ASYMPTOTY

PRIAMKU V TVARE $x = a$, $a \in R$ NAZÝVAME ASYMPTOTOU BEZ SMERNICE (ABS) KEĎ $\lim\limits_{x \to a} f(x) = \pm\infty$

PRIAMKA $y = kx + q$ JE ASYMPTOTA SO SMERNICOU KU GRAFU $f(x)$

$$\lim_{x \to \pm\infty} \left[\frac{f(x)}{x} - (kx + q) \right] = 0$$

$$k_1 = \lim_{x \to \infty} \frac{f(x)}{x} \qquad k_2 = \lim_{x \to -\infty} \frac{f(x)}{x}$$

$$q_1 = \lim_{x \to \infty} \left(f(x) - k_1 x \right) \qquad q_2 = \lim_{x \to -\infty} \left(f(x) - k_2 x \right)$$

PR: $f(x) = \frac{1}{x^2 - 1}$ $\qquad D(f) = \{ \forall x \in R \ ; \ x^2 - 1 \neq 0 \} = R - \{\pm 1\}$

ABS:

$$\lim_{x \to -1^-} \frac{1}{\underbrace{x^2 - 1}_{\to 0^+}} = \infty \qquad \lim_{x \to -1^+} \frac{1}{\underbrace{x^2 - 1}_{\to 0^-}} = -\infty$$

$$\lim_{x \to 1^-} \frac{1}{x^2 - 1} = -\infty \qquad \lim_{x \to 1^+} \frac{1}{x^2 - 1} = \infty$$

$x = -1$

$x = 1$

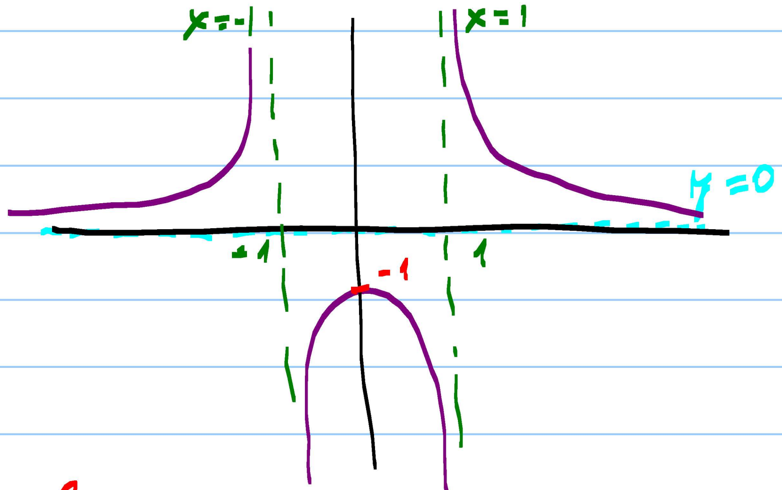
ASS:

$$k_1 = \lim_{x \to \infty} \frac{\frac{1}{x^2 - 1}}{x} = \lim_{x \to \infty} \frac{1}{x^3 - x} = 0$$

$$k_2 = \lim_{x \to -\infty} \frac{1}{x^3 - x} = 0$$

$$q_1 = \lim_{x \to \infty} \left(\frac{1}{x^2-1} - 0 \cdot x \right) = 0 = q_2$$

$$y = 0 \cdot x + 0 = 0 \Rightarrow y = 0$$



PR: $y = \frac{x^3}{x^2+4}$ $\qquad D(f) = R \Rightarrow \nexists$ ABS

ABS: $k_1 = \lim_{x \to \infty} \frac{\frac{x^3}{x^2+4}}{x} = \lim_{x \to \infty} \frac{1 \cdot x^3}{1 \cdot x^3 + 4x} = \frac{1}{1} = 1 = k_2$ *

$$q_1 = \lim_{x \to \infty} \left(\frac{x^3}{x^2+4} - 1 \cdot x \right) = \lim_{x \to \infty} \frac{x^3 - x^3 - 4}{x^2+4} = 0 = q_2$$

