Note Title 11/10/2022

PR.1 POMOCOU DIFERENCIÁLU PRIBLIŽNE URČITE HODNOTU

$$df(x) = f'(x_0)(x - x_0) \quad \text{DIFERENCIÁL}$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

PR: $\sqrt{80}$ $\Rightarrow x = 80 \quad x_0 = 81 \qquad (\sqrt{x})' = \frac{1}{2\cdot\sqrt{x}}$

$$\sqrt{80} \approx \sqrt{81} + \frac{1}{2\sqrt{81}}(80 - 81) = 9 + \frac{1}{2\cdot 9}(-1) =$$

$$= 9 - \frac{1}{18} = \frac{162 - 1}{18} = \frac{161}{18}$$

PR $\sqrt{382}$ $\quad x = 382 \quad x_0 = 400 \qquad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$$\sqrt{382} \approx \sqrt{400} + \frac{1}{2\sqrt{400}}(382 - 400) = 20 + \frac{1}{\cancel{40}_{20}}(-\cancel{18}^{9}) =$$

$$= \frac{400 - 9}{20} = \frac{391}{20}$$

PR $2^{1.9}$ $\quad x = 2^{1.9} \quad x_0 = 2^2 \qquad (2^x)' = 2^x \ln 2$

$$2^{1.9} \approx 2^2 + 2 \ln 2 (1.9 - 2)$$

PR $\operatorname{arctg}(1.1)$ $\quad x = 1.1 \quad x_0 = 1 \quad (\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$$\operatorname{arctg}(1.1) \approx \operatorname{arctg} 1 + \frac{1}{1+1^2}(1.1 - 1) = \frac{\pi}{4} + \frac{1}{2}\cdot\frac{1}{10} = \frac{\pi}{4} + \frac{1}{20} =$$

$$= \frac{5\pi + 1}{20}$$

PR 2. NÁJDITE TAYLOROV POLYNÓM n-TÉHO STUPŇA V BODE x_0 PRE $f(x)$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n +$$

$$+ R_n(x) \quad \text{KDE} \quad R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1} \quad \xi \text{ JE}$$

MEDZI x A x_0

a) $f(x) = \ln x \quad x_0 = 1 \quad n = 4$

$f(1) = \ln 1 = 0 \qquad f'(x) = \frac{1}{x} \qquad f'(1) = \frac{1}{1} = 1$

$f''(x) = \frac{-1}{x^2} \qquad f''(1) = \frac{-1}{1^2} = -1$

$f'''(x) = \frac{2}{x^3} \qquad f'''(1) = \frac{2}{1^3} = 2$

$f^{IV}(x) = (2 \cdot x^{-3})' = -6 \cdot x^{-4} = \frac{-6}{x^4} \qquad f^{IV}(1) = \frac{-6}{1^4} = -6$

$$f(x) = 0 + \frac{1}{1!}(x-1)^1 + \frac{-1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 + \frac{-6}{4!}(x-1)^4 =$$

$$= \frac{1}{1}(x-1)^1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$$

n-TÝ ČLEN RADU BY SA DAL ZAPÍSAŤ AKO:

$$(-1)^{n+1}\frac{1}{n}(x-1)^n \qquad n \in N$$

LIMITY POMOCOU L'HOSPITALA :

AK MÁME LIMITY TYPU $\frac{\pm\infty}{\pm\infty}$ ALEBO $\frac{0}{0}$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$\frac{\infty}{\infty}$ PR: $\lim_{x\to\infty}\frac{x^2+4x+1}{3x^2-1}\overset{L'}{=}\lim_{x\to\infty}\frac{2x+4}{6x}\overset{L'}{=}$

$$=\lim_{x\to\infty}\frac{2}{6}=\frac{1}{3}$$

$\frac{0}{0}$ PR: $\lim_{x\to 0}\frac{\sqrt{1-2x}-1}{x}\overset{L'}{=}\lim_{x\to 0}\frac{\frac{1}{2\sqrt{1-2x}}\cdot(-2)}{1}=$

$$=\frac{\frac{1}{\cancel{2}\cdot\sqrt{1}}(-\cancel{2})}{1}=-1$$

$\frac{\infty}{\infty}$ PR: $\lim_{x\to\frac{\pi}{2}}\frac{\operatorname{tg}3x}{\operatorname{tg}x}\overset{L'}{=}\lim_{x\to\frac{\pi}{2}}\frac{\frac{1}{\cos^2 3x}\cdot 3}{\frac{1}{\cos^2 x}}=$

