"I pledge my honor that I have abided by the Stevens Honor System."
- Cindy Zhang

HOMEWORK 1

MA 222 Probability and Statistics

Problem One.

Definition: Sample Space of an experiment is the set of all possible **outcomes.**

2-3 Subsystem Sample Space S = {SSS, SSF, SFS, SFF, FSS, FSF, FFS, FFF}

a. Which outcomes are contained in the event A that exactly two out of the three components function? (In other words, event A contains two S's)

   Event A = {SSF, SFS, FSS}

b. Which outcomes are contained in the event B that at least two of the components function? (In other words, event B contains two or more S's)

   Event B = {SSS, SSF, SFS, FSS}

c. Which outcomes are contained in the event C that the system functions? (In other words, event C must have at least two S's one being at the front.}

   Event C = {SSS, SSF, SFS}

d. List outcomes in
   a. C' (Complement of Event C or Every outcome not in C)

      Event C' = {SFF, FSS, FSF, FFS, FFF}

   b. A U C

      Event A U C = {SSS, SSF, SFS, FSS}

   c. A N C

      Event A N C = {SSF, SFS}

   d. B U C

      Event B U C = {SSS, SSF, SFS, FSS}

   e. B N C

      Event B N C = {SSS, SSF, SFS}

Problem Two.

a. List the outcomes in S.

   Library Sample Space S = {123, 124, 125, 213, 214, 215, 13, 14, 15, 23, 24, 25, 3, 4, 5}

b. Let A denote the event that exactly one book must be examined. What outcomes are in A?

   Event A = {3,4,5}

c. Let B be the event that book 5 is the one selected. What outcomes are in B?

   Event B = {125, 215, 15, 25, 5}

d. Let C be the event that book 1 is not examined. What outcomes are in C?

   Event C = {23, 24, 25, 3, 4, 5}

Problem Three.

Money-Market Fund	Money-Market	20%
Bond Funds	Short Bond	15%
	Intermediate Bond	10%
	Long Bond	05%
Stock Funds	High-Risk Stock	18%
	Moderate-Risk Stock	25%
Balanced Funds	Balanced	07%

a. What is the probability that the selected individual owns shares in the balance fund?

   Balance Funds = Balanced = 7%

b. What is the probability that the individual owns shares in a bond fund?

   Bond Funds = Short Bond + Intermediate Bond + Long Bond
   = 15% + 10% + 5% = 30%

c. What is the probability that the selected individual does not own shares in the stock fund?

   Not Stock Funds = Everything – High-Risk Stock – Moderate-Risk Stock
   = 100% - 18% - 25%
   = 57%

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Problem Four.

X	P(X)
A1	0.22
A2	0.25
A3	0.28
A1 N A2	0.11
A1 N A3	0.05
A2 N A3	0.07
A1 N A2 N A3	0.01

Proposition: P(A U B) = P(A) + P(B) – P(A N B)

a. A1 U A2

P (A1 U A2) = P (A1) + P (A2) – P (A1 N A2)
= 0.22 + 0.25 – 0.11
= 0.36

b. A1’ N A2’ OR (A1 U A2)’

P (A1 U A2)’ = 1 – P (A1 U A2)
= 1 – 0.36
= 0.64

c. A1 U A2 U A3

P (A1 U A2 U A3) = P (A1) + P (A2) + P (A3) – P (A1 N A2) – P (A1 N A3) – P (A2 N A3) + P (A1 N A2 N A3)
= 0.22 + 0.25 + 0.28 – 0.11 – 0.05 – 0.07 + 0.01
= 0.53

d. A1’ N A2’ N A3’

P (A1’ N A2’ N A3’) = P ((A1 U A2 U A3)’)
= 1 – P (A1 U A2 U A3)
= 1 – 0.53
= 0.47

e. A1’ N A2’ N A3

P (A1’ N A2’ N A3) = P (A3) – P(A1 N A3) – P(A2 N A3) + P(A1 N A2 N A3)
= 0.28 – 0.05 + 0.07 + 0.01
= 0.17

f. (A1’ N A2’) U A3

P((A1’ N A2’) U A3) = P (A1’ N A2’ N A3) + P(A3)
= 0.47 + 0.28
= 0.75

Problem Five.

		Unsafe Conditions	Unrelated to Condition
Shift	**Day**	10%	35%
	Swing	8%	20%
	Night	5%	22%

a. What are the simple events?

Definition: An event is simple if it consists of exactly one outcome.

Let us have:

- Event A1 = {Day Shift}
- Event A2 = {Swing Shift}
- Event A3 = {Night Shift}
- Event B1 = {Unsafe Conditions}
- Event B2 = {Unrelated to Conditions}

Simple Events are combinations of all the events above:

S = {{A1,B1}, {A2,B1}, {A3,B1}, {A1,B2}, {A2,B2}, {A3,B2}}

b. What is the probability that the selected accident was attributed to unsafe conditions?

P(B1) = P({A1,B1}, {A2,B1}, {A3,B1})
= P({A1,B1}) + P({A2,B1}) + P({A3,B1})
= 10% + 8% + 5%
= 23%

c. What is the probability that the selected accident did not occur on the day shift?

P(A1') = 1 – P(A1)
= 1 – P({A1,B1}, {A1,B2})
= 1 – [P({A1,B1}, {A1,B2})]
= 1 – 10% - 35%
= 55%

Problem 6.

a. How many such chain molecules are there? [Hint: If the three A's were distinguishable from one another—A1, A2, A3—and the B's, C's, and D's were also, how many molecules would there be? How is this number reduced when the subscripts are removed from the A's?]

Since we have 3 A's, 3 B's, 3 C's and 3 D's, that is 12 places. In a chain molecule, **order** is important.

Definition: A permutation is an **order** subset. The proposition:

n = individuals in a group = 12
k = number of permutations = 12
$P_{k,n}$ = n! / (n – k)!
= 12! / (12 – 12)!
= 479,001,600 possible chain molecules

If we remove the subscripts:

n = 3
k = 3
$P_{k,n}$ = n! / (n – k)!
= 3! / (3 – 3)!
= 6

That means that each group of 6 becomes 1 outcome when we remove the subscript.

12! / 3! (For getting rid of A's subscripts)

12! / 3!3!3!3! (For getting of all subscripts)

Using the equation above, there are 369,600 chain molecules.

b. Suppose a chain molecule of the type described is randomly selected. What is the probability that all three molecules of each type end up next to one another (such as in BBBAAADDDCCC)?

P (A) = Favorable Outcomes / Sample Space Outcomes

Favorable Outcomes: $P_{4,4}$ = 4! = 24

Sample Space Outcome: From Part (a), 369,600

P (A) = 24 / 369,600 = 0.00006494