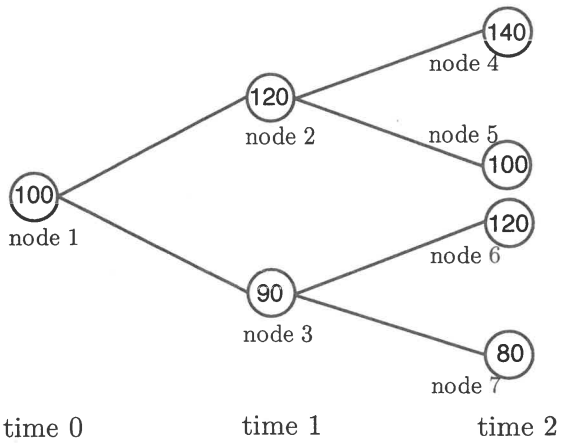
Test 1 of Financial Mathematics

**Question 1.**

An investor deposits $X$ at $t = 0$ and another $2X$ at $t = 2$. The investor will receive $Z$ at $t = 3$ and another $5Z$ at $t = 5$. The continuously compound interest rate is $r$. Express your answers to the following questions in terms of $r$ and $X$.
(a) (6 pts) What is the net present value of this investment at $t = 0$?
(b) (6 pts) What is the future value of this investment at the end of the 5th year?
(c) (8 pts) What is the minimum value for Z such that the return is positive for this investment?

**Question 2.**
A bank offers a saving account which has a continuous compound interest rate of $r_1$ in the first year and a continuous compound interest rate of $r_2$ starting from the beginning of the second year. A person invests 500 dollars in that account at $t = 0$. Express your answers to the following questions in terms of $r_1$ and $r_2$.
(a) (6 pts) What is the total wealth in that bank account at the end of the 5-th year?
(b) (7 pts) What is the effective constant continuously compound interest rate at the end of 5-th year?
(c) (7 pts) What is the effective annually compound constant interest rate at the end of 5-th year?

**Question 3.**
The price of a stock evolves according to the following binomial tree (whose time ranges from 0 to 2).




(a) (7 pts) Calculate the risk-neutral probability at each edge.
(b) (7 pts) Smith bought a European put at $t = 0$. The European put has strike price 132 and will mature at time 2. What is the price of the European put at $t = 0$?
(c) (6 pts) Smith wants to hedge the European put at node 2. How many shares of the underlying stock Smith should hold at node 2?