

Vorkurs Mathematik Lösungen

FaRaFIN Vorkurs-Team

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2 Basismathematik

2.1 Bruchrechnung Lösung

Autor: Katja Matthes

Aufgabe 1

1. $\frac{20}{6} = \frac{10}{3}$

3. $\frac{360}{25} = \frac{72}{5}$

2. $\frac{92}{4} = 23$

4. $\frac{1716}{308} = \frac{39}{7}$

Aufgabe 2

1. $\frac{56}{65} \cdot 12 \cdot \frac{5}{7} \cdot \frac{13}{16} = 6$

3. $\left(\frac{3}{5} - \frac{1}{4}\right) : \frac{3}{4} = \frac{7}{15}$

2. $1 : \left(\frac{2}{9} + \frac{1}{7}\right) = \frac{63}{23}$

Aufgabe 3

1. $\frac{\frac{8}{9}}{\frac{16}{27}} = \frac{3}{2}$

3. $\frac{5\frac{1}{2}}{\frac{11}{12}} = 6$

2. $\frac{2\frac{1}{3}}{1\frac{1}{6}} = 2$

4. $\frac{\frac{99}{100}}{\frac{9}{10}} = \frac{11}{10}$

Aufgabe 4

1. $\frac{5}{6} \cdot \frac{2}{3} - \frac{2}{9} + \frac{3}{4} \cdot 1\frac{7}{9} = \frac{5}{3}$

2. $3\frac{5}{12} - 2\frac{5}{6} + 1\frac{1}{3} : \frac{4}{9} - 2\frac{1}{6} \cdot \frac{1}{2} = \frac{5}{2}$

Aufgabe 5

2 Basismathematik

1. $\left(\frac{2}{3} - \frac{1}{6}\right) \cdot \left(\frac{9}{11} - \frac{3}{7}\right) = \frac{15}{77}$
2. $\left(\frac{1}{8} + \frac{7}{12}\right) : \left(5 - \frac{3}{4}\right) = \frac{1}{6}$
3. $\frac{4}{7} \cdot \left(\left(1\frac{1}{2} - \frac{5}{9}\right) : 4\frac{1}{4}\right) = \frac{8}{63}$
4. $\frac{4}{5} : \left[\left(\frac{5}{8} - \frac{1}{3}\right) \cdot 12\right] = \frac{8}{35}$
5. $\frac{3}{4} \cdot \left(2\frac{1}{2} : 1\frac{1}{4}\right) = \frac{3}{2}$

Aufgabe 6

1. $\frac{\frac{3}{8} \cdot \frac{2}{7}}{\frac{5}{14}} = \frac{3}{10}$
2. $\frac{1\frac{3}{4} + \frac{5}{6}}{\frac{1}{4}} = \frac{31}{3}$
3. $\frac{\frac{8}{9}}{3\frac{1}{3} + \frac{1}{6}} = \frac{16}{63}$
4. $\frac{(\frac{3}{5} - \frac{5}{10}) : \frac{2}{5}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{3}$

2.2 Potenzen Lösung

Autor: Katja Matthes

Aufgabe 1

1. $3x^4 - x^4 - x^3(x+2) = x^4 - 2x^3$
2. $-12a^2 + 3a(a+1) = -9a^2 + 3a$
3. $ax^n + 4x^n = (a+4)x^n$
4. $(1-t)^2 - \frac{1}{2}(1-t)^2 = \frac{1}{2}(1-t)^2$
5. $a(x+t)^k - b(x+t)^k = (a-b)(x+t)^k$
6. $tx^3 - 3x^2 + 2tx^3 - 4x^2 = 3tx^3 - 7x^2$
7. $t^3 \cdot t^4 - t^5(t^2 + 1) = -t^5$
8. $x^2 \cdot x^3 \cdot x^4 = x^9$
9. $3a^k \cdot a^{k-1} \cdot a = 3a^{2k}$
10. $b^n \cdot b^{2n+1} = b^{3n+1}$

