Priebeh funkcie

Note Title 18/10/2022

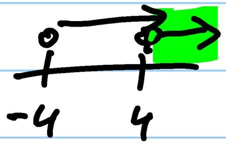
1. D(f) A NULOVÉ BODY
2. PÁRNOSŤ A NEPÁRNOSŤ ✓
3) SPOJITOSŤ A ABS
4) ASS
5) MONOTÓNNOSŤ ✓
6) EXTRÉMY
7) KONVEXNOSŤ ✓ KONKÁVNOSŤ ✓
8) INFLEXNÉ BODY
9) GRAF

PR: $f(x) = \ln(x^2-16)$

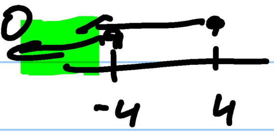
1) $D(f) = \{\forall x \in R \,;\, x^2-16>0\} = (-\infty,-4) \cup (4,\infty)$

$x^2-16>0 \qquad (x-4)(x+4)>0 \Leftrightarrow$

$x-4>0 \wedge x+4>0$



$x-4<0 \wedge x+4<0$



NB $f(x)=0$

$\ln(x^2-16)=0 \Leftrightarrow x^2-16=1 \Leftrightarrow$

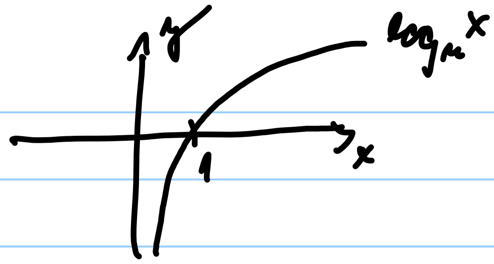
$x^2=17 \Leftrightarrow x=\pm\sqrt{17}$

2) PÁRNOSŤ ✓ NEPÁRNOSŤ ✓

$f(-x) = \ln((-x)^2-16) = \ln(x^2-16) = f(x)$

PÁRNA NEPERIODICKÁ $\nexists\, k \in R \,;\, f(x) = f(x+kp)$

p - PERIÓDA



3) SPOJITOSŤ A ABS

$$\left.\begin{array}{l} \lim\limits_{x\to -4^-} \ln(x^2-16) = -\infty \\ \lim\limits_{x\to 4^+} \ln(x^2-16) = -\infty \end{array}\right\} \Rightarrow \text{ABS } \begin{array}{l} x=-4 \\ x=4 \end{array}$$

4) ABS

$$k_1 = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\ln(x^2-16)}{x} \overset{L'}{=} \lim_{x\to\infty} \frac{\frac{1}{x^2-16}\cdot 2x}{1} \overset{L'}{=}$$

$$\left.\begin{array}{l} = \lim\limits_{x\to\infty} \dfrac{2}{2x} = 0 = k_2 \\ q_1 = \lim\limits_{x\to\infty} \left(\ln(x^2-16) - 0\cdot x\right) = \infty \end{array}\right\} \Rightarrow \nexists \text{ ABS}$$

5) MONOTÓNNOSŤ

$f'(x) > 0$ RASTÚCA

$f'(x) < 0$ KLESAJÚCA

$$f'(x) = \frac{1}{x^2-16}\cdot 2x$$

	$(-\infty,-4)$	$(4,\infty)$
$2x$	$-$	$+$
x^2-16	$+$	$+$
	$-$	$+$

$(-\infty,-4)$ - KLESÁ

$(4,\infty)$ - RASTIE

6) EXTRÉMY (LOKÁLNE) $f'(x)=0 \Rightarrow$ KANDIDÁT NA STAC. BOD A

AK $f''(A) < 0 \Rightarrow$ V A JE LOK MAX

$f''(A) > 0 \Rightarrow$ — $\|$ — MIN

$$f'(x)=0 \Leftrightarrow \frac{2x}{x^2-16} = 0 \Leftrightarrow 2x = 0 \Leftrightarrow x=0$$

$0 \notin D(f) \Rightarrow \nexists$ EXTRÉM

*) KONVEXNOSŤ $\cup$, KONKÁVNOSŤ $\cap$

$f''(x) > 0 \Rightarrow f(x)$ KONVEXNÁ

$f''(x) < 0 \Rightarrow f(x)$ KONKÁVNA

$f''(x) = 0 \Rightarrow$ KANDIDÁT NA INFLEXNÝ BOD

$$f''(x) = \left(\frac{2x}{x^2-16}\right)' = \frac{2\cdot(x^2-16) - 2x(2x)}{(x^2-16)^2} = \frac{2x^2-4x^2-32}{(x^2-16)^2} =$$

$$= \frac{-2x^2-32}{(x^2-16)^2}$$

	$(-\infty,-4)$	$(4,\infty)$
$-2x^2-32$	$-$	$-$
$(x^2-16)^2$	$+$	$+$
	$\ominus$	$\ominus$
	$\cap$	$\cap$

