Matematická analýza cvičenie 3 ZS 2022-2023
piatok 7.9. 8:00 Live

Note Title 07/10/2022

DERIVÁCIA

$$(x^n)' = n\,x^{n-1}$$

$$(c)' = 0$$

PR: $y = x^2 + 2\sqrt{x} + \frac{1}{x^3} + 4$

$$y' = 2\cdot x^{2-1} + 2\cdot\frac{1}{2}\,x^{\frac{1}{2}-1} + (-3)\cdot x^{-3-1} + 0 =$$

$$= 2x + \frac{1}{\sqrt{x}} - 3\,\frac{1}{x^4}$$

$$(f\cdot g)' = f'\cdot g + f\cdot g'$$

PR: $y = \underbrace{(x^2+4x-1)}_{f}\,\underbrace{\sin x}_{g}$

$$y' = \underbrace{(2x+4)}_{f'}\cdot\underbrace{\sin x}_{g} + \underbrace{(x^2+4x-1)}_{f}\cdot\underbrace{\cos x}_{g'}$$

$$\left(\frac{f}{g}\right) = \frac{f'\cdot g - f\cdot g'}{g^2}$$

PR: $y = \dfrac{\cos x}{3\sqrt{x}+1}$

$$y' = \frac{(-\sin x)(3\sqrt{x}+1) - \cos x\left(3\frac{1}{2\sqrt{x}}\right)}{(3\sqrt{x}+1)^2}$$

$$(f(g))' = f'(g)\cdot g'$$

PR: $y = \ln(2x^3-7x)$

$$y' = \frac{1}{2x^3-7x}\cdot(6x^2-7)$$

$$(f^g)' = \left(e^{\ln f^g}\right)' = \left(e^{g\ln f}\right)' = e^{g\ln f}(g\cdot\ln f)' =$$

$$= f^g\left(g'\ln f + g\frac{1}{f}\cdot f'\right)$$

$$y = x^{\sqrt{x}}$$

$$y' = (2x)^{\sqrt{x}}(\sqrt{x}\cdot\ln 2x)' =$$

$$\left(\frac{1}{2\sqrt{x}}\cdot\ln 2x + \sqrt{x}\,\frac{1}{2x}\cdot 2\right)$$

PR: $y = 2\sqrt{1-x^2}\cdot\arccos 2x + \ln\frac{1+3^x}{x}$

$$y' = 2\,\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x)\arccos 2x + 2\sqrt{1-x}\cdot\frac{-1}{\sqrt{1-(2x)^2}}\cdot 2 +$$

$$+\frac{1}{\frac{1+3^x}{x}}\cdot\frac{3^x\ln 3\cdot x-(1+3^x)\cdot 1}{x^2}$$

PR: $y=\sqrt{3}\ln\left(\cos\left(2^x\right)\right)-2\,\mathrm{arctg}\,\frac{x\sqrt{x}}{1-x^2}+(1+\sin x)^x$

$$y'=\sqrt{3}\,\frac{1}{\cos 2^x}\cdot\left(-\sin 2^x\right)\cdot 2^x\ln 2-2\,\frac{1}{1+\left(\frac{x\sqrt{x}}{1-x^2}\right)^2}\cdot$$

$$\frac{\frac{3}{2}x^{\frac{1}{2}}(1-x^2)-x^{\frac{3}{2}}(-2x)}{(1-x^2)^2}+(1+\sin x)^x\cdot\left(1\ln(1+\sin x)+\right.$$

$$\left.+x\cdot\frac{1}{\sin x}\cdot\cos x\right.$$

PR: PODĽA DEFINÍCIE DERIVÁCIE ZDERIVUJ $f=\frac{1}{x}$ V BODE $a=2$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}=\lim_{x\to 2}\frac{\frac{2-x}{2x}}{x-2}=$$

$$=\lim_{x\to 2}\frac{-\cancel{(-2+x)}}{2x\cancel{(x-2)}}=\lim_{x\to 2}\frac{-1}{2x}=\frac{-1}{4}$$

