NOVA IMS Information Management School

Post-Graduation in Data Science for Finance

Asset Pricing & Portfolio Management

Prof. Jorge Bravo

Empirical Analysis of Financial Market Returns and Performance of Alternative Portfolio Investment Strategies

Authors:

Afonso Casanova, 20240795

Francisco Perestrello, 20241560

João Pardal, 20240796

Nuno Vieira, 20241111

NOVA Information Management School November 2024

1. Introduction

In the pursuit of maximizing returns and managing risk in financial investments, it is essential to understand the behavior of financial market returns and to assess the effectiveness of various portfolio strategies. This report conducts an empirical analysis of the statistical properties of financial market returns, alongside a performance evaluation of alternative portfolio investment strategies. By systematically investigating these properties, we aim to uncover insights into critical market dynamics such as volatility clustering, fat tails, and potential deviations from normality that may influence portfolio performance.

To achieve these objectives, we have selected a diverse set of financial assets representing a wide range of industries and asset classes, ensuring a comprehensive market coverage. This selection includes stocks, bonds, and other assets from sectors such as Cryptoassets, Commodities, Healthcare, Financials, Consumer Electronics, Semiconductors, Software, and Industrials. Our asset list[1] features notable constituents like SPY, GOOGL, AAPL, and TSLA, as well as representatives from alternative investments such as GLD (gold) and GBTC (Bitcoin Trust), allowing us to explore the performance of both traditional and alternative investments within the same portfolio framework.

This report’s structure proceeds by first analyzing the historical return characteristics of the chosen assets. We then evaluate the performance of alternative portfolio strategies, leveraging these empirical insights to assess portfolio dynamics. By comparing various approaches, we aim to highlight potential trade-offs in return and risk across diverse portfolio configurations.

[1] SPY - SPDR S&P 500 ETF Trust; GOOGL - Alphabet Inc. (Class A); NFLX - Netflix, Inc.; AAPL - Apple Inc.; AMZN - Amazon.com, Inc.; META - Meta Platforms, Inc.; GLD - SPDR Gold Shares; NVDA - NVIDIA Corporation; IBM - International Business Machines Corporation; TXN - Texas Instruments Incorporated; ASML - ASML Holding N.V.; DECK - Deckers Outdoor Corporation; V - Visa Inc.; MA - Mastercard Incorporated; SNPS - Synopsys, Inc.; JPM - JPMorgan Chase & Co.; AVGO - Broadcom Inc.; AMAT - Applied Materials, Inc.; TLT - iShares 20+ Year Treasury Bond ETF; NVO - Novo Nordisk A/S; LLY - Eli Lilly and Company; AXON - Axon Enterprise, Inc.; XYL - Xylem Inc.; GBTC - Grayscale Bitcoin Trust; NTES - NetEase, Inc.; NEE - NextEra Energy, Inc.; JNJ - Johnson & Johnson; PG - The Procter & Gamble Company; LMT - Lockheed Martin Corporation; TSLA - Tesla, Inc.; LRCX - Lam Research Corporation

2. Financial Assets Overview

This section provides an overview of the financial assets analyzed in this study, focusing on data preparation, transformation, and initial summary statistics that inform the empirical properties of asset returns.

Data Cleaning and Preparation

For this analysis, daily adjusted close prices for the selected assets were sourced from Yahoo Finance, covering the period from January 1, 2010, to October 31, 2024. To maintain data continuity, missing values (NAs) were interpolated wherever possible, ensuring a complete dataset for further analysis. After interpolation, any remaining NAs were removed. As a result, our dataset only includes data starting in May 2015, when all assets had available price data. This approach ensures that our analysis uses a complete dataset with no gaps, thereby improving the reliability and consistency of the statistical analyses.

Frequency Transformation and Return Computation

The daily market data was transformed into three separate frequencies: daily, weekly, and monthly. Weekly and monthly data were derived using the daily adjusted close prices on the last trading day of each week or month. This transformation enables a comparative analysis of asset behavior across different time scales, offering insights into both short and long-term trends. From these adjusted price series, we computed the daily, weekly, and monthly log returns. Log returns are uniquely valuable in empirical finance due to their additive properties over time, simplifying cumulative return calculations and volatility analysis.

Summary Statistics

A preliminary analysis[2] of the selected assets highlighted several notable stocks in terms of annualized return, annualized volatility, and annualized Sharpe ratio (with a risk-free

[2] Tables 1, 2, and 3 in the Appendix

rate assumed at 0%). For instance, NVDA exhibited the highest annualized return in all three frequencies, with a notable 62.22% return using weekly data, reflecting significant growth potential over the analysis period. However, NVDA is also among those with the highest annualized volatilities, up to 48.91% in the daily frequency, indicating high price fluctuations and potential risk. In this metric, GBTC topples every other asset, showing an annualized volatility of 93.29% in the monthly frequency, down to 85.81% in the daily frequency.

In contrast, SPY showed a lower annualized return of 11.26% but demonstrated a relatively stable performance, with an annualized volatility of 17.90%, ranking it among the lowest in the sample using daily data. Notably, the second-least volatile asset, TLT, was the only asset exhibiting negative annualized returns across the sample (-1.59% using daily data).

Finally, LLY, SNPS, AVGO, and NVDA, with their robust annualized Sharpe ratios (especially the latter with a Sharpe Ratio of 1.38 in the weekly frequency), suggest a favorable risk-return trade-off, outperforming other assets in the dataset on a risk-adjusted basis.

This preliminary evaluation of return, volatility, and Sharpe ratio across assets provides foundational insights into their individual risk and return profiles. These characteristics will inform the portfolio optimization models and influence the selection of alternative portfolio strategies, as detailed in subsequent sections of this report.

3. Empirical Analysis of Financial Market Returns

3.1. Absence of Auto-Correlation

Autocorrelation is a statistical metric that measures the relationship between a variable's current value and its past values. It evaluates the degree of correlation in a time series with its lagged versions across different time intervals, known as lags.

In time series analysis, autocorrelation is especially important because many economic and financial series display patterns that depend on their historical values. Recognizing autocorrelation can greatly impact trading strategies and risk assessments, providing valuable insights for informed decision-making.

Mathematically, the autocorrelation function for a given time series is defined as:

$$p_k = \frac{\sum_{t=k+1}^{n}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2}$$

- p_k: Autocorrelation at lag k
- Y_t: Value of the series at time t
- Y_t: Mean of the series
- n: Number of observation

To determine whether the selected assets exhibit autocorrelation, the Ljung-Box test was conducted on their daily, weekly, and monthly log returns. This statistical method is designed to detect autocorrelation within a time series across various lag intervals.

By analyzing daily, weekly, and monthly volatility for the selected assets, we gain insights into the temporal dependencies present in their volatilities. The test produces a p-value, and, if below 0.05, it indicates a rejection of the null hypothesis.

– Null Hypothesis (H0): Indicates that there is no autocorrelation present at any of the specified lags.

– Alternative Hypothesis (H1): Suggests that at least one of the autocorrelation coefficients is significantly different from zero.

In conducting the Ljung-Box test, it is essential to emphasize that a lag of 30 was utilized for the analysis of daily, weekly, and monthly returns. The selection of the lag length is a critical consideration in time series analysis, as it can significantly influence both the results and their interpretation. In the context of the Ljung-Box test, a lag of 30 indicates that we are evaluating autocorrelation across 30 distinct lagged intervals. This approach enables an examination of the correlation between the current value and values up to 30 periods prior, thereby allowing for the identification of longer-term dependencies within the data.

	daily	weekly	monthly
SPY	0.0000	0.9784	0.2718
GOOGL	0.0000	0.1155	0.0324
NFLX	0.0239	0.0419	0.2810
AAPL	0.0000	0.7467	0.2595
AMZN	0.0546	0.1695	0.7092
META	0.0047	0.0476	0.6098
GLD	0.1203	0.6092	0.7281
NVDA	0.0000	0.6596	0.7828
IBM	0.0000	0.2848	0.2690
TXN	0.0000	0.1307	0.0606
ASML	0.0000	0.9537	0.7251
DECK	0.0003	0.8454	0.9976
V	0.0000	0.1334	0.0044
MA	0.0000	0.0273	0.0000
SNPS	0.0000	0.2013	0.3654
JPM	0.0000	0.0353	0.7620
AVGO	0.0000	0.6789	0.0060
AMAT	0.0000	0.3000	0.6808
TLT	0.0000	0.2345	0.8843
NVO	0.2754	0.0580	0.8459
LLY	0.0004	0.8940	0.5447
AXON	0.6614	0.4101	0.7534
XYL	0.0000	0.6727	0.5592
GBTC	0.7705	0.7126	0.1692
NTES	0.4816	0.5006	0.5961
NEE	0.0000	0.0139	0.0705
JNJ	0.0000	0.5267	0.1004
PG	0.0000	0.0162	0.1726
LMT	0.0001	0.3174	0.6521
TSLA	0.0262	0.3513	0.0600
LRCX	0.0000	0.0853	0.7400

Table 1: Ljung-Box p-values

	Daily	Weekly	Monthly
Autocorrelation	80,65%	19,35%	12,90%

A significant proportion of the daily observations exhibit autocorrelation, with 80.65% of assets indicating that past returns are predictive of current returns. This suggests that daily price movements may reveal intrinsic patterns shaped by factors such as market dynamics, investor behavior, or news-driven events. This finding is a significant opportunity for short-term trading strategies that leverage these predictable patterns.

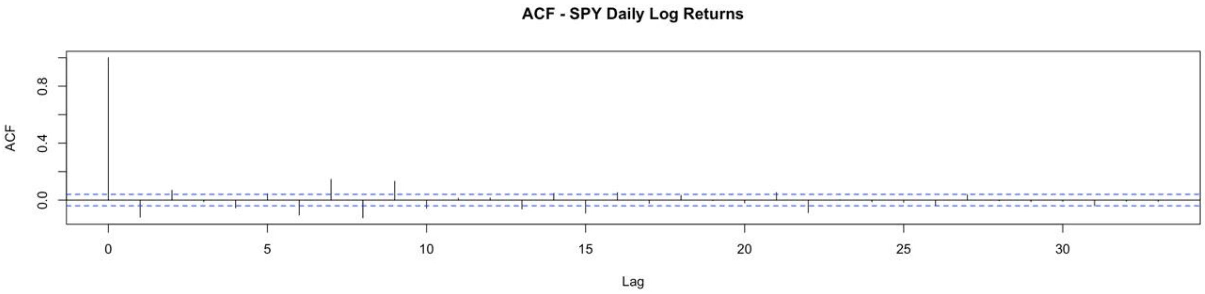
In contrast, the presence of autocorrelation drops markedly in weekly observations, with only 19.35% of the weekly returns exhibiting significant autocorrelation. This reduction showcases the short-term momentum observed in daily data dissipates over longer time frames, possibly due to market corrections or the waning influence of immediate news events as the time frame extends.

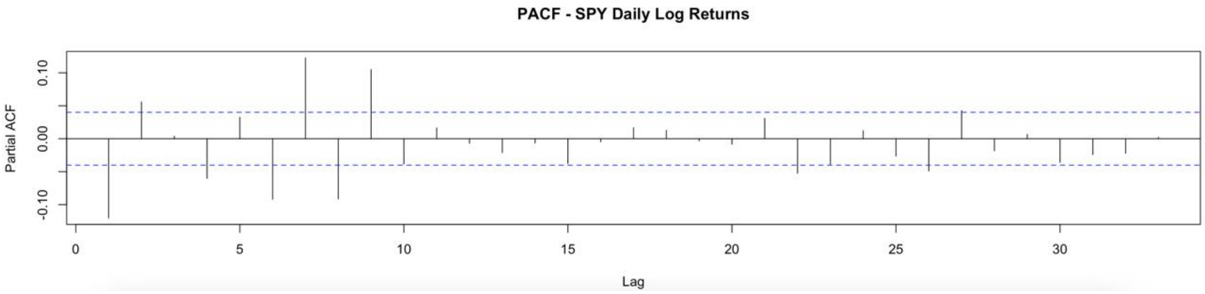
The analysis of monthly data reveals, as expected, the lowest percentage of autocorrelation, with only 12.90% of observations indicating significant autocorrelation. This finding suggests that over a longer time horizon, the predictive power of past returns diminishes significantly. These results are consistent with the efficient market hypothesis, which asserts that in well-functioning markets, past prices have limited predictive capability movements for future prices (as the market incorporates new information consistently, the potential for historical data to influence prices diminishes as the effects of this information have already been factored into current prices). As such, the lower level of autocorrelation in monthly returns underscores the reduced relevance of historical data in forecasting future market behavior over extended periods.

To demonstrate this effect, we decided to plot the ACF (Autocorrelation Function) and PACF (Partial Autocorrelation Function) for SPY. This provides us with a visual representation that enhances our understanding of autocorrelation behavior.

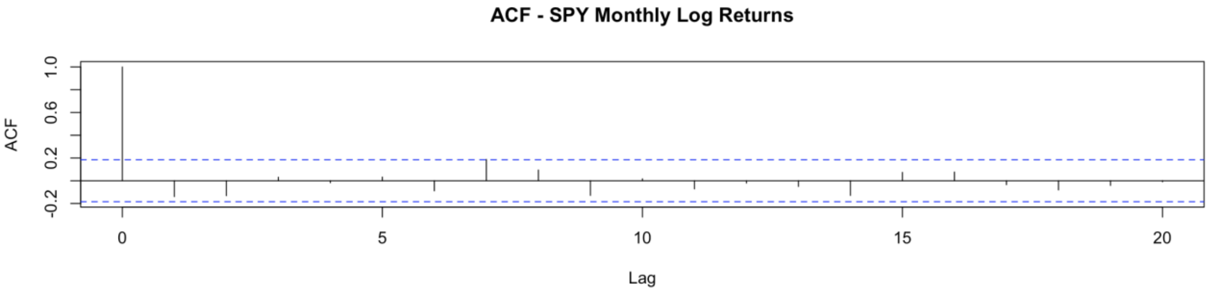
The ACF measures the correlation between a time series and its own lagged values, indicating how past values relate to current values over various time lags. In contrast, the PACF removes the influence of intermediate lags, focusing solely on the direct correlation between the time series lagged values.

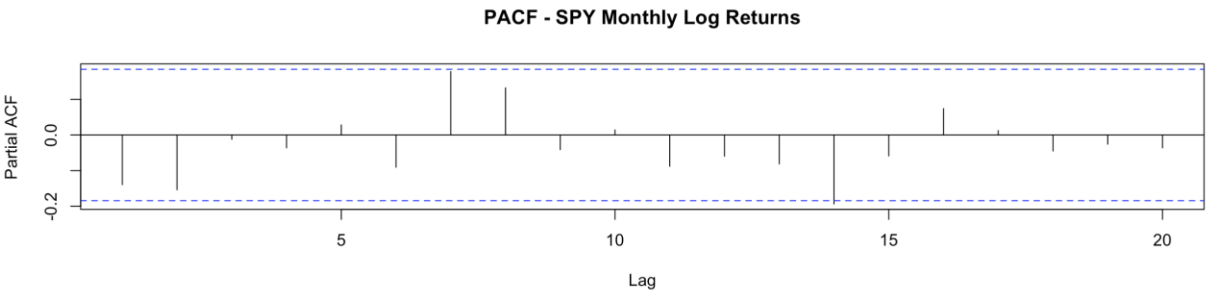
When interpreting the plots, it is important to note that ACF or PACF values exceeding the confidence intervals (represented by blue lines) suggest a statistically significant correlation. This will help inform our subsequent analysis and model selection processes.





Based on the Ljung-Box test, SPY daily log returns show significant autocorrelation, and this is further supported by the ACF and PACF results. The plots reveal several values that exceed the confidence intervals, demonstrating a statistically significant autocorrelation. It is also important to note that when examining autocorrelation with a lag of one day, SPY shows negative partial autocorrelation, suggesting that daily returns tend to move in the opposite direction of the returns from the previous day.





For monthly returns, the ACF and PACF plot also further confirms our Ljung-Box test results – no lags exceed the confidence intervals. This indicates that SPY's monthly log returns do not exhibit a statistically significant correlation among themselves.

3.2. Unconditional Normal Distribution / Fat Tails

In financial markets, it is common to assume that the daily returns of assets follow a normal distribution. However, empirical evidence suggests that this assumption often does not hold true, which has significant implications for risk assessment and investment strategies.

One notable characteristic of the actual distribution of daily returns is the presence of "fat tails", which signifies a greater likelihood of extreme events compared to the normal distribution. This phenomenon can lead to an underestimation of risks when conventional models are employed.

To rigorously evaluate whether the unconditional distribution of daily returns adheres to normality, we employed the Jarque-Bera test at a significance level of 5% on the daily, weekly, and monthly log returns of the selected assets. This statistical method assesses the divergence of a sample distribution from a normal distribution by analyzing its skewness and kurtosis.

$$JB = \frac{n}{6}\left(S + \frac{(K-3)^2}{4}\right)$$

Where n is the sample size, S is the sample skewness, and K is the sample kurtosis.

The Jarque-Bera test provides a test statistic that follows a chi-squared distribution with two degrees of freedom. A significantly high-test statistic, resulting in a low p-value ($p < 0.05$), allows us to reject the null hypothesis in favor of the alternative hypothesis.

