

MATH2033 Mathematical Statistics

Assignment 4

Due Date: **24/Mar/2024(Sunday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.
 - **Late submissions, answers without details, or unrecognizable handwritings** will NOT be graded.
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1. Consider simple random sampling with replacement.

(a) Show that

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

is an unbiased estimate of σ^2 .

(b) Is s an unbiased estimate of σ ?

(c) Show that $n^{-1}s^2$ is an unbiased estimate of σ_X^2 .

2. True or false? Briefly state why if you think it is false

(a) If a sample from a population is large, a histogram of the values in the sample will be approximately normal, even if the population is not normal.

(b) The center of a 95% confidence interval for the population mean is a random variable.

(c) A 95% confidence interval for μ contains the sample mean with probability .95.

(d) A 95% confidence interval contains 95% of the population.

(e) Out of one hundred 95% confidence intervals for μ , 95 will contain μ .

3. Given a random sample of size n from a gamma population, use the method of moments to obtain formulas for estimating the parameters α and β .

4. Suppose that $Y_1 = 0.42$, $Y_2 = 0.10$, $Y_3 = 0.65$ and $Y_4 = 0.23$ is a random sample of size 4 from the pdf

$$f_Y(y|\theta) = \theta y^{\theta-1}, \quad 0 \leq y \leq 1$$

Find the method of moments estimate for θ .