Time Value of Money

Chapter 2

Outline

- Cash flow patterns
- Future values and present values
- Annual percentage rate (APR) and effective annual rate (EAR)
- Amortized loan
- References: BF Chap 7; PF Chap 9

Time Value of Money

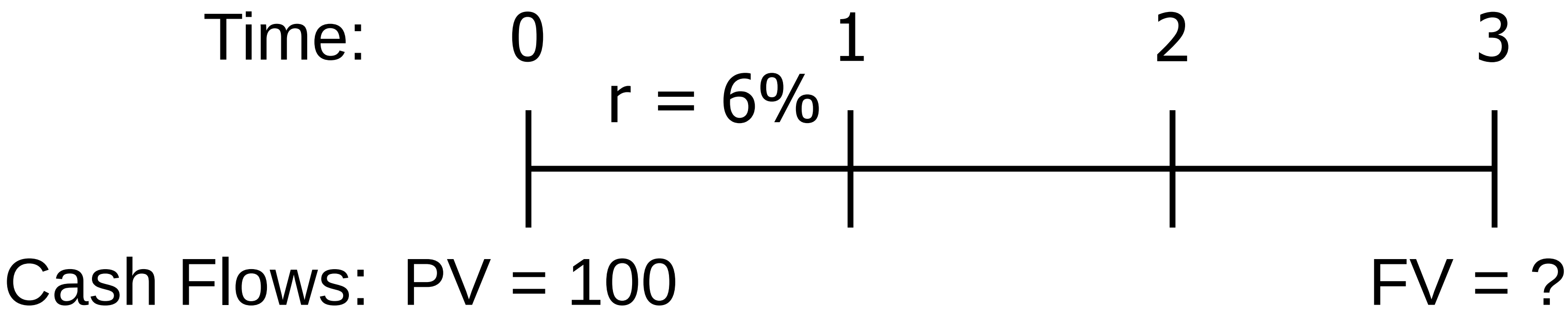
- The principles and computations used to revalue cash payoffs at different times so they are stated in dollars of the same time period
- The most important concept in finance used in nearly every financial decision
  - Business decisions
  - Personal finance decisions

Cash Flow Patterns

- Lump-sum amount
  - A single payment paid or received in the current period or some future period
- Annuity
  - A series of equal payments that occur at equal time intervals
- Uneven cash flow stream
  - Multiple payments that are not equal, do not occur at equal intervals, or both conditions exist

Cash Flow Timelines

Graphical representations used to show timing of cash flows:



Time 0 is today, Time 1 is the end of Period 1 (beginning of Period 2), and so forth.

Future Value

- How much would you have at the end of one year if you deposit $700 in a bank account that pays 10% interest each year?