- Null Hypothesis (H0): The returns are normally distributed
- Alternative Hypothesis (H1): The returns are not normally distributed

	daily	weekly	monthly
SPY	0	0.0000	0.0200
GOOGL	0	0.0000	0.5740
NFLX	0	0.0000	0.0000
AAPL	0	0.0000	0.3911
AMZN	0	0.0000	0.0995
META	0	0.0000	0.0000
GLD	0	0.0000	0.5609
NVDA	0	0.0000	0.0270
IBM	0	0.0000	0.0005
TXN	0	0.0168	0.5426
ASML	0	0.0000	0.8107
DECK	0	0.0000	0.8441
V	0	0.0000	0.7237
MA	0	0.0000	0.0726
SNPS	0	0.0000	0.9637
JPM	0	0.0000	0.0023
AVGO	0	0.0000	0.0958
AMAT	0	0.0000	0.8422
TLT	0	0.0000	0.8450
NVO	0	0.0000	0.0842
LLY	0	0.0000	0.7593
AXON	0	0.0000	0.1792
XYL	0	0.0000	0.1680
GBTC	0	0.0000	0.0000
NTES	0	0.0141	0.3969
NEE	0	0.0000	0.0044
JNJ	0	0.0000	0.6586
PG	0	0.0000	0.7174
LMT	0	0.0000	0.0149
TSLA	0	0.0000	0.1230
LRCX	0	0.0000	0.3980

Table 2: Ljung-Box p-values

	Daily	Weekly	Monthly
Normality	0,00%	0,00%	70,97%

All assets display non-normal log returns at daily and weekly frequencies, indicating that their returns significantly deviate from a normal distribution. Daily and weekly data include a greater number of individual observations within a limited period. While this enhances data resolution, it also amplifies the impact of outliers and fluctuations, leading to a less normal distribution.

Additionally, daily and weekly returns are more sensitive to external factors (such as news, earnings announcements, geopolitical events, etc.), which can cause substantial price swings. This conditional behavior is reflected in the returns through the presence of high and low volatility periods, contributing to heavier tails and skewness, and resulting in deviation from normality.

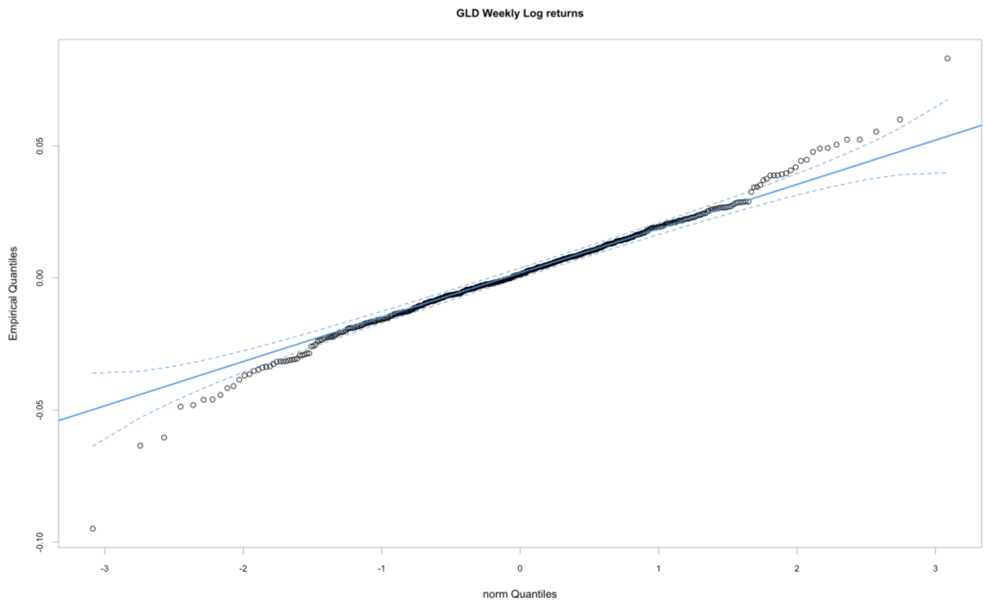
At a monthly frequency, the Jarque-Bera test exhibits a high normality rate, with 22 out of our 30 assets' monthly log returns following a normal distribution, representing 70.97% of assets. The aggregation of daily and weekly returns into monthly data promotes further smoothing of the return distribution. This stabilization effect diminishes the impact of daily volatility and extreme variations, resulting in a more symmetrical and bell-shaped distribution. Monthly log returns allow the market sufficient time to process information, decreasing the likelihood of erratic behavior typical of shorter time frames. Consequently, monthly returns tend to display more consistent pricing trends and patterns.

The lower rate of non-normality suggests that monthly log returns are more compatible with traditional statistical assumptions utilized in modeling. This alignment enables analysts to apply standard techniques that assume normality, making monthly data especially valuable for long-term forecasting and strategic decision-making.

To further evaluate the outcomes of the Jarque-Bera test, we have chosen to utilize visual representations of the log returns of our assets. This approach aims to examplify these normality and non-normality dynamics.

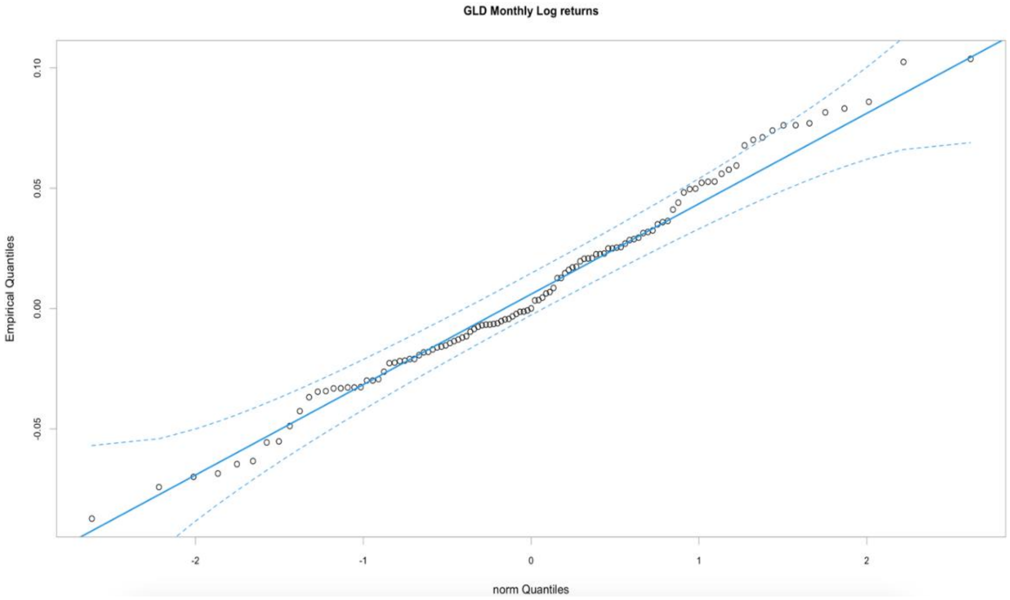
The Quantile-Quantile (QQ) plot was selected as the most effective visualization tool. This graphical method assesses whether a data set follows a particular distribution, commonly the normal distribution. It does this by plotting the quantiles of the sample data against the quantiles of the theoretical distribution. In this case, we will compare the quantiles of the weekly and monthly log return of GLD (SPDR Gold Shares), an exchange-traded fund that tracks the price of gold bullion, against a normal distribution.

	Weekly	Monthly
GLD p-values	0.0000	0.5609



The QQ plot illustrates that the actual quantiles of the GLD weekly log returns are noticeably different from the quantiles expected under a normal distribution. Specifically, the plot displays pronounced deviation in both the upper and lower tails. This suggests that while most returns may cluster around an average value, the extremes – representing both extremely high and low returns – occur with greater frequency than what would be expected if the returns followed a normal distribution. The presence of "fat tails" in the distribution signifies that extreme market events, such as abrupt price changes or volatility spikes, are more common than a normal distribution would predict. Financial markets often exhibit this characteristic due to various factors previously explained. In the context of GLD, this suggests that factors affecting gold prices, such as geopolitical events or macroeconomic data releases, can lead to larger-than-expected fluctuations.

The analysis of the QQ plot clearly indicates that GLD weekly log returns are not normally distributed, showcasing the findings from the Jarque-Bera test. Given the non-normality of the returns, traditional risk measures, which typically assume normality, may underestimate the probability of extreme losses or gains.



The GLD monthly log returns closely adhere to the quantiles of a normal distribution, indicating a tendency towards normality. While the graph shows two outliers that deviate from this normal distribution, these outliers remain relatively close to the expected values which, thanks to the Jarque-Bera test, we can conlude are not statistically significant.

It is also evident in the plot that the tails deviate slightly from the normal line, particularly in the case of extremely positive returns. However, these values still fall within the expected range for normality. This observation highlights the smoothing effects of monthly data aggregation, which reduces volatility in the asset.

3.3. Distribution Asymetry

Mean and standard deviation are useful tools, but they can be misleading when it comes to extreme events. An alternative indicator is Skewness - it provides a more accurate view of an investment's risk profile, considering both the upside and downside potential.

Resorting to the Skewness analysis, we can quantify to which extent a distribution deviates from a normal distribution (mean), providing insights into the likelihood of extreme events happening, both positive and negative. The skewness results can be assessed in the following way:

- **Positive Skewness (>0):** Suggests that the distribution has a longer right tail, implying more frequent small losses but larger extreme gains.
- **Negative Skewness (<0):** Indicates a longer left tail, implying more frequent small gains and a higher probability of extreme losses.
- **Near Zero:** Indicates symmetry and suggests that returns behave closer to a normal distribution.

Skewness is crucial for portfolio optimization, as it provides insights into the probability of extreme events. By incorporating skewness into portfolio optimization, we can refine our investment strategies, considering a more nuanced view of risk that accounts for tail behavior.

	SPY	GOOGL	NFLX	AAPL	AMZN	META	GLD
mean	0.0004945045	0.0007789036	0.0009186247	0.0008773350	0.0009168265	0.0008508755	3.433764e-04
variance	0.0001269714	0.0003232213	0.0007760680	0.0003274404	0.0004176888	0.0005873925	7.752594e-05
skewness	-0.8081396654	-0.0019568108	-1.5872147653	-0.2069952235	-0.0321169968	-1.2713324440	-1.438327e-01
kurtosis	16.6641626108	9.3537013840	33.1905758414	8.4923174138	8.0760587592	30.2199714556	5.960551e+00
Excess.Kurtosis	13.6641626108	6.3537013840	30.1905758414	5.4923174138	5.0760587592	27.2199714556	2.960551e+00

	NVDA	IBM	TXN	ASML	DECK	V	MA
mean	0.0023630656	0.0002658330	0.0006669901	0.0008098160	0.001090231	0.0006298977	0.0007429675
variance	0.0009483954	0.0002295441	0.0003209406	0.0005472232	0.000631826	0.0002410740	0.0002889761
skewness	0.2106846078	-0.7057649363	-0.0841077968	-0.5213335633	-0.018321681	-0.1014996983	-0.0111902225
kurtosis	9.7267962118	13.8051767643	7.8253091560	8.4744496689	11.065069050	12.7898708753	12.3123514399
Excess.Kurtosis	6.7267962118	10.8051767643	4.8253091560	5.4744496689	8.065069050	9.7898708753	9.3123514399

	SNPS	JPM	AVGO	AMAT	TLT	NVO	LLY
mean	0.0010099492	0.0006270145	0.0012293541	0.0009856530	-1.206082e-05	0.0006558188	1.101212e-03
variance	0.0003551266	0.0002977559	0.0005222989	0.0006584608	9.223634e-05	0.0002997930	3.077243e-04
skewness	-0.1681505940	-0.1059379085	-0.4502435191	-0.4412404057	2.927013e-02	-0.3274589766	6.934008e-01
kurtosis	7.7734278835	16.4557781126	11.5839127688	8.1489347972	7.755947e+00	12.0046155045	1.296978e+01
Excess.Kurtosis	4.7734278835	13.4557781126	8.5839127688	5.1489347972	4.755947e+00	9.0046155045	9.969777e+00

	AXON	XYL	GBTC	NTES	NEE	JNJ	PG
mean	1.082705e-03	0.0005800747	0.0019468719	0.0005430177	0.0005841463	0.0003038125	0.0004165990
variance	8.577549e-04	0.0002871446	0.0029229189	0.0007308052	0.0002392106	0.0001312619	0.0001375108
skewness	3.933052e-01	-0.3563341193	-0.0002915059	0.0054175161	-0.5268189495	-0.3555744400	-0.0159429262
kurtosis	1.303774e+01	10.2840047261	6.9227051148	8.4523004867	13.9728646883	13.0029915154	14.8503740191
Excess.Kurtosis	1.003774e+01	7.2840047261	3.9227051148	5.4523004867	10.9728646883	10.0029915154	11.8503740191

	LMT	TSLA	LRCX
mean	0.0005481571	0.001165934	0.0012720023
variance	0.0002013271	0.001304073	0.0007151569
skewness	-0.6338602959	-0.085273768	-0.0389610187
kurtosis	18.7576226555	7.450431131	9.9205707299
Excess.Kurtosis	15.7576226555	4.450431131	6.9205707299

Table 3: Daily Returns Moments

When considering daily returns, focusing on skewness, we highlight a few stand-out performers:

1) Positive Skewness: We identified that **NVDA** and **LLY** have positive skewness, hinting at potential for large daily gains.

2) Negative Skewness: The data showed that **META and GLD** exhibit significant negative skewness. These suggest a greater likelihood of sudden, extremely negative daily returns, which could increase the downside risk for the portfolio.

By considering both skewness and kurtosis, we can gain a more comprehensive understanding of the risk profiles of our assets. A combination of high positive skewness and high kurtosis, for instance, may indicate significant upside potential but also increase the risk of extreme losses. Conversely, a combination of negative skewness and high kurtosis suggests a higher likelihood of significant downside risk.

1) **High Kurtosis and Negative Skewness: NFLX** shows extremely high kurtosis combined with high negative skewness, indicating not only a high potential for extreme values but that these extremes are more likely to be negative, implying a fat-tailed distribution with a tendency towards sharp negative returns.
2) **Balanced Assets: GOOGL** and **TLT** have relatively low skewness and moderate excess kurtosis. These assets may behave more predictably, with fewer extreme movements compared to others with more pronounced values.

Recurring to the diversification method, by including positively skewed stocks like **LLY** can provide exposure to potential upside, complementing more stable assets and offsetting the downside risk from negatively skewed stocks.

In summary, the assets that show significant negative skewness signal the need for risk mitigation strategies in portfolios that include these stocks. Inversely, assets that show positive skewness, can provide potential for gains and be a balancing element in a diversified portfolio.

	SPY	GOOGL	NFLX	AAPL	AMZN	META	GLD
mean	0.0023699220	0.003757864	0.004356650	0.004196831	0.004458670	0.004046381	0.0015877047
variance	0.0005600402	0.001444372	0.003622895	0.001433744	0.001702828	0.002393021	0.0003707099
skewness	-0.8884302269	0.338925782	-0.847628676	-0.362528564	-0.344771117	-0.412094169	-0.1335833038
kurtosis	10.2368245745	5.959265727	11.623974539	5.222329278	4.741393416	7.763293912	5.0212525375
Excess.Kurtosis	7.2368245745	2.959265727	8.623974539	2.222329278	1.741393416	4.763293912	2.0212525375

	NVDA	IBM	TXN	ASML	DECK	V	MA
mean	0.011344030	0.001258267	0.003177141	0.003864952	0.005352877	0.0030214994	0.003571741
variance	0.003903183	0.001116306	0.001137756	0.002449950	0.002935785	0.0009132555	0.001185903
skewness	0.079686017	-0.456475970	-0.136908045	-0.089408601	-0.213027100	-0.4824300513	-1.024651345
kurtosis	4.184108128	6.180800109	3.567601774	5.305032277	7.115995309	7.7182859859	10.507898863
Excess.Kurtosis	1.184108128	3.180800109	0.567601774	2.305032277	4.115995309	4.7182859859	7.507898863

	SNPS	JPM	AVGO	AMAT	TLT	NVO	LLY
mean	0.004860502	0.003013929	0.005853511	0.004730413	-9.281692e-05	0.003147257	0.005318397
variance	0.001547607	0.001353523	0.002317265	0.002972641	3.879307e-04	0.001391173	0.001389016
skewness	-0.132042971	-0.331292503	0.113730860	-0.329180905	-2.797201e-01	-0.491203035	0.096761680
kurtosis	4.617907125	7.960763184	5.792872955	5.312660481	4.252175e+00	5.274843685	4.625394048
Excess.Kurtosis	1.617907125	4.960763184	2.792872955	2.312660481	1.252175e+00	2.274843685	1.625394048

	AXON	XYL	GBTC	NTES	NEE	JNJ	PG
mean	0.005240691	0.002799464	0.009954915	0.002438150	0.00277756	0.0014414994	0.0019917367
variance	0.004280684	0.001363942	0.013361539	0.003087129	0.00114379	0.0005613561	0.0005805914
skewness	0.329482723	-0.600365732	0.313047327	-0.021686788	-0.82309297	-0.3412437735	-0.3205000814
kurtosis	5.615530693	5.738267995	6.816712740	3.641870334	10.04967094	5.2175837756	6.1577030046
Excess.Kurtosis	2.615530693	2.738267995	3.816712740	0.641870334	7.04967094	2.2175837756	3.1577030046

	LMT	TSLA	LRCX
mean	0.0026264792	0.005551526	0.006125140
variance	0.0009927432	0.006366600	0.003445277
skewness	-0.0574754284	0.119300871	-0.205854550
kurtosis	8.8014148609	4.227854495	6.781160315
Excess.Kurtosis	5.8014148609	1.227854495	3.781160315

Table 4: Weekly Returns Moments

Analyzing weekly skewness offers a broader perspective on return asymmetry than daily skewness. We see cases where the skewness changes from a positive value in daily returns to a negative value in weekly returns, as is the example for NTES, and this implies a shift in the distribution shape depending on the time horizon.

More broadly, SPY, MA, and NFLX are highlighted as risky assets with pronounced negative skewness, signaling a need for careful portfolio inclusion due to potential for significant weekly downturns. GOOGL, AXON, and GBTC could contribute upside potential with their positive skewness, providing a balance for portfolios seeking growth. NTES and LMT offer more neutral skewness, acting as stabilizing assets in a portfolio.

