

DIVISION S-6—SOIL AND WATER MANAGEMENT AND CONSERVATION

Physical Basis of the Length-slope Factor in the Universal Soil Loss Equation¹

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ABSTRACT

The length-slope factor in the universal soil loss equation (USLE) is a purely empirical relationship that was derived from an extensive data base. A physically based length-slope factor was independently derived in this paper by using unit stream power theory to describe the erosion processes associated with sheet and rill flow on hillslopes. It was shown that the two length-slope factors are equivalent. Therefore, the USLE length-slope factor is a measure of the sediment transport capacity of runoff from the landscape, but fails to fully account for the hydrological processes that affect runoff and erosion. The strength of the theoretically derived length-slope factor is that it explicitly accounts for the dual phenomena of catchment convergence and rilling. The empirically derived factor can not account for changes in either surface flow or erosion processes, nor slope geometry, and this may explain why the values derived for other factors in the USLE, especially soil erodibilities, have been found to be inconsistent.

Additional Index Words: soil loss prediction, stream power, rill erosion, sheet erosion.

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OVERLAND FLOW on complex hillslopes, produced by rainfall excess, is readily observed to erode more soil from certain points in the landscape than from others. The universal soil loss equation (USLE) is currently the soil loss model most commonly used to predict soil loss rates from the landscape. The equation uses empirically determined multiplicative factors to account for the effect on erosion of rainfall energy (R), surface condition (CP), soil erodibility (K), and for the combined effect of slope and slope-length (LS). The last three factors, CP , K , and LS , interact because the hydrological and erosion processes at work in the landscape affect all three factors. Therefore, it is essential that all hydrological and erosion processes are correctly described when applying the USLE across different slope geometries and soil conditions.

The initial derivation of the USLE effectively required the LS -factor to be constrained to plane slopes, and even with this constraint considerable variation existed in the soil loss data base (Smith and Wischmeier, 1957). Inconsistencies that have appeared when using the equation are often attributed to a failure of the LS -factor to fully account for all transport mechanisms (Loch, 1984), especially when sheet flow

is replaced by rilling and when the slope geometry changes.

The aim of this paper is to investigate these problems by independently deriving a physically based length-slope factor that incorporates within it the processes of soil detachment and transport by overland flow. The objective is to then compare the two LS -factors and so evaluate the capability of the USLE LS -factor to account for different soil loss mechanisms. A potential also exists to explore the use of the theoretically derived LS -factor in the USLE: (i) to reduce observed inconsistencies in the equation, (ii) to account for complex slope geometries, and (iii) to describe soil transport by sheet and rill flow.

SOIL DETACHMENT AND TRANSPORT BY FLOWING WATER

Water on the soil surface has potential energy by virtue of its elevation above some arbitrary datum. This energy becomes available to detach and transport soil particles as the water moves down slope. Yang (1971, 1972) defined the time rate of potential energy expenditure per unit weight of water as the unit stream power, P .

$$P = \frac{dY}{dt} = \frac{dx}{dt} \frac{dY}{dx} = V_x \tan \theta \quad [1]$$

where Y is the elevation above a datum (and also the potential energy per unit weight of water above a datum), x is the longitudinal distance, t is time, V_x is the flow velocity in the longitudinal direction, and θ is the angle of the energy grade line with the horizontal. If the reference coordinates are rotated so that x is parallel to the soil surface and kinematic flow is assumed so that the energy gradient can be replaced by the slope of the soil surface, then

$$P = V \sin \theta = V s \quad [2]$$

where $s = \sin \theta$.

