Chapter 8 solution

1.

For AR(1) model, we have the variance of sample autocorrelation of residuals $\sqrt{Var(\hat{r_1})} \approx \sqrt{\phi^2/n} = \sqrt{0.5^2/100} = 0.05$ so that would expect the lag 1 sample autocorrelation of the residuals to be within ś0.1.The residual autocorrelation of 0.5 is most unusual.

2.

According to the third paragraph on page 183 of our textbook, we know that:
The results analogous to those for AR models hold for MA models. For MA(1) model, the variance of sample autocorrelation of residuals $\sqrt{Var(r_1)} \approx \sqrt{\theta^2/n} = \sqrt{0.5^2/100} = 0.05$. So again a lag 1 residual autocorrelation of 0.5 is most unusual.

3.

For AR(2) model, we have the variance of sample autocorrelation of residuals:

$$\sqrt{Var(\hat{r_1})} \approx \sqrt{\phi_2^2/n} = \sqrt{(-0.8)^2/200} = 0.057$$

so that $\hat{r_1} = 0.13$ is "too large".

$$\sqrt{Var(\hat{r_2})} \approx \sqrt{(\phi_2^2 + \phi_1^2(1+\phi_2)^2)/n} = 0.059$$

so that $\hat{r_2} = 0.13$ is also "too large".

For $k > 2$,

$$\sqrt{Var(\hat{r_k})} \approx \sqrt{1/200} = 0.12$$

so that $\hat{r_3} = 0.12$ is OK.

The Ljung-Box statistic,

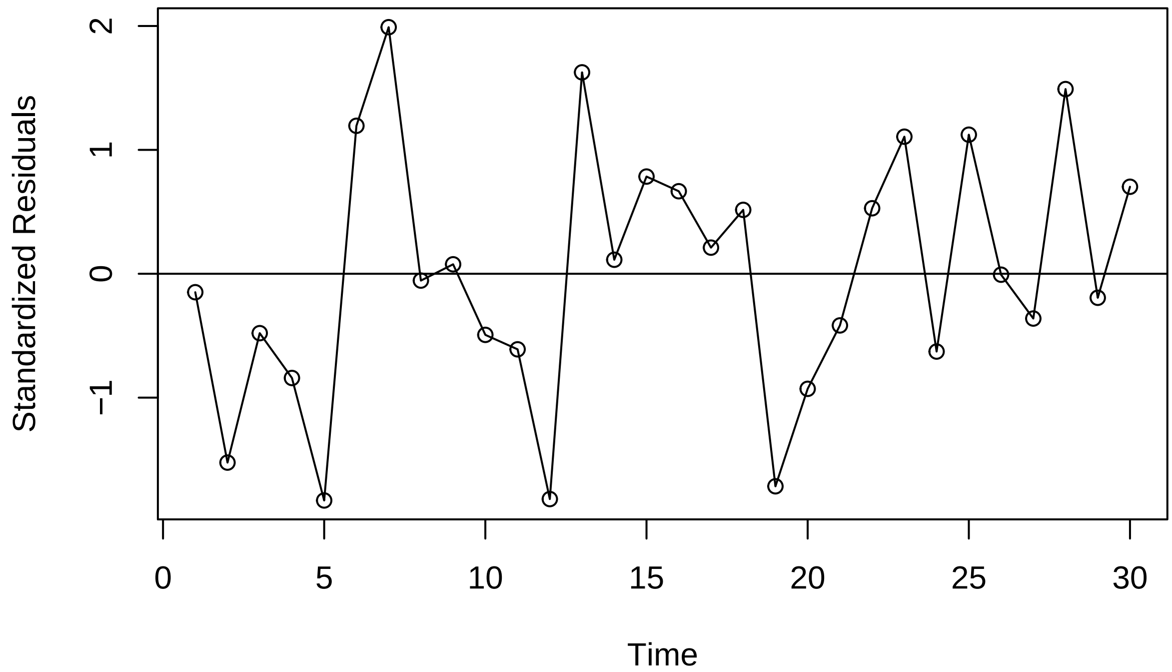
$$Q_* = n(n+2)(\frac{\hat{r_1}^2}{n-1} + \frac{\hat{r_2}^2}{n-2} + \frac{\hat{r_3}^2}{n-3}) = 200(202)(\frac{0.13^2}{199} + \frac{0.13^2}{198} + \frac{0.12^2}{197}) = 9.83$$

