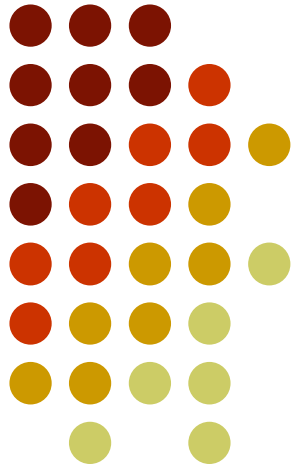
Bond Pricing and Yields

Chapter 8



Outline

- Bond characteristics
- Bond pricing
- Bond yields
- References: BF Chaps 12-13; PF Chap 10

Terminology

- Bond
  - Security that obligates the issuer to make specified payments to the bondholder
- Face value (par value or principal value)
  - Payment at the maturity of the bond
- Coupon
  - The interest payments made to the bondholder
- Coupon rate
  - Annual interest payment, as a percentage of face value

PV Formula to Value a Bond

$$V_d = \frac{INT}{(1+r)^1} + \frac{INT}{(1+r)^2} + \cdots + \frac{INT + M}{(1+r)^N}$$

$$= INT\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right] + \frac{M}{(1+r)^N}$$

Bond value = PV of an annuity of interest

\+ PV of a lump-sum payment at maturity

PV Formula to Value a Bond

- Example 1
  - In October 2014 you purchase 100 euros of bonds in France that pay a 4.25% coupon every year. If the bond matures in 2018 and the yield to maturity (YTM) is 0.15%, what is the value of the bond?

$$V_d = \frac{4.25}{1.0015} + \frac{4.25}{1.0015^2} + \frac{4.25}{1.0015^3} + \frac{104.25}{1.0015^4} = 116.34\ euros$$

PV Formula to Value a Bond

- Example 2
  - If today is October 1, 2015, what is the value of the following bond? An IBM bond pays $115 every September 30 for 5 years. In September 2020 it pays an additional $1000 and retires the bond. The bond is rated AAA (WSJ AAA YTM is 7.5%)

$$V_d = \frac{115}{1.075} + \frac{115}{1.075^2} + \frac{115}{1.075^3} + \frac{115}{1.075^4} + \frac{1{,}115}{1.075^5} = \$1{,}161.84$$

