Extracting Yield and Uncertainty from a Gaussian Function

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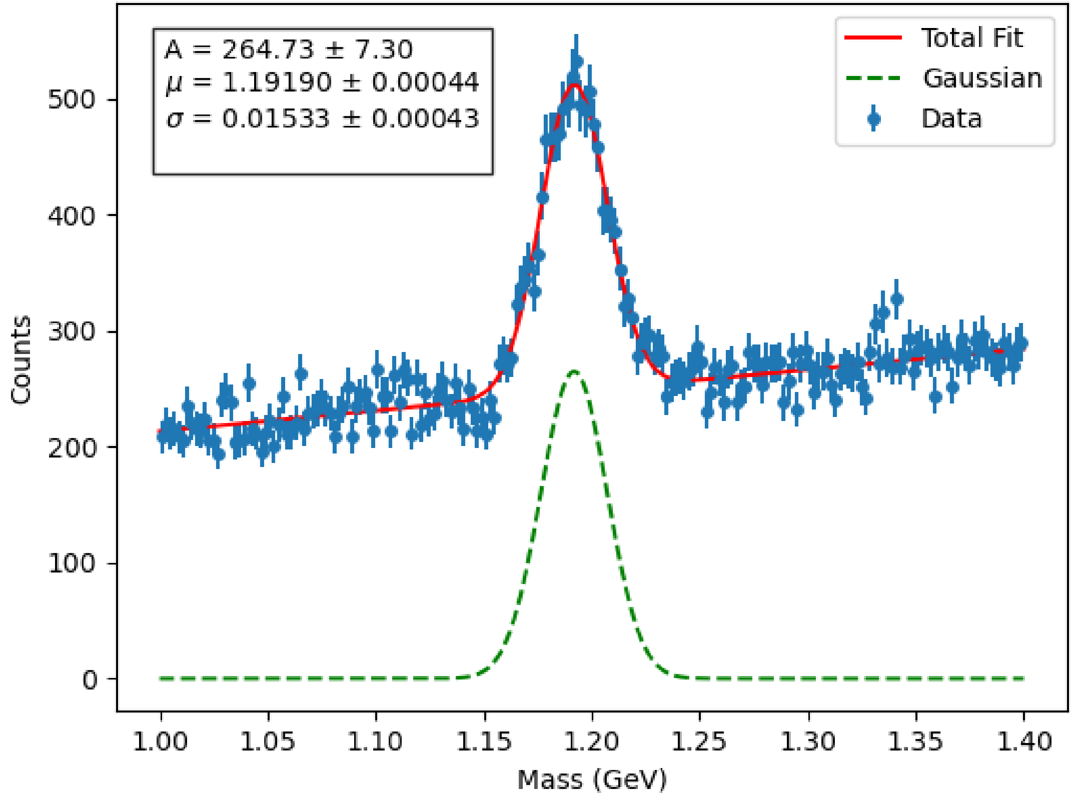
Yield: Gaussian Signal

- Integral of a Gaussian function
  - $I = \int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}}$
- In terms that we're interested in,
  - $G = Ae^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- Relating these two forms, we get
  - $a = \frac{1}{2\sigma^2}, \quad b = -\mu$
- Then
  - $I = A\sqrt{\frac{\pi}{a}} = I = \sqrt{2\pi}A\sigma$



Propagation of Errors

- $\sigma_x^2 \cong \sigma_u^2 (\frac{\partial x}{\partial u})^2 + \sigma_v^2 (\frac{\partial x}{\partial v})^2 + \cdots + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right) + \cdots$
  - Derived from as Taylor Series expansion of the definition of variance:
    - $\sigma^2 = \lim_{N \to \infty} \left[\frac{1}{N} \sum (x_i - \mu)^2\right]$
- Note: *Error Propagation Equation is an approximation*
  - Ignores higher order terms in expansion

Uncertainty of Gaussian Integral by Propagation of Errors

- $I = \sqrt{2\pi} * A * \sigma$

$$\begin{bmatrix} Var(A) & Cov(A\mu) & Cov(A\sigma) \\ Cov(\mu A) & Var(\mu) & Cov(\mu\sigma) \\ Cov(\sigma A) & Cov(\sigma\mu) & Var(\sigma) \end{bmatrix}$$

- $\delta_I^2 = \delta_A^2 (\frac{\partial I}{\partial A})^2 + \delta_\sigma^2 (\frac{\partial I}{\partial \sigma})^2 + 2\delta_A \delta_\sigma \frac{\partial I}{\partial A}\frac{\partial I}{\partial \sigma}$
  - Or, $\delta_I^2 = Var(A)(\frac{\partial I}{\partial A})^2 + Var(\sigma)(\frac{\partial I}{\partial \sigma})^2 + 2Cov(A\sigma)\frac{\partial I}{\partial A}\frac{\partial I}{\partial \sigma}$
    - $\frac{\partial I}{\partial A} = \sqrt{2\pi}\sigma, \ \frac{\partial I}{\partial \sigma} = \sqrt{2\pi}A$
- $\delta_I = \sqrt{(Var(A) * 2\pi\sigma^2) + (Var(\sigma) * 2\pi A^2) + 4\pi Cov(A\sigma)A\sigma}$