If the AR(2) specification is correct, then Q_* has (approximately) a chi-square distribution with $3-2=1$ degree of freedom. However, $Pr[\chi_1^2 > 9.83] = 0.0017$ so that these residual autocorrelation are jointly too large to support AR(2) model.

4.

(a)

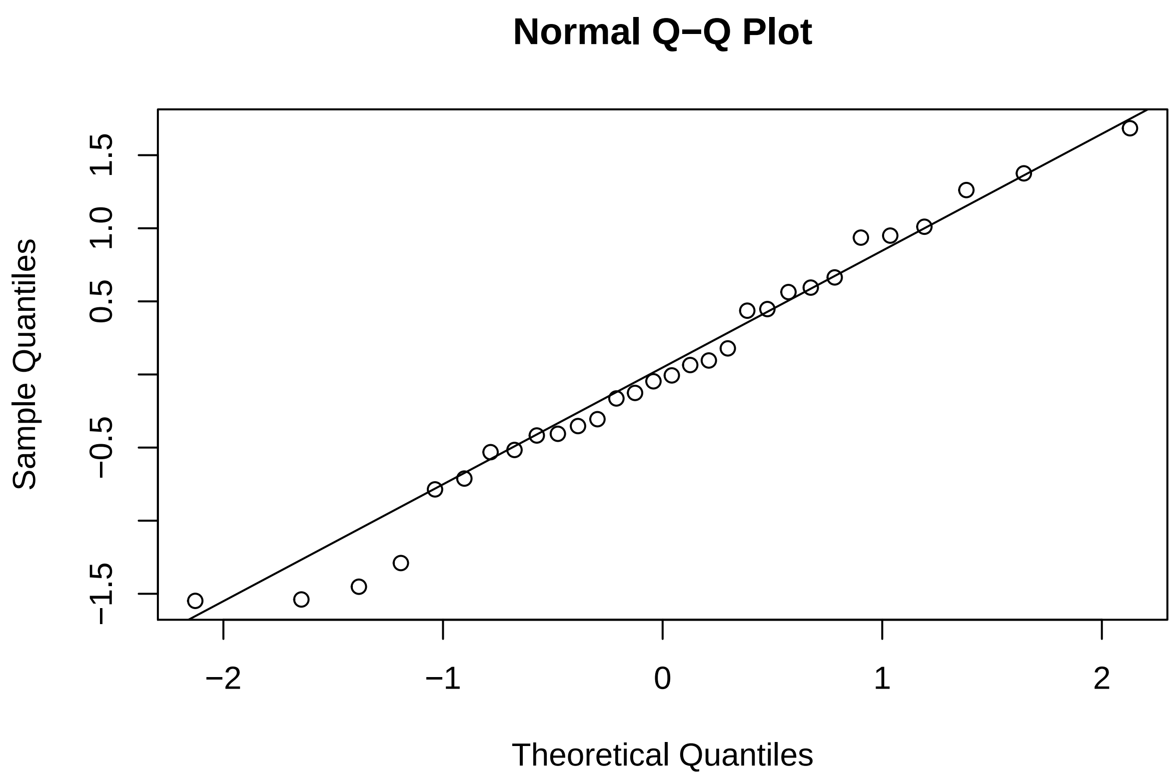
```r
set.seed(12347); series=arima.sim(n=30,list(ar=0.5))
model=arima(series,order=c(1,0,0))
plot(rstandard(model),ylab ='Standardized Residuals', type='o')
abline(h=0)
```



These standardized residuals look fairly "random" with no particular patterns.

