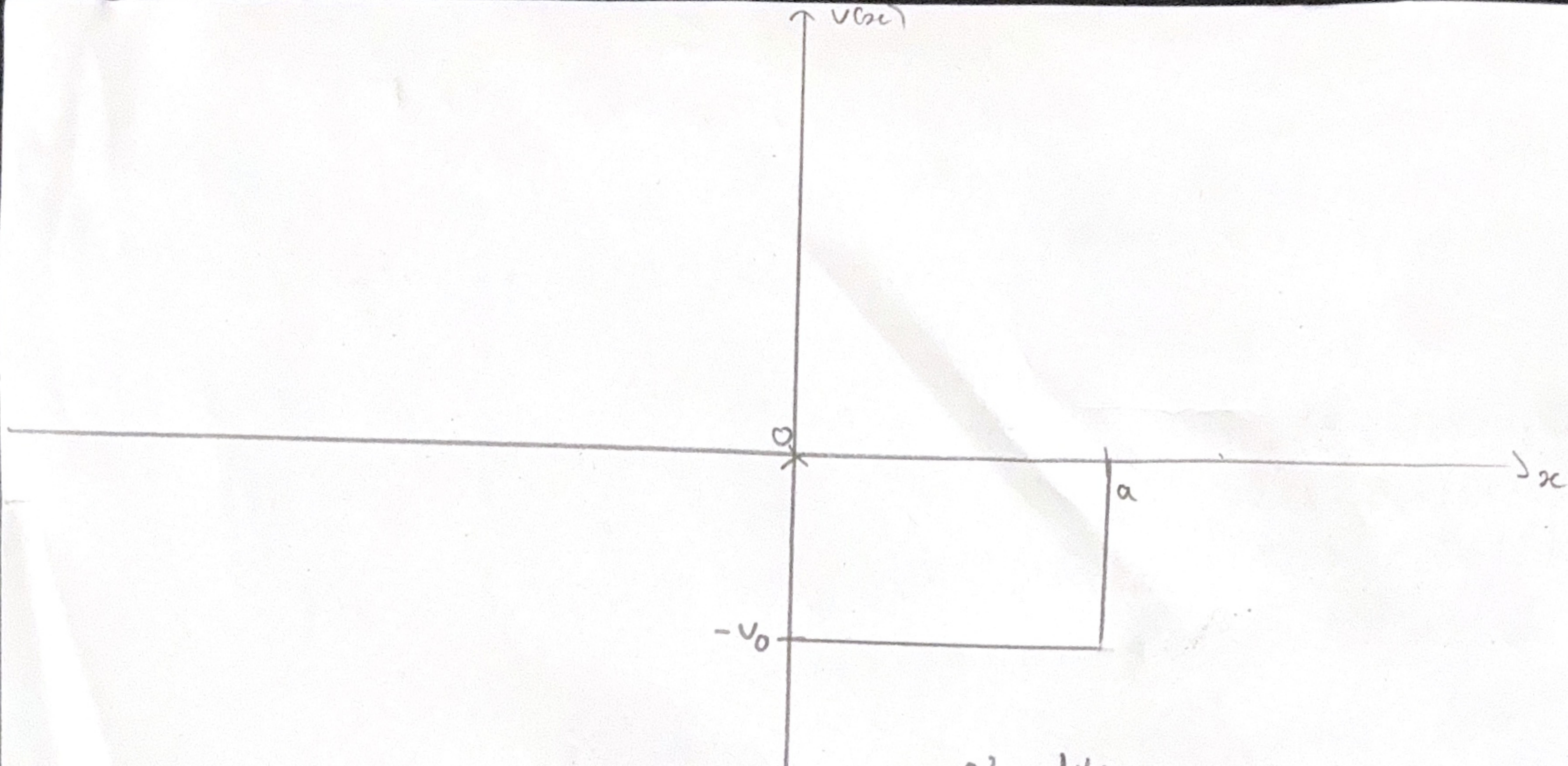

On pose l'équation de Schrödinger : $\frac{-\hbar^2}{2m}\frac{d\Psi}{dx^2} + V(x)\Psi = E\Psi$

Cas I ($x < -a$) :

$$\frac{-\hbar^2}{2m}\frac{d^2\Psi_1}{dx^2} = E\Psi_1$$

$$\Leftrightarrow \frac{d^2\Psi_1}{dx^2} = \frac{2mE\Psi}{-\hbar^2}$$

$$\Leftrightarrow \frac{d^2\Psi_1}{dx^2} + k_1^2\Psi_1 = 0, \quad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \Psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

Cas II ($-a \leq x \leq a$) :

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_2}{dx^2} - V_0\Psi_2 = E\Psi_2$$

$$\Leftrightarrow -\frac{\hbar^2}{2m}\frac{d^2\Psi_2}{dx^2} - (E+V_0)\Psi_2 = 0$$

$$\Leftrightarrow \frac{d^2\Psi_2}{dx^2} + \frac{2m(E+V_0)}{\hbar^2}\Psi_2 = 0$$

$$\Leftrightarrow \frac{d^2\Psi_2}{dx^2} + k_2^2\Psi_2 = 0, \quad k_2 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

$$\Rightarrow \Psi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$$

Cas III ($a < x$) :