Yang (1972, 1973, 1984) and Yang and Stall (1976) showed that in gravel and alluvial channels the total sediment concentration or transport capacity, C_t , in mg L^{-1} was related to P in the following way:

$$\log C_t = G + \beta \log(P/\omega - P_{cr}/\omega) \quad [3]$$

where ω is the terminal fall velocity of sediment particles, P_{cr} is the critical unit stream power required to initiate movement of the sediment particles, and G and β are complex functions of ω , the kinematic viscosity, the shear velocity, and the median particle size of the bed material. This equation can be rewritten as

$$C_t = g(P/\omega - P_{cr}/\omega)^\beta \quad [4]$$

where $G = \log(g)$. Yang (1972) demonstrated that P_{cr}/ω could be set to zero without significantly affecting the values of g (or G) and β in Eq. [3] and [4], providing $C_t > 100 \text{ mg L}^{-1}$. Therefore, C_t can be assumed to be approximately proportional to P , i.e.,

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$$C_i \propto P^\beta \quad [5]$$

SEDIMENT DETACHMENT AND TRANSPORT BY STEADY-STATE SHEET FLOW

The steady-state discharge per unit width, q , across a contour element of length b produced by a constant and uniform rainfall excess rate, i , is

$$q = A (i/b) = Vy \quad [6]$$

where A is the partial catchment area above b (see Fig. 1), V is the average overland (sheet) flow velocity across the contour element, and y is the depth of flow. Substituting Eq. [2] into Eq. [6] and solving for P yields

$$P = (Ai/b)(s/y) \quad [7]$$

If uniform turbulent kinematic sheet flow is assumed then Manning's equation can be used to define the average flow velocity, so that

$$qb = Ai = (1/n) s^{1/2} y^{5/3} b \quad [8]$$

or

$$y = (niA/bs^{1/2})^{3/5} \quad [9]$$

where n is Manning's roughness coefficient, which is related to the condition of the soil surface (i.e., related to the CP factor in the USLE), and s is the sine of the slope of the soil surface. Substitution of Eq. [9] into Eq. [7] yields

$$P = (Ai/b)^{0.4} (s^{1.3}/n^{0.6}) \quad [10]$$

A catchment shape parameter a can be defined as

$$a = A/bl \quad [11]$$

where l is the distance along a streamline from the most remote part of the partial catchment area to the contour element b , as shown in Fig. 1 ($a = 1$ for a rectangular catchment; $a < 1$ for a diverging catchment; and $a > 1$ for a converging catchment). Equation [10] can, therefore, be written as

$$P = (ali)^{0.4} (s^{1.3}/n^{0.6}) \quad [12]$$

If the rainfall excess rate, i , and the roughness, n , are assumed constant over the partial catchment area A (which is the first and simplest approximation that is possible), then

$$P \propto (al)^{0.4} s^{1.3} \quad [13]$$

Moss (1979) stated that the "basic mechanisms of erosion, transportation, and deposition vary little from rivers to overland flow." Recall from Eq. [4] and [5] that Yang (1972) showed that C_i is approximately proportional to P for alluvial streams provided $C_i > 100 \text{ mg L}^{-1}$. Yang (1972, 1973) also showed that Eq. [4] is applicable over a wide range of flow depths, including depths as low as 11.3 mm. Therefore, if it can be assumed that the sediment transport capacity of overland flow is also related to unit stream power in the same way, then

$$C_i \propto [(al)^{0.4} s^{1.3}]^\beta \quad [14]$$

If ρ is the density of water, I is the total depth of rainfall excess (= total runoff volume) from the partial catchment area, A , and Y_s is the sediment yield per unit area, then for steady-state conditions

$$Y_s = C_i \rho I \quad [15]$$

so that

$$Y_s \propto [(al)^{0.4} s^{1.3}]^\beta \quad [16]$$

Therefore, sediment yield per unit area is now propor-

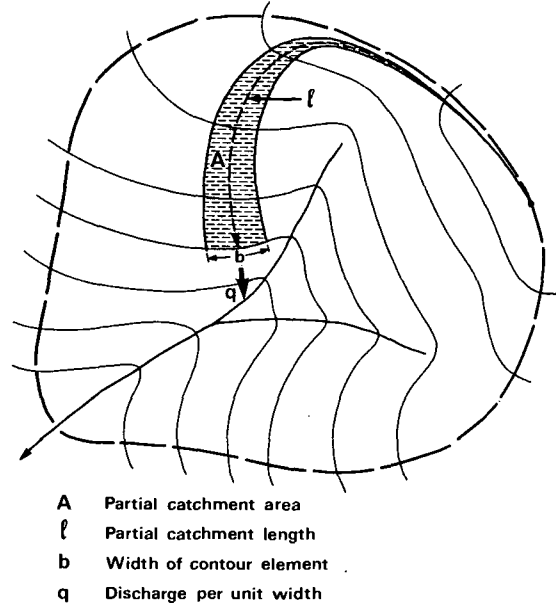


Fig. 1. Definition diagram of runoff from a partial catchment.