PV Formula to Value a Bond

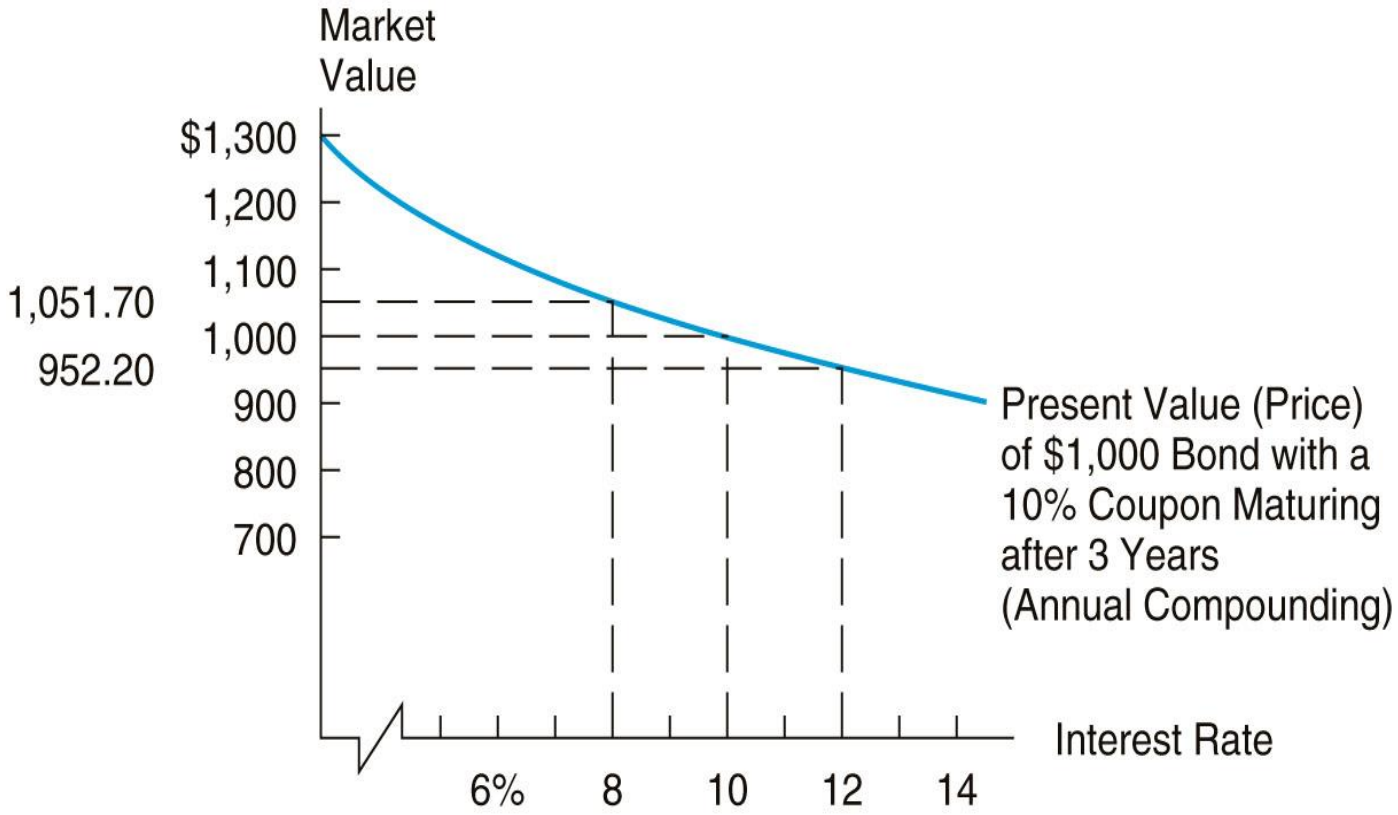
- Example 3
  - In February 2012 you purchase a three-year U.S. government bond. The bond has an annual coupon rate of 11.25%, paid semiannually. If investors demand a 0.085% semiannual return, what is the price of the bond?

$$V_d = \frac{56.25}{1.00085} + \frac{56.25}{1.00085^2} + \frac{56.25}{1.00085^3} + \frac{56.25}{1.00085^4} + \frac{56.25}{1.00085^5} + \frac{1{,}056.25}{1.00085^6}$$

$$= \$1{,}331.40$$

Bond Prices Vary with Interest Rates

- The inverse relationship between bond prices and interest rates



Current Yield

- The current yield (CY) is the percentage that you earn annually.

$$CY = \frac{Annual\ interest\ payment}{Price\ of\ the\ bond}$$

  - Example: If a bond has a coupon rate of 6% and sells for $948.62, CY = $60/$948.62 = 6.3%

Yield to Maturity (YTM)

- YTM is the average rate of return earned on a bond if it is held to maturity
  - Example: Investors expect the bond's dollar payoff to include $70 interest at the end of each year for the next 19 years and a $1,000 principal payment when the bond matures in 19 years. The bond is currently worth $821.

$$\frac{\$70}{(1+YTM)^1}+\frac{\$70}{(1+YTM)^2}+\cdots+\frac{\$70+\$1{,}000}{(1+YTM)^{19}}=\$821$$

Yield to Maturity

- Spreadsheet Calculation
  - YIELD(settlement date, maturity date, annual coupon rate, bond price, redemption value as percent of par value, number of coupon payments per year)

