Priebeh funkcie

Note Title 23/10/2022

1. $D(f)$ A NULOVÉ BODY
2. PÁRNOSŤ A NEPÁRNOSŤ
3. SPOJITOSŤ A ABS
4. ASS
5. MONOTÓNNOSŤ
6. EXTRÉMY
7. KONVEXNOSŤ KONKÁVNOSŤ
8. INFLEXNÉ BODY
9. GRAF

$$f(x)=\frac{2x^3}{x^2-1}$$

1) $D(f)=\{\forall x\in R;\ x^2-1\neq 0\} \Rightarrow x^2\neq 1 \Leftrightarrow x\neq \pm 1 = R-\{\pm 1\}$

NB $f(x)=0 \Leftrightarrow 2x^3=0 \Leftrightarrow x=0$

2) PÁRNOSŤ, NEPÁRNOSŤ

$f(-x)=f(x)$ PÁRNA

$f(-x)=-f(x)$ NEPÁRNA

$$f(-x)=\frac{2(-x)^3}{(-x)^2-1}=\frac{-2x^3}{x^2-1}=-f(x) \Rightarrow \text{NEPÁRNA}$$

3) SPOJITOSŤ $f(x)$ JE NESPOJITÁ V $x=-1$ a $x=1$

ABS:

$$\lim_{x\to -1^-}\frac{\overset{\to -1}{2x^3}}{\underset{\to 0^+}{x^2-1}}=\frac{-1}{0^+}=-\infty \qquad \lim_{x\to -1^+}\frac{\overset{\to -1}{2x^3}}{\underset{\to 0^-}{x^2-1}}=\frac{-1}{0^-}=\infty$$

$$\lim_{x\to 1^-}\frac{\overset{\to 1}{2x^3}}{\underset{\to 0^-}{x^2-1}}=\frac{1}{0^-}=-\infty \qquad \lim_{x\to 1^+}\frac{\overset{\to 1}{2x^3}}{\underset{\to 0^+}{x^2-1}}=\frac{1}{0^+}=\infty$$

4) ASS $k_1=\lim_{x\to\infty}\frac{f(x)}{x}$ $\qquad q_1=\lim_{x\to\infty}(f(x)-q_1x)$

$k_2=\lim_{x\to-\infty}\frac{f(x)}{x}$ $\qquad q_2=\lim_{x\to-\infty}(f(x)-q_2x)$

$y_1=k_1x+q_1$ $\qquad y_2=k_2x+q_2$

$$k_1 = \lim_{x\to\infty} \frac{\frac{2x^3}{x^2-1}}{x} = \lim_{x\to\infty} \frac{2x^3}{x^3-x} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{2\frac{x^3}{x^3}}{\frac{x^3}{x^3} - \frac{x}{x^3}} = \frac{2\cdot 1}{1-0} = 2$$

$$k_2 = \lim_{x\to-\infty} \frac{\frac{2x^3}{x^2-1}}{x} = \lim_{x\to-\infty} \frac{2x^3}{x^3-x} = 2 = k_1$$

$$q_1 = \lim_{x\to\infty} \left(\frac{2x^3}{x^2-1} - 2\cdot x\right) = \lim_{x\to\infty} \frac{2x^3 - 2x(x^2-1)}{x^2-1} =$$

$$= \lim_{x\to\infty} \frac{2x^3 - 2x^3 + 2x}{x^2-1} = \lim_{x\to\infty} \frac{2x}{x^2-1} = 0$$

$$q_2 = \lim_{x\to-\infty} \left(\frac{2x^3}{x^2-1} - 2\cdot x\right) = \lim_{x\to\infty} \frac{2x}{x^2-1} = 0$$

$$y_1 = k_1 x + q_1 = 2x$$

5) MONOTÓNNOSŤ

$f'(x) > 0$ RASTÚCA

$f'(x) < 0$ KLESAJÚCA

$f'(x) = 0$ STACIONÁRNY BOD

$$f'(x) = \left(\frac{2x^3}{x^2-1}\right)' = \frac{2\cdot 3x^{3-1}\cdot(x^2-1) - 2x^3(2x^{2-1}-0)}{(x^2-1)^2} = \frac{6x^2(x^2-1) - 2x^3\cdot 2x}{(x^2-1)^2} =$$

$$= \frac{6x^4 - 6x^2 - 2x^4}{(x^2-1)^2} = \frac{2x^4 - 6x^2}{(x^2-1)^2} = \frac{2x^2(x^2-3)}{(x^2-1)^2}$$

$$f'(x) = 0 \Leftrightarrow 2x^2(x^2-3) = 0 \Leftrightarrow x = 0 \lor |x| = \sqrt{3}$$

