This paper presents the solution to the **inverse kinematic problem** and the **forward kinematic problem** for a 4-wheel mecanum vehicle.

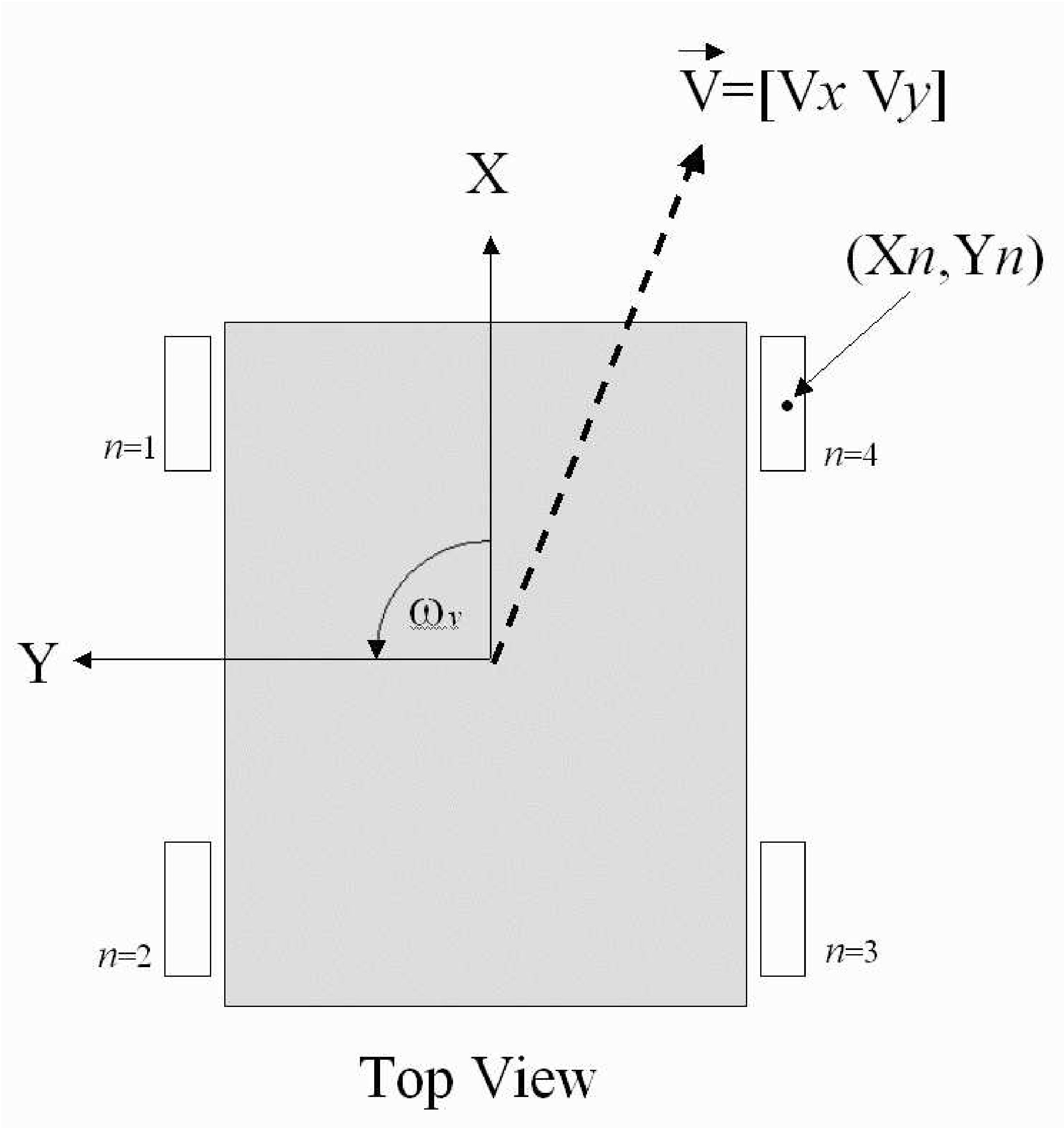
The inverse kinematic equations allow you to calculate the four independent wheel angular velocities required to produce a desired vehicle velocity and rotation.

The forward kinematic equations predict the vehicle motion, given the four wheel angular velocities.

## Definitions

See Figure 1 below. Define a top-view coordinate system on the vehicle with origin equidistant from the four wheels and X-axis pointing forward and Y-axis pointing to the left.

Let [V] be the 3x1 matrix $[V_x\ V_y\ \omega_v]'$ which represents the desired translational and rotational velocity of the vehicle at an instant in time. $\omega_v$ is positive anticlockwise.




Number the 4 wheels 1, 2, 3, 4 starting at the front port wheel and proceeding anticlockwise (as viewed from above).

