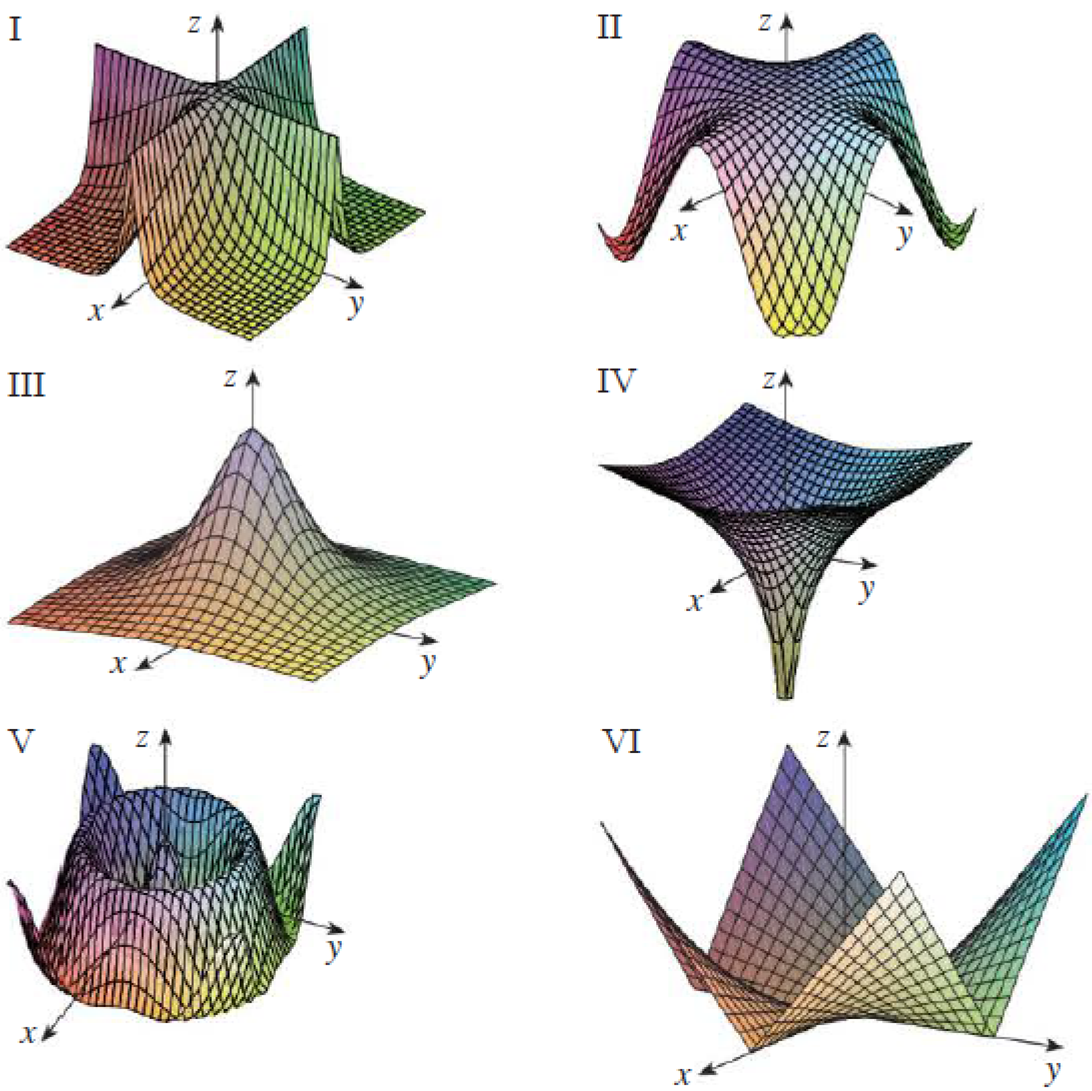
2021-22 First Semester
MATH1083 Calculus II (1002&1003)

Assignment 8

Due Date: 11:30am 10/Mar/2021 (Monday)
[Please pay attention to the change of deadline]

- Write down your **Chinese name** and **student number**. Write neatly on **A4-sized** paper and **show your steps**.
- **Late submissions or answers without details will not be graded.**

