Parámetro a estimar $\theta = \mu$ $\qquad X_1, X_2, \ldots, X_n \underset{iid}{\sim} N(\mu, \sigma)$

$$\hat{\theta} = \overline{X} = \frac{\sum X_i}{n} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Estad. pivote:

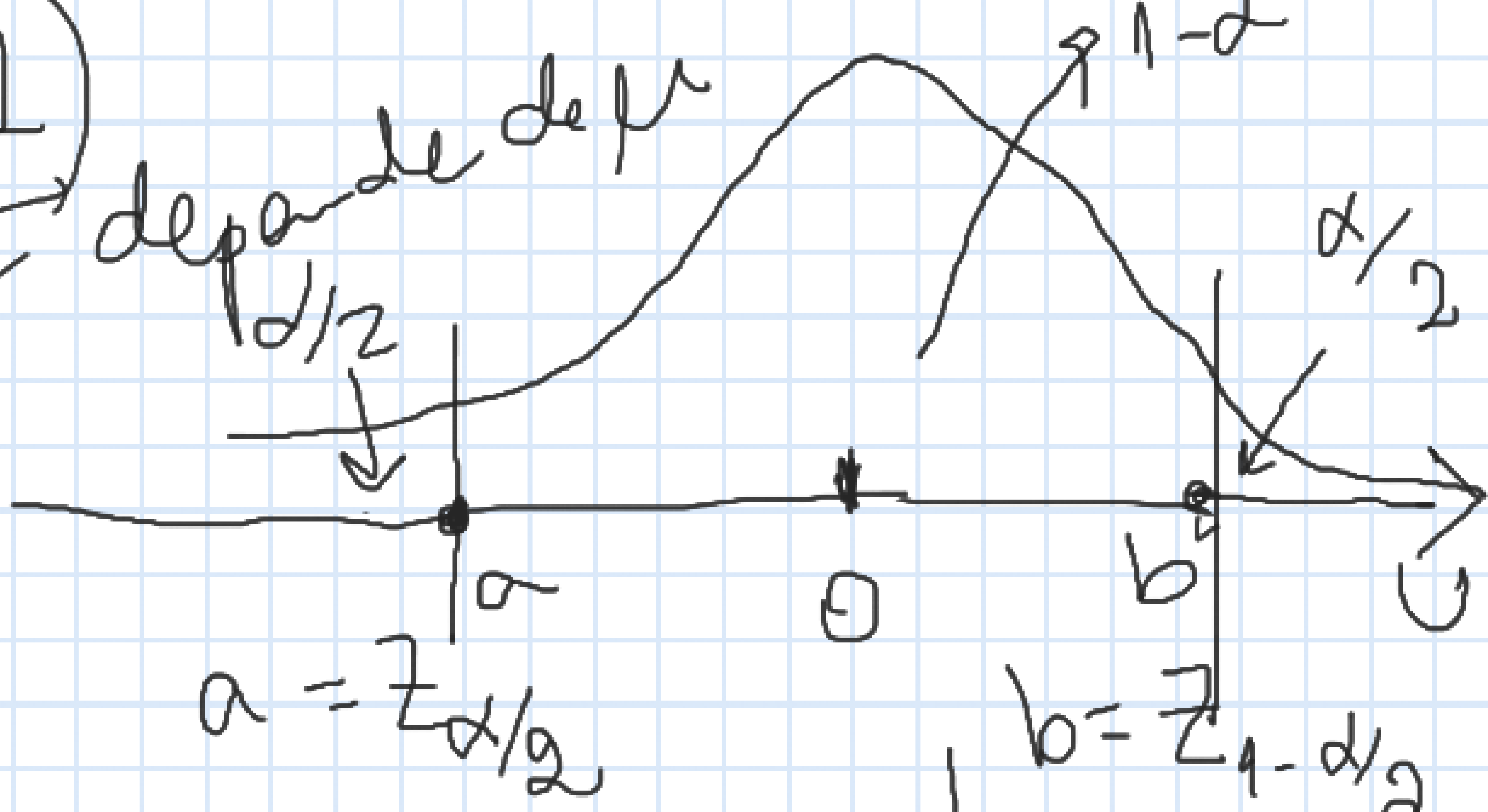
$$U = g(\underset{\sim}{X}, \mu) = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim \underbrace{N(0,1)}_{\text{no}} \rightarrow \text{depende de } \mu$$

