Overview

In this work, we will analyze the performance of portfolios created upon a given signal, in this case, the Highest 5 Days of Return – rmax5_21d. As its name mentions, it is constructed in a rather simple way. Each month, the highest 5 days of return are selected, and an average is computed, giving us our signal.

To explain the economic motivation for why this may or may not be a useful predictor of security returns, let us take a look at T. I. Jensen, B. Kelly, and L. H. Pedersen (2021)[1]. In this paper, the authors observe at a set of economic factors, which they group into clusters based on hierarchical agglomerative clustering (Murtagh and Legendre, 2014)[2], and compute the correlations between the factors based on the CAPM-residual returns of the factors. This algorithm assigned our signal – Highest 5 Days of Return – to a theme cluster called Low Risk.

To find out which factors are the most impactful anomalies in economic terms, the authors looked at which factors matter most from an investment performance standpoint. We see that the Highest 5 Days of Return has a factor alfa of around 0.5% per month, and is highly significant. This could contribute to increase this signal's usefulness in working as a predictor of security returns.

As the CAPM alpha does not control for duplicate behavior other than through the market factor, the authors tried to understand how clusters contribute to their alpha while controlling for all other clusters. Thus, they estimated cluster weights in a tangency portfolio that invests jointly in all cluster-level portfolios. They also tested the significance of the estimated weights. A high weight in the tangency portfolio means that the cluster matters for an investor, even when controlling for all other factors. Looking at the results, however, the authors show that when controlling for all other clusters, our signal's theme cluster – Low Risk – does seem to lose importance, falling among the bottom 6 themes. If we were to discriminate between regions, however, we see that Low-Risk is among the top weights in the tangency portfolio for the USA, while being amongst the bottom in both the rest of the Developed Regions as well as in Emerging Markets, both with weights around 0%. Doing this discrimination through stock sizes, we see that Low Risk has one of the largest weights in Mega stocks, it loses a bit of weight for Large stocks, and becomes rather irrelevant for both Small, Micro and Nano stocks.

To conclude, we believe that, based on economic motivation, our signal – Highest 5 Days of Return – could work as a somewhat fine predictor of security returns. The paper analyzed exposed some interesting statistics about this signal, which contribute for its usefulness as a predictor. However, when controlling for duplicate behavior, they exposed some limitations on our signal. Thus,

[1] T. I. Jensen, B. Kelly, and L. H. Pedersen, *Is There a Replication Crisis in Finance?* (2021)
[2] Murtagh, F. and P. Legendre, *Ward's hierarchical agglomerative clustering method: which algorithms implement ward's criterion?* (2014) Journal of classification 31 (3), 274–295.

I reckon it could work as a useful predictor, but only under the right circumstances, such as when analyzing Mega or Large stocks in the USA, for example.

Strategy Analysis

Let us start by analyzing the performance of strategies built upon the signal above as stand-alone strategies. In this section, we will create terciles of our stocks' monthly returns based on their Highest 5 Days of Return (the signal), with the bottom tercile representing the worst expected performance stocks and the top tercile looking to gather the stocks with the best expected performance. Following this process, in an attempt to make the analysis more meaningful, we will both turn these portfolios' returns into excess returns, as well as transform our terciles into value-weighted portfolios, with stocks with larger market cap having a larger weight on each corresponding portfolio. Finally, we will look to build two strategies with what we have in hand. First, a strategy that only goes long on our top tercile – the expected best performer. Secondly, an approach which goes long on the very same portfolio (top tercile), but also goes short on the bottom tercile – the expected worst performer. To complement this analysis, we will also create both similar portfolios with a constant leverage applied to each so that all have an annualized volatility of 10% over the full sample, as well as create a value-weighted stock market portfolio, from the Ken French data, also known as MktRf. In our discussion about the performance of our strategy, we will compare all approaches to assess how well our strategies perform.

To build these strategies and analyze their performance, we start by loading data and defining important functions that let us determine variate things, such as the investment universe in which we can invest in, i.e. dates for which we have data; our signal, in which a "high" signal predicts high returns and a "low" signal predicts low returns, as it is the Highest 5 Days of Return; the market capitalization, which will allow us to scale our portfolios' stocks according to their value; as well as applying some filters to our data. With these important functions built, we move on to creating our value-weighted terciles, which we do by getting our investment universe, applying filters to it, getting the signals for the whole period, dividing the data into three terciles, creating weights according to each stock's market capitalization, getting the returns of these portfolios, accounting for their stocks' weights, and finally taking the risk-free rate from each return to turn these portfolios' returns into excess returns. From here on, whenever we mention returns, keep in mind that they are all **excess** returns.