(b)

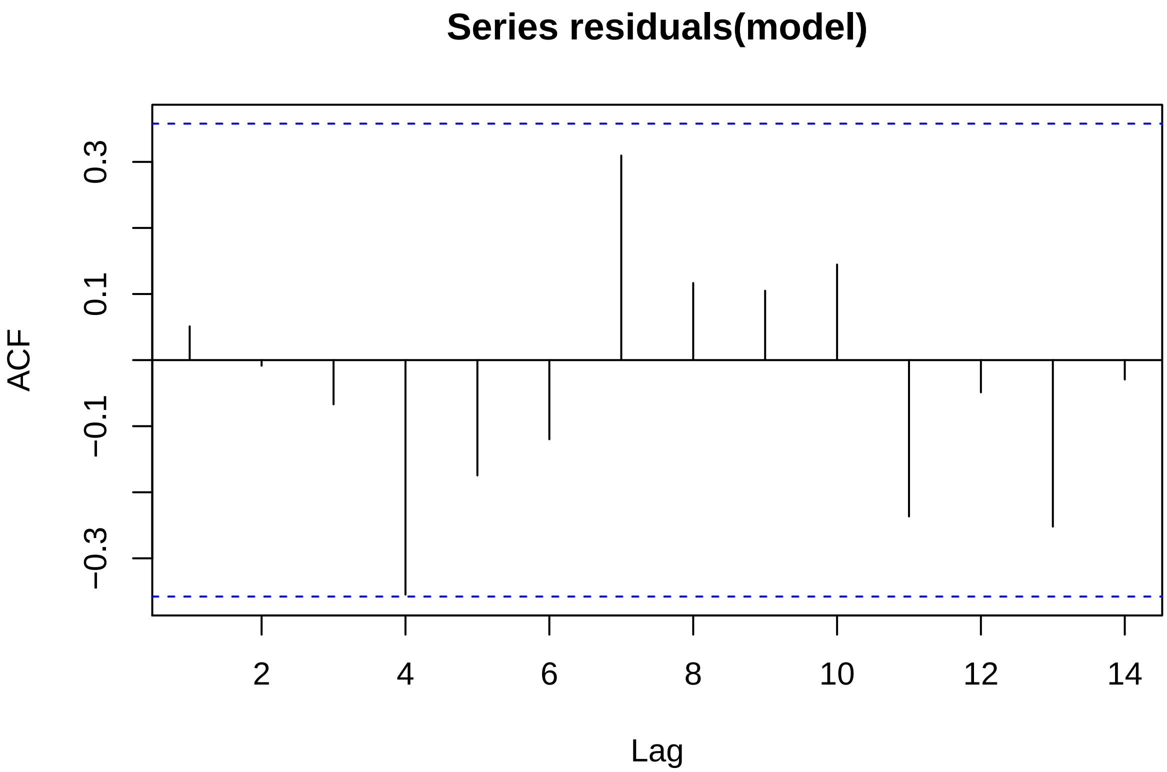
```
qqnorm(residuals(model)); qqline(residuals(model))
```



With a few minor exceptions in the lower tail, the Q-Q plot of the standardized residuals looks reasonably "normal."

(c)

```r
acf(residuals(model))
```



The sample acf at lag 4 is the only individual autocorrelation that comes close to being "significant."

