Chapter Six

Capital Allocation to Risky Assets

Chapter Overview

- Two-step process of portfolio construction
  1. Capital allocation decision
     - Allocation of the overall portfolio to safe assets versus risky assets
  2. Determination of the composition of the risky portion of the complete portfolio

- Demonstration of how risk aversion can be characterized by a “utility function”

Risk and Risk Aversion:
Risk Aversion and Utility Values

- **Risk-averse** investors consider only risk-free or speculative prospects with positive risk premiums

- Portfolio is more attractive when its expected return is higher and its risk is lower
  - What happens when risk increases along with return?

Available Risky Portfolios

Portfolio	Risk Premium	Expected Return	Risk (SD)
L (low risk)	2%	7%	5%
M (medium risk)	4	9	10
H (high risk)	8	13	20

Table 6.1

Available risky portfolios (risk-free rate = 5%)

Risk and Risk Aversion:
Risk Aversion and Utility Values (Continued)

- We assume each investor can assign a welfare, or **utility**, score to competing portfolios

$$U = E(r) - \tfrac{1}{2} A\sigma^2$$

- Utility function
  - U = Utility value
  - $E(r)$ = Expected return
  - A = Index of the investor's risk aversion
  - σ^2 = Variance of returns
  - ½ = Scaling factor

Example

Investor Risk Aversion (A)	Utility Score of Portfolio L [$E(r)$ = .07; σ = .05]	Utility Score of Portfolio M [$E(r)$ = .09; σ = .10]	Utility Score of Portfolio H [$E(r)$ = .13; σ = .20]
2.0	$.07 - ½ \times 2 \times .05^2 = .0675$	$.09 - ½ \times 2 \times .1^2 = .0800$	**$.13 - ½ \times 2 \times .2^2 = .09$**
3.5	$.07 - ½ \times 3.5 \times .05^2 = .0656$	**$.09 - ½ \times 3.5 \times .1^2 = .0725$**	$.13 - ½ \times 3.5 \times .2^2 = .06$
5.0	$.07 - ½ \times 5 \times .05^2 = .0638$	**$.09 - ½ \times 5 \times .1^2 = .0650$**	$.13 - ½ \times 5 \times .2^2 = .03$

Utility scores of alternative portfolios for investors with varying degrees of risk aversion

Investor Types

- **Risk-averse** investors consider risky portfolios only if they provide compensation for risk via a risk premium
  - $A > 0$
- **Risk-neutral** investors find the level of risk irrelevant and consider only the expected return of risk prospects
  - $A = 0$
- **Risk lovers** are willing to accept lower expected returns on prospects with higher amounts of risk
  - $A < 0$

