$\boxed{Ej\ 1}$ $X$ = nº de aciertos en 10 tiros

$$X \sim Bi(n=10, p=1/5) \Rightarrow \underset{x=0,10}{\underline{p_X(x)}} = \binom{10}{x} \cdot \left(\frac{1}{5}\right)^x \cdot \left(\frac{4}{5}\right)^{10-x}$$

$Y$ = nº de aciertos en el 1er tiro

$$Y \sim Be(p=1/5) \Rightarrow p_Y(y) = \left(\frac{1}{5}\right)^y \cdot \left(\frac{4}{5}\right)^{1-y}, \quad y=0,1.$$

$$Y|X=x \sim p_{Y|X=x}(y) = ? \quad \forall x \in \{0,1,\dots,10\}$$

Si $X=0 \Rightarrow P(Y=1/X=0)=0$ y $P(Y=0/X=0)=1$

$$\Rightarrow \boxed{Y|X=0 \sim Be(p=0)}$$

Si $X=10 \Rightarrow P(Y=1/X=10)=1$ y $P(Y=0|X=10)=0$

$$\boxed{Y/X=10 \sim Be(p=1)}$$

Si $X=1 \Rightarrow P(Y=1/X=1) = \dfrac{P(Y=1, X=1)}{P(X=1)} = \dfrac{\cancel{\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^9}}{\binom{10}{1}\cancel{\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^9}} = \dfrac{1}{10}$

$\boxed{1}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \ldots\ \underline{0}$

$\frac{1}{5}\ \ \frac{4}{5}\ \ \frac{4}{5}\ \ldots\ \frac{4}{5}$

$\Rightarrow P(Y=0/X=1) = \dfrac{9}{10}$

$\Rightarrow \boxed{Y/X=1 \sim Be\left(p=\frac{1}{10}\right)}$

Si $X=2 \Rightarrow P(Y=1|X=2) =$

$= \dfrac{P(Y=1, X=2)}{P(X=2)} = \dfrac{\frac{1}{5}\cdot P(W=1)}{\binom{10}{2}\cdot\left(\frac{1}{5}\right)^2\cdot\left(\frac{4}{5}\right)^8} = \dfrac{\cancel{\frac{1}{5}}\cdot\binom{9}{1}\cdot\cancel{\left(\frac{1}{5}\right)^1}\cancel{\left(\frac{4}{5}\right)^8}}{\binom{10}{2}\cancel{\left(\frac{1}{5}\right)^2}\cdot\cancel{\left(\frac{4}{5}\right)^8}} = \dfrac{\binom{9}{1}}{\binom{10}{2}}$

$\boxed{1}\ 1\ 0\ 0 \ldots 0$

$\frac{1}{5}\ \ 0\ 1\ 0 \ldots 0$

$\underbrace{0\ 0\ 1 \ldots 0}_{P(W)=1}$

$= \dfrac{\frac{9!}{1!\,\cancel{8!}}}{\frac{10!}{2!\,\cancel{8!}}} = \dfrac{\cancel{9!}}{\frac{10\cdot\cancel{9!}}{2}} = \dfrac{2}{10}$

