Matematická analýza cvičenie 3 ZS 2022-2023
piatok 7.9. 10:00 Live

Note Title 07/10/2022

DERIVÁCIE

$$(ax^n)' = a n x^{n-1} \qquad a, x, n \in \mathbb{R}$$

$$(a)' = 0$$

PR: $y = 6x^5 + 3\sqrt{x} + \frac{2x^3}{\sqrt{x}} - \frac{1}{2}x + 4$

$$y' = 5 \cdot 6 \cdot x^{5-1} + \frac{1}{2} 3 x^{\frac{1}{2}-1} + 2 \cdot \frac{5}{2} x^{\frac{5}{2}-1} - \frac{1}{2} \cdot 1 x^{1-1} + 0$$

$$\boxed{\frac{2x^3}{\sqrt{x}} = 2x^{3-\frac{1}{2}} = 2x^{\frac{5}{2}}} \qquad = 30x^4 + \frac{3}{2\sqrt{x}} + 5x^{\frac{3}{2}} - \frac{1}{2}$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

PR: $y = \underbrace{\operatorname{tg} x}_{f} \cdot \underbrace{\arcsin x}_{g}$

$$y' = \underbrace{\frac{1}{\cos^2 x}}_{f'} \cdot \underbrace{\arcsin x}_{g} + \underbrace{\operatorname{tg} x}_{f} \cdot \underbrace{\frac{1}{\sqrt{1-x^2}}}_{g'}$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f g'}{g^2}$$

PR: $y = \frac{e^x}{x^2 + 4x}$

$$y' = \frac{e^x \cdot (x^2+4x) - e^x(2x+4)}{(x^2+4x)^2}$$

$$(f(g))' = f'(g) \cdot g'$$

Př: $y = \ln \sqrt{2x}$

$$y' = \frac{1}{\sqrt{2x}} \cdot \left(\sqrt{2x}\right)' = \frac{1}{\sqrt{2x}} \cdot \left(\cancel{\tfrac{1}{2}}(2x)^{-\frac{1}{2}} \cdot \cancel{2}\right)$$

$$= \frac{1}{\sqrt{2x}} \cdot \frac{1}{\sqrt{2x}} = \frac{1}{2x}$$

$$(f^g)' = \left(e^{\ln f^g}\right)' = e^{\ln f^g}\left(\ln f^g\right)' = f^g\left(g \ln f\right)' =$$

$$= f^g\left(g' \cdot \ln f + g \cdot \frac{1}{f} \cdot f'\right)$$

Př: $y = (\cos x)^{\ln x}$

$$y' = (\cos x)^{\ln x}\left(\ln x \cdot \ln \cos x\right)' =$$

$$= (\cos x)^{\ln x}\left(\frac{1}{x} \cdot \ln \cos x + \ln x \cdot \frac{1}{\cos x} \cdot (-\sin x)\right)$$

PR: $y = \sqrt{x^2-4} \cdot \sin 2x$

$$y' = \frac{1}{2}(x^2-4)^{-\frac{1}{2}} \cdot 2x \cdot \sin 2x + \sqrt{x^2-4} \cdot \cos 2x \cdot 2$$

$y = \cos^2 \frac{x}{2} \cdot \ln(1-x)$

$$y' = 2\cos\frac{x}{2}\left(-\sin\frac{x}{2}\right) \cdot \frac{1}{2} \cdot \ln(1-x) + \cos^2\frac{x}{2} \cdot \frac{1}{1-x} \cdot (-1)$$

PR: $y = \dfrac{e^{\sin 3x}}{\operatorname{arccotg}\left(\frac{x+3}{2x^3}\right)}$

PR:

$y = x^{x^x}$

$$y' = x^{x^x}\left(x^x \ln x\right)' = x^{x^x}\left(x^x \cdot (x \ln x)' \cdot \ln x + x^x \cdot \frac{1}{x}\right)$$

$$= x^{x^x} x^x \left(1 \cdot \ln x + x \cdot \frac{1}{x} + \frac{1}{x}\right) = x^{x^x} x^x \left(\ln x + 1 + \frac{1}{x}\right)$$