V BODE a VŠEOBECNE

$$\lim_{x\to a}\frac{\frac{1}{x}-\frac{1}{a}}{x-a}=\lim_{x\to a}\frac{\frac{a-x}{xa}}{x-a}=\lim_{x\to a}-\frac{1}{ax}=$$

$$=\frac{-1}{a^2}$$

PŘ: $f(x) = \begin{cases} x \sin x & x \leq 0 \\ x^2 & x > 0 \end{cases}$ v $a = 0$

$$\lim_{x \to 0^-} \frac{\overbrace{\cancel{x} \sin x}^{f(x)} - \overbrace{0 \cdot \sin 0}^{f(a)}}{\underbrace{\cancel{x}}_{x} - \underbrace{0}_{a}} = \lim_{x \to 0} \frac{\cancel{x} \sin x}{\cancel{x}} = 0$$

$$\lim_{x \to 0^+} \frac{x^2 - 0^2}{x - 0} = \lim_{x \to 0^+} \frac{x^2}{x} = \lim_{x \to 0^+} x = 0$$

POZN.: L'HOSPITAL

$\frac{0}{0}$ $$\lim_{x \to 0} \frac{\sin x}{x} \overset{L'}{=} \lim_{x \to 0} \frac{(\sin x)'}{x'} = \lim_{x \to 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

$$\lim_{x \to 0} \frac{\sin nx}{nx} \overset{L'}{=} \lim_{x \to 0} \frac{\cos nx \cdot n}{n \cdot 1} = \frac{1 \cdot n}{n \cdot 1} = 1$$

PŘ: $$\lim_{x \to \infty} \frac{x^2 + 4x + 5}{2x^2 - 1} \overset{L'}{=} \lim_{x \to \infty} \frac{2x + 4}{4x} \overset{L'}{=} \lim_{x \to \infty} \frac{2}{4} = \frac{1}{2}$$

DOTYČNICA A NORMÁLA KU GRAFU FUNKCIE

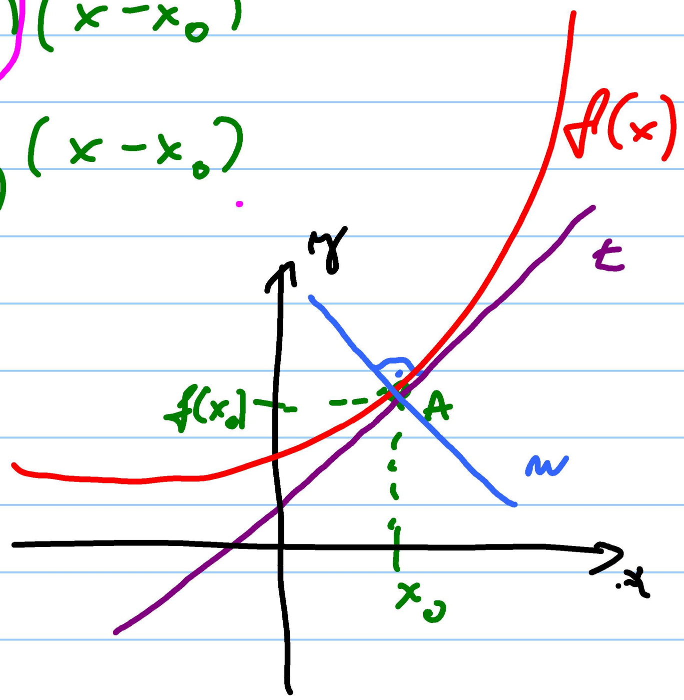
PR. NÁJDI t a n PRE $f(x) = tg\, 2x$ $t \parallel p$

$p: \; y - 2x + 4 = 0$

normála

$t: \; y - f(x_0) = f'(x_0)(x - x_0)$

$n: \; y - f(x_0) = -\frac{1}{f'(x_0)}(x - x_0)$



$f(x_0) = ?$

$f'(x_0) = ?$

$k_p = k_t$ AK $p \parallel t$

$f'(x) = (tg\, 2x)' = \frac{1}{\cos^2 2x} \cdot 2$

$p: \; y - 2x + 4 = 0 \Rightarrow y = 2x - 4$ SMERNICOVÝ TVAR

$\frac{2}{\cos^2 2x} = 2$

OR: $\cos^2 2x \neq 0 \Leftrightarrow$

$2x \neq \frac{\pi}{2}$

$x \neq \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$

$$2 = 2\cos^2 2x \quad /:2$$

$$1 = \cos^2 2x \iff x = 0 + k\pi \quad k \in \mathbb{Z}$$

$A[0, ?]$

$$f(0) = \operatorname{tg} 2\cdot 0 = \operatorname{tg} 0 = 0$$

$A[0,0]$

$$f'(x) = \frac{2}{\cos^2 2x} \qquad f'(0) = 2$$

$$t: \quad y - 0 = 2\cdot(x-0) \Rightarrow y = 2x$$

$$n: \quad y - 0 = -\frac{1}{2}(x-0) \Rightarrow y = -\frac{1}{2}x$$