

# HW2\_2

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1

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1)

$$\begin{array}{ll} \text{maximize} & x_1 + x_2 \\ \text{subject to} & x_1 + 2x_2 \leq 4 \\ & -x_1 + x_2 \leq 1 \\ & 4x_1 + 2x_2 \leq 12 \\ & x_1, x_2 \geq 0 \end{array}$$

2)

$$\begin{array}{ll} & y_1 \times \quad x_1 + 2x_2 \leq 4 \\ & y_2 \times \quad -x_1 + x_2 \leq 1 \\ & y_3 \times \quad 4x_1 + 2x_2 \leq 12 \\ \text{yields} & (y_1 - y_2 + 4y_3)x_1 + (2y_1 + y_2 + 2y_3)x_2 \leq 4y_1 + y_2 + 12y_3 \\ \text{we want} & z = x_1 + x_2 \leq (y_1 - y_2 + 4y_3)x_1 + (2y_1 + y_2 + 2y_3)x_2 \\ \text{so dual problem should be} & \text{minimize} \quad 4y_1 + y_2 + 12y_3 \\ & \text{subject to} \quad y_1 - y_2 + 4y_3 \geq 1 \\ & \quad \quad \quad 2y_1 + y_2 + 2y_3 \geq 1 \\ & \quad \quad \quad y_1, y_2, y_3 \geq 0 \end{array}$$

2

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- (a) GP
- (b) LP
- (c) SDP
- (d) QP

3

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$$\begin{aligned} L(x, \nu) &= f(x) + \nu(2x_1 - x_2 - 5) = x^T H x + \nu^T (Ax - b) \\ \text{where } H &= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, A = \begin{bmatrix} 2 \\ -1 \end{bmatrix}^T, b = [5] \\ \nabla_x L(x, \nu) &= x + A^T \nu = 0 \Rightarrow x = -A^T \nu \\ g(\nu) &= L(-A^T \nu, \nu) = -\frac{1}{2} \nu^T A A^T \nu - b^T \nu \end{aligned}$$

$$\nu \in \mathbb{R},^1 \text{ the dual problem should be QP} \quad \text{maximize} \quad -\frac{1}{2} \nu^T A A^T \nu - b^T \nu = -\frac{5}{2} \nu^2 - 5\nu$$

4

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**1)**

13

**2)**

$\{s, a, b, c, d\}$  and  $\{t\}$