“I pledge my honor that I have abided by the Stevens Honor System.”
- Cindy Zhang

HOMEWORK 3

MA 222 Probability and Statistics

Problem One.

Plane has

- 50 seats
- 55 passengers.

Y	45	46	47	48	49	50	51	52	53	54	55
P(Y)	.05	.10	.12	.14	.25	.17	.06	.05	.03	.02	.01

a. What is the probability that the flight will accommodate all ticketed passengers who show up?

Y has to less than or equal to 50, because there are 50 seats:

$$\begin{aligned} P(Y \leq 50) &= P(Y = 45) + P(Y = 46) + P(Y = 47) + P(Y= 48) + P(Y = 49) + P(Y = 50) \\ &= p(45) + p(46) + p(47) + p(48) + p(49) + p(50) \\ &= 0.05\ 0.1 + 0.12 + 0.12 + 0.14 + 0.25 + 0.17 \\ &= 0.83 \end{aligned}$$

b. What is the probability that not all ticketed passengers who show up can be accommodated?

Complement of part a, where Y is greater than 50:

$$\begin{aligned} P(Y > 50) &= 1 – P(Y \leq 50) \\ &= 1 – 0.83 \\ &= 0.17 \end{aligned}$$

c. If you are the first person on the standby list (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodate

   a. What is the probability that you will be able to take the flight?

   Y has to be less than or equal to 49. This leaves 1 last seat for you:

$$\begin{aligned} P(Y \leq 49) &= P(Y = 45) + P(Y = 46) + P(Y = 47) + P(Y= 48) + P(Y = 49) \\ &= p(45) + p(46) + p(47) + p(48) + p(49) \\ &= 0.05\ 0.1 + 0.12 + 0.12 + 0.14 + 0.25 \\ &= 0.66 \end{aligned}$$

   b. What is this probability if you are the third person on the standby list?

$$\begin{aligned} P(Y \leq 47) &= P(Y = 45) + P(Y = 46) + P(Y = 47) \\ &= p(45) + p(46) + p(47) \\ &= 0.05\ 0.1 + 0.12 + 0.12 \\ &= 0.27 \end{aligned}$$

Problem Two.

Metropolitan Are:

- 25% Insured against Earthquake damage.

Four homeowners are selected at random. Let X denote the number who have earthquake insurance.

a. Find the probability of X.

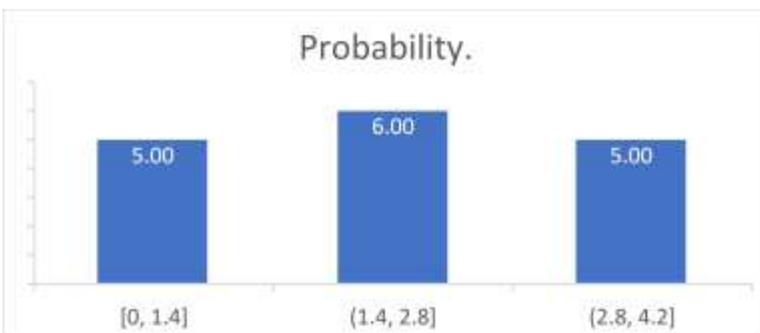
Use Binomial Distribution, P(X = k) = $\frac{4!}{k!(4-k)!} * 0.25^k(1-0.25)^{4-k}$

Sample Space:

{SSSS, SSSF, SSFS, SSFF,
SFSS, SFSF, SFFS, SFFF,
FSSS, FSSF, FSFS, FSFF,
FFSS, FFSF, FFFS, FFFF}

X = 4 → {SSSS} → 0.0039
X = 3 → {SSSF, SSFS, SFSS, FSSS} → 0.0469
X = 2 → {SSFF, SFSF, SFFS, FSSF, FSFS, FFSS} → 0.2109
X = 1 → {FFFS, FFSF, FSFF, SFFF} → 0.4219
X = 0 → {FFFF} → 0.3164

b. Draw the corresponding probability histogram.



X	F(x)
0	0.3164
1	0.4219
2	0.2109
3	0.0469
4	0.0039

c. What is the most likely value for X?

X = 1. Its probability is the highest.

d. What is the probability that at least two of the four selected have earthquake insurance?

P(X ≥ 2) = P(X = 2) + P(X = 3) + P(X = 4)
= 0.2109 + 0.0469 + 0.0039
= 0.2617

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Problem Three.

NBC News reported on May 2, 2013.

- 1 in 20 children in U.S. have some food allergy
- Consider selecting a random sample of 25 children
- Let X be the number in the sample who have a food allergy.

