Note Title 15/10/2022

PR1 POMOCOU DIFERENCIÁLU PRIBLIŽNE URČITE

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

a) $\sqrt{382} \Rightarrow x_0 = 400$ - NAJLEPŠIE PRIBLÍŽENIE K 382

$f(400) = \sqrt{400} = 20$

$f'(x) = (\sqrt{x})' = \frac{1}{2} x^{-\frac{1}{2}}$ $\quad f'(400) = \frac{1}{2 \cdot 20} = \frac{1}{40}$

$\sqrt{382} = 20 + \frac{1}{40}(382 - 400) = 20 + \frac{1}{40} \cdot (-18) = 20 - \frac{9}{20} = \frac{391}{20}$

b) $\sqrt[5]{36} \Rightarrow x_0 = 32$ $\quad f(x_0) = \sqrt[5]{32} = 2$

$f'(x) = (\sqrt[5]{x})' = \frac{1}{5} \cdot x^{-\frac{4}{5}}$ $\quad f'(x_0) = f'(32) = \frac{1}{5\sqrt[5]{32^4}} = \frac{1}{5 \cdot 16} = \frac{1}{80}$

$\sqrt[5]{36} = 2 + \frac{1}{80}(36 - 32) = 2 + \frac{1}{80} \cdot 4 = 2 + \frac{1}{20} = \frac{41}{20}$

c) $2^{1.9} \Rightarrow x_0 = 2 \Rightarrow f(2) = 2^2 = 4$

$f'(x) = (2^x)' = 2^x \ln 2$ $\quad f'(2) = 2^2 \ln 2 = 4 \ln 2$

$2^{1.9} = 4 + 4 \ln 2 (1.9 - 2) = 4 - \frac{4}{10} \ln 2$

d) $\operatorname{arctg}(1.1) \Rightarrow x_0 = 1$ $\quad f(1) = \operatorname{arctg} 1 = \frac{\pi}{4}$

$f'(x) = (\operatorname{arctg} x)' = \frac{1}{1+x^2}$ $\quad f'(1) = \frac{1}{1+1^2} = \frac{1}{2}$

$\operatorname{arctg}(1,1) = \frac{\pi}{4} + \frac{1}{2}(1.1 - 1) = \frac{\pi}{4} + \frac{1}{2} \cdot \frac{1}{10} = \frac{\pi}{4} + \frac{1}{20} = \frac{5\pi}{2}$.

d) $\arcsin(0{,}2) \Rightarrow x_0 = 0 \Rightarrow f(0) = \arcsin 0 = 0$

$$f'(x) = (\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad f'(0) = \frac{1}{\sqrt{1-0^2}} = 1$$

$$\arcsin(0.2) = 0 + 1(0{,}2 - 0) = 0{,}2$$

PR 2. NÁJDITE TAYLOROV POLYNÓM n-TÉHO STUPŇA V x_0 PRE $f(x)$

$$T_n(f(x), x_0, x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

a) $f(x) = \ln x \qquad x_0 = 1 \qquad n = 4$

$$f(1) = \ln 1 = 0 \qquad f' = \frac{1}{x} \qquad f'(1) = \frac{1}{1} = 1$$

$$f''(x) = (x^{-1})' = -1\,x^{-2} = \frac{-1}{x^2} \qquad f''(1) = -\frac{1}{1} = -1$$

$$f'''(x) = (-x^{-2})' = 2\cdot x^{-3} = \frac{2}{x^3} \qquad f'''(1) = \frac{2}{1^3} = 2$$

$$f^{IV}(x) = (2x^{-3})' = -6x^{-4} = \frac{-6}{x^4} \qquad f^{IV}(1) = -\frac{6}{1^4} = -6$$

$$T_4(\ln x, 1, x) = 0 + \frac{1}{1!}(x-1) + \frac{-1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3 + \frac{-6}{4!}(x-1)^4$$