tional to three readily definable landscape parameters, a catchment shape parameter (a), a slope-length parameter (l), and a slope parameter (s). The parameters encompass all the original elements of the USLE length-slope factor, but with an important addition that accounts for catchment shape.

From 65 sets of field data Yang (1973) calculated mean values of β (the parameter in Eq. [3], [4], [5], [14], and [15]) ranging from about 0.82 to 1.12, with standard errors ranging from about 0.10 to 0.24. Therefore, a value of $\beta = 1$ is probably a reasonable assumption for field applications. For laboratory flume data the mean values of β were found to be slightly higher (averaging about 1.2).

LS FACTOR IN THE UNIVERSAL SOIL LOSS EQUATION

In the USLE (Wischmeier and Smith, 1978), the sediment yield per unit area, Y_s , is proportional to the length-slope factor, LS (Smith and Wischmeier, 1957; Wischmeier et al., 1958; Wischmeier and Smith, 1965, 1978), so that

$$Y_s \propto LS \quad [17]$$

where

$$LS = \left(\frac{l}{22.13} \right)^m \left(\frac{0.043x^2 + 0.3x + 0.43}{6.613} \right) \quad [18]$$

and l is the slope length in metres, x is the slope in percent, and $m = 0.3$ for $x \leq 3\%$, $m = 0.4$ for $x = 4\%$, and $m = 0.5$ for $x \geq 5\%$. The reference condition at which $LS = 1$ is for $l = 22.13 \text{ m}$ and $x = 9\%$.

Eq. [16] with $\beta = 1$ can be written in a form similar to Eq. [17] and [18], yielding a length-slope factor, LS_p , that is derived from unit stream power theory. This length-slope factor can be written as

$$LS_p = (al/22.13)^{0.4} (s/0.0896)^{1.3} \quad [19]$$

For the standard reference condition $l = 22.13 \text{ m}$ and $x = 9\%$ ($\sin \theta = 0.0896$), $LS_p = 1$.

RELATIONSHIP BETWEEN LS_p AND LS

The relationship between LS_p and LS was investigated by assuming $a = 1$ (a rectangular catchment) and a uniform

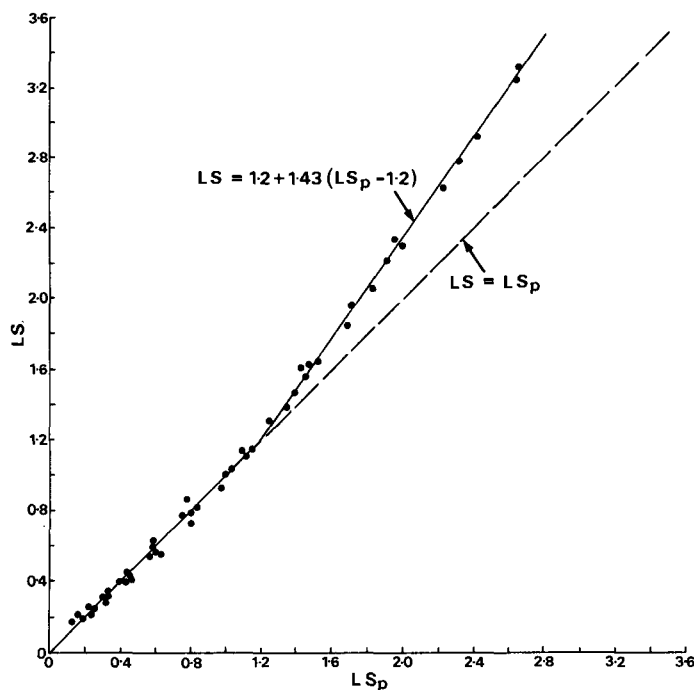


Fig. 2. Comparison of length-slope factors predicted by unit stream power theory (LS_p) and the universal soil loss equation (LS) for planar rectangular catchments. The points represent values of slope, x , and slope-length, l , ranging from 3 to 20% and 5 to 80 m, respectively.