	$(-\infty, -\sqrt{3})$	$(-\sqrt{3}, 0)$	$(0, \sqrt{3})$	$(\sqrt{3}, \infty)$
$2x^2$	+	+	+	+
x^2-3	+	−	−	+
$(x^2-1)^2$	+	+	+	+
	+	−	−	+
	↗	↘	↘	↗

6) EXTRÉMY $\quad f''(SB) > 0 \Rightarrow$ LOKÁLNE MINIMUM

$f''(SB) < 0 \Rightarrow$ LOKÁLNE MAXIMUM

$f''(SB) = 0 \Rightarrow$ KANDIDÁT NA INFL. BOD

$$f'' = \frac{(8x^3-12x)(x^2-1)^{\cancel{2}} - (2x^4-6x^2)\,2\cdot\cancel{(x^2-1)}\cdot 2x}{((x^2-1)^2)^{\cancel{2}\,3}} =$$

$$= \frac{\cancel{8x^5} - 8x^3 - 12x^3 + 12x - \cancel{8x^5} + 24x^3}{(x^2-1)^3} = \frac{4x^3+12x}{(x^2-1)^3} =$$

$$= \frac{4x(x^2+3)}{(x^2-1)^3}$$

$$f''(-\sqrt{3}) = \frac{4\cdot(-\sqrt{3})(3+3)}{(3-1)^3} = \frac{-24\sqrt{3}}{8} = -3\sqrt{3} < 0 \quad \text{LOK. MAX}$$

$$f''(\sqrt{3}) = \frac{4(\sqrt{3})(3+3)}{(3-1)^3} = \frac{24\sqrt{3}}{8} = 3\sqrt{3} > 0 \quad \text{LOK MIN}$$

$$f(-\sqrt{3}) = \frac{2(-\sqrt{3})^3}{((-\sqrt{3})^2-1)} = \frac{-6\sqrt{3}}{2} = -3\sqrt{3} \qquad MAX[-\sqrt{3}, -3\sqrt{3}]$$

$$f(\sqrt{3}) = \frac{2(\sqrt{3})^3}{(\sqrt{3})^2-1} = \frac{6\sqrt{3}}{2} = 3\sqrt{3} \qquad MIN[\sqrt{3}, 3\sqrt{3}]$$

7) KONVEXNOSŤ KONKÁVNOSŤ $\quad f''(x) > 0$ KONVEXNÁ

$f''(x) < 0$ KONKÁVNA

$f''(x) = 0$ KANDIDÁT NA IB

$$f''(x) = 0 \Leftrightarrow 4x(x^2+3) = 0 \Leftrightarrow x = 0$$

	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
$4x$	$-$	$-$	$+$	$+$
x^2+3	$+$	$+$	$+$	$+$
	$-$	$-$	$+$	$+$
	$\cap$	$\cap$	$\cup$	$\cup$

8) INFLEXNÝ BOD $\quad f^{(n)}(x_{IB}) \neq 0 \quad n \geq 2$

n - NEPÁR => IB

n - PÁRNE => EXTR

$$f'''(x) = \frac{(8x^2+12)(x^2-1)^{\cancel{3}} - (4x^3+12x)\cdot 3\cancel{(x^2-1)^2}\cdot 2x}{\left((x^2-1)^3\right)^{\cancel{2}\,3}} =$$

