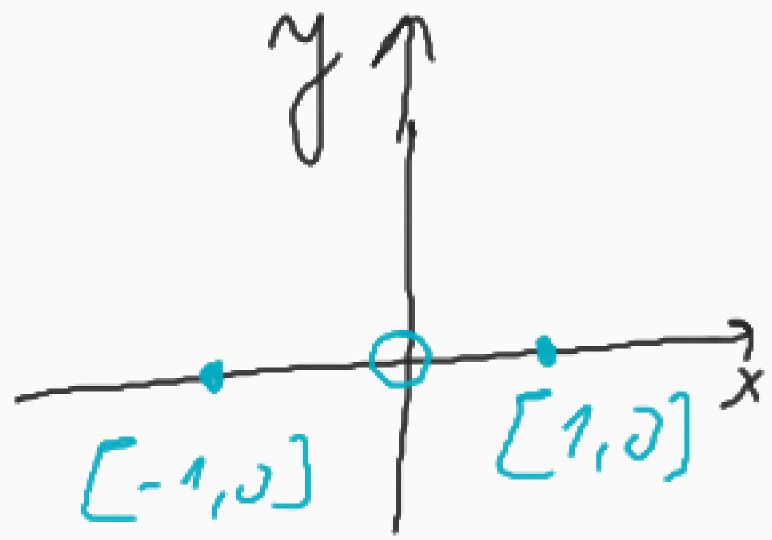
② NIE JE PERIODICKÁ; JE SPOJITÁ

$$f(x) = x \ln(x^2)$$

$$f(-x) = -x \ln((-x)^2) = -x \ln(x^2)$$

$$f(x) = -f(-x) \Rightarrow$$ **FUNKCIA JE NEPÁRNA**

③ $f(x) = x \ln(x^2)$ $\qquad x \neq 0$



$$y = 0 \iff 0 = x \ln(x^2)$$

$$0 = \ln(x^2)$$

$$\ln 1 = \ln(x^2)$$

$$x^2 = 1 \Rightarrow x = \pm 1$$

$NB_1[-1,0]$

$NB_2[1,0]$

④ $\boxed{\text{ABS}}$

$0\ (-\infty)$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x \ln(x^2) =$$

$$= \lim_{x \to 0^+} \frac{\ln(x^2)}{x^{-1}} \overset{L'H}{=} \lim_{x \to 0^+} \frac{\frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x}}{-1x^{-2}} =$$

$$= \lim_{x \to 0^+} \frac{\frac{2}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} \frac{-2x^{\cancel{2}}}{\cancel{x}} = \underline{\underline{0}} \quad \cancel{\exists}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x \ln(x^2) =$$

$$= \lim_{x \to 0^-} \frac{\ln x^2}{x^{-1}} = \lim_{x \to 0^-} -2x = \underline{\underline{0}} \quad \cancel{\exists}$$

$\boxed{\text{ASS}}$

$$\boxed{y = kx + q}$$

$$k_1 = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{\cancel{x} \ln(x^2)}{\cancel{x}} = \infty \quad \nexists$$

$$k_2 = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{\cancel{x} \ln(x^2)}{\cancel{x}} = \infty \quad \nexists$$

5

$$f(x) = x\ln(x^2)$$

$$\boxed{f'(x)} = 1\ln(x^2) + \cancel{x}\cdot\frac{1}{\cancel{x^2}}\cdot 2\cancel{x} = \boxed{2+\ln(x^2)}$$

$$f'(x) > 0 \iff 2+\ln(x^2) > 0$$

$$\ln(x^2) > -2$$

$$e^{\ln(x^2)} > e^{-2}$$

$$x^2 > \frac{1}{e^2} \Rightarrow |x| > \frac{1}{e}$$

+		−		−		+
↗	$-\frac{1}{e}$	↘	0	↘	$\frac{1}{e}$	↗

FUNKCIA JE RÝDZO RASTÚCA NA INTERVALOCH

$\left(-\infty, -\frac{1}{e}\right)$ a $\left(\frac{1}{e}, \infty\right)$ A RÝDZO KLESAJÚCA

$\left(-\frac{1}{e}, 0\right)$ a $\left(0, \frac{1}{e}\right)$.

