On-The-Fly, Data-Driven Reachability Analysis and Control of Unknown Systems: An F-16 Aircraft Case Study

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ABSTRACT

We describe data-driven algorithms, DaTaReach and DaTaControl, for reachability analysis and control of systems with a priori unknown nonlinear dynamics. The resulting algorithms provide provable performance guarantees while satisfying real-time constraints. To this end, they merge data from a single finite-horizon trajectory and, if available, various forms of side information derived from laws of physics and qualitative properties of the system. Specifically, DaTaReach constructs a differential inclusion that contains the unknown vector field. Then, it over-approximates the reachable set through interval Taylor-based methods applied to systems with dynamics described as differential inclusions. DaTaControl achieves near-optimal and convex-optimization-based control of the system through the computed over-approximations and the receding horizon framework. We empirically demonstrate that DaTaControl outperforms, in terms of optimality of the control and computation time, state-of-the-art control approaches based on system identification and contextual optimization. Finally, using the scenario of an F-16 aircraft diving towards the ground, we show how DaTaControl prevents a ground collision using only the measurements obtained during the dive and elementary laws of physics as side information.

CCS CONCEPTS

• **Computing methodologies → Online learning settings**; **Motion path planning**; *Planning under uncertainty*.

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1 INTRODUCTION

Consider a scenario in which significant and unexpected changes in the dynamics of a system occur. The changes in the dynamics are such that the a priori known model cannot be used, and there is a need to learn the new dynamics on the fly. In such a scenario, the system has access to data from only its current trajectory and needs to retain a certain degree of control. We investigate the problem of data-driven, on-the-fly control of systems with unknown nonlinear dynamics under severely limited data.

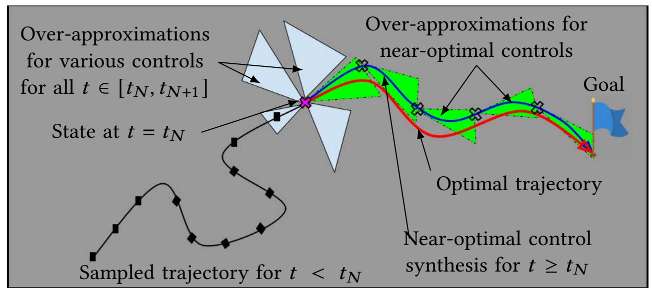

Figure 1: We use limited data and side information for on-the-fly control of systems with unknown dynamics. At each sampling time, we compute an over-approximation of the set of states the system may reach and construct one-step optimal controllers via convex optimization.

In [1], we develop algorithms, DaTaReach and DaTaControl, for the reachability analysis and control of systems with a priori unknown dynamics. DaTaReach exploits the data from a given trajectory of the system to compute an over-approximation of the set of states it may reach. DaTaControl incorporates the computed over-approximation into a constrained optimal control problem, which is then solved on the fly.

Specifically, DaTaReach and DaTaControl can work with data from only a single finite-horizon trajectory of the system and take advantage of various forms of side information on the dynamics. The data include finite samples of the states, the states' derivatives, and the control signals applied. The side information may be a priori knowledge of the regularity of the dynamics, bounds on the vector field locally, monotonicity of the vector field, decoupling in the dynamics among the states, algebraic constraints on the states, or knowledge of parts of the dynamics.

DaTaReach over-approximates the reachable set of the system through a constructed data-driven differential inclusion. It first uses the available data and side information to construct and refine a differential inclusion that contains the unknown vector field. Such refinement process leverages interval-based contractors and the streaming data to prune out from the current differential inclusion, vector fields' values that are guaranteed to be dynamically infeasible. Then, DaTaReach builds on interval Taylor-based methods [4] to over-approximate the reachable set of dynamics described by the differential inclusion. The Taylor expansion can enforce constraints from the side information, and the more data along the trajectory and side information is available, the tighter the over-approximations of the reachable set.

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DaTaControl enables convex-optimization-based, near-optimal control of unknown dynamical systems through DaTaReach. Specifically, we seek to sequentially minimize a given one-step cost function in a discrete-time setting. The one-step cost function, which encodes the system's desired behavior, has to be optimized in a black-box manner since it is typically a function of the unknown state's value at the next time step. DaTaControl computes approximate solutions to this one-step optimal control problem through convex optimization relaxations. The convex relaxations are obtained by replacing the unknown future state with a control-affine linearization of the corresponding over-approximating set. Theoretically, we establish a bound on the suboptimality of the approximate solution with respect to the optimal solution in the case where the dynamics were known. Then, we show that the obtained bound becomes tighter with more data and side information.