With our terciles built, we assign a long-only strategy where we go long on our expected top performing portfolio – the top tercile, and also a long-short strategy which goes long on the very same portfolio, but simultaneously goes short on the expected worst performing portfolio. Now that we have our strategies, we can start analyzing each strategies' performance, as well as compare them to the performance of the classic value-weighted stock market portfolio – MktRf.

As can be seen below in Figure 1, neither of our strategies seem to perform well when compared to the value-weighted stock market portfolio, with our long-only strategy and our long-short strategy showing a full sample cumulative return of around -40% and -80%, respectively, while the market portfolio shows a cumulative return of around 230%.

One thing to note on the graph is that the three different strategies have very distinct volatilities, which may harm our perception of which is in fact better according to each investors' tolerance of risk. Thus, to have a better sense for which portfolio is better per unit of volatility, we will apply a constant leverage to each portfolio to make them have an annualized volatility of 10%.

Let us first recall the effect leverage has on a portfolio's volatility and returns by looking at a portfolio composed of stocks **x** and **y**. The variance of such portfolio is, as we know, $\boldsymbol{\sigma^2 = w^2_x\sigma^2_x + w^2_y\sigma^2_y + 2w_xw_ycov(r_x,r_y)}$. If one of these assets, say **y**, is the risk-free asset, both its variance and covariance with any other asset will be zero, thus leaving us with a portfolio standard deviation of $\boldsymbol{\sigma = w_x\sigma_x}$. Similarly, the portfolio return equation, $\boldsymbol{r_p=\Sigma^N_{i=1}w_ir_i}$, turns into $\boldsymbol{r_{levered}-r_f = w_p*(r_{unlevered}-r_f)}$ if we're working with excess returns, which we are.

We can control volatility by taking on leverage (borrowing from the risk-free rate) and investing in each portfolio accordingly in an attempt to reach our preferred volatility. To achieve a 10% annualized volatility, each portfolio must take on a specific constant leverage, which can be obtained by dividing the target volatility by the annualized standard deviation of each of our portfolios. By computing new returns for each portfolio which consider each portfolio's leverage, we come to a new graph, which shows the cumulative returns for each of these adjusted portfolios. Figure 2 allows us to see that, although the value-weighted market portfolio still clearly outperforms the others (around 120% cumulative return), our long-only strategy now yields positive cumulative returns (around 10%), while the long-short strategy still shows a negative performance (around -42%).

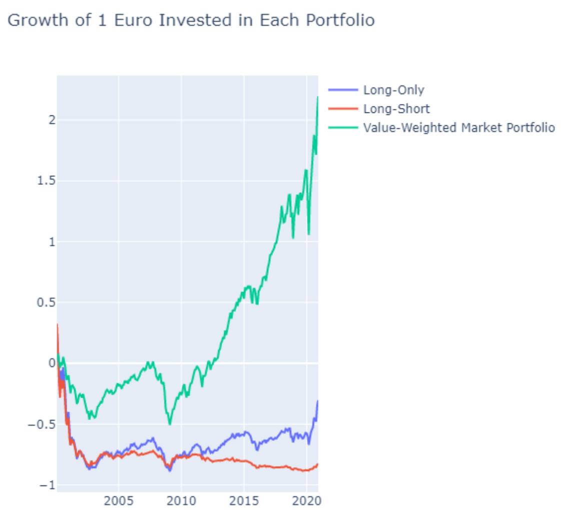

Figure 1 - Growth of 1 Euro Invested in Each Portfolio

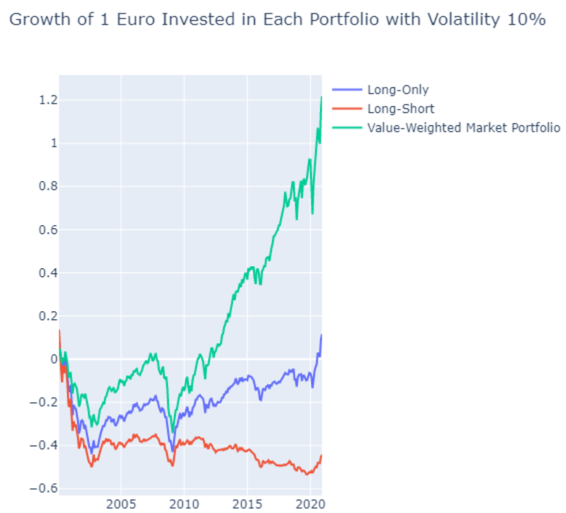

Figure 2 - Growth of 1 Euro Invested in Each Portfolio with Targeted Volatility of 10%

