Assignment 9 solution

1. a.

$$\begin{aligned}\hat{Y}_t(1) &= E_t(Y_{t+1}) \\ &= E_t(\mu + \phi(Y_t - \mu) + e_t) \\ &= \mu + \phi(Y_t - \mu) \\ &= 10.8 + (-0.5)(12.2 - 10.8) \\ &= 10.1\end{aligned}$$

   b.

$$\begin{aligned}\hat{Y}_t(2) &= E_t(Y_{t+2}) \\ &= E_t(\mu + \phi(Y_{t+1} - \mu) + e_t) \\ &= \mu + \phi[\hat{Y}_t(1) - \mu] \\ &= 10.8 + (-0.5)(10.1 - 10.8) \\ &= 11.15\end{aligned}$$

   c.

$$\begin{aligned}\hat{Y}_t(10) &= E_t(Y_{t+10}) \\ &= \mu + \phi^{10}(Y_t - \mu) \\ &= 10.8 + (-0.5)^{10}(12.2 - 10.8) \\ &= 10.801367 \approx \mu\end{aligned}$$

2. a.

$$\hat{Y}_{2007}(1) = 5 + 1.1Y_{2007} - 0.5Y_{2006} = 5 + 1.1(10) - 0.5(11) = 10.5$$
$$\hat{Y}_{2007}(2) = 5 + 1.1\hat{Y}_{2007}(1) - 0.5Y_{2007} = 5 + 1.1(10.5) - 0.5(10) = 11.55$$

   b.

$$(1 - 1.1B + 0.5B^2)Y_t = 5 + e_t$$

   Assume $Y_t = \psi_0 e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + \ldots$, then $\psi_1 - \phi_1\psi_0 = 0$ in parity of e_{t-1} on the both sides. Hence, $\psi_1 = \phi_1\psi_0 = 1.1$.

   c. Using $Var(e_t(\iota)) = \sigma_e^2 \sum_{j=0}^{\iota-1} \Psi_j^2$, the prediction limits are $\hat{Y}_{2007}(1) \pm 2[\sqrt{\sigma_e^2}]$ or $10.5 \pm 2\sqrt{2}$ which is 10.5 ± 2.83. We are 95% confident that the 2008 value will be between 7.67 and 13.33 .

   d. $\hat{Y}_{2008}(1) = \hat{Y}_{2007}(2) + \Psi[Y_{2008} - \hat{Y}_{2007}(1)] = 11.55 + 1.1[12 - 10.5] = 13.2$

3. a.

```
> model=arima(series,order=c(1,0,1)); model
```

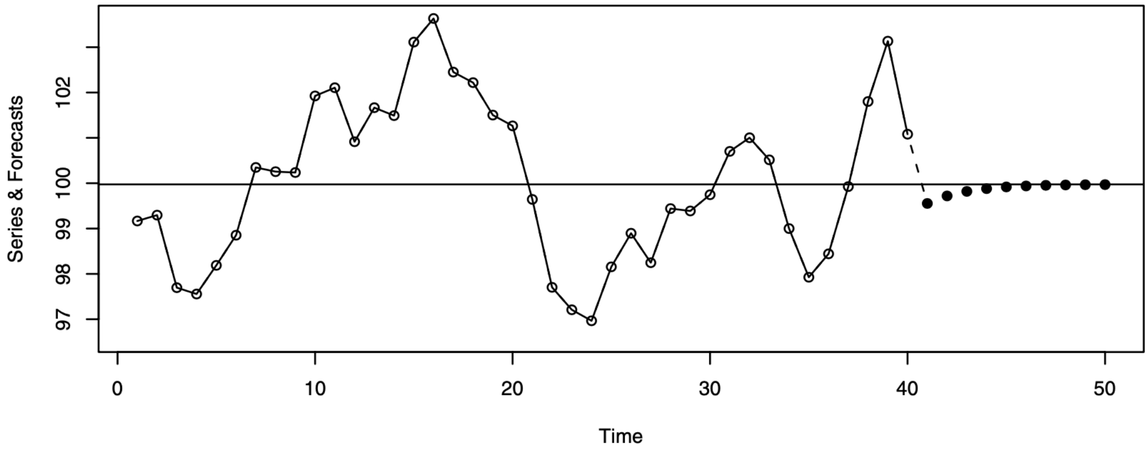
```
Call:
arima(x = series, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.6048  0.6907    99.9745
s.e.  0.1585  0.2522     0.5846

sigma^2 estimated as 0.8162:  log likelihood = -53.6,  aic = 113.19
```

Taking the standard errors into account, the maximum likehood estimates are reasonably close to the true values in this simulation.

b.




```
> result=plot(model,n.ahead=10,ylab='Series & Forecasts',col=NULL,pch=19)
> abline(h=coef(model)[names(coef(model))=='intercept'])
```

The forecfasts approach the series mean fairly quickly.