b) $x^4 - 5x^3 + 2x - 3 \qquad x_0 = -1 \qquad n = 4$

$$f(-1) = (-1)^4 - 5(-1)^3 + 2(-1) - 3 = 1 + 5 - 2 - 3 = 1$$

$$f'(x) = 4x^3 - 15x^2 + 2 \qquad f'(-1) = 4(-1)^3 - 15(-1)^2 + 2 = -17$$

$$f''(x) = 12x^2 - 15x \qquad f''(-1) = 12(-1)^2 - 15(-1) = 27$$

$$f'''(x) = 24x \qquad f'''(-1) = 24(-1) = -24$$

$$f^{IV}(x) = 24 \qquad f^{IV}(-1) = 24$$

$$T_4(x^4 - 5x^3 + 2x - 3, -1, x) = 1 + \frac{-17}{1!}(x+1) + \frac{27}{2!}(x+1)^2 + \frac{-24}{3!}(x+1)^3 + \frac{24}{4!}(x+1)^4$$

c) $f(x) = e^{2x} \sin x \quad x_0 = 0 \quad n = 3$

$f(0) = e^{2\cdot 0} \sin 0 = 1\cdot 0 = 0$

$f'(x) = 2e^{2x}\sin x + e^{2x}\cos x \quad f'(0) = 2e^{2\cdot 0}\sin 0 + e^{2\cdot 0}\cos 0 = 0+1 = 1$

$f''(x) = 2\cdot 2e^{2x}\sin x + 2e^{2x}\cos x + 2e^{2x}\cos x + e^{2x}(-\sin x)$

$f''(0) = 4\cdot 1\cdot 0 + 2\cdot 1\cdot 1 + 2\cdot 1\cdot 1 + 1\cdot 0 = 4$

$f''(x) = 3e^{2x}\sin x + 4e^{2x}\cos x$

$f'''(x) = 2\cdot 3e^{2x}\sin x + 3e^{2x}\cos x + 2\cdot 4e^{2x}\cos x + 4e^{2x}(-\sin x)$

$f'''(0) = 6\cdot 1\cdot 0 + 3\cdot 1\cdot 1 + 8\cdot 1\cdot 1 + 4\cdot 1\cdot 0 = 11$

$$T_3(e^{2x}\sin x, 0, x) = 0 + \frac{1}{1!}(x-0) + \frac{4}{2!}(x-0)^2 + \frac{11}{3!}(x-0)^3 =$$
$$= x + 2x^2 + \frac{11}{6}x^3$$

PR 3 POMOCOU MACLAURINOVHO RADU VYPOČÍTAJTE PRIBLIŽNÚ HODNOTU e^2 S CHYBOU MENŠOU AKO 0,14.

$x_0 = 0 \qquad f(x) = e^x \qquad f(0) = e^0 = 1$

$f'(x) = e^x \qquad f'(0) = e^0 = 1$

$\vdots$

$f^{n}(x) = e^x \qquad f^{n}(0) = e^0 = 1$

$f^{n+1}(x) = e^x \qquad f^{n+1}(0) = e^0 = 1$

$$R_n = \frac{f^{n+1}(\xi)}{(n+1)!}(x-x_0)^{n+1} = \frac{e^{\xi}}{n+1}x^{n+1}$$

$$T_n(e^x, 0, x) = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \dots + \frac{1}{n!}x^n + \underbrace{\frac{e^{\xi}}{n+1}x^{n+1}}_{R_n}$$

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$e^x = e^2 \Rightarrow x = 2 \Rightarrow \xi \in (0,2)$

$e^2 \approx 1 + \frac{1}{1!}2 + \frac{1}{2}2^2 + \dots \qquad \frac{e^2}{(n+1)!} < 0{,}14$

$$\frac{2{,}7^2}{(n+1)!} < 0{,}14 \Rightarrow \frac{7{,}29}{0{,}14} < (n+1)! \Rightarrow 52{,}0714 < (n+1)!$$

$n!: \quad 1! = 1 \quad 2! = 2 \quad 3! = 6 \quad 4! = 24 \quad 5! = 120 \Rightarrow n = 4$

$$e^2 \doteq T_4(e^x, 0, x) = 1 + 1\cdot 2 + \frac{1}{2}2^2 + \frac{1}{6}2^3 + \frac{1}{24}2^4 =$$

$$= 1 + 2 + 2 + \frac{8}{6} + \frac{16}{24} = \frac{6+12+12+8+4}{6} = \frac{42}{6} = 7$$

$$e^2 \approx 7$$

PR 4. POMOCOU TAYLOROVHO POLYNÓMU 3. STUPŇA PRIBLIŽNE VYPOČÍTAJTE $\cos(61°)$ A VYPOČÍTAJTE AJ MAX. CHYBU VAŠEJ APROXIMÁCIE

