1 Electrons in matals: Drude theory

One import property of metals is that metals condcut electricity. In the chapter, we explore why electrons are mobile is some materials but not in others, given that all materials have electrons in them.

We make the following assumption

1. Electron have a scattering time τ. The probability of scattering within a time interval dt is $dt\ /d\tau$
2. Once scattering occurs, electrons have a momentum of $p = 0$
3. Between scattering events, electrons with charge $-e$ respond to electric field E and magnetic field B.

Consider an electron at time t with momentum $p(t)$, its momentum at time $t + dt$ is:

$$\begin{aligned} p(t + dt) = (\text{Probability that scattering occurs})(\text{Momentum of } 0) + \\ (\text{Probability that scattering does not occur})(\text{Momentum}) \end{aligned} \tag{1}$$

The probability of scattering is $dt\ /\tau$, so the probability that no scattering occur is simply $1 = dt\ /\tau$, and the change in momentum between $[t, t + dt]$ is equal to impulse Fdt. Thus, $p(t + dt)$ is:

$$p(t + dt) = \left(1 - \frac{dt}{\tau}\right)(p(t) + Fdt) + 0 \tag{2}$$

Then:

$$\frac{dp}{dt} = F - \frac{p}{\tau} \tag{3}$$

where F is the Lorentz force acting on electrons.

Solving the differential equation, we get;

$$p(t) = p_0 e^{-\frac{t}{\tau}} \tag{4}$$

2 Electrons in fields

2.1 Electrons in electric field

We start by considering the case where electric field $E \neq 0$ but $B = 0$. The lorentz force $F = qE$. Then,

$$\frac{dp}{dt} = -eE - \frac{p}{\tau} \tag{5}$$

In steady state, $dp/dt = 0$, so:

$$\begin{aligned} eE = -\frac{p}{\tau} \\ p = mv = -e\tau E \end{aligned} \tag{6}$$

Now, if there is a density n of electrons in the metal each with charge $-e$, and they are all moving at velocity v, then the electrical current is given by Conductivity $\sigma = j/E$ is:

$$\sigma = \frac{e^2 \tau n}{m} \tag{7}$$

Note that $\sigma = 1\ /\rho$. However, because ρ is a matrix, the equation does not mean a reciprocal relation, so

$$\sigma_{xx} \neq \frac{1}{\rho_{xx}} \quad \sigma_{xy} \neq \frac{1}{\rho_{xy}} \tag{8}$$

Rather, we use the determinant, giving:

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \tag{9}$$

$$\sigma_{xy} = -\frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \tag{10}$$

2.2 Electrons in electric and magnetic fields

Now, we consider the case when electrons experience both non-zero electric and magnetic fields, with a transport equaiton of:

$$\frac{dp}{dt} = -e(E + v \times B) - \frac{p}{\tau} \tag{11}$$

Setting this to zero in steady state and using $p = mv$ and $j = -newv$, we obtain

$$E = \left(\frac{1}{ne} j \times B + \frac{m}{ne^2\tau} j\right) \tag{12}$$

We define the three by three resistivity matrix ρ, such that components of the matrix are given by:

$$E = \rho j \tag{13}$$

$$\rho_{xx} = \rho_{yy} = \rho_{zz} = \frac{m}{ne^2\tau} \tag{14}$$

Let $B = B\hat{z}$, we have:

$$\rho_{xy} = \rho_{yx} = \frac{B}{ne} \tag{15}$$

and all other components of ρ are zero. The off diagonal terms given by Equation 15 is known as the Hall resistivity,

The hall coefficient is defined as:

$$R_H = \frac{\rho_{yx}}{|B|} = -\frac{1}{ne} \tag{16}$$

3 Thermal transport

The thermal conductivity κ is:

$$\kappa = \frac{1}{3} n c_v \langle v \rangle \lambda \tag{17}$$

where c_v is specific heat capacity per particle, $\langle v \rangle$ is the average thermal velocity, and $\lambda = \rangle v \rangle \tau$ is the scattering length. By equipartition theorem(can also be derived explicitly), we know that for a monoatomic particle, the specific heat capacity c_v is:

$$c_v = \frac{3}{2} k_B \tag{18}$$

and

$$\langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}} \tag{19}$$

Using Equation 17, κ is:

$$\kappa = \frac{4}{\pi} \frac{n \tau k_B^2 T}{m} \tag{20}$$

The ratio of thermal and electrical conductivities is:

$$L = \frac{\kappa}{T \sigma} \tag{21}$$

4 References

- Chpater 3 of Basis of Solid State, Oxford, Steve Simon
- Chapter 1 of Solid State, ashcroft