Post-Graduation in Data Science for Finance

Fixed Income Securities

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Individual Project

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a) Analysis of the Correlation Matrix of Spot Rates

The correlation matrix of spot rates reveals a generally high degree of correlation across maturities, suggesting that interest rate movements are largely driven by common underlying factors. However, these correlations are not perfect, indicating that additional forces beyond a single common factor contribute to the evolution of the yield curve.

Maturity	1Y	2Y	3Y	4Y	5Y
1Y	1	0.986	0.960	0.936	0.913
2Y		1	0.993	0.980	0.966
3Y			1	0.997	0.989
4Y				1	0.998
5Y					1

Table 1 – Spot Rates Correlation Matrix

The imperfect correlations challenge the core assumptions of single-factor term structure models, which typically assume that interest rate movements are driven by a single source of risk, usually shifts in the level of rates. In reality, the presence of non-perfect correlations implies that changes in the slope and curvature of the yield curve also play a significant role in explaining interest rate dynamics. This finding underscores the need for a more nuanced approach, such as Principal Component Analysis (PCA), to extract the key drivers of yield curve shifts.

Further, the correlation matrix highlights an important pattern: correlations tend to be highest among nearby maturities and decline as maturities become more distant. This suggests that short-term and long-term interest rates are not perfectly synchronized, reinforcing the idea that multiple factors - such as shifts in the level, slope, and curvature - are necessary to capture the full range of yield curve movements.

b) Principal Component Analysis and Explained Variance

Principal Component Analysis (PCA) is a statistical technique used to transform correlated variables into a smaller set of uncorrelated components, known as principal components. Each principal component captures a specific pattern of variation in the data, with the first component explaining the highest possible variance, followed by subsequent components capturing progressively smaller proportions. In the context of fixed income, PCA is particularly useful for identifying the key factors that drive changes in the yield curve.

By performing PCA on the covariance matrix of spot rates, we find that the first principal component is overwhelmingly dominant, explaining 92.8% of the total variance in spot rates. This confirms that the majority of interest rate movements can be attributed to this common factor.

PC1	PC2	PC3	PC4	PC5
0.982	0.065	0.007	0.000	0.000

Table 2 – Principal Components Proportion of Variance Explained

The second and third principal components account for 6.5% and 0.7% of the variance, respectively. These components capture additional variations in the shape of the yield curve and, as expected, the first three components together effectively describe the entire dynamics of the yield curve, while the fourth and fifth components contribute negligibly to the total variance, reinforcing their lack of significance in explaining interest rate movements.

These results are clearly illustrated in the Proportion of Variance Explained (PVE) and Cumulative PVE plots below, which demonstrate how the first three components essentially capture all the meaningful variation in spot rates.