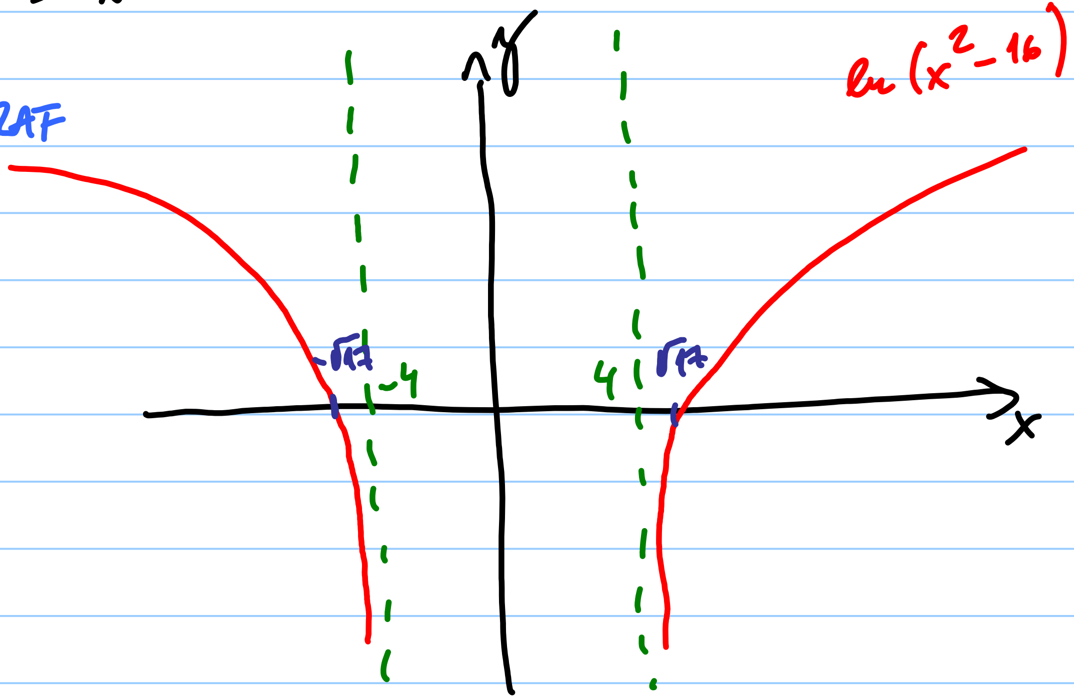
8) INFLEXNÝ BOD

$$f''(x)=0 \iff \frac{-2x^2-32}{(x^2-16)^2}=0 \iff -2x^2-32=0$$

$$x^2=-16 \quad \text{NIKDY V } R$$

$\Rightarrow \nexists$ IB

9) GRAF



PR: $f(x)=\frac{x}{\ln x}$

1) $D(f)=\{\forall x\in R;\ x>0;\ \ln x\neq 0\}=(0,1)\cup(1,\infty)$

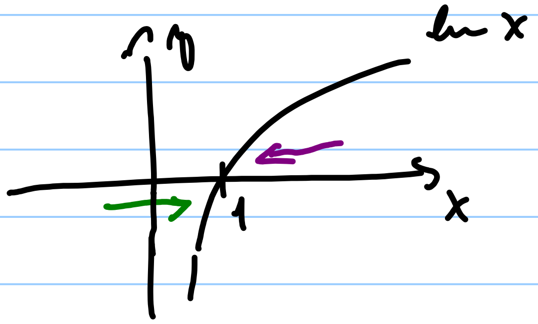
NB $f(x)=0 \iff \frac{x}{\ln x}=0 \iff x=0 \quad 0\notin D(f)$

$$\lim_{x\to 0^+}\frac{x}{\ln x}=\frac{0^+}{-\infty}=0$$

2) PÁR. NEPÁR

D(f) NIE JE SYMETRICKÝ => ANI ANI

3) SPOJITOSŤ A ABS



$$\lim_{x\to 0^+} f(x) = 0$$

$$\lim_{x\to 1^-} \frac{x}{\underbrace{\ln x}_{\to 0^-}} = -\infty$$

$$\lim_{x\to 1^+} \frac{x}{\underbrace{\ln x}_{\to 0^+}} = \infty$$

=> ABS $x = 1$

4) ASS

$$k_1 = \lim_{x\to\infty} \frac{\frac{\cancel{x}}{\ln x}}{\cancel{x}} = \lim_{x\to\infty} \frac{1}{\ln x} = 0$$

