2023-24 First Semester

MATH2023 Ordinary and Partial Differential Equations (1002)

Assignment 8

Due Date: **3/Nov/2023(Thursday), on or before 16:00, in tutorial class.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without details will not be graded.**

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1. For the following differential equation

$$x^2y'' + (x^2 + \frac{1}{4})y = 0, \quad x_0 = 0$$

   i Show that the given differential equation has a regular singular point at the given point x_0.

   ii Determine the indicial equation, the recurrence relation, and the roots of indicial equation.

   iii Find the series solution ($x > 0$) corresponding to the larger root.

2. The Chebyshev equation is

$$(1-x^2)y'' - xy' + \alpha^2 y = 0,$$

   where α is a constant.

   (a) Show that $x = 1$ and $x = -1$ are regular singular points, and find the exponents at each of these singularites.

   (b) Let $\alpha = 0$ and find the first solution $y_1(x)$ corresponding to the larger exponent about $x = 1$.

3. Solve the homogeneous problem

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}.$$