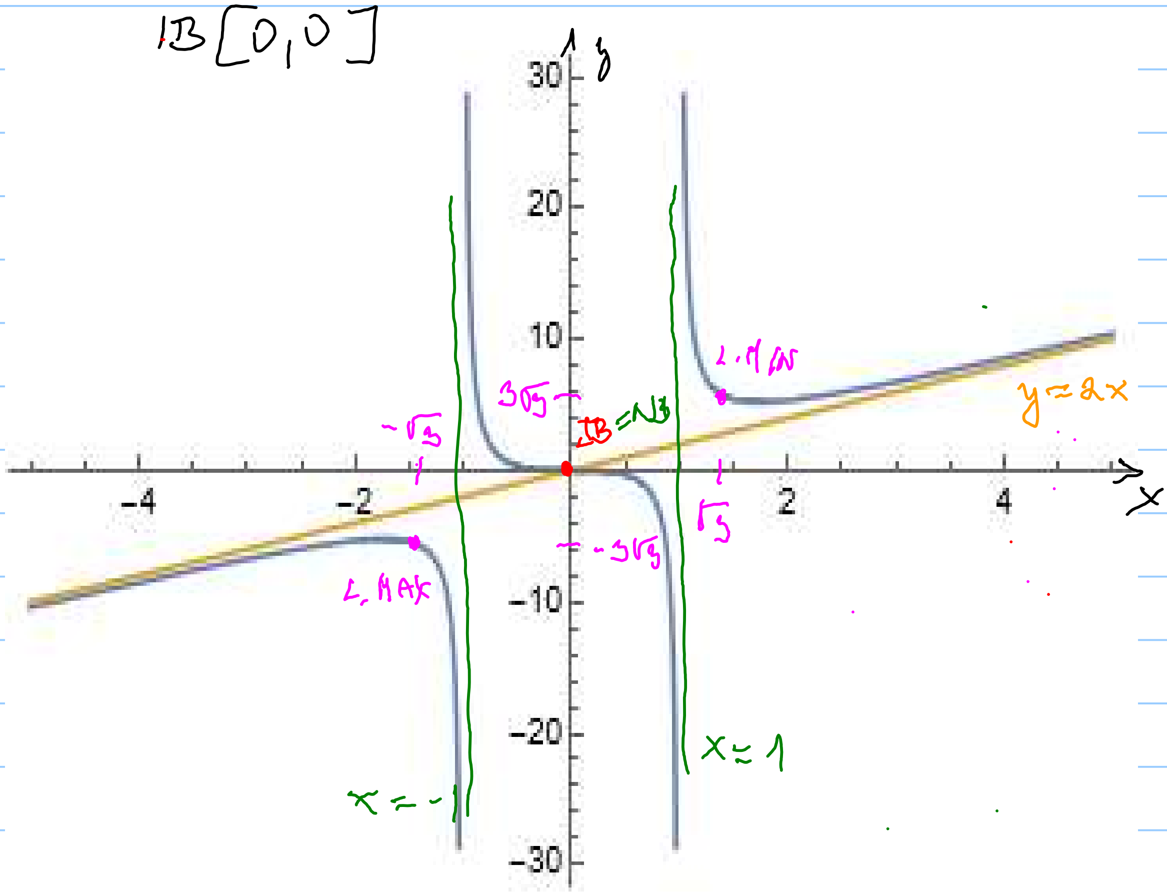
$$= \frac{8x^4+12x^2-8x^2-12-24x^4-72x}{(x^2-1)^3} =$$

$$= \frac{-16x^4+4x^2-72x-12}{(x^2-1)^3}.$$

$$f'''(0) = \frac{-0+0-0-12}{(0-1)^3} = 12 \neq 0 \Rightarrow IB$$

$$f(0) = \frac{2\cdot 0}{(0^2-1)} = 0$$

IB $[0,0]$



PR:

$f(x) = 16x(x-1)^3$

1) $D(f) = \{\forall x \in R ; -\} = R$

NB: $f(x) = 0 \iff x = 0 \vee x = 1$

$NB_1[0,0]$ $NB_2[1,0]$

2) PÁR NEPÁR

$f(-x) = 16(-x)(-x-1)^3 = -16x(-1)(x+1) =$

$= 16x(x+1)^3 \neq f(x)$
$\neq -f(x)$ } ANI ANI.

3) SPOJITOST $f(x)$ SPOJ NA R $\Rightarrow \nexists$ ABS

4) ASS

$k_1 = \lim_{x\to\infty} \frac{16x(x-1)^3}{x} = \lim_{x\to\infty} 16(x-1)^3 = \infty$

$k_2 = \lim_{x\to-\infty} \frac{16(x-1)^3}{x} = \lim_{x\to-\infty} 16(x-1)^3 = -\infty$

$\Rightarrow \nexists$ ASS

5) MONOTÓNNOSŤ

$f'(x) = 16(x-1)^3 + 16x \cdot 3(x-1)^2 \cdot 1 =$

$= (x-1)^2(16x - 16 + 48x) = (x-1)^2\, 16(4x-1)$

SB: $f' = 0 \iff x = 1 \vee x = \frac{1}{4}$

	$(-\infty; \frac{1}{4})$	$(\frac{1}{4}, 1)$	$(1, \infty)$
$(x-1)^{-2}$	+	+	+
$4x-1$	−	+	+
	−	+	+
	↘	↗	↗

6) EXTRÉMY

$$f'' = 2(x-1)16(4x-1) + (x-1)^2 16 \cdot 4 = (x-1)(128x - 32 + 64x - 64) =$$
$$= (x-1)16(12x-6)$$

$$f''\left(\tfrac{1}{4}\right) = \left(\tfrac{1}{4}-1\right)16\left(12 \cdot \tfrac{1}{4} - 6\right) = -\tfrac{3}{4} \cdot 16(-3) = 54 > 0 \text{ LOK. MIN}$$

$$f''(1) = (1-1) \cdot 16(12 \cdot 1 - 6) = 0$$

$$f\left(\tfrac{1}{4}\right) = 16 \cdot \tfrac{1}{4}\left(\tfrac{1}{4}-1\right)^3 = 4 \cdot \tfrac{-27}{64} = \tfrac{-27}{16} \qquad \text{MAX}\left[\tfrac{1}{4}, -\tfrac{27}{16}\right]$$

