2022-23 First Semester

MATH1063 Linear Algebra II (1003)

Assignment 6

Due Date: **14/Apr/2023 (Friday), 09:00 in tutorial class.**

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. For each of the matrices, **find all real eigenvalues**, with their **algebraic multiplicities**. For each eigenvalue, **find as many linearly independent eigenvectors as possible**. Show your work. Do not use technology.

   (a). $A = \begin{bmatrix} -3 & -4 \\ -1 & 0 \end{bmatrix}$; (b). $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$; (c). $C = \begin{bmatrix} k & k & k \\ k & k & k \\ k & k & k \end{bmatrix}$, $k \neq 0$.

2. Based on Problem-1,
   (a) Determine which matrices are diagonalizable.
   (b) Based on part (a), find a diagonal matrix D and a nonsingular matrix P for each diagonalizable matrix so that it can be factorized into a product PDP^{-1}.
   (c) For each diagonalizable matrix M you found in part(b), use PDP^{-1} factorization to compute M^3 and M^n, for $n \in \mathbb{Z}^+$.

3. Prove that $\lambda = 0$ is an eigenvalue of A if and only if A is singular.

4. Show that A and A^T have the same eigenvalues. Do they necessarily have the same eigenspace? Give proofs or a counter-example to support your answer.

5. For which values of constants a are the following matrices diagonalizable?

$$A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \quad , \quad B = \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix}.$$