$$P(a(\underset{\sim}{X}) \leq U \leq b(\underset{\sim}{X})) = 1 - \alpha$$

$$P\left(-b \leq \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} \leq b\right) = 1 - \alpha$$

$$P\left(-b \cdot \frac{\sigma}{\sqrt{n}} \leq \overline{X} - \mu \leq b \cdot \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

$$P\left(-\overline{X} - b \cdot \frac{\sigma}{\sqrt{n}} \leq -\mu \leq -\overline{X} + b \cdot \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$



$a = -b$

$$P\left(\bar{X} - b\cdot\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + b\cdot\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

$$\underbrace{P\left(\bar{X} - Z_{1-\frac{\alpha}{2}}\cdot\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{1-\frac{\alpha}{2}}\cdot\frac{\sigma}{\sqrt{n}}\right)}_{} = 1-\alpha$$

Una vez observada $S(\hat{x})$ la muestra:

$$IC_{\mu,(1-\alpha)} = \left(\underbrace{\bar{X}}_{\text{estimador puntual}} \pm \underbrace{Z_{1-\frac{\alpha}{2}}\cdot\frac{\sigma}{\sqrt{n}}}_{E}\right)$$

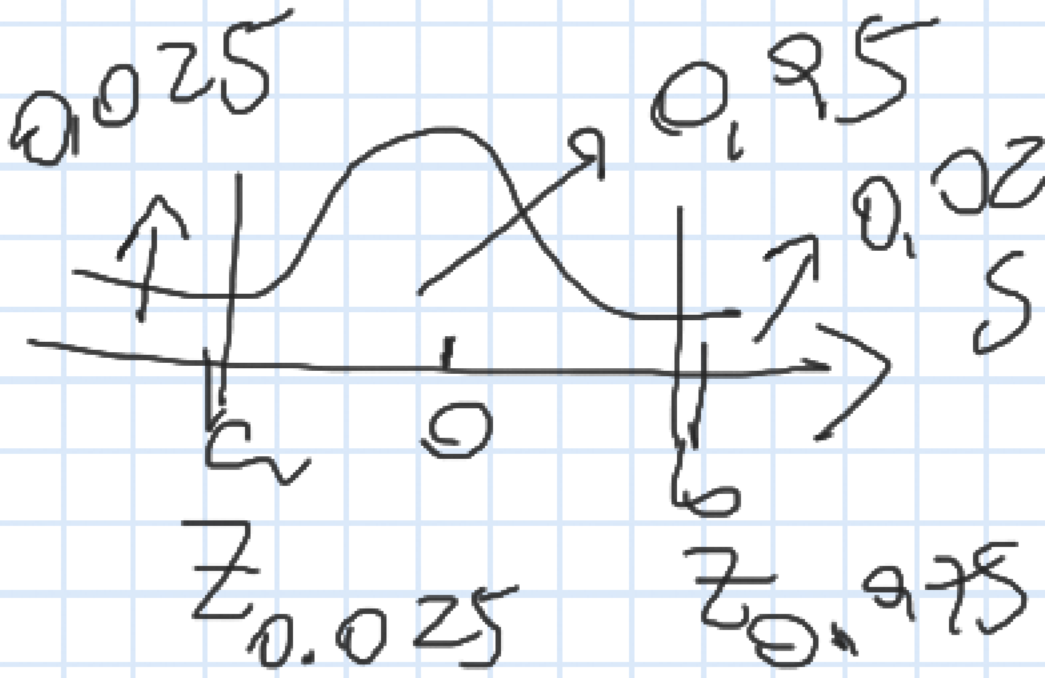
E = error de estimación

Si $(1-\alpha)\uparrow$ (n fijo)

$\Rightarrow Z_{1-\frac{\alpha}{2}}\uparrow \Rightarrow E\uparrow$ (IC + amplio) $\Rightarrow \downarrow$ precisión

