ECON2113 Macroeconomics

Chapter 3 Exercises

Solutions

1.

a. According to Equation (2), the growth of output is equal to the growth in labor times the share of labor plus the growth of capital times the share of capital plus the growth rate of total factor productivity, that is,

$\Delta Y/Y = (1 - \theta)(\Delta N/N) + \theta(\Delta K/K) + \Delta A/A$, where

$1 - \theta$ is the share of labor (N) and θ is the share of capital (K). In this example $\theta = 0.3$; therefore, if output grows at 3% and labor and capital grow at 1% each, we can calculate the change in total factor productivity in the following way

$3\% = (0.7)(1\%) + (0.3)(1\%) + \Delta A/A$

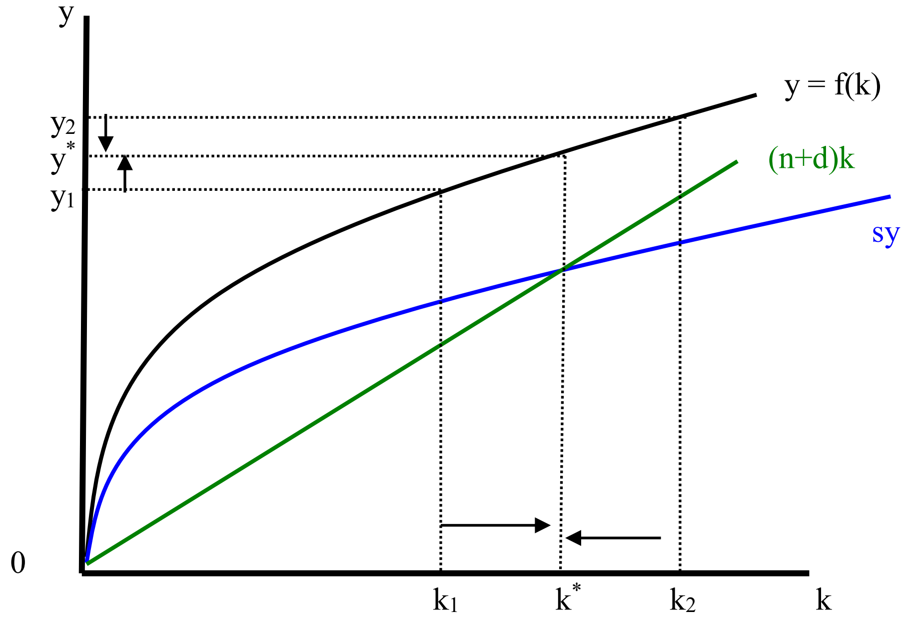
$\Longrightarrow \Delta A/A = 3\% - 1\% = 2\%$,

that is, the growth rate of total factor productivity is 2%.

b. If both labor and the capital stock are fixed, that is, $\Delta N/N = \Delta K/K = 0$, and output grows at 3%, then all the growth has to be attributed to the growth in total factor productivity, that is, $\Delta A/A = 3\%$.

2.

The figure shows output per head as a function of the capital-labor ratio, that is, $y = f(k)$, the savings function, that is $sy = sf(k)$, and the investment requirement, that is, the $(n + d)k$-line. At the intersection of the savings function with the investment requirement, the economy is in a steady-state equilibrium. Now let us assume for simplicity that the earthquake does not affect peoples' savings behavior and that the economy is in a steady-state equilibrium before the earthquake hits, that is, the capital-labor ratio is currently k^*.



If the earthquake destroys one quarter of the capital stock but less than one quarter of the labor force, then the capital-labor ratio will fall from k^* to k_1 and per-capita output will fall from y^* to y_1. Now saving is greater than the investment requirement, that is, $sy_1 > (d + n)k_1$, and the capital stock and the level of output per capita will grow until the steady state at k^* is reached again.