	SPY	GOOGL	NFLX	AAPL	AMZN	META	GLD
mean	0.010414867	0.016447151	0.01889125	0.018244038	0.019425041	0.01782589	0.007203048
variance	0.001990279	0.004761794	0.01490029	0.006378822	0.007602322	0.01016782	0.001597535
skewness	-0.571247873	-0.229656813	-1.62585391	-0.215069633	-0.092858798	-0.89708096	0.202365657
kurtosis	3.596822288	3.157652891	11.48128156	2.537693129	3.972532574	6.19662239	2.713931865
Excess.Kurtosis	0.596822288	0.157652891	8.48128156	-0.462306871	0.972532574	3.19662239	-0.286068135

	NVDA	IBM	TXN	ASML	DECK	V	MA
mean	0.04921332	0.005687071	0.013840250	0.016797281	0.023920525	0.013322968	0.015706122
variance	0.01782998	0.004632193	0.004080564	0.007765469	0.008183273	0.003320618	0.004323932
skewness	-0.55737635	-0.460261970	-0.199506303	-0.118065988	-0.085737167	-0.123729728	-0.504657192
kurtosis	3.53974245	4.537611188	2.682969470	2.817261430	3.206346286	2.724083563	3.309029434
Excess.Kurtosis	0.53974245	1.537611188	-0.317030530	-0.182738570	0.206346286	-0.275916437	0.309029434

	SNPS	JPM	AVGO	AMAT	TLT	NVO	LLY
mean	0.021018556	0.013189382	0.024291708	0.02066664	-0.000486913	0.01380101	0.022550399
variance	0.005260905	0.004880528	0.006083707	0.01001075	0.001563275	0.00416238	0.005189349
skewness	0.058606001	-0.465091910	-0.203737243	-0.09141468	0.132370920	-0.49028741	0.114647471
kurtosis	2.955503724	4.305736068	3.911128610	3.19874966	3.038263661	3.29911433	2.746200455
Excess.Kurtosis	-0.044496276	1.305736068	0.911128610	0.19874966	0.038263661	0.29911433	-0.253799545

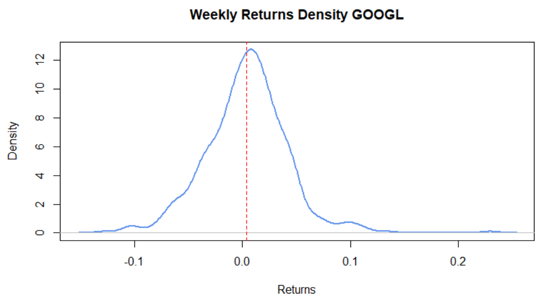
	AXON	XYL	GBTC	NTES	NEE	JNJ	PG	LMT
mean	0.02329182	0.012267262	0.04476820	0.01061753	0.012057590	0.006426329	0.009002550	0.011605966
variance	0.01625918	0.004782796	0.07252241	0.01141873	0.003775881	0.002069754	0.002080211	0.003662034
skewness	0.28389459	-0.423667216	0.89767571	-0.30227388	-0.534466265	-0.191157542	-0.174131084	0.152067887
kurtosis	3.63861330	3.199137424	5.84714581	2.83542501	4.076151630	3.176807487	2.859356860	4.302042352
Excess.Kurtosis	0.63861330	0.199137424	2.84714581	-0.16457499	1.076151630	0.176807487	-0.140643140	1.302042352

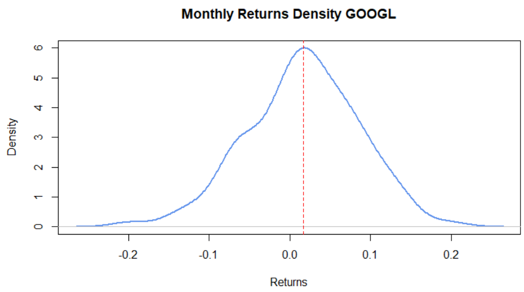
	TSLA	LRCX
mean	0.02420007	0.026390090
variance	0.02761274	0.009771255
skewness	0.36976830	-0.129797070
kurtosis	3.58598140	2.441053757
Excess.Kurtosis	0.58598140	-0.558946243

Table 5: Monthly Returns Moments

Analyzing both Skewness and Kurtosis for the monthly returns, we highlight noteworthy assets.

1) **High Positive Skewness and High Kurtosis:** GBTC not only has a long right tail but is also prone to extreme positive returns, indicating potential for significant gains but also some risk of sharp movements.
2) **High Negative Skewness and High Kurtosis:** NFLX’s distribution stands with a long-left tail and heavy tails, signaling potential for extreme negative returns, thus being pprone to sudden, significant losses, which is critical for understanding downside risk.
3) **Low Skewness and High Kurtosis:** Indicates a symmetric distribution with frequent extreme returns. LMT fits this role and shows small positive skewness but still a risk of extreme events on both sides.
4) **Low Skewness and Low Kurtosis:** This implies a more stable distribution with fewer extreme returns. Assets like DECK show relatively normal behavior, indicating lower tail risk and more predictable returns.





In conclusion, sustained by the previous skewness observations across different timeframes and the above Returns Density graphics, we observe that GOOGL's overtime returns follow the time frequency effect. From the plots, it becomes clear that the skewness tends towards a normal distribution, meaning it approximates zero, thus becoming a more stable distribution with less extreme returns.

3.4. Volatility Clustering

Volatility Clustering refers to the occasion where periods of high market volatility are followed by further high volatility, and periods of low volatility are followed by continued low volatility, this is normally shown in the form of clusters.

This pattern suggests that volatility is not randomly distributed over time but rather exhibits[3] (Bollerslev, 1986; Engle, 1982), or what we call in statistical analysis a tendency. Those volatility periods can be called autoregressive conditional heteroskedasticity. In fact, conditional heteroskedasticity can be useful exploited for forecasting the variance of future periods.

Volatility clustering is crucial for risk management, as it helps investors anticipate periods of sustained risk. Recognizing clustering can improve return modeling by highlighting

---

3 Bollerslev, Tim. 1986. "Generalized Autoregressive Conditional Heteroskedasticity." Journal of Econometrics 31 (3): 307–27.

Engle, Robert. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." Econometrica 50 (4): 987–1007.

the limitations of models assuming constant volatility, like CAPM, and emphasizing the need for adaptable models such as GARCH. It also enhances predictive accuracy, as past volatility patterns can inform future forecasts, supporting more robust financial planning and risk assessment.

The presence of volatility clustering in a specific asset can have significant economic implications. Notably, it can influence asset pricing dynamics, as periods of heightened volatility may lead to increased risk premiums. Investors often demand higher returns to compensate for increased uncertainty during such periods. Furthermore, volatility clustering can be linked to the leverage effect, where negative asset returns tend to be followed by increases in volatility. This phenomenon is particularly important to consider for assets that exhibit this characteristic, as it can impact both short-term trading strategies and long-term investment decisions.

In what concerns to the interpretation in portfolio management, recognizing assets with strong volatility clustering can help allocate resources more effectively by preparing for risk-adjusted positions. In addition, hedging strategies can be tailored for periods identified with clustered volatility, allowing investors to reduce exposure during periods of expected high volatility.

So, in order to identify the existence of volatility clustering we could look at the p-values for the ARCH test (which tests for autoregressive conditional heteroskedasticity). Typically, a p-value below a threshold (often 0.05) indicates that the null hypothesis of "no volatility clustering" is rejected, suggesting the presence of volatility clustering.

With that in mind, according to the table below, we identify assets showing statistically significant volatility clustering for daily, weekly, and monthly returns.

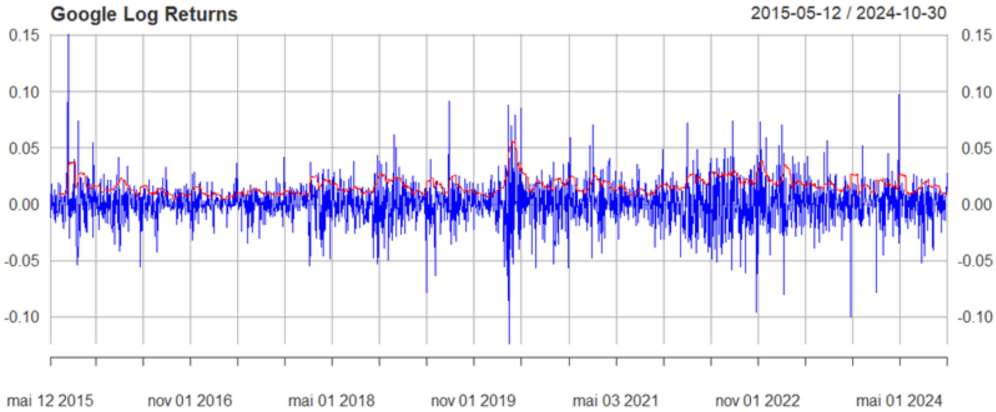
	daily	weekly	monthly
SPY	0.0000	0.0000	0.0472
GOOGL	0.0000	0.0766	0.6708
NFLX	0.8646	0.9875	0.2994
AAPL	0.0000	0.0000	0.1441
AMZN	0.0000	0.0012	0.0473
META	0.1970	0.0004	0.1243
GLD	0.0000	0.0000	0.9013
NVDA	0.0000	0.7543	0.9159
IBM	0.0000	0.0000	0.7080
TXN	0.0000	0.0902	0.4408
ASML	0.0000	0.0003	0.0403
DECK	0.0000	0.0000	0.9056
V	0.0000	0.0000	0.0566
MA	0.0000	0.0000	0.1002
SNPS	0.0000	0.0000	0.2935
JPM	0.0000	0.0000	0.6269
AVGO	0.0000	0.0083	0.6488
AMAT	0.0000	0.0005	0.9327
TLT	0.0000	0.0000	0.2612
NVO	0.0003	0.0556	0.9160
LLY	0.0000	0.1622	0.2486
AXON	0.0001	0.7290	0.6193
XYL	0.0000	0.0000	0.2096
GBTC	0.0000	0.2993	0.1175
NTES	0.0000	0.2387	0.3484
NEE	0.0000	0.0000	0.1778
JNJ	0.0000	0.0000	0.7784
PG	0.0000	0.0000	0.3309
LMT	0.0000	0.0000	0.8796
TSLA	0.0000	0.0000	0.1180
LRCX	0.0000	0.0000	0.7112

Table 6: ARCH p-values

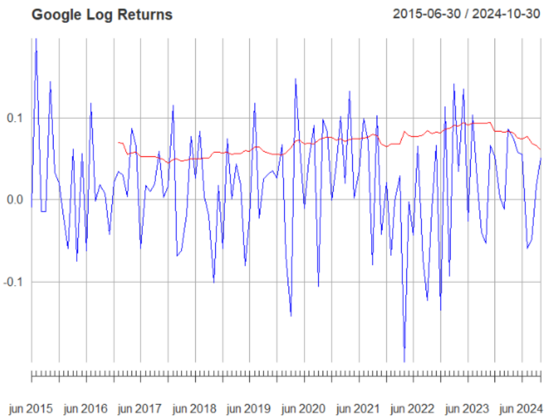
In Daily Volatility Clustering, a strong Clustering is seen, where 93% of assets have p-values below 0.05 for daily returns, indicating significant volatility clustering at this frequency. Notably, only a few assets, such as **NFLX and META**, have p-values greater than 0.05, suggesting no significant clustering at the daily frequency. Most assets exhibit strong volatility clustering at the daily level, which is consistent with typical financial data, where daily returns often display high volatility clustering due to frequent market reactions to news and events.

In turn, Weekly Volatility Clustering shows some clustering, fewer assets (70%) show significant clustering at the weekly level compared to daily (93%). The no clustering evidence shows several assets that do not show significant clustering weekly, such as **NFLX, AXON, and NVDA.** Based on data, we can conclude that weekly returns exhibit less volatility clustering than daily returns, suggesting that some short-term noise is smoothed out, though asset-specific clustering persists.

Finally, concerning Monthly Volatility Clustering, we identify limited clustering, with only 10% of assets displaying significant volatility clustering on a monthly basis. Allowing us to conclude that monthly returns exhibit minimal clustering, as short-term volatility patterns are averaged out over longer timeframes.



Plot 1: Daily GOOGL Returns and 20-day Rolling Volatility



Plot 2: Monthly GOOGL Returns and 20-month Rolling Volatility

To exemplify the time frequency effect, we selected GOOGL as a case study. This stock demonstrates significant daily volatility clustering, which is less apparent on a monthly scale. This finding highlights the potential for short-term volatility to be smoothed out over longer time frames.

Regarding the log returns exhibit 1 shows us daily, while exhibit 2 shows the monthly returns for GOOGL, each represented by the blue line. As we can see, it fluctuates around zero, as is typical for financial returns, with both positive and negative spikes. The larger spikes indicate days of higher volatility, while smaller fluctuations represent periods of lower volatility.

To better visualize and understand the volatility clustering in Google's returns, we have incorporated a 20-period rolling standard deviation, represented by the red line in the chart. This rolling volatility measure smooths out short-term fluctuations and provides a clearer picture of intermediate-term volatility trends. By using a 20-period window, we strike a balance between capturing sufficient data to identify clustering patterns and remaining responsive to recent changes in volatility.

In what concerns to the evidence of volatility clustering - tendency for periods of high volatility to be followed by high volatility, and low volatility to follow low volatility- a certain pattern is visible in the plot. In periods where there are several consecutive days with large spikes in returns (both positive and negative), rolling volatility tends to increase, reflecting higher average volatility during these times. In turn, periods with more stable returns result in lower volatility values, showing reduced volatility.

Overtime, the observation of the clusters of high volatility shows that in certain time periods, such as around 2018-2019 and 2020 (likely due to macroeconomic events, e.g., market corrections or the COVID-19 pandemic), there’s a display of clusters of high volatility with visible spikes (blue and red), with especial attention to a sharp increase in the red line. Conversely, some periods (e.g., parts of 2016-2017 and 2021-2022) show relatively stable, smaller fluctuations in returns, and the red line is closer to zero, indicating low volatility.

Volatility clustering implies that periods of high volatility in Google's stock returns might predict subsequent high-volatility periods, while low-volatility periods tend to be followed by continued stability. Under risk management, for investors and portfolio managers, this pattern means that Google’s stock might require closer monitoring during high-volatility periods, as these tend to persist (as explained by the conditional heteroskedasticity). Observing these patterns could provide insights into how Google's stock responds to external factors over time.

In summary, in what concerns the exhibit of volatility clustering, the daily chart showcases a more dynamic and volatile pattern, while the monthly chart presents a smoother and more stable picture, indicating nonexistence of volatility clustering. These differences highlight the importance of considering the time frame when analyzing volatility clustering and its implications for investment strategies.

3.5. Leverage Effects

The leverage effect is a phenomenon mainly seen in stock markets, where stock volatility tends to rise when stock prices fall. This effect arises because, as a company's stock price decreases, its debt-to-equity ratio effectively grows, making the company appear more indebted and thus riskier.

In other words, when stock prices drop, the market value of the company’s equity declines relative to its debt, increasing the perception of leverage. This higher leverage generally leads to greater price fluctuations, or volatility, in the stock. As a result, the leverage effect implies that declines in stock prices are often followed by periods of increased volatility.

We are running a the leverage effect test on the dataset to test if we can observe indeed the leverage effect on the different assets. Using daily, weekly and monthly returns, our output will return the correlation value, which will suggest or not the leverage effect. Running the test to see if we can witness the leverage effects on the assets, we can clearly observe the negative correlation between most, suggesting that negative returns are associated with higher future volatility, and this is seen in all time frequencies.

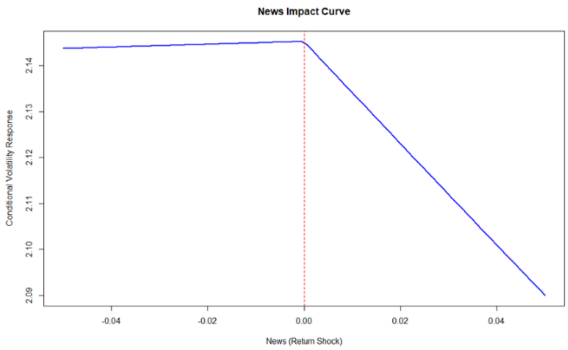
Indeed, the portfolio presents leverage effect in the image below, in other words when the stock prices fall future volatility tends to rise, it's also a common saying in the financial markets that "the stock market rises in a ladder but when it falls it falls like an elevator". It also implies that if there is future volatility rising the portfolio becomes riskier, and from a risk forecasting perspective recognizing the leverage effects helps portfolio managers anticipate increasing levels of risk.

	daily	weekly	monthly
SPY	-0.0885	-0.3320	-0.4158
GOOGL	-0.0073	-0.0838	-0.1905
NFLX	-0.0035	-0.0676	-0.0970
AAPL	-0.0434	-0.1835	-0.0537
AMZN	-0.0348	-0.0677	-0.1300
META	-0.0092	-0.0606	-0.2250
GLD	0.0225	0.0762	-0.0526
NVDA	-0.0614	-0.1779	-0.1626
IBM	-0.0763	-0.1523	-0.0879
TXN	-0.0608	-0.0810	-0.2115
ASML	-0.0060	-0.1459	-0.0970
DECK	-0.0460	-0.1999	-0.2302
V	-0.0770	-0.1910	-0.4120
MA	-0.0483	-0.2400	-0.4056
SNPS	-0.0517	-0.1663	-0.0895
JPM	-0.0430	-0.0966	-0.1838
AVGO	-0.0059	-0.1847	-0.0517
AMAT	0.0071	-0.1487	-0.0930
TLT	0.0445	0.1077	-0.1993
NVO	-0.0157	-0.0758	-0.1092
LLY	-0.0671	-0.0996	0.0406
AXON	0.0303	-0.0414	-0.1073
XYL	-0.0893	-0.1273	-0.2276
GBTC	-0.0127	0.0894	0.0511
NTES	0.0439	-0.0143	-0.1643
NEE	-0.1234	-0.2376	0.0070
JNJ	-0.1473	-0.0741	-0.0882
PG	-0.1892	-0.2611	-0.0806
LMT	0.0020	-0.1369	-0.1713
TSLA	-0.0179	0.0203	-0.0409
LRCX	0.0130	-0.2259	-0.0535

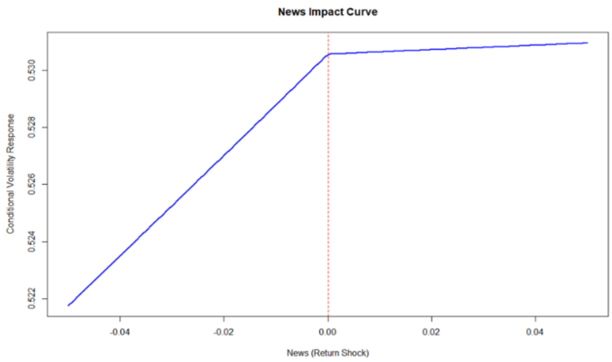
Table 7: Returns and Conditional Volatility Correlations

We choose JNJ as an example of the test because of the daily correlation values and because its an interesting stock due to its business model as a pharmaceutical company, which usually pharmaceuticals stocks have more volatility than other industries.