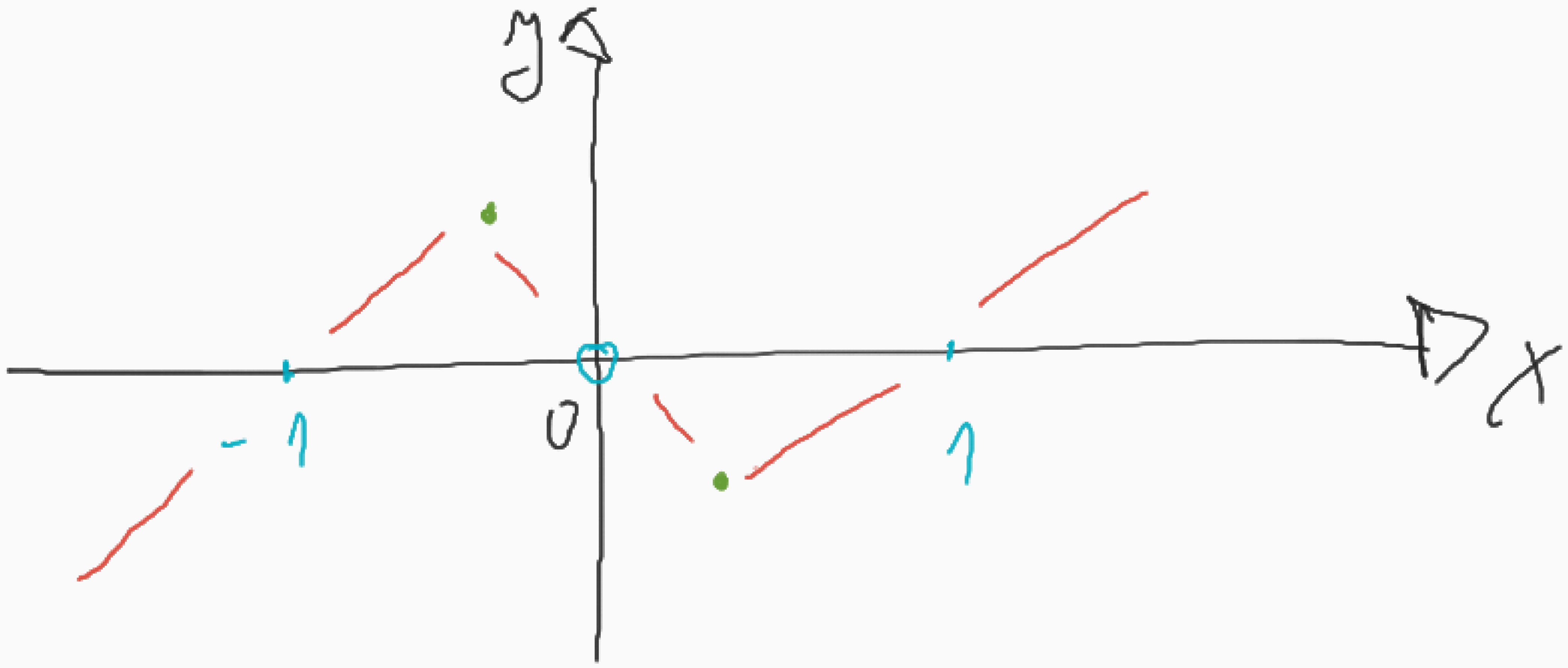
$$f\left(-\frac{1}{e}\right) = -\frac{1}{e} \ln\left(\left(-\frac{1}{e}\right)^2\right) = -\frac{1}{e} \cdot \ln e^{-2} = -\frac{1}{e} \cdot (-2) =$$

$$= \frac{2}{e} \quad \text{A} \quad \boxed{\left[-\frac{1}{e}, \frac{2}{e}\right] \Rightarrow \text{LOK MAX}}$$