$$11. \quad (x+1)^{n-1} \cdot (x+1)^{n+1} = (x+1)^{2n}$$

$$12. \quad \left(\frac{x}{3}\right)^4 \cdot \left(\frac{x}{3}\right)^2 = \left(\frac{x}{3}\right)^6$$

$$13. \quad t^2 \cdot x^2 \cdot t^n \cdot x^{n-1} = t^{2+n} x^{n+1}$$

$$14. \quad a \cdot b^k \cdot a^{2n} \cdot b^{k-3} = a^{2n+1} \cdot b^{2k-3}$$

$$15. \quad (x-2)^n \cdot (x-2)^{1-n} = x-2$$

$$16. \quad 0,3^6 \cdot \left(\frac{10}{3}\right)^6 = 1$$

$$17. \quad 2^x \cdot \left(\frac{5}{2}\right)^x \cdot 5 = 5^{x+1}$$

$$18. \quad 2^5 \cdot \left(\frac{1}{2}\right)^4 = 2$$

$$19. \quad \left(\frac{x}{4}\right)^4 \cdot 4^6 = 4^2 x^4$$

$$20. \quad 2^n \cdot \left(\frac{x}{2}\right)^n \cdot x = x^{n+1}$$

$$21. \quad 9 \cdot 3^{n+1} = 3^{n+3}$$

$$22. \quad (a-b)^9 \cdot (a-b) = (a-b)^{10}$$

$$23. \quad \left(\frac{a-b}{c}\right)^{2k} \cdot \left(\frac{c}{a-b}\right)^{2k} = 1$$

Aufgabe 2

$$1. \quad \frac{a^6}{a^3} = a^3$$

$$7. \quad \frac{81}{3^{x+3}} = 3^{1-x}$$

$$2. \quad \frac{x^{2n+1}}{x^n} = x^{n+1}$$

$$8. \quad \frac{(a-b)^3}{(a-b)^{n-1}} = (a-b)^{4-n}$$

$$3. \quad \frac{15e^{x+1}}{5e^x} = 3e$$

$$9. \quad \frac{(ab)^3}{x^2 y} \cdot \frac{(xy)^2}{a^4 b^2} = \frac{by}{a}$$

$$4. \quad \frac{x^4}{x^7} = x^{-3}$$

$$10. \quad \frac{a^{n+1}}{a^n} = a$$

$$5. \quad \frac{2a^{1-2n}}{4a^{n+1}} = \frac{1}{2} a^{-3n}$$

$$11. \quad \frac{10^3}{2^3} = 5^3$$

$$6. \quad \frac{a^4 b^{4n+3}}{a^n b^{2n-1}} = a^{4-n} b^{2n+4}$$

$$12. \quad \frac{2,5^4}{0,5^4} = 5^4$$

13. $\frac{(10ab)^k}{(4b)^k} = \left(\frac{5}{2}a\right)^k$
14. $\left(\frac{a}{b}\right)^n \cdot \frac{a}{b} = \left(\frac{a}{b}\right)^{n+1}$
15. $\left(\frac{-1}{a-b}\right)^3 = -(a-b)^{-3}$
16. $\left(\frac{x}{2}\right)^3 : \left(\frac{x}{3}\right) = \frac{3}{8}x^2$
17. $(-5^2)^3 = -5^6$
18. $3(c^4)^3 - 6c^{12} = -3c^{12}$
19. $(3b^2c^{n-1})^4 = 81b^8c^{4n-4}$
20. $\left(\frac{7a^2}{49b^3}\right)^2 = \frac{a^4}{49b^6}$
21. $\left(\frac{-1}{c^3}\right)^{2n} = \frac{1}{c^{6n}}$
22. $(3b^{n+1} \cdot c^{n-1})^2 = 9b^{2n+2}c^{2n-2}$
23. $(x^2y^3z^2)^5 = x^{10}y^{15}z^{10}$
24. $(0,5e^{x+2})^2 = 0,25e^{2x+4}$
25. $\left(\frac{2}{x^2}\right)^5 - \left(\frac{3}{x^5}\right)^2 = \frac{23}{x^{10}}$
26. $\left[\left(-\frac{3}{t}\right)^3\right]^4 \cdot \frac{t^9}{81} = \frac{3^8}{t^3}$
27. $\frac{(ab)^2}{x^3y} \cdot \frac{x^5y^2}{a^2b} = bx^2y$
28. $\left(\frac{4-12x}{64}\right)^3 = 1-3x^3$
29. $\frac{(2x-4)^5}{(2-x)^3} = -32(2-x)^2$
30. $\frac{(4ab)^4}{(6a^2)^4} \cdot \frac{5}{b^4} = \frac{80}{81}a^{-4}$
31. $(a-b^2) \cdot (a-b^2)^n = (a-b^2)^{n+1}$

