Investments
FIN-405

Working results of Assignment 1

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Problem 1

a)

Let A_t denote the assets of t, R_t^A and r_t^A denote their simple returns and continuously compounded annual returns, L_t denote the liabilities of t, R_t^L and r_t^L denote their simple returns and continuously compounded annual returns.

Therefore, after 5 years the assets will be

$$A_5 = A_0 \prod_{t=1}^{5}(1 + R_t^A) = A_0 e^{(\sum_{t=1}^{5} log(1+R_t^A))} = A_0 e^{\sum_{t=1}^{5} r_t^A}$$

Meanwhile, we have assumed that $r_t^A \sim N(\mu_A, \sigma^2)$ and is serially uncorrelated,

Thus, we can conclude that,

$$E[\sum_{t=1}^{5} r_t^A] = 5\mu_A \qquad Var[\sum_{t=1}^{5} r_t^A] = 5\sigma_A^2 \qquad \sum_{t=1}^{5} r_t^A \sim N(5\mu_A, 5\sigma_A^2)$$

$$\Rightarrow A_5 = A_0 e^{5\mu_A + \sqrt{5}\sigma_A X}$$

where $X \sim N(0, 1)$.

Besides, we have

$$L_5 = L_0 \prod_{t=1}^{5}(1 + L_t^A) = L_0 e^{\sum_{t=1}^{5} r_t^L} = L_0 e^{5\mu_L}$$

where μ_L is r_t^L.

Hence, the probability that the fund;s assets will be less that its liabilities in 5 years is

$$\begin{aligned}
P\{A_5 < L_5\} &= P\{A_0 e^{5\mu_A + \sqrt{5}\sigma_A X} < L_0 e^{5\mu_L}\} \\
&= P\{\frac{A_0}{L_0} e^{5(\mu_A - \mu_L) + \sqrt{5}\sigma_A X} < 1\} \\
&= P\{\frac{A_0}{L_0} e^{5(\mu_A - \mu_L) + \sqrt{5}\sigma_A X} < 1\} \\
&= P\{X < \frac{log\frac{L_0}{A_0} - 5(\mu_A - \mu_L)}{\sqrt{5}\sigma_A}\} \\
&= \Phi(-2.657) = 0.394\%
\end{aligned}$$

because $X \sim N(0, 1)$, $L_0 = 1.5$ billion, $A_0 = 2$ billion, $\mu_A = 1\%$, $\sigma_A = 4\%$, $\mu_L = 2\%$.

b)

Similarly, if $r_t^L \sim N(\mu_L, \sigma^2)$,

$$L_5 = L_0 e^{5\mu_L + \sqrt{5}\sigma_L Y}$$

where $Y \sim N(0,1)$ and independent of X.

Therefore, we can conclude that,

$$\begin{aligned}P\{A_5 < L_5\} &= P\{\frac{A_0}{L_0} e^{5(\mu_A - \mu_L) + \sqrt{5}(\sigma_A X - \sigma_L Y)} < 1\} \\ &= P\{\sigma_A X - \sigma_L Y < \frac{log\frac{L_0}{A_0} - 5(\mu_A - \mu_L)}{\sqrt{5}}\}\end{aligned}$$

What is more, $\sigma_A X - \sigma_L Y \sim N(0, \sigma_A^2 + \sigma_L^2)$, thus, $\frac{\sigma_A X - \sigma_L Y}{\sqrt{\sigma_A^2 + \sigma_L^2}} \sim N(0,1)$ and

$$\begin{aligned}P\{A_5 < L_5\} &= P\{\frac{\sigma_A X - \sigma_L Y}{\sqrt{\sigma_A^2 + \sigma_L^2}} < \frac{\log\frac{L_0}{A_0} - 5(\mu_A - \mu_L)}{\sqrt{5(\sigma_A^2 + \sigma_L^2)}}\} \\ &= \Phi(-2.377) = 0.873\%\end{aligned}$$

because $X \sim N(0,1)$, $Y \sim N(0,1)$, $L_0 = 1.5$ billion, $A_0 = 2$ billion, $\mu_A = 1\%$, $\sigma_A = 4\%$, $\mu_L = 2\%$, $\sigma_L = 2\%$.

