Note Title 11/3/2022

POKRAČOVANIE SUBSTITUČNEJ METÓDY

(b) $\int (x+3)\sqrt{x^2+6x+1}\,dx = \left| \begin{array}{l} t = x^2+6x+1 \\ dt = (2x+6)\,dx \\ dx = \dfrac{dt}{2(x+3)} \end{array} \right| = \int \cancel{(x+3)}\sqrt{t}\,\dfrac{dt}{2\cancel{(x+3)}} =$

$= \frac{1}{2}\int t^{\frac{1}{2}}dt = \frac{1}{\cancel{2}}\,\frac{t^{\frac{3}{2}}}{\frac{3}{\cancel{2}}} + C = \frac{1}{3}(x^2+6x+1)^{\frac{3}{2}} + C$

(d) $\int x\cdot e^{1-x^2}dx = \left| \begin{array}{l} t = 1-x^2 \\ dt = -2x\,dx \\ x\,dx = \dfrac{dt}{-2} \end{array} \right| = \int e^t\frac{dt}{-2} = -\frac{1}{2}\int e^t dt =$

$= -\frac{1}{2}e^t + C = -\frac{1}{2}e^{1-x^2} + C$

(e) $\int \frac{x}{x+16}dx = \int \frac{x+16-16}{x+16}dx = \int 1 - \frac{16}{x+16}\,dx = x - 16\int\frac{1}{x+16}dx$

$\left| \begin{array}{l} t = x+16 \\ dt = dx \end{array} \right| \Rightarrow -16\int\frac{1}{t}dt = -16\ln|t| \quad = x - 16\ln|x+16| + C$

(f) $\int \frac{x}{x^2+16}dx = \left| \begin{array}{l} t = x^2+16 \\ dt = 2x\,dx \\ x\,dx = \dfrac{dt}{2} \end{array} \right| = \int \frac{1}{t}\,\frac{dt}{2} = \frac{1}{2}\int\frac{1}{t}dt =$

$= \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln(x^2+16) + C$

(g) $\int \frac{x}{x^4+16}dx = \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \\ x\,dx = \dfrac{dt}{2} \end{array} \right| = \int \frac{1}{t^2+16}\,\frac{dt}{2} =$

$x^4 \Rightarrow (x^2)^2$, $t = x^2$

$$\boxed{\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a} + C}$$

$= \frac{1}{2}\int\frac{1}{t^2+4^2}dx = \frac{1}{2}\cdot\frac{1}{4}\,\mathrm{arctg}\,\frac{t}{4} + C = \frac{1}{8}\,\mathrm{arctg}\,\frac{x^2}{4} + C$

(h) $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx = \left| \begin{array}{l} t=\sqrt{x+1}=(x+1)^{\frac{1}{2}} \\ dt=\frac{1}{2}(x+1)^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x+1}}\,dx \end{array} \right| =$

$\frac{dx}{\sqrt{x+1}} = 2dt$

$= \int e^{t}\, 2dt = 2e^{t}+c = \underline{2e^{\sqrt{x+1}}+c}$

METÓDA PER PARTES $\qquad \int u'v = u.v - \int uv'$

(a) $\int x \sin x\,dx = \left| \begin{array}{ll} v=x & v'=1 \\ u'=\sin x & u=-\cos x \end{array} \right| =$

$= -x\cos x - \int 1.(-\cos x)\,dx = -x\cos x + \int \cos x\,dx =$

$= \underline{-x\cos x + \sin x + c}$

2PP

(b) $\int x^2 e^{3x}dx = \left| \begin{array}{ll} v=x^2 & v'=2x \\ u'=e^{3x} & u=\frac{e^{3x}}{3} \end{array} \right| = \frac{x^2e^{3x}}{3} - \frac{2}{3}\int x e^{3x}dx =$

$= \left| \begin{array}{ll} v=x & v'=1 \\ u'=e^{3x} & u=\frac{e^{3x}}{3} \end{array} \right| = \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \int \frac{e^{3x}}{3}dx\right] =$

$= \frac{x^2e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2}{9}\cdot\frac{e^{3x}}{3} + c = \underline{\frac{x^2e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + c}$

(c) $\int x\ln(x^2)\,dx \qquad \cancel{\left| \begin{array}{ll} v=x & v'=1 \\ u'=\ln(x^2) & \end{array} \right|} \qquad \left| \begin{array}{ll} v=\ln(x^2) & v'=\frac{1}{x^2}\cdot 2x \\ u'=x & u=\frac{x^2}{2} \end{array} \right|$

$= \frac{x^2}{2}\ln(x^2) - \int \frac{\cancel{2}x}{\cancel{x^2}}\cdot\frac{\cancel{x^2}}{\cancel{2}}\,dx = \underline{\frac{x^2}{2}\ln(x^2) - \frac{x^2}{2} + c}$

(d) $\int 1.\mathrm{arctg}\,x\,dx = \left| \begin{array}{ll} v=\mathrm{arctg}\,x & v'=\frac{1}{1+x^2} \\ u'=1 & u=x \end{array} \right| = x\,\mathrm{arctg}\,x - \int \frac{x}{1+x^2}dx$

$= x\,\mathrm{arctg}\,x - \frac{1}{2}\int \frac{2x}{1+x^2}dx = \underline{x\,\mathrm{arctg}\,x - \frac{1}{2}\ln(1+x^2)+c}$

(e) $\int 1.\ln(x^2+1)\,dx = \left| \begin{array}{ll} v=\ln(x^2+1) & v'=\frac{1}{x^2+1}\cdot 2x \\ u'=1 & u=x \end{array} \right|$

$$= x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}\,dx = x\ln(x^2+1) - 2\int \frac{x^2}{x^2+1}\,dx$$

$$\cancel{\left|\begin{array}{l} x^2 = t \\ 2x\,dx = dt \\ dx = \frac{dt}{2x} \end{array}\right. \Rightarrow \int \frac{t}{t+1}\,\frac{dt}{2x}}\ \Bigg| = x\ln(x^2+1) - 2\int \frac{x^2+1-1}{x^2+1}\,dx$$

$$= x\ln(x^2+1) - 2\int 1 - \frac{1}{x^2+1}\,dx = \underline{\underline{x\ln(x^2+1) - 2x + 2\,\mathrm{arctg} + c}}$$

f) $\underline{\underline{\overbrace{\int e^{2x}\cos x\,dx}^{I}}} = \left|\begin{array}{ll} u = e^{2x} & u' = e^{2x}\cdot 2 \\ v' = \cos x & v = \sin x \end{array}\right| =$

$\boxed{2PP}$

$$= e^{2x}\sin x - 2\int e^{2x}\sin x\,dx = \left|\begin{array}{ll} u = e^{2x} & u' = e^{2x}\cdot 2 \\ v' = \sin x & v = -\cos x \end{array}\right| =$$

$$= e^{2x}\sin x - 2\left[-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx\right] =$$

$$\underline{\underline{= e^{2x}\sin x + 2e^{2x}\cos x - 4\overbrace{\int e^{2x}\cos x\,dx}^{I}}}$$

$$\Rightarrow I = \underbrace{e^{2x}\sin x + 2e^{2x}\cos x}_{} - 4I$$

$$5I = \quad -\,''\,-$$

$$\int e^{2x}\cos x\,dx = \frac{e^{2x}\sin x + 2e^{2x}\cos x}{5} + C$$