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi_3}{dx^2} = E\Psi_3$$

$$\Leftrightarrow \frac{d^2\Psi_3}{dx^2} = \frac{2mE\Psi_3}{-\hbar^2}$$

$$\Leftrightarrow \frac{d^2\Psi_3}{dx^2} + k_3^2\Psi_3 = 0, \quad k_3 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \Psi_3(x) = Ee^{ikx} + Fe^{-ikx}$$

avec $F = 0$ car pas de réflexion à droite

$$\begin{cases} \psi_1(-a) = \psi_2(-a) \\ \psi_1'(-a) = \psi_2'(-a) \\ \psi_3(a) = \psi_2(a) \\ \psi_3'(a) = \psi_2'(a) \end{cases} \Leftrightarrow \begin{cases} A + B = C + D \quad (1) \\ ik_1 A - ik_1 B = ik_2 C - ik_2 D \quad (2) \\ E e^{ik_3 a} = C e^{ik_2 a} + D e^{-ik_2 a} \quad (3) \\ ik E e^{ika} = ik_2 C e^{ik_2 a} - ik_2 D e^{-ik_2 a} \quad (4) \end{cases}$$

$$k_1 = k_3 = k$$

$k_1(1) \Rightarrow k_1(A+B) = k_1(C+D) \; (5)$

$(5)+(2) \Rightarrow 2k_1 A = C(k_1+k_2) + D(k_1-k_2)$

$(5)-(2) \Rightarrow 2k_1 B = C(k_1-k_2) + D(k_1+k_2)$

$$\Rightarrow B = C\frac{(k_1-k_2)}{2k_1} + D\frac{(k_1+k_2)}{2k_1}$$

$$\Rightarrow A = C\frac{(k_1+k_2)}{2k_1} + D\frac{(k_1-k_2)}{2k_1}$$

$k_2(3) \Rightarrow k_2 E e^{ika} = k_2(C e^{ik_2 a} + D e^{-ik_2 a}) \; (6)$

$(6)-(3) \Rightarrow (k_2 - k) E e^{ika} = 2k_2 D e^{-ik_2 a}$

$$\Rightarrow D = \frac{(k_2 - k)}{2k_2} E e^{ia(k+k_2)}$$

$(6)+(3) \Rightarrow 2k_2 C e^{ik_2 a} = (k_2+k) E e^{ika}$

$$\Rightarrow C = \frac{(k_2+k)}{2k_2} E e^{ia(k-k_2)}$$

L'onde transmise T s'écrit:

$$T = \left|\frac{E}{A}\right|^2$$

$$A = E\left(\frac{(k_1+k_2)^2}{4k_1k_2} e^{ia(k-k_2)} - \frac{(k_1-k_2)^2}{4k_1k_2} e^{ia(k_1+k_2)}\right)$$

$$= \frac{E e^{iak}}{2k_1 k_2}\left(\frac{(k_1+k_2)^2}{2} e^{-iak_2} - \frac{(k_1-k_2)^2}{2} e^{iak_2}\right)$$

$$= \frac{E e^{iak}}{2k_1 k_2}\left(\frac{(k_1+k_2)^2}{2}\left(\cos(ak_2) - i\sin(ak_2)\right) - \frac{(k_1-k_2)^2}{2}\left(\cos(ak_2) + i\sin(ak_2)\right)\right)$$

$$\Rightarrow \Delta = \frac{E e^{iak_1}}{2k_1 k_2}\left(\cos(ak_2)\left(\frac{(k_1+k_2)^2-(k_1-k_2)^2}{2}\right) - i\sin(a k_2)\left(\frac{(k_1+k_2)^2+(k_1+k_2)^2}{2}\right)\right)$$

$$= \frac{E e^{iak}}{4k_1 k_2}\left(\cos(ak_2)\times 4kk_2 - i\sin(k_2 a)\left(2k_1^2+2k_2^2\right)\right)$$

$$= E e^{iak}\left(\cos(ak_2) - i\sin(k_2 a)\left(\frac{k_1^2+k_2^2}{2k_1 k_2}\right)\right)$$

Or, $T = \left|\frac{E}{\Delta}\right|^2 = \frac{|E|^2}{|\Delta|^2}$

Donc, $T = \frac{1}{\left(\cos^2(ak_2) + \left(\frac{k_1^2+k_2^2}{2k_1 k_2}\right)\sin^2(ak_2)\right)}$

Pour avoir l'effet ramsauer, on veut le coefficient de transmission le plus grand.

Pour $T=1$,

$\sin(ak_2) = 0$ et $\cos(ak_2) = 1$

$\Rightarrow ak_2 = n\pi \Rightarrow k_2 = \frac{n\pi}{a}$

On a:

$$k_2 = \sqrt{\frac{2m(V_0+E)}{\hbar^2}} \Leftrightarrow \left(\frac{n\pi}{a}\right)^2 = \frac{2m(V_0+E_n)}{\hbar^2}$$

$$\Leftrightarrow E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} - V_0$$