To have a better grasp of the numbers, a performance analysis was conducted. Firstly, we computed the average annualized excess returns and the annualized Sharpe Ratios for each portfolio. Secondly, we took the common approach of estimating each portfolio's return adjusted for risk using factor models with tradable risk factors, such as the CAPM and the Fama-French 3-Factor Model, and finding its alpha – the average return of the portfolio that is not explained by the models' factors, that is, the risk-adjusted return. This adjustment involves running regressions on portfolios returns to estimate both the betas on different risk factors, which constitute the exposure of said portfolio to said risk factor, as well as the alpha. Finally, we computed the Information Ratio, which tells us the risk-adjusted return per unit of volatility of the residuals of the CAPM. Figure 3 allows us to look at our portfolios' performances over the full sample. It is important to note that, on the tables below, Portfolio 1 corresponds to the Long-Only Portfolio, Portfolio 2 to the Long-Short Portfolio, and Portfolio 3 to the Value-Weighted Market Portfolio.

Portfolio	Avg. Ann. Excess Return	Ann. Sharpe Ratio	CAPM Alpha	CAPM Alpha T-Stat	FF3 Alpha	FF3 Alpha T-Stat	Info. Ratio
1	0.03	0.1	-0.01	-2.59	-0.01	-3.61	-0.57
2	-0.05	-0.23	-0.01	-2.83	-0.01	-4.03	-0.62
3	0.07	0.43					

Figure 3 - Full Sample Performance Analysis

The same analysis was conducted for the first half of the sample as a stand-alone investment universe, as well as the second half of the sample. Figures 4 and 5 report the performance of our three strategies over both of these periods, respectively.

Portfolio	Avg. Ann. Excess Return	Ann. Sharpe Ratio	CAPM Alpha	CAPM Alpha T-Stat	FF3 Alpha	FF3 Alpha T-Stat	Info. Ratio
1	-0.06	-0.16	0	-0.72	0	-0.92	-0.22
2	-0.09	-0.29	-0.01	-1.09	-0.01	-1.38	-0.34
3	-0.01	-0.06					

Figure 4 - 1st Half of Sample Performance Analysis

Portfolio	Avg. Ann. Excess Return	Ann. Sharpe Ratio	CAPM Alpha	CAPM Alpha T-Stat	FF3 Alpha	FF3 Alpha T-Stat	Info. Ratio
1	0.12	0.54	-0.01	-2.63	-0.01	-2.91	-0.85
2	-0.02	-0.14	-0.01	-2.62	-0.01	-2.91	-0.85
3	0.15	1.02					

Figure 5 - 2nd Half of Sample Performance Analysis

From the above tables, we see that the alphas for all three strategies for both models are statistically significant for a 95% confidence level for both the full sample and the second half of the sample, as their t-statistic values all fall onto the rejection zone, i.e. all are lower than -1.96. As pointed out earlier, this is in concordance with the paper discussed. For the first half of the sample, however, we see that the alphas for both models are not statistically significant (all t-statistics are between -1.96 and 1.96). These alphas show us that both our strategies, long-only and long-short, don't seem to have large risk-adjusted returns, but rather slightly negative, which although different from the literature, this difference could be explained by several factors such as investment periods and investment universes. Additionally, we see that indeed the value-weighted market portfolio outperforms our two strategies in all periods, presenting not only larger average annual excess returns, but also higher annualized Sharpe Ratios.

Strategy as part of a diversified portfolio

To try to identify ways to improve our strategies' poor performances, we will, in the following analysis, incorporate each of our strategies into their own tangency portfolio, that is, the portfolio that is fully invested in risky assets that has the maximum achievable Sharpe Ratio. In doing so, we will combine ETFs with our strategies, finding portfolio weights (between the ETFs and our portfolios) through algebra that solves for the portfolio that has covariances with each asset equal to the asset's risk premium. Finally, we re-lever weights to have a total portfolio weight equal to 1 – this is our tangency portfolio for each strategy.

We begin by finding our desired ETFs' (VBR, VUG, VTI, and BND) excess returns, followed by an estimation of the means and covariances between the ETFs themselves as well as between them and each of our strategies – long-only or long-short. To estimate the means and variances, the full data set was used, while to find correlations and covariances, only mutually overlapping data was used.