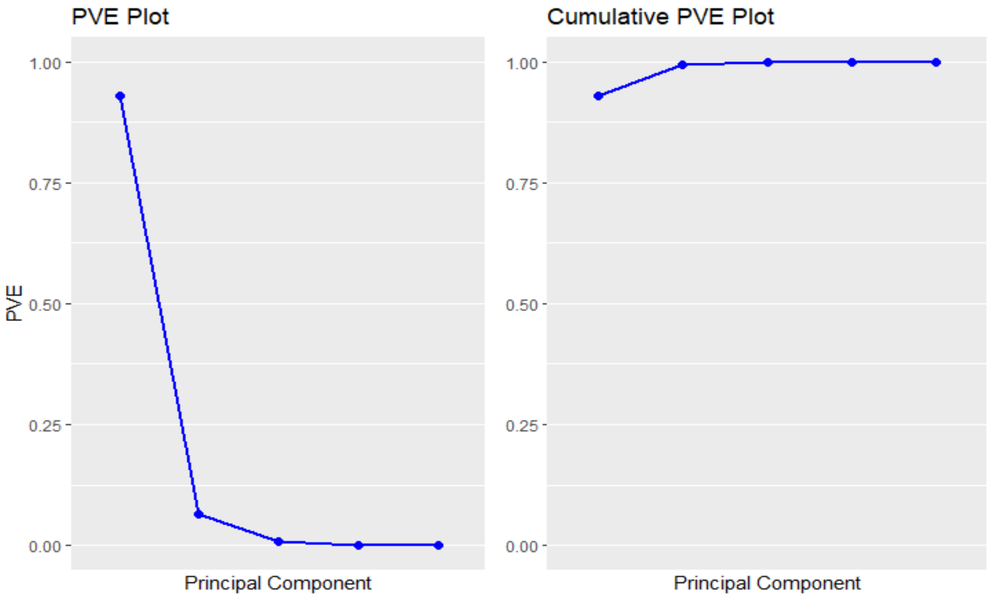

Figure 1 – Proportion of Variance Explained (Simple and Cumulative)

c) Factor Loadings

Figure 3 displays the factor loadings for the five principal components as a function of interest rate maturities. As previously discussed, we will focus on the first three components, as they effectively explain the full dynamics of the yield curve.

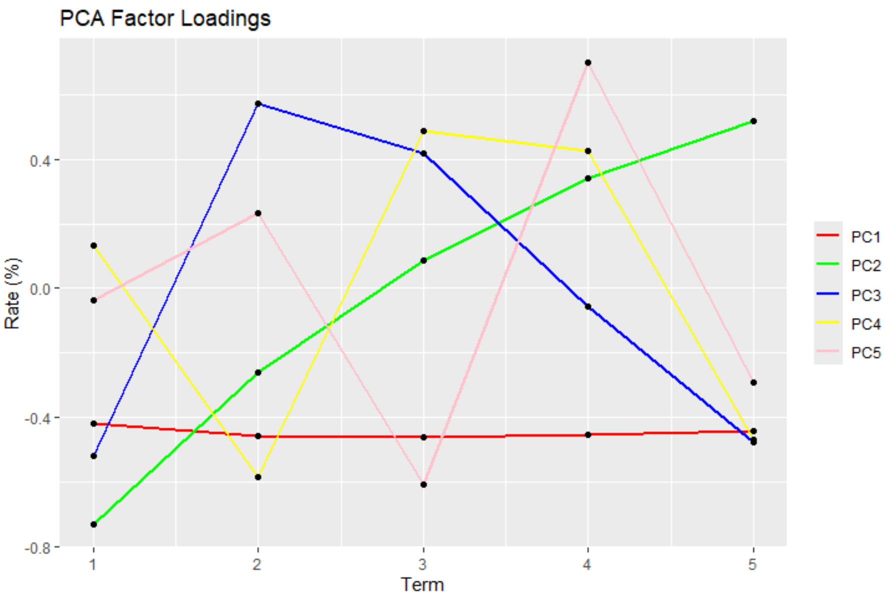

Figure 2 – Factor Loadings

An analysis of the plot reveals that shocks to the first principal component cause relatively uniform changes in yields across all maturities, indicating that this factor primarily represents parallel shifts in the yield curve. Given its broad and consistent impact, this component is commonly referred to as the level factor, as it captures fluctuations in the overall interest rate environment.

The second principal component exhibits an ascending shape across maturities, with shocks to this factor leading to a decrease in short-term rates and an increase in long-term rates. Notably, the effect is close to zero around the three-year maturity, serving as the rotation axis for the yield curve steepening. This pattern aligns with changes in the slope of the yield curve, meaning this component can be interpreted as the slope (in this case steepening) factor.

Lastly, the third principal component influences intermediate and extreme maturities differently. The plot shows that a shock to this factor results in lower yields for both short- and long-term maturities while increasing yields at intermediate terms (particularly for the second and third maturity points). This pattern effectively increases the concavity of the term structure, making this factor representative of the curvature factor.

Together, these three factors - level, slope, and curvature - form the fundamental drivers of interest rate movements, providing a more comprehensive understanding of yield curve dynamics beyond simple parallel shifts.

d) Principal Component Scores Over Time

Figure 4 displays the time series of the three most dominant factors extracted from the spot rate covariance matrix. These principal component scores represent the magnitudes of the factor-driven shocks affecting the yield curve over time.