$$f\left(\frac{1}{e}\right)=\frac{1}{e}\ln\left(\frac{1}{e}\right)^2=\frac{1}{e}\ln e^{-2}=\frac{1}{e}\cdot(-2)\ln e$$

$$=-\frac{2}{e}\qquad B\left[\frac{1}{e},-\frac{2}{e}\right]$$

$0.3 \quad -0,7$



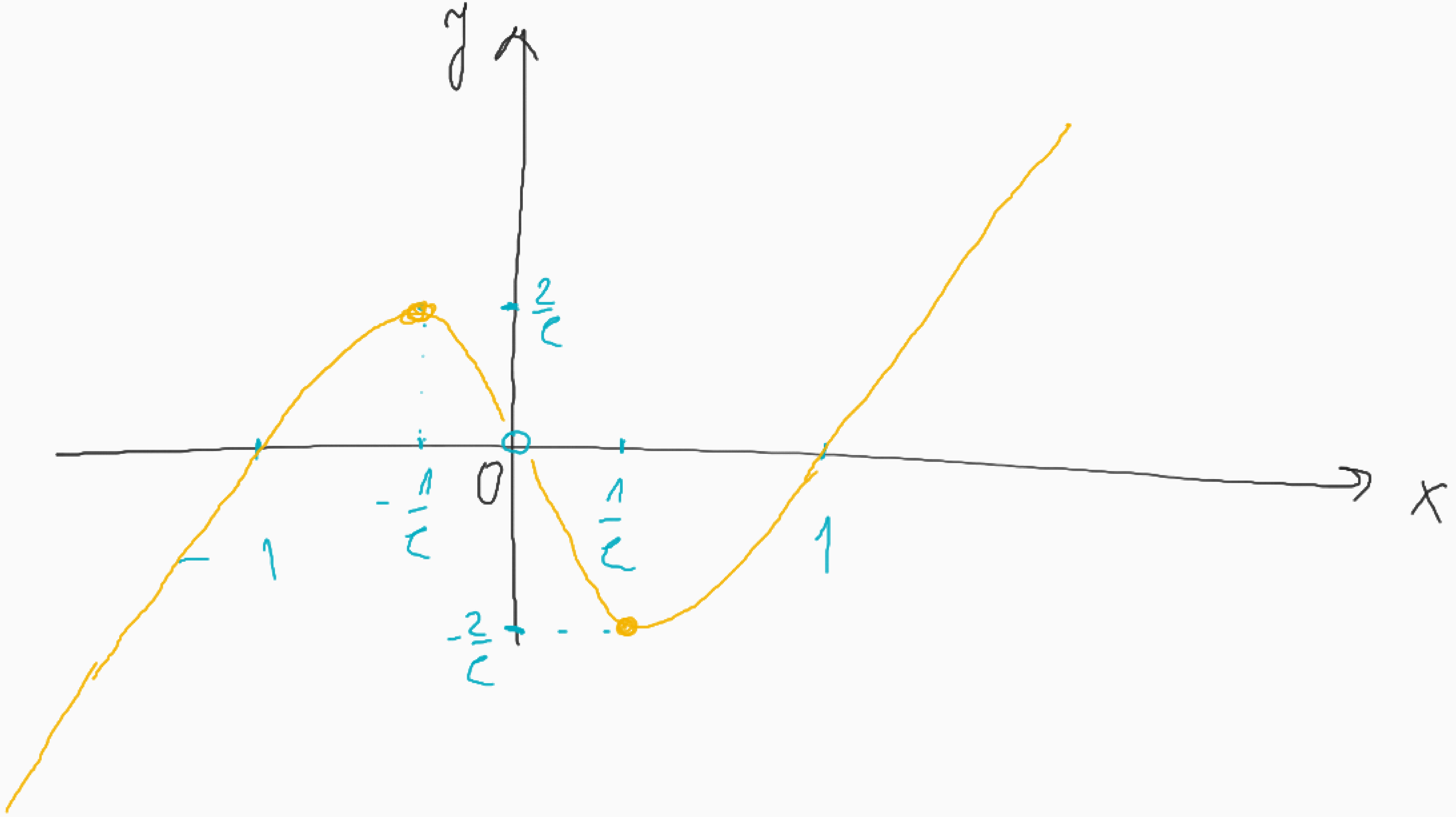
(6) $f''(x) = \left[2+\ln(x^2)\right]' = \frac{1}{x^2}\cdot 2x = \frac{2}{x}$

$(-\infty,0)$	$(0,\infty)$
$-$	$+$
$\cap$	$\cup$

FUNKCIA JE KONVEXNÁ NA $(0,\infty)$ A KONKÁVNA $(-\infty,0)$.