The News Impact plots below showcase how conditional volatility responds to both positive and negative returns. When we compute for monthly data, the negative returns will correspond to an increase in future volatility, but when we compute for a weekly basis the same happens but for positive returns. It's a good example of a stock that shows different conditional volatility responses dependent on the time frequency applied.



Plot 3: Monthly Data News Impact Curve



Plot 4: Weekly Data News Impact Curve

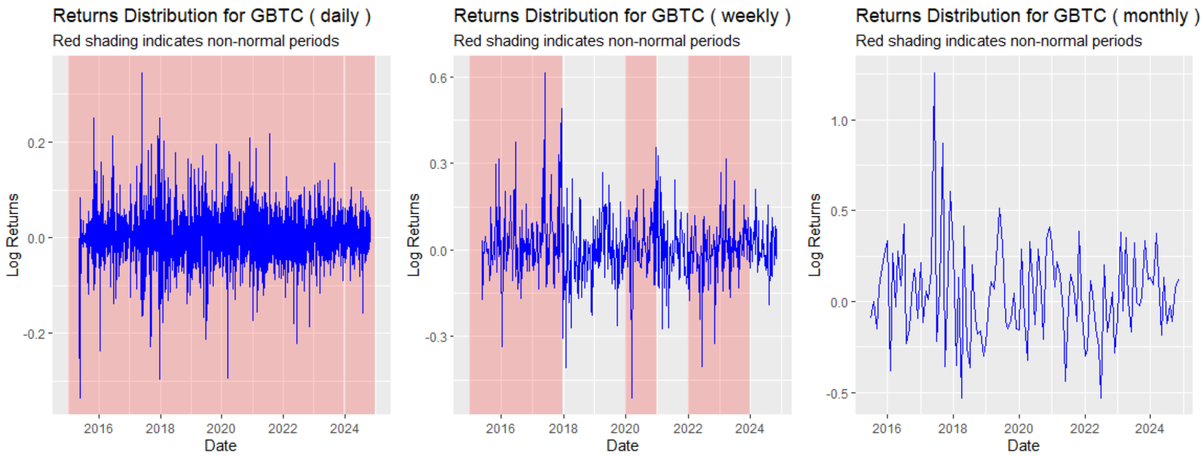
3.6. Conditional Non-Normality

Conditional non-normality in financial market returns refers to the phenomenon where returns exhibit non-normal distribution properties depending on specific conditions or time intervals. This characteristic implies that, rather than having a consistent, stable distribution, financial returns may deviate from normality under certain conditions, such as during periods of high market volatility or specific economic events. Understanding conditional non-normality is essential for accurately modeling risks, as it challenges the assumption of normality often applied in traditional financial theories.

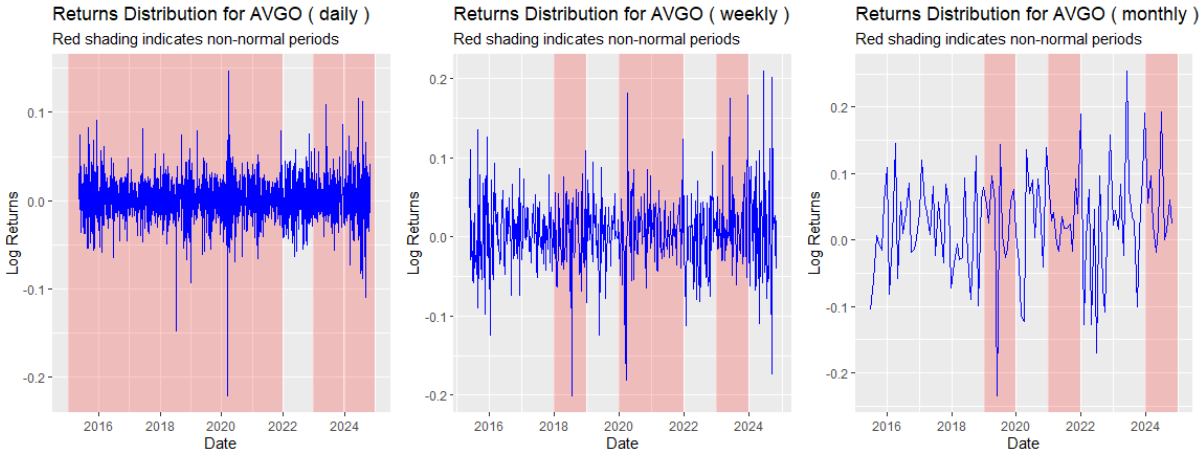
To investigate conditional non-normality, we split the data into one-year intervals and applied the Shapiro-Wilk test to each one-year sample for each asset. The Shapiro-Wilk test assesses whether a sample is likely drawn from a normally distributed population, using the null hypothesis that the sample follows a normal distribution. If the p-value is below a specified significance level (commonly 0.05), we reject the null hypothesis, indicating that the sample likely does not follow a normal distribution. This approach provides an advantage by enabling us to observe normality fluctuations across time, offering a year-by-year view of non-normality patterns rather than a single, aggregate measure.

Our results show that the frequency of data significantly impacts normality testing outcomes. Using daily data, 86.77% of one-year periods across all assets rejected the null hypothesis of normality, suggesting widespread non-normality in high-frequency returns. However, with weekly data, only 29.35% of one-year periods rejected normality, and at the monthly frequency, 95.48% of one-year periods could not reject normality, indicating an increased likelihood of normally distributed returns at lower frequencies.

This relationship between data frequency and normality outcomes is clearly demonstrated by GBTC. As illustrated in the plot of GBTC's return distributions below, we observe that GBTC shows no normal periods at the daily frequency, as indicated by the red shading marking non-normal periods. However, when we examine the weekly frequency, we see an increase in the number of normal periods, and at the monthly frequency, GBTC displays only normal periods. This shift illustrates how lower frequencies smooth out extreme price variations, making normality more likely.



Conversely, AVGO displayed the smallest impact from frequency changes. Even at the monthly frequency, AVGO retained non-normality during several periods—specifically in 2019, 2021, and 2024. This persistence of non-normality at lower frequencies for AVGO could stem from factors like high sensitivity to market-specific risks or substantial volatility in its underlying sector, which may lead to return distributions with fat tails or skewed profiles, even at coarser time intervals.



In conclusion, the conditional non-normality of returns varies substantially by asset and data frequency. While higher frequencies like daily data reveal pronounced non-normality across most assets, lower frequencies smooth out these effects, resulting in distributions that align more closely with normality. Recognizing this conditionality provides valuable insight for investors, emphasizing the importance of data frequency in risk modeling and the potential pitfalls of assuming normally distributed returns across all time horizons.

4. Performance of Investment Strategies

In this section, we will examine the performance of alternative investment strategies through a vanilla portfolio walk-forward backtesting approach applied to daily data. To capture variability in performance across different market conditions and asset selections, we generated 100 randomized datasets, each covering a two-year period with a unique combination of 10 randomly chosen securities. For each backtest, the optimization, rebalancing, and lookback periods were set to a semiannual frequency, short-selling was not allowed, and transaction costs were assumed to be zero.

It is important to note that results may vary with different datasets and specific market conditions. For consistency, we will analyze the performance based on one dataset - Dataset 1 - which spans from October 15, 2019, to October 13, 2021. In addition to examining this dataset, we will conduct a broader analysis by assessing the distribution of metrics across all simulations for all portfolios, allowing us to gain insights into each strategy's performance across varied market scenarios.

Finally, each strategy will also be mapped onto the mean-variance efficient frontier (whose construction will be explained in the Mean-Variance Portfolio section), providing a benchmark for evaluating their positions as either optimal or suboptimal. This setup will enable a performance comparison across different strategies and give context to their behavior under a consistent risk-return framework.

4.1. Equally weighted portfolio

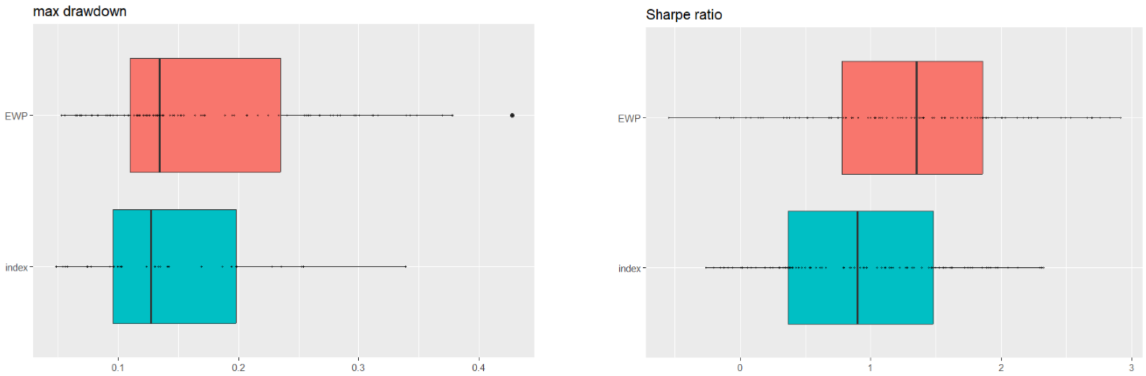
The Equally Weighted Portfolio (EWP) is an investment strategy where each asset in the portfolio is allocated an equal weight, regardless of its market capitalization or other factors.

An Equally weighted portofolio have a vary set of characteristics. In first place, this approach allows to set a diversification strategy; by assigning equal weight to each asset, EWP provides a diverse exposure across various sectors and industries, reducing the impact of individual stock performance on the overall portfolio. Normally, EWP tends to outweigh smaller-cap stocks, which have historically outperformed larger-cap stocks over the long term. This value tilt can contribute to superior returns. And finally, the existence of a rebalancing discipline attached to it, where regular rebalancing is essential to maintain equal weights. This disciplined approach can lead to buying low and selling high, potentially boosting returns.

A simple way to achieve diversification is by allocating the capital equally across all the assets, where 1/N portfolio, uniform portfolio, or maximum deconcentration portfolio, such as

$$\mathbf{w} = \frac{1}{N}$$

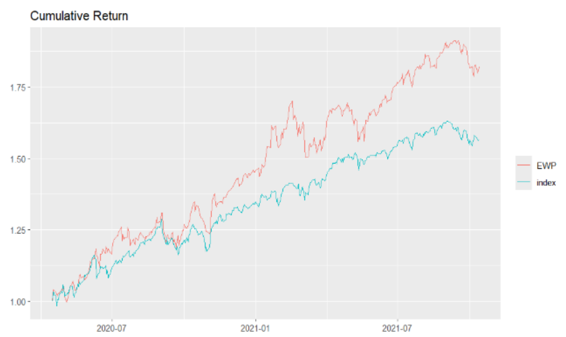
EWPs offer a simple yet effective approach to investing. By diversifying across a wide range of assets and incorporating a value tilt, EWPs can provide attractive long-term returns.

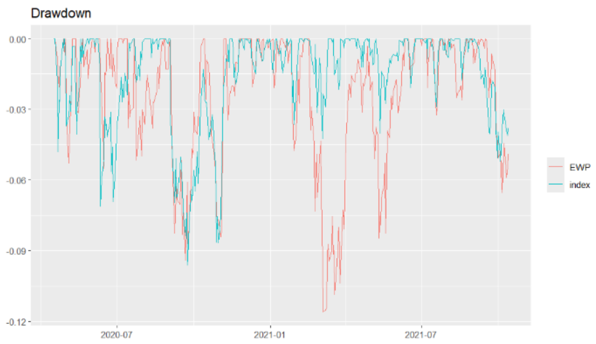


The boxplots provide a compelling visual comparison of the EWP and Index strategies. The EWP demonstrates a clear disadvantage in terms of risk management, where the median maximum drawdown for the EWP strategy appears to be slightly higher than that of the Index. The interquartile range for the EWP strategy is also slightly larger, indicating more variability in its maximum drawdowns. Overall, the boxplot suggests that the EWP strategy, while potentially offering higher returns, may also be associated with higher risk, as evidenced by the larger maximum drawdowns.

Additionally, the EWP's Sharpe ratio appears to be higher than that of the Index. This suggests that, on average, the EWP has generated higher returns relative to its risk. The interquartile range for the EWP strategy is also larger. This implies that the EWP's performance can fluctuate more significantly over time.

These findings collectively suggest that the EWP offers a compelling investment proposition, particularly for investors seeking a balance between risk and return.





The EWP significantly outperformed the S&P 500 over the sample period. Starting with similar values, the EWP ended significantly higher, indicating superior cumulative returns. Additionally, we can state that EWP strategy records consistent growth, especially in the later part of the period. This might indicate that the EWP strategy is more resilient to market fluctuations, possibly due to diversification or the balanced weightings across assets.

What can be inferred from this plot is that both the EWP and Index exhibit similar maximum drawdowns - drawdowns represent the decline from a peak in cumulative return, essentially capturing the depth and duration of losses before recovery - which can reach levels of about -10% to -12% during some of the more severe market downturns. This indicates that neither portfolio is completely immune to large market shocks.

In terms of frequency, both portfolios experience frequent drawdowns, with drawdowns occurring almost consistently across the timeline. This frequency implies that both the EWP and Index are exposed to regular market fluctuations. While in terms of recovery patterns, it’s watchable some periods where the EWP recovers more quickly than the Index, or experiences less severe declines. For instance, during certain drawdown events, the EWP's line (red) bounces back faster or remains closer to the zero line than the Index's line (blue). This may suggest that the EWP's equal-weighting structure allows it to stabilize faster than the Index, which could be more affected by specific sectors or large-cap stocks.

In summary, this drawdown analysis suggests that the EWP might be more resilient to short-term losses, experiencing shallower and less frequent severe drawdowns compared to the Index. While both portfolios are exposed to significant market downturns, the EWP appears to recover more steadily, possibly due to its equal-weighted structure that mitigates over-reliance on any single stock or sector. This makes the EWP a potentially attractive choice for investors seeking to balance return with reduced downside volatility.

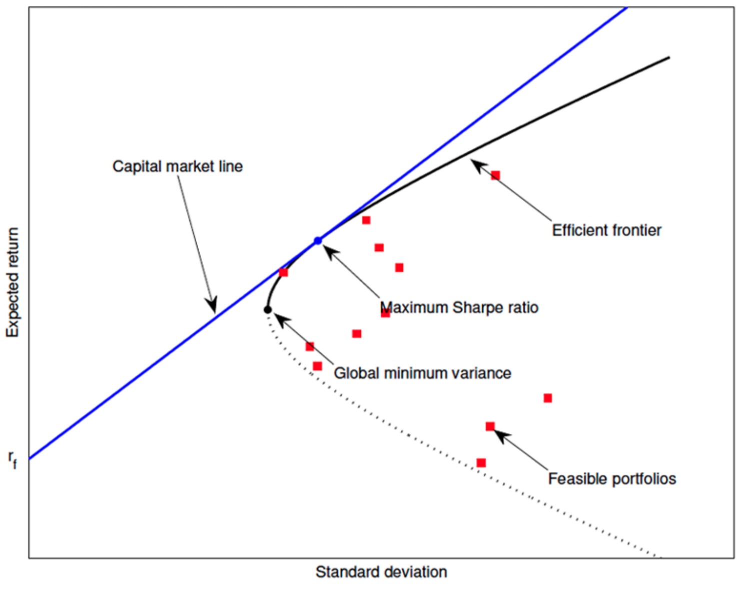
Furthermore, the EWP's median risk-adjusted metrics revealed its strong performance[4]. It ranked third highest in the median Sharpe ratio, demonstrating substantial efficiency in returns relative to risk. Although the good results in the previous median risk-adjusted metrics, ranked among the low performers in median annual volatility. The strategy recorded a considerably good median maximum drawdown, a value that follows the index performance for the same metric in study. It achieved the third-highest median Sortino ratio and median Omega ratio, indicating efficient downside risk management and favorable returns over the risk-free rate, and its median Sterling ratio also placed as the top performer strategy, reflecting its ability to control drawdowns while achieving consistent performance.

[4] Table 4 in the Appendix

4.2. Markowitz's mean-variance portfolio (MVP)

In a world full of risks, rising tensions between major economies and so much uncertainty towards the future, it has become harder and harder to hedge a portfolio against the micro and macro risks.

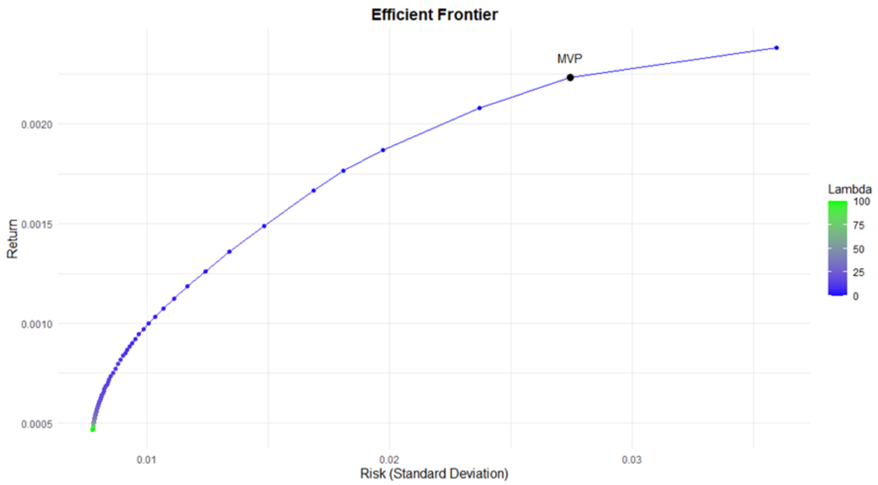
The MVP is a foundational concept that brings optimized returns for a certain level of risk, in other words it balances the idea of expected returns and risk levels, by creating an efficient frontier that shows us the most efficient portfolios. This portfolio theory encourages diversification, spreading investments across multiple assets with low correlation, reducing overall risk. The "optimal" portfolio on the efficient frontier with our assumption of no short selling is the one with the maximum Sharpe Ratio or the minimum variance given a target return.



