ECON2103 Microeconomics

Chapter 6 Exercises

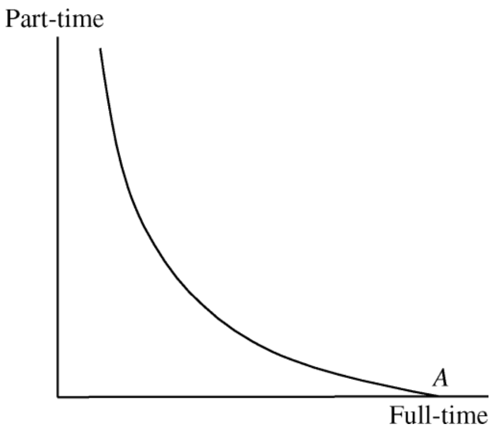
Solutions

1.

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	—	—
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	–15	225

2.

a. Place part-time workers on the vertical axis and full-time workers on the horizontal. The slope of the isoquant measures the number of part-time workers that can be exchanged for a full-time worker while still maintaining output. At the bottom end of the isoquant, at point *A*, the isoquant hits the full-time axis because it is possible to produce with full-time workers only and no part-timers. As we move up the isoquant and give up full-time workers, we must hire more and more part-time workers to replace each full-time worker. The slope increases (in absolute value) as we move up the isoquant. The isoquant is therefore convex and there is a diminishing marginal rate of technical substitution.



b. The marginal rate of technical substitution measures the number of units of capital that can be exchanged for a unit of labor while still maintaining output. If the firm can always trade two units of labor for one unit of capital then the *MRTS* of labor for capital is constant and equal to 1/2, and the isoquant is linear.

c. This firm operates under a fixed proportions technology, and the isoquants are L-shaped. The firm cannot substitute any labor for capital and still maintain output because it must maintain a fixed 2:1 ratio of labor to capital. The *MRTS* is infinite (or undefined) along the vertical part of the isoquant and zero on the horizontal part.

3.

a. This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 10. If L is 4 and K is 4 then q is 20. When the inputs are doubled, output will double. Each marginal product is constant for this production function. When L increases by 1, q will increase by 3. When K increases by 1, q will increase by 2.

b. This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 2.8. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output increases by less than double. The marginal product of each input is decreasing. This can be determined using calculus by differentiating the production function with respect to one input while holding the other input constant. For example, the marginal product of labor is

$$\frac{\partial q}{\partial L} = \frac{2}{2(2L+2K)^{\frac{1}{2}}}.$$

Since L is in the denominator, as L gets bigger, the marginal product gets smaller. If you do not know calculus, you can choose several values for L (holding K fixed at some level), find the corresponding q values and see how the marginal product changes. For example, if $L = 4$ and $K = 4$ then $q = 4$. If $L = 5$ and $K = 4$ then $q = 4.24$. If $L = 6$ and $K = 4$ then $q = 4.47$. Marginal product of labor falls from 0.24 to 0.23. Thus, MP_L decreases as L increases, holding K constant at 4 units. The same is true for changes in K holding L constant.

c. This function exhibits increasing returns to scale. For example, if L is 2 and K is 2, then q is 24. If L is 4 and K is 4 then q is 192. When the inputs are doubled, output more than doubles. Notice also that if we increase each input by the same factor λ then we get the following:

$$q' = 3(\lambda L)(\lambda K)^2 = \lambda^3 3LK^2 = \lambda^3 q.$$

Since λ is raised to a power greater than 1, we have increasing returns to scale.

The marginal product of labor is constant and the marginal product of capital is increasing. For any given value of K, when L is increased by 1 unit, q will go up by $3K^2$ units, which is a constant number since we're holding K constant. Using calculus, the marginal product of capital is $MP_K = 6LK$. As K increases holding L constant, MP_K increases.

d. This function exhibits constant returns to scale. For example, if L is 2 and K is 2 then q is 2. If L is 4 and K is 4 then q is 4. When the inputs are doubled, output will exactly double. Notice also that if we increase each input by the same factor, λ, then we get the following:

$$q' = (\lambda L)^{\frac{1}{2}}(\lambda K)^{\frac{1}{2}} = \lambda L^{\frac{1}{2}}K^{\frac{1}{2}} = \lambda q.$$

Since λ is raised to the power 1, there are constant returns to scale.

The marginal product of labor is decreasing and the marginal product of capital is decreasing. Using calculus, the marginal product of capital is

$$MP_K = \frac{L^{\frac{1}{2}}}{2K^{\frac{1}{2}}}.$$

For any given value of L, as K increases, MP_K will decrease. You can do the same thing for labor.

e. This function exhibits decreasing returns to scale. For example, if L is 2 and K is 2 then q is 13.66. If L is 4 and K is 4 then q is 24. When the inputs are doubled, output increases by less than double.

The marginal product of labor is decreasing and the marginal product of capital is constant. For any given value of L, when K is increased by 1 unit, q goes up by 4 units, which is a constant. To see that the marginal product of labor is decreasing, fix $K = 1$ and choose values for L. If $L = 1$ then $q = 8$, if $L = 2$ then $q = 9.66$, and if $L = 3$ then $q = 10.93$. The marginal product of the second unit of labor is $9.66 - 8 = 1.66$, and the marginal product of the third unit of labor is $10.93 - 9.66 = 1.27$. Marginal product of labor is diminishing.

4.

a. Let q_1 be the output of DISK, Inc., q_2, be the output of FLOPPY, Inc., and X be the same equal amounts of capital and labor for the two firms. Then according to their production functions,

$$q_1 = 10X^{0.5}X^{0.5} = 10X^{(0.5 + 0.5)} = 10X$$

and

$$q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6 + 0.4)} = 10X.$$

Because $q_1 = q_2$, both firms generate the same output with the same inputs. Note that if the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same levels of output. In fact, if $K > L$ then $q_2 > q_1$, and if $L > K$ then $q_1 > q_2$.

b. With capital limited to 9 machine hours, the production functions become $q_1 = 30L^{0.5}$ and $q_2 = 37.37L^{0.4}$. To determine the production function with the highest marginal productivity of labor, consider the following table:

L	***q*** **Firm 1**	***MP_L*** **Firm 1**	***q*** **Firm 2**	***MP_L*** **Firm 2**
0	0.00	—	0.00	—
1	30.00	30.00	37.37	37.37
2	42.43	12.43	49.31	11.94
3	51.96	9.53	57.99	8.68
4	60.00	8.04	65.06	7.07

For each unit of labor above 1, the marginal productivity of labor is greater for the first firm, DISK, Inc.