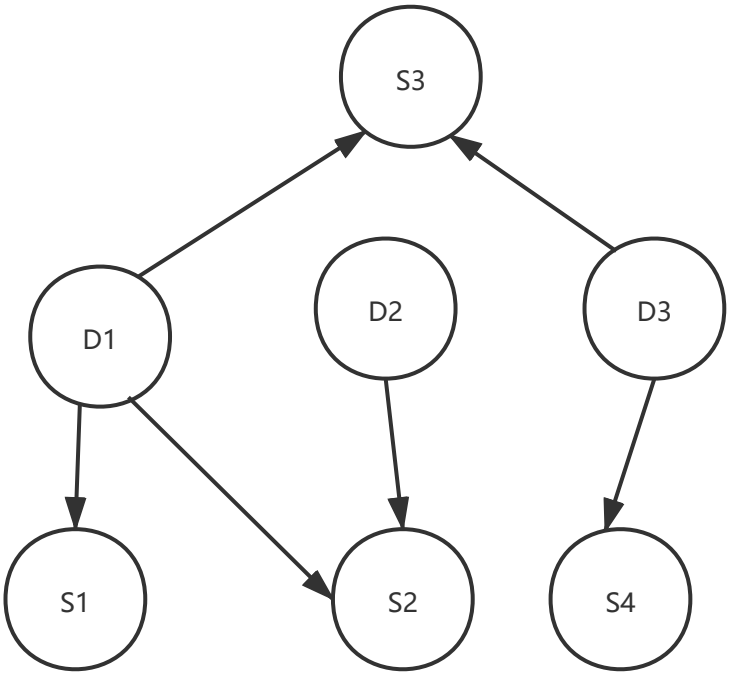
Homework 3

Problem 1

1)



2)

$$p(S1, S2, S3, S4, D1, D2, D3) = p(S1|D1)p(S2|D1, D2)p(S3|D1, D3)p(S4|D3)p(D1)p(D2)p(D3)$$

3)

$$D1, D2$$

4)

$$\begin{aligned} P(D_3 = 1|S_4 = 1) &= \frac{P(S_4 = 1|D_3 = 1)P(D_3 = 1)}{P(S_4 = 1)} \\ &= \frac{P(S_4 = 1|D_3 = 1)P(D_3 = 1)}{P(S_4 = 1|D_3 = 1)P(D3 = 1) + P(S_4 = 1|D_3 = 0)P(D_3 = 0)} \\ &= \frac{0.9 * 0.1}{0.9 * 0.1 + 0.6 * 0.9} \\ &= \frac{1}{7} \end{aligned}$$

Problem 2

$$P(X=0)=\frac{1}{3}$$
$$P(X=1)=\frac{1}{2}$$
$$P(X=2)=\frac{1}{6}$$
$$P(Y=0)=\frac{1}{4}$$
$$P(Y=1)=\frac{5}{12}$$
$$P(Y=2)=\frac{1}{3}$$

$$P(Y=0|X=0)=\frac{P(X=0,Y=0)}{P(X=0)}=\frac{\frac{1}{12}}{\frac{1}{3}}=\frac{1}{4}$$
$$P(Y=0|X=1)=\frac{P(X=1,Y=0)}{P(X=1)}=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$$
$$P(Y=0|X=2)=\frac{P(X=2,Y=0)}{P(X=2)}=\frac{0}{\frac{1}{6}}=0$$
$$P(Y=1|X=0)=\frac{P(X=0,Y=1)}{P(X=0)}=\frac{\frac{1}{6}}{\frac{1}{3}}=\frac{1}{2}$$
$$P(Y=1|X=1)=\frac{P(X=1,Y=1)}{P(X=1)}=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$$
$$P(Y=1|X=2)=\frac{P(X=2,Y=1)}{P(X=2)}=\frac{\frac{1}{12}}{\frac{1}{6}}=\frac{1}{2}$$
$$P(Y=2|X=0)=\frac{P(X=0,Y=2)}{P(X=0)}=\frac{\frac{1}{12}}{\frac{1}{3}}=\frac{1}{4}$$
$$P(Y=2|X=1)=\frac{P(X=1,Y=2)}{P(X=1)}=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$$
$$P(Y=2|X=2)=\frac{P(X=2,Y=2)}{P(X=2)}=\frac{\frac{1}{12}}{\frac{1}{6}}=\frac{1}{2}$$

(a)

$$\begin{aligned}H(X)&=-\frac{1}{3}\log_2\frac{1}{3}-\frac{1}{2}\log_2\frac{1}{2}-\frac{1}{6}\log_2\frac{1}{6}\\&\approx 1.45915\\H(Y)&=-\frac{1}{4}\log_2\frac{1}{4}-\frac{5}{12}\log_2\frac{5}{12}-\frac{1}{3}\log_2\frac{1}{3}\\&\approx 1.55459\end{aligned}$$

(b)

$$\begin{aligned}H(X,Y)=&-\frac{1}{12}\log_2\frac{1}{12}-\frac{1}{6}\log_2\frac{1}{6}-\frac{1}{12}\log_2\frac{1}{12}\\&-\frac{1}{6}\log_2\frac{1}{6}-\frac{1}{6}\log_2\frac{1}{6}-\frac{1}{6}\log_2\frac{1}{6}\\&-\frac{1}{12}\log_2\frac{1}{12}-\frac{1}{12}\log_2\frac{1}{12}\\\approx&\ 2.9183\end{aligned}$$