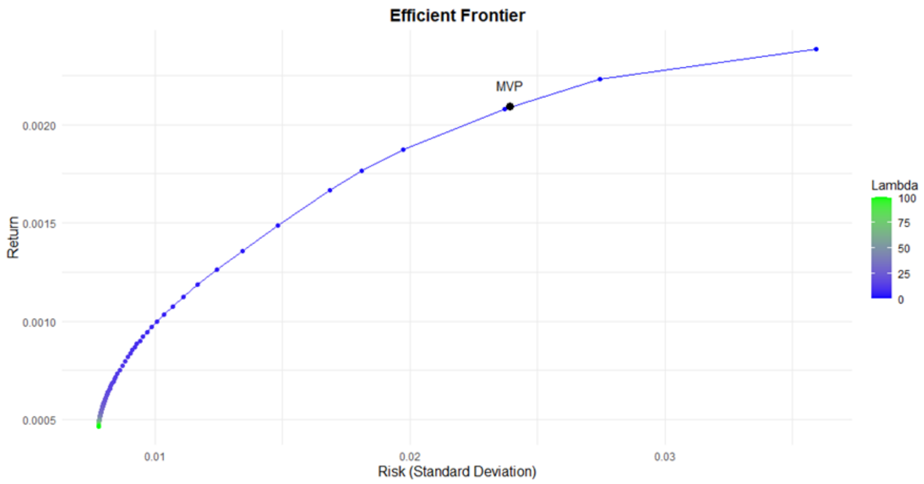
The efficient frontier is a foundational concept in modern portfolio theory that illustrates the optimal balance between risk and return for a given set of investments. Introduced by Harry Markowitz in 1952, the efficient frontier represents a set of portfolios that maximize expected return for a given level of risk or, conversely, minimize risk for a given level of expected return.

Below is the graph that best describes a curve on a risk-return plot, with portfolios along the curve being "efficient" in that no other portfolio offers a higher return for the same risk level. Investors use the efficient frontier to identify portfolios that align with their risk tolerance and investment goals. By understanding the efficient frontier, one can gain insights into the trade-offs between risk and return and the benefits of diversification in achieving optimal portfolio construction.

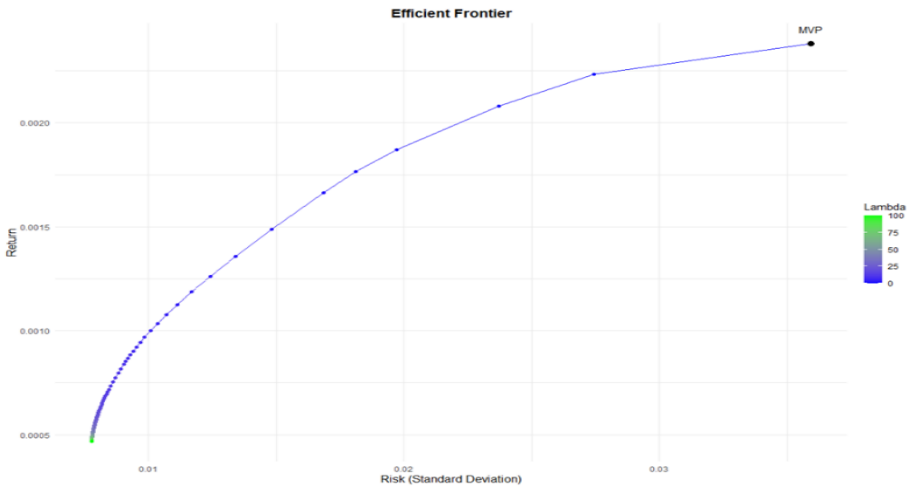
Considering the variable λ as the risk aversion parameter, using λ =0.5, we got the following results in the efficient frontier for the first dataset.



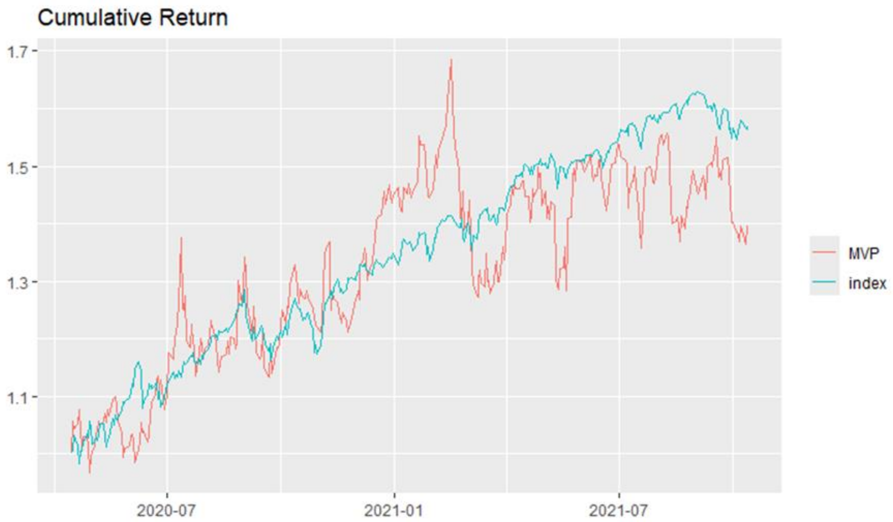
As we test with a λ=0.99 we can see that the dot from the MVP moves to the left meaning we are now more averse to risk and we prefer safer assets, or lower risk assets. Conversely, a lower λ reflects a more aggressive investor who is willing to take on more risk for potentially higher returns.



With a λ= 0.1 we choose as an investor to have a high-risk profile, and the dot of the MVP will move to the right as we choose to optimize for higher returns and attribute less importance to volatility. This moving effect showcases how the Efficient Frontier is built.



Observing the cumulative returns over the period of the dataset, we observe that for a period of around 2 months, it returned more than the S&P 500, however, it underperformed the benchmark eventually.



Testing the summary of the Portfolio performance with the median and max values, we can still reach the same conclusions: It's a riskier portfolio, reaching a drawdown similar to an asset class known for being volatile, for example, cryptocurrencies or options trading. For example, let's assume that we had a portfolio of $10.000 with the max values we would potentially have unrealized losses of upwards of $7000, the annual return would not compensate for the max drawdown.

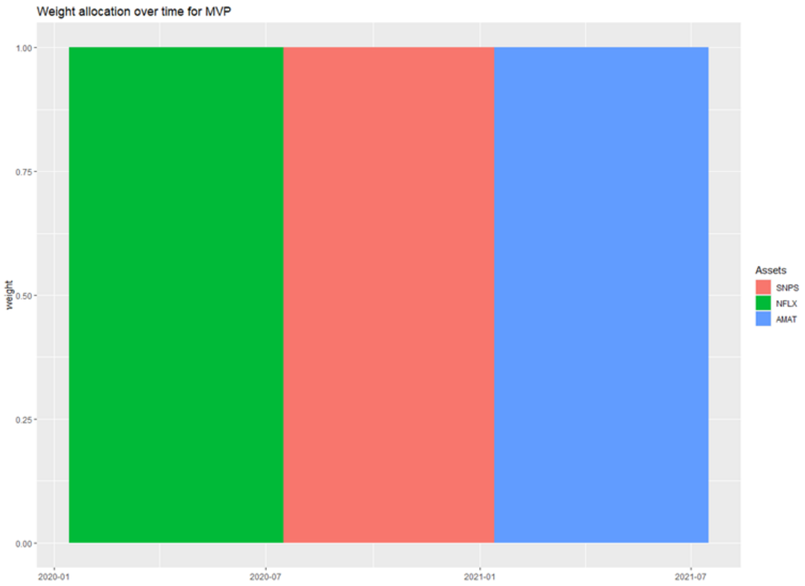
The median values[5] for the MVP show a very poor portfolio strategy with a sharp ratio below 1 which is considered bad or underperforming the market. The max drawdown of 32% is a very high loss for a portfolio manager, with the VaR at 3%, meaning that an investor could lose 3% in the worst day.

In the most extreme cases, we can observe a similar scenario with the median values but with a larger magnitude, with the max drawdown here being more than the double of the worst index maximum drawdown, and creating unrealized losses for an investor of 75% of the portfolio, which is an insane possibility to even think about. The maximum return of 200% is not nearly enough to compensate for the possibility of such max drawdown.

[5] Table 4 in the Appendix

Now let’s observe the drawdown in more detail below. The MVP has larger oscillations to the downside and more aggressive ones, with a clear sharp decline in early 2021 while the benchmark didn’t have a fraction of the same move. The cost of opportunity in holding the MVP vs S&P would be massive.

The image below shows the weight allocation over time of the Portfolio MVP, its interesting to compare in the following portfolios below how the allocations compare to each other. Also noting that using lambda of 0.5 we are basically telling it to compute for chasing return and caring less about variance, causing a greater concentration in the stocks that offer the best returns as we can observe in the plot below.

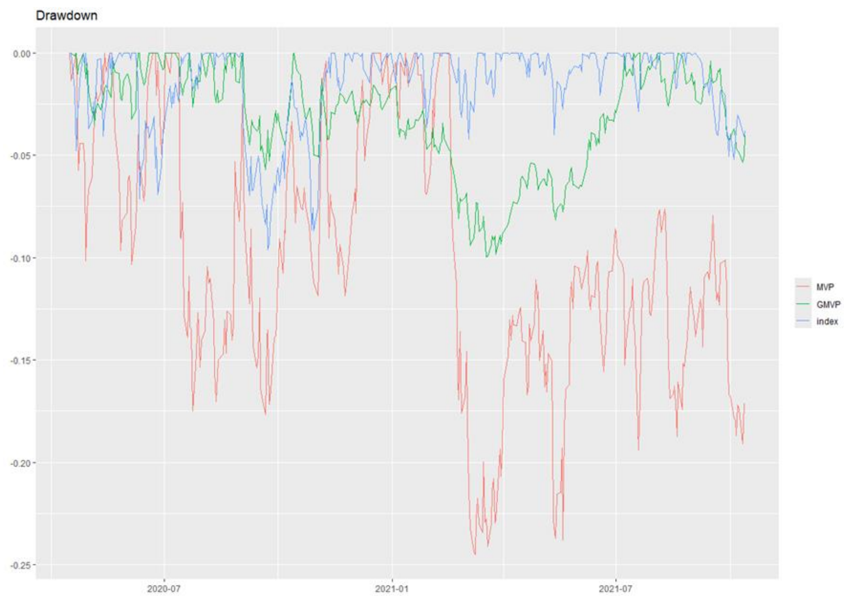


4.3. Global Minimum Variance Portfolio (GMVP)

The Global Minimum Variance Portfolio (GMVP) is a streamlined version of the MVP, focusing solely on volatility and aiming to reduce it as much as possible, resulting in the best protected portfolio against volatility without considering expected returns.

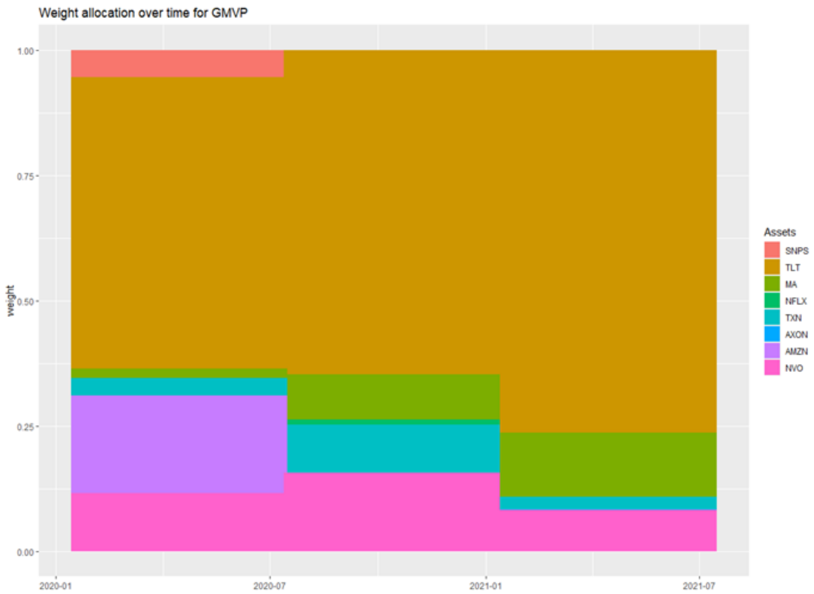
On the efficient frontier, the GMVP sits at the farthest point on the curve, acting as a pivot between desirable portfolios and those to be avoided (offering the same risk as others but with lower returns).

$$\begin{array}{ll} \underset{\mathbf{w}}{\text{minimize}} & \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \\ \text{subject to} & \mathbf{1}^T \mathbf{w} = 1. \end{array}$$



Following the drawdown of the MVP portfolio, let's now test with also plotting the GMVP to see how the drawdown of both compare to the benchmark. The GMVP still has a larger drawdown than the index but far better than the MVP portfolio, still using the same period of sharp decline in early 2021, we can clearly see that just be holding to GMVP we would be much better off than holding a portfolio using MVP as a strategy.

We also plotted the weight allocation over time to see what stocks were owned overtime and their weight and get some sort of data out of it. That said it is interesting to observe that TLT being the 20y+ Bond ETF has a larger weight of the portfolio compared to MVP which was only showing stocks. Noting that 20y+ treasury bond ETF is riskier than the short end of the curve because an investor needs to be properly compensated for locking the money (if not trading on SM) until it reaches maturity, bearing more risk. Still compared to stocks it should be safer (unless the deficit runs out of control which is starting to be the case in the US).



Let's now analyze the MVP, GMVP and the benchmark (S&P 500) performance over the period of the back testing.

It's clear that both Portfolios are underperforming the benchmark by a large margin, still we can clearly see the objective of this portfolio as mentioned in the beginning, the sole purpose of GVMP is to reduce volatility and that is perfectly shown both in the plot and in the table below. In the same 2 months analyzed previously in the MVP performance we can see the massive drawdown that followed the spike in returns, at the same time, GMVP has shown much more resilience.

Analyzing the median values table[6], we can clearly observe that the GMVP has some of the lowest ratios compared to other portfolios, with a sharp ratio of 0.93 and a sortino ratio of 1.38. The annual return is also below the index and the max drawdown is almost the same as the annual return, which an investor would see it as almost a 1 to 1 ratio in a trade, where he can make 1000 or lose 1000 euros, so its obviously not a good risk reward opportunity.

Clearly with median values we can immediately notice that the sharp ratio of this Portfolio is below 1, meaning that it is sub-optimal and for any investor it would be a poor choice of investment. In the extreme cases, the portfolio continues to show sub-optimal ratios, as the annual return is equal to the max drawdown, meaning that if the max drawdown happened the annual return would not be sufficient to compensate for the unrealized losses.

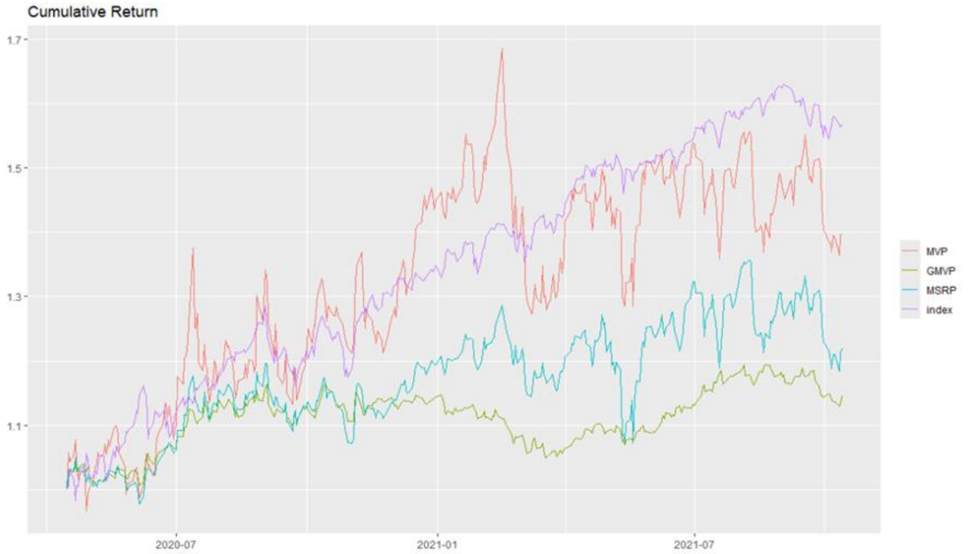
4.4. Maximum Sharpe ratio portfolio (MSRP)

The Maximum Sharpe Ratio, originating from William F. Sharpe's Modern Portfolio Theory, is a useful measure for gauging how well a portfolio performs given the amount of risk taken. It seeks to determine the best balance between returns and risk, capturing how returns stack up against the portfolio’s volatility. To calculate it, you divide the portfolio’s return by its level of volatility, providing a snapshot of how effectively returns are generated relative to the risk level.