$$FV_n = FV_1 = PV + INT$$

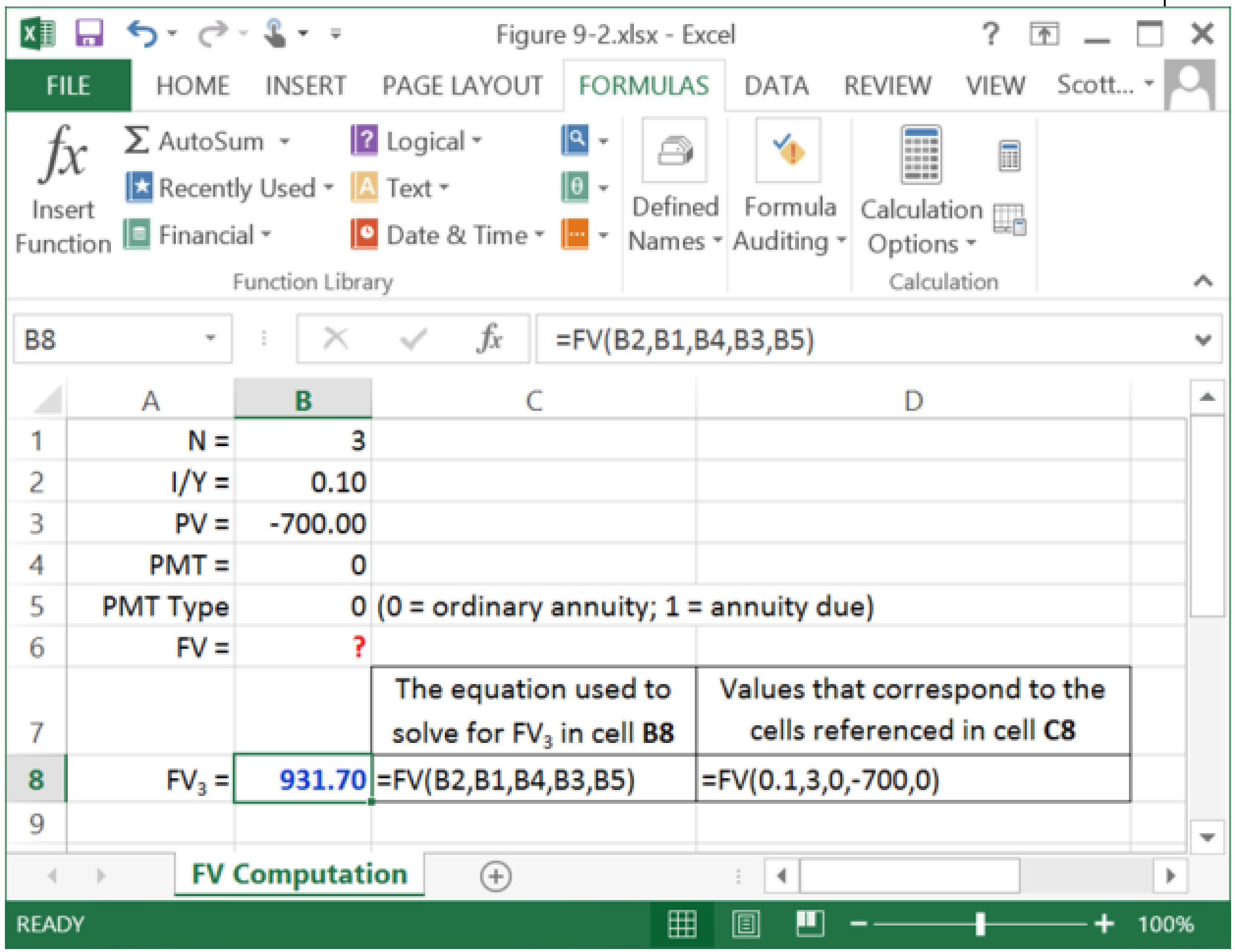
$$= PV\,(1 + r)$$

$$= \$700(1 + 0.10) = \$100(1.10) \boxed{= \$770}$$

In general, $FV_n = PV(1 + r)^n$

Spreadsheet Solution

The input values must be entered in a specific order: I/Y, N, PMT, PV, and PMT type (not used for this problem).