7) KONVEX. KONKÁVN.

$$f'' = 0 \Leftrightarrow (x-1) = 0 \Rightarrow x = 1 \quad \vee \quad 12x - 6 = 0 \Rightarrow x = \tfrac{1}{2}$$

	$(-\infty, \frac{1}{2})$	$(\frac{1}{2}, 1)$	$(1, \infty)$
$x-1$	−	−	+
$12x-6$	−	+	+
	+	−	+
	∪	∩	∪

8) INFL. BOD

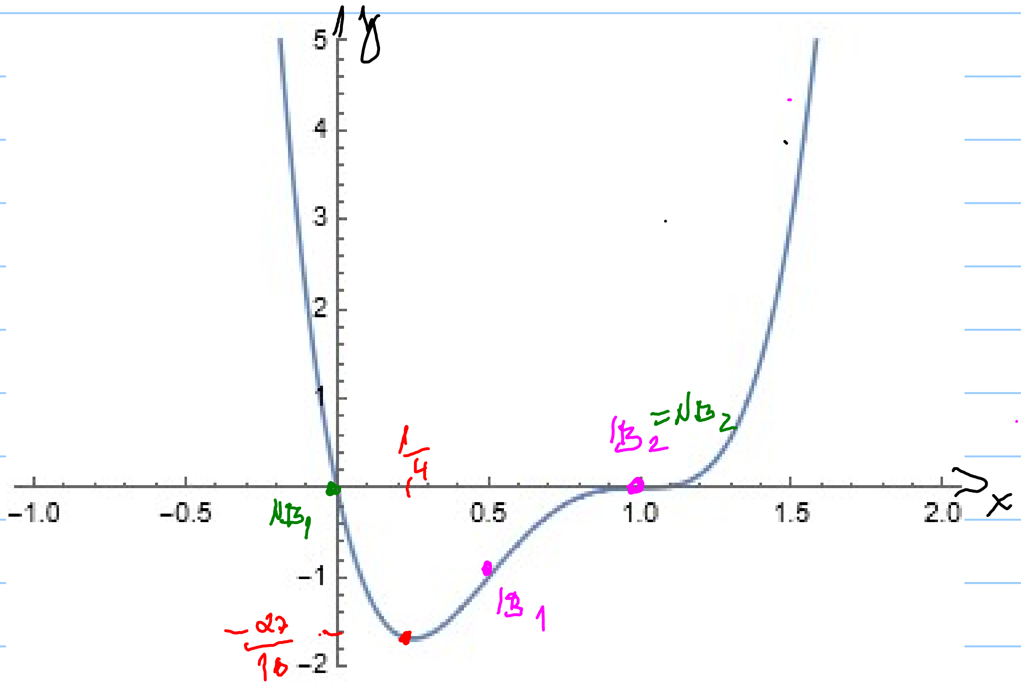
$$f''' = 1 \cdot 16(12x-6) + (x-1)16 \cdot 12 = 192x - 96 + 192x - 192$$
$$= 384x - 288$$

$$f'''\left(\tfrac{1}{2}\right) = 192 - 288 = -96 \neq 0$$

$$f'''(1) = 384 - 288 = 98 \neq 0$$

$$f\left(\tfrac{1}{2}\right) = 16\cdot\tfrac{1}{2}\left(\tfrac{1}{2}-1\right)^3 = 8\cdot\frac{-1}{8} = -1 \qquad IB_1\left[\tfrac{1}{2},-1\right]$$

$$f(1) = 16\cdot 1\cdot(1-1)^3 = 0 \qquad IB_2[1,0]$$



PR $f(x) = \dfrac{\ln x}{\sqrt{x}}$

1) $D(f) = \{\forall x \in R,\ x > 0\} = (0,\infty)$

NB $f(x) = 0 \iff \ln x = 0 \iff x = 1 \qquad NB[1,0]$

2) PÁR, NEPÁR - ANI PÁR, ANI NEPÁR, LEBO D(f) NESYMETR.

3) SPOJ. ABS

$$\lim_{x\to 0^+} \frac{\ln x}{\sqrt{x}} \stackrel{L'}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to 0^+} \frac{2\sqrt{x}}{x} =$$

$$= \lim_{x\to 0^+} \frac{2}{\sqrt{x}} = \infty$$

4) ASS: $k_1 = \lim_{x\to\infty} \frac{\frac{\ln x}{\sqrt{x}}}{x} \overset{L}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1\cdot\sqrt{x} + x\cdot\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2}{3x\sqrt{3}}$

$= 0$

$q_1 = \lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} \overset{L}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x} =$

$= \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0 \Rightarrow$ ASS: $y = 0$

5) MONOT.

