"I have pledged my honor that I have abided by the Stevens Honor System."
Cindy Zhang

HOMEWORK 10

MA 222 Probability and Statistics

Problem One. Assume that the helium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with true standard deviation 0.75.

a. Compute a 95% CI for the true average porosity of a certain seam if the average porosity for 20 specimen from the seam was 4.85.

Given:

- Standard Devation = 0.75
- n = 20
- x = 4.85
- α = 0.05
- $z_{0.05/2} = z_{0.025} = 1.96$
- $P(Z > z_{0.025}) = 0.025$

$$\left(x - z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}, x + z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(4.85 - 1.96 * \frac{0.75}{\sqrt{20}}, 4.85 + 1.96 * \frac{0.75}{\sqrt{20}}\right)$$

$$= (4.52, 5.18)$$

b. Compute a 98% CI for the true average porosity of a certain seam if the average porosity for 20 specimen from the seam was 4.56.

Given:

- Standard Devation = 0.75
- n = 16
- x = 4.56
- α = 0.02
- $z_{0.02/2} = z_{0.01} = 2.33$
- $P(Z > z_{0.01}) = 0.01$

$$\left(x - z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}, x + z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(4.56 - 2.33 * \frac{0.75}{\sqrt{16}}, 4.56 + 2.33 * \frac{0.75}{\sqrt{16}}\right)$$

$$= (4.12, 5)$$

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c. How large a sample size is necessary if the width of the 95% interval is to be 0.40?

$$\left(x - z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}, x + z_{\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{n}}\right)$$

$$n = \left(2z_{\frac{\alpha}{2}} * \frac{\sigma}{w}\right)^2$$

Given:

- w = 0.4
- $z_{0.025}$ = 1.96

$$n = \left(2z_{\frac{\alpha}{2}} * \frac{\sigma}{w}\right)^2 = \left(2 * 1.96 * \frac{0.75}{0.4}\right)^2 = 55$$

d. What sample size is necssary to estimate the true porosity to within 0.2 with 99% confidentce?

Given:

- w = 2 * 0.2 = 0.4
- $z_{0.005}$ = 2.58

$$n = \left(2z_{\frac{\alpha}{2}} * \frac{\sigma}{w}\right)^2 = \left(2 * 1.96 * \frac{0.75}{0.4}\right)^2 = 94$$

Problem Two. By how much must the sample size n be increase if the width of the CI (7.5) is to be halved? If the sample size is increased by a factor of 25, what effect will this have on the width of the interval? Justify your assertions.

Normal Population → 100(1- α)% confidence interval for the mean when the variance is known:

$$\left(x - z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}, x + z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}\right)$$

For the confidence interval to have with w, the sample size n is calculate with:

$$n = \left(2z_{\frac{\propto}{2}} * \frac{\sigma}{w}\right)^2$$

This means, the smaller the width the larger the sample size, and the larger the standard deviation, the larger the sample size. This imples that with can be calculate with this equation:

$$w = 2z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}$$

Therefore when the new width is halved or w/2, the sample size can be found by either equation, we will adjust the sample size to be 4 times of it's original inorder to obtain the w/2:

$$w' = 2z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n'}} = w = 2z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{4n}} = \frac{1}{2}\left[2z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}\right] = \frac{w}{2}$$

The sample size n needs to be increase by 4 times.

If sample size increase by 25 or n' = 25n then the width would be decreasing by a factor of 5:

$$w' = 2z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n'}} = w = 2z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{25n}} = \frac{1}{5}\left[2z_{\frac{\propto}{2}} * \frac{\sigma}{\sqrt{n}}\right] = \frac{w}{5}$$

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Problem Three. The superintendent of a large school district, having once had a course in probability and statistics, believes that the number of teachers absent on any given day has a Poisson distribution with parameters mu. Use the accompanying data on absences for 50 days to obtain a large sample CI for mu. [Hint: The mean and the variance of a Poisson variable both equal mu, so has approximately a standard normal distribution. Now proceed as in the derivation of the interval for p by making a probability statement (with probability 1 – α) and solving the resulting inequalities for mu.]

