

# Assignment 6

## 2 Eigenvectors

### 2.1

$$\begin{aligned} M\psi_n(x) &= \frac{d^2}{dx^2} \psi_n(x) \\ &= \frac{d}{dx} (\sqrt{2}\pi n \cos(\pi n x)) \\ &= -\sqrt{2}\pi^2 n^2 \sin(\pi n x) \\ &= \lambda_n \psi_n(x) \end{aligned}$$

### 2.2

$$\begin{aligned} \int_0^1 \psi_n(x) \psi_m(x) dx &= \int_0^1 2 \sin(\pi n x) \sin(\pi m x) dx \\ &= \int_0^1 (\cos(\pi(m-n)x) - \cos(\pi(m+n)x)) dx \\ &= \begin{cases} \int_0^1 (1 - \cos(2\pi n x)) dx & \text{if } n = m \\ \int_0^1 (\cos(\pi(m-n)x) - \cos(\pi(m+n)x)) dx & \text{otherwise} \end{cases} \\ &= \begin{cases} f(1) - f(0) & \text{if } n = m \\ g(1) - g(0) & \text{otherwise} \end{cases} \\ f(x) &= x - \frac{1}{2\pi n} \sin(2\pi n x) \\ f(1) &= 1 - \frac{1}{2\pi n} \sin(2\pi n) = 1 \\ f(0) &= 0 - \frac{1}{2\pi n} \sin(0) = 0 \\ g(x) &= \frac{1}{(m-n)\pi} \sin(\pi(m-n)x) - \frac{1}{(m+n)\pi} \sin(\pi(m+n)x) \\ g(1) &= \frac{1}{(m-n)\pi} \sin(\pi(m-n)) - \frac{1}{(m+n)\pi} \sin(\pi(m+n)) = 0 \\ g(0) &= \frac{1}{(m-n)\pi} \sin(0) - \frac{1}{(m+n)\pi} \sin(0) = 0 \\ \text{hence, } \int_0^1 \psi_n(x) \psi_m(x) dx &= \begin{cases} 1 & \text{if } n = m \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

### 2.3

$$\begin{aligned} y(x) &= \psi_1(x) \\ &= \sqrt{2} \sin(\pi x) \\ My &= \frac{d^2}{dx^2} y(x) \\ &= -\sqrt{2}\pi^2 \sin(\pi x) \\ \text{let } k &= \pi, \text{ we have } \frac{d^2}{dx^2} y(x) + k^2 y(x) = 0 \end{aligned}$$

本题中使用了 `numpy.allclose()` 进行验证

```

1 def exe_2_3():
2     N = 99
3     h = 0.01
4     x = np.arange(0, 1, h)
5     x = np.delete(x, 0, None)
6     y = psi_func(x, 1)
7     A = matrix_laplacian(N)
8     M = np.power(h, -2) * A
9     zero = np.linspace(0, 0, 99)
10    print(np.allclose(zero, np.dot(M, y) + np.pi ** 2 * y, atol=1e-02))

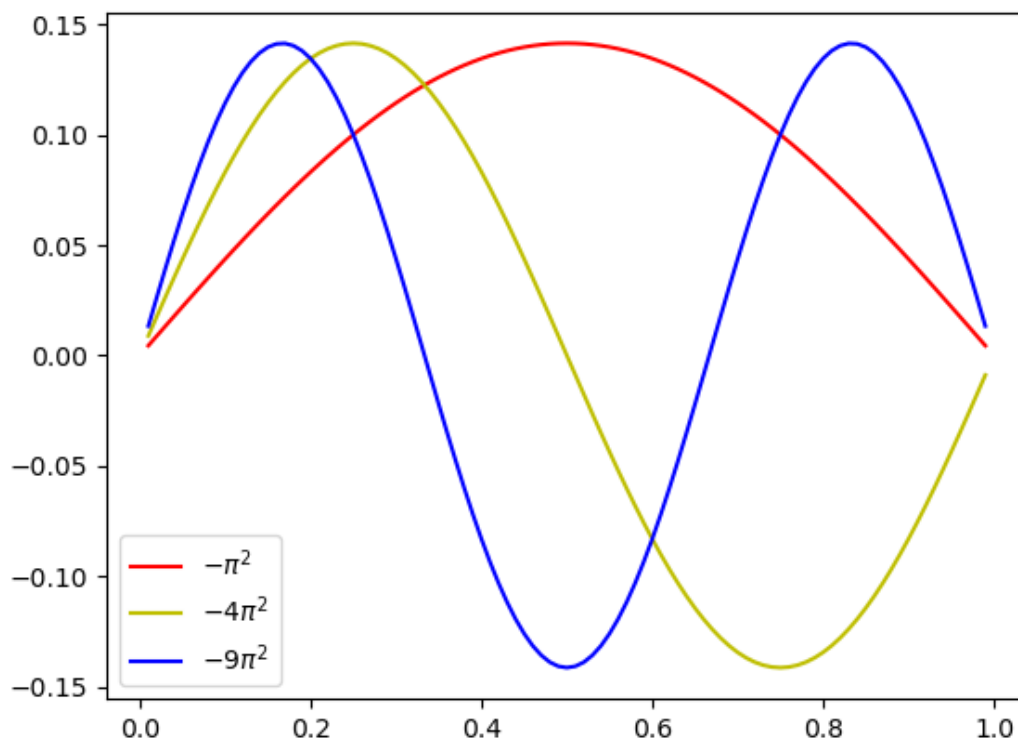
```

