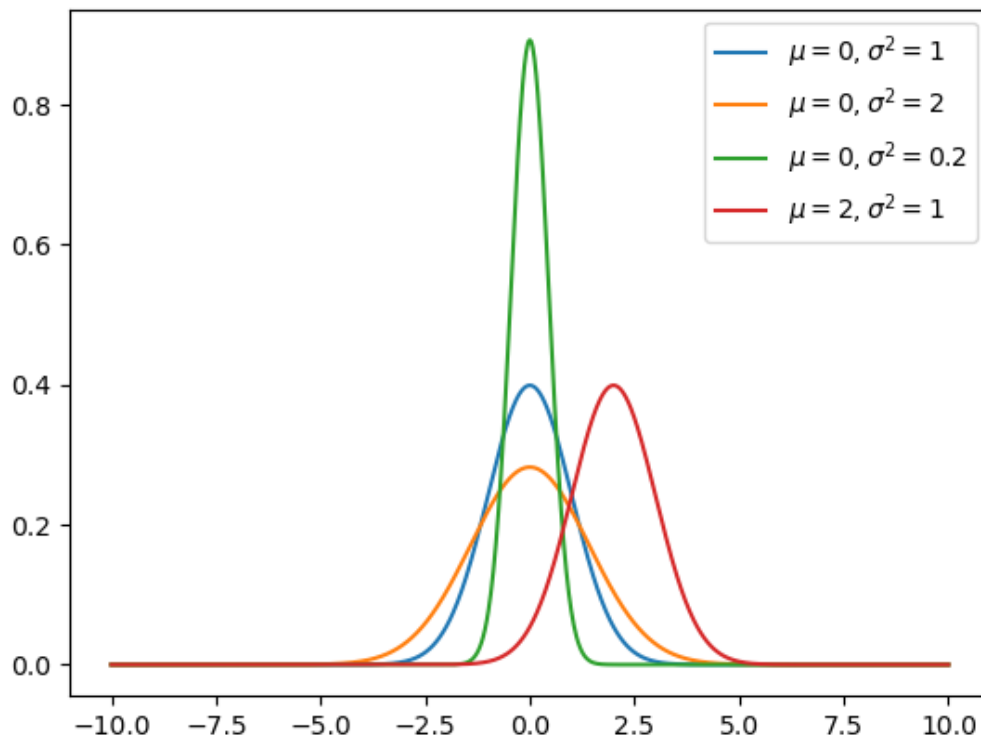


Assignment 5

2 NumPy Warm-up

2.1



2.2

验证代码

```
1 def gaussian(x, mean, variance):
2     zeta = math.sqrt(2 * math.pi * variance)
3     return 1. / zeta * np.exp(-np.power(x - mean, 2) / (2 * variance))
4
5
6 def verify(mean, variance):
7     v, err = integrate.quad(gaussian, -10, 10, args=(mean, variance))
8     print('mean = {0}, variance = {1} :'.format(mean, variance), end='')
9     # call numpy.isclose to check value
10    print(np.isclose([v], [1.0]))
```