Aufgabe 3

1. $\left(\frac{1}{2}x^2\right)^5 + \frac{1}{8}(x^2)^5 + (2x^5)^2 = \frac{133}{32}x^{10}$
2. $\frac{1}{4} \cdot 2^4(2^2)^3 = 2^8$
3. $(3^{n+1})^2 = 3^{2n+2}$
4. $(3x^2 - 5x)(1 - x^3) + (x^2 + 3x^4)x^3 = 3x^7 - 2x^5 + 5x^4 + 3x^2 - 5x$
5. $a^{2r}b^r(a^{2r} - a^rb^{r+1} + b^{2r+2}) = a^{4r}b^r - a^{3r}b^{2r+1} + a^{2r}b^{3r+2}$

Aufgabe 4

1. $-3x^3 \cdot x^2 + 5x \cdot x^4 = 2x^5$
2. $4t^{n-4}t^3 - t \cdot t^{n-2} = 3t^{n-1}$
3. $2x^5y^3y - 4x^3y^2x^2y^2 = -2x^5y^4$
4. $\frac{4x^5+6x^4-12x^2}{2x^2} = 2x^3 + 3x^2 - 6$

5. $(9 \cdot 3^n - 3^{n+1}) : 3^{n-1} = 18$ 7. $\frac{5a-20}{4a-16} = \frac{5}{4}$
6. $(2x+6)^2 + (x+3)^2 = 5(x+3)^2$ 8. $(3t^2 - 3t^3)^2 = 9t^4(1-t)^2$

Aufgabe 5

1. $3a^2 + 6a^3 = 3a^2(1 + 2a)$ 5. $x^4 + 2x^3 = x^3(x + 2)$
2. $\frac{1}{2}e^x - \frac{1}{4}e^{x+1} = \frac{1}{4}e^x(2 - e)$ 6. $x^{n+3} - 4x^{n+2} = x^{n+2}(x - 4)$
3. $a^{5b} + 3a^b = a^b(a^{4b} + 3)$ 7. $-6t^{n+2} + 18t^{2-n} = 6t^2(-t^n + 3t^{-n})$
4. $2^x + 2^{x+1} = 3 \cdot 2^x$ 8. $e^x - e^{3x} = e^x(1 - e^{2x})$

Aufgabe 6

1. $\frac{x^4 - x^3}{x^2 - x} = x^2$ 3. $\frac{a^7 b^3 - a b^7}{a^5 b - a^2 b^4} = \frac{a^6 b^2 - b^6}{a^4 - a b^3}$
2. $\frac{e^{3x} + e^{2x}}{e^{2x}} = e^x + 1$ 4. $\frac{32}{2^{n+5}} + \frac{2^{-n+3}}{8} = \frac{1}{2^{n-1}}$

