CVIČENIE MA 2. ZS 2022 - 2023

DELENIE POLYNÓMOV:

PR: $(x^3-1):(x+1) = x^2-x+1$

$$
\begin{array}{l}
-x^3 \mp x^2 \\
\hline
0 - x^2 \\
\quad + x^2 + x \\
\hline
\quad 0 + x - 1 \\
\quad\quad - x \mp 1 \\
\hline
\quad\quad 0 - 2
\end{array}
$$

$$= x^2 - x + 1 - \frac{2}{x+1}$$

HORNEROVA SCHÉMA

PR: $x^3 - x^2 - 8x + 12 = 0$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

	1	−1	−8	12
−1	$\square$	−1	2	6
	1	−2	−6	18

	1	-1	-8	12
2	□	2	2	-12
	1	1	-6	0
2	□	2	6	
	1	3	0	

$$(x-2)(x-2)\cdot(x+3)=0$$

92: $4x^3-4x^2+7x-3=0$

$\pm 1 \quad \pm 2 \quad \pm 4 \qquad\qquad \pm 1 \quad \pm 3$

$$k=\{-3,-1,1,3,\pm\tfrac{1}{2},\pm\tfrac{3}{2},\pm\tfrac{1}{4},\pm\tfrac{3}{4}\}$$

	4	-4	+7	-3
$\frac{1}{2}$	□	2	-1	3
	4	-2	6	0

$$\left(x-\tfrac{1}{2}\right)\left(4x^2-2x+6\right)=0$$

$$2\left(x-\tfrac{1}{2}\right)\left(2x^2-x+3\right)=0$$

PARCIÁLNE ZLOMKY :

PR:
$$\frac{2x-5}{x^2-5x+6} = \frac{2x-5}{(x-3)(x-2)} = \frac{A}{(x-3)} + \frac{B}{(x-2)} =$$

$$2x-5 = A(x-2) + B(x-3)$$

$$2x - 5 = Ax - 2A + Bx - 3B$$

$$2 = A + B \quad / \cdot 2$$

$$-5 = -2A - 3B$$

$$4 = 2A + 2B$$

$$-5 = -2A - 3B$$

$$-1 = 0 - B \Rightarrow B = 1$$

DOSAD'

$$2 = A + 1 \Rightarrow A = 1$$

$$= \frac{1}{(x-3)} + \frac{1}{(x-2)}$$

SK:
$$\frac{x-2+x-3}{(x-3)(x-2)} = \frac{2x-5}{(x-3)(x-2)} \checkmark$$

LIMITA FUNKCIE

DÔLEŽITÉ LIMITY

LIMITY V NEKONEČNE

$$\lim_{x \to \pm\infty} \frac{1}{x} = 0$$

$$\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to \infty} \sqrt[x]{x} = 1 \qquad x > 0$$

LIMITY V PEVNOM BODE

$$\lim_{x \to 0} \sin \frac{1}{x} = \nexists$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \ln e = 1$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a \qquad a > 0$$

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$

Typ $a^2 - b^2$

$\infty \cdot 0$ 9Z: $\lim\limits_{x \to \infty} x\left(\sqrt{x^2+4} - x\right) \cdot \frac{\sqrt{x^2+4}+x}{\sqrt{x^2+4}+x} =$

$$= \lim_{x\to\infty} x \frac{\left(\left(\sqrt{x^2+4}\right)^2 - x^2\right)}{\sqrt{x^2+4}+x} =$$

$$= \lim_{x\to\infty} \frac{x\left(\cancel{x^2}+4\cancel{-x^2}\right)}{\sqrt{x^2+4}+x} =$$

$$= \lim_{x\to\infty} \frac{4x}{\sqrt{x^2+4}+x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} =$$

$$= \lim_{x\to\infty} \frac{\frac{4x}{x} \to 4}{\sqrt{\frac{x^2}{x^2} + \frac{4}{x^2}} + \frac{x}{x} \to 1} = \frac{4}{\sqrt{1}+1} = 2$$

Typ: $\lim\limits_{x\to\infty} (\alpha x)^{\beta x} = A^B$

a^∞ $\lim\limits_{x\to\infty}\left(\frac{x+1}{2x+1}\right)^{x^2} = \left(\lim\limits_{x\to\infty} \frac{x+1}{2x+1}\right)^{\lim\limits_{x\to\infty} x^2} =$

$$= \left(\frac{1}{2}\right)^\infty = \frac{1}{2^\infty} = 0$$

TYP $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

Př: $\lim\limits_{x\to\infty}\left(\frac{x-1}{x+1}\right)^x = \lim\limits_{x\to\infty}\left(\frac{x+1-1-1}{x+1}\right)^x =$

$1 \qquad \frac{-2}{x+1}$

$= \lim\limits_{x\to\infty}\left(1-\frac{2}{x+1}\right)^x = \lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x+1}{-2}}\right)^x =$

$\frac{x+1}{-2} = t$

$x \to \infty$

$t \to -\infty$

$x = -2t-1$

$= \lim\limits_{t\to-\infty}\left(1+\frac{1}{t}\right)^{-2t-1} =$

$= \lim\limits_{t\to-\infty}\left(\left(1+\frac{1}{t}\right)^t\right)^{\frac{1}{t}(-2t-1)} =$

$\to e$

$= e^{\lim\limits_{t\to-\infty}\frac{-2t-1}{t}} = e^{\lim\limits_{t\to-\infty} -2-\frac{1}{t}} = e^{-2} = \frac{1}{e^2}$

$\frac{1}{t} \to 0$

LIMITY V PEVNOM BODE

$\frac{0}{0}$

$\lim\limits_{x\to 1}\frac{x-1}{x^2-1} = \lim\limits_{x\to 1}\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} =$

$= \lim\limits_{x\to 1}\frac{1}{x+1} = \frac{1}{2}$

9R. $\lim_{x\to\infty}\left(\frac{1}{x}\right)^{\frac{1}{\ln x}} = \lim_{x\to\infty}\left(e^{\ln\frac{1}{x}}\right)^{\frac{1}{\ln x}} =$

$= \lim_{x\to\infty}\left(e^{\ln x^{-1}}\right)^{\frac{1}{\ln x}} = \lim_{x\to\infty}\left(e^{-\ln x}\right)^{\frac{1}{\ln x}} =$

$\lim_{x\to\infty} e^{\frac{-\ln x}{\ln x}} = e^{-1}$