Si $n \uparrow \Rightarrow \frac{\sigma}{\sqrt{n}} \downarrow \Rightarrow E \downarrow \Rightarrow$ IC + estrecho

$\rightarrow \uparrow$ precisión $(1-\alpha$ fijo$)$



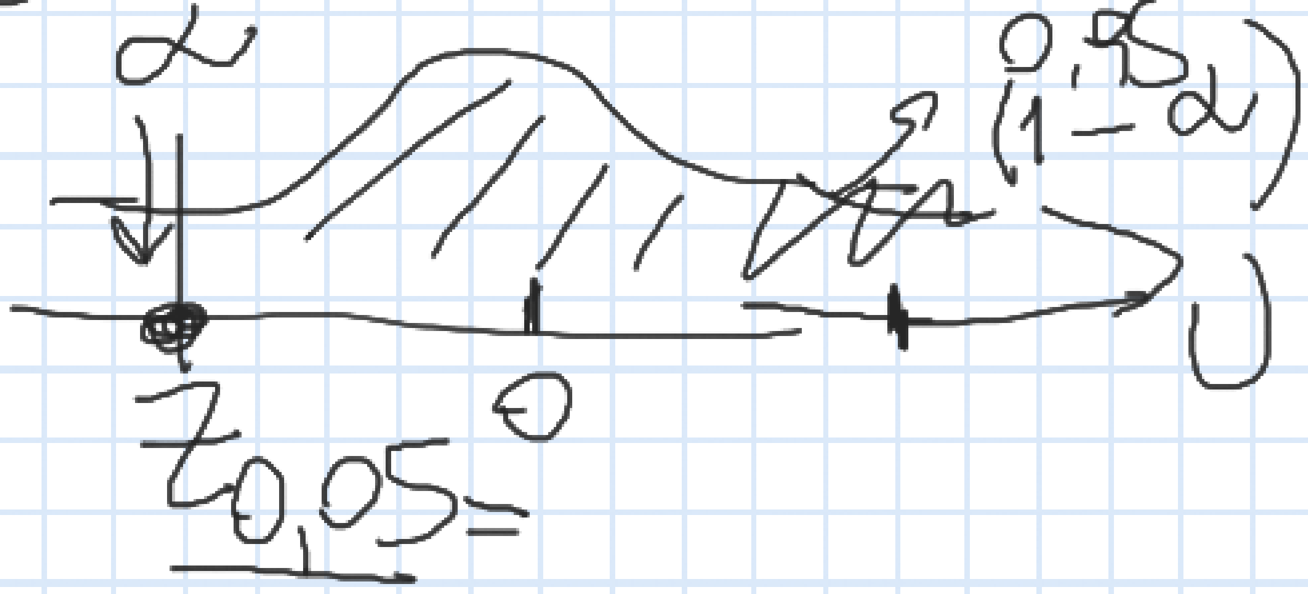
$$IC_{\mu, 95\%} = \left(\bar{X} \pm z_{0,975} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

