Quantitative Risk Management
MATH-471

Working results of Assignment 3

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Question 1

Following figure shows the daily loss, $VaR_{0.95}$ and $VaR_{0.99}$ respectively. The date that the loss is higher than value of risk are marked as green and yellow dots:

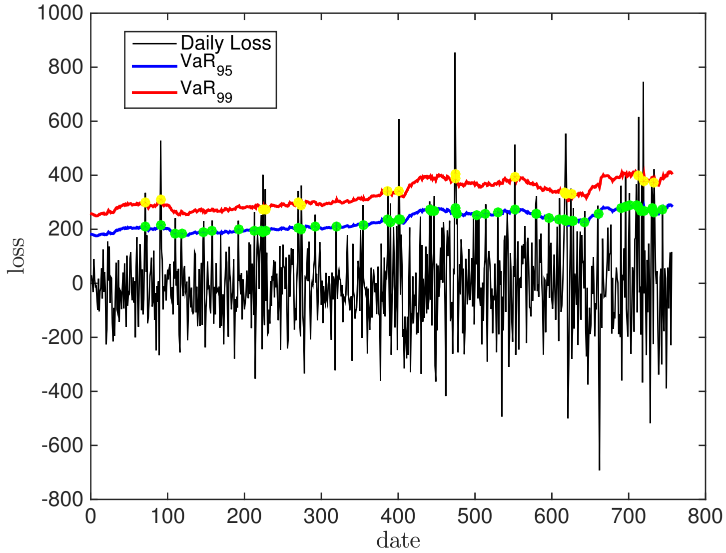

Figure 1: $VaR_{0.95}$, $VaR_{0.99}$ and loss from March 11, 2013 to March 10, 2016

The total number of dates for evaluation is 756 (3 years). The number of $VaR_{0.95}$ breaches is 52 while the number of $VaR_{0.99}$ breaches is 18. In other words, the percentage of breaches is 6.9% and 2.4% during the three years evaluation. Hence we believe that this estimation is good and this model is valid.

Question 2

1. L has geometric distribution with parameter p. Here 0 is included in the support.

Given $p = 0.5$, then $VaR_{0.95} = 4$ which can be calculated via calling *geoinv* function in MATLAB.

Following Figure 2 in next page shows the plot of VaR_α for values of α ranging from 0.9 to 0.99 and $p = 0.5$.

2. X and Y are independent with Poisson distributions with $\lambda_X = 1$ and $\lambda_Y = 2$.

Given $L = X + Y$, we can conclude that L follows the Poisson distribution with $\lambda_L = 1 + 2 = 3$. Following Figure 3 shows values of $VaR_\alpha X$, $VaR_\alpha Y$ and $VaR_\alpha L$ of α ranging from 0.9 to 0.99:

