

**2021-22 First Semester**  
**MATH1083 Calculus II (1002)**

Assignment 7

Due Date: 11:30am 5/Apr/2021(Wed).

- Write down your **Chinese name** and **student number**. Write neatly on **A4-sized** paper and **show your steps**.
- **Late submissions or answers without details will not be graded.**

1. Find the limit

(a)

$$\lim_{(x,y) \rightarrow (3,2)} \frac{x^2y + xy^2}{x^2 - y^2}$$

(b)

$$\lim_{(x,y) \rightarrow (\pi, \pi/2)} \frac{\cos y - \sin 2y}{\cos x \cos y}$$

2. Show that the limit does not exist

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2 \cos y}{x^2 + y^4}$$

(b)

$$\lim_{(x,y) \rightarrow (1,0)} \frac{xy - y}{(x - 1)^2 + y^2}$$

3. Use the Squeeze Theorem to find the limit

(a)

$$\lim_{(x,y) \rightarrow (0,0)} xy \sin \frac{1}{x^2 + y^2}$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$$

4. Determine the set of points at which the given function is continuous.

(a)

$$f(x, y, z) = \frac{x^3}{y} + \sin z$$

(b)

$$f(x, y) = \begin{cases} \frac{\sin \sqrt{1-x^2-y^2}}{\sqrt{1-x^2-y^2}} & \text{if } x^2 + y^2 \neq 1 \\ 1 & \text{if } x^2 + y^2 = 1 \end{cases}$$

(c)

$$f(x, y, z) = \sqrt{y - x^2} \ln z$$

5. Use polar coordinates to find the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2}$$

6. Find the first partial derivatives of the functions

(a)

$$f(x, y) = \frac{x}{(x + y)^2}$$

(b)

$$R(p, q) = \tan^{-1} (pq^2)$$

(c)

$$z = x \sin(xy)$$