$$= (3,26 ; 5,01)$$

1,96

$$P\left(\mu > \bar{X} - z_{1-\alpha} \frac{\sigma}{\sqrt{n}}\right) = 0,95$$

$$S(\underset{\sim}{x}) = (2,71 ; +\infty)$$



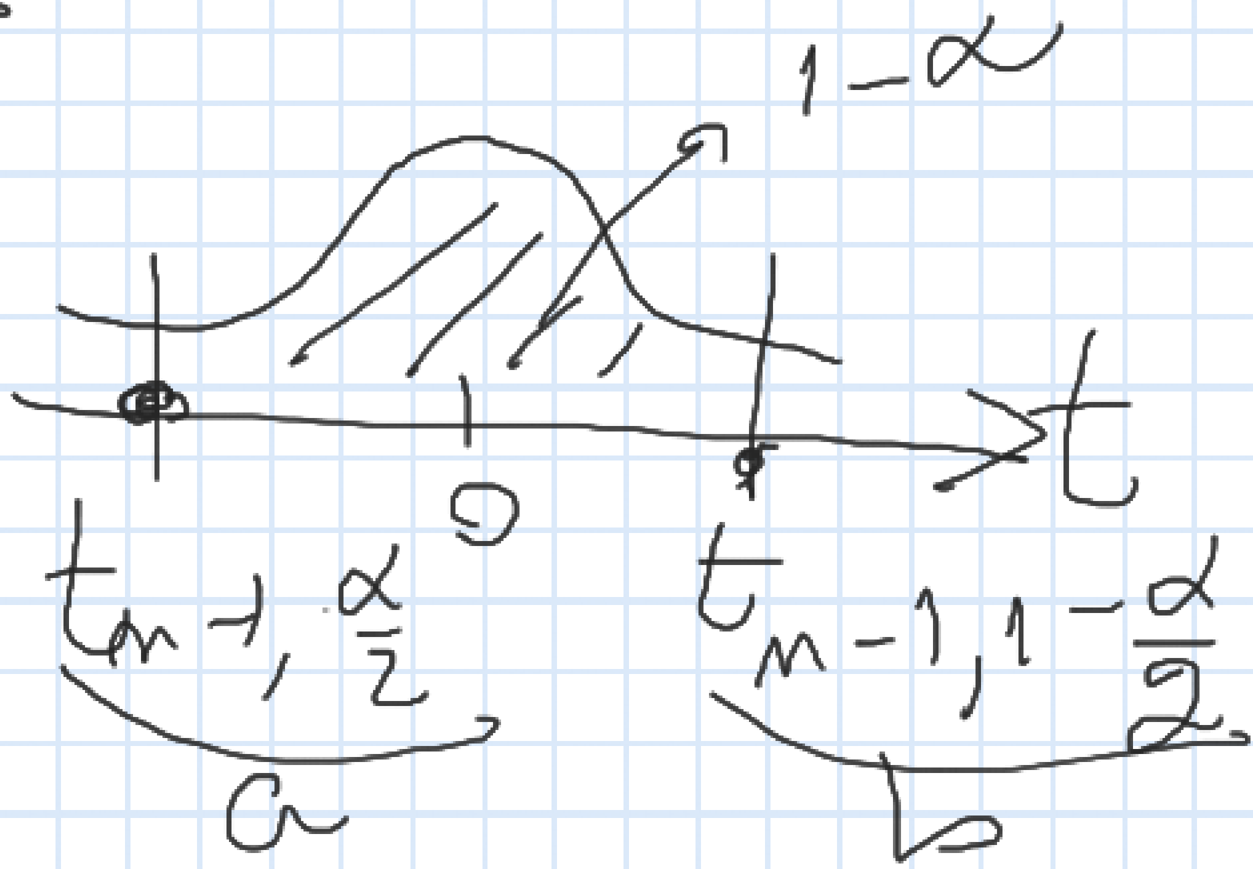
Parámetro a estimar $\theta = \mu$ (σ desconocido)

$X_1, X_2, \ldots, X_n \sim N(\mu = 2, \sigma = 3)$, $n = 50$

$$U = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \qquad gl = n - 1 = 49$$

$$P\left(a(\underset{\sim}{X}) \leq U \leq b(\underset{\sim}{X})\right) = 1 - \alpha$$

$$P\left(a(\underset{\sim}{X}) \leq \frac{\bar{X} - \mu}{S/\sqrt{n}} \leq b(\underset{\sim}{X})\right) = 1 - \alpha$$