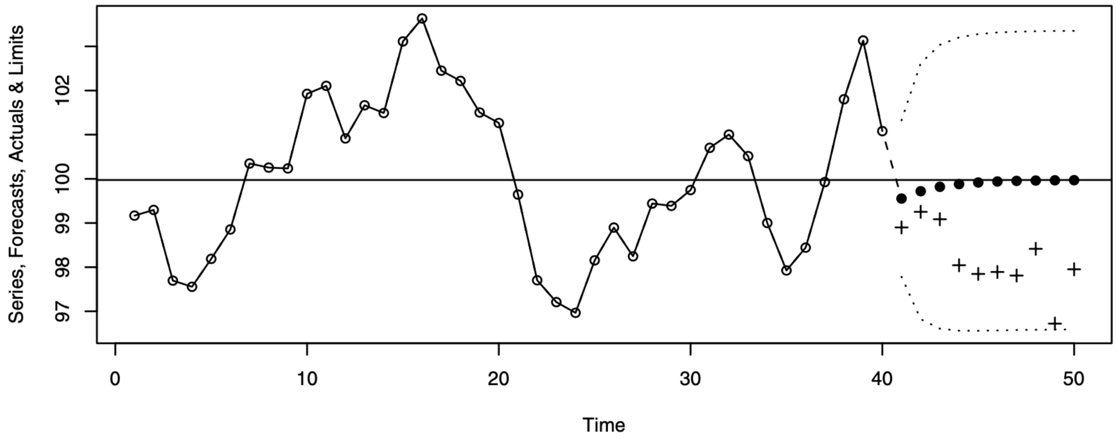
c.

```
> forecast=result$pred; cbind(actual,forecast)
```

```
Time Series:
Start = 41
End = 50
Frequency = 1
     actual forecast
41 98.90034 99.55443
42 99.25304 99.72043
43 99.08626 99.82082
44 98.04358 99.88154
45 97.84692 99.91826
46 97.89159 99.94047
47 97.81065 99.95391
48 98.41574 99.96203
49 96.72142 99.96694
50 97.95263 99.96992
```

See part (d) for a graphical comparison.

d.




```
> plot(model,n.ahead=10,ylab='Series, Forecasts, Actuals & Limits',pch=19)
> points(x=(41:50),y=actual,pch=3)
> abline(h=coef(model)[names(coef(model))=='intercept'])
```

This series is quite erratic but the actual series values are contained within the forecast limits. The forecasts decay to the estimated process mean rather quickly and the prediction limits are quite wide.

```
> set.seed(127456); series=arima.sim(n=35,list(order=c(0,1,1),ma=-0.8))[-1]
> # delete initial term as it is always = 0
> actual=window(series,start=31); series=window(series,end=30)
```