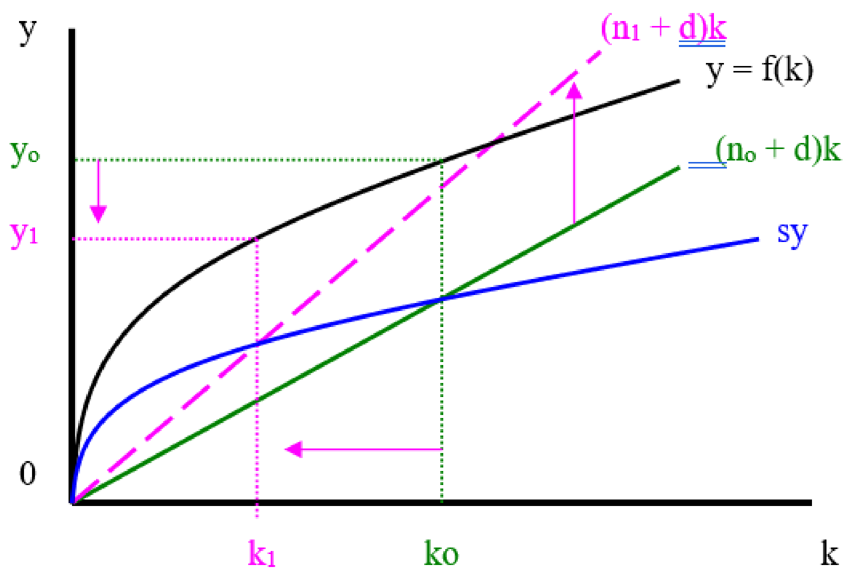
However, if the earthquake destroys one quarter of the capital stock but more than one quarter of the labor force, then the capital-labor ratio will increase from k^* to k_2. Saving (and gross investment) now will be less than the investment requirement and thus the capital-labor ratio and the level of output per capita will fall until the steady state at k^* is reached again.

If exactly one quarter of both the capital stock and the labor stock are destroyed, then the steady state will be maintained, that is, the capital-labor ratio and the output per capita will not change.

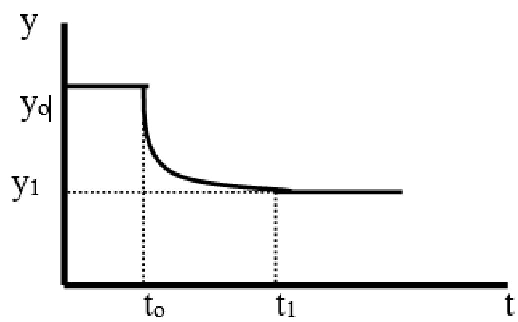
If the severity of the earthquake has an effect on peoples' savings behavior, the savings function sy = sf(k) will move either up or down, depending on whether the savings rate (s) increases (if people save more, so more can be invested later in an effort to rebuild) or decreases (if people save less, since they decide that life is too short not to live it up). But in either case, a new steady-state equilibrium will be reached.

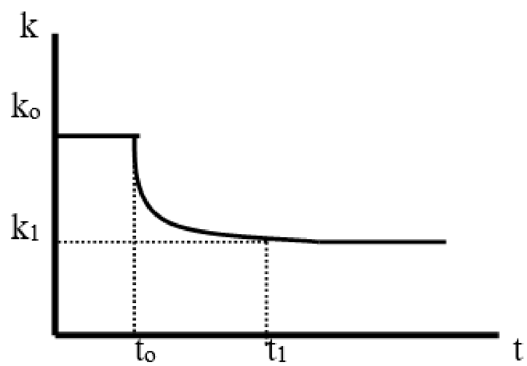
3.

a. An increase in the population growth rate (n) affects the investment requirement, that is, as n gets larger, the (n + d)k-line gets steeper. As the population grows, more needs to be saved and invested to equip new workers with the same amount of capital that existing workers already have. Since the population will now be growing faster than output, income per capita (y) will decrease and a new optimal capital-labor ratio will be determined by the intersection of the sy-curve and the new $(n_1 + d)k$-line. Since per-capita output will fall, we will have a negative growth rate in the short run. However, the steady-state growth rate of output will increase in the long run, since it will be determined by the new and higher rate of population growth $n_1 > n_o$.



b. Starting from an initial steady-state equilibrium at a level of per-capita output y_o, the increase in the population growth rate (n) will cause the capital-labor ratio to decline from k_o to k_1. Output per capita will also decline, a process that will continue at a diminishing rate until a new steady-state level is reached at y_1. The growth rate of output will gradually adjust to the new and higher level n_1.





4.

a. If the production function is of the form $Y = K^{1/2}(AN)^{1/2}$,

and A is normalized to 1, we have $Y = K^{1/2}N^{1/2}$.

In this case, capital's and labor's shares of income are both 50%.

b. This is a Cobb-Douglas production function.

c. A steady-state equilibrium is reached when $sy = (n + d)k$.