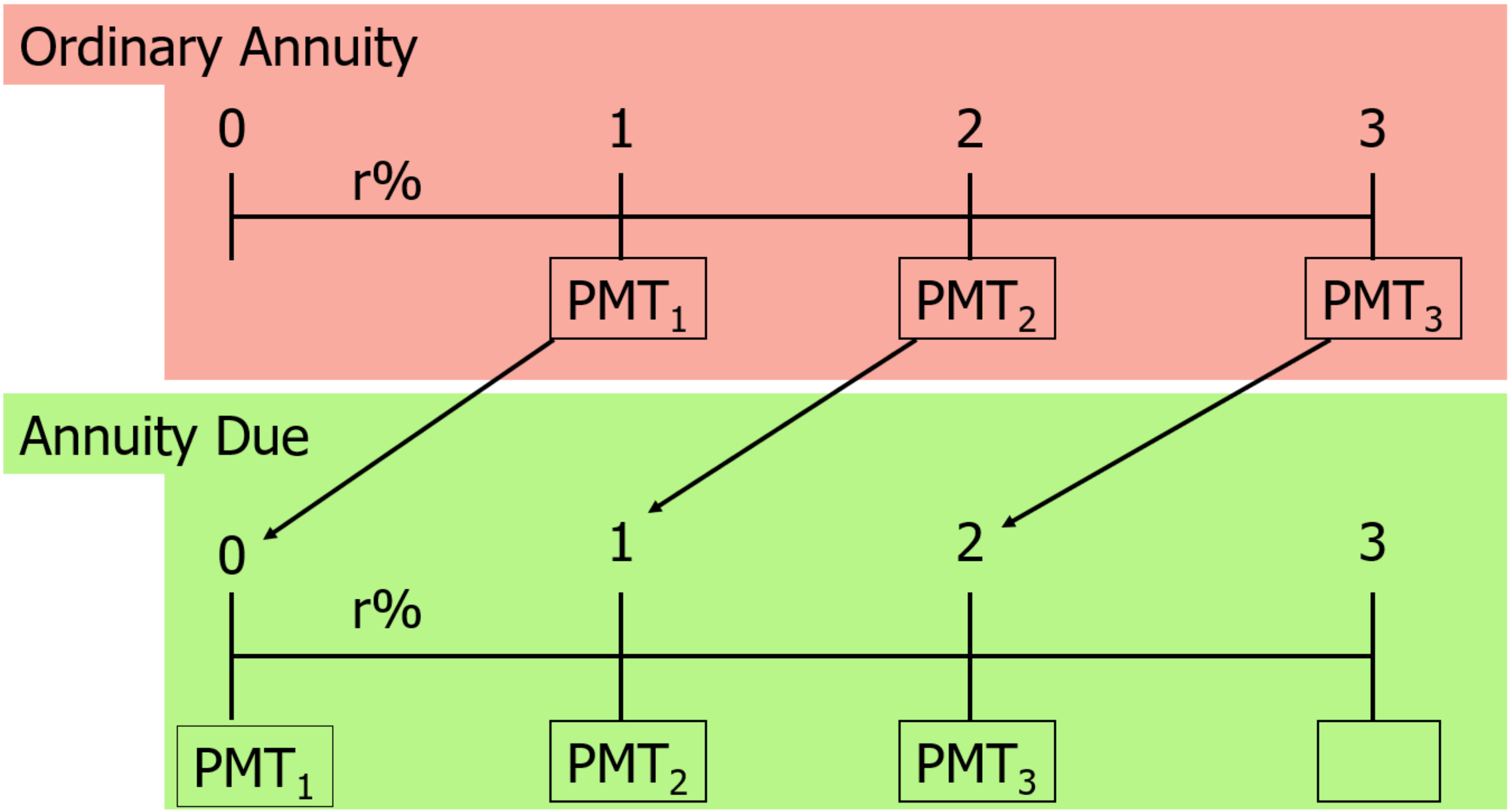


	A	B	C	D
1	N =	3		
2	I/Y =	0.10		
3	PV =	-700.00		
4	PMT =	0		
5	PMT Type	0	(0 = ordinary annuity; 1 = annuity due)	
6	FV =	?		
7			The equation used to solve for FV_3 in cell **B8**	Values that correspond to the cells referenced in cell **C8**
8	FV_3 =	931.70	=FV(B2,B1,B4,B3,B5)	=FV(0.1,3,0,-700,0)
9				

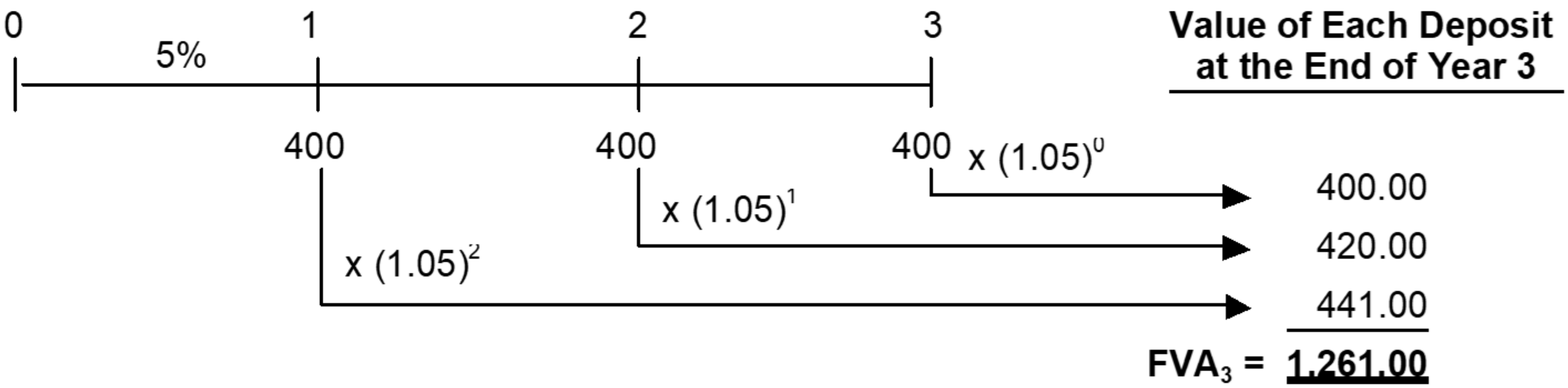
Future Value of an Annuity

- Annuity
  - A series of payments of equal amounts at equal intervals for a specified number of periods
- Ordinary (deferred) annuity
  - An annuity whose payments occur at the ***end*** of each period.
- Annuity due
  - An annuity whose payments occur at the ***beginning*** of each period.

