

## PT Assignment 2

1. Let  $\Omega$  be a set that is not empty. Suppose  $A \subset \Omega$  and  $A$  is neither  $\Omega$  nor  $\emptyset$ . Show that

$$\mathcal{A} = \{\emptyset, A, A^c, \Omega\}$$

is a  $\sigma$ -algebra over  $\Omega$ .

2. Let  $A, B, C$  be events. Suppose  $P(A) = 1/2$ ,  $P(B) = 1/3$ ,  $P(C) = 1/5$ ,  $P(A \cap B) = 1/10$ ,  $P(A \cap C) = 1/15$ ,  $P(B \cap C) = 1/20$  and  $P(A \cap B \cap C) = 1/30$ . Find  $P(A \cup B)$ ,  $P(A^c \cap B^c)$ ,  $P(A \cup B \cup C)$ ,  $P(A^c \cap B^c \cap C^c)$ ,  $P(A^c \cap B^c \cap C)$  and  $P((A^c \cap B^c) \cup C)$ .  
[Hint: the inclusion-exclusion principle for three events is as follows:  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$ .]

3. Four red, 8 blue, and 5 green balls are randomly arranged in a line.

- (a) What is the probability that the first 5 balls are blue?
- (b) What is the probability that none of the first 5 balls is blue?
- (c) What is the probability that the final 3 balls are of different colors?
- (d) What is the probability that all the red balls are together?

4. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. The classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students who are in both Spanish and French, 4 who are in both Spanish and German, and 6 who are in both French and German. In addition, there are 2 students taking all 3 classes.

- (a) If a student is chosen randomly, what is the probability that he or she is not in any of the language classes?
- (b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?
- (c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?

5. Two cards are chosen at random from a deck of 52 playing cards. What is the probability that they

- (a) are both aces?
- (b) have the same value?

6. An instructor gives her class a set of 10 problems with the information that the final exam will consist of a random selection of 5 of them. If a student has figured out how to do 7 of the problems, what is the probability that he or she will answer correctly
- (a) all 5 problems?
  - (b) at least 4 of the problems?
7. From a group of 3 first-year students, 4 sophomores, 4 juniors, and 3 seniors, a committee of size 4 is randomly selected. Find the probability that the committee will consist of
- (a) 1 from each class;
  - (b) 2 sophomores and 2 juniors;
  - (c) only sophomores or juniors.
8. Three married couples take seats around a table at random. Compute  $P(\text{no wife sits next to her husband})$ .