Trade-Off Between Risk and Return

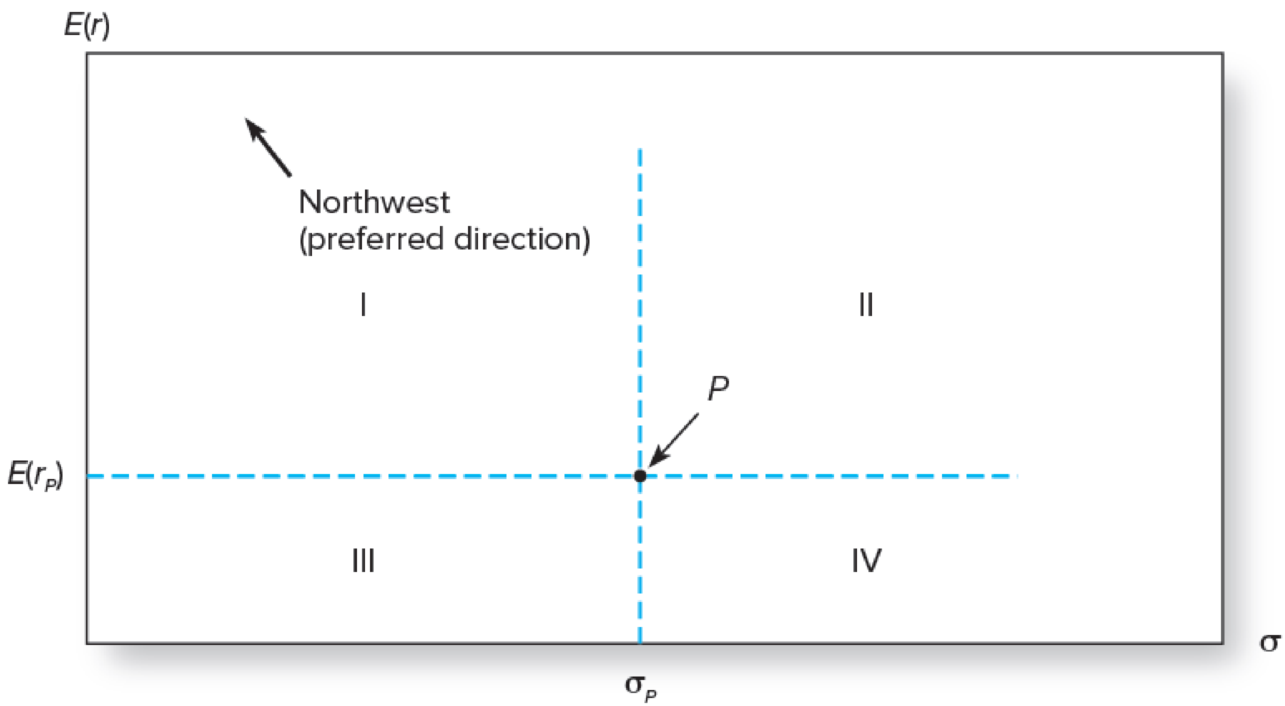


Figure 6.1 The trade-off between risk and return of a potential investment portfolio, *P*

Mean-Variance (M-V) Criterion

Mean-variance (M-V) criterion

- The selection of portfolios based on the means and variances of their returns
- The choice of the highest expected return portfolio for a given level of variance or the lowest variance portfolio for a given expected return

Requirements for Portfolio A to dominate Portfolio B

- $E(r_A) \geq E(r_B)$
- $\sigma_A \leq \sigma_B$
- At least one inequality is strict (to rule out indifference between the two portfolios)

Indifference Curves

Equally preferred portfolios lie in the mean–standard deviation plane on an **indifference curve,** which connects all portfolio points with the same utility value