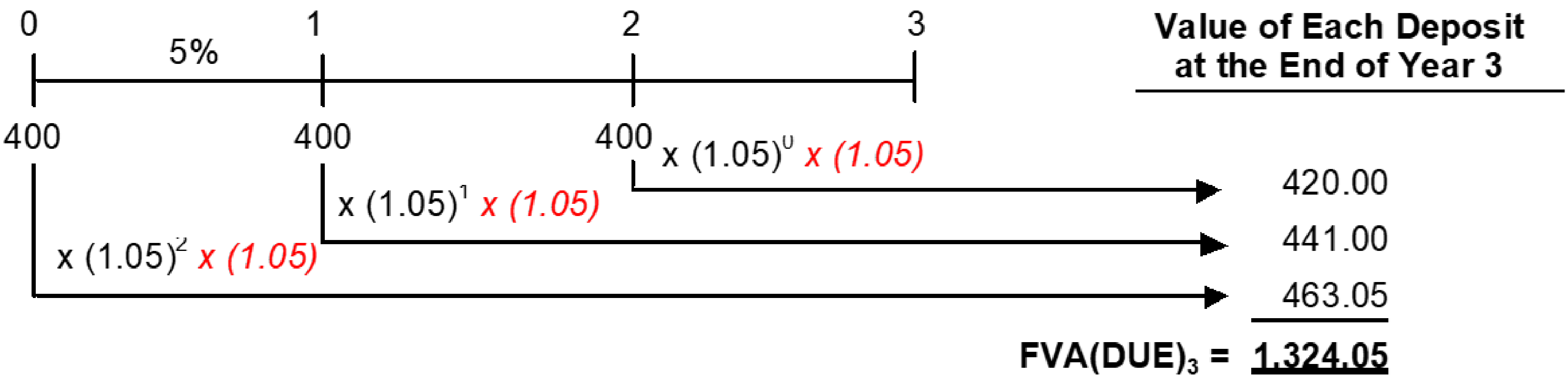
Ordinary Annuity versus Annuity Due



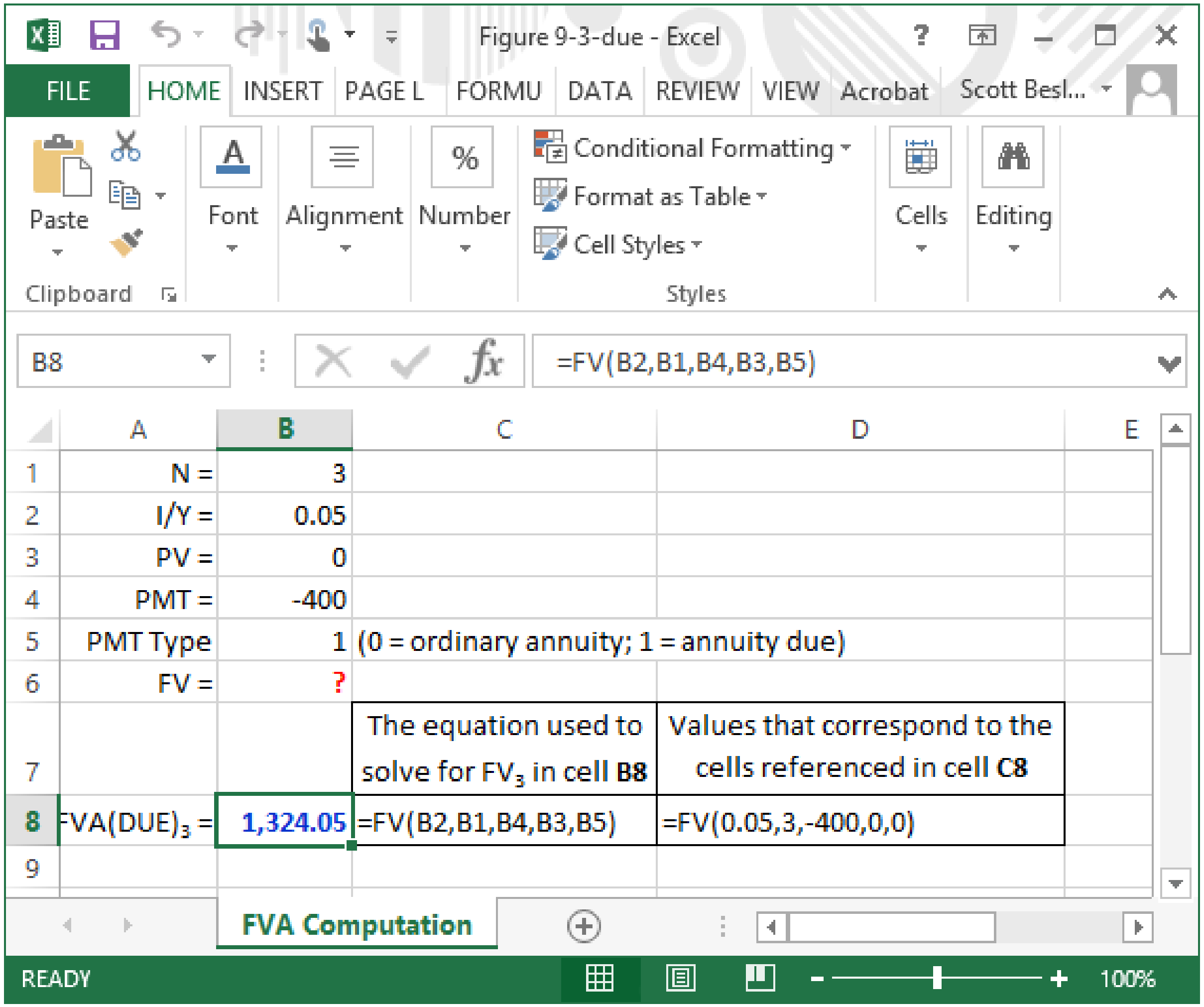
FV of a 3-year Ordinary Annuity of $400 at 5%