$f(x) = \cos x \qquad x = 61° \qquad x_0 = 60° = \frac{\pi}{3}$

$f(x_0) = \cos\frac{\pi}{3} = \frac{1}{2}$

$f'(x) = -\sin x \qquad f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$f''(x) = -\cos x \qquad f''\left(\frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$f'''(x) = \sin x \qquad f'''\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$$T_3\left(\cos x, \frac{\pi}{3}, x\right) = \frac{1}{2} + \frac{-\frac{\sqrt{3}}{2}}{1!}\left(x - \frac{\pi}{3}\right) + \frac{-\frac{1}{2}}{2!}\left(x - \frac{\pi}{3}\right)^2 + \frac{\frac{\sqrt{3}}{2}}{3!}\left(x - \frac{\pi}{3}\right)^3$$

$$T_3(\cos 61°) = \frac{1}{2} - \frac{\sqrt{3}}{2}\left(\frac{61-60}{180}\pi\right) - \frac{1}{4}\left(\frac{\pi}{180}\right)^2 + \frac{\sqrt{3}}{12}\left(\frac{\pi}{180}\right)^3$$

$$0{,}5 - 0{,}0048\pi - 0{,}00003\pi^2 +$$

$f^{IV}(x) = \cos x \qquad R_n = \frac{f^{n+1}(\xi)}{(n+1)!}(x - x_0)^{n+1}$

$R_4 = \frac{\cos\frac{\pi}{3}}{4!}\left(\frac{\pi}{180}\right)^4$ CHYBA APROXIMÁCIE.

L'HOSPITALOVO PRAVIDLO

PRE LIMITY TYPU $\frac{\infty}{\infty}, \frac{0}{0}$ $\qquad \lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$

POUŽÍVA SA AJ PRE LIMITY TYPU $0^0 \quad 1^\infty \quad 0^\infty \quad \infty^0 \quad \infty-\infty$

$\frac{0}{0}$

a) $$\lim_{x\to\infty}\frac{\frac{\pi}{2}-\operatorname{arctg} x}{\ln\sqrt{\frac{x-1}{x+1}}} \overset{L'}{=} \lim_{x\to\infty}\frac{-\frac{1}{1+x^2}}{\frac{1}{\sqrt{\frac{x-1}{x+1}}}\cdot\frac{1}{2}\left(\frac{x-1}{x+1}\right)^{-1/2}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}} =$$

$$= \lim_{x\to\infty}\frac{-\left(\frac{x-1}{x+1}\right)(x+1)^2}{\frac{2}{2}(1+x^2)} = \lim_{x\to\infty}\frac{(x-1)(x+1)}{1+x^2} = \lim_{x\to\infty}\frac{-(x^2-1)}{x^2+1} = -1$$

$\frac{-\infty}{-\infty}$

b) $$\lim_{x\to 0^+}\frac{\ln(\sin 3x)}{\ln(\sin 5x)} \overset{L'}{=} \lim_{x\to 0^+}\frac{\frac{1}{\sin 3x}\cdot 3\cos 3x}{\frac{1}{\sin 5x}\cdot 5\cos 5x} =$$

$$= \lim_{x\to 0^+}\frac{3\sin 5x\cos 3x}{5\sin 3x\cos 5x}\cdot\frac{\frac{1}{5x}}{\frac{1}{3x}}\cdot\frac{5}{3} =$$

$$= \lim_{x\to 0^+}\frac{\cancel{3}\cdot\frac{\sin 5x}{5x}\cdot\cos 3x}{\cancel{5}\cdot\frac{\sin 3x}{3x}\cdot\cos 5x}\cdot\frac{\cancel{5}}{\cancel{3}} = 1$$

$\to 1 \quad \to 1 \quad \to 1 \quad \to 1$

$$\lim_{x\to 0^+}\frac{\sin 5x}{5x} \overset{L'}{=} \lim_{x\to 0^+}\frac{\cos 5x\cdot 5}{5} = 1$$