Problem 2

a)

$$u(W) = -e^{-aW} \Rightarrow u'(W) = ae^{-aw},\ u''(W) = -a^2e^{-aw}$$

$$ARA(W) = -\frac{u''(W)}{u'(W)} = -\frac{-a^2e^{-aw}}{ae^{-aw}} = a,\ RRA(W) = W \times ARA(W) = aW$$

It's constant absolute risk aversion

b)

$$u(W - \lambda) = -e^{-a(w-\lambda)} = -e^{-aW + a\lambda} = e^{a\lambda}u(W)$$

$$E[u(W + \varepsilon)] = E[-e^{-a(w+\varepsilon)}] = E[-e^{-aw} \times e^{-a\varepsilon}] = u(W)E[e^{-a\varepsilon}]$$

$$u(W - \lambda) = E[u(W + \varepsilon)] \Rightarrow e^{a\lambda} = E[e^{-a\varepsilon}] \Rightarrow \lambda = \frac{1}{a}\ln E[e^{-a\varepsilon}]$$

c)

Under the assumption we have $\varepsilon \sim N(0,\, \sigma^2)$

$$\Rightarrow E[e^{-a\varepsilon}] = e^{-a*0 + \frac{1}{2}a^2\sigma^2} = e^{\frac{1}{2}a^2\sigma^2}$$

$$\Rightarrow \lambda = \frac{1}{a}\ln E[e^{-a\varepsilon}] = \frac{\frac{1}{2}a^2\sigma^2}{a} = \frac{1}{2}a\sigma^2$$

d)

Risk premium is uncorrelated with initial wealth W, it's not very reasonable in reality. Usually, they should have a positive relationship. $\lambda = \frac{1}{2}a\sigma^2$, we can see it increases in a, which means more risk averse will result in larger risk premium. This is within our expectation. Also, risk premium increases in σ^2. More risky, more need to be compensated.

Problem 3

a)

Following Table 1 displays the statistics of monthly returns of all the 7 countries or regions:

Table 1: Statistical values of monthly returns

	US	Japan	Switzerland	Germany	Asia	Latin	EU-ME
Arithmetic Mean	0.0089	0.0022	0.0097	0.0087	0.0081	0.0152	0.0055
Geometric Mean	0.0080	0.0004	0.0084	0.0065	0.0056	0.0112	0.0022
Standard Deviation	0.0417	0.0599	0.0493	0.0659	0.0699	0.0880	0.0819
Skewness	-0.5564	0.2372	-0.2426	-0.4098	-0.2099	-0.4875	0.0251
Kurtosis	4.1235	3.9612	3.7646	4.5166	3.9121	4.5028	5.4654
95% VaR	-0.0658	-0.0904	-0.0886	-0.1044	-0.1145	-0.1324	-0.1357

And the histogram of returns from all markets are showed as follow:

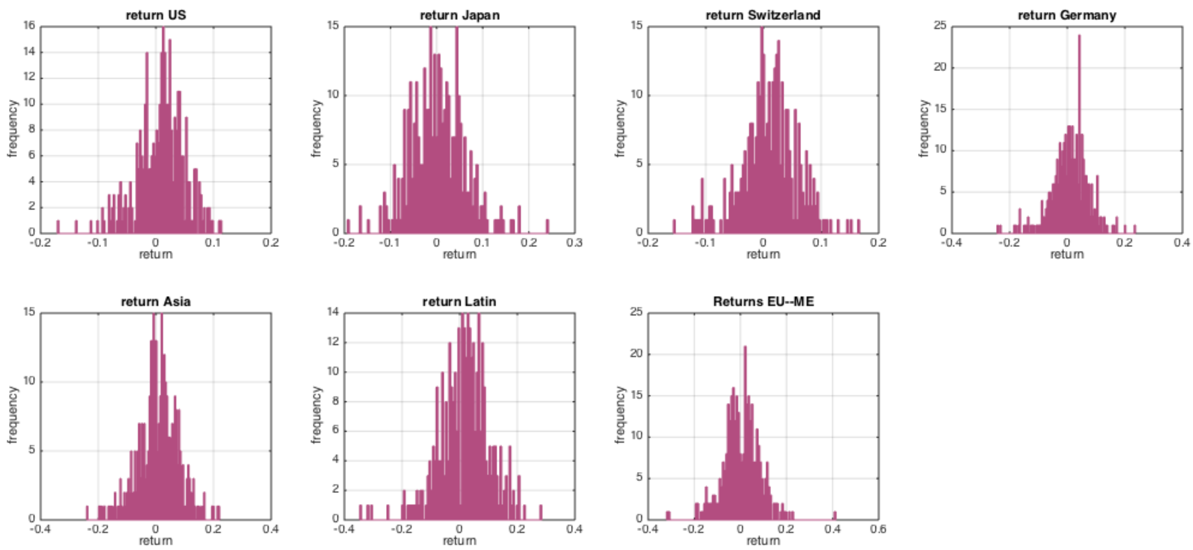

Figure 1: histogram of returns in each market

We then can see that investing in emerging markets (EM) is riskier than that in developed markets. The reason is that EM have higher standard deviations and VaR than developed markets, also, EM have higher extremes (not shown in table but can obtained using `min()` and `max()`). Hence investing in EM is riskier.

b)

We show the differences between geo_mean and $ari\_mean - 1/2 * variance$ in all markets in following table:

Table 2: Verification of rule of thumb

	US	Japan	Switzerland	Germany	Asia	Latin	EU-ME
Differences	-0.000030	0.000014	-0.000024	-0.000052	-0.000034	-0.000139	-0.000029

According to above Table 2, we can safely say that the rule of thumb is verified as the differences are really small and close to 0 in all markets.

c)

Following two table show the correlation matrix for returns in developed markets and correlation matrix for returns in Switzerland and EM regions respectively:

Table 3: Correlation of returns in developed countries

correlation	US	Japan	**Switzerland**	Germany
US	1.0000	0.4385	0.6326	0.7062
Japan	0.4385	1.0000	0.5052	0.4149
Switzerland	0.6326	0.5052	1.0000	0.7143
Germany	0.7062	0.4149	0.7143	1.0000

Table 4: Correlation between Switzerland and EM

correlation	**Switzerland**	Asia	Latin	EU-ME
Switzerland	1.0000	0.4888	0.3985	0.4967
Asia	0.4888	1.0000	0.5845	0.5639
Latin	0.3985	0.5845	1.0000	0.5667
EU-ME	0.4967	0.5639	0.5667	1.0000

From above two tables, we can observe that the return correlation between Switzerland and EM (especially Latin) are smaller than correlation between Switzerland and developed countries. Hence for a swiss investor, he/she should choose EM markets to diversify portfolios.

d)

Following Figure 2 (next page) illustrates the time series of the average correlation among all markets. Observing the figure we can conclude that the average correlation increases along with time. We believe the reason is that the markets are increasingly interconnected and there are increasing number of international investors. Hence the effects of diversification may be reduced gradually.

We see that the average correlation peaks during the financial crisis within 2008-2009. As a result, we can say that during the financial crisis, the benefits of diversification is not significant and investments may still suffer significant loss.

Figure 2: time series of average return

Experiments codes

For commented and runnable codes for above problems, please refer to:

https://github.com/FINEPFL/Investments-FIN-405/blob/master/w1/w1.m