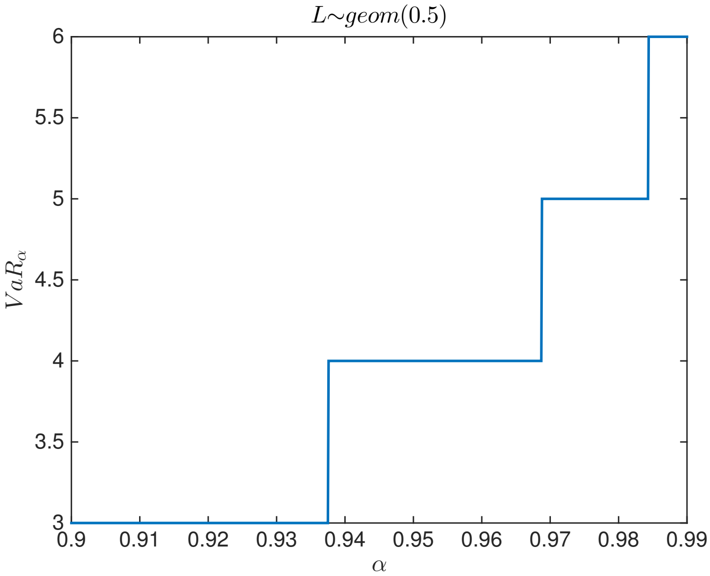

Figure 2: VaR_α when $p = 0.5$

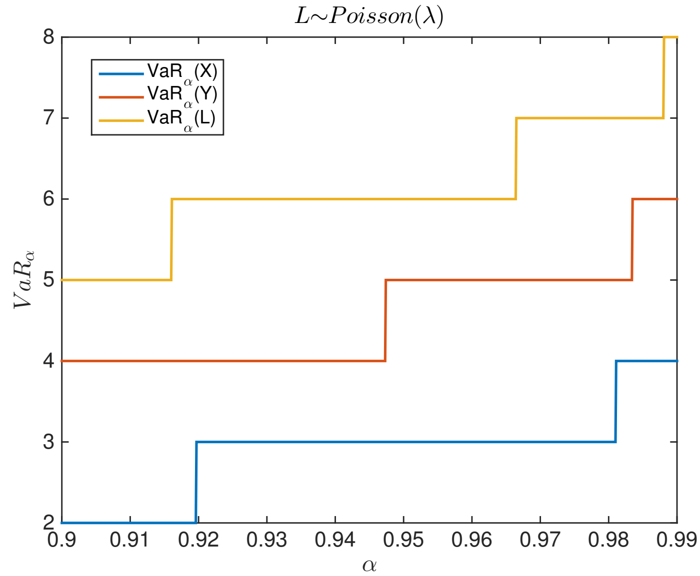

Figure 3: VaR_α for X, Y and L

Queation 3

Suppose such a distribution that L_i is are independent Bernoulli random variables, where $i = 1, 2$, with the distribution that,

$$P[L_i = 0] = \alpha + \delta$$
$$P[L_i = 1] = 1 - (\alpha + \delta)$$

Obviously, $VaR_\alpha(L_i) = 0$, where $i = 1, 2$. However, as for the distribution of $L = L_1 + L_2$, it will be,

$$\begin{aligned} P[L = 0] &= (\alpha + \delta)^2 \\ P[L = 1] &= 2(\alpha + \delta)(1 - \alpha - \delta) \\ P[L = 2] &= (1 - \alpha - \delta)^2 \end{aligned}$$

Therefore, we can have that,

$$VaR_\alpha(L) > 0 = VaR_\alpha(L_1) + VaR_\alpha(L_2)$$

if $(\alpha + \delta)^2 < \alpha$, which means that VaR_α is not subadditive.

Question 4

1

Based on the problem, we have already known that, the current value of each bond is 1000CHF, which means that $V_{i,0} = 1000$. Therefore, if the bond will not default at $t = 1$, its value will be $V_{i,1} = 1050$, otherwise, its value will be $V'_{i,1} = 0$. Thus, we can easily obtain the loss L_i,

$$\begin{aligned} L_i &= -(V_{i,1} - V_{i,0}) \\ &= -[1050(1 - I_i)] - 1000 \\ &= 1050 I_i - 50 \end{aligned}$$

2

Based on the problem, we have already known that the probability that the bond will default is equal to 0.02, which means that the probability distribution of I_i is,

$$\begin{aligned} P[I_i = 0] &= 0.98 \\ P[I_i = 1] &= 0.02 \end{aligned}$$

Hence, the probability distribution of L_i is,

$$\begin{aligned} P[L_i = 1000] &= 0.02 \\ P[L_i = -50] &= 0.98 \end{aligned}$$

3

Based on the problem, we can easily acquire that, as for portfolio a, its loss will be $L_a = 100(L_i) = 100(1050I_i - 50)$, since it consists of 100 unit of a single bond. Meanwhile, its probability distribution is same as one bond, which is,

$$\begin{aligned} P[L_a = 100000] &= P[L_i = 1000] = 0.02 \\ P[L_a = -5000] &= P[L_i = -50] = 0.98 \end{aligned}$$

Thus, as for VaR_α, it can be easily find that,

$$
\begin{aligned}
VaR_{0.95}[L_a] &= -5000 \\
VaR_{0.99}[L_a] &= 100000
\end{aligned}
$$

However, as for portfolio b, which is the sum of 100 independent different bonds, the loss of this portfolio will be,

$$
\begin{aligned}
L_b &= \sum_{i=1}^{100} 1050 I_i - 100 \times 50 \\
&= 1050 \sum_{i=1}^{100} I_i - 5000 \\
&= 1050B - 5000
\end{aligned}
$$

where B means the number of bonds which will default. Based on the problem, we know the probability distribution of I_i, which is obvious independent Bernoulli distribution, therefore, we can conclude that B's probability distribution will be Binomial distribution with parameters $n = 100, p = 0.02$. Therefore, to compute $VaR_{0.95}$ and $VaR_{0.99}$, we need to compute the quantiles of B in 0.95 and 0.99, which are,

$$
\begin{aligned}
B^{-1}(0.95, 100, 0.02) &= 5 \\
B^{-1}(0.99, 100, 0.02) &= 6
\end{aligned}
$$