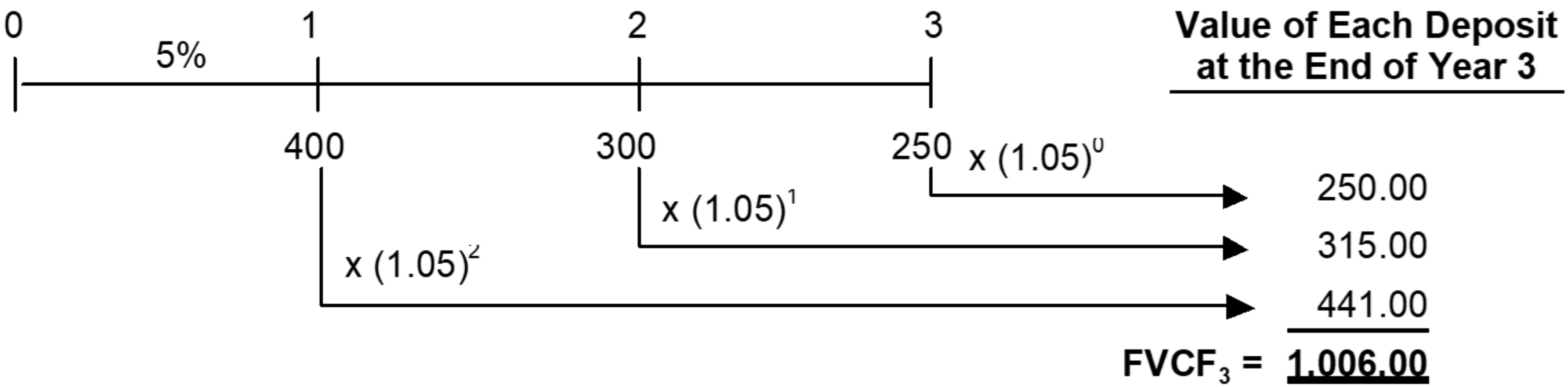
FV of a 3-year Annuity Due of $400 at 5%



Spreadsheet Solution – FVA(DUE)



Future Value of an Uneven Cash Flow



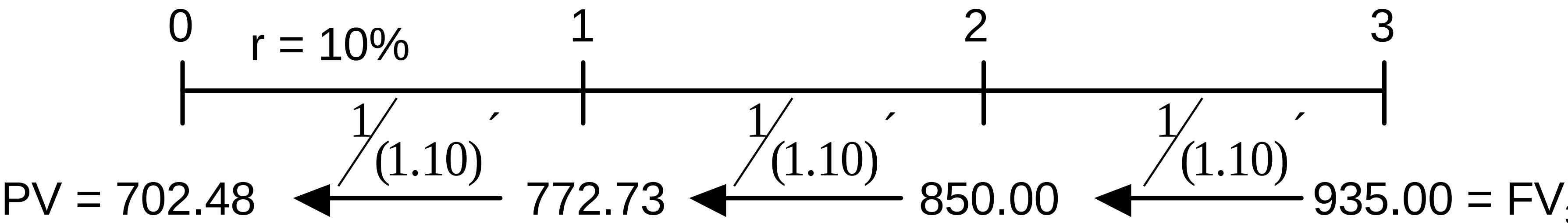
$$FVCF_n = CF_1(1+r)^{n-1} + \cdots + CF_n(1+r)^0 = \sum_{t=1}^{n} CF_t(1+r)^{n-t}$$

Present Value

- **Present value** is the value today of a future cash flow or series of cash flows.

- **Discounting** is the process of finding the present value of a future cash flow or series of future cash flows; it is the reverse of compounding.

PV of \$935 due in three years if r = 10%



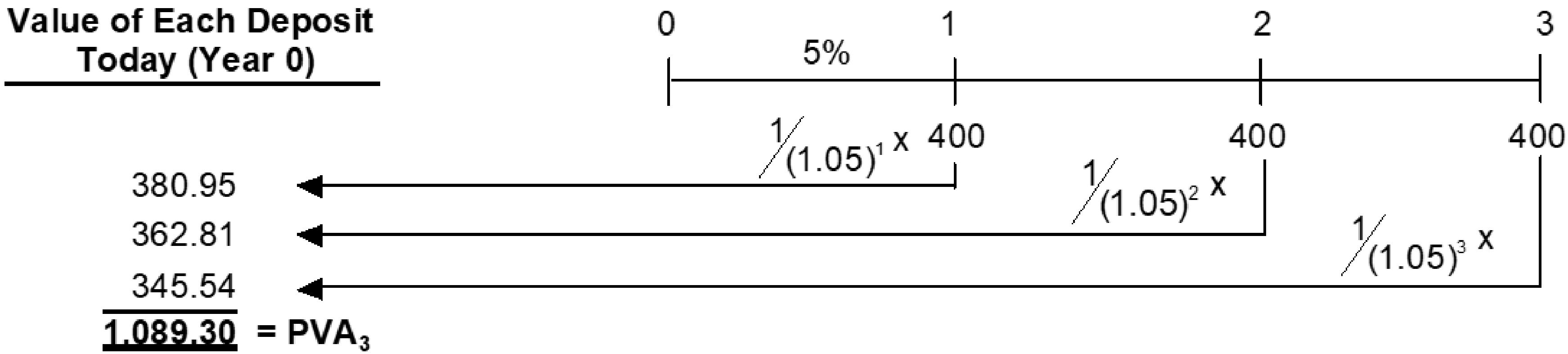
$$PV = \frac{FV_n}{(1+r)^n} = \frac{\$935}{1.10^3} = \$702.48$$