$W =$ nº de aciertos en 9 tiros; $W \sim Bi(n=9, p=1/5)$

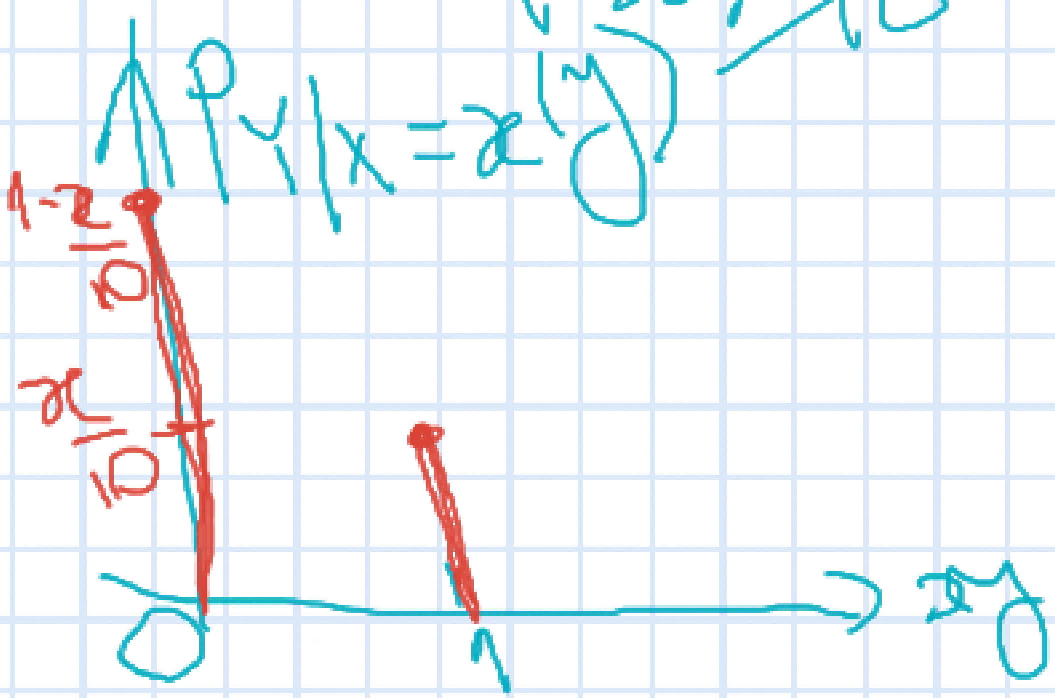
$$P(Y=1 \mid X=2) = \frac{2}{10} \Rightarrow P(Y=0 \mid X=2) = \frac{8}{10}$$

$$\Rightarrow \boxed{Y \mid X=2 \sim Be\left(p = \frac{2}{10}\right)}$$

$$P(Y=1 \mid X=x) = \frac{1/5 \cdot P(W=x-1)}{P(X=x)} = \frac{\left(\frac{1}{5}\right)\cdot\binom{9}{x-1}\cdot\left(\frac{1}{5}\right)^{x-1}\cdot\left(\frac{4}{5}\right)^{9-(x-1)}}{\binom{10}{x}\cdot\left(\frac{1}{5}\right)^{x}\cdot\left(\frac{4}{5}\right)^{10-x}} =$$

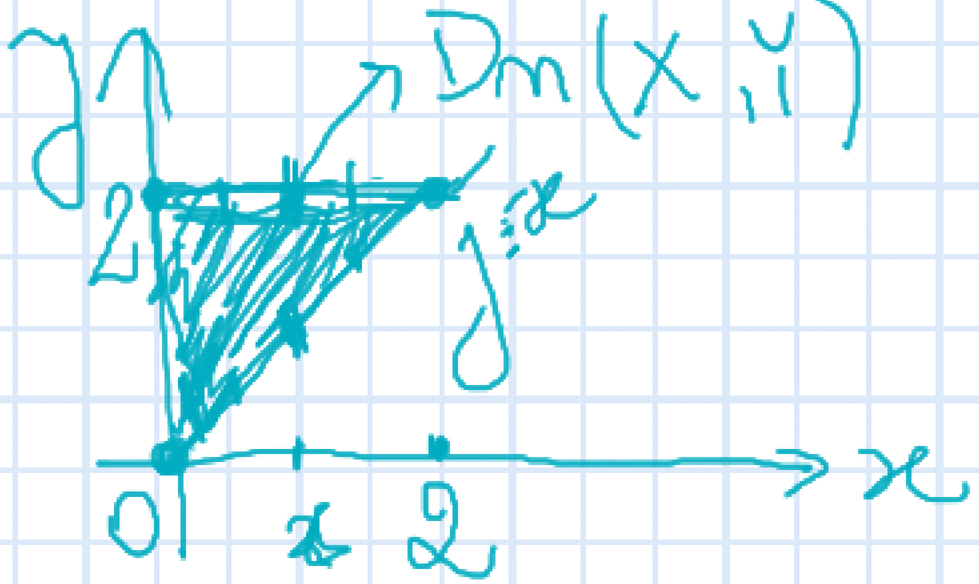
$$= \frac{\cancel{\left(\frac{1}{5}\right)^{x}}\cdot\binom{9}{x-1}\cdot\cancel{\left(\frac{4}{5}\right)^{10-x}}}{\binom{10}{x}\cdot\cancel{\left(\frac{1}{5}\right)^{x}}\cdot\cancel{\left(\frac{4}{5}\right)^{10-x}}} = \frac{x}{10}, \quad \forall x \in \{0,1,2,\dots,10\}$$

