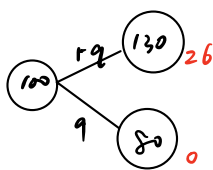


1



European call $K=104$ $r_f = 6\%$ $\delta = 0\% \rightarrow 2\%$ $\Delta t = \frac{1}{2}$ yr
(continuous)

$$q = \frac{S_+ - S_0 e^{(r-\delta)\Delta t}}{S_+ - S_-} = \frac{130 - 100 e^{(0.06-0.02) \times \frac{1}{2}}}{130 - 80} = \frac{130 - 100 e^{(0.06-0.02) \times 0.5}}{50}$$

For $\delta = 0$,

$$q_1 = \frac{130 - 100 e^{0.03}}{50} = 0.5391$$

For $\delta = 0.02$,

$$q_2 = \frac{130 - 100 e^{0.02}}{50} = 0.5596$$

$$\Delta V = (1-q_2)V_+ - (1-q_1)V_+ e^{-r\Delta t}$$

$$= ((1-0.5596) - (1-0.5391)) \times (130-104) \times e^{-0.06 \times \frac{1}{2}} = -0.5174$$

2. $r_f = 5\%$ $\delta = 2\%$ $1-q = 0.55$ $\frac{u+d}{2} = 1$ European call $K=90$ $\Delta t = \frac{1}{2}$ yr.

$$\frac{u+d}{2} = 1 \Rightarrow d = 2-u$$

$$q = \frac{u - e^{(r-\delta)\Delta t}}{u-d} = \frac{u - e^{(0.05-0.02) \times \frac{1}{2}}}{2u-2} = 0.45 \Rightarrow u = \frac{e^{0.015} - 0.9}{0.1} = 1.1511$$

$$d = 2-u = 0.8489$$

$$V = q(K - S(0)d) \times e^{-r\Delta t}$$

$$= 0.45(90 - 80 \times 0.8489) \times e^{-5\% \times \frac{1}{2}} = 9.9148$$

3. $\Delta t = \frac{1}{2}$ yr $S(0) = 20$ $r_f = 3\%$ $\delta = 1\%$ European call $K=20$ $b=0.4$

Put-call parity

$$C - P + K e^{-r\Delta t} = S e^{-\delta\Delta t}$$

$$P = C - S e^{-\delta\Delta t} + K e^{-r\Delta t}$$

$$= C - e^{-0.005} S + K e^{-0.015}$$

The put option consists of a call minus $e^{-0.005}$ shares of stock plus $K e^{-0.015}$ in bonds.

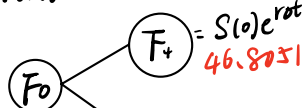
$$\text{Sol is } 0.4 - e^{-0.005} = -0.5950$$

4. $u = e^{0.2} = 1.2214$ $d = e^{-0.2} = 0.8187$ $r_f = 5\%$ $S(0) = 200$ $\delta = 0$. European call $\Delta t = 1$ yr $K = 210$.

$$S(0) = 200 \quad \delta = 0$$

$$1-q = \frac{1-d}{u-d} = 0.4502$$

Futures contract



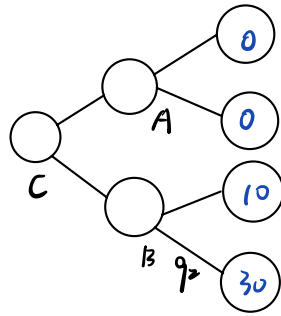
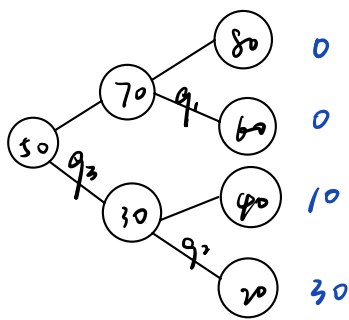
$$S(0)e^{r\Delta t} = 200 \times e^{5\% \times 1} = 210.2542$$

$$F_+ = S(0)e^{r\Delta t}u = 200 \times e^{5\%} \times 1.2214 = 256.8051$$

$$F_- = S(0)e^{r\Delta t}d = 200 \times e^{5\%} \times 0.8187 = 171.1416$$

$$V = (1-q)(F_+ - K) e^{-r\Delta t} = 0.4502 \times (256.8051 - 210) \times e^{-5\% \times 1} = 20.0425$$