Spreadsheet Solution

	A	B	C	D
1	N =	3		
2	1/Y =	0.1		
3	PV =	?		
4	PMT =	0		
5	PMT Type	0	(0 = ordinary annuity; 1= annuity due)	
6	FV =	935		
7				
8				
9	PV =	-702.48	=PV(B2,B1,B4,B6,B5)	=PV(0.1,3,0,935,0)

Present Value of an Annuity

- PVA$_n$ = the present value of an annuity with n payments



Numerical Solution

$$PVA_n = PMT\left[\sum_{t=1}^{n}\frac{1}{(1+r)^t}\right] = PMT\left[\frac{1 - \frac{1}{(1+r)^n}}{r}\right]$$

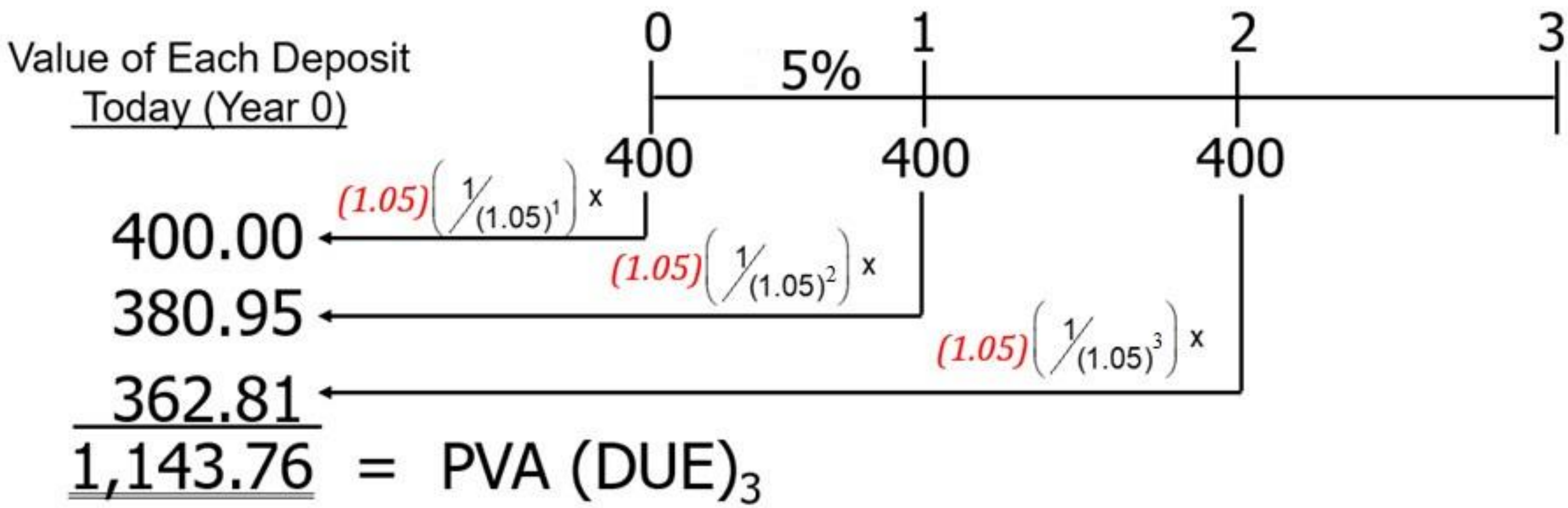
$$PVA_3 = \$400\left[\frac{1 - \frac{1}{(1.05)^3}}{0.05}\right]$$