$\frac{0}{0}$ $$=\lim_{x\to\frac{\pi}{2}}\frac{3\cos^2 x}{\cos^2 3x}\overset{L'}{=}\lim_{x\to\frac{\pi}{2}}\frac{\cancel{3}\cdot\cancel{2}\cos x\cdot(-\sin x)}{\cancel{2}\cos 3x\cdot(-\sin 3x)\cdot\cancel{3}}\overset{L'}{=}$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\underbrace{(-\sin x)(-\sin x)}_{\to 1}+\underbrace{\cos x\cdot(-\cos x)}_{\to 0}}{\underbrace{(-\sin 3x\cdot 3)(-\sin 3x)}_{\to 1\cdot 3}+\underbrace{\cos 3x\cdot(-\cos 3x\cdot 3)}_{\to 0}}=\frac{1}{3}$$

∞^0 **PR:** $\lim\limits_{x\to 0}\left(\frac{1}{2x}\right)^{tg x} = \lim\limits_{x\to 0} e^{\ln\left(\frac{1}{2x}\right)^{tg x}} =$

$= \lim\limits_{x\to 0} e^{tg x \ln\frac{1}{2x}} = e^{\lim\limits_{x\to 0} tg x \ln\frac{1}{2x}} =$ $0\cdot\infty$

$\frac{\infty}{\infty}$ $\lim\limits_{x\to 0}\frac{\ln\frac{1}{2x}}{\frac{1}{tg x}} \overset{L'}{=} \lim\limits_{x\to 0}\frac{\frac{1}{\frac{1}{2x}}\cdot(-1)\frac{1}{2x^2}}{-\frac{1}{tg^2 x}\cdot\frac{1}{\cos^2 x}} =$

$= \lim\limits_{x\to 0}\frac{\cancel{4}^{2}\cancel{x}\, tg^2 x\cdot\cos^2 x}{\cancel{2}x^{\cancel{2}}} = \lim\limits_{x\to 0}\frac{2\, tg^2 x\cdot\cos^2 x}{x} =$

$= \lim\limits_{x\to 0}\frac{2\frac{\sin^2 x}{\cancel{\cos^2 x}}\cdot\cancel{\cos^2 x}}{x} = \lim\limits_{x\to 0}\frac{2\sin^2 x}{x} \overset{L'}{=}$

$\lim\limits_{x\to 0}\frac{2\cdot 2\cdot\sin x\cdot\cos x}{1} = 4\cdot 0\cdot 1 = 0$

$= e^0 = 1$ $\left(\frac{1}{2x}\right)' = \left(\frac{1}{2}\cdot x^{-1}\right)' = \frac{1}{2}x^{-2}(-1) = -\frac{1}{2x^2}$

1^∞ **PR** $\lim\limits_{x\to 0}(\cos 2x)^{\frac{1}{x^2}} = \lim\limits_{x\to 0} e^{\ln(\cos 2x)^{\frac{1}{x^2}}} =$

$= \lim\limits_{x\to 0} e^{\frac{1}{x^2}\ln(\cos 2x)} = e^{\lim\limits_{x\to 0}\frac{\ln(\cos 2x)}{x^2}} \overset{L'}{=}$ $\frac{0}{0}$

$e^{\lim\limits_{x\to 0}\frac{\frac{1}{\cos 2x}\cdot(-\sin 2x)\cdot 2}{2x}} = e^{\lim\limits_{x\to 0}\frac{-2\sin 2x}{2x\cos 2x}} =$

$$\overset{L'}{=} e^{\lim\limits_{x\to 0} \frac{-2(\cos 2x)\cdot 2}{2(\cos 2x) + (2x\cdot(-\sin 2x)\cdot 2)}} = \frac{-2\cdot 1\cdot 2}{2\cdot 1 + 0} = -2$$

$\infty - \infty$ PŘ: $\lim\limits_{x\to\infty} \ln x - x = \lim\limits_{x\to\infty} x\left(\frac{\ln x}{x} - 1\right) =$

$$\lim_{x\to\infty} x \cdot \lim_{x\to\infty}\left(\frac{\ln x}{x} - 1\right) = \infty \cdot \,? \;\; \heartsuit =$$

$$\lim_{x\to\infty}\left(\frac{\ln x}{x} - 1\right) = \lim_{x\to\infty}\left(\frac{\ln x - x}{x}\right) \overset{L'}{=}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x} - 1}{1} = -1$$

$$\heartsuit = \infty\cdot(-1) = -\infty$$

0^0 PŘ: $\lim\limits_{x\to 1^+} \left(\cos\frac{\pi}{2}x\right)^{\log x} = e^{\lim\limits_{x\to 1^+} \ln\left(\cos\frac{\pi}{2}x\right)^{\log x}} =$

