Physics 24A – Lab 2 Thursday, 25 January 2024

Solving Differential Equations by Separation of Variables. Making Equations Dimensionless.

Solo Problem 1 – Boat in a Stream

A boat is floating in a stream where water flow speed is increasing as it descends downhill according to equation

$$v(x) = \alpha\sqrt{x - x_0}$$

where x is the boat position and α and x_0 are constants with appropriate units. Find the position of the boat as a function of time if it starts at $x(0) = 0$.

Group Problem 1 – Falling Droplet

A small droplet of mass m, falling through air under the influence of gravity, experiences a drag force proportional to its velocity. The equation of motion for this droplet is therefore

$$F = m\frac{dv}{dt} = mg - Cv,$$

where C is a constant that depends on the properties of air and the size of the droplet. Assuming that it starts at rest at $t = 0$, find the velocity of the droplet as a function of time. What is it terminal velocity, as $t \to \infty$?

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Group Problem 2 – Viscous Drag II

A body of mass m moving through air, experiences a drag force proportional to its velocity, i.e. $\vec{F}_v = -C\vec{v}$, where C is a constant that depends on the properties of air and the size of the moving body. If gravity can be neglected and the body starts out with speed v_0, we found that its subsequent speed is given by equation

$$v(t) = v_0 e^{-Ct/m}$$

Define natural physical units for the relevant variables in this system and use them to make this equation dimensionless.

Group Problem 3 – Falling Droplet II

A small droplet of mass m falling through air under the influence of gravity, experiences a drag force proportional to its velocity. The equation of motion for this droplet is

$$F = m\frac{dv}{dt} = mg - Cv,$$

where C is a constant that depends on the properties of air and the size of the droplet. Define natural physical units for the relevant variables in this system and use them to make this differential equation dimensionless.

Solo Problem 2 – Turbulent Drag

A large body of mass m moving through air, experiences a drag force proportional to the square of its speed. The equation of motion for this body is then

$$F = m\frac{dv}{dt} = mg - Bv^2,$$

where B is a constant that depends on the density of air and the size of the body. Define natural physical units for the relevant variables in this system and use them to make this differential equation dimensionless.

If you are feeling ambitious, solve the resulting differential equation. *Hint*: use the fact that

$$\frac{1}{1-x^2} = \frac{1}{2}\left(\frac{1}{1-x} + \frac{1}{1+x}\right)$$