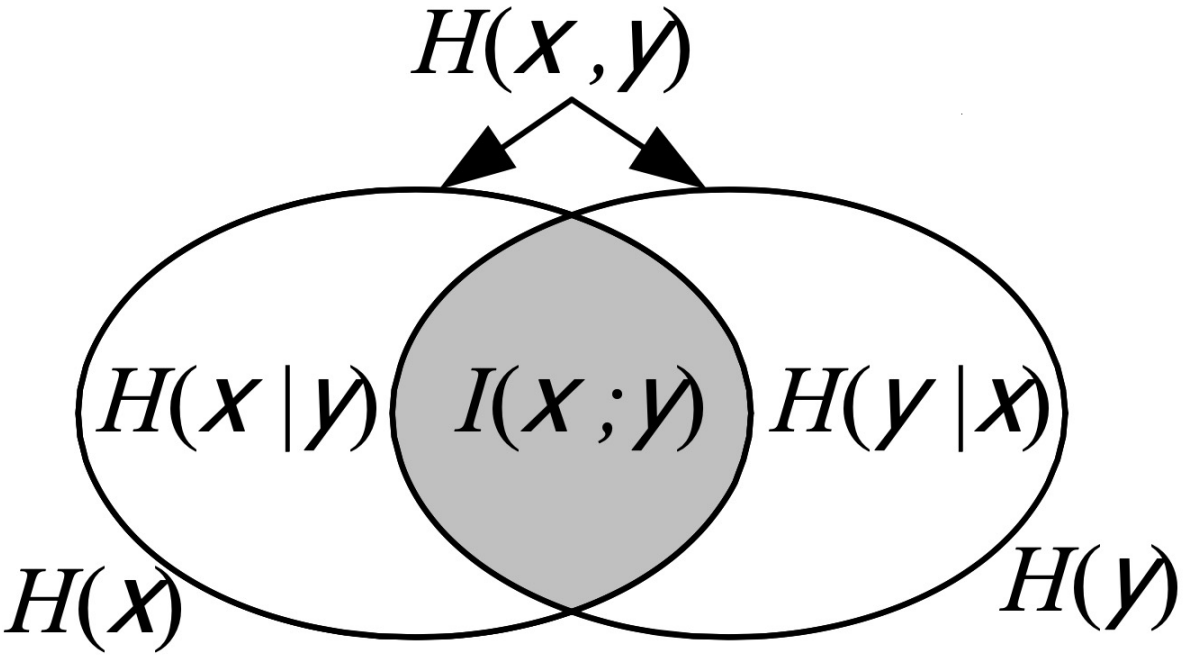
(c)

$$\begin{aligned} H(Y|X) = &-\frac{1}{12}\log_2\frac{1}{4} - \frac{1}{6}\log_2\frac{1}{3} - 0 \\ &- \frac{1}{6}\log_2\frac{1}{2} - \frac{1}{6}\log_2\frac{1}{3} - \frac{1}{12}\log_2\frac{1}{2} \\ &- \frac{1}{12}\log_2\frac{1}{4} - \frac{1}{6}\log_2\frac{1}{3} - \frac{1}{12}\log_2\frac{1}{2} \\ &\approx 1.45915 \end{aligned}$$

(d)

$$\begin{aligned} I(X;Y) = &\frac{1}{12}\log_2\frac{\frac{1}{12}}{\frac{1}{3}\times\frac{1}{4}} + \frac{1}{6}\log_2\frac{\frac{1}{6}}{\frac{1}{3}\times\frac{5}{12}} + \frac{1}{12}\log_2\frac{\frac{1}{12}}{\frac{1}{3}\times\frac{1}{3}} \\ &+ \frac{1}{6}\log_2\frac{\frac{1}{6}}{\frac{1}{2}\times\frac{1}{4}} + \frac{1}{6}\log_2\frac{\frac{1}{6}}{\frac{1}{2}\times\frac{5}{12}} + \frac{1}{6}\log_2\frac{\frac{1}{6}}{\frac{1}{2}\times\frac{1}{3}} \\ &+ 0 + \frac{1}{12}\log_2\frac{\frac{1}{12}}{\frac{1}{6}\times\frac{5}{12}} + \frac{1}{12}\log_2\frac{\frac{1}{12}}{\frac{1}{6}\times\frac{1}{3}} \\ &\approx 0.095437 \end{aligned}$$

(e)



Problem 3

$$
\begin{aligned}
P(Y=-) = \frac{1}{4}, P(Y=+) &= \frac{3}{4} \\
P(A=1) = 1, P(A=0) &= 0 \\
P(B=1) = \frac{1}{2}, P(B=0) &= \frac{1}{2} \\
P(Y=-|A=1) = \frac{1}{4}, P(Y=+|A=1) &= \frac{3}{4} \\
P(Y=-|A=0) = 0, P(Y=+|A=0) &= 0 \\
P(Y=-|B=1) = 0, P(Y=+|B=1) &= 1 \\
P(Y=-|B=0) = \frac{1}{2}, P(Y=+|B=0) &= \frac{1}{2} \\
H(Y) &= -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4} \\
&\approx 0.811278 \\
H(Y|A) &= -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4} \\
&\approx 0.811278 \\
H(Y|B) &= \frac{1}{2} \times (\log_2 1) + \frac{1}{2} \times (-\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}) \\
&= 0.5 \\
I(Y;A) = 0, &\qquad I(Y;B) \approx 0.311278
\end{aligned}
$$

So choose option 2 i.e. B.