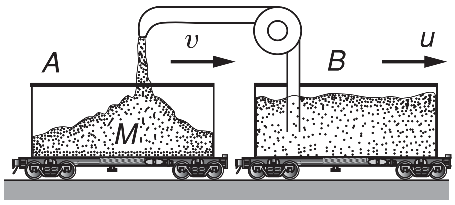
Physics 24A – Problem Set 5

Name ______________________________

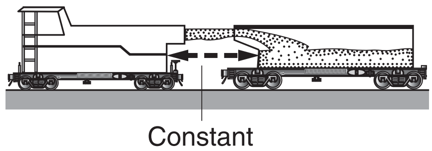
Due: Monday, 19 February 2024

For full credit, include words explaining your reasoning, diagrams, as well as calculations. You must provide the full problem statement with your solution.

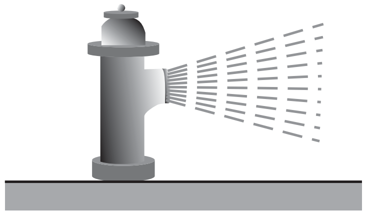
Problem 1 – Two carts and sand (KK 4.12) Material is blown into cart A from cart B at a rate b (kilograms per second), as shown. The material leaves the chute vertically downward, so that it has the same horizontal velocity u as cart B. At the moment of interest, cart A has mass M and velocity v. Find dv/dt, the instantaneous acceleration of A.



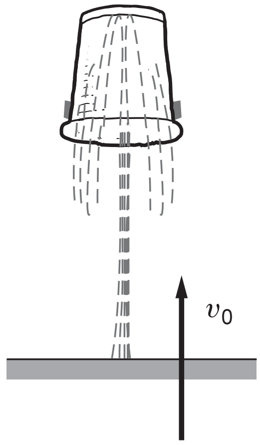
Problem 2 – Sand sprayer (KK 4.13) A sand-spraying locomotive sprays sand horizontally into a freight car as shown in the sketch. The locomotive and freight car are not attached. The engineer in the locomotive maintains his speed so that the distance to the freight car is constant. The sand is transferred at a rate $dm/dt = 10\,\mathrm{kg/s}$ with a velocity $u = 5\,\mathrm{m/s}$ relative to the locomotive. The freight car starts from rest with an initial mass of $m_0 = 2000\,\mathrm{kg}$. Find its speed after $100\,\mathrm{s}$.



Problem 3 – Fire hydrant (KK 4.22) Water shoots out of a fire hydrant having nozzle diameter D, with nozzle speed V_0. What is the reaction force on the hydrant? Assume that the water makes a 90° turn to get out of the hydrant, and take the density of water to be ρ.



Problem 4 – Suspended garbage can* (KK 4.23) An inverted garbage can of mass M is suspended in air by water from a geyser. The water shoots up from the ground with a speed v_0, at a constant rate K (kg/s). The problem is to find the maximum height at which the garbage can rides. Assume that the water bounces elastically off the bottom of the can and neglect any interactions between the falling water and the can. *Note:* The hint at the back of the book is incorrect. It should read "if $\left(\frac{Mg}{2K}\right)^2 = \frac{v_0^2}{2}$, $h = \frac{v_0^2}{4g}$."



Problem 5 – Growing raindrop (KK 4.24) A raindrop of initial mass M_0 starts falling from rest under the influence of gravity. Assume that the drop gains mass from the cloud at a rate proportional to the product of its instantaneous mass and its instantaneous velocity:

$$\frac{dM}{dt} = kMV,$$

where k is a constant. Show that the speed of the drop eventually becomes effectively constant, and give an expression for the terminal speed. Neglect air resistance.

Problem 6 – Rocket in interstellar cloud (KK 4.26) A cylindrical rocket of diameter $2R$ and mass M is coasting through empty space with speed v_0 when it encounters an interstellar cloud. The number density of particles in the cloud is n (particles per unit volume). Each particle has mass $m \ll M$, and they are initially at rest.

(a) Assume that each cloud particle bounces off the rocket elastically, and that the collisions are so frequent they can be treated as continuous. Prove that the retarding force has the form bv^2, where v is the instantaneous rocket speed, and determine constant b. Assume that the front cone of the rocket subtends angle $\alpha = \pi/2$, as shown. *Hint*: Start by explaining why the front cone of the rocket sweeps out a volume $dV = \pi R^2 v dt$ in time dt.

(b) Find the speed of the rocket in the cloud, $v(t)$.