Let ($Xn$,$Yn$) be the coordinates of the center of the $n^{th}$ wheel.

Let $\theta_n$ be the anticlockwise angle that the axis of the mecanum roller of wheel_$n$ in contact with the floor makes with the X axis. Assume all four wheels are parallel to the X axis. Let the radius of each wheel be r.

**Inverse Kinematic Problem**

The inverse kinematic problem is to find the 4x1 matrix

$[\Omega] = [\omega_1\ \omega_2\ \omega_3\ \omega_4]'$ which represents the rotational velocities of the 4 mecanum wheels which produce the desired [V].

In other words, we are looking for a 4x3 matrix [R] such that

$[\Omega] = (1/r)[R][V]$    Equation(1)

[R] allows us to compute $[\Omega]$, given [V].

Proceed as follows:

Assuming the vehicle is a rigid body, the $V_x$ and $V_y$ vehicle translational velocity components are present at each wheel center.

Each wheel also has an *additional* X and Y velocity component due to the vehicle's rotational velocity $\omega v$ given by

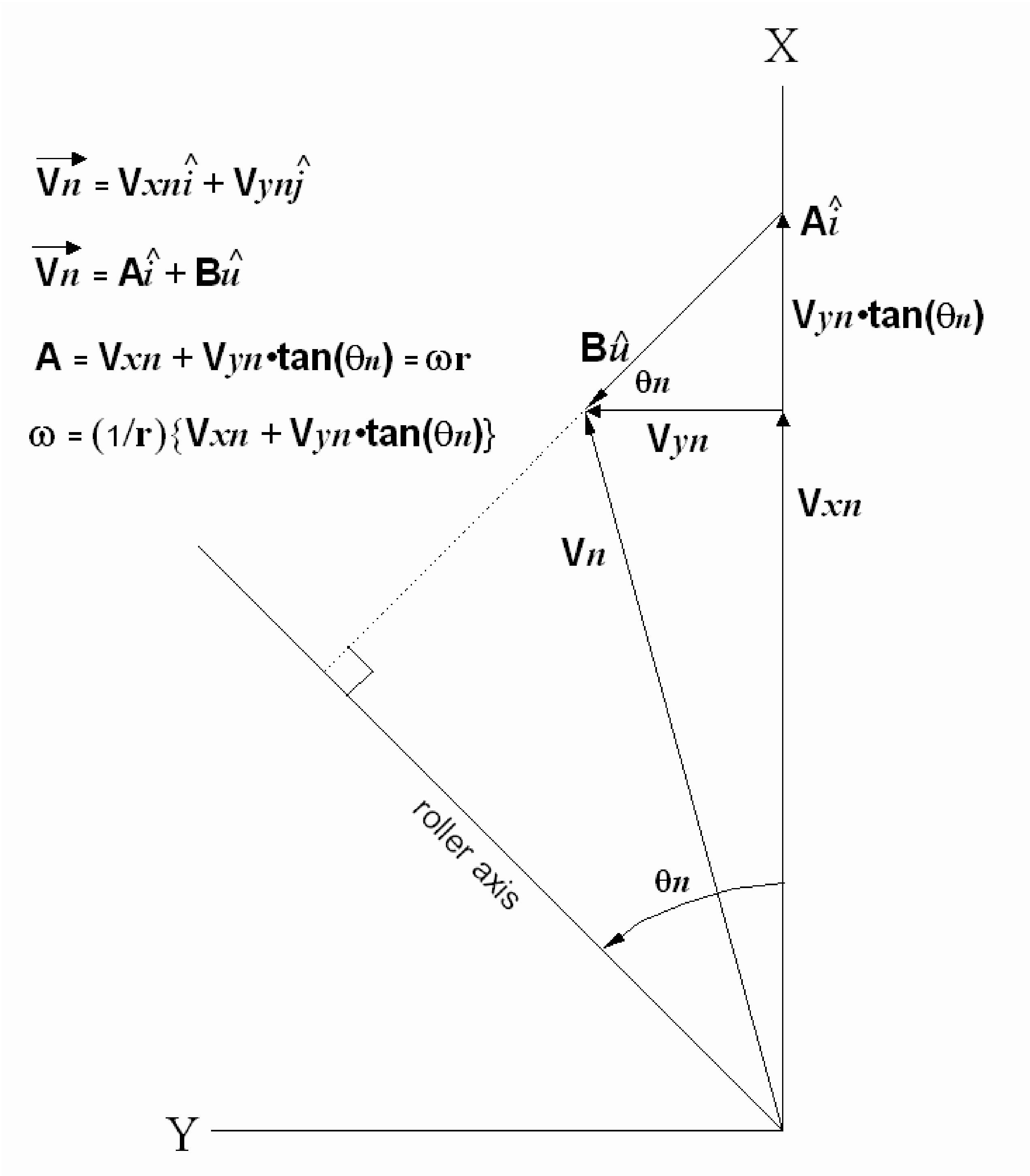
$V_{xrn} = -Y_n \cdot \omega v$    $V_{yrn} = X_n \cdot \omega v$    Equation(2)