Aufgabe 7

1. $y = \frac{1}{4}x^4 - 2tx^3 + \frac{9}{2}t^2x^2$ mit $x = 3t \Rightarrow y = \frac{27}{4}t^4$
2. $y = e^{3x-t^2} + 3e^{5t-(t-x)}$ mit $x = -t \Rightarrow y = 1 + 3e^{3t}$
3. $y = \frac{3}{2t^2}x^4 - \frac{4}{t}x^3 + 3x^2 - 4$ mit $x = \frac{1}{3}t \Rightarrow y = \frac{11}{54}t^2 - 4$
4. $y = \frac{e^{3tx} + 4e^3}{tx-4}$ mit $x = \frac{1}{t} \Rightarrow y = -\frac{5}{3}e^3$
5. $y = \frac{tx^3}{2(x+t)^2}$ mit $x = -3t \Rightarrow y = -\frac{27}{8}t^2$

Aufgabe 8

1. $a^n + a^{4-n} + a^{2n} = a^{2n}(a^{-n} + a^{4-3n} + 1)$

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$$2. \quad a^3 + a^{1-n} + a^{n+4} = a^{n+3}(a^{-n} + a^{-2n-2} + a)$$

$$3. \quad \frac{3}{2}x^4 + \frac{3}{4}x^3 + \frac{1}{8}x^2 = \frac{1}{8}x^2(12x^2 + 6x + 1)$$

$$4. \quad e^{3x} - 2e^{-x} = e^{-x}(e^{4x} - 2)$$

$$5. \quad te^{2x} - 2e^{x+1} = e^x(te^x - 2e)$$

Aufgabe 9

$$1. \quad \frac{1}{4} \cdot 2^{-4} \cdot (2^2)^3 = 1$$

$$2. \quad (e^x - e^{-x} + 5)e^x = e^{2x} + 5e^x - 1$$

$$3. \quad 2^x(2^{-1} + 2^x) = 2^{x-1} + 2^{2x}$$

$$4. \quad (x^4 + x^{-2})(x^3 - x^{-3}) = x^7 - x^{-5}$$

Aufgabe 10

$$1. \quad a^2 \cdot (a^2)^{-2} + 3a \left(\frac{1}{a}\right)^3 = 4a^{-2}$$

$$5. \quad \left(\frac{2}{x}\right)^3 + \left(\frac{1}{x}\right)^3 = \frac{9}{x^3}$$

$$2. \quad \frac{1}{18} \cdot (3^2)^2 + \frac{1}{2} \cdot 3^3 \cdot \left(\frac{1}{3}\right)^2 = 6$$

$$6. \quad \frac{1}{e^{2x}} + 3(e^{-x})^2 - \left(\frac{2}{e^x}\right)^2 = 0$$

$$3. \quad (x^2 \cdot x^{-3})^{-2} + \left(\frac{3}{x^2}\right)^{-1} = \frac{4}{3}x^2$$

$$7. \quad e^{-x} \cdot e^{-x+2} \cdot e^{2x-3} = e^{-1}$$

$$4. \quad a^5 \cdot a^{-2} + 4a^2 \cdot a = 5a^3$$

$$8. \quad 6x^3 \cdot x^{-1} - 8x^4 \cdot x^{-2} = -2x^2$$

$$9. \quad (t^7 - t^4) \cdot t^{-3} = t^4 - t$$

Aufgabe 11

$$1. \quad \frac{-2^3 - 2 \cdot 4}{2 \cdot 2^3} = -1$$

$$3. \quad \frac{e^{3x+1}}{e^{-x+2}} = e^{4x-1}$$

$$2. \quad \frac{(1-x)^2}{(x-1)} = x - 1$$

$$4. \quad \frac{1,5e^{3x} - e^x}{1,5e^{3x}} = 1 - \frac{2}{3}e^{-2x}$$

Aufgabe 12

1. $a^4 \cdot a^{-6} - 3a^3 \cdot a^{-5} + a^2 = -2a^{-2} + a^2$
2. $(a^{n+2} - 4a^n - 2a^{2-n}) \cdot \frac{a^{-2}}{2} = \frac{1}{2}a^n - 2a^{n-2} - a^{-n}$
3. $4x^{-4}x^7 - 0,5x^4x^{-1} + \left(\frac{1}{x^2}\right)^{1,5} = 3,5x^3 + \frac{1}{x^3}$
4. $\frac{a^{n+1}}{a} + \frac{a^{2n-1}}{a^{n+2}} + (a^{n-1})^2 \cdot a^{2-n} = 2a^n + a^{n-3}$
5. $\frac{2^{2k}}{8} \cdot 2^{3-k} + 2 \cdot 2^{k-1} = 2^{k+1}$

