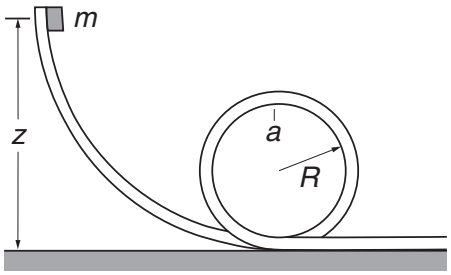
Physics 24A – Problem Set 6

Name _____

Due: Monday, 26 February 2024

For full credit, include words explaining your reasoning, diagrams, as well as calculations. You must provide the full problem statement with your solution.

Problem 1 – Loop-the-loop (KK 5.01, modified) A small block of mass m starts from rest and slides along a frictionless loop-the-loop as shown.

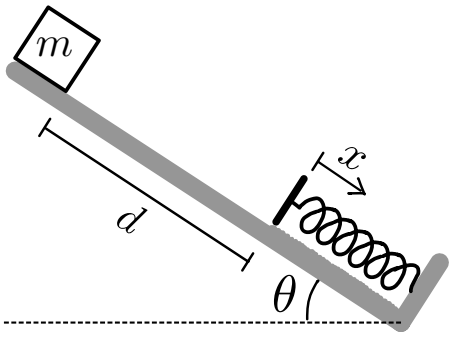


(a) Find the initial height z such that at the top of the track (at point a) m experiences effective gravity equal to g.

(b) What should be the minimal initial height z_0, so that m is able to make it to the top of the track?

(c) What happens to the mass if it starts at $z < z_0$? *Hint:* Be careful, since there are two cases to consider here.

Problem 2 – Incline, spring and friction (KK 5.02, modified) Consider a block sitting on a frictionless inclined plane that makes an angle θ relative to the horizontal. At the bottom of the incline is a massless spring, with spring constant k, aligned parallel to the incline. The end of the spring is a distance d along the incline from the block. The block is released from rest, slides down the incline, and impacts the spring at $x = 0$. At the same time, it begins to experience an increasing frictional force with a coefficient of friction $\mu = bx$, where b is constant.



(a) How long does it take for the block to slide down the incline and contact the end of the spring?

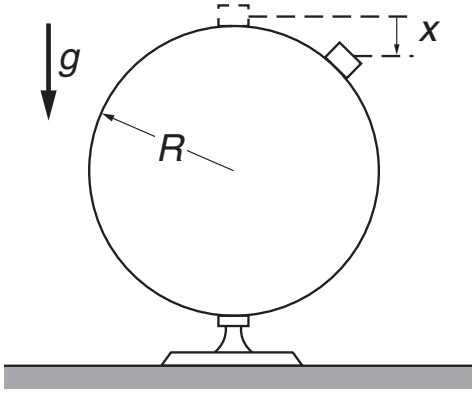
(b) Find the maximum distance that the spring is compressed.

Problem 3 – Work on a whirling mass (KK 5.05) Mass m whirls on a frictionless table, held to circular motion by a string which passes through a hole in the table. The string is pulled so that the radius of the circle changes from r_i to r_f .

(a) Show that the quantity $L = mr^2\dot{\theta} = mr^2\omega$ remains constant. *Hint:* Start with the equation of motion along $\hat{\theta}$ and solve for $\omega(r)$.

(b) Show that the work in pulling the string equals the increase in kinetic energy of the mass.

Problem 4 – Block sliding on a sphere* (KK 5.06, modified) A small block slides from rest from the top of a frictionless sphere of radius R, as shown on the figure.



(a) At what distance below the top, x_0, does it lose contact with the sphere? The sphere does not move.

(b) If the block starts instead with an arbitrary initial horizontal speed v_0, find x at which it will leave the sphere. What is the minimum value of v_0 for which the block leaves the sphere right at the top?