Therefore the total linear velocity $V_n$ at each wheel center is given by the vector components

$V_{xn} = V_x + V_{xrn} = V_x - Y_n \cdot \omega v$

and

$V_{yn} = V_y + V_{yrn} = V_y + X_n \cdot \omega v$    Equation(3)




See Figure 2 above. The linear velocity vector V*n* = [V*xn* V*yn*] at each wheel is resolved into two vectors, one parallel to the X axis (**A*î***) and one perpendicular to the axis of the roller in contact with the floor (**B*û***). The magnitude of the component parallel to the X axis is

$\omega_n \cdot r = V_{xn} + V_{yn} \cdot \tan(\Theta_n)$ Equation(4)

Substituting $V_{xn}$ and $V_{yn}$ from Equation(3) gives

$\omega_n \cdot r = (V_x - Y_n \cdot \omega_v) + (V_y + X_n \cdot \omega_v)\tan(\Theta_n)$ Equation(5)

and therefore the rotational velocity of each wheel is given by

$\omega_n = (1/r)\{(V_x - Y_n \cdot \omega_v) + (V_y + X_n \cdot \omega_v)\tan(\Theta_n)\}$ Equation(6)

Equation (6) gives each wheel rotational velocity $\omega_n$ (the 4 elements of matrix [Ω]) as a linear function of [V] and the constants r, $X_n$, $Y_n$, and $\Theta_n$, and so the matrix [R] is readily observed to be:

$$
\begin{matrix}
1 & \mathrm{TAN}(\Theta_1) & (X_1 \cdot \mathrm{TAN}(\Theta_1) - Y_1) \\
1 & \mathrm{TAN}(\Theta_2) & (X_2 \cdot \mathrm{TAN}(\Theta_2) - Y_2) \\
1 & \mathrm{TAN}(\Theta_3) & (X_3 \cdot \mathrm{TAN}(\Theta_3) - Y_3) \\
1 & \mathrm{TAN}(\Theta_4) & (X_4 \cdot \mathrm{TAN}(\Theta_4) - Y_4)
\end{matrix}
$$

Assuming $\Theta_2$ & $\Theta_4$ = 45 degrees, and $\Theta_1$ & $\Theta_3$ = -45 degrees, this simplifies to:

$$\begin{matrix} 1 & -1 & -X_1-Y_1 \\ 1 & 1 & X_2-Y_2 \\ 1 & -1 & -X_3-Y_3 \\ 1 & 1 & X_4-Y_4 \end{matrix}$$

Assume all $Xn$ have the same magnitude and all $Yn$ have the same magnitude, and let $K=abs(Xn)+abs(Yn)$. Then the matrix simplifies to:

$$\begin{matrix} 1 & -1 & -K \\ 1 & 1 & -K \\ 1 & -1 & K \\ 1 & 1 & K \end{matrix}$$

**Forward Kinematic Problem**

Now consider the forward kinematic problem for the above special case, i.e., find the 3x4 matrix [F] such that

$$[F][\Omega](r)=[V]$$

...in other words, given the four wheel rotational velocities [Ω], find the resulting vehicle motion [V].

This problem, in general, has no solution, since it represents an overdetermined system of simultaneous linear equations. The physical meaning of this is: if four arbitrary rotational velocities are chosen for the four wheels, there is in general no vehicle motion which does not involve some wheel "scrubbing" (slipping) on the floor. However, a matrix [F] which generates a "best fit" least squares solution can be found:

Start with the inverse kinematic equation:

$$[\Omega](r)=[R][V]$$

multiply both sides by the transpose of [R]:

$$[R]'[\Omega](r)=[R]'[R][V]$$

multiply both sides by the inverse of [R]'[R]:

$$(([R]'[R])^{-1})[R]'[\Omega](r)=(([R]'[R])^{-1})([R]'[R])[V]$$

the right-hand side of the above equation is just [V], so:

$(([R]'[R])^{-1})[R]'[\Omega](r)=[V]$

Let $[F]=(([R]'[R])^{-1})[R]'$ and:

$[F][\Omega](r)=[V]$

which is the forward kinematic equation.

Using the simplified inverse matrix [R], the forward matrix [F] is readily computed to be:

| | | | |
|---|---|---|---|
| 1/4 | 1/4 | 1/4 | 1/4 |
| -1/4 | 1/4 | -1/4 | 1/4 |
| -1/(4K) | -1/(4K) | 1/(4K) | 1/(4K) |