$0\cdot\infty$

c) $$\lim_{x\to 2}\frac{x^2-4}{x^2}\operatorname{tg}\left(\frac{\pi x}{4}\right) = \lim_{x\to 2}\frac{x^2-4}{x^2}\cdot\frac{\sin\frac{\pi}{4}x}{\cos\frac{\pi}{4}x} \overset{L'}{=} \quad \frac{0}{0}$$

$$= \lim_{x\to 2}\frac{2x\cdot\sin\frac{\pi}{4}x+(x^2-4)\cdot\cos\frac{\pi}{4}x\cdot\frac{\pi}{4}}{2x\cdot\cos\frac{\pi}{4}x+x^2\left(-\sin\frac{\pi}{4}x\right)\frac{\pi}{4}} = \frac{2\cdot 2\cdot 1}{2^2(-1)\cdot\frac{\pi}{4}} = -\frac{4}{\pi}$$

$\to 0 \quad \to 0$

d) $\lim\limits_{x\to 0} \dfrac{\sin x - x}{\arcsin x - x} \overset{L'}{=} \lim\limits_{x\to 0} \dfrac{\cos x - 1}{\frac{1}{\sqrt{1-x^2}} - 1} = \lim\limits_{x\to 0} \dfrac{\sqrt{1-x^2}\,(\cos x - 1)}{1-\sqrt{1-x^2}} \overset{L'}{=}$

$$\lim_{x\to 0} \frac{\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x)\cdot(\cos x - 1) + \sqrt{1-x^2}\cdot(-\sin x)}{-\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)} =$$

$$= \lim_{x\to 0} \frac{\dfrac{(-2x)(\cos x - 1) + \left(\sqrt{1-x^2}\right)^2(-\sin x)}{\cancel{\sqrt{1-x^2}}}}{-\dfrac{-\cancel{2}x}{\cancel{2}\cancel{\sqrt{1-x^2}}}} =$$

$$= \lim_{x\to 0} \frac{(-2x)(\cos x - 1) + (1-x^2)(-\sin x)}{-x} \overset{L'}{=}$$

$$= \lim_{x\to 0} \frac{\underbrace{-2(\cos x - 1)}_{\to 0} + \underbrace{(-2x)(-\sin x)}_{\to 0} + \underbrace{(-2x)(-\sin x)}_{\to 0} + \underbrace{(1-x^2)}_{\to 1}\underbrace{(-\cos x)}_{-1}}{+1} =$$

$$= \frac{-1}{1} = -1$$

e) $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} x - x}{x - \sin x} \overset{L'}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{\cos^2 x} - 1}{1-\cos x} =$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{\cos^2 x\,(1-\cos x)} = \lim_{x\to 0} \frac{(\cancel{1-\cos x})(1+\cos x)}{\cos^2 x\,(\cancel{1-\cos x})} =$$

$$= \lim_{x\to 0} \frac{1+\cos x}{\cos^2 x} = \frac{1+1}{1^2} = 2$$

f) $\lim\limits_{x\to 0^+} (e^x - 1)\operatorname{cotg} x = \lim\limits_{x\to 0^+} \dfrac{\operatorname{cotg} x}{\frac{1}{e^x - 1}} \overset{L}{=}$

$$= \lim_{x\to 0} \frac{\frac{-1}{\sin^2 x}}{-\frac{1}{(e^x-1)^2}\cdot e^x} = \lim_{x\to 0} \frac{(e^x-1)^2}{\sin^2 x \cdot e^x} \overset{L}{=}$$

$$= \lim_{x\to 0} \frac{2(e^x-1)\cdot \cancel{e^x}}{2\sin x\cos x\,\cancel{e^x}+\sin^2 x\,\cancel{e^x}} =$$

$$= \lim_{x\to 0} \frac{\cancel{2}e^x}{\cancel{2}\underbrace{(\cos x\cdot\cos x)}_{\downarrow 1}+\underbrace{2\sin x(-\sin x)}_{\downarrow 0}+\underbrace{2\sin x\cdot\cos x}_{\downarrow 0}} = \frac{1}{1}=1$$

g) $\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) = \lim_{x\to 1^+} \frac{x\ln x-(x-1)}{(x-1)\ln x} \overset{L}{=}$