$$= \$400(2.72325) = \$1089.30$$

Spreadsheet Solution

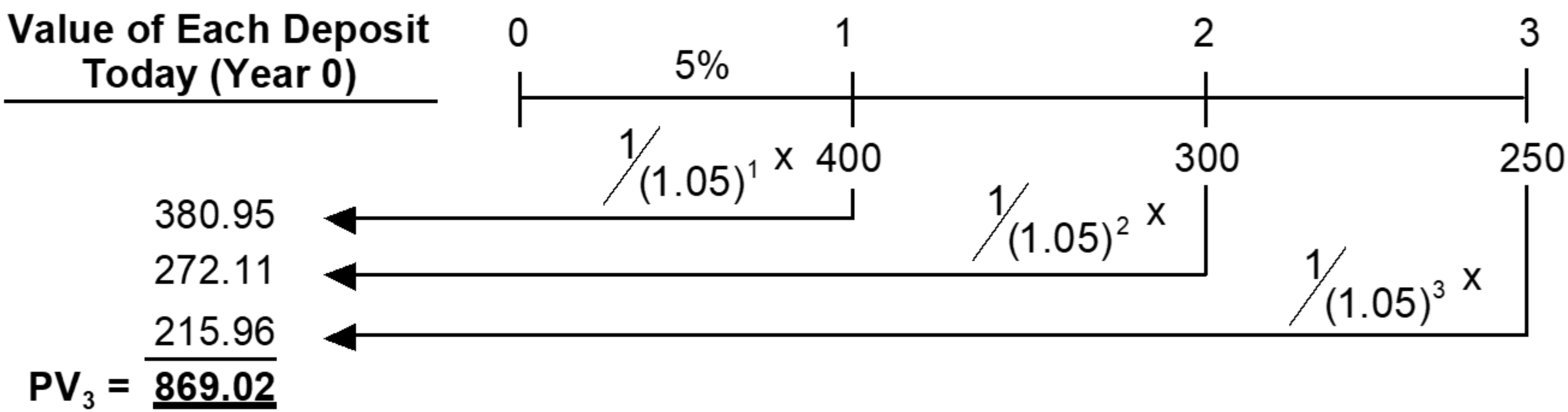
	A	B	C	D
1	N =	3		
2	1/Y =	0.05		
3	PV =	?		
4	PMT =	-400		
5	PMT Type	0	(0 = ordinar annuity; 1= annuity due)	
6	FV =	0		
7				
8				
9	PV =	1089.3	=PV(B2,B1,B4,B6,B5)	=PV(0.05,3,400,0,0)

Present Value of an Annuity Due



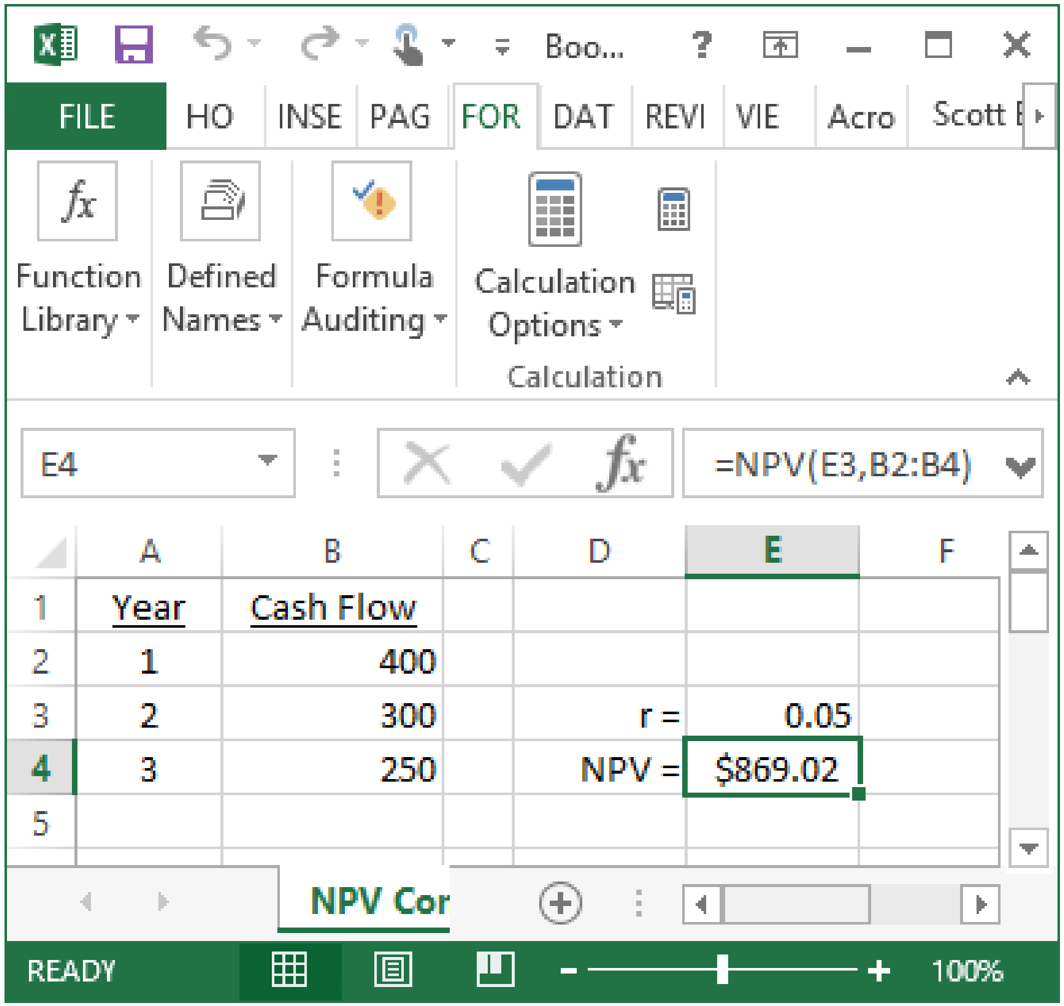
Uneven Cash Flow Streams

- A series of cash flows in which the amount varies from one period to the next.