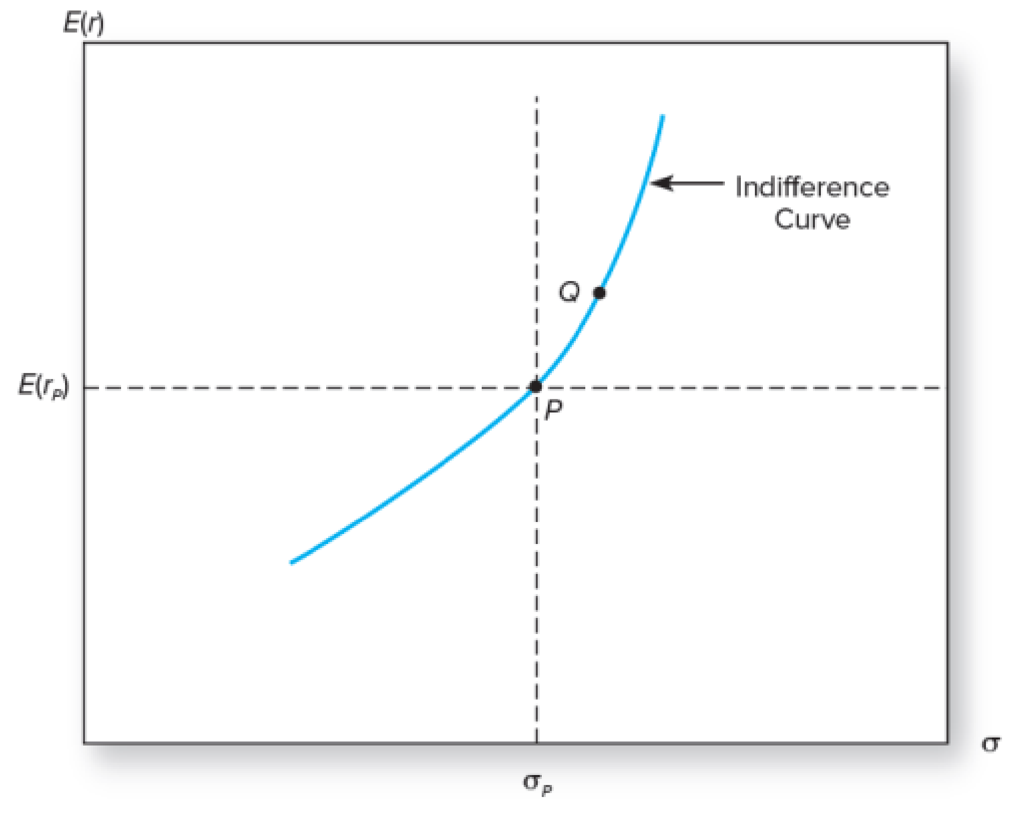


Figure 6.2 The indifference curve

Capital Allocation Across Risky and Risk-Free Portfolios

- Most basic asset allocation choice is risk-free money market securities versus other risky asset classes

- Simplest way to control risk is to manipulate the ratio of risky assets to risk-free assets

Basic Asset Allocation Example
(1 of 2)

Total market value	$300,000
Risk-free money market fund	$90,000
Equities	$113,400
Bonds (long-term)	$96,600
Total risky assets	$210,000

$$W_E = \frac{\$113,400}{\$210,000} = 0.54 \qquad W_B = \frac{\$96,600}{\$210,00} = 0.46$$

Basic Asset Allocation Example

(2 of 2)

Let

- y = Weight of the risky portfolio, P, in the complete portfolio
- $(1-y)$ = Weight of risk-free assets

$$y = \frac{\$210{,}000}{\$300{,}000} = 0.7 \qquad 1 - y = \frac{\$90{,}000}{\$300{,}000} = 0.3$$

$$E: \frac{\$113{,}400}{\$300{,}000} = .378 \qquad B: \frac{\$96{,}600}{\$300{,}000} = .322$$

The Risk-Free Asset

- T-bills viewed as “the” **risk-free asset**
- Broad range of money market instruments are considered effectively risk-free assets

Portfolios: Risky Asset and Risk-Free Asset

- It's possible to create a complete portfolio by splitting investment funds between safe and risky assets

Let

- y = Portion allocated to the risky portfolio, P
- $(1 - y)$ = Portion to be invested in risk-free asset, F

One Risky Asset and a Risk-Free Asset: Example (1 of 2)

$$E(r_P) = 15\%$$

$$\sigma_p = 22\%$$

$$r_f = 7\%$$

- Expected return on the complete portfolio (in percentage)

$$E(r_C) = yE(r_P) + (1 - y)r_f$$
$$= r_f + y[E(r_P) - r_f] = 7 + y(15 - 7)$$

- Risk of the complete portfolio (in percentage)

$$\sigma_C = y\sigma_P = 22y$$

One Risky Asset and a Risk-Free Asset: Example (2 of 2)