slope. Slopes ranging from 3% to 20% and slope-lengths ranging from 5 to 80 m were included, which roughly corresponds to the range of l and x values in the data from which the LS factor in the USLE was derived. The results of the comparison are presented in Fig. 2. This figure shows that there is a strong one-to-one correspondence between LS and LS_p for $LS < 1.2$. At $LS = 1.2$ there is a distinct break point, after which

$$LS = 1.2 + 1.43(LS_p - 1.2) \text{ for } LS_p > 1.2 \quad [20]$$

For slopes of 5, 10, and 20% this break point occurs at slope-lengths of approximately 150, 23, 6.5, and 2.5 m, respectively. The possible reason for the occurrence of this break point is discussed later.

Recently McCool (K. Saxton, 1985, personal communication) found the LS factor in the USLE to be in error for longer and steeper slopes. This is consistent with the relationship between LS and LS_p described above.

Slope Relationships

The source data upon which Eq. [18] is based are not readily available. However, it is informative to reexamine the regression equations that various researchers have developed from this data base. Zingg (1940), analyzing data from Kansas, Alabama, and Missouri concluded that soil loss per unit area varied as the 1.4 power of percent slope. Smith and Whitt (1947) analyzed data from flatter slopes on a Putnam (Mollic Albaqualfs) soil and determined that soil loss varied as the 1.33 power of slope. Musgrave (1947) calculated an exponent of 1.35. These values compare to an exponent of 1.3 derived from unit stream power theory (Eq. [19]).

Smith and Wischmeier (1957) analysed sets of data from Wisconsin (3–18% slopes), Illinois, Missouri, and Ohio and derived the equation

$$Y_s \propto 0.043x^2 + 0.3x + 0.43, \quad [21]$$

which is a component of Eq. [18]. Their analysis of data from Virginia (5–25% slopes) yielded an equation of similar form, but with different coefficients.

These relationships are compared to that predicted from unit stream power theory (Eq. [19]) in Fig. 3. All equations were adjusted to give unit relative soil loss on a 3% slope so that the curves are the same as those presented in Smith and Wischmeier (1957). Figure 3 shows that predictions of relative soil loss (as a function of slope alone) based on unit stream power theory lie within the range of experimentally derived relationships.

Slope-length Relationships

Zingg (1940) concluded from his analysis of experimental data from Missouri, Oklahoma, Wisconsin, and Texas that soil loss per unit area was proportional to the 0.6 power of the slope-length, l . Musgrave (1947) derived an average exponent of 0.35. Smith and Wischmeier (1957), analyzing 1360 plot-yr of data estimated a weighted exponent of 0.46, but had individual values that ranged from 0 to 0.9. They recommended values of the slope-length exponent of 0.5 ± 0.1 (Smith and Wischmeier, 1957; Wischmeier *et al.*, 1958). The soil loss vs. slope-length data from all three studies exhibited a high degree of variability. In a later publication, Wischmeier and Smith (1978) recommended values of the exponent of 0.3 for $x \leq 3\%$, 0.4 for $x = 4\%$, and 0.5 for $x \geq 5\%$, but noted that values > 0.5 are possible on steep-slopes susceptible to rill formation. The exponent predicted using unit stream power theory (0.4) lies within the range of published values, but large differences between the published values strongly suggests that possible changes in soil transport mechanisms are occurring that remain unidentified.