Aufgabe 131. n gerade:

$$\begin{aligned}
 (a-b)^n + (b-a)^n &= (a-b)^n + (-1)^n \cdot (a-b)^n \\
 &= (a-b)^n + (a-b)^n \\
 &= 2(a-b)^n
 \end{aligned}$$

 n ungerade:

$$\begin{aligned}
 (a-b)^n + (b-a)^n &= (a-b)^n + (-1)^n \cdot (a-b)^n \\
 &= (a-b)^n - (a-b)^n \\
 &= 0
 \end{aligned}$$

2. n gerade:

$$\begin{aligned}
 &(x-2)^n + (2x-4)^n - (2-x)^n \\
 &= (x-2)^n + (2x-4)^n - (-1)^n \cdot (x-2)^n \\
 &= (x-2)^n + (2x-4)^n - (x-2)^n \\
 &= (2x-4)^2
 \end{aligned}$$

 n ungerade:

$$\begin{aligned}
 &(x-2)^n + (2x-4)^n - (2-x)^n \\
 &= (x-2)^n + 2^n \cdot (x-2)^n - (-1)^n \cdot (x-2)^n \\
 &= (x-2)^n + 2^n \cdot (x-2)^n + (x-2)^n \\
 &= 2(x-2)^n + 2^n(x-2) \\
 &= (2+2^n)(x-2)^n
 \end{aligned}$$

2.3 Binomische Formeln Lösung

Autor: Katja Matthes

Aufgabe 1

1. $(4x + 3y^3)^2 = 16x^2 + 24xy^3 + 9y^6$
2. $-(x^4 - 2)^2 = -x^8 + 4x^4 - 4$
3. $(x^2 - x^3)(x^2 + x^3) = x^4 - x^6$
4. $(3x^2 + 2t)^2 = 9x^4 + 12x^2t + 4t^2$
5. $-\frac{1}{2}(x^2 - 4)^2 = -\frac{1}{2}x^4 + 4x^2 - 8$
6. $\left(-\frac{1}{2}(x^2 - 4)\right)^2 = \frac{1}{4}x^4 - 2x^2 + 4$
7. $x^2y^2(x^4 + 2x^2y + y^2) = (x^3y + xy^2)^2$

Aufgabe 2

1. $(x - 3)^n \cdot (x + 3)^n = (x^2 - 9)^n$
2. $\frac{(a^2 - b^2)^3}{(a - b)^3} = (a + b)^3$
3. $\frac{(4 - x^2)^n}{(2 - x)^n} = (2 + x)^n$
4. $\frac{(c-1)^{n-1}}{(c^2-1)^{n-1}} = \frac{1}{(c+1)^{n-1}}$
5. $\frac{(a^{2n} - b^{2n})^2}{(a^n - b^n)^2} = (a^n + b^n)^2$
6. $(a^3 - ab^2)(a + b)^2 = a(a - b)(a + b)^3$
7. $\frac{[(x-y)^2]^k}{(x^2-y^2)^k} = \left(\frac{x-y}{x+y}\right)^k$
8. $(a + b)^4(a - b)^4(a^2 - b^2)^5 = (a^2 - b^2)^9$

Aufgabe 3

1. $(3x - 6) \left(\frac{1}{4}x^2 - x + 1 \right) = \frac{3(x-2)^3}{4}$
2. $a^2 - 2a^3 + a^4 = a^2(1 - a)^2$
3. $3a^3 - 12a^9 = 3a^2(1 - 2a^3)(1 + 2a^3)$
4. $x^4 - a^2 = (x^2 - a)(x^2 + a)$
5. $3 - x^2 = (\sqrt{3} - x)(\sqrt{3} + x)$
6. $x^{2n} + 4x^n + 4 = (x^n + 2)^2$
7. $x^{n+2} - 6x^{n+1} + 9x^n = x^n(x - 3)^2$
8. $e^{2x} - 1 = (e^x - 1)(e^x + 1)$
9. $x^2e^x + 2xe^x + e^x = e^x(x + 1)^2$