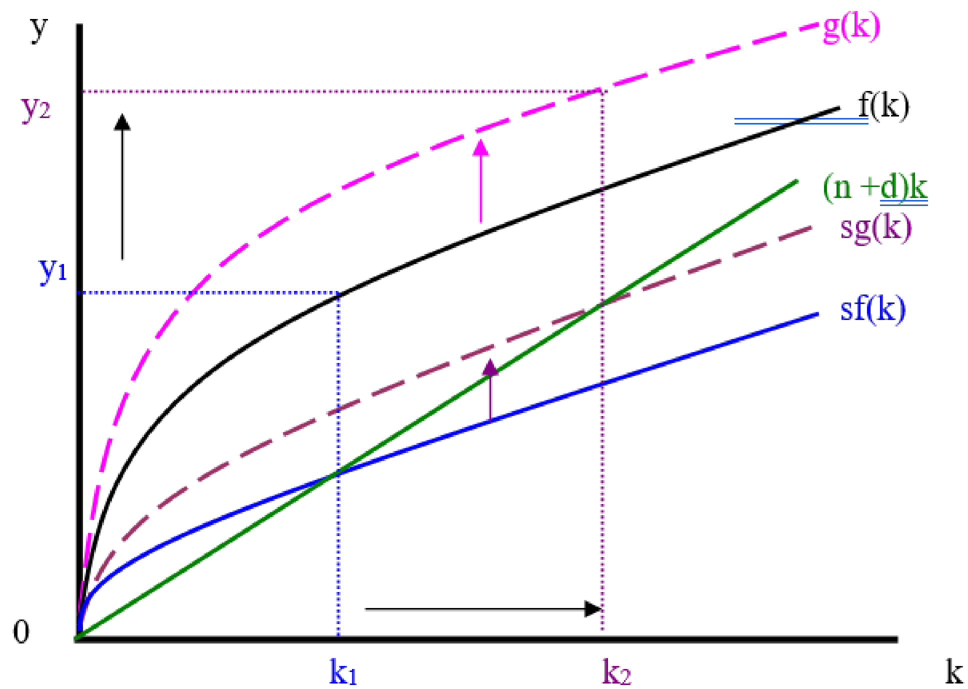
From $Y = K^{1/2}N^{1/2} \Longrightarrow Y/N = K^{1/2}N^{-1/2} \Longrightarrow y = k^{1/2} \Longrightarrow sk^{1/2} = (n + d)k$

$\Longrightarrow k^{-1/2} = (n + d)/s = (0.07 + 0.03)/(.2) = 1/2 \Longrightarrow k^{1/2} = 2 = y \Longrightarrow k = 4$.

d. At the steady-state equilibrium, output per capita remains constant, since total output grows at the same rate as the population (7%), as long as there is no technological progress, that is, $\Delta A/A = 0$. But if total factor productivity grows at $\Delta A/A = 2\%$, then total output will grow faster than population, that is, at 7% + 2% = 9%, so output per capita will grow at 2%.

5.

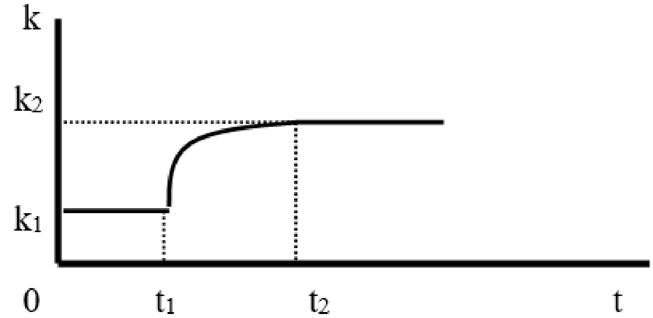
a.



If technological progress occurs, then the level of output per capita for any given capital-labor ratio increases. The function y = f(k) increases to y = g(k), and therefore the savings function increases from sf(k) to sg(k).

b. Since g(k) > f(k), it follows that sg(k) > sf(k) for each level of k. Therefore, the intersection of the sg(k)-curve with the (n + d)k-line is at a higher level of k. The new steady-state equilibrium will now be at a higher level of saving and output per capita, and at a higher capital-labor ratio.

c. After the technological progress occurs, the level of saving and investment will increase until a new and higher optimal capital-labor ratio is reached. The investment ratio will increase in the transition period, since more investment will be required to reach the higher optimal capital-labor ratio.


k
k2
k1
0
t1
t2
t