$f'(x) = \frac{\frac{1}{x}\cdot\sqrt{x} - \ln x \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = \frac{2-\ln x}{2\sqrt{x}\cdot x}$

SB: $f' = 0 \Leftrightarrow 2 - \ln x = 0 \Leftrightarrow x = e^2$

	$(0, e^2)$	(e^2, ∞)
$2-\ln x$	$-$	$+$
$2\sqrt{x}\cdot x$	$+$	$+$
	$-$	$+$
	$\searrow$	$\nearrow$

6) $f'' = \frac{-\frac{1}{\cancel{x}}(2\sqrt{x}\cdot\cancel{x}) - (2-\ln x)2\cdot\frac{3}{2}x^{\frac{1}{2}}}{(2\sqrt{x}\cdot x)^2} =$

$= \frac{-\sqrt{x}(2+6-3\ln x)}{4x^3} = \frac{-\sqrt{x}(8-3\ln x)}{4x^3}$

$f''(e^2) = \frac{-\sqrt{e}(8-6)}{4e^3} = \frac{-2\sqrt{e}}{4e^3} = \frac{-1}{2e^{5/2}} < 0$ LOC MAX

$f(e^2) = \frac{\ln e^2}{\sqrt{e^2}} = \frac{2}{e}$ MAX $[e^2, \frac{2}{e}]$

7) KONV. KONK:

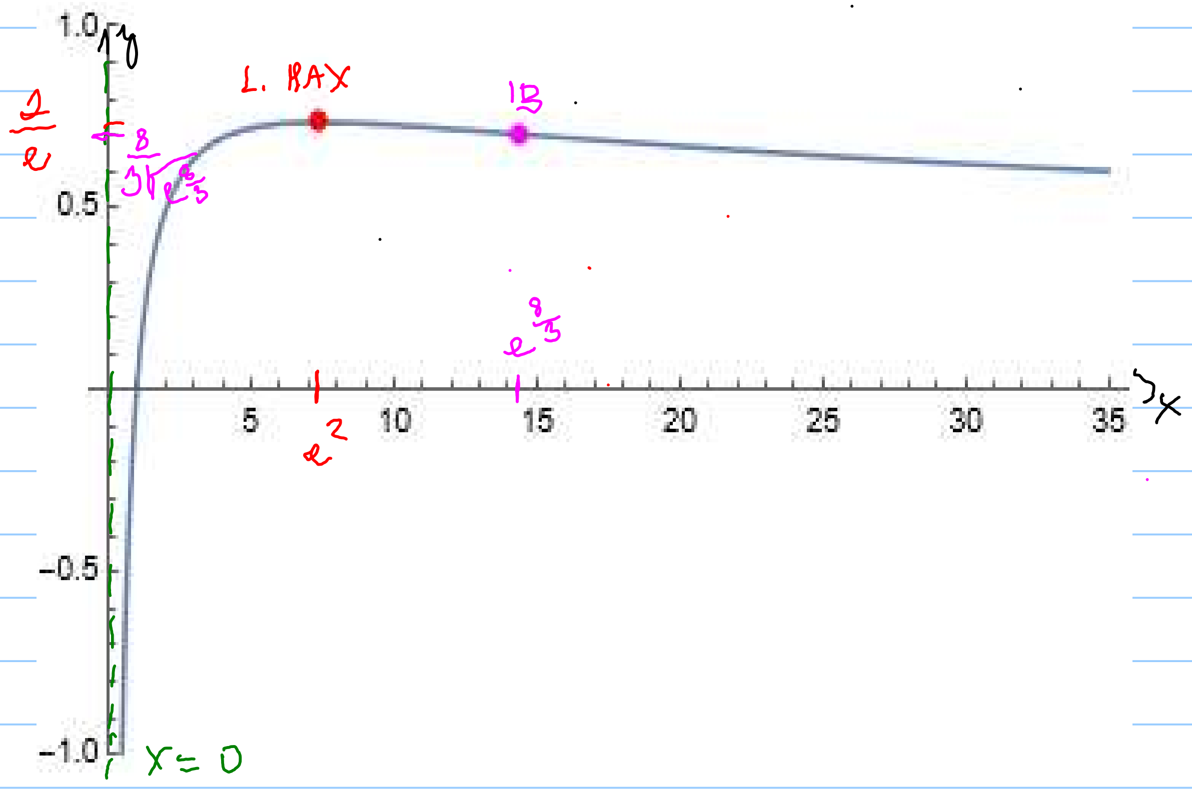
$f''=0 \Leftrightarrow -\sqrt{x}\,(8-3\ln x)=0 \Leftrightarrow -\sqrt{x}=0 \Leftrightarrow$

$x=0 \notin D(f) \vee 8-3\ln x=0 \Leftrightarrow \ln x=\frac{8}{3} \Leftrightarrow x=e^{\frac{8}{3}}$