Aufgabe 4

1. $\frac{a^3 + 2a^2b + ab^2}{(a+b)^2} = a$
2. $\frac{a^4 - a^2b^2}{ab - a^2} = -a(a + b)$
3. $\frac{t^3 + 6t^2 + 9t}{t^2 - 9} = \frac{t(t+3)}{t-3}$
4. $\frac{x^{2n} - 10x^n + 25}{x^{2n} - 25} = \frac{x^n - 5}{x^n + 5}$
5. $\frac{x^6 - t^2}{x^4 + tx} = \frac{x^3 - t}{x}$
6. $\frac{x^{n+3} - x^{n+1}}{x^{n+1} + x^n} = x(x - 1)$
7. $\frac{(x^2 + 8xy + 16y^2)}{(2x - 3y)^{-2}} : \frac{x^2 - 16y^2}{2x - 3y} = \frac{(x + 4y)(2x - 3y)^3}{x - 4y}$
8. $\frac{4t^2 - 4}{t^2 + 2t + 1} = \frac{4(t-1)}{t+1}$

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$$9. \quad \frac{x^{n-1} - x^n}{x^n - x^{n+2}} = \frac{1}{x(1+x)}$$

$$10. \quad \frac{2(a^2+b^2)^2}{a^5-ab^4} = \frac{2(a^2+b^2)}{a(a^2-b^2)}$$

$$11. \quad \frac{x^4-x^3}{x^4-x^2} = \frac{x}{x+1}$$

$$12. \quad \frac{x^3y-xy^5}{x^3y^2-x^2y^4} = \frac{x+y^2}{xy}$$

$$13. \quad \frac{am-an+bm-bn}{a^2-b^2} = \frac{m-n}{a-b}$$

Aufgabe 5

$$1. \quad (e^x + e^{-x})^2 = e^{2x} + e^{-2x} + 2$$

$$2. \quad (a^2 - a^{-2})^2 = a^4 - 2 + a^{-4}$$

$$3. \quad (x^{-2} - 3x)(x^{-2} + 3x) = x^{-4} - 9x^2$$

$$4. \quad (2^{-x} + 2^x)(2^{-x} - 2^x) = 2^{-2x} - 2^{2x}$$

Aufgabe 6

$$1. \quad \frac{e^{2x} - e^{-2x}}{e^x - e^{-x}} = e^x + e^{-x}$$

$$2. \quad \left(\frac{x-y}{a-b}\right)^5 \cdot \left(\frac{x-y}{5}\right)^{-2} \cdot \frac{(a-b)^2}{(x^2-y^2)} = \frac{25(x-y)^2}{(x+y)(a-b)^3}$$

2.4 Polynomdivision Lösungen

Autor: Marko Rak

$$1. \left(\begin{array}{r} x^3 \\ -x^3 - x^2 \\ \hline -x^2 \\ x^2 + x \\ \hline x + 1 \\ -x - 1 \\ \hline 0 \end{array} + 1 \right) : (x + 1) = x^2 - x + 1$$

$$2. \left(\begin{array}{r} x^4 \\ -x^4 - x^3 - x^2 \\ \hline -x^3 - x^2 - x \\ x^3 + x^2 + x \\ \hline \end{array} - x + 1 \right) : (x^2 + x + 1) = x^2 - x + \frac{1}{x^2 + x + 1}$$

$$3. \left(\begin{array}{r} x^2 \\ -x^2 - 3x \\ \hline -3x - 9 \\ 3x + 9 \\ \hline 0 \end{array} - 9 \right) : (x + 3) = x - 3$$