$$= e^{\lim\limits_{x\to 1^+} \log x \cdot \ln\left(\cos\frac{\pi}{2}x\right)} = e^{\lim\limits_{x\to 1} \frac{\log x}{\frac{1}{\ln\left(\cos\frac{\pi}{2}x\right)}}} \overset{L'}{=} \quad \frac{0}{0}$$

$$e^{\lim\limits_{x\to 1^+} \frac{\frac{1}{x\cdot\ln 10}}{-\frac{1}{\ln^2\left(\cos\frac{\pi}{2}x\right)} \cdot \frac{1}{\cos\frac{\pi}{2}x} \cdot \left(-\sin\frac{\pi}{2}x\right)\cdot\frac{\pi}{2}}} =$$

$$e^{\lim\limits_{x\to 1^+} \frac{-\ln^2\left(\cos\frac{\pi}{2}x\right)\cdot\cos\frac{\pi}{2}x}{x\cdot\ln 10\cdot\left(-\sin\frac{\pi}{2}x\right)\cdot\frac{\pi}{2}}} = e^{\frac{0}{1\cdot\ln 10\cdot 1\cdot\frac{\pi}{2}}} = e^0 = 1$$

$-\infty \cdot 0$

(*) $$\lim_{x\to 1^+} -\ln^2\left(\cos\frac{\pi}{2}x\right)\cdot\cos\frac{\pi}{2}x = \lim_{x\to 1^+}\frac{-\ln^2\left(\cos\frac{\pi}{2}x\right)}{\frac{1}{\cos\frac{\pi}{2}x}} \overset{L'}{=} \frac{-\infty}{\infty}$$

$$= \lim_{x\to 1^+}\frac{-2\ln\left(\cos\frac{\pi}{2}x\right)\cdot\frac{1}{\cancel{\cos}\frac{\pi}{2}x}\cdot\cancel{\frac{\pi}{2}}}{-\frac{1}{\cos^{\cancel{2}}\frac{\pi}{2}x}\cdot\left(-\sin\frac{\pi}{2}x\right)\cdot\cancel{\frac{\pi}{2}}} =$$

$$\lim_{x\to 1^+}\frac{-2\ln\left(\cos\frac{\pi}{2}x\right)\cdot\cos\frac{\pi}{2}x}{-\sin\frac{\pi}{2}x} = \frac{0}{-1} = 0$$

(*)

$$\lim_{x\to 1^+} -2\ln\left(\cos\frac{\pi}{2}x\right)\cdot\cos\frac{\pi}{2}x =$$

$$= \lim_{x\to 1^+}\frac{-2\ln\left(\cos\frac{\pi}{2}x\right)}{\frac{1}{\cos\frac{\pi}{2}x}} \overset{L'}{=} \lim_{x\to 1^+}\frac{-2\cdot\frac{1}{\cos\frac{\pi}{2}x}\cdot\left(\cancel{-\sin\frac{\pi}{2}x}\right)\cdot\cancel{\frac{\pi}{2}}}{-\frac{1}{\cos^2\frac{\pi}{2}x}\cdot\left(\cancel{-\sin\frac{\pi}{2}x}\right)\cdot\cancel{\frac{\pi}{2}}}$$

$$= \lim_{x\to 1^+}\frac{-2\cos^2\frac{\pi}{2}x}{\cos\frac{\pi}{2}x} = \lim_{x\to 1^+} -2\cos\frac{\pi}{2}x = 0$$

$\frac{-\infty}{-\infty}$ PR! $$\lim_{x\to 0^+}\frac{\ln(\sin 3x)}{\ln(\sin 5x)} \overset{L'}{=} \lim_{x\to 0^+}\frac{\frac{1}{\sin 3x}\cdot\cos 3x\cdot 3}{\frac{1}{\sin 5x}\cdot\cos 5x\cdot 5} =$$

$\frac{0}{0}$ $$= \lim_{x\to 0^+}\frac{\sin 5x\cdot\cos 3x\cdot 3}{\sin 3x\cdot\cos 5x\cdot 5} \qquad \frac{\frac{1}{5x}}{\frac{1}{3x}} \quad \frac{5}{3}$$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} \overset{L'}{=} \lim_{x\to 0}\frac{\cos x}{1} = 1}$$

$$= \lim_{x \to 0^+} \frac{\overbrace{\frac{\sin 5x}{5x}}^{1} \cdot \overbrace{\cos 3x}^{1} \cdot \cancel{3} \cdot \cancel{5}}{\underbrace{\frac{\sin 3x}{3x}}_{1} \cdot \underbrace{\cos 5x}_{1} \cdot \cancel{5} \cdot \cancel{3}} = 1$$