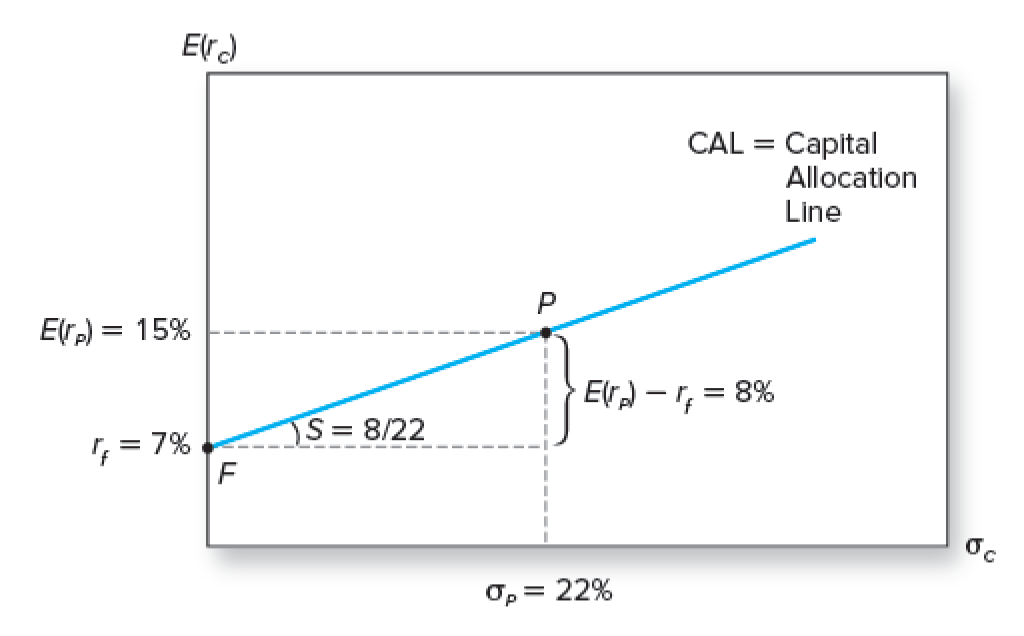
- The slope of the line is:

$$\text{Slope} = \frac{E(r_P) - r_f}{\sigma_P} = \frac{8}{22}$$

- Expected return-standard deviation tradeoff for the complete portfolio is:

$$E(r_C) = 7\% + \frac{8}{22}\sigma_C$$

The Investment Opportunity Set



One Risky Asset and a Risk-Free Asset Portfolios

- *Investment opportunity set* offers feasible expected return and standard deviation pairs of all portfolios resulting form different values of *y*

- Graph showing all feasible risk-return combination of a risky and risk-free asset is the **capital allocation line (CAL)**

- **Reward-to-volatility ratio** (aka **Sharpe ratio**)
  - Ratio of expected excess return to portfolio standard deviation

Risk Tolerance and Asset Allocation

- Investor must choose one optimal portfolio, *C*, from the set of feasible choices
  - Expected return of the complete portfolio:

$$E(r_c) = r_f + y \times \left[E(r_p) - r_f \right]$$

  - Variance:

$$\sigma_c^2 = y^2 \times \sigma_p^2$$

Utility Levels for Various Positions in Risky Assets

(1) y	(2) $E(r_C)$	(3) σ_C	(4) $U = E(r) - \frac{1}{2}A\sigma^2$
0	0.070	0	0.0700
0.1	0.078	0.022	0.0770
0.2	0.086	0.044	0.0821
0.3	0.094	0.066	0.0853
0.4	0.102	0.088	0.0865
0.5	0.110	0.110	0.0858
0.6	0.118	0.132	0.0832
0.7	0.126	0.154	0.0786
0.8	0.134	0.176	0.0720
0.9	0.142	0.198	0.0636
1.0	0.150	0.220	0.0532

Table 6.4

Utility levels for various positions in risky assets (y) for an investor with risk aversion $A = 4$

Utility as a Function of Allocation to the Risky Asset, *y* (1 of 2)