$$4. \left(\begin{array}{r} 6x^3 - 5x^2 - 36x + 35 \\ -6x^3 + 14x^2 \\ \hline 9x^2 - 36x \\ -9x^2 + 21x \\ \hline -15x + 35 \\ 15x - 35 \\ \hline 0 \end{array} \right) : (3x - 7) = 2x^2 + 3x - 5$$

$$5. \left(\begin{array}{r} x^5 \\ -x^5 + x^4 - 2x^3 + 3x^2 - x \\ \hline x^4 - 2x^3 + 2x^2 - 3x + 1 \\ -x^4 + x^3 - 2x^2 + 3x - 1 \\ \hline -x^3 \end{array} - x^2 - 2x + 1 \right) : (x^4 - x^3 + 2x^2 - 3x + 1) = x + 1 + \frac{-x^3}{x^4 - x^3 + 2x^2 - 3x + 1}$$

$$6. \left(\begin{array}{r} x^5 - x^3 + x^2 + x - 2 \\ -x^5 + x^3 \\ \hline x^2 + x - 2 \\ -x^2 + 1 \\ \hline x - 1 \end{array} \right) : (x^2 - 1) = x^3 + 1 + \frac{x - 1}{x^2 - 1}$$

$$\begin{array}{r}
 7. \quad \left(\begin{array}{r} 3x^3 + 2x^2 + 4x + 9 \\ -3x^3 - 5x^2 \end{array} \right) : (3x + 5) = x^2 - x + 3 + \frac{-6}{3x + 5} \\
 \hline
 \begin{array}{r} -3x^2 + 4x \\ 3x^2 + 5x \end{array} \\
 \hline
 \begin{array}{r} 9x + 9 \\ -9x - 15 \end{array} \\
 \hline
 -6
 \end{array}$$

$$\begin{array}{r}
 8. \quad \left(\begin{array}{r} 2x^5 + 8x^4 + x^3 - x^2 + 12x + 3 \\ -2x^5 - 8x^4 - 2x^3 \end{array} \right) : (x^2 + 4x + 1) = 2x^3 - x + 3 + \frac{x}{x^2 + 4x + 1} \\
 \hline
 \begin{array}{r} -x^3 - x^2 + 12x \\ x^3 + 4x^2 + x \end{array} \\
 \hline
 \begin{array}{r} 3x^2 + 13x + 3 \\ -3x^2 - 12x - 3 \end{array} \\
 \hline
 x
 \end{array}$$

$$\begin{array}{r}
 9. \quad \left(\begin{array}{r} x^6 - 2x^5 + 9x^4 - 8x^3 + 15x^2 \\ -x^6 + x^5 - 5x^4 \end{array} \right) : (x^2 - x + 5) = x^4 - x^3 + 3x^2 \\
 \hline
 \begin{array}{r} -x^5 + 4x^4 - 8x^3 \\ x^5 - x^4 + 5x^3 \end{array} \\
 \hline
 \begin{array}{r} 3x^4 - 3x^3 + 15x^2 \\ -3x^4 + 3x^3 - 15x^2 \end{array} \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 10. \quad \left(\begin{array}{r} 2x^7 - x^6 + 3x^5 - \frac{1}{2}x^4 + x^3 \\ -2x^7 + x^6 - 2x^5 \end{array} \right) : (2x^3 - x^2 + 2x) = x^4 + \frac{1}{2}x^2 \\
 \hline
 \begin{array}{r} x^5 - \frac{1}{2}x^4 + x^3 \\ -x^5 + \frac{1}{2}x^4 - x^3 \end{array} \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 11. \quad \left(\begin{array}{r} x^7 - 6x^5 + x^4 - 11x^2 - 3x + 1 \\ -x^7 - 2x^4 \end{array} \right) : (x^3 + 2) = x^4 - 6x^2 - x + \frac{x^2 - x + 1}{x^3 + 2} \\
 \hline
 \begin{array}{r} -6x^5 - x^4 - 11x^2 \\ 6x^5 + 12x^2 \end{array} \\
 \hline
 \begin{array}{r} -x^4 + x^2 - 3x \\ x^4 + 2x \end{array} \\
 \hline
 x^2 - x
 \end{array}$$

