ešte k spojitosti

$$f(x)=\frac{x}{\sqrt{x+1}-1}$$

$\sqrt{x+1}-1\neq 0$ $\qquad x+1\geq 0$

$x+1\neq 1$ $\qquad x\geq -1$ $\qquad D(f):\langle -1,\infty)-\{0\}$

$x\neq 0$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}\overset{\frac{0}{0}}{=}\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}=$$

$$=\lim_{x\to 0}\frac{\cancel{x}\,(\sqrt{x+1}+1)}{\cancel{x}+\cancel{1}-\cancel{1}}=\boxed{\lim_{x\to 0}\sqrt{x+1}+1}=\underline{\underline{2}} \Rightarrow g(x)=\sqrt{x+1}+1$$

$$\boxed{f(x)\begin{cases}\dfrac{x}{\sqrt{x+1}-1} & x\in\langle -1,\infty)-\{0\}\\ 2 & x=0\end{cases}}$$

DERIVÁCIE

$$f(x)=3\cdot 4^x+2\log x$$

$$f'(x)=\cancel{0\cdot 4^x}+3\cdot 4^x\ln 4+\cancel{0\cdot\log x}+2\cdot\frac{1}{x\ln 10}$$

$$f(x)=\frac{e^x+\sin x}{2}=\frac{1}{2}\left(e^x+\sin x\right)$$

$$f'(x)=\frac{1}{2}\cdot\left(e^x+\cos x\right)$$

12) $f(x)=x\ln x-4x \Rightarrow f'(x)=1\cdot\ln x+\cancel{x}\cdot\frac{1}{\cancel{x}}-4=\underline{\underline{\ln x-3}}$

13) $f(x)=\dfrac{\sin x}{1-\cos x}\Rightarrow f'(x)=\dfrac{\cos x\,(1-\cos x)-\sin x\,(\sin x)}{(1-\cos x)^2}=$

$$=\frac{\cos x-\cos^2x-\sin^2x}{(1-\cos x)^2}=\frac{\cos x-1}{(1-\cos x)^2}=\frac{-\cancel{(1-\cos x)}}{(1-\cos x)^{\cancel{2}}}=\frac{-1}{1-\cos x}$$

14) $f(x)=\dfrac{\sin x}{\cos x}$ tgx $\Rightarrow f'(x)=\dfrac{\cos x\cdot\cos x-\sin x\cdot(-\sin x)}{\cos^2x}=\underline{\underline{\dfrac{1}{\cos^2x}}}$

15) $f(x)=\dfrac{\cos x}{\sin x}$ cotgx $\Rightarrow f'(x)=\dfrac{-\sin x\cdot\sin x-\cos x\cdot\cos x}{\sin^2x}=\underline{\underline{\dfrac{-1}{\sin^2x}}}$

18) $f(x)=4^x+x^4\Rightarrow f'(x)=4^x\ln 4+4x^3$

19) $f(x)=\arcsin x-\dfrac{\operatorname{arctg}x}{\sqrt{x}}\Rightarrow f'(x)=\dfrac{1}{\sqrt{1-x^2}}-\dfrac{\frac{1}{1+x^2}\cdot\sqrt{x}-\operatorname{arctg}x\,\frac{1}{2\sqrt{x}}}{x}$

DERIVACIA ZLOZENEJ FUNKCIE

1) $f(x) = (2+3x)^{17} \Rightarrow f'(x) = 17(2+3x)^{16}\cdot 3 = 51(2+3x)^{16}$

2) $f(x) = \sin(x^{-5}) \Rightarrow f'(x) = \cos(x^{-5})\cdot(-5)x^{-6}$

3) $f(x) = e^{3x} \Rightarrow f'(x) = e^{3x}\cdot 3$

4) $f(x) = x^2(x^3-1)^2 \Rightarrow f'(x) = 2x(x^3-1)^2 + x^2\cdot 2(x^3-1)^1\cdot 3x^2$

5) $f(x) = \sqrt[3]{(2x+3)^2} = (2x+3)^{\frac{2}{3}} \Rightarrow f'(x) = \frac{2}{3}(2x+3)^{-\frac{1}{3}}\cdot 2 = \frac{4}{3\sqrt[3]{2x+3}}$

6) $f(x) = 4^{3x} + 3^{x^3} \Rightarrow f'(x) = 4^{3x}\cdot\ln 4\cdot 3 + 3^{x^3}\cdot\ln 3\cdot 3x^2$

8) $f(x) = \sqrt{\sin\left(\frac{2x}{3}\right)} \Rightarrow f'(x) = \frac{1}{2}\left(\sin\left(\frac{2x}{3}\right)\right)^{-\frac{1}{2}}\cdot\cos\left(\frac{2x}{3}\right)\cdot\frac{2}{3}$

9) $f(x) = \ln\left(\sqrt{\frac{x-2}{x+2}}\right) \Rightarrow f'(x) = \frac{1}{\sqrt{\frac{x-2}{x+2}}}\cdot\frac{1}{2}\left(\frac{x-2}{x+2}\right)^{-\frac{1}{2}}\cdot\frac{1\cdot(x+2)-(x-2)\cdot 1}{(x+2)^2} =$

$= \frac{1}{\sqrt{\frac{x-2}{x+2}}}\cdot\frac{1}{\cancel{2}}\,\frac{1}{\sqrt{\frac{x-2}{x+2}}}\cdot\frac{\cancel{4}\,2}{(x+2)^2} = \frac{\cancel{x+2}}{(x-2)}\cdot\frac{2}{(x+2)^{\cancel{2}}} = \frac{2}{x^2-4}$