	A	B	C	D
1		**Semiannual coupons**		**Annual coupons**
2				
3	Settlement date	2000/12/31		2000/12/31
4	Maturity date	2019/12/31		2019/12/31
5	Annual coupon rate	0.07		0.07
6	Bond price (flat)	82.1		82.1
7	Redemption value (% of face value)	100		100
8	Coupon payments per year	2		1
9				
10	**Yield to maturity (decimal)**	**0.0898**		**0.0900**
11				
12	The formula entered here is =YIELD(B3,B4,B5,B6,B7,B8)			

Yield to Maturity (YTM)

- Approximate YTM

$$Approximate\ YTM = \frac{Annual\ interest + Accrued\ capital\ gains}{Average\ value\ of\ bond}$$

$$= \frac{INT + \left(\frac{M - V_d}{N}\right)}{\left|\frac{2(V_d) + M}{3}\right|}$$

  - Using previous example

$$YTM \approx \frac{\$70 + \left(\frac{\$1{,}000 - \$821}{19}\right)}{\left|\frac{2(\$821) + \$1{,}000}{3}\right|} = 0.0902 = 9.0\%$$

Current Yield and YTM

- Premium bonds: bonds selling above par value
  - Coupon rate > Current yield > YTM
- Discount bonds: bonds selling below par value
  - Coupon rate < Current yield < YTM
- The premium reduces the yield to maturity
- The discount increase the yield to maturity