## 2.4

```

1 The 3 eigenvalues with the smallest magnitude are
2 [ -9.86879269 -39.46543143 -88.76070794]

```



## 3 SVD Decomposition

### 3.1

singular value

```

1 [1.26937750e+02 2.07874619e+01 1.60450118e+01 1.26918863e+01
2  1.15486524e+01 1.08934857e+01 9.00529303e+00 8.04440859e+00
3  6.62368843e+00 6.22811669e+00 5.25236858e+00 4.80801403e+00
4  4.71712918e+00 4.56192984e+00 4.28531542e+00 4.20400629e+00
5  4.10885462e+00 3.63585620e+00 3.39320579e+00 3.33575421e+00
6  3.21885036e+00 3.12013675e+00 2.84107542e+00 2.62259591e+00
7  2.58455060e+00 2.53083528e+00 2.39023583e+00 2.37065172e+00]

```

|    |                |                |                |                 |
|----|----------------|----------------|----------------|-----------------|
| 8  | 2.23939409e+00 | 2.17416334e+00 | 2.06789065e+00 | 2.06118874e+00  |
| 9  | 2.02593012e+00 | 1.98634931e+00 | 1.96619070e+00 | 1.93862002e+00  |
| 10 | 1.82450658e+00 | 1.79764444e+00 | 1.77490203e+00 | 1.71101152e+00  |
| 11 | 1.66017545e+00 | 1.63313052e+00 | 1.58449441e+00 | 1.55865054e+00  |
| 12 | 1.50628418e+00 | 1.45734653e+00 | 1.41785301e+00 | 1.38427515e+00  |
| 13 | 1.37644974e+00 | 1.34822142e+00 | 1.32540873e+00 | 1.30171470e+00  |
| 14 | 1.24365759e+00 | 1.22734081e+00 | 1.19185335e+00 | 1.18211798e+00  |
| 15 | 1.15415721e+00 | 1.14174033e+00 | 1.10366020e+00 | 1.07849266e+00  |
| 16 | 1.04871461e+00 | 1.04455363e+00 | 1.02182217e+00 | 1.00101058e+00  |
| 17 | 9.75636387e-01 | 9.68706971e-01 | 9.60102933e-01 | 9.42246881e-01  |
| 18 | 9.12538013e-01 | 8.88586193e-01 | 8.64584445e-01 | 8.47820551e-01  |
| 19 | 8.16031747e-01 | 8.12829597e-01 | 8.04620613e-01 | 7.88752373e-01  |
| 20 | 7.83485733e-01 | 7.62528113e-01 | 7.55958574e-01 | 7.41815432e-01  |
| 21 | 7.26122498e-01 | 7.12033588e-01 | 7.03490055e-01 | 6.89692742e-01  |
| 22 | 6.83519885e-01 | 6.70412202e-01 | 6.52977047e-01 | 6.41234036e-01  |
| 23 | 6.26481323e-01 | 6.24022605e-01 | 6.01337662e-01 | 5.95852407e-01  |
| 24 | 5.85441474e-01 | 5.73743710e-01 | 5.70486319e-01 | 5.60258168e-01  |
| 25 | 5.47198720e-01 | 5.37745714e-01 | 5.29038478e-01 | 5.19542952e-01  |
| 26 | 5.10675530e-01 | 5.00953343e-01 | 4.86424677e-01 | 4.75470323e-01  |
| 27 | 4.68507212e-01 | 4.57734745e-01 | 4.52586755e-01 | 4.45288488e-01  |
| 28 | 4.41215625e-01 | 4.36422750e-01 | 4.27892180e-01 | 4.15745633e-01  |
| 29 | 3.99831969e-01 | 3.96315631e-01 | 3.88879579e-01 | 3.81194466e-01  |
| 30 | 3.71542325e-01 | 3.68432957e-01 | 3.64547502e-01 | 3.57837906e-01  |
