# Brief Solution to Assignment 1

1. A stock price is currently \$100. It is known that at the end of two months it will be either \$125 or \$90. The interest rate is 5% per annum with continuous compounding. What is the value of a two-month European call option with a strike price of \$105?

   **Solution:** The risk-neutral probability of a up movement is $p$.

$$\begin{aligned} p &= \frac{100 \times e^{\frac{0.05}{6}} - 90}{125 - 90} = 0.3096 \\ V_0 &= [0 \times (1-p) + 20 \times p] \times e^{-\frac{0.05}{6}} = 6.1411 \end{aligned}$$

2. A stock price is currently \$120. It is known that at the end of four months it will be either \$100 or \$130. The interest rate is 3% per annum with continuous compounding. What is the value of a four-month European put option with a strike price of \$110.

   **Solution:** The risk-neutral probability of a up movement is $p$.

$$\begin{aligned} p &= \frac{120 \times e^{0.03 \times \frac{4}{12}} - 100}{130 - 100} = 0.7069 \\ V_0 &= [0 \times p + 10 \times (1-p)] \times e^{-0.03 \times \frac{4}{12}} = 2.9022 \end{aligned}$$

3. A stock price is currently \$150. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The interest rate is 7% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of \$150.

   **Solution:** The risk-neutral probability of a up movement is $p$.

$$\begin{aligned} p &= \frac{e^{\frac{0.07}{2}} - 0.90}{1.10 - 0.90} = 0.6781 \\ V_0 &= (150 \times 1.1 \times 1.1 - 150) \times p \times p \times e^{-0.07} = 13.5050 \end{aligned}$$

4. For the situation considered in Exercise 3, what is the value of a one-year European put option with a strike price of \$150? Verify that the European call and European put prices satisfy the put-call parity formula.

   **Solution:**

   From Question 3, we have $p = 0.6781$.

$$V_0 = \Big[1.5 \times [(1-p) \times p + 1.5 \times p \times (1-p)] + 28.5 \times (1-p) \times (1-p)\Big] \times e^{-0.07} = 3.3641$$

   Call-put parity: $C + Ke^{-r(T-t)} = P + S_t$

$$\begin{aligned} \text{left} &= 13.5050 + 150e^{-0.07} = 153.3641 \\ \text{right} &= 3.3641 + 150 = 153.3641 = \text{left} \end{aligned}$$