"I have pledged my honor that I have abided by the Stevens Honor System."
Cindy Zhang

TEST 2

MA 222 Probability and Statistics

Problem 1. It is a fact that deaths by traffic accidents occur at a rate of 8 per hour. Assuming independence,

Given:

Rate 8 deaths per hour.

Proposition: $P_k(t) = e^{-at} * (at)^k / k!$

a) compute the probability that one hour will pass without any death

a = 8 (deaths per hour)
t = 1 (hour)
k = 0
put in formula:

$$e^{-8(1)} \cdot \frac{(8(1))^0}{0!}$$

$$= 0.000335462627903$$

b) a 15-minute period would pass with no deaths

a = 8 (deaths per hour)
t = 0.25 (hour)
k = 0
put in formula:

$$e^{-8(0.25)} \cdot \frac{(8(0.25))^0}{0!}$$

$$= 0.135335283237$$

c) four consecutive 15-minute periods would pass with no deaths.

a = 8 (deaths per hour)
t = 0.25 * 4 (hour)
k = 0
put in formula:

$$e^{-8(1)} \cdot \frac{(8(1))^0}{0!}$$

$$= 0.000335462627903$$

Problem 2. The length (body tail) in inches (no decimals) of an adult squirrel in the forests of Somerset may be modeled with a Gaussian random variable with mean 16 inches and standard deviation of 1 inch. If you catch one of these squirrels what is the probability that it will be

Given:

Mean = 16 inches

Standard Deviation = 1 inch

a) at least 14 inches long?

Proposition: Let Z be a continuous rv with cdf $\Phi(z)$. Then for any a:

$P(a < Z) = 1 - \Phi(a)$

$P(Z < a) = \Phi(a)$

$P(X \leq 14)$:

$\frac{X-16}{1} \leq \frac{14\text{-}16}{1} \rightarrow X\text{-}16 < \text{-}2$

$P(X \leq 14) = 1\text{-}\ 0.2868 =$ 0.7133

b) between 12 and 15 inches long?

Proposition: Let Z be a continuous rv with cdf $\Phi(z)$. Then for any a and b with a < b,

$P(a < Z < b) = \Phi(b) - \Phi(a)$

$P(12 < X < 15)$:

$\frac{12-16}{1} < \frac{X-16}{1} < \frac{15-16}{1} \rightarrow \frac{-3}{1} < \frac{X-16}{1} < \frac{-1}{1}$

$P(12 < X < 15) = \Phi(0.2721) - \Phi(0.6875) =$ -0.0428

c) suppose you catch one of those squirrels with a length of 26 inches. Comment on this fact in view of the modeling assumptions.

Very Unlikely, its 10 standard deviations away.

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Problem 3. Suppose there are 2000 units of which 10% are known to be defective.

a) Find the exact probability that no more than 2 will be obtained in a sample (without replacement) of size 10.

Given:

N = 2000
n = 10
r = 200

$$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

P(X = 0) = p(0) = 0

$$P(X = 1) \rightarrow p(1) = \frac{\binom{200}{1}\binom{2000-200}{10-1}}{\binom{2000}{10}} =$$

P(X < 2) = P(0) + P(1) = 0 +

b) Find a good approximation to that exact probability. Check that the conditions to the approximation are satisfied.

Problem 4: X is a continuous random variable having the Uniform probability distribution between 5 and 10. A sample of four independent observations of X is taken, and the following gambling game is played: If all the observations are larger than 8, player wins $75. Otherwise, player loses $2. What is the expected gain of the player on a single play of this game?