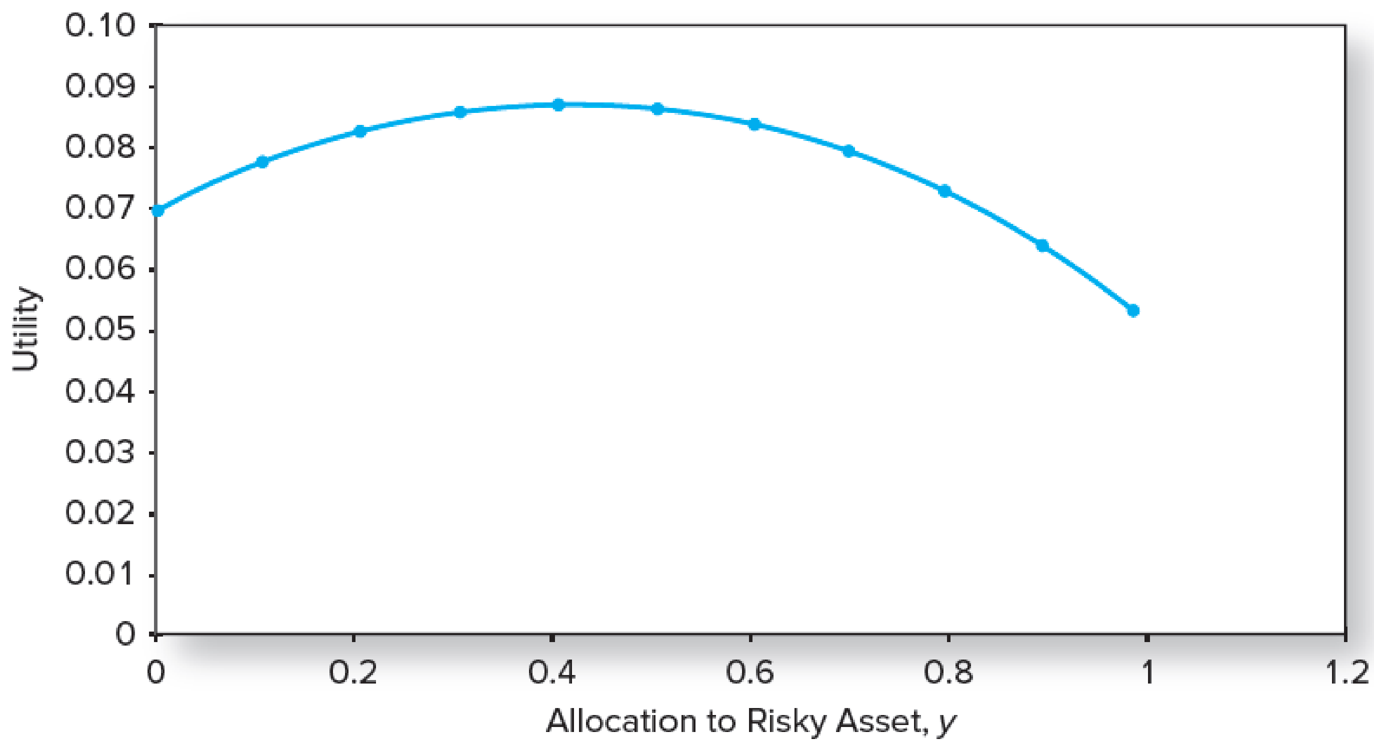


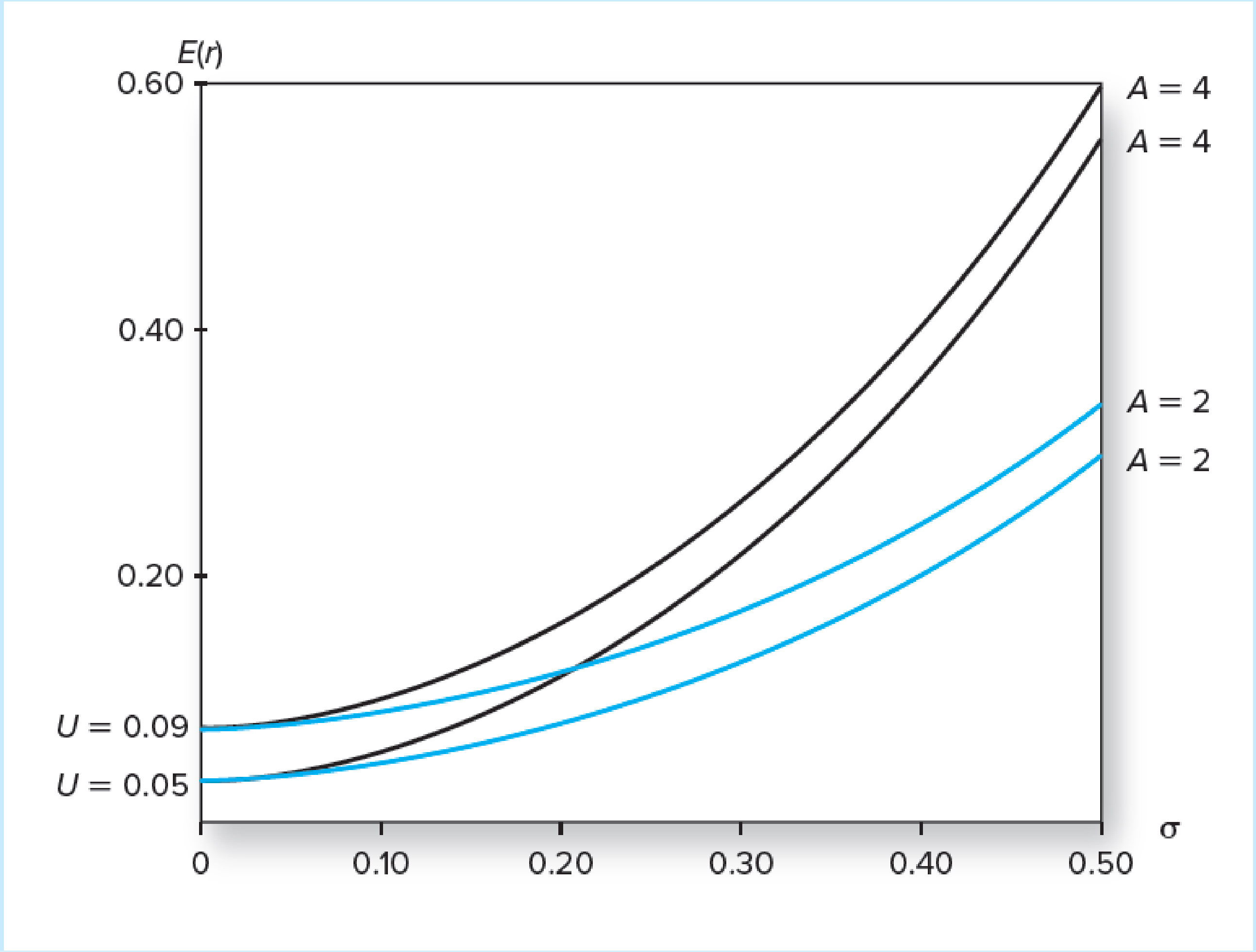
Figure 6.5 Utility as a function of allocation to the risky asset, *y*

Utility as a Function of Allocation to the Risky Asset, *y* (2 of 2)

