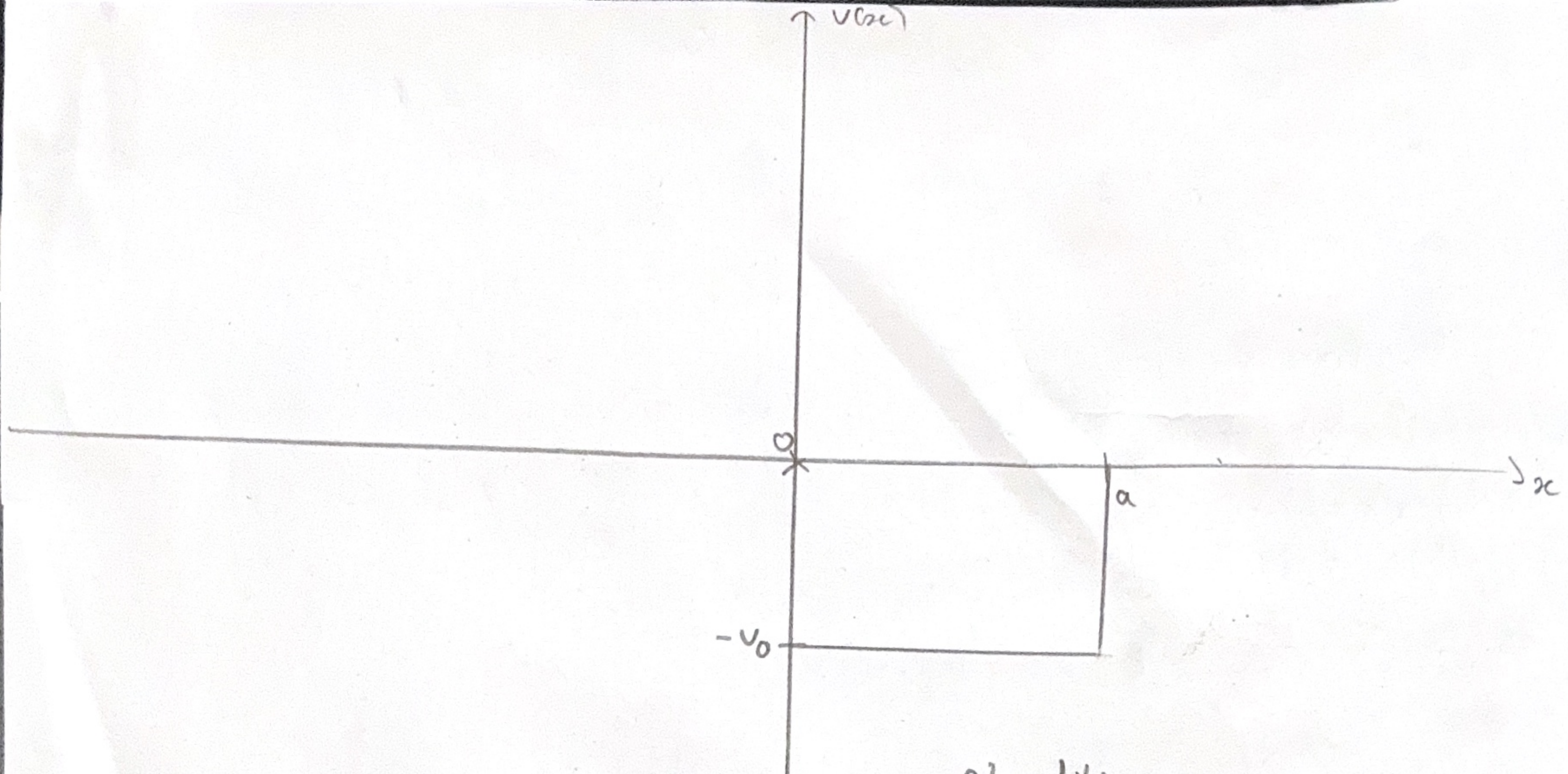

On pose l'équation de Schrödinger : $-\frac{\hbar^2}{2m}\frac{d\psi}{dx^2} + V(x)\psi = E\psi$

Cas I ($x < -a$) :

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} = E\psi_1$$

$$\Leftrightarrow \frac{d^2\psi_1}{dx^2} = \frac{2mE\psi}{-\hbar^2}$$

$$\Leftrightarrow \frac{d^2\psi_1}{dx^2} + k_1^2\psi_1 = 0, \quad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

Cas II ($-a \leq x \leq a$) :

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} - V_0\psi_2 = E\psi_2$$

$$\Leftrightarrow -\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} - (E+V_0)\psi_2 = 0$$

$$\Leftrightarrow \frac{d^2\psi_2}{dx^2} + \frac{2m(E+V_0)}{\hbar^2}\psi_2 = 0$$

$$\Leftrightarrow \frac{d^2\psi_2}{dx^2} + k_2^2\psi_2 = 0, \quad k_2 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

$$\Rightarrow \psi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$$

Cas III ($a < x$) :