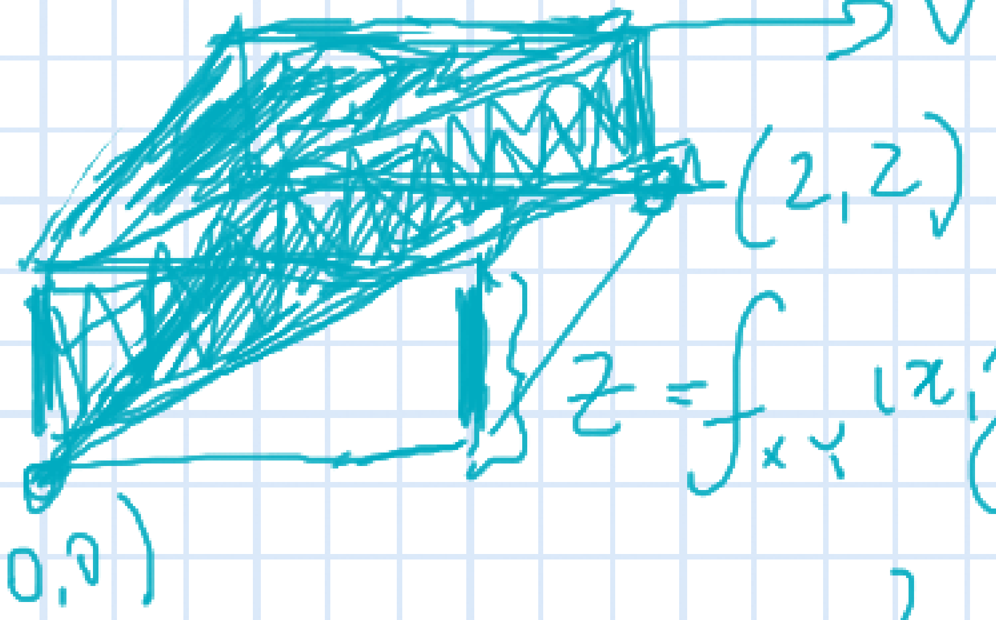
$$\Rightarrow \boxed{Y \mid X=x \sim Be\left(p = \frac{x}{10}\right) \quad \forall x = \overline{0,10}}$$




$$\Rightarrow E[Y \mid X=x] = ?$$

Ej 2: $Y|X=x \sim f_{Y|X=x}(y) = \, ?$




$V = 1 \Rightarrow C = \frac{1}{2}$

$z = f_{XY}(x,y) = C$

$$f_{XY}(x,y) = \frac{1}{2} \cdot \mathbb{1}\{0<x<y<2\}$$

$$f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{\cancel{\frac{1}{2}} \cdot \mathbb{1}\{0<x<y<2\}}{\cancel{\frac{1}{2}}(2-x)\cdot \mathbb{1}\{0<x<2\}} = \frac{1}{2-x} \cdot \mathbb{1}\{0<x<y<2\}$$

$$f_X(x) = \int_y f_{XY}(x,y)\,dy = \int_x^2 \frac{1}{2} \cdot \mathbb{1}\{0<x<y<2\}\,dy = \frac{1}{2}\cdot(2-x)\cdot \mathbb{1}\{0<x<2\}$$