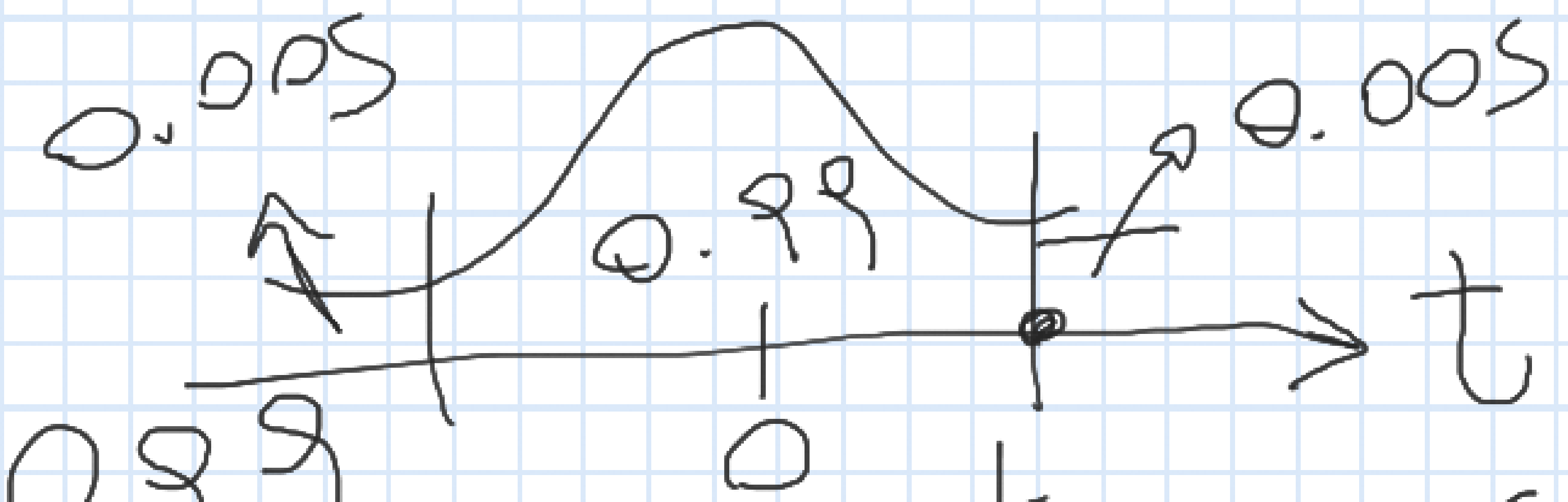


$$P\left(-b \leq \frac{\bar{X} - \mu}{S/\sqrt{n}} \leq b\right) = 1 - \alpha$$

$a = -b$

$$P\left(\bar{X} - t_{n-1,1-\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1,1-\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

$$1-\alpha = 0{,}99$$



$$\underbrace{t_{49,\,0.995}}_{2.68}$$

$$P\left(\bar{X} \pm 2{,}68 \cdot \frac{S}{\sqrt{n}}\right) = 0{,}99$$

Una vez tomada la muestra

$$IC_{\mu,\,99\%} = (0{,}9\ ;\ 3{,}15)$$

X = nº de caras en un total de 50 ensayos.

$y_1 \; y_2 \; y_3 \; \dots \; y_{50}$

$1 \; 0 \; 1 \; 1 \; \dots$

$X \sim Bi(n = 50, \; p = P(\text{"cara"}))$

Parámetro a estimar $= p = \theta$

$X = \sum_{i=1}^{50} y_i = 30$

$\hat{\theta} = \hat{p} = \frac{X}{n} \underset{\substack{n \to +\infty \\ \text{xTCL}}}{\sim} N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$

$E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{\cancel{n}} \cancel{n} p \quad (n = 50 \geq 30)$