$$y = 1 \cdot x + 0 \Rightarrow y = x$$

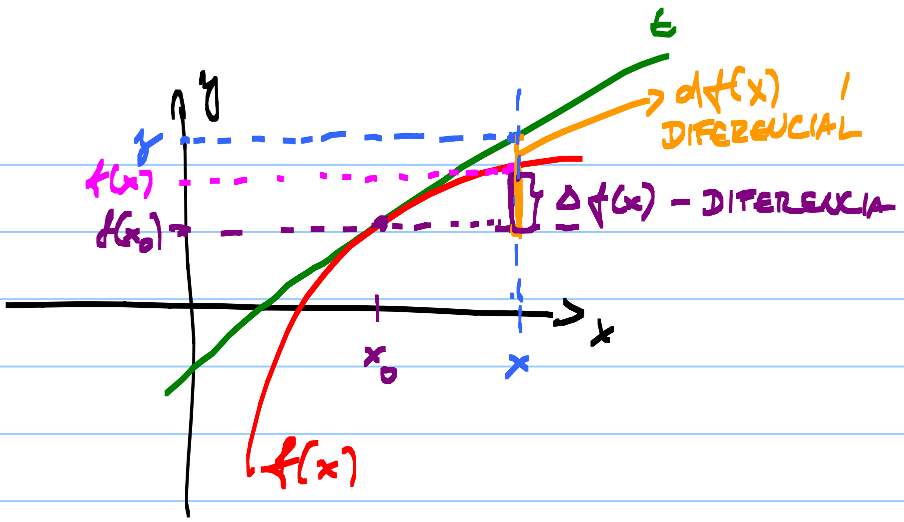
$$* \quad \lim_{x \to \infty} \frac{x^3}{x^3+4x} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \to \infty} \frac{\overbrace{\frac{x^3}{x^3}}^{\to 1}}{\underbrace{\frac{x^3}{x^3}}_{\to 1} + \underbrace{\frac{4x}{x^3}}_{\to 0}} = \frac{1}{1+0} = 1$$

DU: $f(x) = \sqrt{x^2-1}$ $\qquad f(x) = \sqrt{1-x^2}$

$f(x) = \ln \frac{x+1}{x-1}$ $\qquad f(x) = x + e^{-x}$

$f(x) = x \operatorname{arctg} x$

DIFERENCIÁL :



$$\Delta f(x) = f(x) - f(x_0)$$

$$df(x) = f'(x_0)(x - x_0) \qquad y - f(x_0) = \underbrace{f'(x_0)(x - x_0)}_{df(x_0)}$$

PR: POMOCOU DIFERENCIÁLU VYPOČÍTAJ $\sqrt[5]{36}$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$\left(\sqrt[5]{x}\right)' = \frac{1}{5} x^{-\frac{4}{5}}$$

$x_0 = 32$ (NAJLEPŠIE PRIBLÍŽENIE K 36)

$$\sqrt[5]{36} \approx \sqrt[5]{32} + \frac{1}{5}(32)^{-\frac{4}{5}}(36 - 32) = 2 + \frac{1}{5} \cdot \frac{1}{16}(4) =$$

$$= 2 + \frac{1}{20} = \frac{41}{20}$$

TAYLOROV POLYNÓM

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0)^1 + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

$$+ R_n(x)$$

$R_n(x)$ - ZVYŠOK T.P. $= \dfrac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$ KDE ξ JE

HODNOTA MEDZI x A x_0

PR: NÁJDI T.P. n-TÉHO STUPŇA V x_0 PRE $f(x)$

$f(x) = \ln x \qquad x_0 = 1 \qquad n = 4$

$f(1) = \ln 1 = 0$

$f'(x) = (\ln x)' = \frac{1}{x} \qquad f'(1) = \frac{1}{1} = 1$

$f''(x) = -\frac{1}{x^2} \qquad f''(1) = -1$

$f'''(x) = +\frac{2}{x^3} \qquad f'''(1) = 2$

$f^{IV}(x) = -\frac{6}{x^4} \qquad f^{IV}(1) = -6$

$$T(\ln x, 0, 4) = 0 + \frac{1}{1!}(x-1)^1 + \frac{-1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 +$$

$$+ \frac{-6}{4!}(x-1)^4 = x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 +$$