| 31 | 3.48151659e-01 | 3.46570584e-01 | 3.38876184e-01 | 3.35561588e-01  |
| 32 | 3.26529780e-01 | 3.20740385e-01 | 3.15486152e-01 | 3.08146033e-01  |
| 33 | 3.00431297e-01 | 2.94669977e-01 | 2.90311234e-01 | 2.87227600e-01  |
| 34 | 2.76299306e-01 | 2.73721266e-01 | 2.71232088e-01 | 2.61697226e-01  |
| 35 | 2.58033074e-01 | 2.56859055e-01 | 2.48103025e-01 | 2.44099814e-01  |
| 36 | 2.39841043e-01 | 2.30561270e-01 | 2.26842498e-01 | 2.20374621e-01  |
| 37 | 2.17519476e-01 | 2.15351075e-01 | 2.08439776e-01 | 1.99546649e-01  |
| 38 | 1.97046957e-01 | 1.93262238e-01 | 1.92175462e-01 | 1.86585621e-01  |
| 39 | 1.81777429e-01 | 1.79415734e-01 | 1.75762779e-01 | 1.74273437e-01  |
| 40 | 1.71873104e-01 | 1.67890941e-01 | 1.60378741e-01 | 1.55756882e-01  |
| 41 | 1.50242108e-01 | 1.50073698e-01 | 1.43851844e-01 | 1.39399933e-01  |
| 42 | 1.35961506e-01 | 1.34923292e-01 | 1.32997546e-01 | 1.27368687e-01  |
| 43 | 1.24339670e-01 | 1.21380297e-01 | 1.15995090e-01 | 1.14762576e-01  |
| 44 | 1.14068525e-01 | 1.12288331e-01 | 1.07015813e-01 | 1.05780747e-01  |
| 45 | 1.01519484e-01 | 1.01304653e-01 | 9.91456744e-02 | 9.60435628e-02  |
| 46 | 9.32498564e-02 | 9.10576853e-02 | 8.70498094e-02 | 8.35005109e-02  |
| 47 | 8.24886471e-02 | 8.19722466e-02 | 8.00461176e-02 | 7.47326661e-02  |
| 48 | 7.40872397e-02 | 7.08024417e-02 | 6.98458991e-02 | 6.72360609e-02  |
| 49 | 6.40895123e-02 | 6.06327092e-02 | 5.94645321e-02 | 5.65462600e-02  |
| 50 | 5.61618664e-02 | 5.52418327e-02 | 5.41166944e-02 | 5.30290151e-02  |
| 51 | 4.87814269e-02 | 4.69029581e-02 | 4.43088409e-02 | 4.28233365e-02  |
| 52 | 4.04756882e-02 | 3.97658938e-02 | 3.67501588e-02 | 3.57115675e-02  |
| 53 | 3.54381170e-02 | 3.32277891e-02 | 3.29052123e-02 | 2.95517634e-02  |
| 54 | 2.89600747e-02 | 2.73630552e-02 | 2.48336307e-02 | 2.43931793e-02  |
| 55 | 2.34743228e-02 | 2.28863178e-02 | 2.12830293e-02 | 1.95905824e-02  |
| 56 | 1.81609515e-02 | 1.78154111e-02 | 1.65508616e-02 | 1.52140051e-02  |
| 57 | 1.49334074e-02 | 1.38957387e-02 | 1.32042674e-02 | 1.27542407e-02  |
| 58 | 1.25275831e-02 | 1.16408761e-02 | 1.09618519e-02 | 9.58762123e-03  |
| 59 | 9.54969964e-03 | 8.87367821e-03 | 8.44558638e-03 | 7.62071949e-03  |
| 60 | 7.00600024e-03 | 6.87810074e-03 | 6.50185160e-03 | 5.93504986e-03  |
| 61 | 5.50772400e-03 | 5.28097803e-03 | 4.99857721e-03 | 4.55829959e-03  |
| 62 | 4.12636484e-03 | 3.74701488e-03 | 3.37481039e-03 | 2.98115057e-03  |
| 63 | 2.62623026e-03 | 2.36444779e-03 | 1.92816906e-03 | 1.33287788e-03  |
| 64 | 1.17163227e-03 | 8.32607758e-04 | 2.62305623e-04 | 1.65859499e-04] |