4. a.

```
> model=arima(series,order=c(0,1,1)); model
```

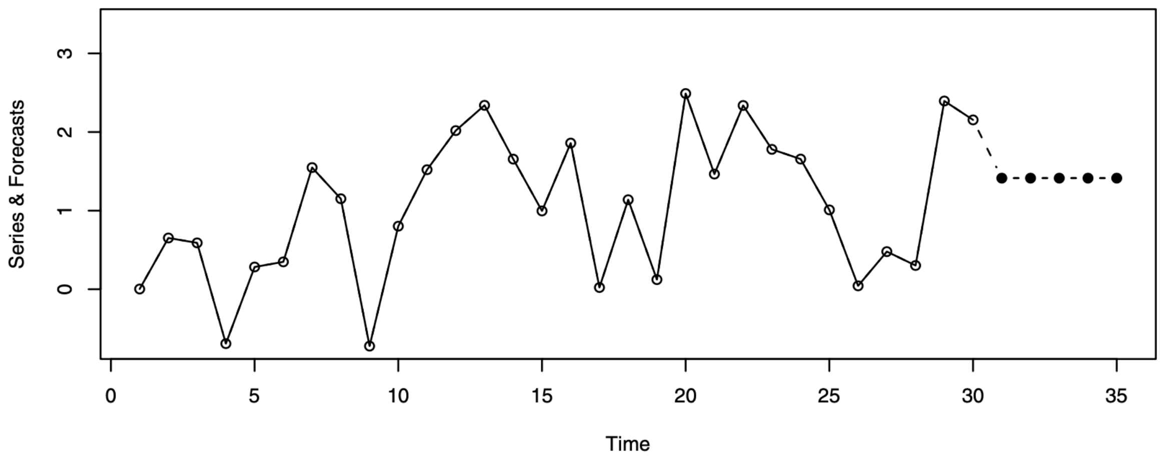
```
Call:
arima(x = series, order = c(0, 1, 1))

Coefficients:
          ma1
      -0.7696
s.e.   0.1832

sigma^2 estimated as 0.845:  log likelihood = -39.15,  aic = 80.31
```

Taking the standard errors into account, the maximum likehood estimate is quite close to the true value in this simulation.

b.




```
> result=plot(model,n.ahead=5,ylab='Series & Forecasts',col=NULL,pch=19)
```

For this model the forecasts are constant for all lead times.

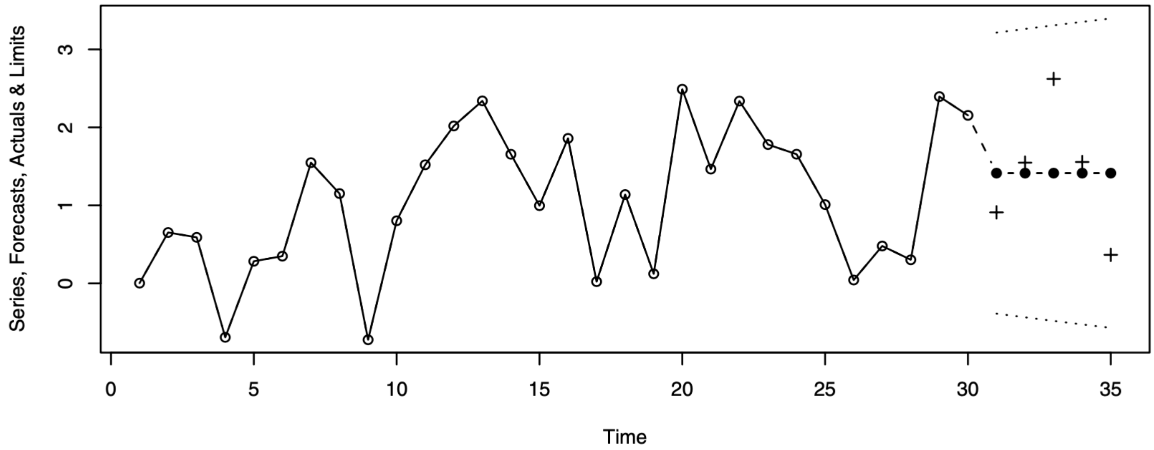
c.

```
> forecast=result$pred; cbind(actual,forecast)
```

```
Time Series:
Start = 31
End = 35
Frequency = 1
      actual forecast
31 0.9108642 1.413627
32 1.5476147 1.413627
33 2.6211930 1.413627
34 1.5560880 1.413627
35 0.3657792 1.413627
```

For this model the foecasts are the same at all lead times. See part (d) for a graphical comparison.

d.




```
> plot(model,n.ahead=5,ylab='Series, Forecasts, Actuals & Limits',pch=19)
> points(x=(31:35),y=actual,pch=3)
```

The forecat limits contain all of the actual values but they are quite wide.

e.

Repeat parts (a) through (d) with a new simulated series using the same values of the parameters and same sample size.

5.