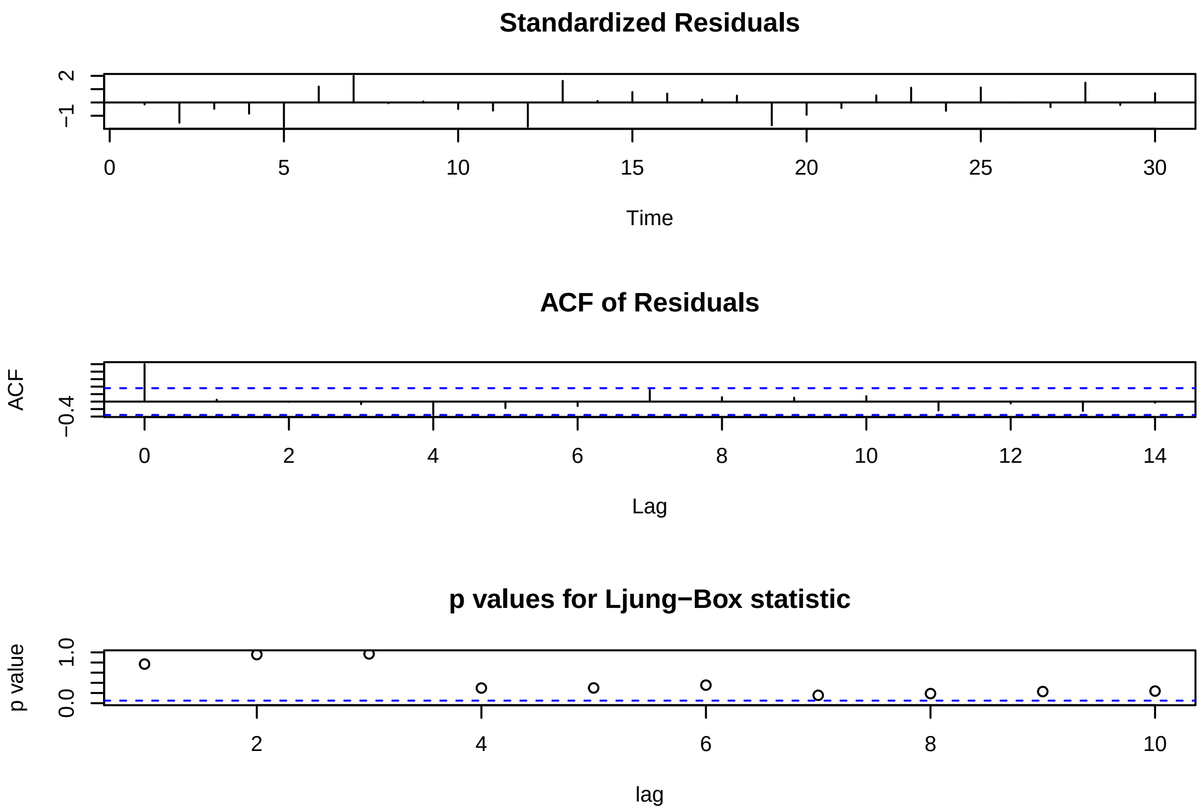
(d)

```r
LB.test(model,lag=8)
```

```
##
##  Box-Ljung test
##
## data:  residuals from  model
## X-squared = 11.24, df = 7, p-value = 0.1285
```

This test does not reject randomness of the error terms based on the first eight autocorrelations of the residuals.

```r
tsdiag(model)
```



The bottom display shows the p-values of the Ljung-Box test for a variety of values of the "K" parameter—the highest lag used in the sum. The top display will flag potential outliers, if any, using the Bonferroni criteria.

5.

```
data(robot); mod1=arima(robot,order=c(1,0,0)); res1=rstandard(mod1); mod1
```

```
##
## Call:
## arima(x = robot, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.3074     0.0015
## s.e.  0.0528     0.0002
##
## sigma^2 estimated as 6.482e-06:  log likelihood = 1475.54,  aic = -2947.08
```