5. $r=0.1$ $\delta=0.05$ American put $K=50$.



$$q = \frac{S_+ - S_0 e^{(r-\delta)t}}{S_+ - S_-}$$

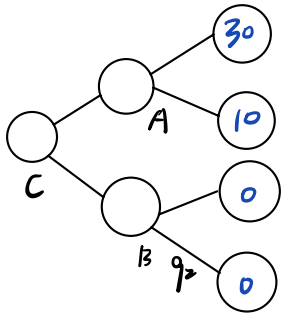
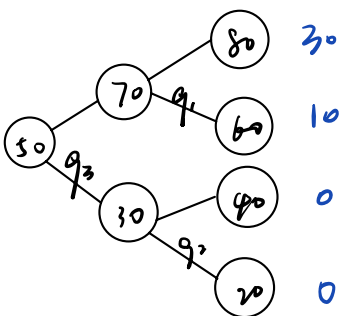
$$\begin{cases} q_1 = \frac{80-70e^{(0.1-0.05) \times 1}}{80-60} = 0.3206 \\ q_2 = \frac{40-30e^{(0.1-0.05) \times 1}}{40-20} = 0.4231 \\ q_3 = \frac{70-50e^{(0.1-0.05) \times 1}}{70-30} = 0.4359 \end{cases}$$

for node A, $V_+ = 0$

$$\begin{aligned} \text{for node B, } V_- &= \max \{ K - S_-, (q_2 \times 30 + (1-q_2) \times 10) e^{-rt} \} \\ &= \max \{ 50 - 30, (0.4231 \times 30 + 0.5769 \times 10) e^{-0.1 \times 1} \} \\ &= \max \{ 20, 16.7051 \} \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{for node C, } V_0 &= \max \{ K - S, (q_3 \times V_- + (1-q_3) V_+) e^{-rt} \} \\ &= \max \{ 0, (0.4359 \times 20 + 0.5641 \times 0) e^{-0.1 \times 1} \} \\ &= 7.8884 \end{aligned}$$

6. $r=0.1$ $\delta=0.05$ American call $K=50$.



$$q = \frac{S_+ - S_0 e^{(r-\delta)t}}{S_+ - S_-}$$

$$\begin{cases} q_1 = \frac{80-70e^{(0.1-0.05) \times 1}}{80-60} = 0.3206 \\ q_2 = \frac{40-30e^{(0.1-0.05) \times 1}}{40-20} = 0.4231 \\ q_3 = \frac{70-50e^{(0.1-0.05) \times 1}}{70-30} = 0.4359 \end{cases}$$

for node B, $V_- = 0$

$$\begin{aligned} \text{for node A, } V_+ &= \max \{ S_+ - K, (q_1 \times 10 + (1-q_1) \times 30) e^{-rt} \} \\ &= \max \{ 70 - 50, (0.3206 \times 10 + 0.6794 \times 30) e^{-0.1 \times 1} \} \\ &= \max \{ 20, 21.3433 \} \\ &= 21.3433 \end{aligned}$$

$$\begin{aligned} \text{for node C, } V_0 &= \max \{ S - K, (q_3 \times V_- + (1-q_3) V_+) e^{-rt} \} \\ &= \max \{ 0, (0.4359 \times 0 + 0.5641 \times 21.3433) e^{-0.1 \times 1} \} \\ &= 10.8940 \end{aligned}$$

7. Asian average strike put. $\max\{0, \bar{S} - 5\}$ $r=5\%$ $\delta=5\%$ $dt=1yr.$

