2023-24 First Semester

MATH2023 Ordinary and Partial Differential Equations (1002)

Assignment 5

Due Date: **6/Nov/2023(Monday), on or before 10:00, in lecture.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without details will not be graded.**

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1. Find the general solution to the given differential equations

   (a) $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0$

   (b) $y'' + 9y = 9\sec^2(3t), \quad 0 < t < \pi/6$

2. The given functions y_1 and y_2 are solutions to the corresponding homogeneous equation. Find a particular solution of the given nonhomogeneous equation.

   (a) $t^2y'' - t(t+2)y' + (t+2)y = 2t^3, \quad t > 0; \qquad y_1(t) = t, \quad y_2(t) = te^t.$

   (b) $x^2y'' - 3xy' + 4y = x^2\ln x, \quad x > 0; \qquad y_1(x) = x^2, \quad y_2(x) = x^2\ln x.$

3. In this problem, we indicate an alternate procedure for solving the differential equation

$$y'' + by' + cy = (D^2 + bD + c)y = g(t), \qquad (*)$$

   where b and c are constants, and D denotes differentiation with respect to t. Let r_1 and r_2 be the zeros of the characteristic polynomial of the corresponding homogeneous equation. These roots may be real and different, real and equal, or complex conjugate numbers.

   (a) Verify that Eq. $(*)$ can be written in the factored form

$$(D - r_1)(D - r_2)y = g(t),$$

   where $r_1 + r_2 = -b$ and $r_1r_2 = c$.

   (b) Let $u = (D - r_1)y$. Then show that the solution of Eq. $(*)$ can be found by solving the following two first order equations:

$$(D - r_1)u = g(t), \quad (D - r_2)y = u(t).$$

4. Based on the method illustrated in Problem 3, solve the following differential equations.

   (a) $y'' + 4y = t^2 + 3e^t$

(b) $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0$

5. (Free vibration) The position of a certain spring-mass system satisfies the initial value problem

$$\frac{3}{2}u'' + ku = 0, \quad u(0) = 2, u'(0) = v.$$

If the period and amplitude of the resulting motion are observed to be π and 3, respectively, determine the values of k and v.

6. (Forced Vibration) A mass of 5kg stretches a spring 10cm. The mass is acted on by an external force of $10\sin(t/2)$ N(newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4cm/sec.

   (a) If the mass is set in motion from its equilibrium position with an initial velocity of 3cm/sec, formulate the initial value problem describing the motion of the mass.

   (b) Find the solution of the initial value problem.

   (c) Identify the transient and steady-state parts of the solution.

   (d) If the given external force is replaced by a force $2\cos(\omega t)$ of frequency ω, find the value of ω for which the amplitude of the forced response is maximum.