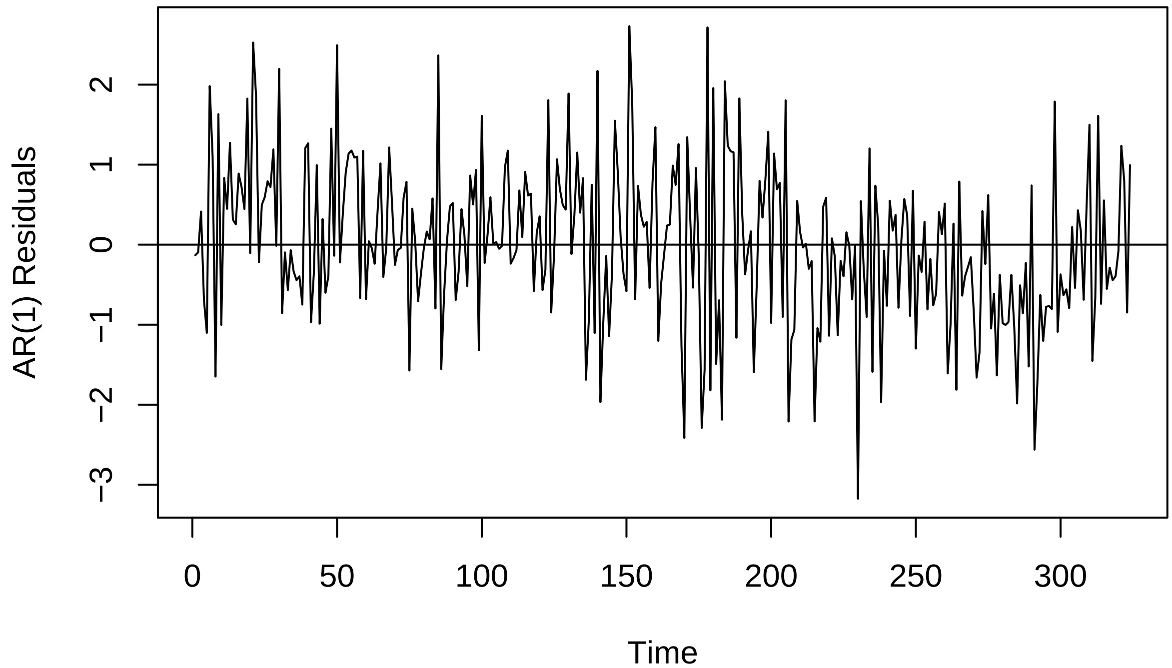
```
mod2=arima(robot,order=c(1,0,1)); res2=rstandard(mod2); mod2
```

```
##
## Call:
## arima(x = robot, order = c(1, 0, 1))
##
## Coefficients:
##          ar1      ma1  intercept
##       0.9472  -0.8062     0.0015
## s.e.  0.0309   0.0609     0.0005
```

```
##
## sigma^2 estimated as 5.948e-06:  log likelihood = 1489.3,  aic = -2972.61
```

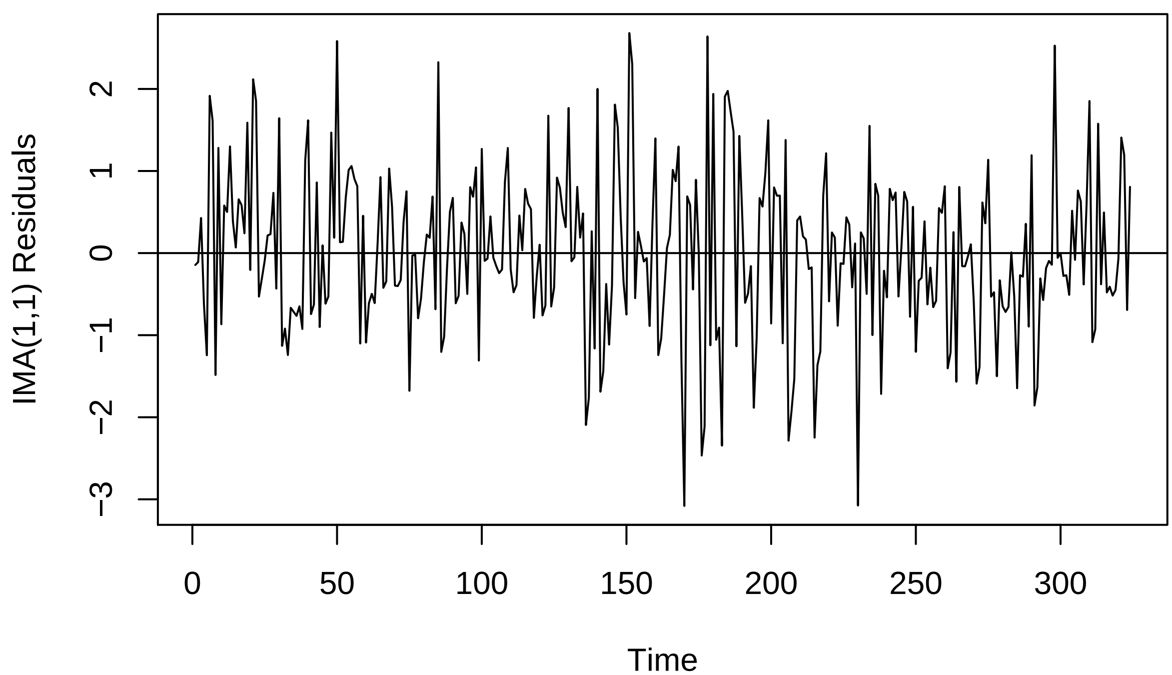
Both models have statistically significant parameter estimates. The log likelihood and AIC values are just a little better in the IMA(1,1) model.

```
plot(res1,ylab='AR(1) Residuals'); abline(h=0)
```



