*"I pledge my honor that I have abided by the Stevens Honor System."*
*- Cindy Zhang*

# HOMEWORK 2

MA 222 Probability and Statistics

**Problem One.**

| | | Event A's | | |
|---|---|---|---|---|
| | | **A1** | **A2** | **A3** |
| Event B's | **B1** | 14% | 20% | 26% |
| | **B2** | 20% | 10% | 10% |

a. What is the probability that the individual purchased a small cup? A cup of decaf coffee?

Event A1 = "Small Cup" → P (A1) = 14% + 20% = 34%

Event B1 = "Decaf Coffee" → P (B1) = 20% + 10% + 10% = 40%

b. If we learn that the selected individual purchased a small cup, what now is probability that he or she chose decaf coffee, and how would you interpret this probability?

We will use the same events from part a. Since A1 has occurred, it will be represented as P (B1 | A1) and using the equation, you will get 59%.

P(B1 | A1) = $\frac{P\ (A1\ N\ B1)}{P\ (A1)} = \frac{20\%}{34\%} = 59\%$

c. If we learn that the selected individual purchase decaf, what now is the probability that a small size was selected, and how does this compare to the corresponding unconditional probability of (a)?

P (A1 | B1) = $\frac{P\ (A1\ N\ B1)}{P\ (B1)} = \frac{20\%}{40\%} = 50\%$

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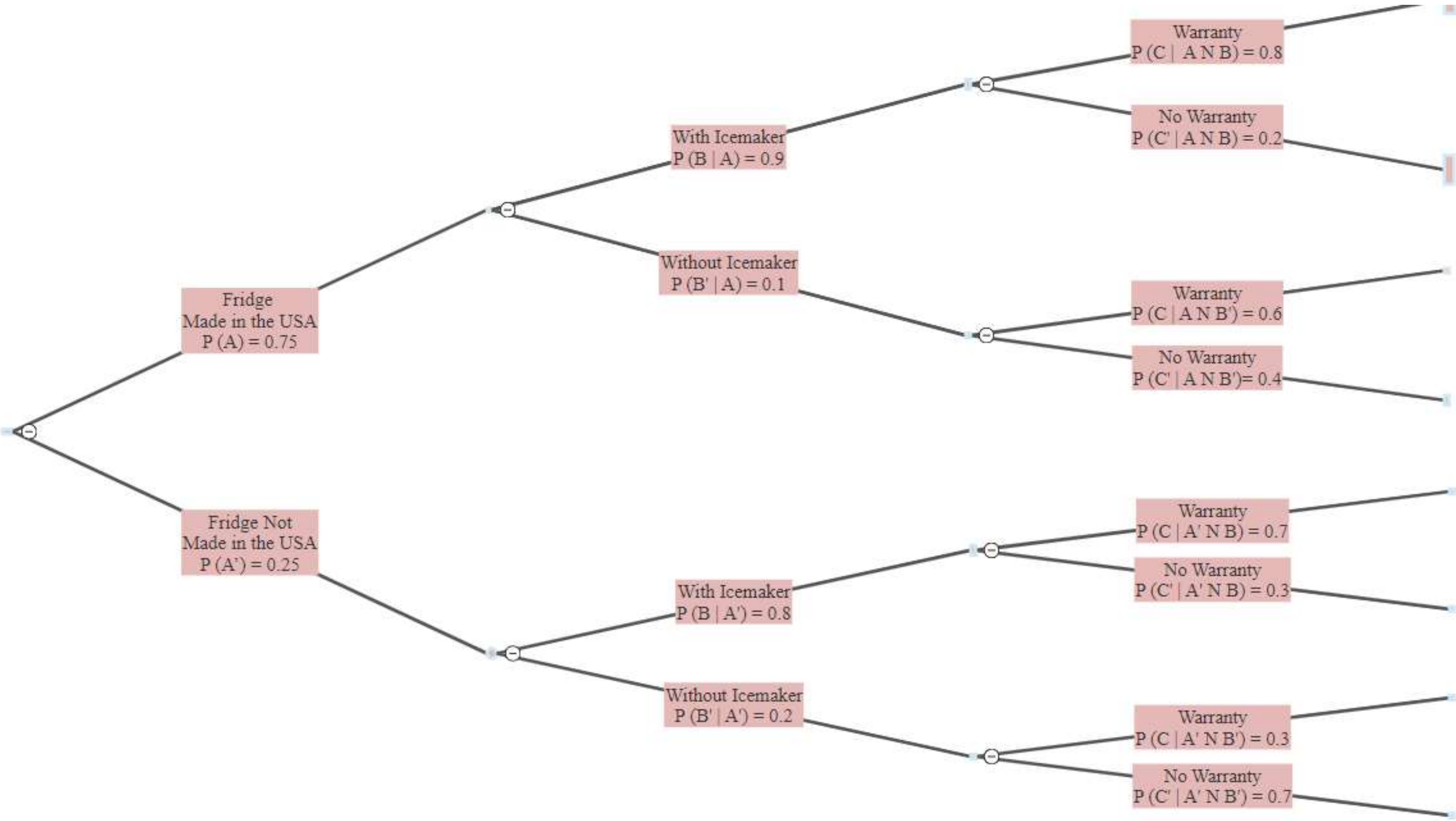
**Problem Two.**