	$(0,e^{\frac{8}{3}})$	$(e^{\frac{8}{3}},\infty)$
$-\sqrt{x}$	$-$	$-$
$8-3\ln x$	$+$	$-$
	$-$	$+$
	$\cap$	$\cup$

8) IB $f(x)$ JE SPOJ NA $(0,\infty) \Rightarrow$ A $\vee$ $x=e^{\frac{8}{3}}$ ZĽAVA JE KONKÁVNA, SPRAVA KONVEXNÁ $\Rightarrow$ JE V $x=e^{3}$ INFLEXNÝ BOD

$f(e^{\frac{8}{3}})=\frac{\ln e^{\frac{8}{3}}}{\sqrt{e^3}}=\frac{8}{3\sqrt{e^{\frac{8}{3}}}}$ IB $\left[e^{\frac{8}{3}},\frac{8}{3\sqrt{e^{8/3}}}\right]$



PŘ $f(x) = \ln(4-x^2)$

1) $D(f) = \{\forall x \in \mathbb{R} ; \ 4-x^2 > 0\} \Rightarrow x^2 < 4 \Leftrightarrow x \in (-2,2)$

NB $f(x)=0 \Leftrightarrow 4-x^2 = 1 \Leftrightarrow x^2 = 3 \Leftrightarrow x = \pm\sqrt{3}$

2) PAŘ, $f(-x) = \ln(4-(-x)^2) = \ln(4-x^2) = f(x) = $ PAŘ

3) SPOJ A ABS

$$\left.\begin{array}{l} \lim\limits_{x\to -2^+} \ln(4-x^2) = -\infty \\ \lim\limits_{x\to 2^-} \ln(4-x^2) = -\infty \end{array}\right\} \text{ABS} \quad \begin{array}{l} x=-2 \\ x=2 \end{array}$$

4) ABS $\nexists$ LEBO $D(f) = (-2,2)$

5) $f'(x) = \dfrac{1}{4-x^2} \cdot 2x = \dfrac{-2x}{4-x^2}$

SB: $f'=0 \Leftrightarrow -2x=0 \Leftrightarrow x=0$

	$(-2,0)$	$(0,2)$
$-2x$	+	−
$4-x^2$	+	+
	+ ↗	− ↘

6) $f'' = \dfrac{-2(4-x^2)+2x(-2x)}{4-x^2} = \dfrac{-8+2x^2-4x^2}{4-x^2} = \dfrac{-2(4+x^2)}{4-x^2}$

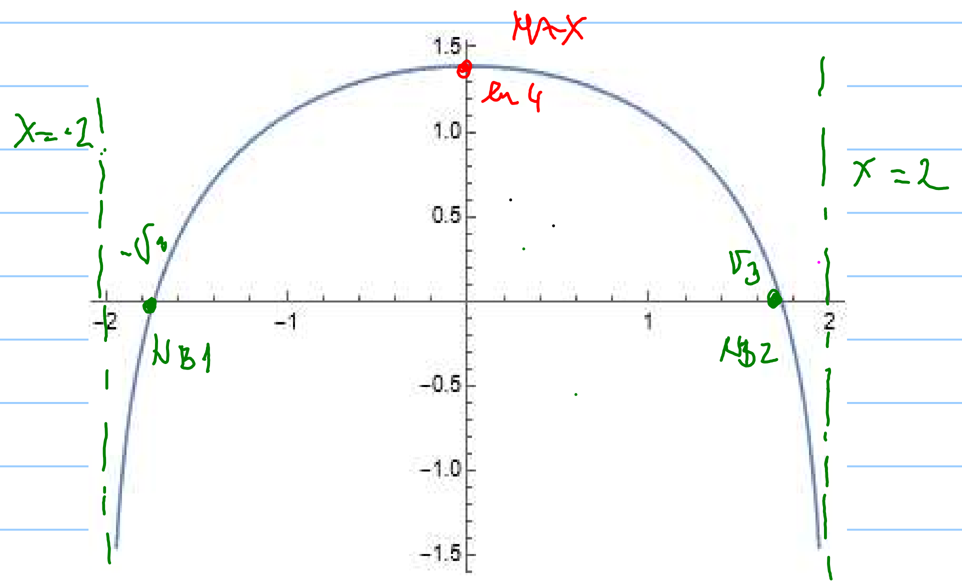
$f''(0) = \dfrac{-2\cdot(4-0^2)}{4-0^2} = -2 < 0$ LOK MAX

$f(0) = \ln(4-0^2) = \ln 4$

7)

$f'' = 0 \iff -2(4+x^2) = 0$ NIKDY $\Rightarrow \nexists$ KIB

$f''(x)$ NA $D(f) < 0 \Rightarrow$ KONKÁVNA



PR $f(x) = x - 2\operatorname{arctg} x$

1 $D(f) = \{\forall x \in \mathbb{R}\} = \mathbb{R}$

NB $f(x) = 0 \iff \operatorname{arctg} x = \frac{x}{2} \iff x = 0$

2 PAR. NEP. $f(-x) = -x - 2\operatorname{arctg}(-x) = -x + 2\operatorname{arctg} x =$

$= -(x - 2\operatorname{arctg} x) = -f(x)$ NEPÁRNA.