$\nexists k_2$

$$q_1 = \lim_{x\to\infty} \frac{x}{\ln x} - 0\cdot x = \lim_{x\to\infty} \frac{x}{\ln x} \overset{\prime}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{x}} =$$

$$= \lim_{x\to} x = \infty$$

5) MONOTÓNNOSŤ

$$f'(x) = \frac{1\cdot \ln x - x\cdot\frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$$

$$f'(x) = 0 \iff \ln x - 1 = 0 \iff \ln x = 1$$

$$\iff x = e \quad \text{SB}$$

	$(0,1)$	$(1,e)$	(e,∞)
$\ln x - 1$	$-$	$-$	$+$
$\ln^2 x$	$+$	$+$	$+$
	$-$	$-$	$+$
	$\searrow$	$\searrow$	$\nearrow$

6) EXTRÉMY

$$f'' = \frac{\frac{1}{x}(\ln^{\cancel{2}} x) - (\ln x - 1)\cdot 2 \cdot \cancel{\ln x} \cdot \frac{1}{x}}{\ln^{\cancel{4}\,3} x} =$$

$$= \frac{\ln x - 2\ln x + 2}{x \ln^3 x} = \frac{-\ln x + 2}{x \ln^3 x}$$

$$f''(e) = \frac{-\ln e + 2}{e \cdot \ln^3 e} = \frac{-1+2}{e \cdot 1} = \frac{1}{e} > 0 \Rightarrow$$

LOK MIN. $f(e) = \frac{e}{\ln e} = e \quad M[e, e]$

7) KONVEX. KONKÁV.

$$f'' = 0 \iff \frac{-\ln x + 2}{x \ln^3 x} = 0 \iff -\ln x + 2 = 0$$

$\Leftrightarrow \quad \ln x = 2 \quad \Leftrightarrow \quad x = e^2$ - KANDIDÁT NA IB

	$(0,1)$	$(1,e^2)$	(e^2,∞)
$-\ln x+2$	$+$	$+$	$-$
$x\cdot\ln^3 x$	$-$	$+$	$+$
	$-$	$+$	$-$
	$\cap$	$\cup$	$\cap$

8, INFL. BOD

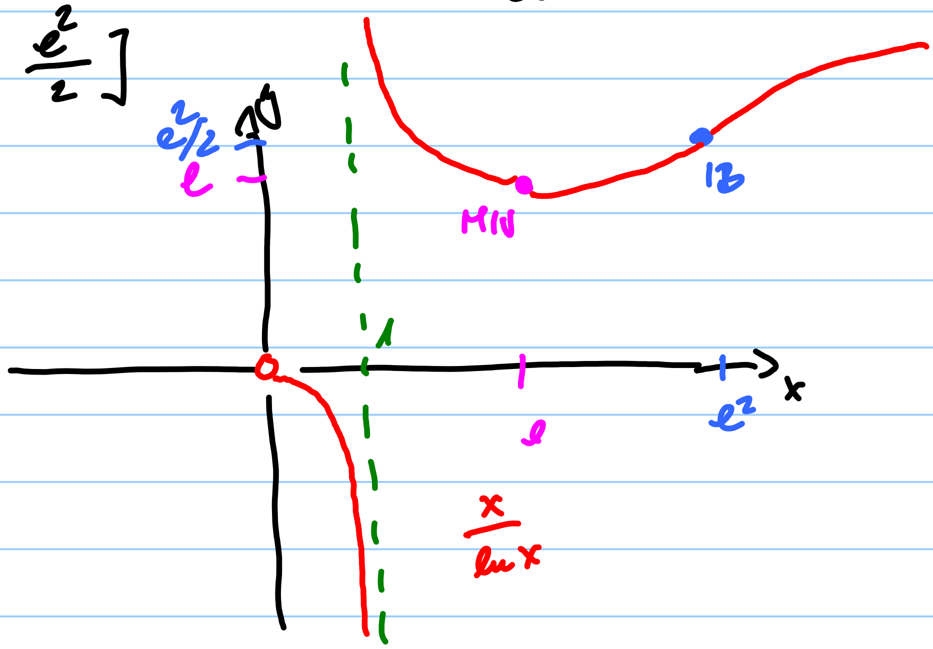
$$f''' = \frac{-\frac{1}{\cancel{x}}(\cancel{x}\ln^3 x) - (-\ln x+2)\cdot\left(1\cdot\ln^3 x + x\cdot 3\ln^2 x\cdot\frac{1}{x}\right)}{(x\ln^3 x)^2}$$

$$f'''(e^2) = \frac{-2^3 - \underbrace{(-2+2)\left(2^3 + e\cdot 3\cdot 2^2\cdot\frac{1}{e}\right)}_{\to 0}}{(e\cdot 2^3)^2} \neq 0$$