PR: $y = (\arcsin 3x^2)^{\sqrt{1-x}}$

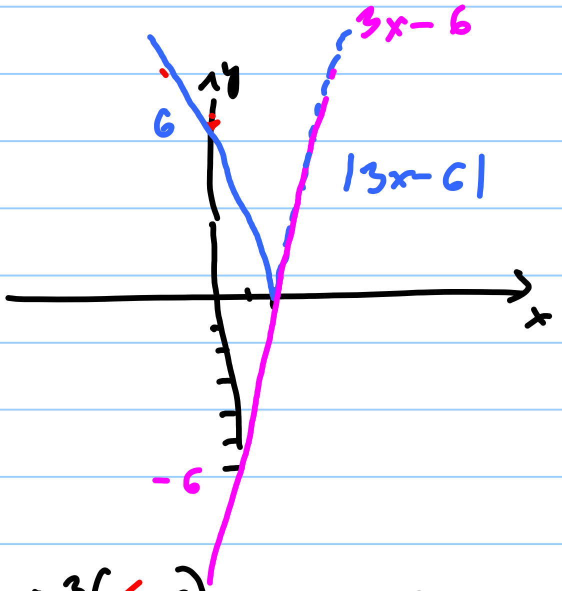
$$y' = (\arcsin 3x^2)^{\sqrt{1-x}}\left(\sqrt{1-x}\,\ln \arcsin 3x^2\right)'$$

$$= (\arcsin 3x^2)^{\sqrt{1-x}}\left(\frac{1\,(-1)}{2\sqrt{1-x}} \ln \arcsin 3x^2 + \right.$$

$$\left. \sqrt{1-x} \cdot \frac{1}{\arcsin 3x^2} \cdot \frac{1}{\sqrt{1-(3x^2)^2}} \cdot 6x\right)$$

PR: 2 NÁJDI DERIVÁCIU FUNKCIE POMOCOU DEFIN. DERIVÁCIE V BODE a.

$f(x) = |3x-6|$ $\quad a=2$



$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to 2^-} \frac{-3x+6-(-3\cdot 2+6)}{x-2} =$$

$$= \lim_{x\to 2^-} \frac{-3x+6-0}{x-2} = \lim_{x\to 2^-} \frac{-3\cancel{(x-2)}}{\cancel{x-2}} = -3$$

$$\lim_{x\to 2^+} \frac{3x-6-(3\cdot 2-6)}{x-2} = \lim_{x\to 2} \frac{3(x-2)}{x-2} = 3$$

$\Rightarrow \nexists$

PR: $f(x) = \begin{cases} x\sin x & x\le 0 \\ x^2 & x>0 \end{cases}$ $\quad a=0$

$$\lim_{x\to 0^-} \frac{x\cdot\sin x - 0\cdot\sin 0}{x-0} = \lim_{x\to 0} \frac{\cancel{x}\sin x}{\cancel{x}} =$$

$$= \lim_{x\to 0} \sin x = 0$$

$$\lim_{x\to 0^+} \frac{x^2-0^2}{x-0} = \lim_{x\to 0^+} \frac{x^2}{x} = \lim_{x\to 0^+} x = 0$$

DOTYČNICA A NORMÁLA KU GRAFU FUNKCIE

PR. NÁJDI t a n $f(x) = x + e^{-x}$ v $A[0, ?]$

$t: \ y - f(x_0) = f'(x_0)(x - x_0)$

$n: \ y - f(x_0) = \dfrac{-1}{f'(x_0)}(x - x_0)$

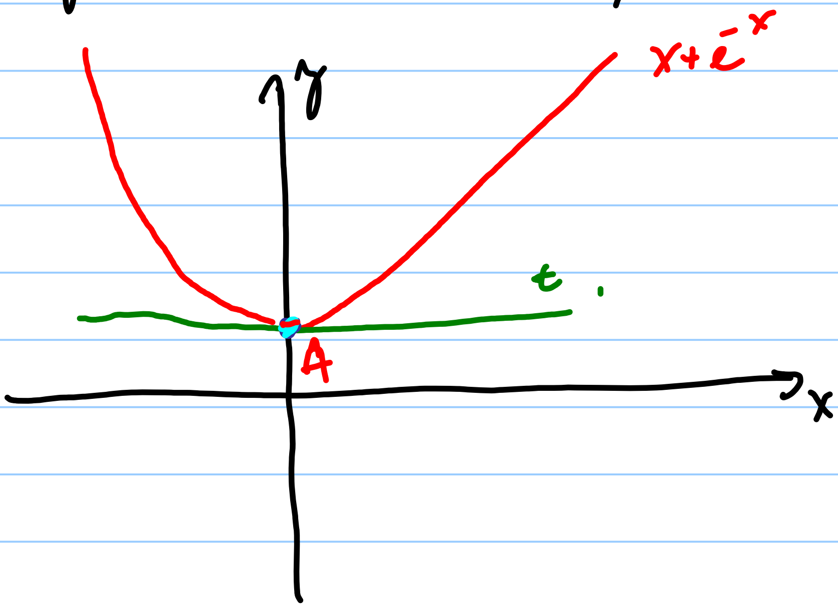
$f(x_0) = ?$ $\qquad f(x_0) = 0 + e^{-0} = 1 \Rightarrow A[0,1]$