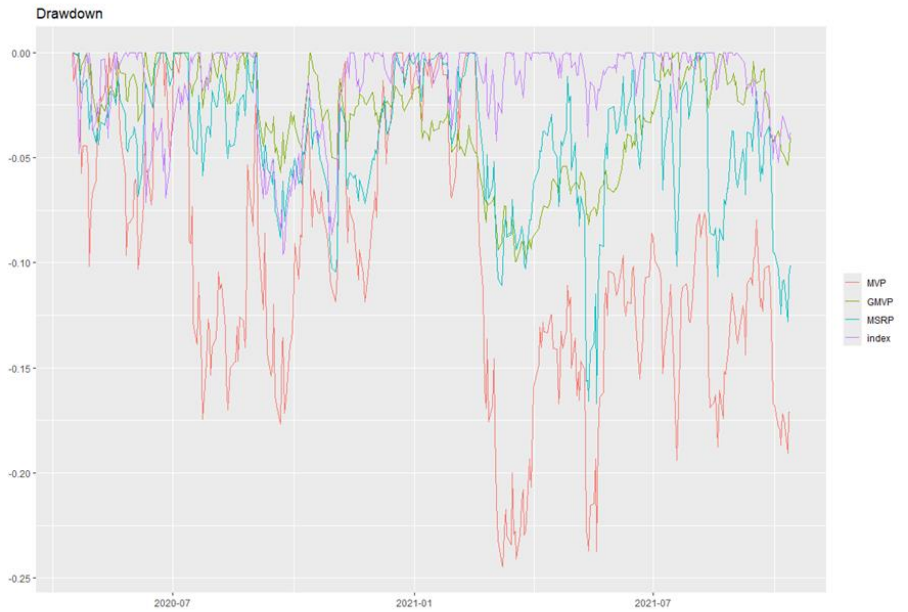
$$SR = (Rp - Rf) / \sigma p$$

Where SR represents the Sharpe Ratio, Rp is the portfolio return, Rf is the risk-free rate, and $\sigma\rho$ is the standard deviation (or volatility) of the portfolio.

[6] Table 4 in the Appendix

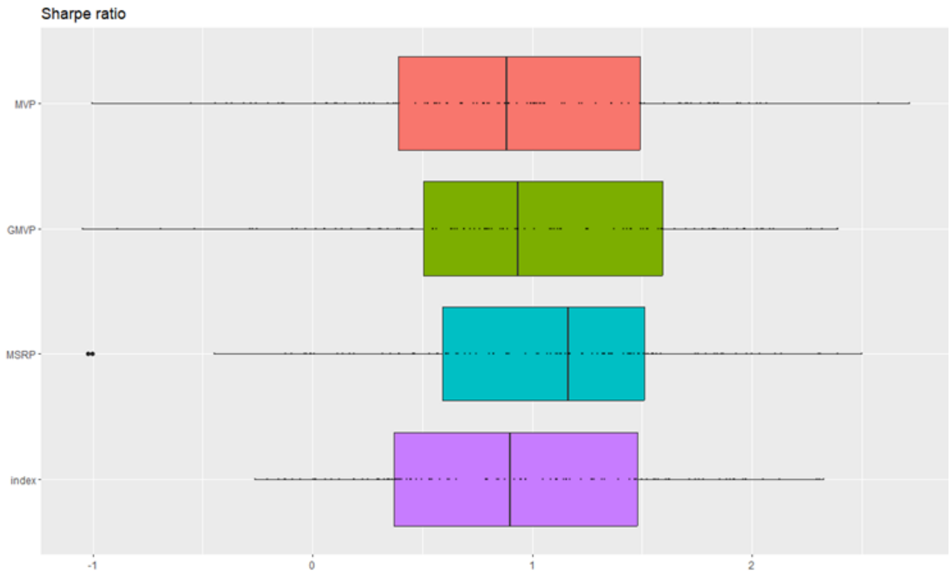


Let’s now analyze the image below the drawdown of MSRP. MSRP is oscillating with smaller but aggressive declines compared to GMVP but still not in the same magnitude of MVP. Both MVP and MSRP have similar declines, so they are some what correlated in the same behavior but with different magnitudes. Getting back to the same period of observation, early 2021, both MSRP and GMVP had a large decline although MSRP was earlier in reaching the local bottom than MVP. We can conclude that from the max drawdown of the 3 Portfolios, GMVP is by far the safest option for a conservative investor who just wants to minimize risk.



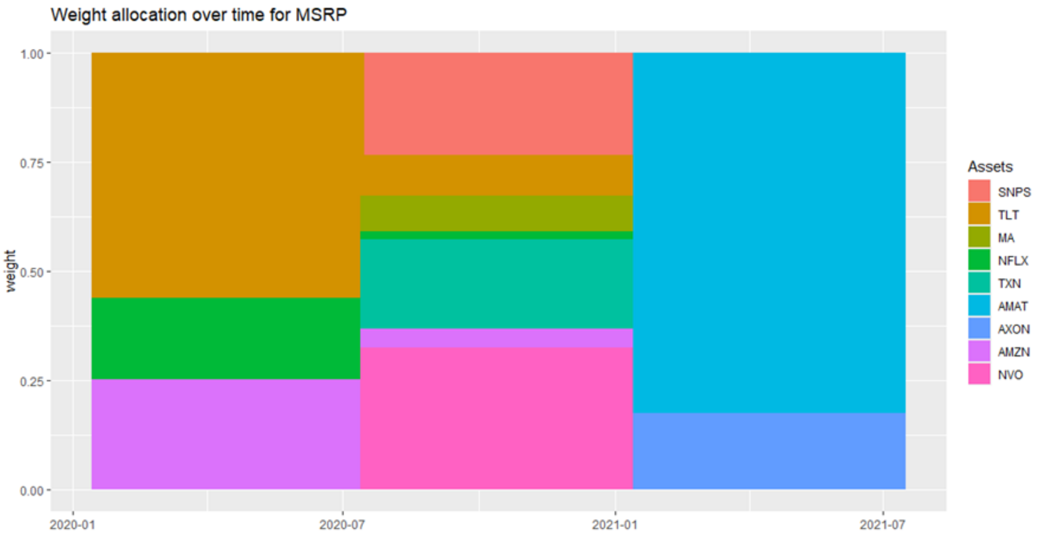
It’s another great portfolio concept to back test different portfolios to provide the highest risk adjusted return. It represents the optimal combination of assets that deliver the highest return beyond the risk-free rate for each additional unit of risk. Investors use the Maximum Sharpe Ratio to find the most efficient way to increase returns while keeping risk under control.

Comparing now the Sharpe Ratio of the 3 Portfolios (MVP, GMVP and MSRP) against the benchmark, we can see that the MSRP has the best Sharpe ratio.



Despite having the sharpest ratio and performing reasonably it still underperforms against the S&P 500. An investor would still have outperformed the previous models by just buying and holding VUAA ETF (tracking S&P in Europe Exchanges). The free-float market capitalization-weighted methodology (S&P 500) outperforms MVP, GMVP and MSRP.

Finally, observing the evolution of weight allocations over the dataset, the MSRP still presents from the 2 previous portfolios models a more diverse weight allocation over the time, having less concentration in one particular asset.

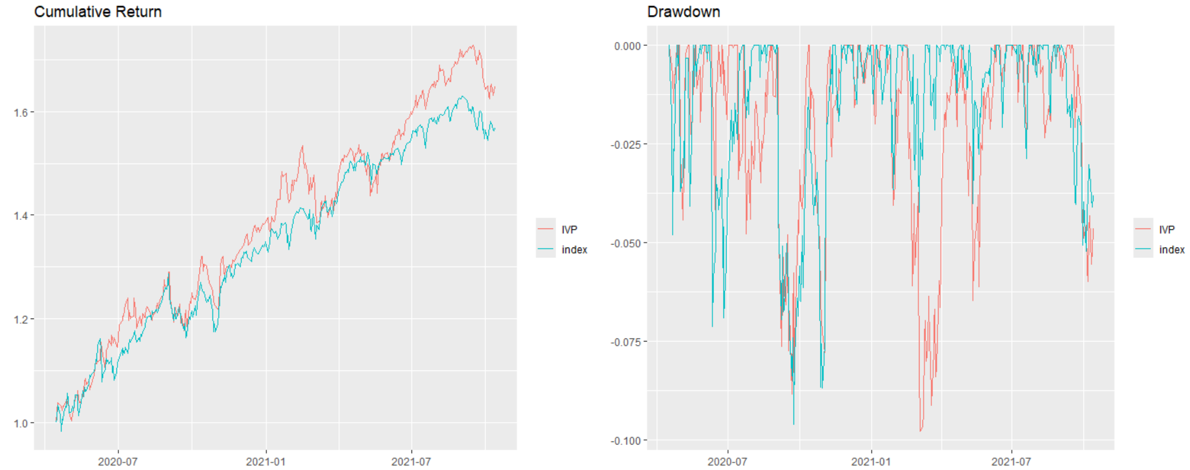


4.5. Inverse Volatility Portfolio (IVP)

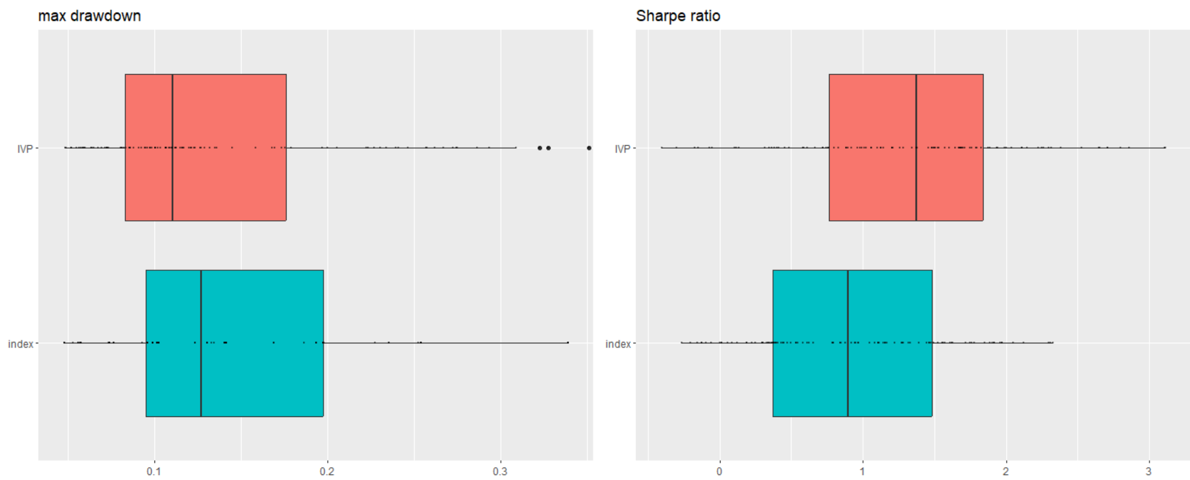
The Inverse Volatility Portfolio (IVP) strategy aims to allocate capital inversely proportional to each asset's historical volatility. This approach favors assets with lower volatility, under the assumption that these assets contribute less to overall portfolio risk. By weighting assets inversely to their volatility, the IVP seeks a balance where riskier assets hold less weight, thus potentially stabilizing returns and mitigating drawdowns. This method is particularly appealing for risk-sensitive investors, as it allocates more heavily to relatively stable assets, aligning portfolio composition with lower risk exposure.

$$w_i = \frac{1}{\sigma_i} \qquad \text{s.t.} \ \sum_{i=1}^{n} w_i = 1$$

Although it is important to recognize that performance may vary with different datasets and, thus, specific market conditions, the IVP strategy showed promising results in the context of Dataset 1. Over the sample period, the IVP consistently outperformed the market index in cumulative returns, suggesting increased value from this simple strategy. Notably, early in the sample period, the market faced significant turbulence, as evidenced by the steep declines in the market index drawdown. During the sample, both the IVP and the market index recorded maximum drawdowns below 10%, indicating that while IVP managed risk well, it behaved similarly to the broader market. This result implies that IVP's volatility-sensitive design allowed it to partially buffer market shocks, demonstrating its effectiveness in managing risk during turbulent times.



Analyzing the broader context with a boxplot of maximum drawdowns from portfolio simulations, the IVP demonstrates lower interquartile drawdown values compared to the market index, indicating a slight edge in risk handling. This shift in interquartile data reveals that IVP can better control extreme negative fluctuations, lending stability to the portfolio during market turmoil. When observed through a Sharpe ratio boxplot, the IVP demonstrates superior risk-adjusted performance. The highest simulation Sharpe ratios for the IVP notably exceed those of the market index, showcasing the strategy's strong efficiency in generating returns relative to its risk.



Moreover, IVP ranked the highest among all strategies in terms of median Sharpe ratio[7], a testament to its consistently robust performance across simulations. Beyond the Sharpe ratio, the IVP also excelled in several other risk-adjusted metrics, highlighting its overall effectiveness. It achieved the highest median Sortino ratio, favoring upside potential while managing downside risk, and maintained one of the lowest median annual volatilities. The strategy also demonstrated its strength by outperforming all other strategies in terms of the median Omega Ratio, which highlights its capability to achieve positive returns above the risk-free rate, while also earning a top ranking in the median Sterling ratio, emphasizing its effectiveness in managing drawdowns and maintaining performance over time.

The results underscore the IVP's strength in handling risk and delivering stable returns in volatile environments, positioning it as a favorable alternative to the market index, particularly for risk-conscious investors seeking risk-adjusted outperformance.

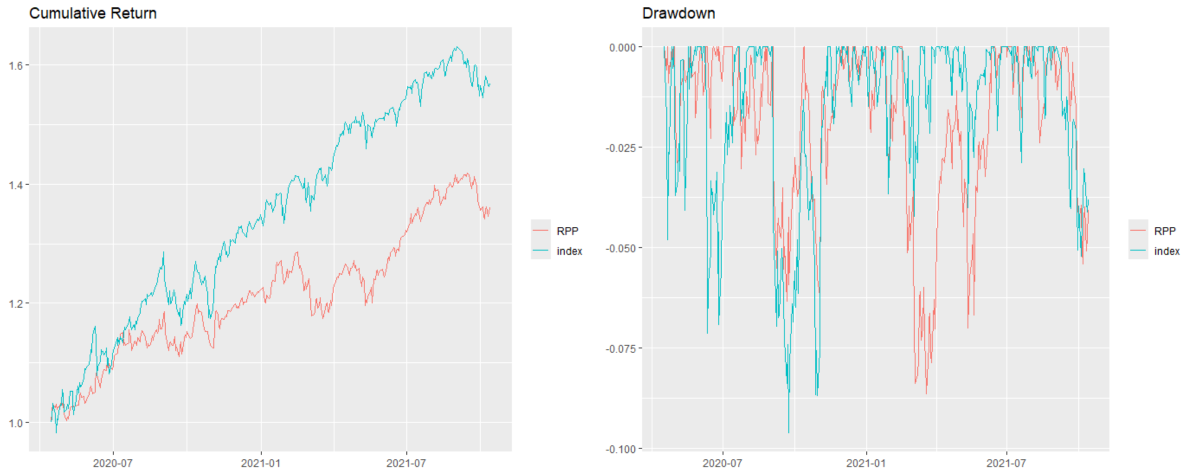
4.6. Risk parity portfolio (RPP)

The Risk Parity Portfolio (RPP) strategy seeks to allocate capital such that each asset contributes equally to the overall portfolio risk. This approach is based on the idea that balancing risk, rather than dollar amounts, leads to a more stable portfolio. By ensuring that no single asset dominates the portfolio's risk profile, the RPP minimizes the impact of highly volatile assets and aims to deliver a smoother return profile, especially in uncertain market environments.

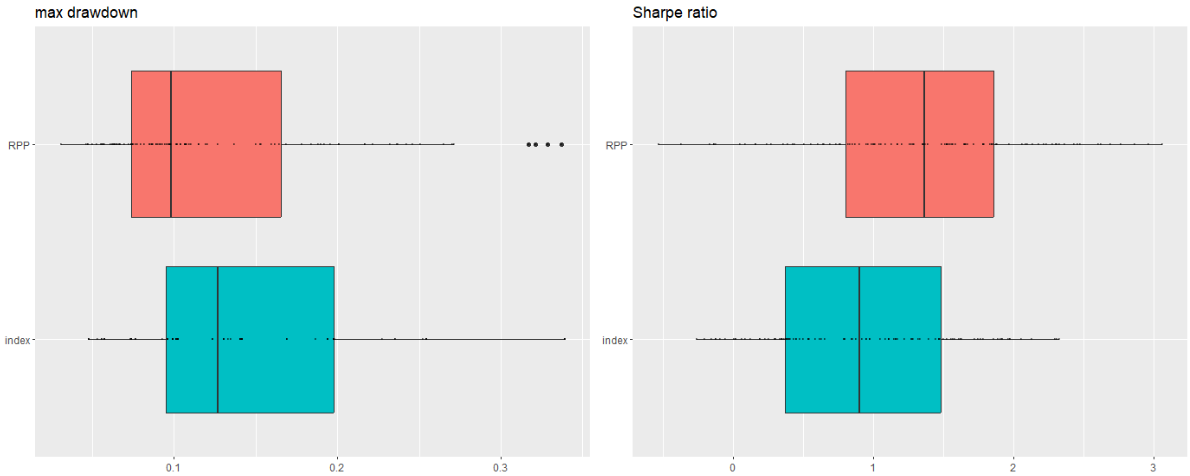
As with any strategy, the results of RPP can vary depending on the dataset and specific market conditions. In this sample, the RPP strategy shows a more conservative return trajectory, underperforming the market index in cumulative returns. A divergence from the market's

[7] Table 4 in the Appendix

movement becomes apparent halfway through the sample, suggesting that while the RPP's approach may limit gains, it provides a layer of risk control. In terms of drawdown, the RPP showed improved risk handling, with a maximum drawdown below 9%, slightly better than the market's maximum drawdown, which approached 10%. This reduced drawdown indicates that RPP's emphasis on equal risk contribution helps to limit the depth of losses, a valuable trait in turbulent markets.



More broadly, we observe that the RPP provides considerable value in managing extreme losses, as shown by the boxplot of portfolio simulations' maximum drawdown. The entire distribution of maximum drawdowns for the RPP is shifted downward, suggesting a lower likelihood of severe losses. Only a few outliers exceed the 30% maximum drawdown, highlighting the strategy's robustness in reducing large drawdowns. In terms of risk-adjusted returns, the Sharpe ratio boxplot reveals a wide distribution for the RPP, indicating considerable variability in risk-adjusted performance across simulations. While the best Sharpe ratios achieved by this strategy considerably exceed those of the market index, the worst Sharpe ratios fall into negative territory, reflecting periods of negative returns in the sample. This variability points to the RPP's potential for high performance in favorable conditions but also highlights a sensitivity to market fluctuations that can lead to lower performance.