$$\Rightarrow \boxed{Y|X=x \sim \mathcal{U}(x,2) \quad \forall x\in(0,2)}$$

Prop: $\boxed{X|Y=y \sim \mathcal{U}(0,y) \quad \forall y \in (0,2)}$

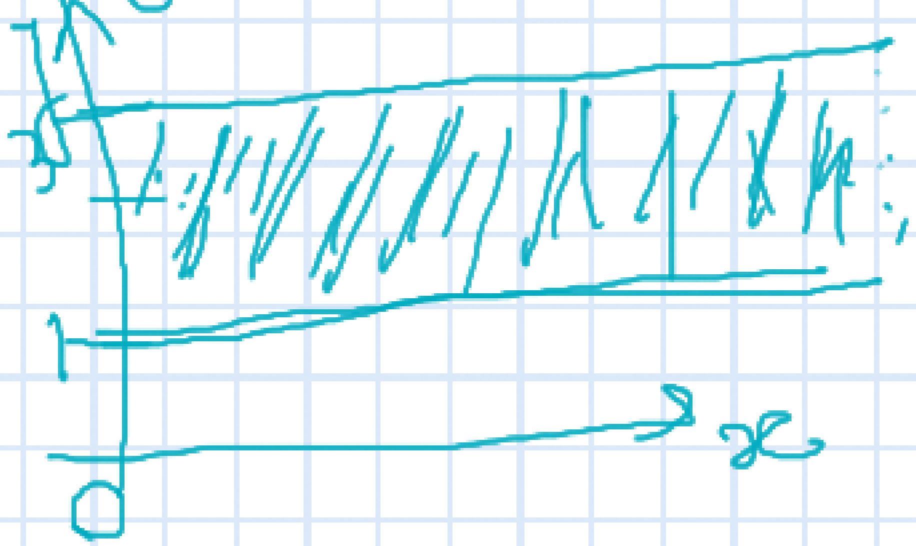
$$P(A\cap B) = P(A/B)\cdot P(B) \quad \text{ó} \quad P(A\cap B) = P(B/A)\cdot P(A)$$

$$\boxed{f_{XY}(x,y) = f_{X|Y=y}(x)\cdot f_Y(y)}$$

ó

$$= f_{Y|X=x}(y)\cdot f_X(x)$$

Ej 3:




$$f_{XY}(x,y) = \frac{e^{-\frac{x}{2y}}}{4y}\cdot \mathbb{1}\{0<x,\ 1<y<3\}$$

$$\boxed{f_{X|Y=y}(x) = \frac{f_{XY}(x,y)}{f_Y(y)}}$$

$$f_Y(y) = \int_x f_{XY}(x,y)\,dx = \int_0^{+\infty} \frac{e^{-\frac{x}{2y}}}{4y}\,\mathbb{1}\ldots\,dx = \frac{1}{4y}\int_0^{+\infty} e^{-\frac{1}{2y}x}\,dx$$

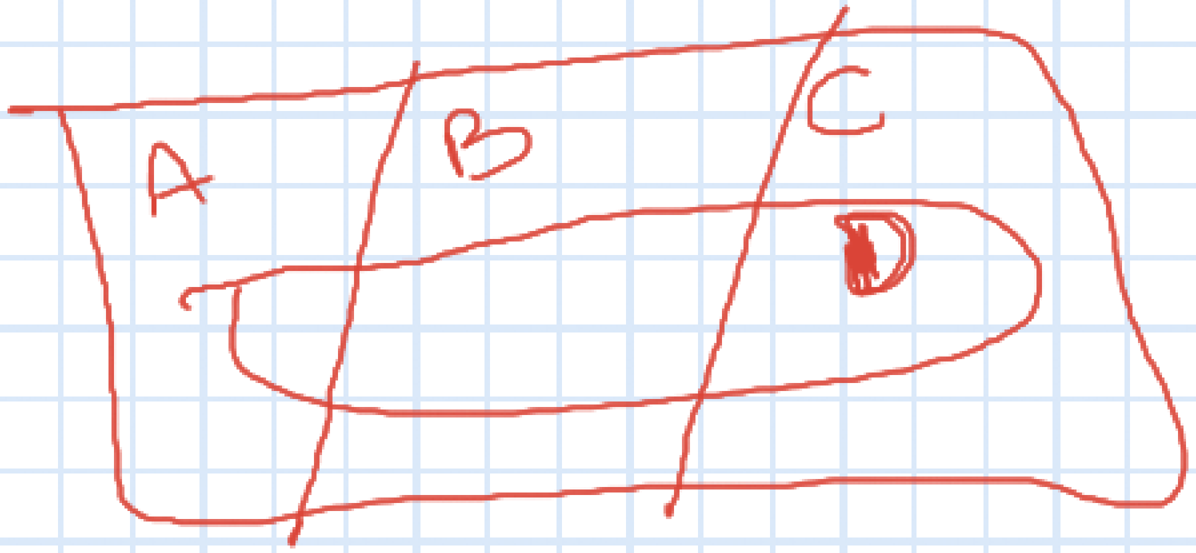
$$\dots = \frac{1}{2}, \quad 1<y<3 \quad \Rightarrow \boxed{\begin{array}{l} f_Y(y) = \frac{1}{2}\cdot \mathbb{1}\{1<y<3\} \\ \Rightarrow Y \sim \mathcal{U}(1,3) \end{array}}$$

