Matematická analýza Cvičenie 8 ZS 8.11.2022 10:00

Note Title 08/11/2022

NEURČITÝ INTEGRÁL

I. SUBSTITÚCIA

PR: $\int \frac{\overbrace{\arcsin^2 x}^{t}}{\sqrt{1-x^2}}\,\overbrace{dx}^{dt}$ $\left| \begin{array}{l} \arcsin x = t \\ \frac{1}{\sqrt{1-x^2}}\,dx = 1\cdot dt \end{array} \right|$ =

$$= \int t^2\,dt = \frac{t^3}{3} = \frac{\arcsin^3 x}{3} + c$$

II. PER PARTES

$$= \left| \begin{array}{ll} u = \arcsin^2 x & u' = 2\arcsin x \cdot \frac{1}{\sqrt{1-x^2}} \\ v' = \frac{1}{\sqrt{1-x^2}} & v = \int \frac{1}{\sqrt{1-x^2}}\,dx \end{array} \right|$$

$\int \sqrt{x}\,\ln(2x)\,dx$ $\left| \begin{array}{ll} u' = \sqrt{x} & w = \int x^{\frac{1}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2\sqrt{x^3}}{3} \\ v = \ln(2x) & v' = \frac{1}{2x}\cdot 2 \end{array} \right|$

$$= \frac{2}{3}\sqrt{x^3}\cdot\ln(2x) - \int \frac{2}{3}\sqrt{x^3}\cdot\frac{1}{x}\,dx =$$

$$= \frac{2}{3}\sqrt{x^3}\cdot\ln(2x) - \frac{2}{3}\int \sqrt{\frac{x^3}{x^2}}\,dx =$$

$$= \frac{2}{3}\sqrt{x^3}\cdot\ln(2x) - \frac{2}{3}\cdot\frac{2}{3}\sqrt{x^3} + c =$$

$$\frac{2}{3}\sqrt{x^3}\left(\ln(2x) - \frac{2}{3}\right) + c$$

Pr.: $I = \int \frac{x^3+3}{x^2-3x}\,dx =$

$(x^3+3) : (x^2-3x) = x+3$

$-x^3+3x^2$

$0+3x^2$

$-3x^2+9x$

$9x+3$ zvyšok

$= \int x+3 + \frac{9x+3}{x^2-3x}\,dx = \underbrace{\int x+3\,dx}_{I.} + \underbrace{\int \frac{9x+3}{x(x-3)}\,dx}_{II.}$

I. $\int x+3\,dx = \frac{x^2}{2} + 3x + c$

II. $\frac{9x+3}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3} \quad / \cdot x(x-3)$

$9x+3 = A(x-3) + Bx$

$9x+3 = Ax - 3A + Bx$

$9 = A+B$

$3 = -3A \Rightarrow A = -1 \Rightarrow 9 = -1+B \Rightarrow B = 10$

$\int \frac{9x+3}{x(x-3)}\,dx = \underbrace{\int \frac{-1}{x}\,dx}_{IV.} + \underbrace{\int \frac{10}{x-3}\,dx}_{III.} =$

III. $\left| \begin{matrix} x-3 = t \\ dx = dt \end{matrix} \right| = 10\int \frac{1}{t}\,dt = 10 \ln|t| = 10\ln|x-3|$

$I = I. + II. = I. + III. + IV. = \underbrace{\frac{x^2}{2}}_{I.} + 3x - \underbrace{\ln|x|}_{IV.} + \underbrace{10\ln|x-3|}_{III.} + c$

$= \frac{x^2}{2} + 3x + \ln\frac{|x-3|^{10}}{|x|} + c$