Furthermore, the RPP's median risk-adjusted metrics revealed its strong performance[8]. It ranked second highest in the median Sharpe ratio, demonstrating substantial efficiency in returns relative to risk. The strategy also recorded the smallest median maximum drawdown and ranked among the top performers in median annual volatility. It achieved the second-highest median Sortino ratio and median Omega ratio, indicating efficient downside risk management and favorable returns over the risk-free rate, and its median Sterling ratio also placed it among the top strategies, reflecting its ability to control drawdowns while achieving consistent performance.

Overall, the RPP showcases a balance between risk control and return generation, providing an appealing option for investors prioritizing stability in volatile markets. While the strategy may lag behind in cumulative returns in some samples, its superior handling of risk, particularly in drawdown management and downside protection, reinforces its value as a resilient alternative to more aggressive portfolios.

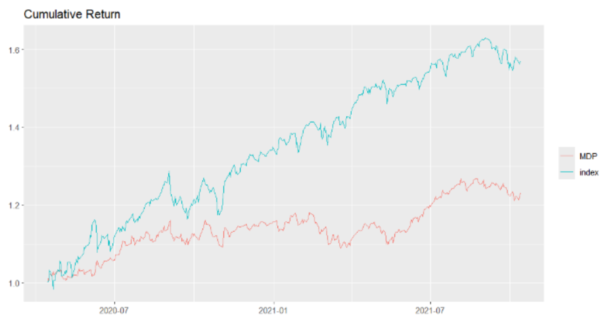
[8] Table 4 in the Appendix

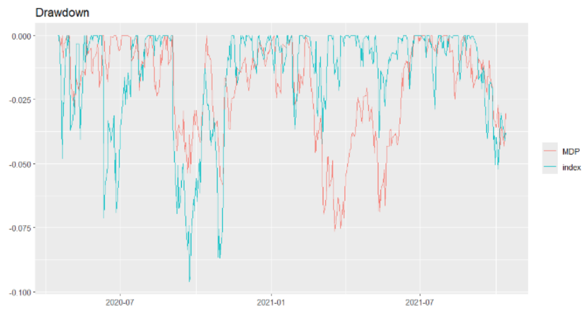
4.7. Most diversified portfolio (MDP)

The MDP strategy aims to maximize portfolio diversification by spreading risk across assets as evenly as possible. Unlike traditional portfolios that focus on risk-return optimization, MDP focuses uniquely on diversification as a means of reducing risk. This approach derives from the idea that, by diversifying broadly across uncorrelated assets, an investor can achieve greater stability and resilience, especially during market downturns.

$$DR(w) = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sigma_P}$$

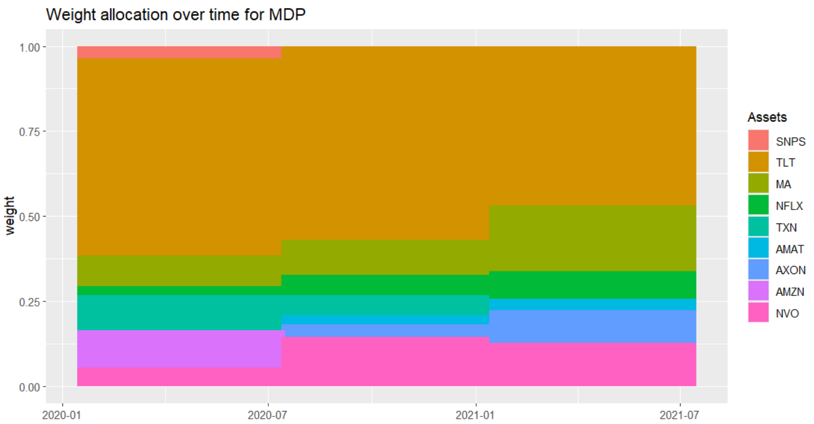
Where: wi = portfolio weight in asset i; σi = the risk of asset i; σp is the total risk of the portfolio.





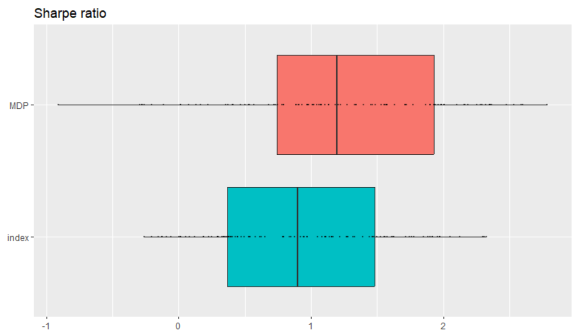
In this sample, the MDP strategy shows a more conservative return trajectory, underperforming the market index in cumulative returns. A divergence from the market's movement becomes apparent almost since the beginning of the sample, suggesting that the MDP’s approach focus steadily gains, while providing a layer of risk control.

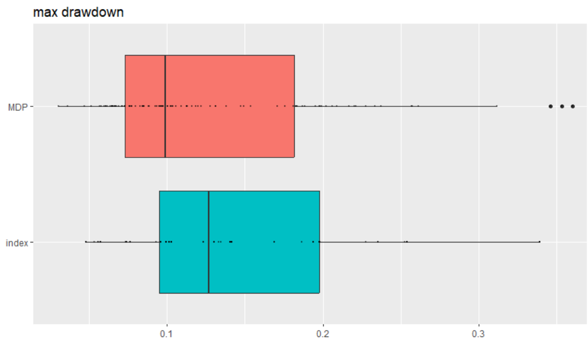
In terms of drawdown, the MDP exhibits significantly lower and less frequent drawdowns compared to the index, showing improved risk handling, with a maximum drawdown below 7,5%, way better than the market’s maximum drawdown, which approached 9%. This suggests that the MDP is more resilient to market downturns and can better protect investors' capital. The smoother drawdown pattern of the MDP indicates a more stable and less volatile investment experience, making it an attractive option for risk-averse investors seeking to mitigate downside risk while still generating returns.



Regarding asset allocation over time for the MDP, the graph above shows how weights for each asset within the portfolio shift from January 2020 to July 2021. The assets are represented by distinct colors, each occupying a portion of the vertical scale, which sums up to a total portfolio weight of 1 (or 100%). It is evident from the chart that TLT dominates the portfolio, maintaining a consistently high weight for the majority of the period. This significant allocation indicates that TLT (a bond ETF) plays a critical role in stabilizing the portfolio, aligning with the MDP strategy's focus on minimizing risk.

Over time, we observe some adjustments in the weights of other assets, such as NVO (in pink), AMZN (in teal), and NFLX (in green). These changes suggest periodic rebalancing to optimize the portfolio’s risk profile in response to shifting market conditions. Notably, assets such as TXN and AXON maintain smaller and more stable weightings throughout the period, indicating they have a more supplementary role in the MDP. This dynamic allocation highlights the strategy’s emphasis on maintaining a low-risk profile, with primary reliance on TLT, while diversifying with smaller positions in other assets to achieve the portfolio's overall risk minimization objective.

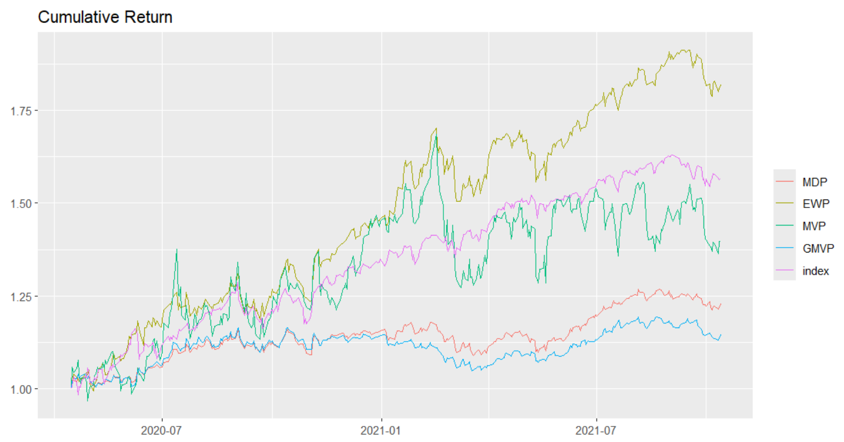




The MDP strategy, as evidenced by the boxplots, offers a compelling blend of risk and return. It exhibits a strong ability to limit maximum drawdowns, with only a few outliers exceeding the 30% maximum drawdown, suggesting resilience to market downturns. The MDP's more controlled drawdown profile makes it a compelling choice for investors seeking a robust and balanced investment strategy.

Additionally, the MDP's Sharpe ratios indicate a favorable risk-adjusted performance, demonstrating its efficiency in generating returns relative to the risk taken. However, it's important to denote that the MDP's performance may be less aggressive, which can potentially generate higher returns but at the cost of increased risk. The MDP's focus on risk mitigation might lead to lower absolute returns during periods of strong market. Therefore, the choice between the MDP and other strategies depends on individual investor risk tolerance and return objectives.

The MDP's conservative, risk-averse approach leads to a relatively modest cumulative return, prioritizing stability and reduced volatility over aggressive growth. as the cumulative return portfolio strategies benchmark graph demonstrates.



The MDP underperforms most strategies analyzed. This highlights the trade-off between risk and return: EWP offers higher potential returns but with increased volatility, while MDP prioritizes stability, providing a smoother, lower-return path for conservative investors.

In summary, MDP's cumulative return, while lower than other strategies, aligns with its primary objective of minimizing risk, offering a reliable but modest growth option within a diversified investment approach.

Additionally, the MDP strategy, as evidenced by the provided data[9], exhibits a strong risk-adjusted performance profile. It secured a median Sharpe ratio of 1.20, indicating a favorable balance between risk and return. Notably, the MDP's median maximum drawdown of 10% underscores its ability to mitigate significant losses during market downturns.

Furthermore, the MDP's median Sortino ratio and Omega ratio suggest effective downside risk management. The Sortino ratio measures the risk-adjusted return relative to downside deviation, while the Omega ratio evaluates the probability of exceeding a specific return threshold. These metrics highlight the MDP's ability to generate positive returns while limiting downside risk. Additionally, the strategy's relatively low volatility further reinforces its appeal to risk-averse investors.

[9] Table 4 in the Appendix

Overall, the MDP strategy offers a compelling blend of risk and return characteristics. Its strong performance metrics, particularly in terms of risk-adjusted returns and downside protection, make it a suitable choice for investors seeking a balanced and resilient investment approach.

4.8. Maximum decorrelation portfolio (MDCP)

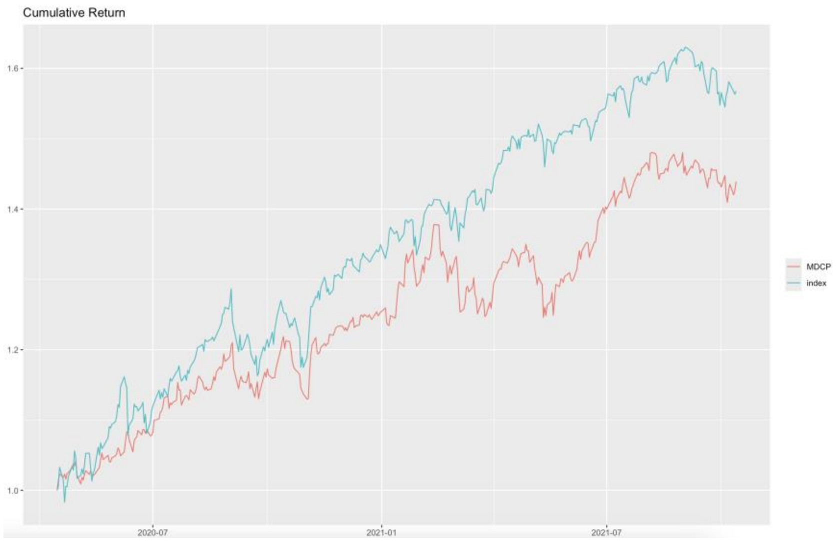
The Maximum Decorrelation Portfolio (MDCP) is an investment approach aimed at optimizing asset allocation by reducing risk through diversification. MDCP achieves this by selecting assets that exhibit low or negative correlation with one another. This approach mitigates overall portfolio volatility, as declines in one asset may be counterbalanced by positive returns in others, especially during periods of market stress.

Unlike traditional portfolio optimization, which minimizes risk based on the covariance matrix, MDCP focuses on the correlation matrix of assets. This shift emphasizes the relationships between assets rather than individual asset volatility, aiming to reduce risk by avoiding highly correlated assets.

In mathematical terms, the formulation of this portfolio optimization is as follows.

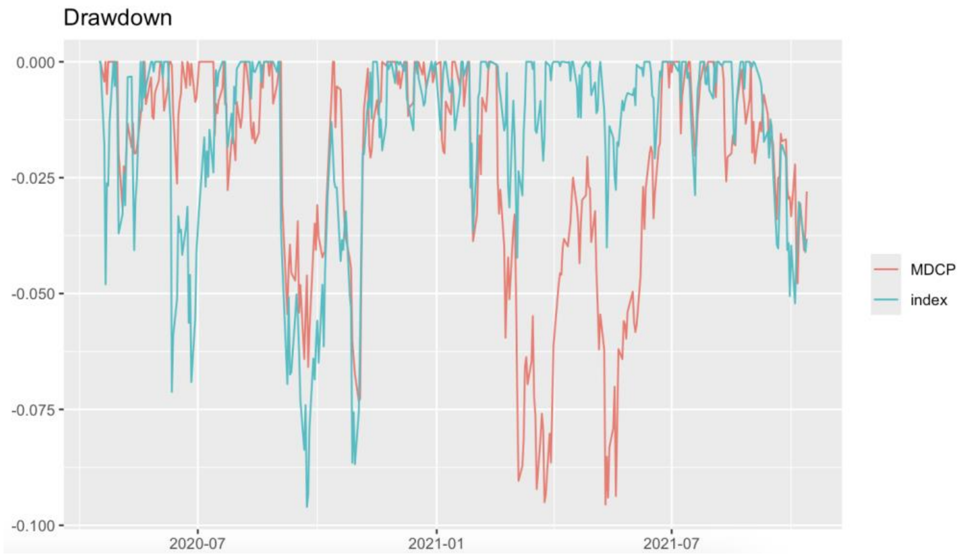
$$\min_{w} w^T C w$$

$$\text{subject to } \sum_{i=1}^{N} w_i = 1, w_i \geq 0, \quad i = 1, \dots, N,$$



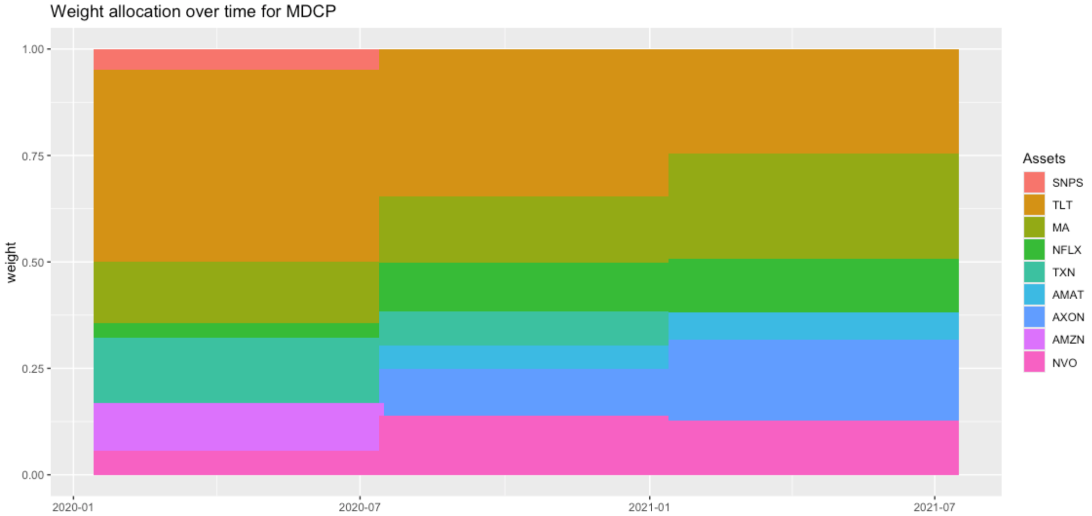
The dataset for sample 1 reflects a positively trending market, as indicated by the strong gains in the index over time. Although MDCP's diversification aims to reduce risk, it appears to underperform during bullish markets, as the index outpaces it. This is explained by the reduced correlation among assets in the MDCP approach, which ends up diluting returns in trending markets, limiting its potential for capitalizing on synchronized upward movements.

Notably, the cumulative return graph reveals that MDCP's performance trends similarly to the index. This correlation suggests that MDCP may not fully achieve true decorrelation and cannot avoid the systematic risks embedded in the overall market.



Both MDCP and the index exhibit similar maximum drawdowns, both remaining below 10%. This indicates that MDCP, despite generating lower returns, still maintains resilience against significant losses, presenting a stable risk profile to the index. Moreover, in the particular market conditions of dataset 1, the MDCP's performance in minimizing drawdowns does not outperform significantly the index, suggesting that, contrary to expectations, the strategy is not fully effective in reducing losses in adverse market conditions.

The aim of MDCP is to minimize volatility through reduced asset correlation, but this strategy does not fully protect against market downturns.