$$f_{X|Y=y}(x) = \frac{e^{-\frac{x}{2y}}\cdot \cancel{2}}{\cancel{4}_{2}\, y}\cdot \mathbb{1}\{0<x,\ 1<y<3\}$$

$$= \frac{1}{2y} e^{-\frac{x}{2y}} \cdot \mathbb{1}\{0<x,\ 1<y<3\} \Rightarrow \boxed{X|Y=y \sim \exp\left(\lambda = \frac{1}{2y}\right)_{y\in(1,3)}}$$

$$P(D) = P(D|A)\cdot P(A) + P(D/B)\cdot P(B) + P(D/C)\cdot P(C)$$

Regla de la prob. total




$$\boxed{F_X(x) = \sum_{m=1}^{n} F_{X/M=m}(x)\cdot P_M(m)}$$

$$F_X(x) = \sum_{m=1}^{n} P(X\leq x / M=m)\cdot P_M(m)$$

$\boxed{\text{Ej. 4:}}$ $T$ = tiempo de viaje (en hs). ; $T \sim f_T(t) = ?$

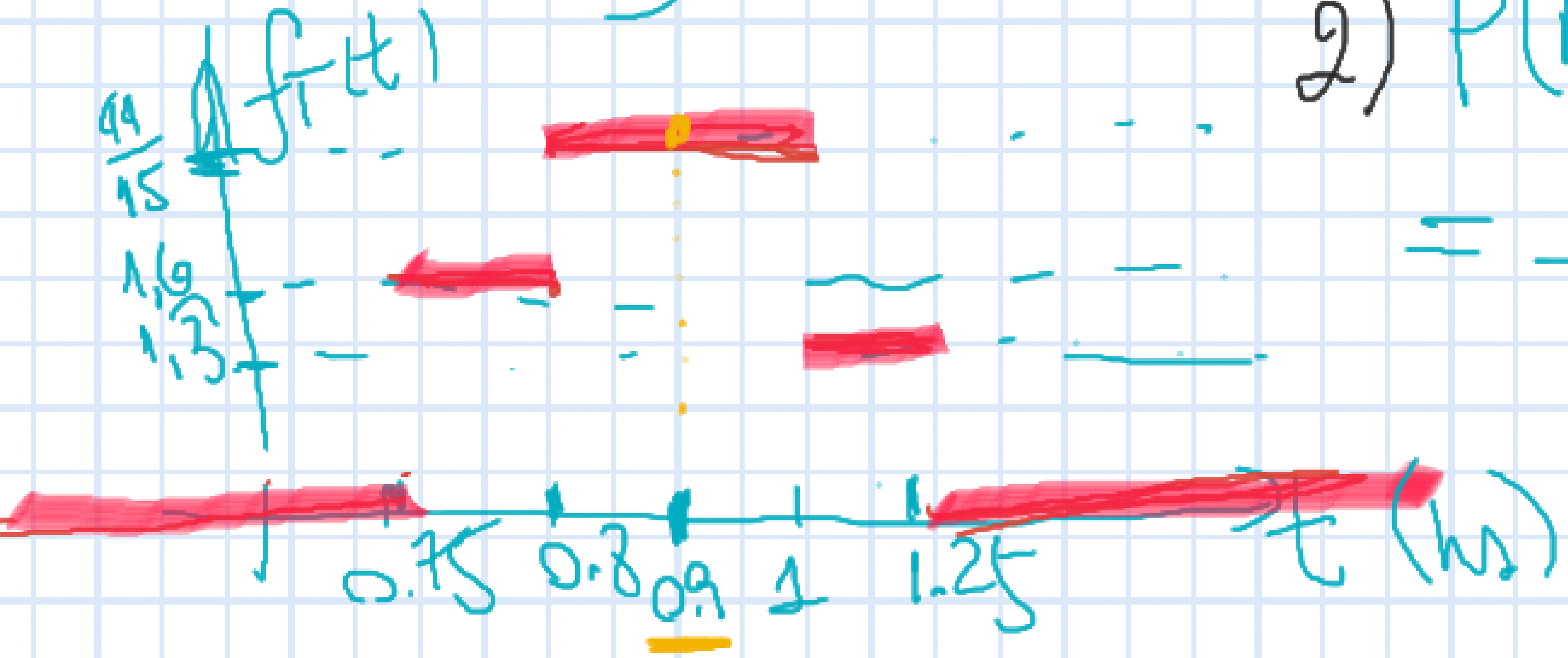
$$M = \begin{cases} 0 & \text{si viaja Tren} \\ 1 & \text{si viaja subte} \end{cases} \qquad \begin{aligned} f_{T/M=0}(t) &\sim \mathcal{U}(0.8\,;\,1.25) \\ f_{T/M=1}(t) &\sim \mathcal{U}(0.75\,;\,1) \end{aligned}$$