$$- \frac{1}{4}(x-1)^4$$

L'HOSPITALOVO PRAVIDLO

LIMITY TYPU $\frac{\infty}{\infty}, \frac{0}{0}, 0^{\infty}, 1^{\infty}, \infty^0, \infty - \infty$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$\frac{\infty}{\infty}$ PR: $\lim\limits_{x\to\infty}\dfrac{x^2+4x}{2x^2-5x+1} \overset{L'}{=} \lim\limits_{x\to\infty}\dfrac{2x+4}{4x-5} \overset{L'}{=} \lim\limits_{x\to\infty}\dfrac{2}{4} = \dfrac{1}{2}$

$\infty-\infty$ PR: $\lim\limits_{x\to\infty} \ln x - x = \lim\limits_{x\to\infty} x\left(\dfrac{\ln x}{x}-1\right) =$

$$= \lim_{x\to\infty} x\cdot\left(\underbrace{\lim_{x\to\infty}\frac{\ln x}{x}} + \lim_{x\to\infty}(-1)\right) = \infty\cdot(0-1) = -\infty$$

$$\lim_{x\to\infty}\frac{\ln x}{x} \overset{L}{=} \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = 0$$

∞^0 PR: $\lim\limits_{x\to0^+}\left(\dfrac{1}{2x}\right)^{\operatorname{tg}x} = \lim\limits_{x\to0^+} e^{\ln\left(\frac{1}{2x}\right)^{\operatorname{tg}x}} = \lim\limits_{x\to0^+} e^{\operatorname{tg}x\cdot\ln\frac{1}{2x}}$ $=$

$0\cdot\infty$ $\Rightarrow \lim\limits_{x\to0^+}\operatorname{tg}x\cdot\ln\dfrac{1}{2x} = \lim\limits_{x\to0^+}\dfrac{\ln\frac{1}{2x}}{\frac{1}{\operatorname{tg}x}} \overset{L'}{\underset{\frac{\infty}{\infty}}{=}}$

$$\lim_{x\to0^+}\frac{\frac{1}{\frac{1}{2x}}\cdot\frac{-1}{2x^2}}{-\frac{1}{\operatorname{tg}^2x}\cdot\frac{1}{\cos^2x}} = \qquad \left(\left(\frac{1}{2x}\right)' = \frac{1}{2}\cdot(-1)x^{-2} = \frac{-1}{2x^2}\right)$$

$$= \lim_{x\to0^+}\frac{-\cancel{2}\cancel{x}\cdot(-\operatorname{tg}^2x)\cdot\cos^2x}{\cancel{2}x^{\cancel{2}}} = \lim_{x\to0^+}\frac{\operatorname{tg}^2x\cdot\cos^2x}{x} \overset{\frac{0}{0}}{=}$$

$$\lim_{x\to0^+}\frac{\frac{\sin^2x}{\cancel{\cos^2x}}\cdot\cancel{\cos^2}x}{x} = \lim_{x\to0}\frac{\sin^2x}{x} \overset{L'}{=} \lim_{x\to0^+}\frac{2\sin x\cdot\cos x}{1} =$$

$= 0$

$= e^0 = 1$

SPOJITOST A DIFERENCOVATELNOST

$$f(x) = \begin{cases} x \operatorname{arctg} x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\lim_{x \to 0^+} x \cdot \operatorname{arctg} x = 0 = \lim_{x \to 0^-} x \operatorname{arctg} x$$ JE SPOJITA

$$f'(x) \left(x \cdot \operatorname{arctg} x\right)' = 1 \cdot \operatorname{arctg} x + x \cdot \frac{1}{1+x^2}$$

$$f'(0) = \operatorname{arctg} 0 + 0 \cdot \frac{1}{1+0^2} = 0$$ JE DIFERENCOVATEL.

$$f(x) = \begin{cases} x \operatorname{arctg} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$0 \cdot \infty$ $$\lim_{x \to 0^-} x \operatorname{arctg} \frac{1}{x} = \lim_{x \to 0^-} \frac{\operatorname{arctg} \frac{1}{x}}{\frac{1}{x}} \overset{L'}{=}$$