Event A = Fridge Manufactured in the U.S
Event B = Fridge has Icemaker
Event C = Purchased Extended Warranty

a. Construct a tree diagram consisting of first-, second, and third-generation branches, and place an event label and appropriate next to each branch.




| | |
|---|---|
| Probability of Event A and A': | P (A) = 0.75 |
| | P (A') = 0.25 |
| Probability of Event B and B' (after A): | P (B \| A) = 0.9 |
| | P (B' \| A) = 0.1 |
| Probability of Event B and B' (after A'): | P (B \| A') = 0.8 |
| | P (B' \| A') = 0.2 |
| Probability of Event C and C' (after A N B): | P ( C \| A N B) = 0.8 |
| | P ( C' \| A N B) = 0.2 |
| Probability of Event C and C' (after A' N B): | P ( C \| A' N B) = 0.7 |
| | P ( C' \| A' N B) = 0.3 |
| Probability of Event C and C' (after A N B'): | P ( C \| A N B') = 0.6 |
| | P ( C' \| A N B') = 0.4 |
| Probability of Event C and C' (after A' N B'): | P ( C \| A' N B') = 0.3 |
| | P ( C' \| A' N B') = 0.7 |

b. Compute P (A N B N C)

P (A N B N C) = P (C | A N B) * P (A N B)
= P (C | A N B) * P (B | A) * P(A)
= (0.75)(0.9)(0.8)

= 0.54

c. Computer P (B N C)

P (B N C) = P (B N C N A) + P (B N C N A')
= 0.54 + (0.25)(0.8)(0.7)
= 0.68

d. Computer P (C)

P (C) = P(A N B N C) + P(A' N B N C) + P(A N B' N C) + P(A' N B' N C)
= 0.54 + (0.75)(0.1)(0.6) + (0.25)(0.8)(0.7) + (0.25)(0.2)(0.3)
= 0.54 + 0.45 + 0.14 + 0.015

=

e. Computer P(A | B N C), the probability of a U.S. purchase given that an ice maker and extended warranty are also purchased.

P(A | B N C) = $\frac{P\ (A\ N\ B\ N\ C)}{P\ (B\ N\ C)} = \frac{0.54}{0.68} = 0.794 = \sim 0.80$

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**Problem Three.**

| | | | | |
|---|---|---|---|---|
| Event A | A1 | Receiver Functions | P(A1) | 0.95 |
| | A2 | Speakers Functions | P(A2) | 0.98 |
| | A3 | CD Functions | P(A3) | 0.80 |

a. What is the probability that all three components function properly throughout the warranty period?

P(A1 N A2 N A3) = P(A1) * P(A2) * P(A3)
= (0.95)(0.98)(0.80)
= 0.7448 = ~0.75

b. What is the probability that at least one component needs service during the warranty period?

P(complement of part a) = 1 – 0.75 = 0.25

c. What is the probability that all three components need service during the warranty period?