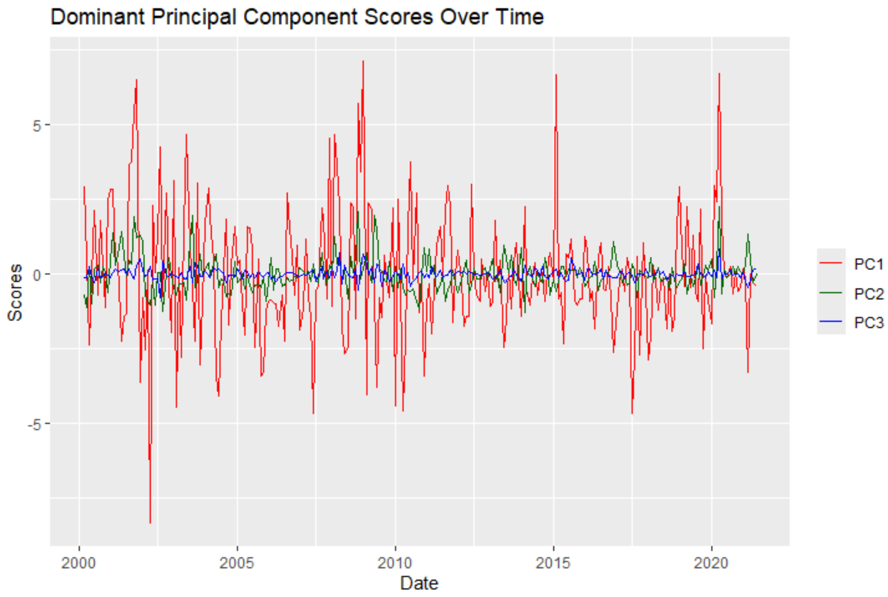

Figure 3 – Dominant Components Over Time

An interesting observation is that their behavior closely resembles that of financial returns. Notably, the first factor exhibits the highest variance, while the third factor shows the least. This is expected because the first principal component captures the most substantial and systematic movements in interest rates, which tend to be more volatile. In contrast, the third component explains only a minor fraction of the total variance, leading to relatively smaller fluctuations over time.

The resemblance of factor scores to financial returns arises from both their constructions. First, the principal components are derived from historical changes in spot rates, rather than their absolute levels. As a result, factor scores reflect unexpected movements in the term structure rather than its actual shape. This is analogous to how financial returns measure percentage changes in asset prices rather than price levels themselves.

Additionally, just as asset returns respond to economic news and policy changes, yield curve movements are also driven by macroeconomic factors such as inflation expectations, central bank actions, and risk sentiment. These shocks influence the factor scores similarly to how news impacts stock returns, further reinforcing the resemblance.

Moreover, the first factor largely reflects systemic interest rate changes, which are comparable to broad market returns, while the second and third factors correspond to localized shifts in slope and curvature, much like sector-specific or idiosyncratic returns in asset pricing. Since these effects are smaller and more transient, their variance is lower, similar to how firm-specific stock returns exhibit lower volatility when compared to market-wide shocks.

In summary, the principal component scores over time resemble returns because they are essentially a transformed view of the same data, capturing the essence of how rates and returns are influenced by underlying economic factors.

e) Simulating Hedging a Portfolio Using PCA

To simulate hedging a bond portfolio using Principal Component Analysis, let us first set some assumptions.

1. The target portfolio is given by the following parameters:

   Face Value = $1.000; **Coupon Rate** = 5%; **Investment Horizon (H)** = 3 Years;

   Compounding = Continuous.

