

PT Assignment 6

1. Suppose that the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{b}{4} & 0 \leq b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \leq b < 2 \\ \frac{11}{12} & 2 \leq b < 3 \\ 1 & 3 \leq b \end{cases}$$

(a) Find $P\{X = i\}$, $i = 1, 2, 3$.

(b) Find $P\{\frac{1}{2} < X < \frac{3}{2}\}$.

2. You are dealt one card at random from a full deck and your opponent is dealt 2 cards (without any replacement). If you get an Ace, he pays you \$10, if you get a King, he pays you \$5 (regardless of his cards). If you have neither an Ace nor a King, but your card is red and your opponent has no red cards, he pays you \$1. In all other cases you pay him \$1. Determine your expected earnings. Are they positive?

3. Let X be such that

$$P\{X = 1\} = p = 1 - P\{X = -1\}$$

Find $c \neq 1$ such that $E[c^X] = 1$.

4. If X is a binomial random variable with expected value 6 and variance 2.4, find $P\{X = 5\}$.
5. An urn contains 11 balls, 3 white, 3 red, and 5 blue balls. Take out 3 balls at random, without replacement. You win \$1 for each red ball you select and lose a \$1 for each white ball you select. Determine the p.m.f. of X , the amount you win.
6. Denote by d the dominant gene and by r the recessive gene at a single locus. Then dd is called the pure dominant genotype, dr is called the hybrid, and rr the pure recessive genotype. The two genotypes with at least one dominant gene, dd and dr , result in the phenotype of the dominant gene, while rr results in a recessive phenotype. Assuming that both parents are hybrid and have n children, what is the probability that at least two will have the recessive phenotype? Each child, independently, gets one of the genes at random from each parent.
7. You and your opponent both roll a fair die. If you both roll the same number, the game is repeated, otherwise whoever rolls the larger number wins. Let N be the number of times the two dice have to be rolled before the game is decided.

- (a) Determine the probability mass function of N .
- (b) Compute EN .
- (c) Compute P (you win).
- (d) Assume that you get paid \$10 for winning in the first round, \$1 for winning in any other round, and nothing otherwise. Compute your expected winnings.