



On pose l'équation de Schrödinger: $-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$

Cas I ($x < -a$):

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_1}{dx^2} = E\psi_1$$

$$\Leftrightarrow \frac{d^2\psi_1}{dx^2} = \frac{2mE\psi_1}{-\hbar^2}$$

$$\Leftrightarrow \frac{d^2\psi_1}{dx^2} + k_1^2\psi_1 = 0, \quad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \psi_1(x) = A e^{i k_1 x} + B e^{-i k_1 x}$$

Cas II ($-a \leq x \leq a$):

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_2}{dx^2} - V_0\psi_2 = E\psi_2$$

$$\Leftrightarrow -\frac{\hbar^2}{2m} \frac{d^2\psi_2}{dx^2} - (E + V_0)\psi_2 = 0$$

$$\Leftrightarrow \frac{d^2\psi_2}{dx^2} + \frac{2m(E + V_0)}{\hbar^2} \psi_2 = 0$$

$$\Leftrightarrow \frac{d^2\psi_2}{dx^2} + k_2^2\psi_2 = 0, \quad k_2 = \sqrt{\frac{2m(E + V_0)}{\hbar^2}}$$

$$\Rightarrow \psi_2(x) = C e^{i k_2 x} + D e^{-i k_2 x}$$

Cas III ($x > a$):

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_3}{dx^2} = E\psi_3$$

$$\Leftrightarrow \frac{d^2\psi_3}{dx^2} = \frac{2mE\psi_3}{-\hbar^2}$$

$$\Leftrightarrow \frac{d^2\psi_3}{dx^2} + k_3^2\psi_3 = 0, \quad k_3 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \psi_3(x) = E e^{i k_3 x} + F e^{-i k_3 x}$$

avec $F = 0$ car pas de réflexion à droite

$$\begin{cases} \psi_1(0) = \psi_2(0) \\ \psi_1'(0) = \psi_2'(0) \\ \psi_3(a) = \psi_2(a) \\ \psi_3'(a) = \psi_2'(a) \end{cases} \Rightarrow \begin{cases} A+B = C+D \quad (1) \\ i k_1 A - i k_1 B = i k_2 C - i k_2 D \quad (2) \\ E e^{i k_1 a} = C e^{i k_2 a} + D e^{-i k_2 a} \quad (3) \\ i k_1 E e^{i k_1 a} = i k_2 C e^{i k_2 a} - i k_2 D e^{-i k_2 a} \quad (4) \end{cases}$$

$$k_1 = k_3 = k$$

$$k_1(1) \Rightarrow k_1(A+B) = k_1(C+D) \quad (5)$$

$$(5) + (2) \Rightarrow 2k_1 A = C(k_1 + k_2) + D(k_1 - k_2)$$

$$(5) - (4) \Rightarrow 2k_1 B = C(k_1 - k_2) + D(k_1 + k_2)$$

$$\Rightarrow B = \frac{C(k_1 - k_2)}{2k_1} + \frac{D(k_1 + k_2)}{2k_1}$$

$$\Rightarrow A = \frac{C(k_1 + k_2)}{2k_1} + \frac{D(k_1 - k_2)}{2k_1}$$

$$k_2(3) \Rightarrow k_2 E e^{i k_1 a} = k_2 (C e^{i k_2 a} + D e^{-i k_2 a}) \quad (6)$$

$$(6) - (3) \Rightarrow (k_2 - k) E e^{i k_1 a} = 2k_2 D e^{-i k_2 a}$$

$$\Rightarrow D = \frac{(k_1 - k)}{2k_2} E e^{i a(k_1 + k_2)}$$

$$(6) + (3) \Rightarrow 2k_2 C e^{i k_2 a} = (k_2 + k) E e^{i k_1 a}$$

$$\Rightarrow C = \frac{(k_2 + k)}{2k_2} E e^{i a(k_2 - k)}$$

L'ordre transmise T s'écrit:

$$T = \left| \frac{E}{A} \right|^2$$

$$A = E \left(\frac{(k_1 + k_2)^2}{4 k_1 k_2} e^{i a(k_1 - k_2)} - \frac{(k_1 - k_2)^2}{4 k_1 k_2} e^{i a(k_1 + k_2)} \right)$$

$$= \frac{E e^{i a k}}{2 k_1 k_2} \left(\frac{(k_1 + k_2)^2}{2} e^{-i a k_2} - \frac{(k_1 - k_2)^2}{2} e^{i a k_2} \right)$$

$$= \frac{E e^{i a k}}{2 k_1 k_2} \left(\frac{(k_1 + k_2)^2}{2} (\cos(a k_2) - i \sin(a k_2)) - \frac{(k_1 - k_2)^2}{2} (\cos(a k_2) + i \sin(a k_2)) \right)$$

$$\begin{aligned}
 \Rightarrow \Delta &= \frac{E e^{i a b_1}}{4 b_1 b_2} \left(\cos(a b_2) \left(\frac{(b_1 + b_2)^2 - (b_1 - b_2)^2}{2} \right) - i \sin(a b_2) \left(\frac{(b_1 + b_2)^2 + (b_1 - b_2)^2}{2} \right) \right) \\
 &= \frac{E e^{i a b_1}}{4 b_1 b_2} \left(\cos(a b_2) \times 4 b_1 b_2 - i \sin(b_2 a) (2 b_1^2 + 2 b_2^2) \right) \\
 &= E e^{i a b_1} \left(\cos(a b_2) - i \sin(b_2 a) \left(\frac{b_1^2 + b_2^2}{2 b_1 b_2} \right) \right)
 \end{aligned}$$

$$\text{On, } T = \left| \frac{E}{\Delta} \right|^2 = \frac{|E|^2}{|\Delta|^2}$$

$$\text{Donc, } T = \frac{1}{\left(\cos^2(a b_2) + \left(\frac{b_1^2 + b_2^2}{2 b_1 b_2} \right) \sin^2(a b_2) \right)}$$

Pour avoir l'effet Ramsauer, on veut le coefficient de transmission le plus grand.

$$\text{Pour } T=1,$$

$$\sin(a b_2) = 0 \text{ et } \cos(a b_2) = 1$$

$$\Rightarrow a b_2 = n\pi \Rightarrow b_2 = \frac{n\pi}{a}$$

Or on a:

$$k_2 = \sqrt{\frac{2m(V_0 + E)}{\hbar^2}} \Rightarrow \left(\frac{n\pi}{a} \right)^2 = \frac{2m(V_0 + E)}{\hbar^2}$$

$$\Rightarrow E_n = \frac{\hbar^2 n^2 \pi^2}{2m a^2} - V_0$$