2. The hedging assets considered are assumed to have annual coupons, face value $100, continuous compounding, and have the following characteristics:

Instruments	**Coupon**	**Maturity**
H2	5.0%	2 Years
H3	4.5%	3 Years
H4	4.0%	4 Years

3. At the time of hedging, the yield curve is well-represented by the following Nelson-Siegel-Svensson parameters:

β0	β1	β2	β3	τ1	τ2
5.90%	-1.60%	-0.50%	1%	5	0.5

In this analysis, we follow a methodology explored by the Bocconi Students Investment Club[1]. This approach involves hedging our portfolio's exposure to the most significant factors driving yield curve movements. Specifically, we conduct three hedging experiments:

1. Hedging against the first principal component (level factor) only;
2. Hedging against the first two principal components (level and slope factors);
3. Hedging against all three principal components (level, slope, and curvature factors).

For each case, the number of hedging instruments used matches the number of principal components being hedged. This ensures that the hedging strategy neutralizes exposure to the selected yield curve factors while keeping the implementation straightforward and allowing for matrix operations.

First Experiment: Hedging the Level Factor

To hedge against the first principal component, we selected H2 as the hedging instrument. The problem was defined as follows:

$$[e_{1,2} * D_{H2}] * [w_{H2}] = [e_{1,3} * -D_P]$$

Where $e_{x,y}$ represents the factor loading associated with the principal component x and the y-years tenor, D_x is the duration of asset x, and w_x is the weight of asset x.

The optimal solution required short-selling 1.556 units of H2 per $100 of portfolio value, resulting in a total short position of 15.56 units of H2 for our portfolio with a face value of $1000.

[1] https://bsic.it/fixed-income-trading-unlocking-risk-reduction-with-ols-and-pca-hedging/

Second Experiment: Hedging the Level and Slope Factors

Next, we extended the hedge to account for the first two principal components. For this, we selected H2 and H3 as hedging instruments. The problem was defined as follows:

$$\begin{bmatrix} e_{1,2} * D_{H2} & e_{1,3} * D_{H3} \\ e_{2,2} * D_{H2} & e_{2,3} * D_{H3} \end{bmatrix} \begin{bmatrix} w_{H2} \\ w_{H3} \end{bmatrix} = \begin{bmatrix} e_{1,3} * -D_P \\ e_{2,3} * -D_P \end{bmatrix}$$

The optimal solution resulted in no position in H2, while short-selling 1.045 units of H3 per $100 of portfolio value, leading to a total short position of 10.45 units of H3.

Third Experiment: Hedging the Level, Slope, and Curvature Factors

In the final experiment, we hedged against all three principal components by incorporating H2, H3, and H4 as hedging instruments. The problem was defined as follows:

$$\begin{bmatrix} e_{1,2} * D_{H2} & e_{1,3} * D_{H3} & e_{2,4} * D_{H4} \\ e_{2,2} * D_{H2} & e_{2,3} * D_{H3} & e_{2,4} * D_{H4} \\ e_{3,2} * D_{H2} & e_{3,3} * D_{H3} & e_{3,4} * D_{H4} \end{bmatrix} \begin{bmatrix} w_{H2} \\ w_{H3} \\ w_3 \end{bmatrix} = \begin{bmatrix} e_{1,3} * -D_P \\ e_{2,3} * -D_P \\ e_{3,3} * -D_P \end{bmatrix}$$

The resulting solution maintained the same weights for H2 and H3 as in the second experiment while requiring an additional minimal short position in H4 to fully neutralize the portfolio's exposure to all three factors.

Performance Comparison After a Yield Curve Shift

Now, let us assume that the yield curve undergoes a shift and is now characterized by the following parameters:

β0	β1	β2	β3	τ1	τ2
6.5%	-1.0%	0.1%	2%	5	0.5

Comparing the three hedging strategies, the first, which only hedged against the level factor, generated the largest profit of 0.97%, while the second and third strategies, which also accounted for slope and curvature changes, returned a lower but stable 0.56% in global portfolio value.

This outcome is expected, as in this specific case, the largest shock was to the level parameter. This meant that the additional hedging costs incurred in the second and third strategies were not compensated by significant improvements in protection. However, while a partial hedge may be optimal in this scenario, a more comprehensive PCA-based hedge provides better stability, particularly in situations where the yield curve experiences more pronounced slope or curvature changes rather than purely parallel shifts.