MATH2033 Mathematical Statistics
Assignment 8

Due Date: **12/May/2024(Sunday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.
- **Late submissions**, **answers without details**, or **unrecognizable handwritings** will NOT be graded.

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1. If $H_0 : \mu = 30$ is tested against $H_1 : \mu > 30$ using $n = 16$ observations (normally distributed) and if $1 - \beta = 0.85$ when $\mu = 34$, what does α equal? Assume that $\sigma = 9$.

2. Let X have a binomial distribution with the number of trials $n = 10$ and with p either $1/4$ or $1/2$. The simple hypothesis $H_0 : p = 1/2$ is rejected, and the alternative simple hypothesis $H_1 : p = 1/4$ is accepted, if the observed value of X_1, a random sample of size 1, is less than or equal to 3.

   (a) Should we consider it as a large sample test or small sample test?

   (b) Find the significance level and the power of the test.

3. Construct a power function $\gamma(\mu)$ for the $\alpha = 0.05$ test of $H_0 : \mu = 60$ versus $H_1 : \mu \neq 60$ if the data consist of a random sample of size 16 from a normal distribution having $\sigma = 4$ by finding $\gamma(56)$, $\gamma(58)$ and $\gamma(60)$.

4. A sample of size 1 from the pdf $f_Y(y) = (1 + \theta)y^\theta$, $0 \leq y \leq 1$, is to be the basis for testing

$$H_0 : \theta = 1 \quad \text{versus} \quad H_1 : \theta < 1$$

   The critical region will be $\{y \leq 1/2\}$. Express $1 - \beta$ as a function of θ.

5. Let $X_1, \ldots, X_n$ be iid samples from the distribution that has the pdf

$$f(x|\theta) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-\theta)^2}{2}\right), \quad -\infty < x < \infty$$

   It is desired to test the simple hypothesis $H_0 : \theta = \theta_0 = 0$ against the alternative simple hypothesis $H_1 : \theta = \theta_1 = 1$.

   (a) Find the best critical region with $\alpha = 0.05$ and $n = 25$.

   (b) Based on your result in part (a), find the power of this best test of H_0 against H_1 when H_1 is true.