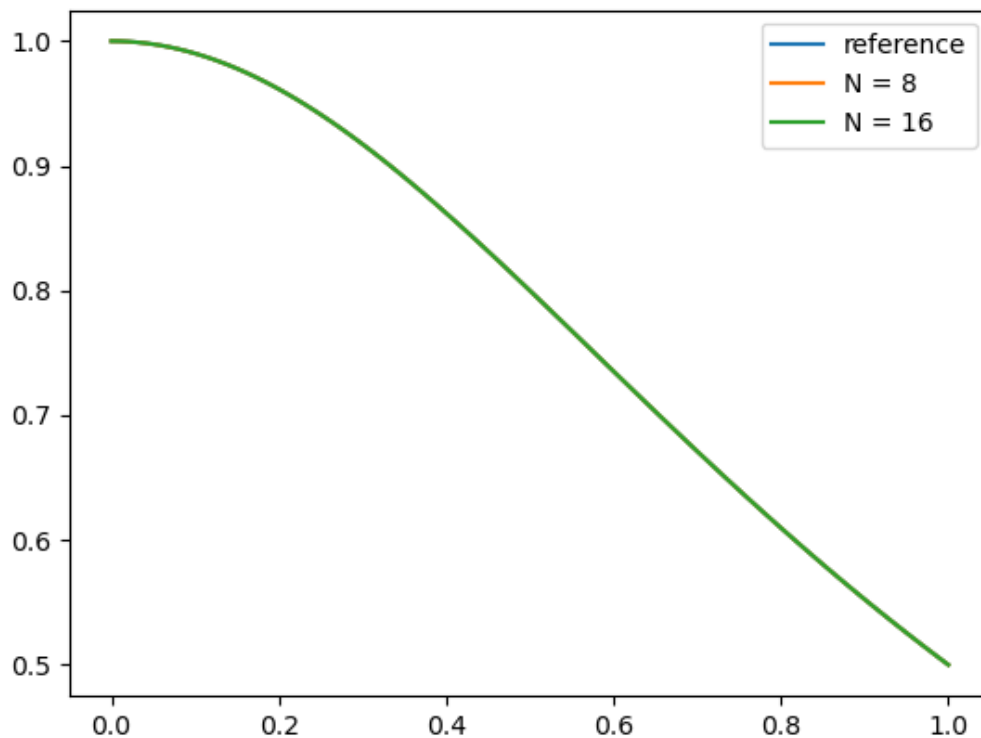
结果

```
1 mean = 0, variance = 1 : [ True]
2 mean = 0, variance = 2 : [ True]
3 mean = 0, variance = 0.2 : [ True]
4 mean = 2, variance = 1 : [ True]
```