$\Rightarrow x = e^2$ JE IB $\quad f(e^2) = \frac{e^2}{\ln e^2} = \frac{e^2}{2}$

IB$\left[e^2, \frac{e^2}{2}\right]$

9, GRAF



PR: $f(x) = x \operatorname{arctg} x$

1) $D(f) = \{\forall x \in R\} = R$

NB = $f(x)=0 \Leftrightarrow x \operatorname{arctg} x = 0 \Leftrightarrow x = 0 \vee \operatorname{arctg} x = 0$

$\Leftrightarrow x = 0$

2) PÁR. NEPÁR

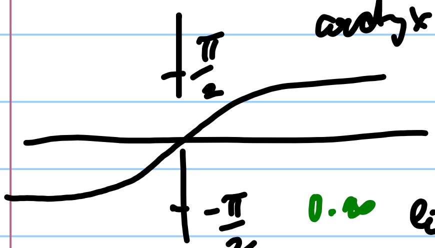
NEPÁRNA

$f(-x) = (-x)\operatorname{arctg}(-x) = (-x)\cdot(-1)\operatorname{arctg} x \quad x \cdot \operatorname{arctg} x = f(x)$

PÁRNA

3) SPOJ A ABS $\quad f(x)$ je SPOJ NA $R \Rightarrow \nexists$ ABS

4) ABS $\quad k_1 = \lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} \frac{\cancel{x}\operatorname{arctg} x}{\cancel{x}} = \frac{\pi}{2}$



$q_1 = \lim\limits_{x\to\infty}\left(x \operatorname{arctg} x - \frac{\pi}{2}x\right) =$

$0\cdot\infty \quad \lim\limits_{x\to\infty} x\cdot\left(\operatorname{arctg} x - \frac{\pi}{2}\right) = \lim\limits_{x\to\infty} \frac{\operatorname{arctg} x - \frac{\pi}{2}}{\frac{1}{x}} \overset{L'}{=} \frac{0}{0}$

$\lim\limits_{x\to\infty} \frac{\frac{1}{1+x^2}}{-x^{-2}} = \lim\limits_{x\to\infty} \frac{-x^2}{1+x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim\limits_{x\to\infty} \frac{-\frac{x^2}{x^2}}{\frac{1}{x^2} + \frac{x^2}{x^2}} = \frac{-1}{1} = -1$

$\to 1 \quad \to 0 \quad \to 1$

$y = k_1 x + q = \frac{\pi}{2}x - 1$

$$k_2 = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x \operatorname{arctg} x}{x} = \lim_{x \to -\infty} \operatorname{arctg} x = \frac{-\pi}{2}$$

$$q_2 = \lim_{x \to -\infty} \left(x \operatorname{arctg} x + \frac{\pi}{2} \right) = \lim_{x \to -\infty} \frac{\operatorname{arctg} x + \frac{\pi}{2}}{\frac{1}{x}} \overset{L'}{=}$$

$$= \lim_{x \to -\infty} \frac{-x^2}{1+x^2} = -1 \qquad y_2 = k_2 x + q_2 = \frac{-\pi}{2} x - 1$$

5) MONOTÓNOSŤ

$$f'(x) = 1 \cdot \operatorname{arctg} x + x \cdot \frac{1}{1+x^2} = \operatorname{arctg} x + \frac{x}{1+x^2}$$

$$f'(x) = 0 \iff \operatorname{arctg} x + \frac{x}{1+x^2} = 0 \iff$$

$$\operatorname{arctg} x = \frac{-x}{1+x^2} \iff x = 0 \text{ SB}$$

	$(-\infty, 0)$	$(0, \infty)$
$\operatorname{arctg} x$	$-$	$+$
x	$-$	$+$
$1+x^2$	$+$	$+$
	$-$	$+$
	$\searrow$	$\nearrow$

6) EXTR.

$$f'' = \frac{1}{1+x^2} + \frac{1 \cdot (1+x^2) - x \cdot 2x}{(1+x^2)^2} =$$

$$= \frac{1+x^2 + x^2 - 2x^2}{(1+x^2)} = \frac{1}{1+x^2} \neq 0$$

$$\Rightarrow f'(0) = \frac{1}{1+0^2} = 1 > 0 \Rightarrow \text{LOK. MIN}$$

$$f(0) = 0 \cdot \text{arctg}\, 0 = 0$$

$M[0,0]$

KONV, KONK.

$$f'' = \frac{1}{1+x^2} > 0 \Rightarrow \text{KONVEXNÁ NA } D(f)$$

$$f'' \neq 0 \Rightarrow \nexists \text{ IB}$$