$$\min f(x)$$

$$f(x_k + d) = \underbrace{f(x_k) + \nabla f(x_k)^T d + \frac{1}{2} d^T \underbrace{\nabla^2 f(x_k)}_{\substack{n \times n \\ O(n^2) \text{ deriv.}}} d}_{m_k(d)} + o(\|d\|^2)$$

Def.: d é direção de descida para f a partir de x se $\exists\ \bar{\alpha} > 0$ t.q.

$$f(x + \alpha d) < f(x), \quad \forall \alpha \in (0, \bar{\alpha}].$$

Teo.: $d^T \nabla f(x) < 0 \Rightarrow d$ é de descida

Lembrete: $d = -\nabla^2 f(x_k)^{-1} \nabla f(x_k)$ p/ $\nabla^2 f(x_k)$ def. pos., então d é de descida.

Ex.: $d_k = -\nabla f(x_k)$ é de descida p/ x_k ñ crítico

$$d_k^T \nabla f(x_k) = -\|\nabla f(x_k)\|^2 < 0$$

Opção 1: Busca Exata

$$\min_{\alpha} f(x_k + \alpha d_k)$$

Futuro: - min. de 1 var. num intervalo
- Seção áurea

Busca com Armijo

$$f(x_k + \alpha d_k) < f(x_k) + \eta \alpha \nabla f(x_k)^T d_k$$

Variante: - Passo constante

$$\alpha = \hat{\alpha}$$

$$x_{k+1} = x_k + \hat{\alpha} d_k$$

- Passo decrescente

$$\{\alpha_k\}; \quad \alpha_{k+1} \leq \alpha_k \qquad \alpha_k = \frac{1}{k}$$

$$m_k(d) = f(x_k) + \nabla f(x_k)^T d + \frac{1}{2} d^T d$$

$$\nabla m_k(d) = \nabla f(x_k) + d \quad ; \quad \nabla^2 m_k(d) = I.$$

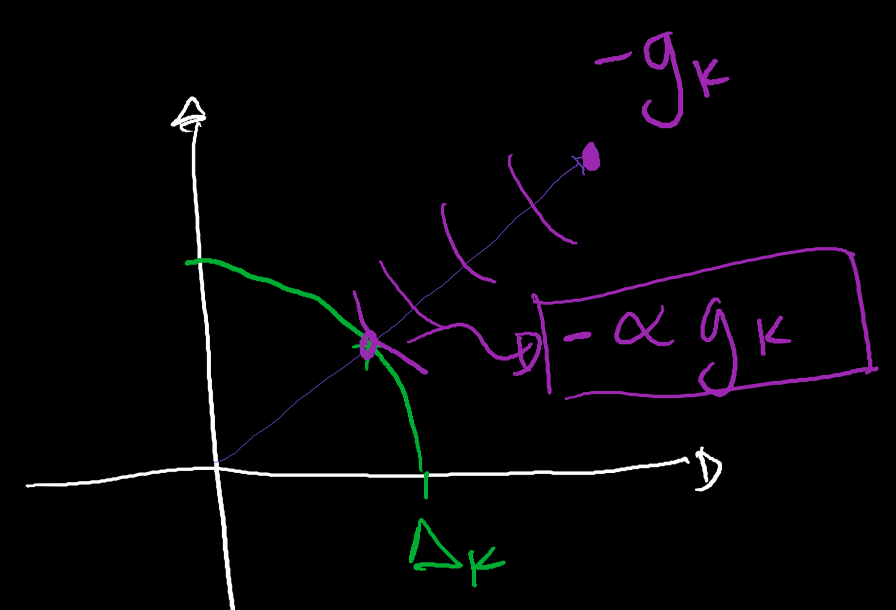
$$\min \{ m_k(d) : \|d\| \leq \Delta \}$$

$g_k = \nabla f(x_k)$; $\quad m_k(d) = f_k + g_k^T d + \frac{1}{2} d^T d$

$f_k = f(x_k)$

$$= \frac{1}{2}(d + g_k)^T (d + g_k) - \frac{1}{2} g_k^T g_k + f_k$$

$$= \frac{1}{2} \|d + g_k\|^2 + cte_k$$



$$\alpha = \begin{cases} \dfrac{\Delta_k}{\|g_k\|}, & \|g_k\| > \Delta_k \\ 1, & \|g_k\| \leq \Delta_k \end{cases}$$