From Fig. 3 it can be seen that Eq. [19] and [21] predict similar relative soil losses for slopes less than about 13%. However, at greater slopes Eq. [21] (and hence Eq. [18]) predicts increasingly greater values of relative soil loss compared to Eq. [19], the unit stream power based equation. Also, the slope-length exponent in Eq. [19] is 0.4, whereas in Eq. [18] it is 0.5 for $x \leq 5\%$. These differences between Eq. [18] and [19] combine to produce a distinct break point in the $LS - LS_p$ relationship at $LS = 1.2$, as shown in Fig. 2.

There exists some uncertainty about the appropriate value of β in Eq. [16]. Yang (1973) calculated mean values ranging from 0.82 to 1.12 derived from field measurements of alluvial streams. If β is not equal to unity then the exponents in Eq. [19] must be adjusted. For example, if β ranges from 0.82 to 1.12 then the length exponent would range from 0.33 to 0.45 and the slope exponent would range from 1.07 to 1.46, respectively. These values are within the range of values found in the experimental data described above.

Effect of Catchment Convergence and Divergence

The variable a in Eq. [19] accounts for the effect of overland flow convergence ($a > 1$) or divergence ($a < 1$) on the length-slope factor derived from unit stream power theory. Equation [19] implies that for catchments of equal area, a converging catchment will produce more sediment than a diverging catchment. This behavior is exactly what is observed in nature (Mosley, 1972, 1974). Hence, from unit stream power theory,

$$Y_s \propto a^{0.4}. \quad [22]$$

The USLE does not take into account catchment convergence or divergence explicitly. Therefore, it is not possible to verify the form of Eq. [22] by comparison to the USLE in the way that the slope and slope-length terms were. The relationship between LS and LS_p for a range of a values is presented in Fig. 4. The line of equal value ($LS = LS_p$) is

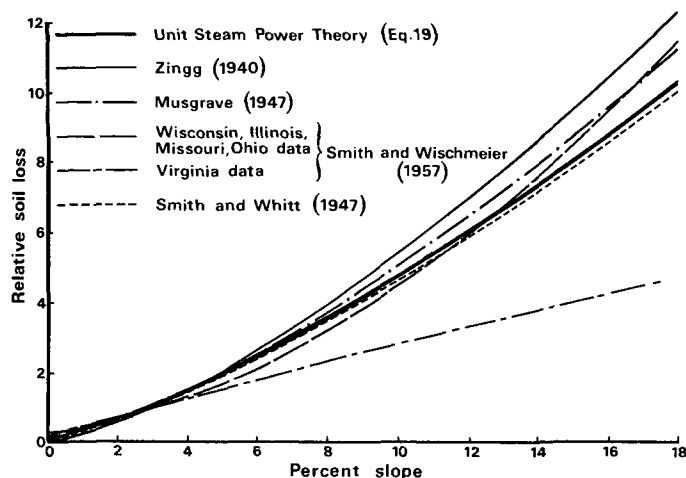


Fig. 3. Comparison of relative soil loss predicted by unit stream power theory and a number of empirical equations proposed in the literature. All equations were adjusted to give unit relative soil loss on a 3% slope [adapted from Smith and Wischmeier (1957)].

shown as the dashed line on this figure. As LS increases, increasing values of catchment convergence (i.e., increasing values of a) would produce $LS = LS_p$. For example, for $LS = 2.2$, $LS = LS_p$ if $a = 1.5$. Alternatively, flow convergence can occur by the progressive formation and merging of rills in the downstream direction and this is discussed below.

Effect of Rilling on the Relationship Between LS_p and LS

Rilling concentrates overland flow, thereby increasing flow depth and flow velocity, which increases the unit stream power. Mosley (1972, 1974) has demonstrated that rilling increases the sediment loss from hillslopes, which is consistent with an increase in unit stream power. Wischmeier and Smith (1978) noted that on long-steep slopes susceptible to rill formation the exponent, m , in Eq. [18] is likely to be considerably >0.5 . This also implies that rilling increases the sediment yield and thus increases the LS factor in the USLE.