Note: Since we assume all parameters are *known*, conditioning on $Y_1, Y_2, \dots, Y_t$ is the same as conditioning on $X_1, X_2, \dots, X_t$. So

$$\begin{aligned}\hat{Y}_t(\ell) &= E(\beta_0 + \beta_1(t+\ell) + X_{t+\ell}|Y_1, Y_2, \dots, Y_t) \\ &= \beta_0 + \beta_1(t+\ell) + E(X_{t+\ell}|Y_1, Y_2, \dots, Y_t) \\ &= \beta_0 + \beta_1(t+\ell) + E(X_{t+\ell}|X_1, X_2, \dots, X_t)\end{aligned}$$

Now, since $\{X_t\}$ is an AR(1) process, $E(X_{t+\ell}|X_1, X_2, \dots, X_t) = \phi^\ell X_t = \phi^\ell X_t = \phi^\ell(Y_t - \beta_0 - \beta_1 t)$ and the desired result is obtained.

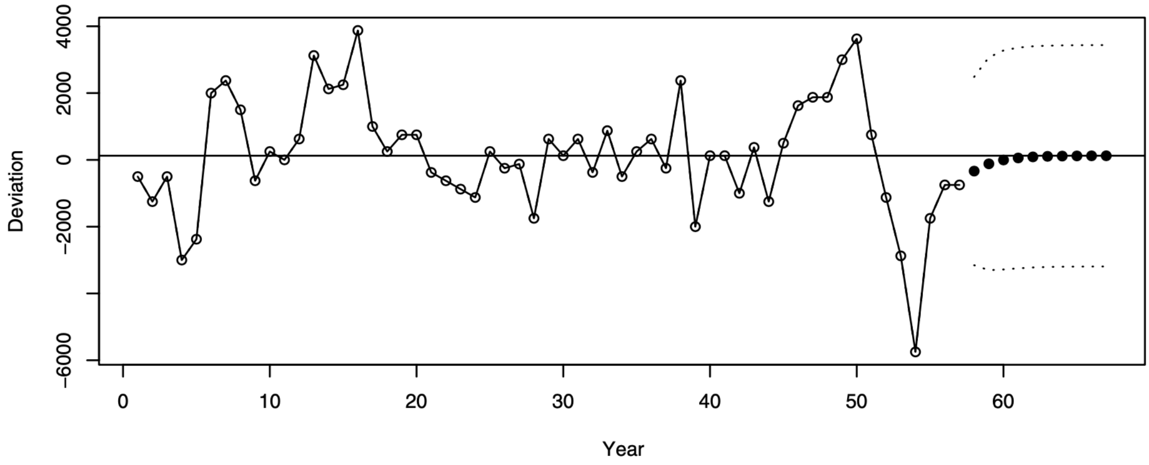
6. a.

```
> data(deere3); model=arima(deere3,order=c(1,0,0)); plot(model,n.ahead=10)$pred
```

```
Time Series:
Start = 58
End = 67
Frequency = 1
 [1] -335.145915 -117.120755   -2.538371   57.680013   89.327581  105.959853
 [7]  114.700888  119.294709  121.708976  122.977786
```

The forecasts are reasonably constant from forecast lead 8 on.

b.




```
> win.graph(width=6.5,height=3,pointsize=8)
> plot(model,n.ahead=10,ylab='Deviation',xlab='Year',pch=19)
> abline(h=coef(model)[names(coef(model))=='intercept'])
```

Since the model does not contain a lot of autocorrelation or other structure, the forecasts, plotted as solid circles, quickly settle down to the mean of the series.

7. a.

