PT

Solution to Assignment 1

1. Let R, G, B respectively represent the red, green and blue marbles.

   (a) The sample space is $\Omega = \{\mathrm{RR}, \mathrm{RG}, \mathrm{RB}, \mathrm{GR}, \mathrm{GG}, \mathrm{GB}, \mathrm{BR}, \mathrm{BG}, \mathrm{BB}\}$. Since all outcomes have the same probability, the probability of each point in the sample space is $\frac{1}{9}$.

   (b) The sample space is $\Omega = \{\mathrm{RG}, \mathrm{RB}, \mathrm{GR}, \mathrm{GB}, \mathrm{BR}, \mathrm{BG}\}$. Since all outcomes have the same probability, the probability of each point in the sample space is $\frac{1}{6}$.

2. The evens can be represented as follows:

   (a) $F \cap E^c \cap G^c$

   (b) $E \cap F \cap G^c$

   (c) $E \cup F \cup G$

   (d) $(E \cap F) \cup (E \cap G) \cup (F \cap G)$

   (e) $E \cap F \cap G$

   (f) $E^c \cap F^c \cap G^c$

   (g) $(E \cap G^c \cap F^c) \cup (E^c \cap G \cap F^c) \cup (E^c \cap G^c \cap F) \cup (E^c \cap F^c \cap G^c)$

   (h) $E^c \cup F^c \cup G^c$.

3. Consider a sample space $\Omega = [0, 1]$ and assign probabilities in such a way that each sub-interval has probability equal to its length:

$$P([a, b]) = b - a,$$

   where $[a, b] \subset [0, 1]$. Hence, $P(\{a\}) = 0$, $a \in [0, 1]$, but $\{a\}$ is not $\emptyset$.

4.

   (a) $\{\text{the circuit works in order}\} = A \cap (B \cup C) \cap D$

   (b) $\{\text{the circuit works out of order}\} = A^c \cup (B^c \cap C^c) \cup D^c$

5. Noting that $A \cap B = \emptyset$, the probabilities are as follows:

   (a) $P(A \cup B) = P(A) + P(B) = 0.8$.

   (b) $P(A \setminus B) = P(A) = 0.3$.

   (c) $P(A \cap B) = P(\emptyset) = 0$.

6. We must have

$$\mathcal{A} = \{\phi, \{1\}, \{2,3\}, \{1,2,3\}\}.$$

   Suppose $\{1,2\} \in \mathcal{A}$. Then $\{2\} = \{1,2\} \backslash \{1\} \in \mathcal{A}$, which contradicts to the assumption.

7. The sample space is $\Omega = \{(i,j) : i,j = 1, \cdots, 6\}$. So $|\Omega| = 36$.

   (a) $A = \{$ exactly one six$\} = \{(6,1),(6,2),\cdots,(6,5),(1,6),\cdots,(5,6)\}$. So $P(A) = \frac{|A|}{|\Omega|} = \frac{10}{36} = \frac{5}{18}$

   (b) $A = \{$ both numbers are odd $\} = \left\{ \begin{array}{c} (1,1),(1,3),(1,5),(3,1),(3,3),(3,5),\} \\ (5,1),(5,3),(5,5) \end{array} \right\}$. So $P(A) = \frac{|A|}{|\Omega|} = \frac{9}{36} = \frac{1}{4}$.

   (c) $A = \{$ sum is 4$\} = \{(1,3),(2,2),(3,1)\}$. So $P(A) = \frac{|A|}{|\Omega|} = \frac{3}{36} = \frac{1}{12}$.

   (d) $A = \{$ sum is divisible by 3$\} = \left\{ \begin{array}{c} (1,2),(2,1),(1,5),(2,4),(3,3), \\ (4,2),(5,1),(3,6),(4,5),(5,4), \\ (6,3),(6,6) \end{array} \right\}$. So $P(A) = \frac{|A|}{|\Omega|} = \frac{12}{36} = \frac{1}{3}$.