A simple analysis is presented below to determine the potential magnitude of the effect of rilling on the LS_p -factor derived from unit stream power theory. It is also aimed at showing that LS_p and LS can be equivalent for large LS values if rilling is taken into account in the LS_p -factor. The following assumptions are made to simplify the analysis: (i) rill width \gg depth of flow in the rill(s); (ii) Manning's n for each rill and the overland flow plane are the same; (iii) a uniform and constant rainfall excess rate over the partial catchment area that produces steady-state runoff; and (iv) the entire discharge across the contour element (Fig. 1) is carried in the rills. A more comprehensive and exact analysis is presented by Moore and Burch (1986) and does not make assumptions (i) and (ii).

The first assumption means that the hydraulic radius, r , and the cross-section area, C_x , for any assumed cross-section shape (i.e., rectangular, triangular, trapezoidal, or parabolic) can be written in the general forms:

$$r = c y \quad [23]$$

and

$$C_x = e(b/J)y \quad [24]$$

where y is the depth of flow in the rill, b is the width of the contour element (Fig. 1), J is the number of rills crossing the contour element, and c and e are constants. With this assumption $c = 1$ for a rectangular cross-section, $c = 2/3$

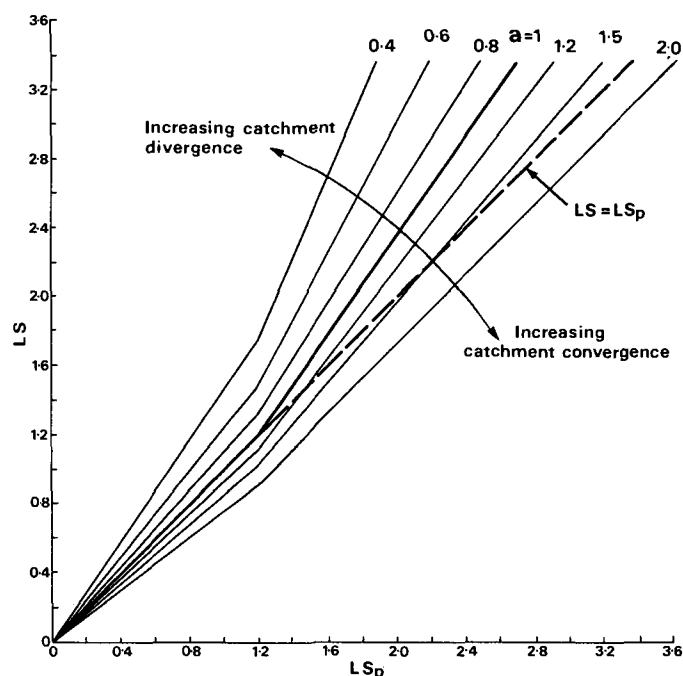


Fig. 4. Effect of catchment convergence and divergence on the relationship between LS and LS_p , where $a (= A/bI)$ is a catchment shape factor.

for a parabolic cross-section, and $c = 1/2$ for a triangular cross-section. In Eq. [24] eb/J is the top width of each rill and J/b is the number of rills per unit length of contour element.

Using these assumptions an equation equivalent to Eq. [12] can be derived that also includes, in a simplistic way, the effect of rilling.

$$P = (ali)^{0.4} s^{1.3} (c/e)^{0.4} / n^{0.6} \quad [25]$$

Therefore, an extended form of Eq. [19] can be written as:

$$LS_p = (l/22.13)^{0.4} (s/0.0896)^{1.3} Z \quad [26]$$

where $Z = (c/e)^{0.4}$ is a rilling factor that modifies the length-slope factor derived from unit stream power theory.

If sheet flow occurs $J = 1$, $c = 1$, and $e = 1$ so that $Z = 1$, and Eq. [26] reduces to Eq. [19]. If there is one parabolic shaped rill per unit length of contour element, with a top width of 20 cm, then $J = 1$, $c = 2/3$, and $e = 0.2$, so that $Z = 1.62$. Finally, if there are five parabolic rills per unit length of contour element, each with a top width of 10 cm, then $J = 5$, $c = 2/3$, and $e = 0.5$, so that $Z = 1.12$. This latter scenario would give $LS_p = LS$ for $LS = 2.2$ on a nonconverging or diverging catchment. These results imply that sheet flow has the lowest sediment transport capacity and that once rilling is initiated, increasing amalgamation of the rills increases the unit stream power and increases the sediment transport capacity.