$$PVCF_n = CF_1\left[\frac{1}{(1+r)^1}\right] + CF_2\left[\frac{1}{(1+r)^2}\right] + \cdots + CF_n\left[\frac{1}{(1+r)^n}\right] = \sum_{t=1}^{n} CF_t\left[\frac{1}{(1+r)^t}\right]$$

Spreadsheet Solution



Compounding

- Will the FV of a lump sum be larger or smaller if we compound more often, holding the stated r constant?
  - If compounding is more frequent than once per year—for example, semiannually, quarterly, or daily—interest is earned on interest. Because interest is compounded more often, the future value will be larger.

Comparison of Different Interest Rates

r_{SIMPLE} = Simple (Quoted) Rate
Used to compute the interest paid each period

APR = Annual Percentage Rate = r_{SIMPLE}
APR is a non-compounded interest rate

EAR = Effective Annual Rate = r_{EAR}
The rate that would produce the same future value if annual compounding had been used

EAR for a simple rate of 10%, compounded semi-annually

$$EAR = r_{EAR} = \left(1 + \frac{r_{SIMPLE}}{m}\right)^{m} - 1$$

$$= \left(1 + \frac{0.10}{2}\right)^{2} - 1.0$$

$$= (1.05)^{2} - 1.0 = 0.1025 = 10.25\%$$

FV after *n* years, compounded *m* times a year

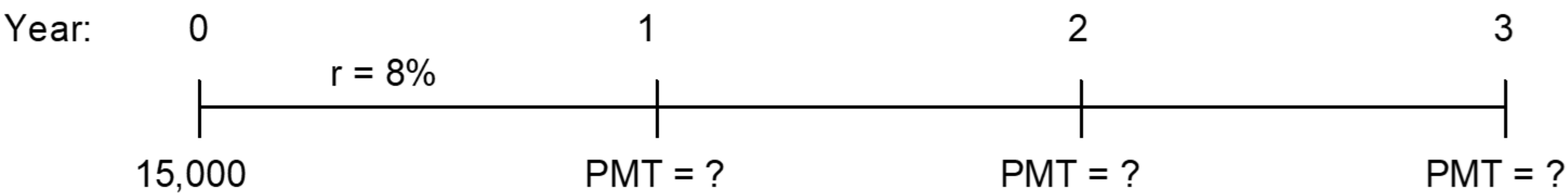
$$FV_n = PV\left(1 + \frac{r_{SIMPLE}}{m}\right)^{m \times n}$$

$$FV_{3\times 4} = \$100\left(1 + \frac{0.10}{4}\right)^{4\times 3}$$

$$= \$100(1.34489) = \$134.49$$

Amortized Loans

- Amortized Loan: A loan that is repaid in equal payments over its life; payment includes both principal repayment and interest



Spreadsheet Solution

	A	B	C	D
1	N =	3		
2	1/Y =	0.08		
3	PV =	15000		
4	PMT =	?	(0 = end of the period; 1= beginning of the period)	
5	PMT Type	0		
6	FV =	0		
7				
8				
9	PMT =	-5820.5	=PMT(B2,B1,B3,B6,B5)	=PMT(0.08,3,15000,0,0)

Loan Amortization Schedule

Year	Beg. of Year Balance (1)	Payment (2)	Interest @ 8% (3) = (1) x 0.08	Repayment of Principal (4) = (2) – (3)	End of Year Balance (5) = (1) – (4)
1	$15,000.00	$5,820.50	$1,200.00	$4,620.50	$10,379.50
2	10,379.50	5,820.50	830.36	4,990.14	5,389.36
3	5,389.36	5,820.50	431.15	5,389.35	0.01

The $0.01 remaining balance at the end of Year 3 results from a rounding difference.

Summary

- Three basic types of cash flow patterns
  - Lump-sum amount
  - Annuity
  - Uneven cash flow stream
- Future values and present values
  - Calculations
    - Ordinary annuity and annuity due
    - Different compounding frequencies

Summary

- APR and EAR
  - APR is a simple non-compounded interest rate quoted on loans
  - EAR is the actual interest (compounded) rate
- Amortized loan
  - A loan paid off in equal payments over a specified period
  - Each payment includes repayment of some principal and payment of interest