Then X ~ Bin(25, 0.05)

a. Determine both $P(X \leq 3)$ and $P(X < 3)$.

$$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$$
$$= \binom{25}{0}(0.05)^0(1-0.05)^{25-0} + \binom{25}{1}(0.05)^1(1-0.05)^{25-1}$$
$$+ \binom{25}{2}(0.05)^2(1-0.05)^{25-2} + \binom{25}{3}(0.05)^3(1-0.05)^{25-3}$$
$$= (1)(1)(0.95)^{25} + (25)(0.05)(0.95)^{24} + (300)(0.0025)(0.95)^{23} + (2300)(0.000125)(0.95)^{22}$$
$$= 0.2774 + 0.3650 + 0.2305 + 0.0930$$
$$= 0.97$$

$$P(X < 3) = P(0) + P(1) + P(2)$$
$$= 0.2774 + 0.3650 + 0.2305$$
$$= 0.87$$

b. Determine $P(X \geq 4)$.

$$P(X \geq 4) = P(4) + P(5) + P(6) + \ldots + P(25)$$
$$= 1 - P(X < 4)$$
$$= 1 - P(X \leq 3)$$
$$= 1 - 0.97$$
$$= 0.03$$

c. Determine $P(1 \leq X \leq 3)$.

$$P(1 \leq X \leq 3) = P(X \leq 3) - P(0)$$
$$= 0.97 - 0.2774$$
$$= 0.69$$

d. What is $E(X)$ and o_x?

$$E(X) = (25)(0.05)$$
$$= 1.25$$
$$O_x = \sqrt{(25)(0.05)(1-0.05)}$$
$$= 1.089$$

e. In a sample of 50 children, what is the probability that none has a food allergy?

$$P(X = 0) = P(0)$$
$$= \binom{50}{0}(0.05)^0(1-0.05)^{50-0}$$
$$= (1)(1)(0.0769)$$
$$= 0.0769$$

Problem Four.

A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified as "seconds"

a. How likely is it that only one is a second?

Theorem: b(x; n, p) = $\begin{cases} \binom{n}{x} p^x (1-p)^{n-x} \\ 0 \end{cases}$, x = 0, 1, 2, …, n

b(1;6, 0, 1) = P(X = 1) = $\binom{6}{1} 0.1^1 (1-0.1)^{6-1} = 0.3543$

b. What is the probability that at least two are seconds?

B(x; n, p) = P(X ≤ x) = $\sum_{y=0}^{x} b(y;\ n, p), x = 0, 1, \ldots, n$

P(X ≥ 2) = 1 – P(X < 2) = 1 – P(X ≤ 1)
= 1 – B(1;6, 0.1) = 1 – b(1;6, 0.1) – b(0;6, 0.1)
= 1 – 0.53 – 0.35
= 0.12

c. If goblet is examined one by one, what is the probability that at most five must be selected to find four that are not seconds?

P(X = 0) = $\binom{4}{0} 0.1^0 (1-0.1)^{4-0} = 0.6561$

Second event is $\binom{4}{1} 0.1^1 (1-0.1)^{4-1} * 0.9 = 0.26244$

0.66 + 0.26 = 0.92

Problem Five.

Each time a component is tested:

- Success (S)
- Failure (F)

List all outcomes corresponding to the five smallest possible values of Y, and state which Y value is associated with each one.

Y = 3 → {SSS}
Y = 4 → {FSSS}
Y = 5 → {FFSSS, SFSSS}
Y = 6 → {SSFSSS, SFFSSS, FSFSSS, FFFSSS}
Y = 7 → {SSFFSSS, SFSFSSS, SFFFSSS, FSSFSSS, FSFFSSS, FFSFSSS, FFFFSSS}

Problem Six.

Game:

- First player tosses three fair coins
- Second, two fair coins.

How players win:

- The winner, who gets all five coins, is the one who scores more heads.

In the case of a tie the game is repeated until there is a decisive result. What is the expectation of winning for each of the players?

First player wins if:	Second player wins if:	Tie if
• 3 heads to 2 heads • 3 heads to 1 head • 3 heads to 0 heads • 2 heads to 1 head • 2 head to 0 heads • 1 head to 0 heads	• 2 heads to 1 head • 2 heads to 0 heads • 1 head to 0 heads	• 2 heads to 2 heads • 1 head to 1 head • 0 head to 0 heads
• 3 → HHH • 2 → HHT, THH, HTH • 1 → HTT, THT, TTH • 0 → TTT	• 2 → HH • 1 → HT, TH • 0 → TT	

Player 1 has a ½ chance of winning.

- 3 heads = 1/8 → 1/8
- 2 heads = 3/8 → (3/8)((1/4)+(2/4)) = 9/32
- 1 head = 3/8 → (3/8)(1/4) = 3/32

Add up to → 16/32, simplifies to 1/2

Player 2 has a

- 2 heads = ¼ → (1/4)((3/8) + (1/4)) = 4/32
- 1 head = ½ → (1/2)(1/4) = 2/32

Add up to → 6/32, simplifies to 3/16

Expectation of Winning (First Player):

E(X) = (n)(p) = (1/2)(2) – (3/16)(3) = 7/16

Expectation of Winning (Second Player):

E(X) = (n)(p) = (3/16)(3) – (1/2)(2)= -7/16