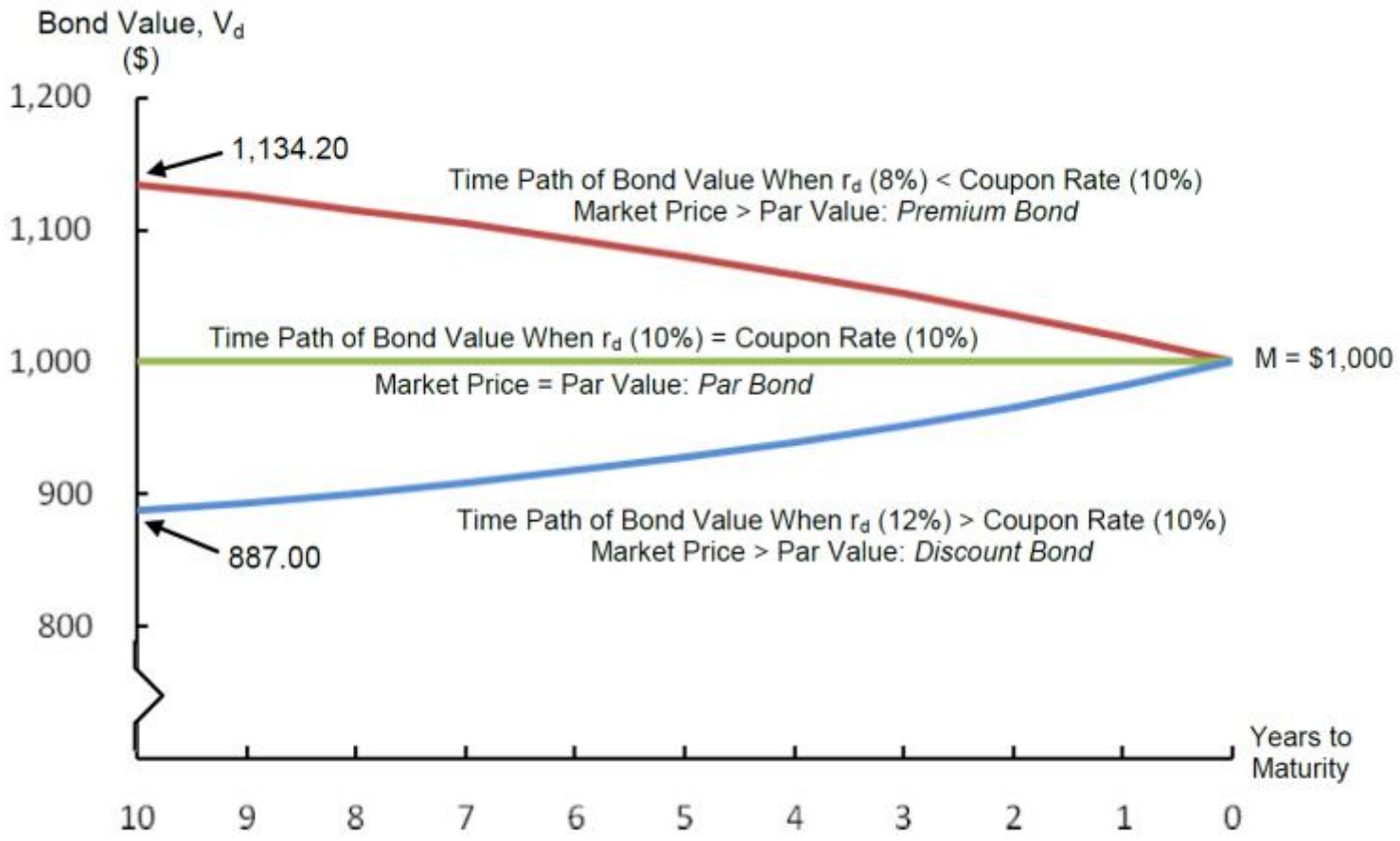
Changes in Bond Values over Time

- When r (market rate) equals the coupon rate, the bond will sell at its par value.
- Interest rates in the economy change continuously.
- As interest rates change, so do the market values of bonds such that the rate of return earned by investing in a bond, i.e., YTM, is the same as the appropriate interest rate in the financial markets.

Changes in Bond Values over Time

- When r > coupon rate, the bond's market price is less than its maturity value, and the bond is said to be selling at a discount.
- When r < coupon rate, the bond's market price is greater than its maturity value, and the bond is said to be selling at a premium.
- The market value of a bond will always approach its par value as its maturity date approaches, provided the firm does not go bankrupt.

Time Path of Value of a 10% Coupon, $1000 Par Value



Changes in Bond Values over Time

- Return (yield) on a bond

$$\text{Bond yield} = \text{Current (interest) yield} + \text{Capital gains yield}$$

$$= \frac{\text{INT}}{V_{d,Begin}} + \frac{V_{d,End} - V_{d,Begin}}{V_{d,Begin}}$$

INT = interest
$V_{d,Begin}$ = beginning value of the bond
$V_{d,End}$ = ending value of the bond

Changes in Bond Values over Time

- Bond yield example
  - r = 8%, V_d = $1,134.20, 10 years to maturity
  - A year from now, V_d = $1,124.94. A decrease of value by $9.26
  - Current yield = 100/1,134.20 = 0.0882 = 8.82%
  - Capital gains yield = -9.26/1,134.20 = -0.0082 = -0.82%
  - Total rate of return (yield) = (100-9.26)/1,134.20 = 8% = r