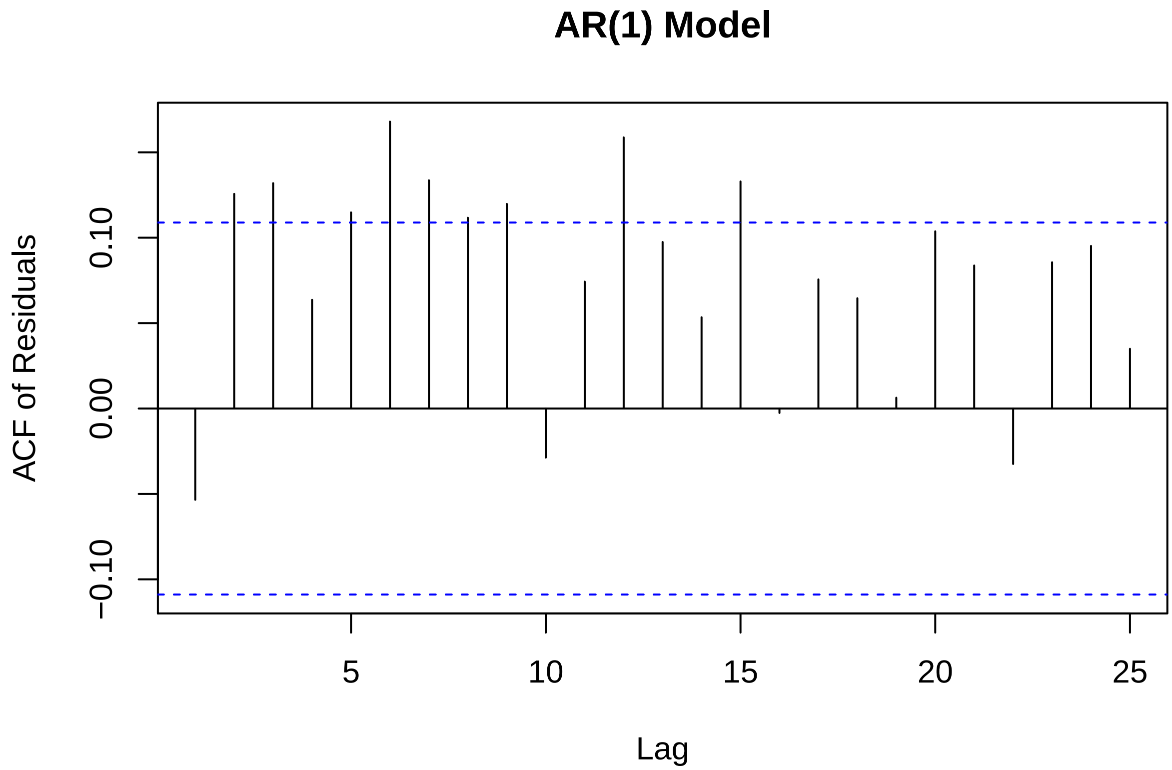
There may be a little drift in these residuals over time. There are more positive residuals in the first half of the series and more negative in the last half.

```
plot(res2,ylab='IMA(1,1) Residuals'); abline(h=0)
```