3 SPOJ. ABS $f(x)$ JE SPOJ NA $\mathbb{R} \Rightarrow \nexists$ ABS

4 ASS $k_1 = \lim_{x\to\infty} \frac{x - 2\operatorname{arctg} x}{x} \overset{L'}{=} \lim_{x\to\infty} \frac{1 - \frac{2}{1+x^2}}{1} = 1 = k_2$

$$q_1 = \lim_{x\to\infty}\left(x - 2\operatorname{arctg} x - 1\cdot x\right) = \lim_{x\to\infty} -2\operatorname{arctg} x = -\pi$$

$$q_2 = \lim_{x\to-\infty}\left(x - 2\operatorname{arctg} x - x\right) = \lim_{x\to-\infty} -2\operatorname{arctg} x = \pi$$

$y_1: 1\cdot x - \pi$ $\quad$ $y_2: = x + \pi$

5) $f'(x) = 1 - \dfrac{2}{1+x^2} = \dfrac{1+x^2-2}{1+x^2} = \dfrac{x^2-1}{1+x^2}$

SB: $f'(x) = 0 \iff x^2 - 1 = 0 \iff x = \pm 1$

	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
x^2-1	+	−	+
$1+x^2$	+	+	+
	+	−	+
	↗	↘	↗

6) $f'' = \dfrac{2x(1+x^2) - (x^2-1)\cdot 2x}{(1+x^2)^2} = \dfrac{2x + 2x^3 - 2x^3 + 2x}{(1+x^2)^2} =$

$= \dfrac{4x}{(1+x^2)^2}$

$f''(-1) = \dfrac{4\cdot(-1)}{(1+(-1)^2)^2} = \dfrac{-4}{4} = -1 < 0$ LOK MAX $[-1, -1+\frac{\pi}{2}]$

$f''(1) = \dfrac{4\cdot 1}{(1+1^2)^2} = \dfrac{4}{4} = 1 > 0$ LOK MIN $[1, 1-\frac{\pi}{2}]$

$f(-1) = -1 - 2\operatorname{arctg}(-1) = -1 + 2\frac{\pi}{4} = -1 + \frac{\pi}{2}$