$$= \lim_{x\to 1^+} \frac{\ln x + x\cdot\cancel{\frac{1}{x}}-\cancel{1}}{1\cdot\ln x+(x-1)\cdot\frac{1}{x}} = \lim_{x\to 1^+}\frac{x\ln x}{x\ln x+x-1} \overset{L'}{=}$$

$$= \lim_{x\to 1^+}\frac{\ln x + x\cdot\frac{1}{x}}{\ln x+1+1} = \frac{0+1}{0+2} = \frac{1}{2} = \lim_{x\to 1^-}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$$

i) $\lim_{x\to 0}\frac{x-\operatorname{arctg} x}{x^3} = \lim_{x\to 0}\frac{1-\frac{1}{1+x^2}}{3x^2} = \lim_{x\to 0}\frac{\cancel{1}+x^2-\cancel{1}}{3x^2(1+x^2)} =$

$$= \lim_{x\to 0}\frac{2\cancel{x}}{6\cancel{x}+12x^{\cancel{2}}} = \lim_{x\to 0}\frac{2}{6+12x^2} = \frac{2}{6}=\frac{1}{3}$$

j) $\lim_{x\to 1^-}(x-1)\ln(1-x) = \lim_{x\to 1^-}\frac{\ln(1-x)}{\frac{1}{x-1}} \overset{L'}{=} \lim_{x\to 1^-}\frac{\frac{1}{1-x}\cdot\cancel{(-1)}}{\cancel{-1}\frac{1}{(x-1)^2}\cdot 1} =$

$$= \lim_{x\to 1^-}\frac{x^2-1}{1-x} = \lim_{x\to 1^-}\frac{\cancel{(x-1)}(x+1)}{-\cancel{(x-1)}} = -2$$

k) $\lim_{x\to\frac{\pi}{2}}\left(\operatorname{tg} x-\frac{1}{\cos x}\right) = \lim_{x\to\frac{\pi}{2}}\frac{\sin x}{\cos x}-\frac{1}{\cos x} = \lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\cos x} \overset{L'}{=}$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\cos x}{-\sin x} = \frac{0}{-1} = 0$$

l) $\lim_{x\to 0^+}(\operatorname{tg} x)^{\sin x} = \lim_{x\to 0^+} e^{\ln(\operatorname{tg} x)^{\sin x}} = \lim_{x\to 0^+} e^{\sin x\ln\operatorname{tg} x} =$

$$= e^{\lim\limits_{x\to 0^+} \frac{\ln \operatorname{tg} x}{\frac{1}{\sin x}}} = e^{\lim\limits_{x\to 0^+} \frac{\frac{1}{\operatorname{tg} x}\cdot\frac{1}{\cos^2 x}}{\frac{-1}{\sin^2 x}\cdot\cos x}} =$$

$$= e^{\lim\limits_{x\to 0^+} \frac{-\sin^3 x}{\cos^4 x}} = e^0 = 1$$

m) $\lim\limits_{x\to 0^+} \left(\frac{1}{x}\right)^{\operatorname{tg} x} = \lim\limits_{x\to 0^+} e^{\ln\left(\frac{1}{x}\right)^{\operatorname{tg} x}} = \lim\limits_{x\to 0^+} e^{\operatorname{tg} x \ln \frac{1}{x}} =$

$$= e^{\lim\limits_{x\to 0^+} \frac{\ln \frac{1}{x}}{\frac{1}{\operatorname{tg} x}}} \overset{L}{=} e^{\lim\limits_{x\to 0^+} \frac{\frac{1}{\cancel{\frac{1}{x}}}\cdot(\cancel{-1})\frac{1}{x^{\cancel{2}}}}{\frac{\cancel{-}1}{\operatorname{tg}^2 x}\cdot\frac{1}{\cos^2 x}}} =$$

$$= e^{\lim\limits_{x\to 0^+} \frac{\operatorname{tg}^2 x \cdot \cos^2 x}{x}} \overset{L}{=}$$

$$\overset{L}{=} e^{\lim\limits_{x\to 0^+} \frac{2\operatorname{tg} x \frac{1}{\cancel{\cos^2 x}}\cdot\cancel{\cos^2 x} + \operatorname{tg}^2 x\cdot 2\cos x\cdot(-\sin x)}{1}} = e^0 = 1$$