Mosley (1972, 1974) showed that as water moves downslope sheet flow is transformed into rill flow in a large number of small rills. Further down slope these small rills merge into a limited number of larger rills as the sediment transport capacity of the hillslope increases. These observations are consistent with the experimental data on which the LS -factor in the USLE is based, and are consistent with the predictions of the LS_p -factor, so that $LS_p = LS$.

DISCUSSION

The LS -factor in the USLE is a purely empirical relationship that was derived from an extensive data

base consisting of over 10 000 plot-yr of data. Generally the USLE experimental plots were rectangular, uniform in slope, and not subjected to rilling processes. However, for the steeper slopes and the longer slope-lengths rilling did occur on the plots, which would have increased the observed sediment losses. Hence, the USLE implicitly accounts for some degree of rilling within the data base used to derive the LS -factor. The problem with the USLE is that neither of the processes of flow convergence or rilling are accounted for parametrically within the equation.

The LS_p -factor, derived solely from the theoretical application of unit stream power theory, is capable of explicitly accounting for the effects of both flow convergence and rilling. For $LS < 1.2$, $LS_p = LS$ if no convergence or rilling is assumed. For $LS > 1.2$, $LS_p = LS$ if some degree of convergence and/or rilling is included, which is appropriate in view of the physical characteristics of the long-steep plots, described above. If no rilling or convergence is assumed then $LS_p < LS$ for $LS > 1.2$.

Based on the foregoing discussion and the results presented, there is strong evidence to suggest that LS_p and LS are equivalent functions, the former being derived from theory and the latter being derived from regression analysis of experimental data. The LS -factor does, therefore, have a physical basis.

By inference from the equivalence between the LS_p and LS -factors, it is clear that the length-slope factor in the USLE is a measure of the sediment transport capacity of flowing water at the point of interest in the landscape. This, in turn, is also related to the rate of dissipation of the potential energy content of the water. The LS -factor attempts to account for the hydrological processes that affect runoff and erosion, though in an empirical and incomplete way. The length-slope factor derived from unit stream power theory explicitly uses the hydrology of the up-slope contributing area in its derivation. Loch (1984) found that USLE soil erodibility factor values, measured experimentally, were strongly dependent on the erosion processes at work, especially as the down-slope discharge (rainfall excess rate) exceeded apparent threshold rates that resulted in the transition from sheet flow to flow in rills. Because the LS -factors in the USLE do not account for this transition, the measured erodibility factors in the USLE may be nonunique. However, the LS_p -factors can take into account the interaction between runoff and erosion (via flow convergence and rilling) and so use of these factors should allow unique values of soil erodibility to be determined. This needs to be evaluated in future erosion research, particularly for use in the development and application of computer models for predicting soil loss.

If the USLE is to be applied to real-world catchments, whether they are large or small, then it is recommended that the length-slope factor derived from unit stream power theory (Eq. [26]) be used rather than the original equation (Eq. [18]). This will allow a greater range of topographic attributes (slope, slope-

length, and catchment convergence) and rilling to be explicitly accounted for within the soil loss calculations.

CONCLUSIONS

A length-slope factor has been independently and theoretically derived using unit stream power theory to describe soil transport in overland flow, which includes both sheet flow and flow in rills. The derived factor has been evaluated by comparing it with its empirically derived counterpart in the USLE. This comparison examined slope, slope-length, catchment geometry, and rilling effects separately and in combination. In all cases the two factors were found to be equivalent functions. However, the theoretically derived factor is physically based and better accounts for complex slope geometries and the effects of rilling. It also offers potential benefits if it is substituted for the original factor when applying the USLE in future soil loss studies.

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