depende de p

$E(\hat{p}) = p$

$Var(\hat{p}) = \frac{p(1-p)}{n}$

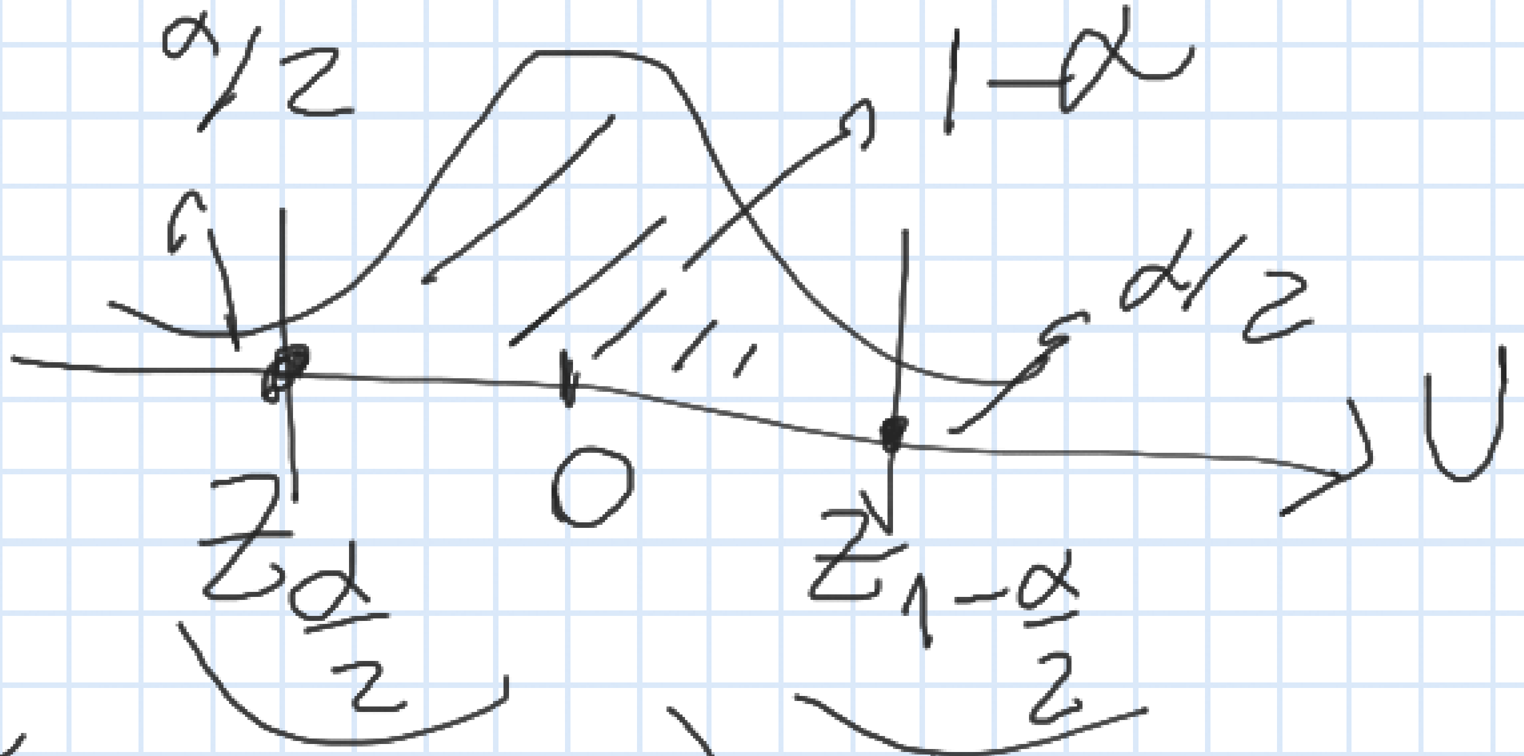
$U = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \underset{\substack{n \to +\infty \\ \text{xTCL}}}{\sim} N(0,1)$

teo de Slustky

Est. Pivote asintótica

$$P(a \leq U \leq b) = 1-\alpha$$

$$P\left(-b \leq \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \leq b\right) = 1-\alpha$$



$$P\left(-z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq \hat{p} - p \leq z \cdot \sqrt{\cdots}\right) = 1-\alpha \qquad a = -b$$

$$\vdots$$

$$P\left(\hat{p} - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 1-\alpha$$

$$P\left(\hat{p} \pm \underbrace{Z_{1-\frac{\alpha}{2}} \cdot \boxed{\sqrt{\frac{\hat{p}\cdot(1-\hat{p})}{n}}}}_{E}\right) = 1-\alpha$$

Est. puntual

Para nuestros datos

$$\hat{p} = \frac{x}{n} = \frac{3}{5} \Rightarrow IC_{p,\,95\%} = \left(\frac{3}{5} \pm \overbrace{\underbrace{Z_{0.975}}_{1{,}96} \cdot \sqrt{\frac{3/5 \cdot 2/5}{50}}}^{E}\right)$$

$$= \left(\frac{3}{5} \pm 0{,}135\right) = \left(\quad\right)$$