$$\begin{array}{r}
 12. \quad \left(\begin{array}{r} 3x^5 + 6x^4 + \frac{11}{3}x^3 + 4x^2 + \frac{20}{3}x \\ -3x^5 - x^4 - 4x^2 \end{array} \right) : (3x^4 + x^3 + 4x) = x + \frac{5}{3} + \frac{2x^3}{3x^4 + x^3 + 4x} \\
 \hline
 \begin{array}{r} 5x^4 + \frac{11}{3}x^3 + \frac{20}{3}x \\ -5x^4 - \frac{5}{3}x^3 - \frac{20}{3}x \end{array} \\
 \hline
 2x^3
 \end{array}$$

$$\begin{array}{r}
 13. \quad \left(\begin{array}{r} \frac{1}{6}x^4 + \frac{11}{36}x^3 - \frac{23}{18}x^2 - \frac{1}{3}x + \frac{2}{3} \\ -\frac{1}{6}x^4 + \frac{4}{9}x^3 - \frac{2}{9}x^2 \end{array} \right) : \left(\frac{1}{2}x^2 - \frac{4}{3}x + \frac{2}{3} \right) = \frac{1}{3}x^2 + \frac{3}{2}x + 1 \\
 \hline
 \begin{array}{r} \frac{3}{4}x^3 - \frac{3}{2}x^2 - \frac{1}{3}x \\ -\frac{3}{4}x^3 + 2x^2 - x \end{array} \\
 \hline
 \begin{array}{r} \frac{1}{2}x^2 - \frac{4}{3}x + \frac{2}{3} \\ -\frac{1}{2}x^2 + \frac{4}{3}x - \frac{2}{3} \end{array} \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 14. \quad \left(\begin{array}{r} \frac{1}{2}x^5 + \frac{5}{4}x^4 + \frac{1}{2}x^3 - \frac{1}{4}x^2 - \frac{1}{2}x \\ -\frac{1}{2}x^5 - x^4 \end{array} \right) : \left(\frac{1}{2}x^2 + x \right) = x^3 + \frac{1}{2}x^2 - \frac{1}{2} \\
 \hline
 \begin{array}{r} \frac{1}{4}x^4 + \frac{1}{2}x^3 \\ -\frac{1}{4}x^4 - \frac{1}{2}x^3 \end{array} \\
 \hline
 \begin{array}{r} -\frac{1}{4}x^2 - \frac{1}{2}x \\ -\frac{1}{4}x^2 + \frac{1}{2}x \end{array} \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 15. \quad \left(\begin{array}{r} \frac{1}{2}x^5 - \frac{3}{4}x^4 - \frac{1}{4}x^3 + \frac{3}{4}x^2 - \frac{15}{4}x + \frac{7}{4} \\ -\frac{1}{2}x^5 + \frac{1}{4}x^4 \end{array} \right) : \left(\frac{1}{2}x - \frac{1}{4} \right) = x^4 - x^3 - x^2 + x - 7 \\
 \hline
 \begin{array}{r} -\frac{1}{2}x^4 - \frac{1}{4}x^3 \\ -\frac{1}{2}x^4 - \frac{1}{4}x^3 \end{array} \\
 \hline
 \begin{array}{r} -\frac{1}{2}x^3 + \frac{3}{4}x^2 \\ -\frac{1}{2}x^3 - \frac{1}{4}x^2 \end{array} \\
 \hline
 \begin{array}{r} \frac{1}{2}x^2 - \frac{15}{4}x \\ -\frac{1}{2}x^2 + \frac{1}{4}x \end{array} \\
 \hline
 \begin{array}{r} -\frac{7}{2}x + \frac{7}{4} \\ -\frac{7}{2}x - \frac{7}{4} \end{array} \\
 \hline
 0
 \end{array}$$