n) $\lim\limits_{x\to\infty} \left(\frac{2}{\pi}\operatorname{arctg} x\right)^x = \lim\limits_{x\to\infty} e^{\ln\left(\frac{2}{\pi}\operatorname{arctg} x\right)^x} =$

$$= e^{\lim\limits_{x\to\infty} x \ln\left(\frac{2}{\pi}\operatorname{arctg} x\right)} = e^{\lim\limits_{x\to\infty} \frac{\ln \frac{\pi}{2}\operatorname{arctg} x}{\frac{1}{x}}} \overset{L}{=}$$

$$= e^{\lim\limits_{x\to\infty} \frac{\frac{1}{\cancel{\frac{2}{\pi}}\operatorname{arctg} x}\cdot\frac{\cancel{\frac{2}{\pi}}}{1+x^2}}{-\frac{1}{x^2}}} = e^{\lim\limits_{x\to\infty} \frac{\overbrace{-x^2}^{\to -1}}{(1+x^2)\underbrace{\operatorname{arctg} x}_{\to\frac{\pi}{2}}}} = e^{-\frac{1}{\frac{\pi}{2}}} = e^{-\frac{2}{\pi}}$$

o) $\lim\limits_{x\to 3} \left(\frac{\sin x}{\sin 3}\right)^{\operatorname{cotg}(x-3)} = \lim\limits_{x\to 3} e^{\ln\left(\frac{\sin x}{\sin 3}\right)^{\operatorname{cotg}(x-3)}} =$

$$e^{\lim\limits_{x\to 3} \operatorname{cotg}(x-3)\ln\left(\frac{\sin x}{\sin 3}\right)} = e^{\lim\limits_{x\to 3} \frac{\ln\left(\frac{\sin x}{\sin 3}\right)}{\frac{1}{\operatorname{cotg}(x-3)}}} \overset{L}{=}$$

$$= e^{\lim\limits_{x\to 3} \frac{\frac{\frac{1}{\sin x}}{\cancel{\sin 3}} \cdot \frac{1}{\cancel{\sin 3}} \cos x}{-\frac{1}{\cot^2(x-3)} \cdot \left(-\frac{1}{\sin^2(x-3)}\right)} \cdot 1} = e^{\lim\limits_{x\to 3} \frac{\cot^2(x-3)\sin^2(x-3)\cos x}{\sin x}} =$$

$$= e^{\lim\limits_{x\to 3} \frac{\frac{\cos^2(x-3)}{\cancel{\sin^2(x-3)}} \cdot \cancel{\sin^2(x-3)} \cos x}{\sin x}} = e^{1 \cdot \frac{\cos 3}{\sin 3}} = e^{\cot 3}.$$

p) $\lim\limits_{x\to 0^+} \left(e^{2x}+x\right)^{\frac{1}{x}} = \lim\limits_{x\to 0^+} e^{\ln(e^{2x}+x)^{\frac{1}{x}}} =$

$$= \lim\limits_{x\to 0^+} e^{\frac{1}{x}\ln(e^{2x}+x)} \overset{L'}{=} e^{\lim\limits_{x\to 0^+} \frac{\frac{1}{e^{2x}+x}\cdot(e^{2x}\cdot 2+1)}{1}} =$$

$$= e^{\frac{1\cdot 2+1}{1+0}} = e^{3}$$

r) $\lim\limits_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{e^x-1}\right) = \lim\limits_{x\to 0} \frac{(e^x-1)-\sin x}{(e^x-1)\sin x} \overset{L'}{=}$

$$= \lim\limits_{x\to 0} \frac{e^x - \cos x}{e^x\cdot\sin x + (e^x-1)\cos x} \overset{L'}{=}$$

$$= \lim\limits_{x\to 0} \frac{e^x + \sin x}{e^x \sin x + e^x\cos x + e^x\cos x + (e^x-1)(-\sin x)} =$$

$$= \frac{1+0}{0+1+1+0} = \frac{1}{2}$$

DZ 2 VYŠETRITE SPOJITOSŤ FUNKCIE

$$f(x)=\begin{cases} \ln x \cdot \log_{10}(1-x) & x\in(0,1) \\ 0 & x=1 \\ x^{\frac{1}{x-1}} & x\in(1,\infty) \end{cases}$$