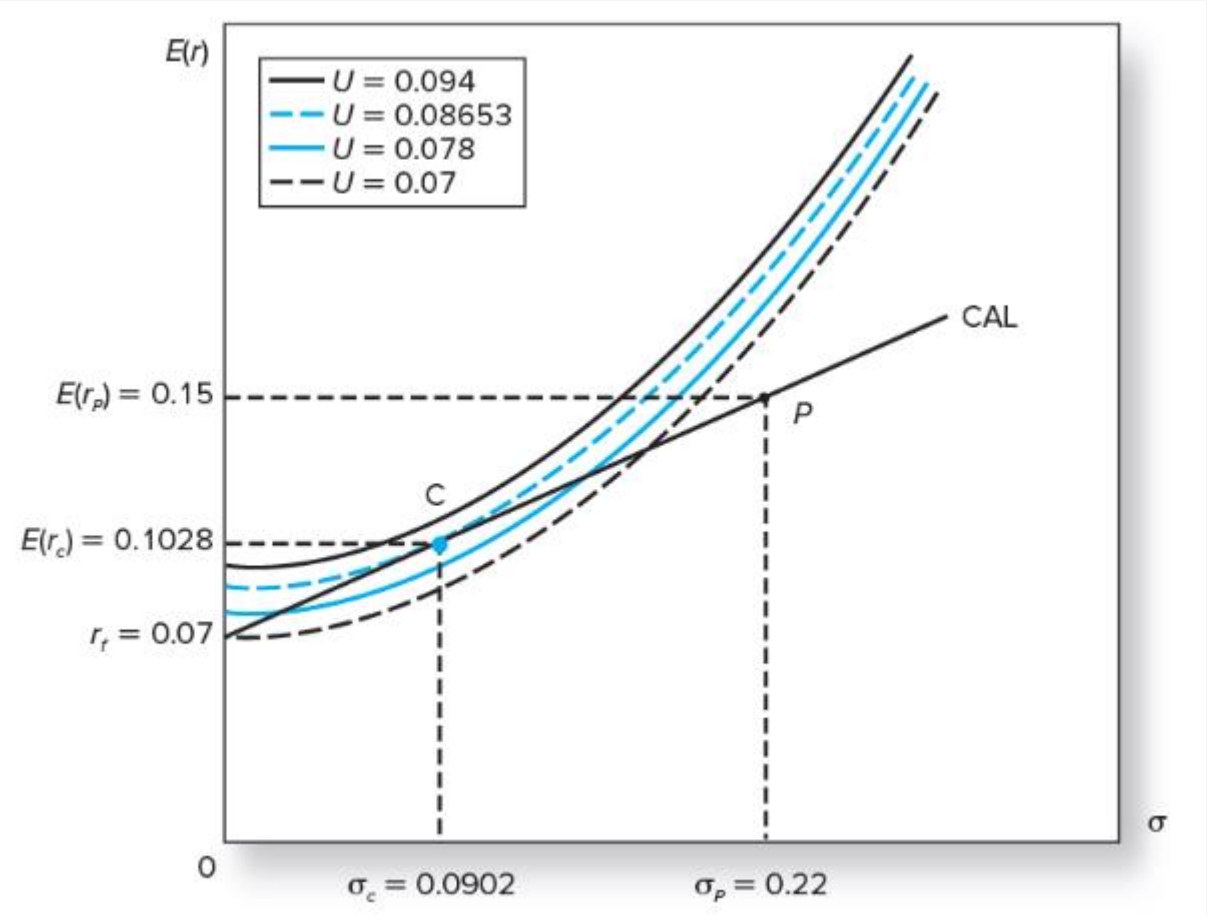
$$\underset{y}{\text{Max}}\, U = E(r_C) - \tfrac{1}{2}A\sigma_C^2 = r_f + y\left[E(r_P) - r_f\right] - \tfrac{1}{2}Ay^2\sigma_P^2$$

$$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2}$$

Indifference Curves for $U = .05$ and $U = .09$ with $A = 2$ and $A = 4$



Finding the Optimal Complete Portfolio



Passive Strategies: The Capital Market Line (1 of 2)

- A **passive strategy** avoids any direct or indirect security analysis

- A natural candidate for a passively held risky asset would a well-diversified portfolio of common stocks

Passive Strategies: The Capital Market Line (2 of 2)

- **Capital market line (CML)** results when using the market index as the risky portfolio

- From 1926 to 2018, the passive risky portfolio offered an average risk premium of 8.34% with a standard deviation of 20.36%, resulting in a reward-to-volatility ratio of .41