n = 50

$$Z = \frac{X - \mu}{\sqrt{\mu/n}}$$

$$Z\sqrt{\frac{\mu}{n}} = X - \mu$$

$$Z\sqrt{\frac{\mu}{n}} + \mu = X$$

$$\mu = X - Z\sqrt{\frac{\mu}{n}}$$

$$\mu = X - z_{\frac{\propto}{2}}\sqrt{\frac{\mu}{n}}$$

$$x = \frac{\sum xf}{n}$$
$$= \frac{(0)(1) + (1)(4) + (2)(8) + (3)(10) + (4)(8) + (5)(7) + (6)(5) + (7)(3) + (8)(2) + (9)(1) + (10)(1)}{1 + 4 + 8 + 10 + 8 + 7 + 5 + 3 + 2 + 1 + 1}$$
$$= \frac{203}{50} = 4.06$$

$$\left(x - z_{\frac{\propto}{2}} * \sqrt{\frac{\mu}{n}}, x + z_{\frac{\propto}{2}}\sqrt{\frac{\mu}{n}}\right)$$

$$\left(4.06 - 1.96\sqrt{\frac{4.06}{50}}, 4.06 + 1.96\sqrt{\frac{4.06}{50}}\right)$$

$$(3.5015, 4.6185)$$

Problem Four. The melting point of each of the 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in x = 94.32. Assume that the distribution of the melting point is normal with σ = 1.20.

a. Test H_0: μ = 95 versus H_a: μ ≠ 95 using a two tailed level 0.01 test.

$$Z = \frac{X - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z = \frac{94.32 - 95}{1.2\sqrt{16}} = -2.27$$

$$P(Z > -z \; and \; Z < z) = P(Z > 2.27 \; and \; Z < -2.27) = 2 * \phi(-2.27)$$
$$= 2 * 0.0116\, m = 0.0232$$

Since the P value 0.0232 is bigger than 0.01, DO NOT reject hypothesis H_0.

b. If a level .01 test is used, what is b(94), the probability of a type II error when μ = 94?

$$\beta(\mu') = \phi\left(z_{\frac{\alpha}{2}} + \frac{\mu_0 - \mu'}{\frac{\sigma}{\sqrt{n}}}\right) - \phi\left(z_{\frac{\alpha}{2}} + \frac{\mu_0 - \mu'}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$\beta(94) = \phi\left(2.58 + \frac{95 - 94}{\frac{1.2}{\sqrt{16}}}\right) - \phi\left(2.58 + \frac{95 - 94}{\frac{1.2}{\sqrt{16}}}\right) = \phi(5.91) - \phi(0.75)$$
$$= 0.9999 - 0.7733 = 0.2266$$

c. What value of n is necessary to ensure that b(94) = .1when α = .01?

$$n = \left[\frac{\sigma(z_{\frac{\alpha}{2}} + z_\beta)}{\mu_0 - \mu'}\right]^2$$

$$n = \left[\frac{1.2(2.58 + 1.28)}{95 - 94}\right]^2 = 21.46$$

$$n = 22$$

Problem Five. Body armor provides critical protection for law enforcement personnel, but it does affect balance and mobility. The article "Impact of Police Body Armour and Equipment on Mobility" (Applied Ergonomics, 2013: 957–961) reported that for a sample of 52 male enforcement officers who underwent an acceleration task that simulated exiting a vehicle while wearing armor, the sample mean was 1.95 sec, and the sample standard deviation was .20 sec. Does it appear that true average task time is less than 2 sec? Carry out a test of appropriate hypotheses using a significance level of .01.

Given:

- n = 52
- x = 1.95
- s = 0.20
- α = 0.01

Given Claim : "Average is less than 2 seconds"

Null Hypothesis (H_0) : $\mu = 2$

Alternative Hypothesis (H_a) : $\mu < 2$

Sample mean : μ

Sample Standard Deviation : $\frac{\sigma}{\sqrt{n}}$

Sample z-value : $Z = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}} = Z = \frac{1.95-2}{\frac{0.2}{\sqrt{52}}} = -1.80$

Sample P-value : $Z = P(Z <= 1.80) = 0.0359$

If P value smaller than the significance level (0.01) then the null hypothesis is rejected but in this case, there is not sufficient evidence to support the claim.