Stephen Leake

6 February 2014

1 Introduction

"Kinematic Analysis of Four-Wheel Mecanum Vehicle" presents equations relating the rotational velocity of Mecanum wheels to the translational and rotational velocity of the robot.

Here we extend that to encoder counts and delta position for straight line and pure rotation motions.

The key equation is (6):

$$\omega_n = \frac{1}{r} \left\{ (V_x - Y_n\ \omega_v) + (V_y - X_n\ \omega_v)\ tan(\theta_n) \right\} \tag{1}$$

where:

V_x	x component of robot velocity
V_y	y component of robot velocity
ω_v	robot rotational velocity
ω_n	rotational velocity of wheel n
r	radius of wheels
X_n	x component of position of wheel n
Y_n	y component of position of wheel n
θ_n	mounting angle of wheel rollers on wheel n

Mecanum rollers are usually mounted at +- 45 degrees, and at the corners of a rectangle. Then we have:

wheel	position	number	roller angle	roller tangent
front left	$-K_x, +K_y$	1	-45	-1
rear left	$-K_x, -K_y$	2	+45	+1
rear right	$+K_x, +K_y$	3	-45	-1
front right	$+K_x, -K_y$	4	+45	+1

(1) assumes the wheels do not slip. In practice, they do, and by different amounts for forward and sideways motion. For motion on a uniform surface, we can incorporate that with a slip scale factor. We need three: $slip_x, slip_y, slip_{theta}$.

2 Straight line motion

Straight line motion is given by $\omega_v = 0$. Then (1) reduces to:

$$
\begin{aligned}
\omega_n &= \frac{1}{r}\left(V_x + V_y tan(\theta_n)\right) \qquad (2)\\
\omega_1 &= \frac{1}{r}\left(V_x - V_y\right)\\
\omega_2 &= \frac{1}{r}\left(V_x + V_y\right)\\
\omega_3 &= \frac{1}{r}\left(V_x - V_y\right)\\
\omega_4 &= \frac{1}{r}\left(V_x + V_y\right)
\end{aligned}
$$

Integrating this over time gives the total wheel rotation θ_n as a function of change in robot position Δ_x, Δ_y:

$$
\begin{aligned}
\theta_n &= \frac{1}{r}\left(\Delta_x + \Delta_y tan(\theta_n)\right) \qquad (3)\\
\theta_1 &= \frac{1}{r}\left(\Delta_x - \Delta_y\right)\\
\theta_2 &= \frac{1}{r}\left(\Delta_x + \Delta_y\right)\\
\theta_3 &= \frac{1}{r}\left(\Delta_x - \Delta_y\right)\\
\theta_4 &= \frac{1}{r}\left(\Delta_x + \Delta_y\right)
\end{aligned}
$$

Solving for Δ_x, Δ_y, and including the slip factors:

$$\begin{aligned}
r\theta_1 &= +\Delta_x - \Delta_y \\
r\theta_2 &= +\Delta_x + \Delta_y \\
r\theta_3 &= +\Delta_x - \Delta_y \\
r\theta_4 &= +\Delta_x + \Delta_y
\end{aligned}$$

$$\begin{aligned}
\Delta_x &= slip_x \frac{r}{4}(+\theta_1 + \theta_2 + \theta_3 + \theta_4) \qquad (4)\\
\Delta_y &= slip_y \frac{r}{4}(-\theta_1 + \theta_2 - \theta_3 + \theta_4)
\end{aligned}$$

3 Pure Rotation

Pure rotation is given by $V_x = V_y = 0$. Then (1) reduces to:

$$\begin{aligned}
\omega_n &= \frac{\omega_v}{r}\left\{Y_n - X_n tan(\theta_n)\right\} \qquad (5)\\
\omega_1 &= \frac{\omega_v}{r}\left(+K_y - K_x\right) \\
\omega_2 &= \frac{\omega_v}{r}\left(-K_y + K_x\right) \\
\omega_3 &= \frac{\omega_v}{r}\left(+K_y + K_x\right) \\
\omega_4 &= \frac{\omega_v}{r}\left(-K_y - K_x\right)
\end{aligned}$$

Integrating over time gives the wheel rotation θ_n as a function of change in orientation $theta_v$:

$$\theta_n = \frac{\theta_v}{r}\left\{Y_n - X_n tan(\theta_n)\right\}$$

We can use any single wheel to find $theta_v$, but it is more robust to average them all. We also include the slip factor:

$$
\begin{aligned}
\theta_v &= \frac{r\ slip_{theta}\ \theta_n}{\{Y_n - X_n tan(\theta_n)\}} \qquad (6) \\
&= \frac{r\ slip_{theta}}{4}\{\frac{\theta_1}{+K_y - K_x} + \\
&\quad \frac{\theta_2}{-K_y + K_x} + \\
&\quad \frac{\theta_3}{+K_y + K_x} + \\
&\quad \frac{\theta_4}{-K_y - K_x}\}
\end{aligned}
$$