1. (2pt) A patient goes to the doctor for a medical condition, the doctor suspects three diseases as the cause of the condition. The three diseases are D_1,D_2,D_3, which are marginally independent from each other. There are four symptoms S_1,S_2,S_3,S_4 which the doctor wants to check for presence in order to find the most probable cause of the condition. The symptoms are conditionally dependent to the three diseases as follows: S_1 depends only on D_1, S_2 depends on D_1 and D_2. S_3 is depends on D_1 and D_3, whereas S_4 depends only on D_3. Assume all random variables are Boolean, they are either 1 or 0.

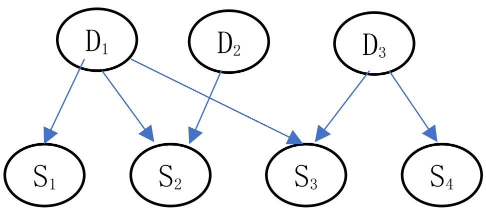
1) (0.5pt) Draw the Bayesian network for this problem. (Hint: the application of medical diagnosis we introduced in class)
2) (0.5pt) Write down the expression for the joint probability distribution as a product of conditional probabilities.
3) (0.5pt) What is the Markov Blanket of variable S_2?
4) (0.5pt) Suppose we are given the conditional probability tables below:

$P(D_3=1)$
0.1

D_3	$P(S_4=1)$
0	0.6
1	0.9

Derive the posterior distribution $P(D_3=1|S_4=1)$ using Bayesian rules.

Solution:

1) 

2) P(D1,D2,D3,S1,S2,S3,S4) = P(D1)P(D2)P(D3)P(S1|D1)P(S2|D1,D2)P(S3|D1,D3)P(S4|D3)
3) D_1, D_2
4) $P(D3=1|S4=1) = \frac{P(D_3=1)P(S_4=1|D_3=1)}{P(S_4=1)} = \frac{P(D_3=1)P(S_4=1|D_3=1)}{P(S_4=1|D_3=1)P(D_3=1)+P(S_4=1|D_3=0)P(D_3=0)} =$

$$\frac{0.1\times0.9}{0.9\times0.1+0.6\times0.9} = 1/7 \approx 0.143$$

2. (2pt) Let p(x, y) be as shown in the table below.

X\Y	0	1	2
0	1/12	1/6	1/12
1	1/6	1/6	1/6
2	0	1/12	1/12

Find

(a) H(X), H(Y),

(b) H(X,Y)

(c) H(Y|X)

(d) I(X;Y)

(e) Draw a Venn diagram for the quantities in (a) through (d)

Solution:

(a) P(x=0)=1/12+1/6+1/12=1/3, p(x=1)=1/6+1/6+1/6=1/2, P(x=3)=1/12+1/12=1/6

So, H(X) = -1/3log1/3 – 1/2log1/2 -1/6log1/6 ≈1.46

P(Y=0) = 1/4, P(Y=1)=5/12, P(Y=2) = 1/3

So, H(Y) = -1/4log1/4 – 5/12log5/12 – 1/3log1/3 ≈1.55

(b) H(X,Y)= -1/12log1/12 -1/6log1/6-1/12log1/12

-1/6log1/6-1/6log1/6-1/6log1/6

-1/12log1/12-1/12log1/12

≈2.918

(c) H(Y|X) = H(X, Y)-H(X) ≈1.459

或者

H(Y|X) = -1/12log1/4-1/6log1/2-1/12log1/4

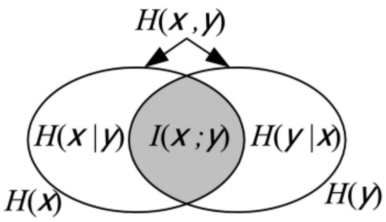
-1/6log1/3-1/6log1/3-1/6log1/3

-0-1/12log1/2-1/12log1/2

≈1.459

(d) I(X;Y) = H(X) + H(Y) – H(X, Y) ≈0.096

(e)



3. (1pt) We have a dataset in the following table where A, B denote attributes and Y denotes labels. We want to build a decision tree to classify them according to Y.

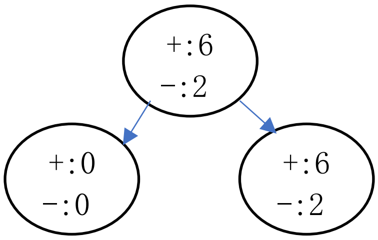
Y	A	B
−	1	0
−	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1

Which attribute should be selected for the next split? Give your explanation.

1) A
2) B
3) A or B (tie)
4) Neither

Solution:

按 A 划分：

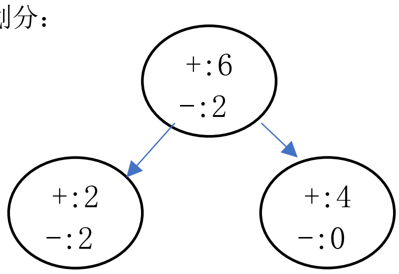


H(Y)=-1/4log1/4-3/4log3/4=0.81

H(Y|left)=0, H(Y|right)=H(Y)=-1/4log1/4-3/4log3/4=0.81

IG(Y; A) = 0.81-0.81x1=0

按 B 划分：



H(Y) = -1/4log1/4-3/4log3/4 = 0.81,

H(Y|left) = -1/2log1/2-1/2log1/2=1, H(Y|right) = 0

IG(Y; B) = 0.81 – (1x1/2+0x1/2)=0.31

IG(Y;B)>IG(Y;A), 所以按 B 划分