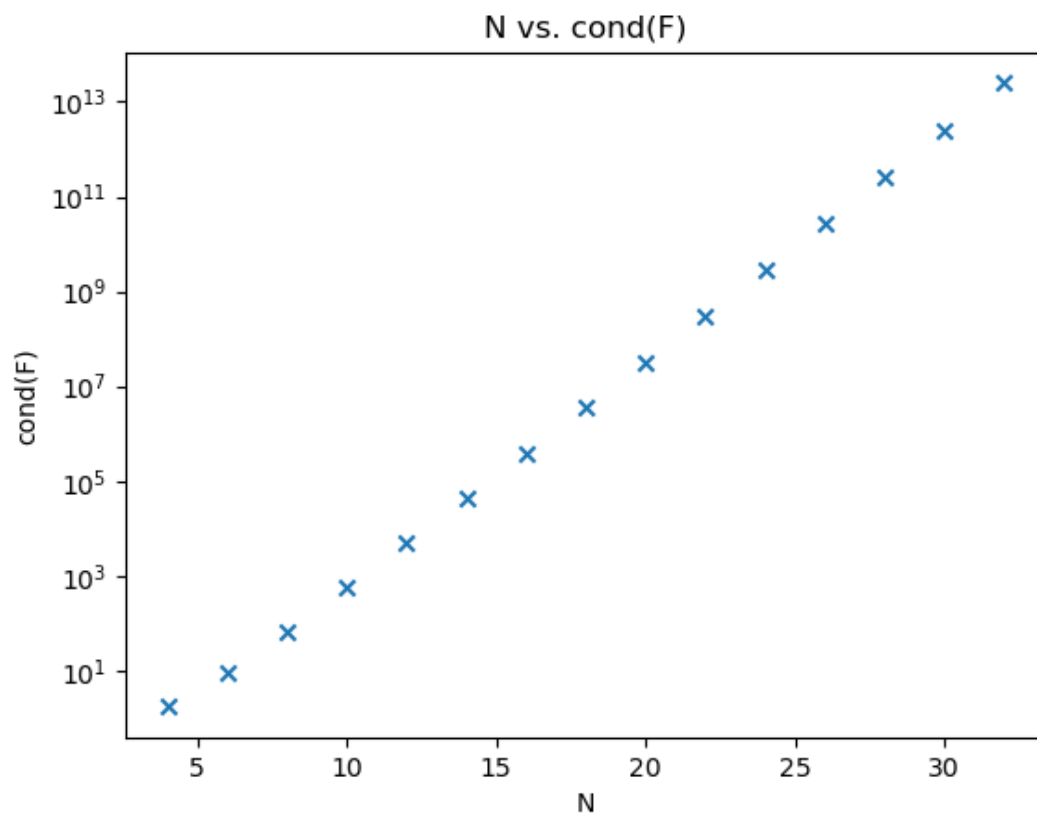
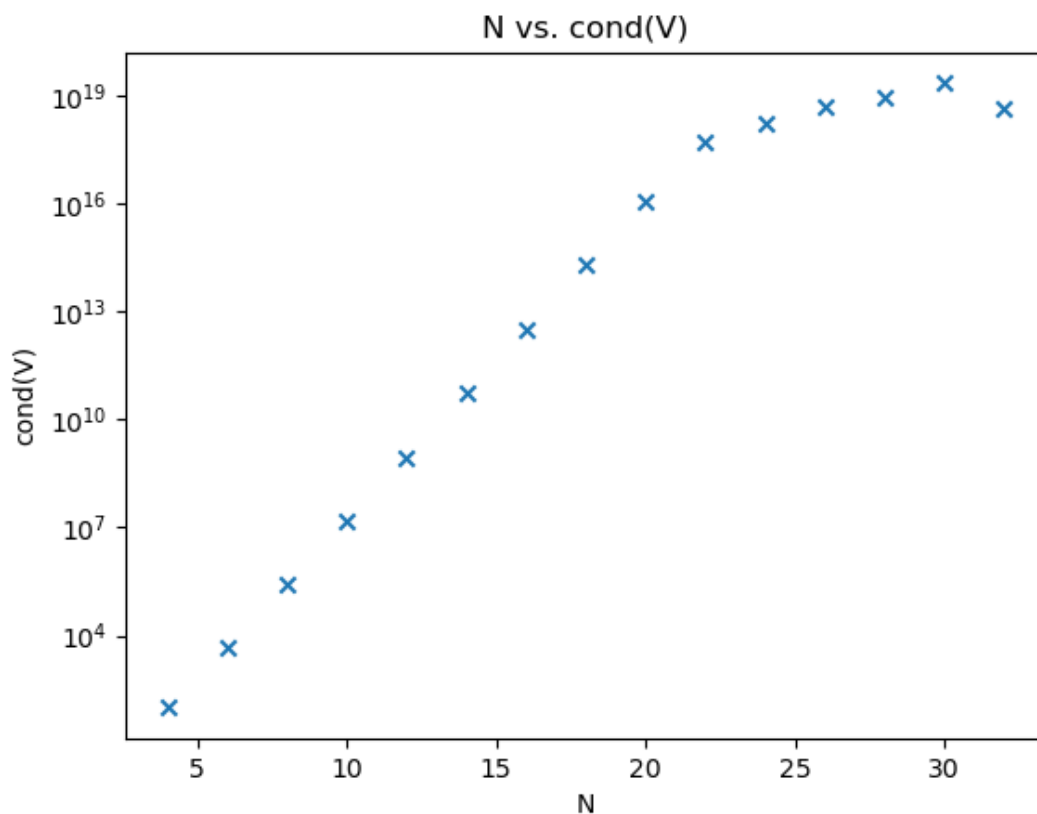
3 Numerics and Linear Algebra