$$\lim_{x\to 1^-} \ln x \log_{10}(1-x) = \lim_{x\to 1^-} \frac{\log_{10}(1-x)}{\frac{1}{\ln x}} \overset{L}{=}$$

$$= \lim_{x\to 1^-} \frac{\frac{1}{(1-x)\cdot \ln 10}\cdot\cancel{(-1)}}{\frac{\cancel{-}1}{\ln^2 x}\cdot\frac{1}{x}} = \lim_{x\to 1^-} \frac{x\ln^2 x}{(1-x)\ln 10} \overset{L}{=}$$

$$= \lim_{x\to 1^-} \frac{\ln^2 x + x\cdot 2\ln x\cdot\frac{1}{x}}{(-1)\ln 10} = \frac{0+1\cdot 2\cdot 0\cdot\frac{1}{1}}{-\ln 10} = 0$$

$$\lim_{x\to 1^+} x^{\frac{1}{x-1}} = e^{\lim_{x\to 1^+} \ln x^{\frac{1}{x-1}}} = e^{\lim_{x\to 1^+} \frac{1}{x-1}\ln x} \overset{L}{=}$$

$$= e^{\lim_{x\to 1^+} \frac{\frac{1}{x}}{1}} = e^1 \qquad \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

$\Rightarrow$ NESPOJITÁ

DZ 3 ZISTETE ČI JE FUNKCIA

$$f(x)=\begin{cases} x\,\mathrm{arctg}\,\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$$

a) SPOJITÁ: $\displaystyle \lim_{x\to 0^-} x\,\mathrm{arctg}\,\frac{1}{x} = \lim_{x\to 0} \frac{\mathrm{arctg}\,\frac{1}{x}}{\frac{1}{x}} =$

$\frac{1}{x} = t \qquad x\to 0 \Rightarrow t=\infty$

$$\lim_{t\to\infty} \frac{\mathrm{arctg}\, t}{t} = 0 \Rightarrow \text{SPOJITÁ}$$

b) DIFERENCOVATEĽNÁ V $a = 0$

$$f'(x) = 1 \cdot \operatorname{arctg} \frac{1}{x} + \cancel{x} \cdot \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot (-1) \frac{1}{x^{\cancel{2}}} =$$

$$= \operatorname{arctg} \frac{1}{x} - \frac{x^2}{x^2+1} \cdot \frac{1}{x}$$

$$D(f') = \{\forall x \in R ;\ x \neq 0\} \Rightarrow f(x) \text{ NIE JE DIFERENCOVATEĽNÁ}$$

PR 4 ZISTITE, ČI JE FUNKCIA

$$f(x) = \begin{cases} (x-1)^2 \cos \frac{1}{x-1} & x \neq 1 \\ 0 & x = 1 \end{cases}$$

a) SPOJITÁ V $a = 0$

$$\lim_{x \to 1^-} (x-1)^2 \cos \frac{1}{x-1} = \lim_{x \to 1^-} \frac{\cos \frac{1}{x-1}}{\frac{1}{(x-1)^2}} \overset{L'}{=}$$

$$\lim_{x \to 1^-} \frac{\sin \frac{1}{x-1} \cdot \frac{\cancel{-}1}{\cancel{(x-1)^2}} \cdot 1}{\frac{\cancel{-}2}{(x-1)^{\cancel{3}}} \cdot 1} = \lim_{x \to 1^-} \frac{\sin \frac{1}{x-1}}{2(x-1)} \overset{L'}{=}$$

$$= \lim_{x \to 1^-} \frac{\cos \frac{1}{x-1} \cdot \frac{-1}{(x-1)^2} \cdot 1}{2} = \frac{0}{2} = 0$$

$$\lim_{x \to 1^+} f(x) = 0$$

$\Rightarrow$ SPOJITÁ

b) DIFERENCOVATELNÁ V $a = 1$

$$f'(x) = 2(x-1)\cdot 1 \cos\frac{1}{x-1} + \cancel{(x-1)^2}\left(\cancel{-}\sin\frac{1}{x-1}\right)\cdot\frac{\cancel{-}1}{\cancel{(x-1)^2}}\cdot 1$$

$$= 2(x-1)\cos\frac{1}{x-1} + \sin\frac{1}{x-1}$$