**k=2**



**k=4**



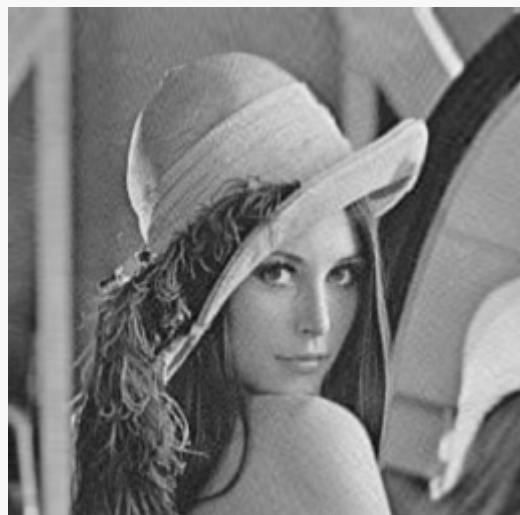
**k=8**



**k=16**



**k=32**



**k=64**

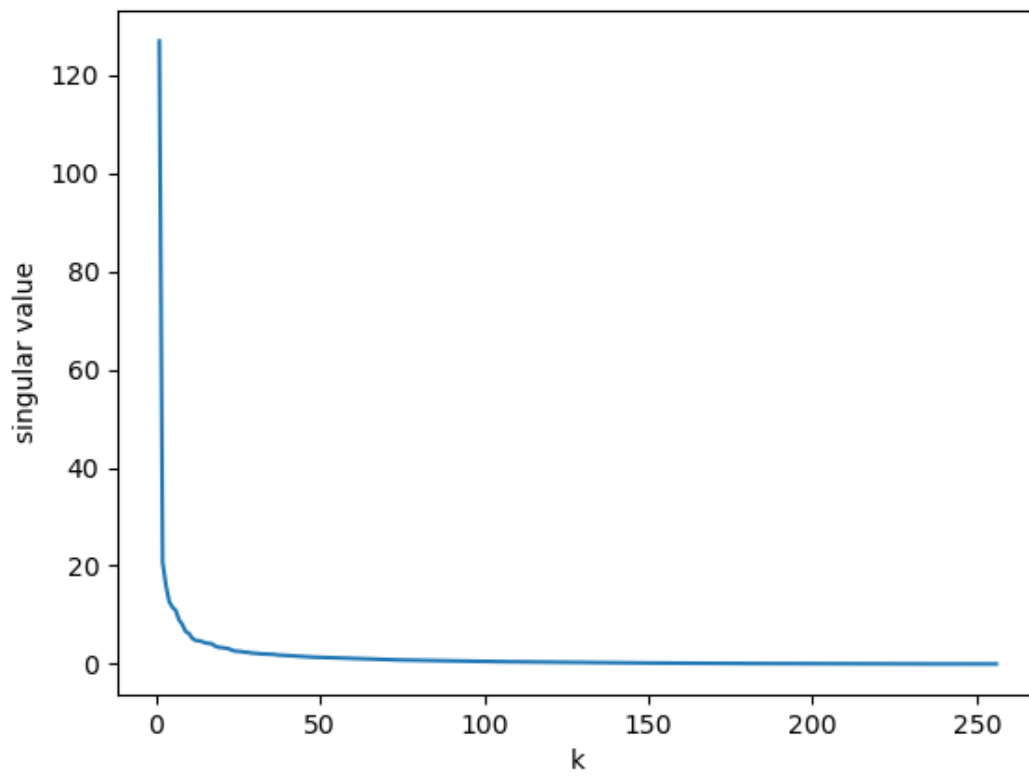


k=128



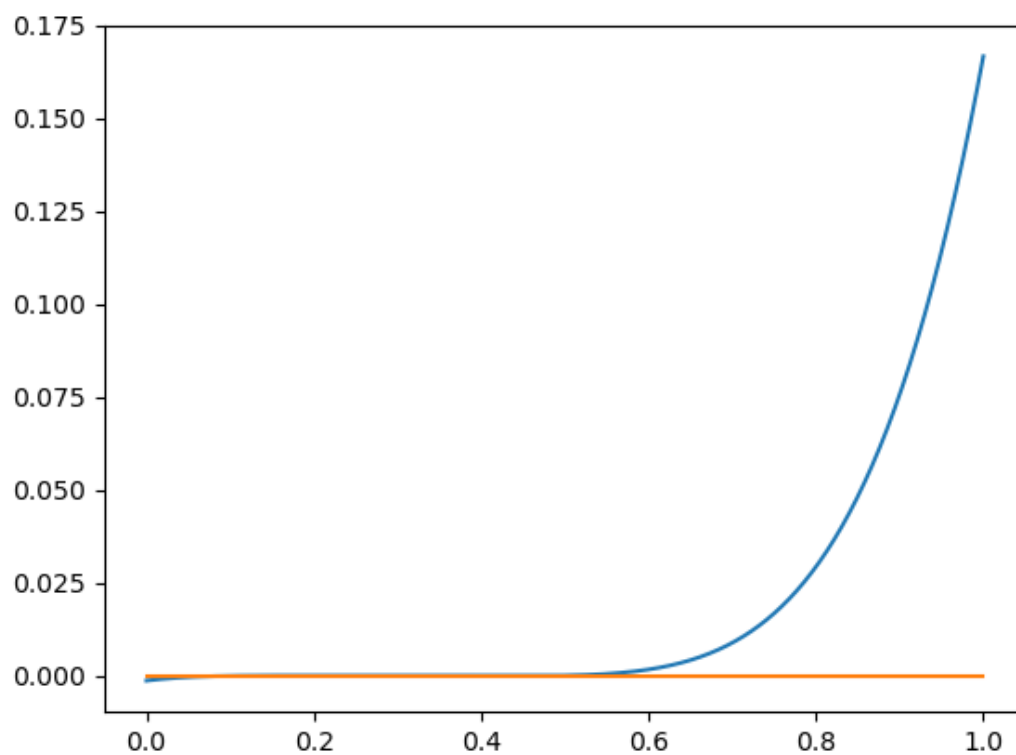
k=256

### 3.3



## 4 Nonlinear System

### 4.1



共5个根

## 4.2

root searched using newton method:0.16666666666666893

## 4.3

root searched using secant method:0.49999999850055993

与 Newton's method 得到的结果不同