AFM Assignment 5

1. Given standard Brownian motion W_t, show whether the following expressions are martingale or not? (λ is a constant)

   (1) $e^{\lambda W_t}$

   (2) $e^{W_t - \frac{1}{2}\lambda^2}$

   (3) $(t + W_t^2)^2$

   (4) $(W_t^2 - t)^2 - 2t^2$

2. Given standard Brownian motion W_t, based on Itô isometry,

$$E\left[\left(\int_0^T f(t, W_t) dW_t\right)^2\right] = E\int_0^T |f(t, W_t)|^2 dt,$$

   evaluate :

   (1) $E\left[\left(\int_0^T (t + 2W_t) dW_t\right)^2\right]$

   (2) $E\left[\left(\int_0^T e^{-W_t^2} dW_t\right)^2\right]$

   (3) $E\left[\left(\int_0^t W_s^2 dW_s\right)^2\right]$

   (4) $E\left[\int_0^t W_s^2 ds\right]$