$f''\left(-\frac{1}{e}\right) = \frac{\frac{2}{1}}{-\frac{1}{e}} = -2e < 0 \Rightarrow$ LOK. MAX

$f''\left(\frac{1}{e}\right) = \frac{\frac{2}{1}}{\frac{1}{e}} = 2e > 0 \Rightarrow$ LOK MIN


y
x
0
$\frac{2}{e}$
$-\frac{2}{e}$
$-\frac{1}{e}$
$\frac{1}{e}$
-1
1

5 $f(x) = e^{-x^2}$

1 $\boxed{D(f) = \mathbb{R}}$

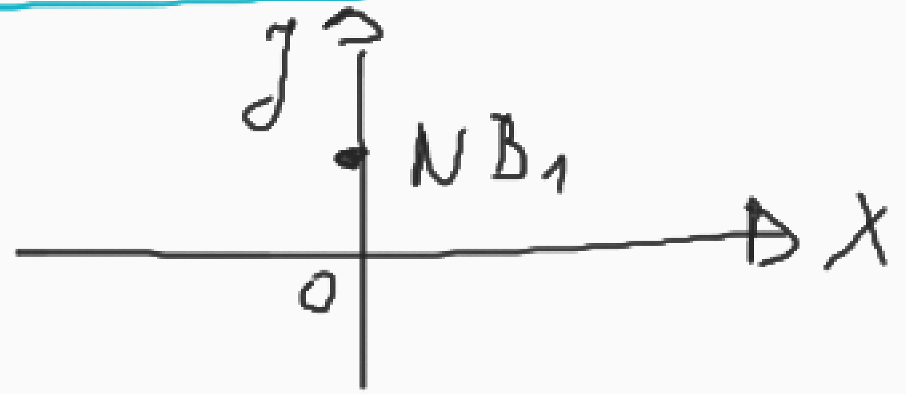
2 $$\left.\begin{aligned} f(x) &= e^{-x^2} \\ f(-x) &= e^{-(-x)^2} = e^{-x^2} \end{aligned}\right\} f(x) = f(-x)$$

$\boxed{\text{PÁRNA}}$, SPOJITÁ

NIE JE PERIODICKÁ

3 $x = 0 \iff y = e^0 = 1$ $\boxed{NB_1\,[0,1]}$

$y = 0 \iff 0 \neq e^{-x^2}$ $\nexists$



4 $\boxed{ABS}$ $\nexists$ (lebo $D(f)$)

$\boxed{ASS}$ $y = kx + q$

$$k_1 = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{e^{-x^2}}{x} = \lim_{x \to \infty} \frac{1}{x e^{x^2}}$$

$$= \underline{\underline{0}}$$

$$q_1 = \lim_{x \to \infty} \left(f(x) - k_1 x \right) = \lim_{x \to \infty} \left(e^{-x^2} - 0 \right) =$$