1. Match the function with its graph.



(a)

$$f(x,y) = \frac{1}{1+x^2+y^2}$$

(b)

$$f(x,y) = \frac{1}{1+x^2y^2}$$

(c)

$$\ln(x^2+y^2)$$

(d)

$$\cos\sqrt{x^2+y^2}$$

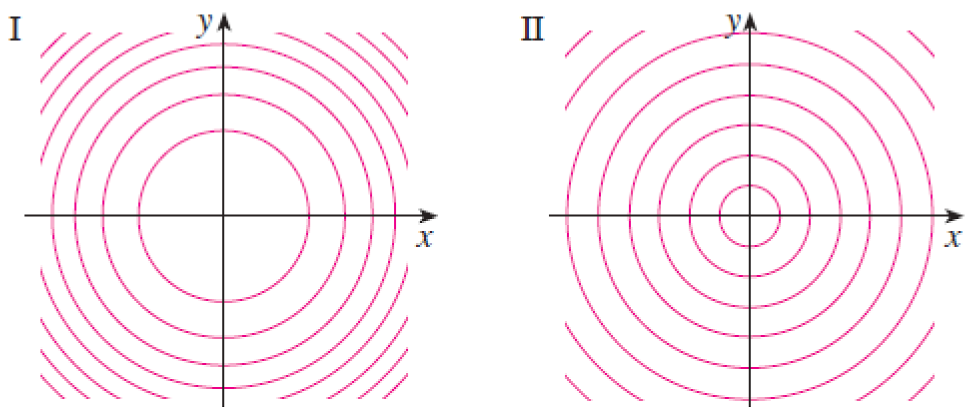
(e)

$$f(x,y) = |xy|$$

(f)

$$\cos(xy)$$

2. Two contour maps are shown. One is for a function f whose graph is a cone. The other is for a function g whose graph is a paraboloid. Which one is which and why?



3. Discuss the continuity of the function

$$f(x,y) = \begin{cases} \frac{\sin xy}{xy} & xy \neq 0 \\ 1 & xy = 0 \end{cases}$$

4. Find the indicated partial derivative

(a)

$$R(s,t) = te^{s/t}; \qquad R_t(0,1)$$

(b)

$$f(x,y,z) = x^{yz}, \qquad f_z(e,1,0)$$

5. Find all the second partial derivatives
$f(x,y) = \ln(ax+by)$

6. Verify that the conclusion of Clairaut's Theorem holds, that is $u_{xy} = u_{yx}$, $u(x,y) = \cos(x^2y)$

7. Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$ for

$$x^2 + 2y^2 + 3z^2 = 1$$

8. If $f(x,y,z) = xy^2z^3 + \arcsin(x\sqrt{z})$, find f_{xyz} in the easist order.

9. If

$$u = e^{a_1x_1 + a_2x_2 + a_3x_3}$$

where $a_1^2 + a_2^2 + a_3^2 = 1$, show that

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} = u$$

10. Find an equation of the tangent plane to the given surface at the specified point
    (a) $z = (x+2)^2 - 2(y-1)^2 - 5$, $(2, 3, 3)$
    (b) $z = \frac{x}{y^2}$, $(-4, 2, -1)$

11. Prove that if f is a function of two variables that is differentiable at (a, b), then f is continuous at (a, b)

12. Find the linearization $L(x, y)$ of the function $f(x, y) = y + \sin(x/y)$ at the point $(0, 3)$.

13. Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at point $(3, 2, 6)$ and use it to approximate the number $\sqrt{3.02^2 + 1.97^2 + 5.99^2}$

14. Use Chain Rule to find dz/dt

$$z = \sqrt{1 + xy}, \qquad x = \tan t, \quad y = \arctan t$$

15. Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$

$$z = \frac{\sin\theta}{r} \qquad r = st, \quad \theta = s^2 + t^2$$

16. If $z = f(x, y)$ where $x = r\cos\theta$, $y = r\sin\theta$
    (a) Find $\partial z/\partial r$ and $\partial z/\partial\theta$
    (b) Show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

17. Find the directional derivative of $f = \sqrt{2x + 3y}$ at the given point $(3, 1)$ in the direction indicated by the angle $\theta = -\frac{\pi}{6}$.

18. For the function $f(x, y) = x^2 e^y$
    (a) Find the gradient of f.
    (b) Evaluate the gradient at point $P(3, 0)$
    (c) Find the rate of change of f at P in the direction of the vector $\vec{u} = \frac{1}{5}(3\vec{i} - 4\vec{j})$