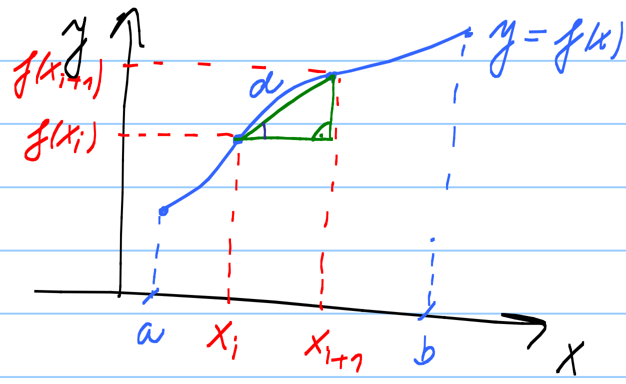
DĹŽKA ROVINNEJ KRIVKY



$$d \approx \sqrt{(x_{i+1}-x_i)^2 + (f(x_{i+1}) - f(x_i))^2}$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

$$\Rightarrow f(x_{i+1}) - f(x_i) = f'(x_i)(x_{i+1}-x_i)$$

$$d \approx \sqrt{(x_{i+1}-x_i)^2 + (f'(x_i))^2 \cdot (x_{i+1}-x_i)^2} = \underbrace{(x_{i+1}-x_i)}_{dx} \cdot \sqrt{1+(f'(x_i))^2}$$

$$D = \sum_{i=1}^{n} \sqrt{1+f'(x_i)^2} \cdot (x_{i+1}-x_i) \approx \boxed{\int_a^b \sqrt{1+f'(x_i)^2}\, dx}$$

PR7 VYPOČÍTAJTE DĹŽKY DANÝCH KRIVIEK

a) $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$; $x \in \langle 0,3 \rangle$

$$y' = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} (x^2+2)^{\frac{1}{2}} \cdot \cancel{2}x = x \sqrt{x^2+2}$$

$$D = \int_0^3 \sqrt{1 + (x\sqrt{x^2+2})^2}\, dx = \int_0^3 \sqrt{1+x^2(x^2+2)}\, dx = \int_0^3 \sqrt{1+x^4+2x^2}\, dx$$

$$= \int_0^3 \sqrt{(1+x^2)^2}\, dx = \int_0^3 1+x^2\, dx = \left[x + \frac{x^3}{3}\right]_0^3 = 3+9 = 12$$

b) $y = \frac{2}{3} x \sqrt{x}$; $x \in \langle 0,1 \rangle$

$$y = \frac{2}{3} x^{\frac{3}{2}} \Rightarrow y' = \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} x^{\frac{1}{2}}$$

$$D = \int_0^1 \sqrt{1 + (x^{\frac{1}{2}})^2}\, dx = \int_0^1 \sqrt{1+x}\, dx = \left| \begin{array}{l} t = 1+x \\ dt = dx \\ x_1 = 0 \Rightarrow t_1 = 1+0 = 1 \\ x_2 = 1 \Rightarrow t_2 = 1+1 = 2 \end{array} \right|$$

$$= \int_1^2 \sqrt{t}\, dt = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^2 = \frac{2}{3} \left[t^{\frac{3}{2}} \right]_1^2 = \underline{\underline{\frac{2}{3} \left[2\sqrt{2} - 1 \right]}} ;$$

ⓒ $y = \ln(\sin x) \qquad x \in \left\langle \frac{\pi}{3} ; \frac{\pi}{2} \right\rangle$

$$y' = \frac{1}{\sin x} \cdot \cos x \qquad D = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{1 + \frac{\cos^2 x}{\sin^2 x}}\, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}\, dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x}\, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x} \cdot \frac{\sin x}{\sin x}\, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1 - \cos^2 x}\, dx =$$

$$= \left| \begin{array}{l} t = \cos x \\ dt = -\sin x\, dx \\ x_1 = \frac{\pi}{3} \Rightarrow t_1 = \frac{1}{2} \\ x_2 = \frac{\pi}{2} \Rightarrow t_2 = 0 \end{array} \right| = \int_{\frac{1}{2}}^{0} \frac{-dt}{1 - t^2} \qquad \boxed{\int \frac{1}{1-x^2}\, dx = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + c}$$

$$= -\frac{1}{2} \left[\ln \left| \frac{1+t}{1-t} \right| \right]_{\frac{1}{2}}^{0} = \overbrace{-\frac{1}{2} \ln 1}^{0} + \frac{1}{2} \ln \left| \frac{\frac{3}{2}}{\frac{1}{2}} \right| =$$

$$= \frac{1}{2} \ln \left| \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{1} \right| = \underline{\underline{\frac{1}{2} \ln 3}} ;$$

ⓓ $y = \frac{x^2}{4} - \frac{1}{2} \ln x \quad ; \quad x \in \langle 1, 2 \rangle$

$$y' = \frac{2x}{4} - \frac{1}{2} \cdot \frac{1}{x} = \frac{x}{2} - \frac{1}{2x} \Rightarrow (y')^2 = \left(\frac{x}{2} - \frac{1}{2x} \right)^2 = \frac{x^2}{4} - \frac{\cancel{2x}}{\cancel{2}} \cdot \frac{1}{\cancel{2x}} + \frac{1}{4x^2}$$

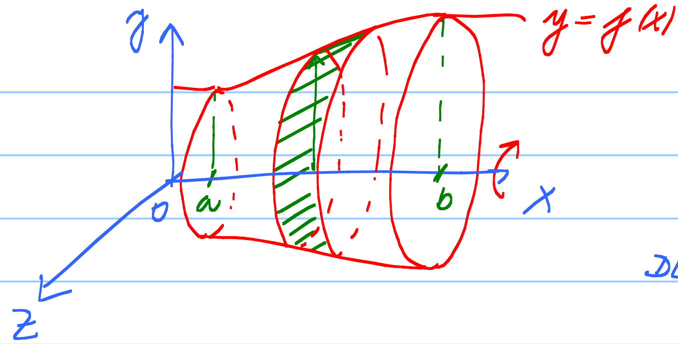
$$= \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}$$

$$D = \int_1^2 \sqrt{1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}}\, dx = \int_1 \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}}\, dx = \int_1 \sqrt{\frac{x^4 + 2x^2 + 1}{4x^2}}\, dx$$

$$= \int_1^2 \frac{\sqrt{(x^2+1)^2}}{2x}\, dx = \int_1^2 \frac{x^2+1}{2x}\, dx = \int_1 \frac{x}{2} + \frac{1}{2x}\, dx =$$

$$= \left[\frac{x^2}{4} + \frac{1}{2} \ln |x| \right]_1^2 = \frac{4}{4} + \frac{1}{2} \ln 2 - \frac{1}{4} - \underbrace{\frac{1}{2} \ln 1}_{0} = \underline{\underline{\frac{3}{4} + \frac{1}{2} \ln 2}} ;$$

OBSAH POVRCHU ROTAČNEJ PLOCHY



OBVOD KRUHU: $O = 2\pi r$

NAŠE "r" JE "$f(x_i)$"

DĹŽKA KRIVKY: $D = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

OBSAH POVRCHU: (OBVOD KRUHU) x (DĹŽKA KRIVKY)

$$\boxed{S = 2\pi \int_a^b |f(x)| \sqrt{1+(f'(x))^2}\,dx}$$