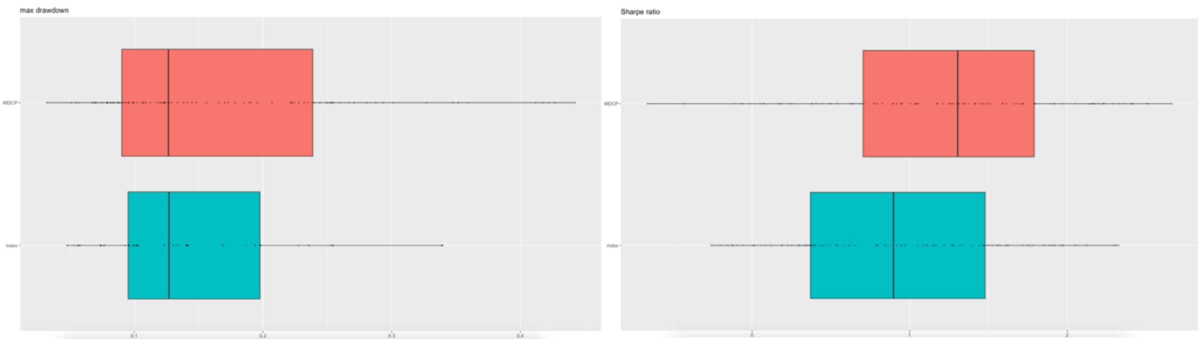
	SNPS	TLT	MA	NFLX	TXN	AMAT	AXON	ASML	AMZN	NVO
SNPS	1.0000000	-0.16770140	0.6243954	0.52256601	0.7185031	0.7208660	0.47923813	0.70421482	0.6408473	0.4723406
TLT	-0.1677014	1.00000000	-0.3392054	-0.06833267	-0.3198648	-0.2369372	-0.08141169	-0.06819265	-0.1171619	-0.1226025
MA	0.6243954	-0.33920541	1.0000000	0.30655303	0.6344473	0.6154820	0.39675921	0.56011328	0.4348765	0.3966834
NFLX	0.5225660	-0.06833267	0.3065530	1.00000000	0.4488643	0.3911637	0.29762702	0.41021875	0.6277245	0.3143098
TXN	0.7185031	-0.31986476	0.6344473	0.44886431	1.0000000	0.8149720	0.45780591	0.75064079	0.5624464	0.4540475
AMAT	0.7208660	-0.23693716	0.6154820	0.39116366	0.8149720	1.0000000	0.47148144	0.83077401	0.4898445	0.4341160
AXON	0.4792381	-0.08141169	0.3967592	0.29762702	0.4578059	0.4714814	1.00000000	0.48443843	0.3601112	0.3011631
ASML	0.7042148	-0.06819265	0.5601133	0.41021875	0.7506408	0.8307740	0.48443843	1.00000000	0.4936640	0.4766039
AMZN	0.6408473	-0.11716188	0.4348765	0.62772447	0.5624464	0.4898445	0.36011116	0.49366397	1.0000000	0.3742088
NVO	0.4723406	-0.12260251	0.3966834	0.31430975	0.4540475	0.4341160	0.30116305	0.47660388	0.3742088	1.0000000

Table 5: Asset Correlation Matrix

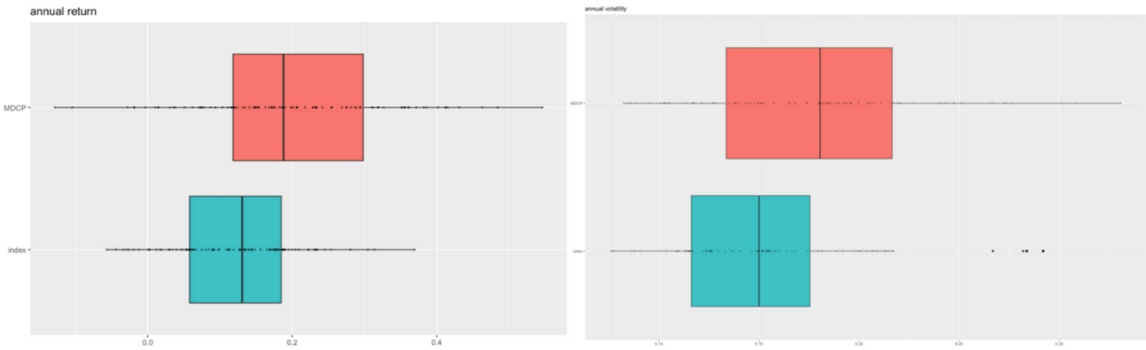
The correlation matrix provided differs from the one calculated for the MDCP. While it is based on Dataset 1, this correlation matrix derives its calculations from the full two years of data generated by the backtesting, rather than being recalculated every six months. Even though it is not identical to the MDCP's correlation matrix, it still offers valuable insights into the relationships among the selected assets.

In the Maximum Decorrelation Portfolio (MDCP) for Dataset 1, the strategy behind asset selection and weighting focuses on reducing overall portfolio correlation to limit volatility.

One possible explanation for the MDCP's underperformance in reducing the maximum drawdown compared to the index is that rebalancing occurs only every six months, while correlations between assets may change more frequently within that period. This suggests that the current rebalancing interval may be too long to accurately capture and adjust for shifts in correlations, potentially warranting a reevaluation of the six-month timeframe for correlation assessment and portfolio adjustment.



In analyzing the MDCP within our backtest results, this strategy shows a positive Sharpe Ratio, outperforming the index regarding risk-adjusted returns.



This might initially suggest that the MDCP is a favorable option for investors seeking a balance between risk and returns, as it appears to offer a better risk-adjusted performance by theoretically minimizing the portfolio's overall risk. However, this conclusion requires a deeper examination.

While the MDCP achieves a higher Sharpe Ratio compared to the index, this outcome does not stem directly from its optimization strategy aimed at reducing volatility by minimizing correlation among assets. Rather, the improved Sharpe Ratio is largely due to higher returns, not from a decrease in portfolio risk. This observation indicates that the MDCP's apparent advantage in risk-adjusted returns does not fully reflect its correlation-minimizing design, calling into question the strategy's effectiveness in genuinely reducing the overall portfolio risk.

Furthermore, the MDCP exhibits greater values than the index in both maximum drawdown and annual volatility[10], suggesting that it carries an increased level of risk. The larger drawdowns indicate that the MDCP is more vulnerable to significant losses during adverse market conditions, while the higher annual volatility points to more substantial fluctuations in portfolio value. Collectively, these factors suggest that, despite a positive Sharpe Ratio, the MDCP is not effectively reducing overall portfolio risk, indicating that the intended minimization of volatility through decorrelation is not functioning as expected.

Rather than lowering the portfolio's risk profile, the MDCP's strategy results in higher risk but achieves through proportionally higher returns. This raises critical questions about whether the decorrelation focus in MDCP effectively achieves the intended risk reduction or merely shifts the risk-return balance towards greater, albeit riskier, potential returns.

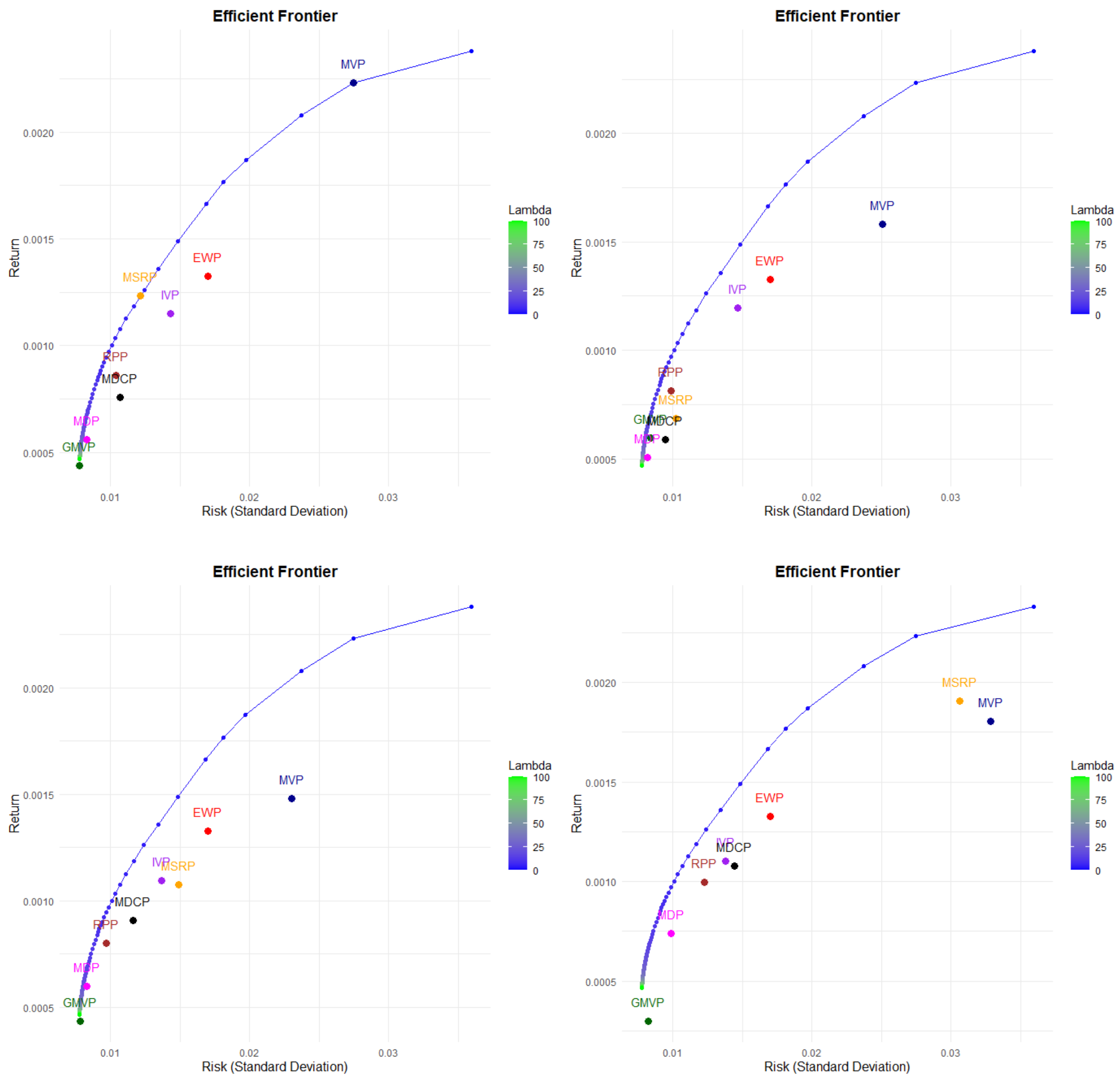
4.9. Efficient Frontier Analysis

The Efficient Frontier analysis provides insights into the performance of each portfolio strategy across four distinct plots. The first plot represents the ex-post optimized portfolios on the dataset's efficient frontier, while the remaining three show the optimized portfolios at each rebalancing point, based on the semiannual lookback period during backtesting, plotted against the same efficient frontier.

In the first plot, we observe that only the MVP, GMVP, and MSRP are positioned as efficient portfolios, meaning they achieve the best possible return for a given level of risk or the lowest risk for a given return. This outcome is expected, as these portfolios are constructed within the same Mean-Variance framework that defines the efficient frontier, while the remaining strategies offer alternative solutions. Given the inherent unpredictability in

[10] Table 4 in the Appendix

forecasting risk and return, these alternative strategies provide valuable practical applications, even if they deviate from strict efficiency, making them a reasonable tradeoff for real-world investors.



This becomes clear in the three subsequent plots where we show each portfolio's behavior over time, for which different optimization frameworks exhibit distinct tendencies. Notably, we see high variability in the positions of the MVP and MSRP in the risk-return space, illustrating how inaccuracies in risk and return predictions can lead to suboptimal allocations. The MDCP also exhibits significant variability, highlighting the challenges of predicting correlation dynamics and the limitations of this method in adjusting to changing relationships.

In contrast, alternative strategies display consistency and robustness throughout the backtest period. The EWP maintains a stable position in the risk-return space due to its straightforward, prediction-independent methodology, offering reliable performance without dependency on forecasts. The IVP, RPP, and MDP show slight variations in their positions, driven by minor fluctuations in the volatility profiles of the dataset's stocks, suggesting that these approaches can offer reliable and consistent options for risk-averse investors in such scenarios. Finally, the GMVP is consistently located at the leftmost end of the plot, highlighting its effectiveness in minimizing portfolio variance throughout the sample, further underscoring the stability of volatility in this dataset.

In conclusion, while many of these strategies are not positioned as efficient on the ex-post efficient frontier, their behavior over time suggests they may provide greater consistency and reliability in real-world applications. The observed movements of these portfolios across the backtest period illustrate that alternative strategies, though not always optimal in a mean-variance framework, can be much more effective in achieving their unique objectives than traditional Markowitz approaches, especially in uncertain and dynamic market conditions.

Appendix

Tables

Table 1: Summary Statistics (daily prices)

	Annualized Return	Annualized Std Dev	Annualized Sharpe (Rf=0%)
SPY	0.1126	0.179	0.6294
GOOGL	0.1658	0.2854	0.5809
NFLX	0.1381	0.4421	0.3123
AAPL	0.1926	0.2873	0.6703
AMZN	0.1981	0.3251	0.6092
META	0.1428	0.3849	0.3709
GLD	0.0774	0.1398	0.5537
NVDA	0.6036	0.4891	1.2342
IBM	0.0403	0.2405	0.1675
TXN	0.1349	0.2844	0.4745
ASML	0.1421	0.3713	0.3827
DECK	0.2058	0.3994	0.5154
V	0.137	0.2464	0.5558
MA	0.1611	0.27	0.5967
SNPS	0.2284	0.2995	0.7626
JPM	0.127	0.2739	0.4636
AVGO	0.2691	0.363	0.7413
AMAT	0.1774	0.4073	0.4356
TLT	-0.0159	0.1525	-0.104
NVO	0.1342	0.2748	0.4884
LLY	0.2651	0.2785	0.9517
AXON	0.1755	0.465	0.3774
XYL	0.1055	0.2699	0.391
GBTC	0.1222	0.8581	0.1425
NTES	0.0433	0.4291	0.101
NEE	0.1212	0.2456	0.4933
JNJ	0.0614	0.1818	0.3377
PG	0.0908	0.1861	0.4877
LMT	0.1189	0.2252	0.528
TSLA	0.1329	0.5732	0.2319
LRCX	0.2562	0.4251	0.6026

Table 2: Summary Statistics (weekly prices)

	Annualized Return	Annualized Std Dev	Annualized Sharpe (Rf=0%)
SPY	0.1126	0.1709	0.6588
GOOGL	0.1689	0.274	0.6164
NFLX	0.1356	0.4345	0.312
AAPL	0.1935	0.2736	0.7072
AMZN	0.2087	0.2982	0.6999
META	0.153	0.353	0.4335
GLD	0.0734	0.1389	0.5282
NVDA	0.6222	0.4513	1.3788
IBM	0.0385	0.2409	0.16
TXN	0.144	0.2435	0.5915
ASML	0.1449	0.3575	0.4053
DECK	0.213	0.3917	0.5439
V	0.1424	0.2182	0.6529
MA	0.165	0.2486	0.6637
SNPS	0.2319	0.2834	0.8184
JPM	0.1279	0.2656	0.4815
AVGO	0.27	0.3476	0.7769
AMAT	0.1807	0.3936	0.4592
TLT	-0.0162	0.1422	-0.1136
NVO	0.1339	0.2693	0.4972
LLY	0.2665	0.27	0.9871
AXON	0.172	0.4725	0.3639
XYL	0.1051	0.2682	0.3918
GBTC	0.1772	0.8342	0.2124
NTES	0.0452	0.4011	0.1126
NEE	0.1179	0.2445	0.4823
JNJ	0.0618	0.171	0.3611
PG	0.0916	0.174	0.5264
LMT	0.1168	0.2275	0.5135
TSLA	0.1266	0.5764	0.2196
LRCX	0.2515	0.4269	0.5891

Table 3: Summary Statistics (monthly prices)

	Annualized Return	Annualized Std Dev	Annualized Sharpe (Rf=0%)
SPY	0.116	0.1547	0.7501
GOOGL	0.1786	0.2389	0.7474
NFLX	0.1182	0.4229	0.2795
AAPL	0.1905	0.2771	0.6873
AMZN	0.2057	0.3024	0.6805
META	0.152	0.3494	0.4351
GLD	0.0768	0.138	0.5569
NVDA	0.5914	0.4619	1.2802
IBM	0.042	0.2355	0.1782
TXN	0.1488	0.2214	0.672
ASML	0.1628	0.3066	0.531
DECK	0.2545	0.3134	0.812
V	0.1482	0.1996	0.7426
MA	0.1719	0.2276	0.7551
SNPS	0.2384	0.251	0.9498
JPM	0.1343	0.2418	0.5555
AVGO	0.2783	0.2707	1.0282
AMAT	0.2015	0.3473	0.5802
TLT	-0.0163	0.137	-0.1189
NVO	0.1468	0.2239	0.6557
LLY	0.2612	0.2505	1.0426
AXON	0.195	0.4413	0.4419
XYL	0.1128	0.2422	0.4657
GBTC	0.1365	0.9329	0.1463
NTES	0.0559	0.3704	0.1509
NEE	0.1248	0.2131	0.5857
JNJ	0.0657	0.1577	0.4166
PG	0.0982	0.1582	0.6205
LMT	0.1228	0.2097	0.5857
TSLA	0.131	0.576	0.2274
LRCX	0.2843	0.3479	0.8171

Table 4: Performance Table (median values)

	Sharpe ratio	max drawdown	annual return	annual volatility	Sortino ratio	downside deviation	Sterling ratio	Omega ratio	VaR (0.95)	CVaR (0.95)
IVP	1.38	11.0%	19.0%	14.0%	1.95	0.10	1.51	1.28	1.0%	2.0%
RPP	1.36	10.0%	17.0%	13.0%	1.94	0.09	1.55	1.27	1.0%	2.0%
EWP	1.35	13.0%	23.0%	18.0%	1.93	0.12	1.60	1.26	2.0%	2.0%
MDCP	1.31	13.0%	19.0%	18.0%	1.85	0.12	1.56	1.25	2.0%	2.0%
MDP	1.20	10.0%	15.0%	13.0%	1.79	0.09	1.36	1.23	1.0%	2.0%
MSRP	1.16	16.0%	19.0%	20.0%	1.66	0.14	1.49	1.23	2.0%	3.0%
GMVP	0.92	10.0%	11.0%	11.0%	1.29	0.08	1.06	1.17	1.0%	1.0%
index	**0.90**	**13.0%**	**13.0%**	**15.0%**	**1.24**	**0.11**	**1.10**	**1.18**	**2.0%**	**2.0%**
MVP	0.88	32.0%	31.0%	38.0%	1.25	0.25	1.19	1.17	3.0%	5.0%