$f'(x_0) = ?$ $\qquad f'(x) = 1 + e^{-x} \cdot (-1)$

$\qquad f'(x_0) = 1 + e^{0} \cdot (-1) = 0$

$t: \ y - 1 = 0 \cdot (x - 0) \Rightarrow y = 1$

$n: \ y - 1 = \dfrac{-1}{0} \cdot (x - 0)$ $\quad \nexists$



PR: $f(x) = 16x(x-1)^3 \quad v \quad A[1, ?]$

PR: $f(x) = x^2 - 7x + 3 \qquad t \perp p \qquad p: 3x - y + 2 = 0$

$p: \; y = 3x + 2 \Rightarrow k_p = 3 \Rightarrow k_t = -\frac{1}{3} \quad t \perp p$

$f'(x) = 2x - 7 = k_t = -\frac{1}{3}$

$$2x - 7 = -\frac{1}{3} \quad / + 7$$

$$2x = -\frac{1}{3} + 7 = \frac{20}{3} \quad / : 2$$

$$x = \frac{20}{6} \quad \Rightarrow A\left[\frac{20}{6}, ?\right]$$

$$f\left(\frac{20}{6}\right) = \left(\frac{20}{6}\right)^2 - 7 \cdot \frac{20}{6} + 3 = \frac{400 - 840 + 108}{36} = \frac{-332}{36}$$

$$= \frac{-166}{18} = \frac{-83}{9} \qquad A\left[\frac{20}{6}, -\frac{83}{9}\right]$$

$$t: \; y + \frac{83}{9} = -\frac{1}{3}\left(x - \frac{20}{6}\right)$$

$$n: \; y + \frac{83}{9} = \frac{-1}{-\frac{1}{3}}\left(x - \frac{20}{6}\right) \Rightarrow y + \frac{83}{9} = 3\left(x - \frac{20}{6}\right)$$

ASYMPTOTY KU GRAFU FUNKCIE:

ASYMPTOTA BEZ SMERNICE $x = a$

$$\left.\begin{aligned}\lim_{x\to a^+} f(x) &= \\ \lim_{x\to a^-} f(x) &= \end{aligned}\right\} \pm\infty \Rightarrow \exists \text{ ABS}$$

ASYMPTOTA SO SMERNICOU. $y = kx + q$

$$k_1 = \lim_{x\to\infty} \frac{f(x)}{x} \qquad k_2 = \lim_{x\to-\infty} \frac{f(x)}{x}$$

$$q_1 = \lim_{x\to\infty} (f(x) - k_1 x) \qquad q_2 = \lim_{x\to-\infty} (f(x) - k_2 x)$$

PR:

$$y = \frac{-x^3 + 2x + 3}{x(x+5)}$$

OR: $x \neq 0 \lor x + 5 \neq 0$

$OR = R - \{0; -5\}$

ABS

$$\left.\begin{aligned}\lim_{x\to -5^-} \frac{\overbrace{-x^3+2x+3}^{\to 118}}{\underbrace{x}_{\to -5}(\underbrace{x+5}_{\to 0^-})} &= +\infty \\ \lim_{x\to -5^+} \frac{\overbrace{-x^3+2x+3}^{\to 118}}{\underbrace{x}_{\to -5}(\underbrace{x+5}_{\to 0^+})} &= -\infty\end{aligned}\right\} \Rightarrow x = -5$$

$$\lim_{x\to 0^-} \frac{-x^3+2x+3}{x(x+5)} = -\infty$$

$-x^3+2x+3 \to 3$, $x \to 0^-$, $x+5 \to 5$

$$\lim_{x\to 0^+} \frac{-x^3+2x+3}{x(x+5)} = \infty$$

$-x^3+2x+3 \to 3$, $x \to 0^+$, $x+5 \to 5$

$\Rightarrow x = 0$

ASS

$$k_1 = \lim_{x\to\infty} \frac{\frac{-x^3+2x+3}{x(x+5)}}{x} = \lim_{x\to\infty} \frac{-x^3+2x+3}{x^3+5x^2} = -1$$

$$q_1 = \lim_{x\to\infty} \left(\frac{-x^3+2x+3}{x(x+5)} - (-1)\cdot x \right) =$$

$$\lim_{x\to\infty} \left(\frac{-x^3+2x+3+x(x(x+5))}{x(x+5)} \right) =$$

$$= \lim_{x\to\infty} \frac{\cancel{-x^3}+2x+3+\cancel{x^3}+5x^2}{x^2+5x} = 5$$

$$\boxed{y = -1\cdot x + 5}$$

$$k_2 = \lim_{x\to-\infty} \frac{f(x)}{x} = -1 \qquad q_2 = q_1$$