2021-22 First Semester
MATH1083 Calculus II (1002)

Assignment 5

Due Date: 11:30am 22/Mar/2021(Wed).

- Write down your **Chinese name** and **student number**. Write neatly on **A4-sized** paper and **show your steps**.
- **Late submissions or answers without details will not be graded.**

1. For two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, with angle θ in between:
   (a) Prove **Cauchy-Schwartz Inequality**
   $$\left|\overrightarrow{a}\cdot\overrightarrow{b}\right| \leq |\overrightarrow{a}|\left|\overrightarrow{b}\right|$$
   (b) Use Cauchy-Schwartz Inequality to prove the **Triangle Inequality**
   $$\left|\overrightarrow{a}+\overrightarrow{b}\right| \leq |\overrightarrow{a}|+\left|\overrightarrow{b}\right|$$
   Hint: *use the fact that*
   $$\left|\overrightarrow{a}+\overrightarrow{b}\right|^2 = \left(\overrightarrow{a}+\overrightarrow{b}\right)\cdot\left(\overrightarrow{a}+\overrightarrow{b}\right)$$
   (c) Prove the **Parallelogram Indentity**:
   $$\left|\overrightarrow{a}+\overrightarrow{b}\right|^2+\left|\overrightarrow{a}-\overrightarrow{b}\right|^2 = 2\left|\overrightarrow{a}\right|^2+2\left|\overrightarrow{b}\right|^2$$
   and give a geometric interpretation of the Parallelogram Indentity.
2. Find the **area of the parallelogram** with vertices $A(-3,0)$, $B(-1,3)$, $C(5,2)$ and $D(3,-1)$. *[Hint:Parallelogram in 2D space, $A=|\overrightarrow{a}|\left|\overrightarrow{b}\right|\sin\theta$]*
3. Find the **area of the parallelogram** with vertices $P(1,0,2)$, $Q(3,3,3)$, $R(7,5,8)$ and $S(5,2,7)$. *[Hint:Parallelogram in 3D space, $A=\left|\left(\overrightarrow{a}\times\overrightarrow{b}\right)\right|$]*
4. Find the volume of the **parallelepiped** determined by the vectors $\overrightarrow{a}=(1,2,3)$, $\overrightarrow{b}=(-1,1,2)$ and $\overrightarrow{c}=(2,1,4)$
5. If $\overrightarrow{a}\times\overrightarrow{b}=(1,2,2)$ and $\overrightarrow{a}\cdot\overrightarrow{b}=\sqrt{3}$, find the **angle** between $\overrightarrow{a}$ and $\overrightarrow{b}$.
6. Show that
   $$\left|\overrightarrow{a}\times\overrightarrow{b}\right|^2 = |\overrightarrow{a}|^2\left|\overrightarrow{b}\right|^2-\left(\overrightarrow{a}\cdot\overrightarrow{b}\right)^2$$
7. Find the **vector equation** and **parametric equations** for the line:
   (a) The line through the point $(4,2,-3)$ and parallel to the vector $2\overrightarrow{i}-\overrightarrow{j}+6\overrightarrow{k}$
   (b) The line through the point $(8,-1,3)$ and $(1,2,3)$
   (c) The line through $(-6,2,3)$ and parallel to line $x=y=\frac{z-1}{6}$
   (d) The line through $(2,1,0)$ and perpendicular to both $\overrightarrow{i}+\overrightarrow{j}$ and $\overrightarrow{j}+\overrightarrow{k}$
8. Find an **equation of the plane**:
   (a) The plane through the point $(3,2,1)$ with normal vector $\overrightarrow{i}-\overrightarrow{j}+2\overrightarrow{k}$
   (b) The plane through the point $(5,-2,4)$ and perpendicular to the vector $-\overrightarrow{i}+2\overrightarrow{j}+3\overrightarrow{k}$
9. **Find the plane** that passes through the point $(6,-1,3)$ and contains the **line** with symmetric equations
   $$\frac{x}{3}=y+4=\frac{z}{2}$$
10. Find the distance
    (a) from the point to the given **line**: $(4,1,-2)$; $x=1+t, \quad y=3-2t, \quad z=4-3t$
    (b) from the point to the given **plane**: $(1,2,4)$, $3x+2y+6z=5$
    (c) between two parallel planes: $2x-3y+z=4, \quad 4x-6y+2z=3$