$f(1) = 1 - 2\operatorname{arctg}(1) = 1 - \frac{\pi}{2}$

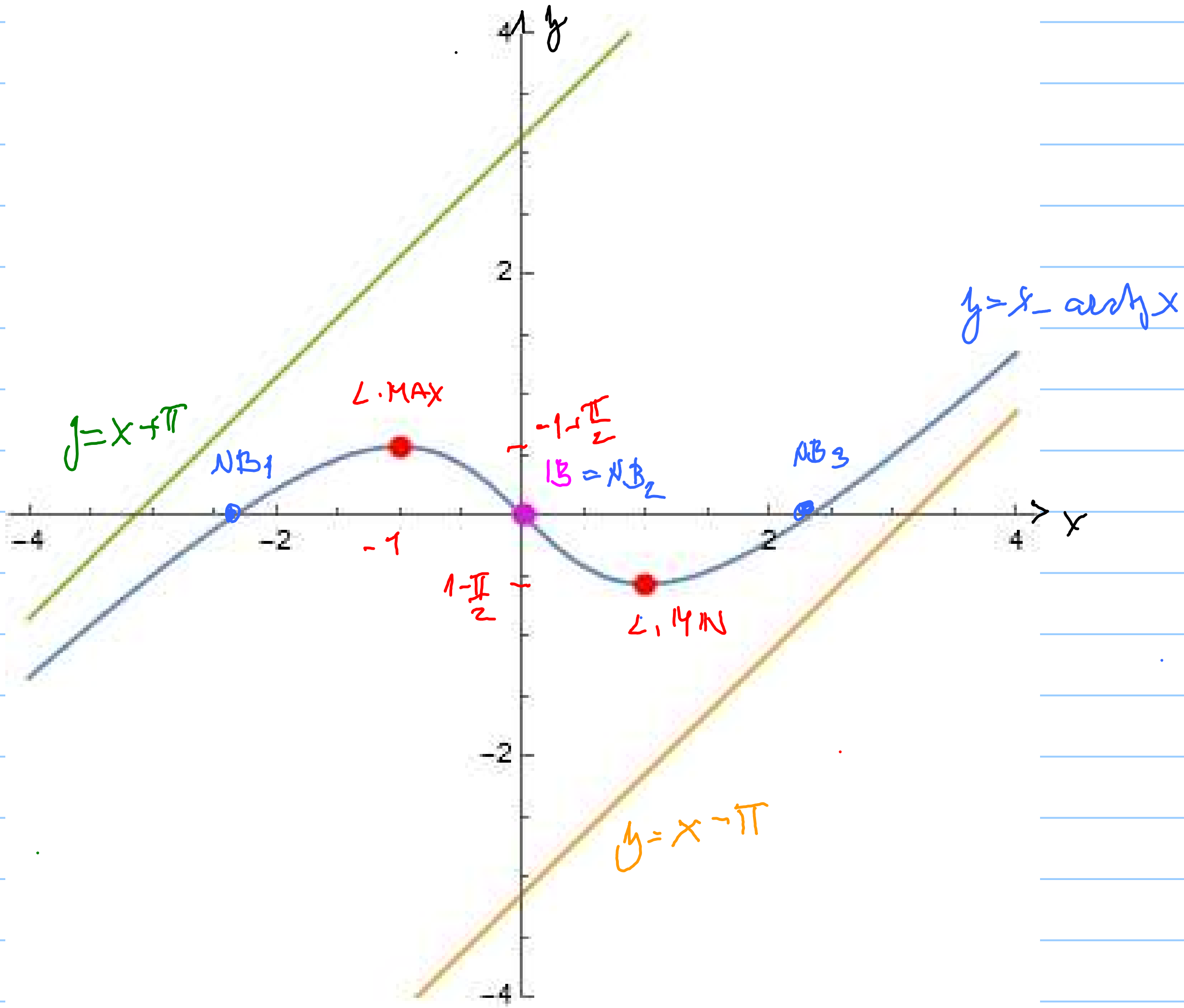
7 $f''(x) = 0 \iff 4x^2 = 0 \iff x = 0$

	$(-\infty, 0)$	$(0, \infty)$
$4x^2$	$-$	$+$
$1+x^2$	$+$	$+$
	$-$	$+$
	$\cap$	$\cup$

8) $$f''' = \frac{4(1+x^2) - 4x(2x)}{(1+x^2)^2} = \frac{4+4x^2-8x^2}{(1+x^2)^2} = \frac{4-4x^2}{(1+x^2)^2}$$

$$f'''(0) = \frac{4-0}{(1+0)^2} = 4 \neq 0 \Rightarrow IB\ [0,0]$$

$$f(0) = 0 - 2\operatorname{arctg} 0 = 0$$



71 $f(x) = \arcsin(\sin x)$

1) $D(f) = \{\forall x \in R\} = R$

NB $f(x) = 0 \iff x = 0$

2) PÁR, NEPÁR,

$$f(-x) = \arcsin(\sin(-x)) = -\arcsin(\sin x)$$

NEPÁRNA

3) SPOJ $\Rightarrow \nexists$ ABS PERIODICKÁ S PERIÓDOU $k = \pi$

4) ASS $k_1 = \lim_{x \to \infty} \frac{\arcsin(\sin x)}{x} = 0$

$q_1 = \lim_{x \to \infty} \arcsin(\sin x) = \nexists$ OSCILUJÚCA

$\Rightarrow \nexists$ ASS

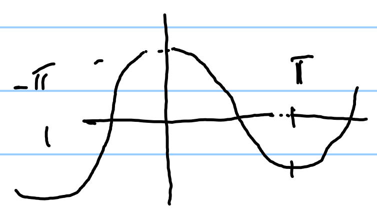
5)
$$f' = \frac{1}{\sqrt{1 - \sin^2 x}} \cdot \cos x = \frac{1}{\sqrt{\cos^2 x}} \cdot \cos x = \frac{\cos x}{|\cos x|}$$

SB $= f' = 0 \iff \cos x = 0 \iff x = +\frac{\pi}{2} + k\pi \quad k \in N$

$f(\frac{\pi}{2}) = \arcsin(\sin \frac{\pi}{2}) = \arcsin 1 = \frac{\pi}{2}$

$f(-\frac{\pi}{2}) = \arcsin(\sin -\frac{\pi}{2}) = -\frac{\pi}{2}$

	$(-\frac{3}{2}\pi, -\frac{\pi}{2})$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$(\frac{\pi}{2}, \frac{3\pi}{2})$
$\cos x$	$-$	$+$	$-$
$\|\cos x\|$	$+$	$+$	$+$
	$\searrow$	$\nearrow$	$\searrow$



6) $$f''(x) = \frac{-\sin x \cdot |\cos x| - \cos x \cdot |-\sin x|}{\cos^2 x}$$

$f'' = 0 \quad \forall x \in D(f) \Rightarrow$ NIE JE KONVEXNÁ ANI KONKÁVNA