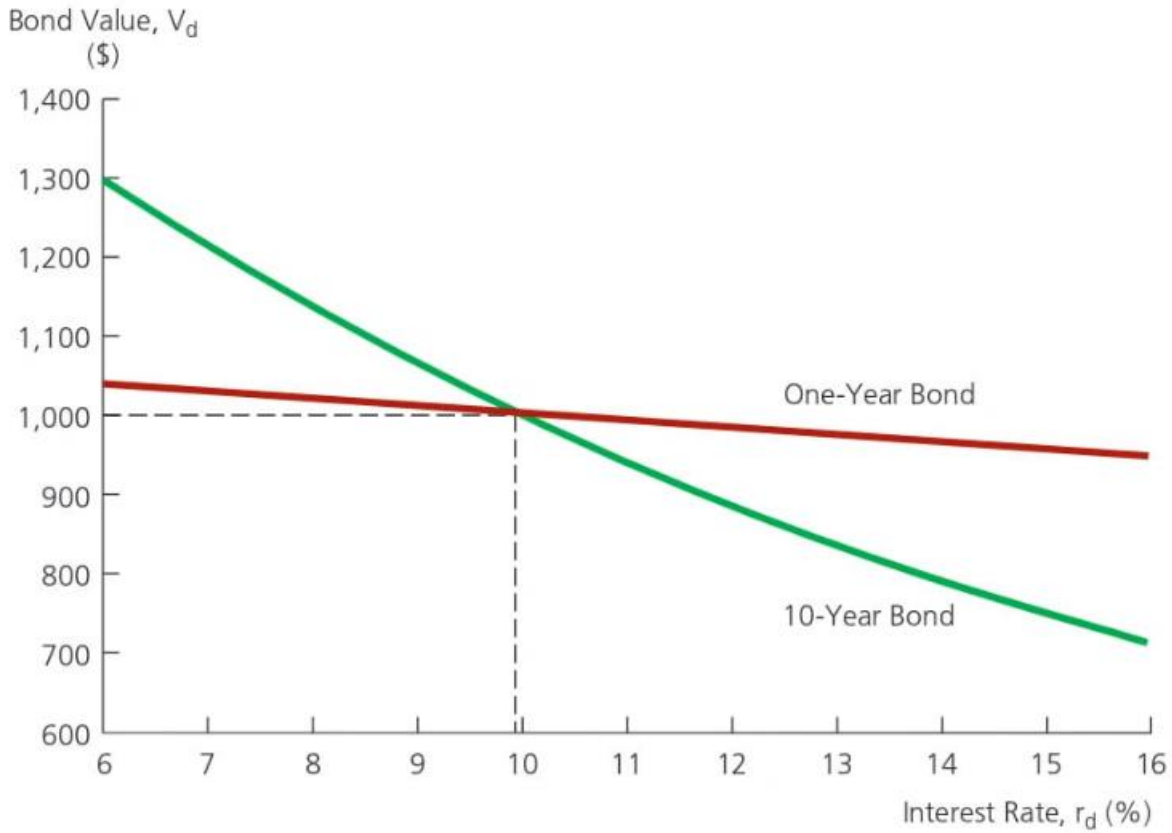
Interest Rate Risk on a Bond

- Interest rate price risk
  - The risk of changes in bond prices to which investors are exposed due to changing interest rates
- Interest rate reinvestment risk
  - The risk that income from a bond portfolio will vary because cash flows have to be reinvested at current market rates
- These two risks tend to offset each other.

Interest Rate Risk on a Bond

- An investor's exposure to interest rate price risk is higher on bonds with long maturities than on those that mature in the near future
- The longer the maturity of the bond, the more significantly its price changes in response to a given change in interest rates.

FIGURE 10-4 Value of Long- and Short-Term 10% Annual Coupon Rate Bonds at Different Market Interest Rates (r_d)



Current Market Interest Rate, r_d	Value of One-Year Bond	Value of 10-Year Bond
6%	$1,037.74	$1,294.40
8	1,018.52	$1,134.20
10	1,000.00	1,000.00
12	982.14	887.00
14	964.91	791.36
16	948.28	710.01

Summary

- Present value formula to value a bond

$$V_d = \frac{INT}{(1+r)^1} + \frac{INT}{(1+r)^2} + \cdots + \frac{INT + M}{(1+r)^N}$$

$$= INT\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right] + \frac{M}{(1+r)^N}$$

  - The inverse relationship between bond prices and interest rates

Summary

- Yields
  - Current yield

$$CY = \frac{Annual\ interest\ payment}{Price\ of\ the\ bond}$$

  - Yield to maturity: average rate of return earned on a bond if it is held to maturity
  - Comparison of the CY and the YTM
    - Premium bonds: Coupon rate > Current yield > YTM
    - Discount bonds: Coupon rate < Current yield < YTM
  - Bond yield = current (interest) yield + capital gains yield

Summary

- Time path of bond values
  - The market value of a bond will always approach its par value as its maturity date approaches, provided the firm does not go bankrupt.
- Interest rate risk
  - Interest rate price risk
  - Interest rate reinvestment risk