The drift observed in the residuals of the AR(1) model does not seem to appear with the IMA(1,1) model residuals. We proceed to look at correlation in the residuals.

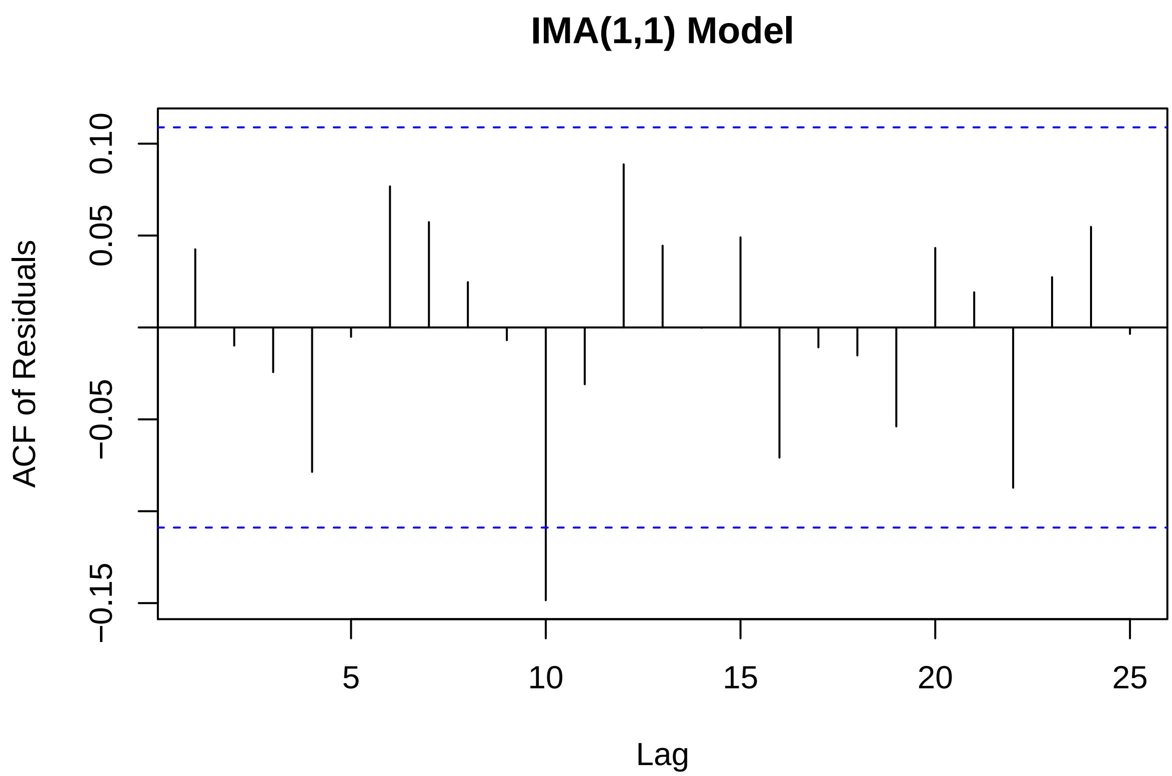
```r
acf(residuals(mod1), main='AR(1) Model',ylab='ACF of Residuals'); LB.test(mod1)
```




```
##
##  Box-Ljung test
##
## data:  residuals from  mod1
## X-squared = 52.512, df = 11, p-value = 2.201e-07
```