3_1

```
1  +-----+-----+-----+-----+-----+-----+-----+-----+
2  | N |                               monomial coefficient                               |
3  | residual L2 Norm |
4  +-----+-----+-----+-----+-----+-----+-----+-----+
5  | 8 | [ 1.00000000e+00 -1.92980788e-03 -9.64341068e-01 -2.55940537e-01 |
6  |   | 3.3306690738754696e-16 |
7  |   | 1.93414812e+00 -1.83752964e+00 7.29012818e-01 -1.03419883e-01] |
8  |   |
9  | 16 | [ 1.00000000e+00 2.66979000e-07 -1.00001351e+00 2.93916601e-04 |
10 |   | 1.7306664177452076e-15 |
11 |   | 9.96295668e-01 3.05534156e-02 -1.17587763e+00 7.32810435e-01 |
12 |   |
13 |   | -1.25243897e+00 5.13167697e+00 -9.56973019e+00 1.01178035e+01 |
14 |   |
15 |   | -6.70183430e+00 2.79541511e+00 -6.78605634e-01 7.36509945e-02] |
16 |   |
17 +-----+-----+-----+-----+-----+-----+-----+-----+
```



3.2



从图中可以看到两个矩阵的 condition number 随 n 而指数级增长，比较直观的解释是随着 n 的增大， V 和 F 列向量之间的夹角越来越小，列与列之间越来越线性相关，因此即使 \vec{f} 变化很小，却会导致系数变化幅度很大

3.3

[illegible]

- 12 is the largest value of N where A_V is positive definite, and the condition number of that V is 883478686.1836076
- 20 is the largest value of N where A_F is positive definite, and the condition number of that F is 32732760.078300484

3 4

```
1 +---+-----+-----+-----+-----+-----+-----+-----+-----+----  
   |                                     V                                     |  
   residual L2 Norm(V) |  
3 +---+-----+-----+-----+-----+-----+-----+-----+-----+----  
   | 8 | [ 1.0000000e+00 -1.9298022e-03 -9.6434115e-01 -2.55940037e-01 |  
   4.2569029120714095e-11 |
```

```

5 | | 1.93414674e+00 -1.83752766e+00 7.29011394e-01 -1.03419477e-01] |
|
6 +---+-----+-----+-----+-----+-----+-----+-----+
|
7
8
9 +---+-----+-----+-----+-----+-----+-----+-----+
|
10 | N | F
| residual L2 Norm(F) |
11 +---+-----+-----+-----+-----+-----+-----+-----+
|
12 | 8 | [ 1.35488014 -0.04116264 -0.35430384 0.0015691 0.26057558
0.83058785 | 1.4895204919483639e-15 |
13 | | -0.01057558 -0.08058785]
| |
14 +---+-----+-----+-----+-----+-----+-----+-----+
|

```

LU 计算出的残差要更小

4 Least Squares Problems and QR

4_1

```

1 +---+---+-----+-----+-----+-----+-----+-----+
|
2 | M | N | monomial coefficient
|
3 +---+---+-----+-----+-----+-----+-----+-----+
|
4 | 16 | 4 | [[ 1.00166564 -0.02698999 -1.01909544 0.54698731]]
|
5 | 16 | 8 | [[ 1.00000188e+00 -1.07143503e-03 -9.75766185e-01 -2.00107692e-01
|
6 | | 1.80128191e+00 -1.67195234e+00 6.25188414e-01 -7.75730913e-
02]] |
7 +---+---+-----+-----+-----+-----+-----+-----+
|

```

```

1 +---+---+-----+-----+-----+-----+-----+-----+
|
2 | M | N | monomial coefficient(F)
|
3 +---+---+-----+-----+-----+-----+-----+-----+
|
4 | 16 | 4 | [[ 1.23673804 -0.02483006 0.24763933 0.53933343]]
|
5 | 16 | 8 | [[ 1.35569612 -0.04267561 -0.36313314 0.00197172 0.26167473
0.83497201 |
6 | | -0.01168661 -0.08810734]]
|
7 +---+---+-----+-----+-----+-----+-----+-----+
|

```

4_2