1)

$$\boxed{f_T(t) = f_{T/M=0}(t)\cdot p_M(0) + f_{T/M=1}(t)\cdot p_M(1)}$$

$$f_T(t) = \frac{1}{0.45}\,\mathbb{1}\{0.8<t<1.25\}\cdot 0{,}60 + \frac{1}{0.25}\cdot\mathbb{1}\{0{,}75<t<1\}\cdot 0{,}40$$

$$f_T(t) = \frac{4}{3}\cdot\mathbb{1}\{0.8<t<1.25\} + \frac{8}{5}\cdot\mathbb{1}\{0.75<t<1\}$$




2) $P(M=1/T=0.9) = p_{M/T=0.9}(1)$

$$= \frac{f_{T/M=1}(0.9)\cdot p_M(1)}{\frac{4}{3}+\frac{8}{5}} = \frac{\frac{1}{0.25}\cdot 0{,}40}{\frac{44}{15}}$$

$$= \frac{8/5}{44/15} = 0{,}54 \neq p_M(1) = 0{,}40$$

M y T no son independ.

Ej 5 y 6:

F. de Regresión

Ej 1: $Y/X=x \sim Be\left(\frac{x}{10}\right) \Rightarrow \varphi(x) = E[Y/X=x] = \frac{x}{10}, \forall x = \overline{0,10}$

$x \in \{0,1,\ldots,10\}$

$E[Y/X] = \frac{X}{10} \rightarrow$ Esp. Condicional

Ej 2: $Y/X=x \sim U(x,2) \Rightarrow \varphi(x) = E[Y/X=x] = \frac{x+2}{2}, \forall x \in (0,2)$

$x \in (0,2)$

$E[Y/X] = \frac{X+2}{2}$

Ej 3: $X/Y=y \sim \exp\left(\frac{1}{2y}\right) \Rightarrow \varphi(y) = E[X/Y=y] = 2y, \; y \in (1,3)$

$E[X/Y] = 2Y$

$Y \sim U(1,3) \Rightarrow E(Y) = 2$

$E[X] = E[E[X/Y]] = E[2Y] = 2 \cdot E(Y) = 2 \cdot 2 = 4$

$\rightarrow = \int_y 2y \cdot f_Y(y)\,dy = 2 \cdot \int_1^3 \frac{1}{2}\,dy = 4$