$$= \lim_{x \to \infty} \frac{1}{e^{x^2}} = 0 \qquad y = k_1 x + q_1 = 0$$

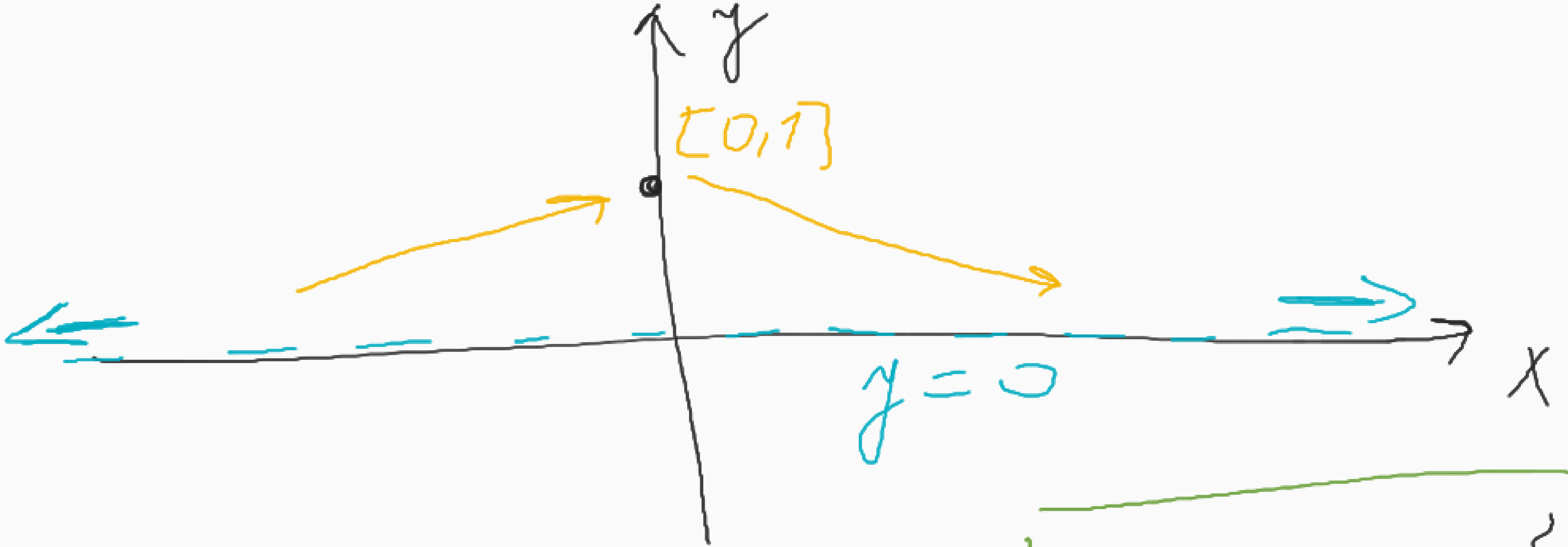
$$\boxed{y = 0}$$

$$k_2 = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{e^{-x^2}}{x} =$$

$$= \lim_{x \to -\infty} \frac{1}{x e^{x^2}} = \underline{\underline{0}}$$

$$\boxed{y = 0}$$

$$q_2 = \lim_{x \to -\infty} (f(x) - k_2 x) = \lim_{x \to -\infty} \frac{1}{e^{x^2}} = \underline{\underline{0}}$$



(5) $f(x) = e^{-x^2}$

$$\boxed{f'(x) = e^{-x^2} \cdot (-2x)}$$

$f'(x) = 0 \Leftrightarrow e^{-x^2} \cdot (-2x) = 0$

$x = 0$

$(-\infty, 0)$	$(0, \infty)$
+	−
↗	↘

V BODE [0,1] MÁ FUNKCIA
LOK. MAX.

⑥ $f''(x) = \left[e^{-x^2} \cdot (-2x)\right]' =$

$= e^{-x^2} \cdot (-2x) \cdot (-2x) + e^{-x^2} \cdot (-2) =$

$= e^{-x^2}(4x^2 - 2) = \boxed{e^{-x^2} \cdot 2(2x^2 - 1)}$

$f''(0) \Leftrightarrow 2x^2 - 1 = 0 \Rightarrow x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

$(-\infty, -\frac{\sqrt{2}}{2})$	$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$	$(\frac{\sqrt{2}}{2}, \infty)$
+	−	+

INF. BOD

$x_1 = -\frac{\sqrt{2}}{2}$

$x_2 = \frac{\sqrt{2}}{2}$

$f''(0) = e^0 \cdot 2(-1) < 0 \Rightarrow$ V BODE $x = 0$ LOK MAX.