Receiver does not function: P(A1') = 0.05
Speakers do not function: P(A2') = 0.02
CD does not function: P(A3') = 0.20

P(all need service) = P(A1' N A2' N A3')
= P(A1') * P(A2') * P(A3')
= 0.05 * 0.02 * 0.20
= 0.0002

d. What is the probability that only the receiver needs service during the warranty period?

P(only receiver needs service) = P(A1' N A2 N A3)
= P(A1') * P(A2) * P(A3)
= 0.05 * 0.98 * 0.80
= 0.0392

e. What is the probability that exactly one of the three components needs service during the warranty period?

P(A1' N A2 N A3) = 0.0392 (from part d)

P(Only Speakers Need Service) = P(A1 N A2' N A3)
= P(A1) * P(A2') * P(A3)
= 0.95 * 0.02 * 0.80
= 0.0152

P(Only CD Player Need Service) = P(A1 N A2 N A3')
= P(A1) * P(A2) * P(A3')
= 0.95 * 0.98 * 0.20
= 0.1862

P(Exactly one of the three) = 0.0392 + 0.0152 + 0.1862 = 0.2406

f. What is the probability that all three components function properly throughout the warranty period but that at least one fails within a month after the warranty expires?

It will either fail or not fail, so there's a 50%.
P (None of the component fail) = (50%)(50%)(50%) = 12.5%
P (One of the components fail) = 1 – 12.5% = 87.5%
P (None of the components fail during warranty) * P(One of the components fail) =
74.48% * 87.5% = 65.17%

**Problem Four.**

Event A1 = Likes Vehicle #1 → P(A1) = 0.55
Event A2 = Likes Vehicle #2 → P(A2) = 0.65
Event A3 = Likes Vehicle #3 → P(A3) = 0.70

P(A1 U A2) = 0.80
P(A2 N A3) = 0.40
P(A1 U A2 U A3) 0.88

a. What is the probability that the individual likes both vehicle #1 and vehicle #2?

Proposition: P(A U B) = P(A) + P(B) – P(A N B)

Solve for P(A N B), where A = A1, B = A2:

P(A1 N A2) = P(A1) + P(A2) – P(A1 U A2)
= 0.55 + 0.65 – 0.80
= 0.40

b. Determine and interpret P (A2 | A3)

Conditional Probability: $P(B\ |A) = \frac{P\ (A\ N\ B)}{P\ (A)}$

Solve for P(B | A), where A = A3, B = A2:

P(A2 | A3) = $\frac{P\ (A2\ N\ A3)}{P\ (A3)} = \frac{0.40}{0.70} = 0.57$

c. Are A2 and A3 independent events? Answer in two different ways.

Answer: No.

Explanation One: Property of Independent Events says that if A and B are independent events then P (B | A) is equal to P(B).
P(A2 | A3) = 0.57 (solved from in part b)
P(A2) = 0.65 (given.)
0.65 not equal to 0.57.

Explanation Two: Multiplication Rule for Independent Events says that if A and B are independent events then P(A N B) = P(A) * P(B)
P(A2 N A3) = 0.40 (given.)
P(A2) * P(A3) = 0.65 * 0.70 = 0.46
0.46 is not equal to 0.40

d. If you learn that the individual did not like vehicle #1, what now is the probability that he or she liked at least one of the other two vehicles?

Complement Rule: P(A') = 1 – P(A)

Solve for P(A'), where A = A1:

P(A1') = 1 – P(A1)
= 1 – 0.55
= 0.45

P ((A2 U A3) N A1') is P (A1 U A2 U A3) without A1. (If I drew a Venn Diagram.)

P((A2 U A3) N A1') = P(A1 U A2 U A3) – P(A1)
= 0.88 – 0.55
= 0.33

Conditional Probability: $P(B\,|A) = \frac{P\,(A\;N\;B)}{P\,(A)}$

Solve for P(B | A), where A = A1', B = A2 U A3:

P(A2 U A3 | A1') = $\frac{P\,((A2\;U\;A3) N\;A1')}{P\,(A1')} = \frac{0.33}{0.45} = 0.73$