"I have pledged my honor that I have abided by the Stevens Honor System."
Cindy Zhang

HOMEWORK 9

MA 222 Probability and Statistics

Problem One. In a random sample of 80 components of a certain type, 12 are found to be defective.

a. Give a point estimate of the porportion of all such components that are not defective.

   Given:

   - 80 Components
   - 12 Defective
   - 68 Not Defective

   P = 68 / 80 = 0.85

b. A system is to be contructed by randomly selecting two of these components and connecting them in series, as shown here.

   The series connection implies that the system will function if and only if neither component is defective (i.e., both component work properly). Estimate the proportion of all such systems that work properly. [Hint: If p denotes the probability that a component works properly, how can P (system works) be expressed in terms of p?]

   P(system functions) = p * p = $p^2 \rightarrow (0.85)^2 =$ 0.723

Problem Two. Each of the 150 newly manufactured items is examined and the number of scratches per item is recorded (the items are suppose to be free of scratches), yeilding the following data:

Let X = the number of scractches on a randomly chosen item, and assume that X has a Posisson Distribution with parameter mu.

a. Find an unbias estimator of mu and compute the estimate for the data . [Hint: E(X) = mu for X Poisson, so E(X) = ?]

   E(X) = V(X) = mu

$$E(X) = E(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n}\sum_{i=1}^{n} E(X_{i)} = \frac{1}{n} * n * E(X1)$$

$$n = 150$$

   x = $\frac{1}{150}(0 * 18 + 1 * 37 + 2 * 42 + 3 * 30 + \ldots + 7 * 1) = 2.11$

b. What is the standard deviation (standard error) of your estimator? Comput the estimated standard error.

   V(X) = mu / n

   $\sigma = \sqrt{\frac{mu}{n}} = \sqrt{\frac{2.11}{150}} = 0.12$

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Problem Three. Let $X_1, X_2, \ldots, X_n$ represent a random sample from a rayleigh distribution with pdf

a. It can be showned that $E(X^2) = 2(\text{theta})$. Use this fact to construct an unbiased estimator of theta.

   $E(X^2) = 2(\text{theta}) \rightarrow \frac{\sum X_i^2}{2n}$

   $E(\text{theta}) = E(\frac{\sum X_i^2}{2n}) = \frac{1}{2n} E(\sum_{i=1}^{n} X_i^2) = \frac{1}{2n} \sum_{i-1}^{n} E(X_i^2) = \theta$

b. Estimate theta from the following n = 10 observations on the vibratory stress of a turbine blade under specified conditions:

   n = 10

$$\sum_{i=1}^{10} x_i^2 = 16.88^2 + 10.23^2 + \ldots + 10.95^2 = 1490.1058$$

$$\theta = \frac{1}{20} * 1490.1058 = 74.51$$

Problem Four. Let X denote the proportion of alloted time that randomly selected student spends working on a certain aptitude test. Suppose the pdf of X is where -1 < theta. A random sample of ten students yields data $x_z = 0.92$, $x_2 = 0.79$

a. Use the method of moments to obtain an estimator of theta, and then compute the estimate for this data.

   Let the random variables have same distribution with pmf or pdf where the parameters are unknown.

   Random Variables = X_1, X_2, X_3, etc..

   X = E(X)

$$E(X) = \int_0^1 x(\theta + 1)x^{\theta}dx = (\theta + 1) * \frac{x^{\theta} + 2}{\theta + 2}\bigg|_0^1 = \frac{\theta + 1}{\theta + 2}$$

$$X = E(X) = \frac{\theta + 1}{\theta + 2} = \frac{\theta + 1 + (1 - 1)}{\theta + 2} = \frac{\theta + 2}{\theta + 2} - \frac{1}{\theta + 2}$$

$$X - 1 = \frac{1}{\theta + 2}$$

$$\theta + 2 = \frac{1}{1 - X}$$

$$\theta = \frac{1}{1 - X} - 2$$

   Sample mean:

$$x = \frac{x1 + x2 + \ldots + xn}{n} = \frac{1}{10}(0.92 + 0.79 + \ldots + 0.88) = 0.8$$

$$\theta = \frac{1}{1 - 0.8} - 2 = 3$$

b. Obtain the maximum likelihood estimator of theta, and then compute the estimate for the given data.

   Let random variables have joint pdf or pmb where parameters are unknown.

   Likelihood Function is when function f is a function of parameters $\theta_i, i = 1,2,3,4\ldots,m$

   Maximum likelihood estimators are obtained by substituting X_i with x_i.

$$f(x_1,x_2,\ldots,x_n;\theta) = (\theta + 1)x_1^{\theta} * (\theta + 1)x_2^{\theta} * \ldots * (\theta + 1)x_n^{\theta}$$

$$= (\theta + 1)^2(x_1 * x_2 * \ldots * x_n)\,\theta$$

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$\ln f(x_1,x_2,\ldots,x_n;\theta) = \ln[(\theta+1)^2(x1 * x2 * \ldots * xn)\,\theta]$

$= n * \ln(\theta+1) + \theta \sum_{i=1}^{n} \ln X_i$

Derivative:

$\frac{d}{d\theta} f(x_1,x_2,\ldots,x_n;\theta) = \frac{d}{d\theta}[2 * \ln(\theta+1) + \theta * \sum_{i=1}^{n} \ln x_i]$

$$= n * \frac{1}{\theta+1} + \sum_{i=1}^{n} \ln(X_i)$$

$$n * \frac{1}{\theta+1} + \sum_{i=1}^{n} \ln(X_i) = 0$$

$$\theta = -\frac{n}{\sum_{i=1}^{n} \ln X_i} - 1$$

Using the formula:

$$\theta = -\frac{10}{-2.4295} - 1 = 3.12$$