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_3}{dx^2} = E\psi_3$$

$$\Leftrightarrow \frac{d^2\psi_3}{dx^2} = \frac{2mE\psi_3}{-\hbar^2}$$

$$\Leftrightarrow \frac{d^2\psi_3}{dx^2} + k_3^2\psi_3 = 0, \quad k_3 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \psi_3(x) = Ee^{ikx} + Fe^{-ikx}$$

avec $F = 0$ car pas de réflexion à droite

$$\begin{cases} \psi_1(-a) = \psi_2(-a) \\ \psi_1'(-a) = \psi_2'(-a) \\ \psi_3(a) = \psi_2(a) \\ \psi_3'(a) = \psi_2'(a) \end{cases} \Leftrightarrow \begin{cases} A + B = C + D \quad (1) \\ ik_1 A - ik_1 B = ik_2 C - ik_2 D \quad (2) \\ E e^{ik_3 a} = C e^{ik_2 a} + D e^{-ik_2 a} \quad (3) \\ ik E e^{ika} = ik_2 C e^{ik_2 a} - ik_2 D e^{-ik_2 a} \quad (4) \end{cases}$$

$k_1 = k_3 = k$

$k_1(1) \Rightarrow k_1(A+B) = k_1(C+D) \; (5)$

$(5)+(2) \Rightarrow 2k_1 A = C(k_1+k_2) + D(k_1-k_2)$

$(5)-(4) \Rightarrow 2k_1 B = C(k_1-k_2) + D(k_1+k_2)$

$\Rightarrow B = C\dfrac{(k_1-k_2)}{2k_1} + D\dfrac{(k_1+k_2)}{2k_1}$

$\Rightarrow A = C\dfrac{(k_1+k_2)}{2k_1} + D\dfrac{(k_1-k_2)}{2k_1}$

$k_2(3) \Rightarrow k_2 E e^{ika} = k_2(C e^{ik_2 a} + D e^{-ik_2 a}) \; (6)$

$(6)-(3) \Rightarrow (k_2 - k) E e^{ika} = 2k_2 D e^{-ik_2 a}$

$\Rightarrow D = \dfrac{(k_2-k)}{2k_2} E e^{ia(k+k_2)}$

$(6)+(3) \Rightarrow 2k_2 C e^{ik_2 a} = (k_2+k) E e^{ika}$

$\Rightarrow C = \dfrac{(k_2+k)}{2k_2} E e^{ia(k-k_2)}$

L'onde transmise T s'écrit:

$$T = \left| \frac{E}{A} \right|^2$$

$$A = E\left(\frac{(k_1+k_2)^2}{4k_1k_2} e^{ia(k-k_2)} - \frac{(k_1-k_2)^2}{4k_1k_2} e^{ia(k_1+k_2)} \right)$$

$$= \frac{E e^{iak}}{2k_1 k_2} \left(\frac{(k_1+k_2)^2}{2} e^{-iak_2} - \frac{(k_1-k_2)^2}{2} e^{iak_2} \right)$$

$$= \frac{E e^{iak}}{2k_1 k_2} \left(\frac{(k_1+k_2)^2}{2} (\cos(ak_2) - i\sin(ak_2)) - \frac{(k_1-k_2)^2}{2} (\cos(ak_2 + i\sin(ak_2)) \right)$$

$$\begin{cases} A+B = D+E \\ k_1(A-B) = k_3(D-E) \quad (2) \\ Ce^{ik_2a} = De^{ik_3a} + Ee^{-ik_3a} \\ ik_2Ce^{ik_2a} = k_3\left(e^{ik_3a}D - Ee^{-ik_3a}\right) \quad (4) \end{cases}$$

$$k_1(A+B) = k_1(D+E) \quad (1)$$

$$(1)-(2) = k_1(2B) = k_1B - k_3B + k_1E - k_3E$$

$$k_1(2B) = B(k_1-k_3) + E(k_1-k_3)$$

$$\Rightarrow B = \frac{k_1-k_3}{2k_1}B + E\frac{k_1-k_3}{2k_1}$$

$$(1)+(2) = \quad k_1(2A) = k_1D + k_1E + k_3D - k_3E$$

$$k_12A = D(k_1+k_3) + E(k_1-k_3)$$

$$A = D\frac{(k_1+k_3)}{2k_1} + E\frac{(k_1-k_3)}{2k_1}$$

$$k_3Ce^{ik_2a} = k_3De^{ik_3a} + k_3Ee^{-ik_3a} \quad (3)$$

$$(3)+(4) \Rightarrow Ce^{ik_2a}(k_2+k_3) = 2k_3De^{ik_3a}$$

$$\Leftrightarrow D = \frac{(k_2+k_3)}{2k_3}e^{ia(k_2-k_3)} \times C$$

$(3)-(4)$

$$\Rightarrow Ce^{ik_2(a)}(k_2-k_3) = 2k_3Ee^{-ik_3a}$$

$$\Rightarrow E = \frac{k_2-k_3}{2k_3}Ee^{ia(k_2+k_3)}$$