To have a better understanding of the performance of our two tangency portfolios, we will compare their behavior to the typical 60/40 Portfolio (60% allocated to VTI and 40% allocated to BND), as well as to the well-known Equal-Weight Portfolio (distributed between the four ETFs). The results are shown in Figure 6, which shows us that, although still underperforming when compared to both the 60/40 and the equal-weight strategies, all portfolios now show a cumulative return greater than 200%.

Similarly, to have a better understanding of the performance of each portfolio per unit of volatility, we apply constant leverages to them so that they all present an annualized volatility of 10%. Adjusting portfolios' returns by the same method as above (weighting returns according to the respective leverages), we get to Figure 7, which shows us that, by targeting a 10% annualized volatility, we increase the performance of both our tangency portfolios in relation to the other two portfolios. This time, both our tangency portfolios show cumulative returns of around 500%, while the 60/40 and equal-weight strategies now yield a bit above 200% cumulative returns.

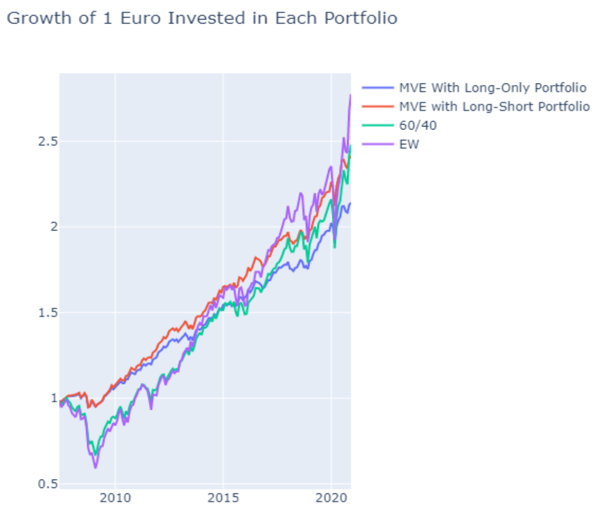

Figure 6 - Growth of 1 Euro Invested in Each Portfolio

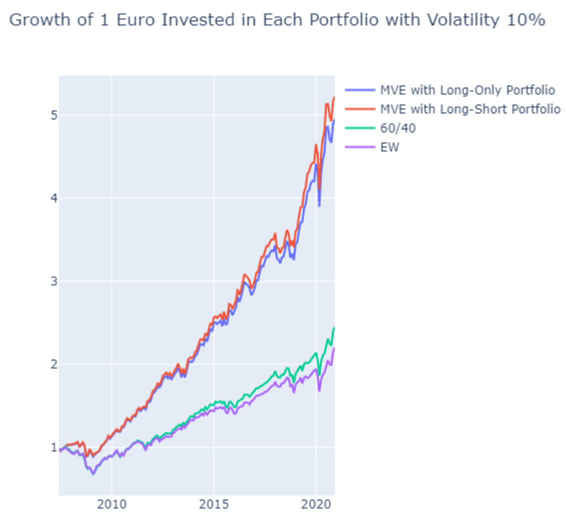

Figure 7 - Growth of 1 Euro Invested in Each Portfolio with Volatility 10%

Applying the same methodology as before, we conducted a performance analysis on these four strategies to have a better understanding of the actual numbers. The following tables report these statistics, although this time we are only evaluating the average annualized excess returns and the annualized Sharpe Ratios over the full sample period. It is important to note once more that, on the tables below, Portfolios 1, 2, 3, and 4 correspond to the MVE with Long-Only Portfolio, the MVE with Long-Short Portfolio**,** the 60/40 Portfolio, and the EW Portfolio, respectively.

Portfolio	Avg. Ann. Excess Return	Ann. Sharpe Ratio
1	0.06	1.23
2	0.07	1.27
3	0.07	0.71
4	0.08	0.63

Figure 8 - Performance Analysis of the Four Portfolios

Portfolio	Avg. Ann. Excess Return	Ann. Sharpe Ratio
1	0.12	1.23
2	0.13	1.27
3	0.07	0.71
4	0.06	0.63

Figure 9 - Performance Analysis of the Four Portfolios with Targeted Volatility 10%

As can be seen, the Sharpe Ratios of our tangency portfolios already hinted that the potential for these strategies was large, as the smaller average annualized excess returns suggest a smaller volatility, producing the larger Sharpe Ratios. Indeed, when controlling for volatility, our tangency portfolios finally outperform the comparison portfolios, showing our strategies are certainly worth including in a diversified portfolio, at least over the short time period analyzed.