The residuals from the AR(1) model clearly have too much autocorrelation.

```r
acf(residuals(mod2), main='IMA(1,1) Model',ylab='ACF of Residuals'); LB.test(mod2)
```

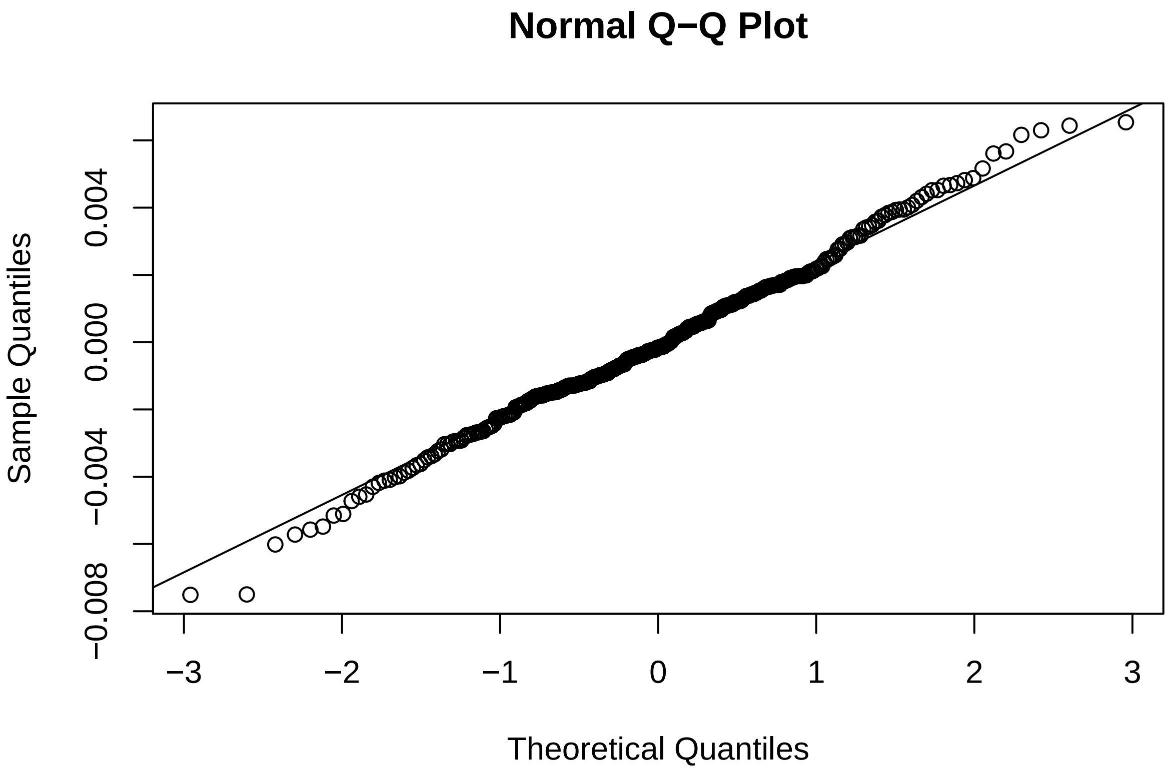



```
##
##  Box-Ljung test
##
## data:  residuals from  mod2
## X-squared = 16.543, df = 10, p-value = 0.0851
```

The residuals from the IMA(1,1) model are much less correlated with only one significant autocorrelation at lag 10. The Ljung-Box test indicates that, jointly, the residual autocorrelations are not too large.

Next we check out normality of the error terms by first displayinga Q-Q plot of the residuals.

```
qqnorm(residuals(mod2)); qqline(residuals(mod2))
```



The Q-Q plot looks good. Let's confirm this with the Shapiro-Wilk test.

```
shapiro.test(residuals(mod2))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(mod2)
## W = 0.99496, p-value = 0.3717
```

Normality looks like a viable assumption for the error terms in the IMA(1,1) model for the robot time series. Finally, let's look at the results from the tsdiag command.