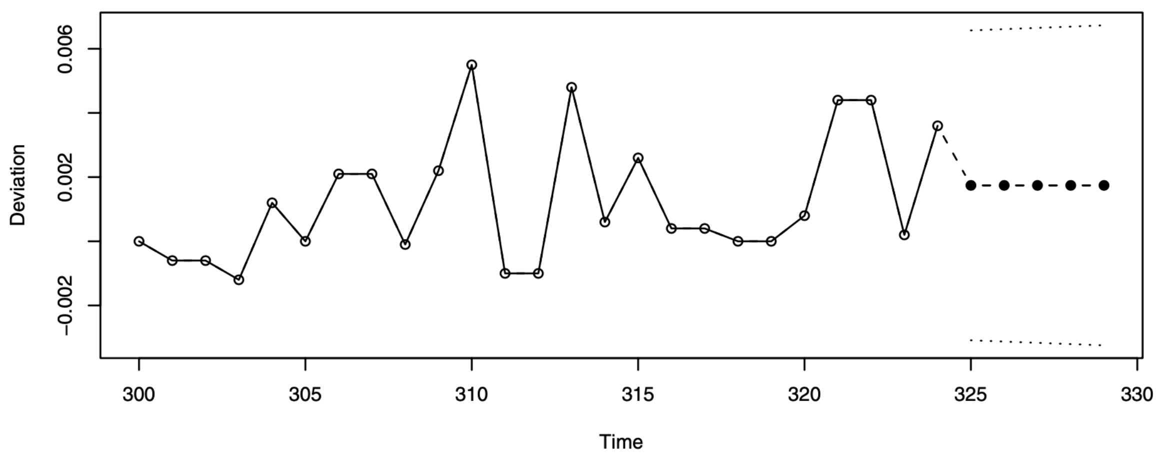
```
> data(robot); model=arima(robot,order=c(0,1,1)); model; plot(model,n.ahead=5)$pred
```

```
Time Series:
Start = 325
End = 329
Frequency = 1
[1] 0.001742672 0.001742672 0.001742672 0.001742672 0.001742672
```

```
> plot(model,n.ahead=5)$upi; plot(model,n.ahead=5)$lpi
```

```
Time Series:
Start = 325
End = 329
Frequency = 1
[1] 0.006669889 0.006710540 0.006750862 0.006790862 0.006830548
Time Series:
Start = 325
End = 329
Frequency = 1
[1] -0.003184545 -0.003225197 -0.003265519 -0.003305518 -0.003345204
```

b.




```
> win.graph(width=6.5,height=3,pointsize=8)
> plot(model,n1=300,n.ahead=5,ylab='Deviation',pch=19)
```

Thew forecast limits are quite wide in this fitted model and the forecasts are relatively constant.

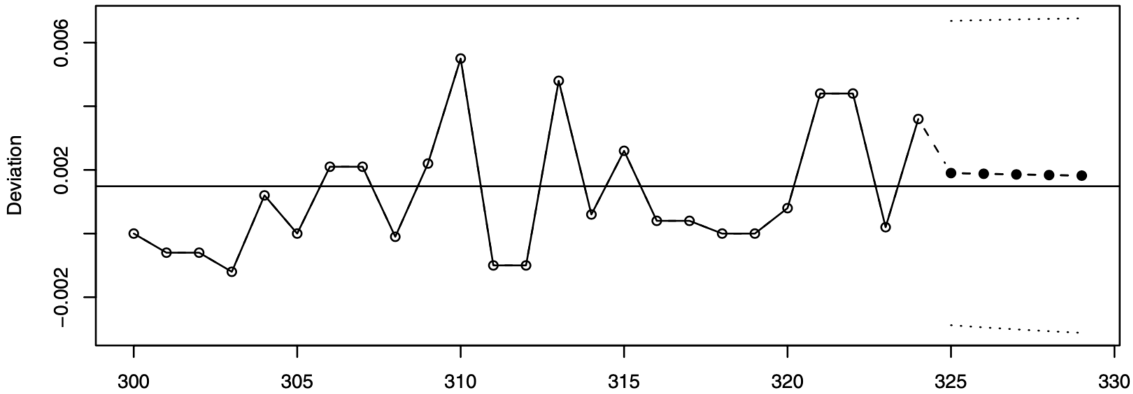
c.

```
> model=arima(robot,order=c(1,0,1)); plot(model,n.ahead=5)$pred
```

```
Time Series:
Start = 325
End = 329
Frequency = 1
[1] 0.001901348 0.001879444 0.001858695 0.001839041 0.001820424
```

```
> plot(model,n.ahead=5)$upi; plot(model,n.ahead=5)$lpi
```

```
Time Series:
Start = 325
End = 329
Frequency = 1
[1] 0.006571344 0.006611183 0.006650699 0.006689898 0.006728790
Time Series:
Start = 325
End = 329
Frequency = 1
[1] -0.003086000 -0.003125839 -0.003165355 -0.003204555 -0.003243446
```




```
> plot(model,n1=300,n.ahead=5,ylab='Deviation',pch=19)
> abline(h=coef(model)[names(coef(model))=='intercept'])
```

Both of these models give quite similar forecasts and forecast limits.