$\Rightarrow 2(x^2-4)^{-1}$

LOGARITMICKÉ DERIVOVANIE:

1) $f(x) = x^x$ $\qquad f(x) = e^{\ln x^x} = \boxed{e^{x\ln x}}$

$f'(x) = e^{x\ln x}\cdot\left(1\cdot\ln x + x\cdot\frac{1}{x}\right) = e^{x\ln x}(\ln x+1)$

$e^{x\ln x} = x^x$

$\boxed{f'(x) = f(x)\cdot(\ln f(x))'}$

$= x^x\cdot(\ln x^x)' = x^x\cdot(x\ln x)' = x^x\cdot\left(1\cdot\ln x + x\cdot\frac{1}{x}\right) = x^x(\ln x+1)$

2) $f(x) = x^{\sin x}$ $\qquad f(x) = e^{\ln x^{\sin x}} = e^{\sin x\cdot\ln x}$

$f'(x) = e^{\sin x\cdot\ln x}\cdot\left(\cos x\cdot\ln x + \sin x\cdot\frac{1}{x}\right)$

$x^{\sin x}$

3) $f(x) = (\sin x)^{\cos x} \Rightarrow f(x) = e^{\ln \sin x^{\cos x}} = e^{\cos x\cdot\ln(\sin x)}$

$f'(x) = e^{\cos x\cdot\ln(\sin x)}\cdot\left(-\sin x\cdot\ln(\sin x) + \cos x\cdot\frac{1}{\sin x}\cdot\cos x\right)$

4) $f(x) = (x^2+1)^{\operatorname{arctg} x} \Rightarrow f'(x) = (x^2+1)^{\operatorname{arctg} x}\cdot\left(\frac{1}{1+x^2}\cdot\ln(x^2+1) + \frac{2x}{x^2+1}\operatorname{arctg} x\right)$

DERIVOVANIE FUNKCIÍ DANÝCH IMPLICITNE:

$y(x) \quad (y(x))^2$

① $x^2 + xy + y^2 - 3 = 0$ ZLOŽENÁ FUNKCIA

AKO SÚČIN

$$2x + \boxed{1 \cdot y + x\frac{dy}{dx}} + 2y\cdot\frac{dy}{dx} - 0 = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx}(x+2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y} \qquad x \neq -2y$$

② $x^2 - 3xy + 4y^2 - 2x + 3y = 0$

$$2x - \left(3y + 3x\cdot\frac{dy}{dx}\right) + 8y\cdot\frac{dy}{dx} - 2 + 3\frac{dy}{dx} = 0$$

$$-3x\frac{dy}{dx} + 8y\frac{dy}{dx} + 3\frac{dy}{dx} = -2x + 3y + 2$$

$$\frac{dy}{dx} = \frac{-2x + 3y + 2}{-3x + 8y + 3}$$

③ $x^2 y^3 - \sin(xy) = 0$

$$2x\cdot y^3 + x^2\cdot 3y^2\cdot\frac{dy}{dx} - \cos(xy)\cdot\left(1y + x\cdot 1\cdot\frac{dy}{dx}\right) = 0$$

$$2xy^3 + x^2 3y^2\frac{dy}{dx} - y\cos(xy) - x\cos(xy)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y\cos(xy) - 2xy^3}{x^2 3y^2 - x\cos(xy)}$$

FUNKCIE DANÉ PARAMETRICKY:

$$\left.\begin{aligned} x &= te^t \\ y &= t^3 + 6t \end{aligned}\right\} \quad \frac{dy}{dx} = \frac{(t^3+6t)'}{(t\cdot e^t)'} = \frac{3t^2+6}{e^t + te^t} = \frac{3(t^2+2)}{e^t(1+t)}$$

$$\left.\begin{aligned} x &= t^{\frac{3}{2}} \\ y &= t^2 \end{aligned}\right\} \Rightarrow \frac{dy}{dx} = \frac{(t^2)}{(t^{\frac{3}{2}})} = \frac{2t}{\frac{3}{2}t^{\frac{1}{2}}} = \frac{4\sqrt{t}}{3}$$

DERIVÁCIE VYŠŠÍCH RÁDOV

$f^{4}(x)$; Ak $f(x) = x^6 + 5x^4 + 2x^3 - x^2$

$f'(x) = 6x^5 + 20x^3 + 6x^2 - 2x$

$f''(x) = 30x^4 + 60x^2 + 12x - 2$

$f'''(x) = 120x^3 + 120x + 12$

$f^{4}(x) = 360x^2 + 120$

$f''(x)$ $f(x) = tg\,x$

$f'(x) = \frac{1}{\cos^2 x} = \cos^{-2} x$

$f''(x) = -2\cos^{-3}x \cdot (-\sin x) = 2\frac{\sin x}{\cos^3 x}$

$f'''(x)$ $f(x) = arctg\,x$

$f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$

$f''(x) = -1(1+x^2)^{-2} \cdot 2x = \frac{-2x}{(1+x^2)^2}$

$f'''(x) = \frac{-2(1+x^2)^{\cancel{2}} + 2x \cdot 2\cancel{(1+x^2)} \cdot 2x}{(1+x^2)^{\cancel{4}3}} = \frac{-2(1+x^2) + 8x^2}{(1+x^2)^3} =$

$= \frac{-2 - 2x^2 + 8x^2}{(1+x^2)^3} = \frac{-2 + 6x^2}{(1+x^2)^3}$