# MATH2033 Mathematical Statistics
# Assignment 5

Due Date: **7/Apr/2024(Sunday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.
- **Late submissions**, **answers without details**, or **unrecognizable handwritings** will NOT be graded.

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1. Suppose that $X$ is a discrete random variable with

$$P(X=0) = \frac{2}{3}\theta$$
$$P(X=1) = \frac{1}{3}\theta$$
$$P(X=2) = \frac{2}{3}(1-\theta)$$
$$P(X=3) = \frac{1}{3}(1-\theta)$$

where $0 \le \theta \le 1$ is a parameter. The following 10 independent observations were taken from such a distribution: $(3, 0, 2, 1, 3, 2, 1, 0, 2, 1)$.

   (a) Find the method of moments estimate of $\theta$.

   (b) Find an approximate **standard error** (in the context of parameter estimation, standard deviation of the estimator is also often called its "standard error") for your estimate.

   (c) What is the maximum likelihood estimate of $\theta$?

2. Suppose that $X$ is a discrete random variable with $P(X=1) = \theta$ and $P(X=2) = 1-\theta$. Three independent observations of $X$ are made: $x_1 = 1, x_2 = 2, x_3 = 2$.

   (a) Find the method of moments estimate of $\theta$.

   (b) What is the likelihood function?

   (c) What is the maximum likelihood estimate of $\theta$?

3. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with density function

$$f(x \mid \sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x|}{\sigma}\right)$$

(a) Find the method of moments estimate of $\sigma$.

(b) Find the maximum likelihood estimate of $\sigma$.

4. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. $N(\mu, \sigma^2)$.

   (a) If $\mu$ is known, what is the mle of $\sigma$?

   (b) If $\sigma$ is known, what is the mle of $\mu$?

5. Suppose $X$ is a random variable with pdf/pmf $\pi(x|\theta)$, where $\theta \in \Theta$ is a parameter. Define the **Fisher Information** of the parameter $\theta$ as follows:

$$I(\theta) \stackrel{\text{def}}{=} \mathbb{E}_\theta\left[\left(\frac{\partial}{\partial\theta}\log\pi(X|\theta)\right)^2\right] = \int\left(\frac{\partial}{\partial\theta}\log\pi(x|\theta)\right)^2\pi(x|\theta)\mathrm{d}x.$$

   (a) Suppose $X$ is a Bernoulli random variable with parameter $p \in [0,1]$, namely, $\mathbb{P}(X = 1) = p, \mathbb{P}(X = 0) = 1 - p$. Find the Fisher information $I(p)$.

   (b) Suppose $X$ is a Poisson random variable with parameter $\lambda > 0$, namely, $\mathbb{P}(X = k) = e^{-\lambda}\lambda^k/k!$. Find the Fisher information $I(\lambda)$.

   (c) Suppose $X$ is a random variable with pdf

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

   where $\theta > 0$ is the parameter. Find the Fisher information $I(\theta)$.