Thus, based on the relation between B and L_b, we can easily obtain that,

$$
\begin{aligned}
VaR_{0.95}(L_b) &= 1050 \times 5 - 5000 = 250 \\
Var_{0.99}(L_b) &= 1050 \times 6 - 5000 = 1300
\end{aligned}
$$

Appendix

Following are our codes for solving the questions of this assignment. In addition, for commented and runnable codes for above problems, please refer to:

https://github.com/FINEPFL/Quant-Risk-Manage-MATH-471

```
clearvars; close all; clc

handle = fopen('data.csv');
raw_data = textscan(handle, '%f %f %s %f', 'delimiter', ',');
fclose(handle);

i = 1;
numMSFT = 0;
numYHOO = 0;
numITEL = 0;
while i <= length(raw_data{1, 3})
    switch(raw_data{1, 3}{i})
        case 'MSFT'
            numMSFT = numMSFT + 1;
        case 'YHOO'
            numYHOO = numYHOO + 1;
        otherwise
            numITEL = numITEL + 1;
    end
    i = i + 1;
end

msft_price = raw_data{1, 4}(1:numMSFT);
itel_price = raw_data{1, 4}(1+numMSFT:numMSFT+numITEL);
yhoo_price = raw_data{1, 4}(1+numMSFT+numITEL:end);

Price = [msft_price itel_price yhoo_price];

%% march 11 2013 to march 10 2016 corresponds to 503 and 1259
getReturn = @(P) log(P(2:end, :)./P(1:end-1, :));
Return    = getReturn(Price);
b_ = [100;
      100*Price(503, 1)/ Price(503, 2);
      100*Price(503, 1)/ Price(503, 3);];

for i=1:(size(Return, 1)-503)
    mu    = mean(Return(i:i+503, :))';
    Sigma = cov(Return(i:i+503, :));
    b = b_ .* Price(i+503, :)';
    L(i) = -b' * Return(i+503, :)';
    VaR_95(i) = -b' * mu + sqrt(b' * Sigma * b) * norminv(0.95);
    VaR_99(i) = -b' * mu + sqrt(b' * Sigma * b) * norminv(0.99);

    breach_95(i) = VaR_95(i) < L(i);
    breach_99(i) = VaR_99(i) < L(i);

end
```

```matlab
figure(1)
xaxis = 1:960;
plot(L, 'k', 'linewidth', 1); hold on;
plot(VaR_95, 'b', 'linewidth', 2); plot(VaR_99, 'r', 'linewidth', 2);
legend('Loss', 'VaR_{95}', 'VaR_{99}')
xlabel('date', 'interpreter', 'latex')
ylabel('loss', 'interpreter', 'latex')
plot(xaxis(breach_95), VaR_95(breach_95), 'g.', 'markersize', 25)
plot(xaxis(breach_99), VaR_99(breach_99), 'y.', 'markersize', 25)
set(gca, 'fontsize', 15)
times95 = sum(double(breach_95))
times99 = sum(double(breach_99))

%% Q2
% 1
clearvars; close all; clc
p = 0.5;
VaR_95 = geoinv(0.95, p)
alpha  = 0.9:0.0001:0.99;
VaR_alpha = geoinv(alpha, p);
plot(alpha, VaR_alpha, 'linewidth', 1.8)
xlabel('$\alpha$', 'interpreter', 'latex')
ylabel('$VaR_\alpha$', 'interpreter', 'latex')
title('$L \sim geom(0.5)$', 'interpreter', 'latex')
set(gca, 'fontsize', 15)
% 2
clearvars; close all; clc
alpha  = 0.9:0.0001:0.99;
VaR_X = poissinv(alpha, 1);
VaR_Y = poissinv(alpha, 2);
VaR_Z = poissinv(alpha, 3);
plot(alpha, VaR_X, alpha, VaR_Y, alpha, VaR_Z, 'linewidth', 1.8)
legend('VaR_\alpha(X)', 'VaR_\alpha(Y)', 'VaR_\alpha(L)')
xlabel('$\alpha$', 'interpreter', 'latex')
ylabel('$VaR_\alpha$', 'interpreter', 'latex')
title('$L \sim Poisson(\lambda)$', 'interpreter', 'latex')
set(gca, 'fontsize', 15)
```