Furthermore, DaTaControl is suitable for real-time control of unknown systems as we establish a bound on the number of primitive operations it requires to terminate. Specifically, we show in [1] that DaTaControl can compute approximate solutions to the one-step optimal control problem with a worst-case time complexity that is linear in the length of the trajectory and quadratic in both the dimension of the control input and state.

Data-driven approaches have been proposed for control and reachable set computation of unknown dynamical systems. They either combine system identification and model predictive control [3] or utilize contextual optimization-based approaches [5] to solve the control problem. These approaches require significantly more data than DaTaControl and DaTaReach, and most of them cannot incorporate side information. Further, our initial results in [1] show that the developed algorithms outperform, in terms of computation time and optimality of the control, the data-driven control approaches SINDYc [3] and CGP-LCB [5].

2 DATA-DRIVEN CONTROL OF AN F-16

2.1 F-16 Flight Control System

The F-16 aircraft's flight control system [2] is described by a hierarchical feedback control loop consisting of an autopilot and a low-level controller. The autopilot performs *higher-level* maneuvers, such as ground collision avoidance, waypoint tracking, and more. In contrast, the low-level control tracks the references from the autopilot and maintains stability by actuating the flight control surfaces appropriately. The control system uses a closed-loop feedback control to actuate the flight control surfaces, including the ailerons, elevators, and rudders, in order to meet the desired flight objectives.

Inside the simulator, the underlying nonlinear dynamics, containing 13-states and 4-control inputs, capture the (6-DOF) movement of an aircraft through the standard aerodynamic equations. The dynamics describe the evolution of the system's states, namely velocity v_t, angle of attack α, sideslip β, altitude h, attitude angles: roll ϕ, pitch θ, yaw ψ, and their corresponding rates p, q, r, engine *power* and two more states for translation along north and east. The plant model is built on several *linearly interpolated lookup tables* that incorporate wind tunnel data describing the engine model, the various coefficients including damping, force and moment coefficients, and the moments due to the control surfaces.

2.2 Ground Collision Scenario

To illustrate the applicability of DaTaControl, we present its performance on a ground collision avoidance scenario. We initialize the simulator such that the plane is diving nose down towards the ground. The autopilot uses a PID law to compute the references on the system's states that DaTaControl must track to avoid the crash. The initial condition X_0 used is $\theta = -85\pi/180$, $v_t = 540$, $h = 3600$, $\phi = \pi/4$, $\psi = -\pi/4$, $\beta = 0$, $\alpha = 2.5\pi/180$, $p = q = r = 0$.

To this end, at each time step of the F-16 simulator, DaTaControl controls the system by using only the measurements of the state, the derivative of the states, and the control signals applied from the beginning of the system run until the current time step. Specifically, DaTaControl takes as side information the knowledge of the rigid-body dynamics of a 6-DOF, while assuming that the effect of the aerodynamics forces and moments are completely *unknown*. That is, the interpolated lookup tables and various aerodynamic coefficients obtained using wind tunnel data are unknown. Thus, the system's response to control inputs is unknown and needs to be learned on the fly. Figure 2 shows that DaTaControl outperforms the tuned linear-quadratic regulator (LQR) of the simulator.

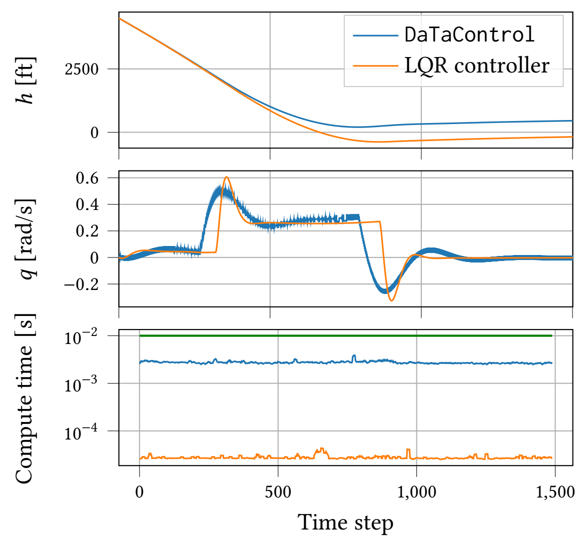

Figure 2: DaTaControl successfully enables the F-16 to avoid the collision with the ground while the embedded, pre-tuned LQR controller fails. Further, DaTaControl can be applied in real time as its compute time is less than the *time-step's value* $\Delta t = 0.01$s enforced by the simulator.

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