FM II Brief Solution to Assignment 2

1. Consider a standard Brownian Motion W_u. Evaluate $E(7W_t - 9W_s)$ for $t > s$. **Solution:**

$$E(7W_t - 9W_s) = 7E(W_t) - 9E(W_s) = 0$$

2. Consider a standard Brownian Motion W_u. Evaluate $E[(11 + 9W_t)^2]$.
Solution:

$$\begin{aligned} E[(11 + 9W_t)^2] &= E(121) + E(198W_t) + E(81W_t^2) \\ &= 121 + 0 + 81t = 121 + 81t \end{aligned}$$

3. Consider a standard Brownian Motion W_u. Evaluate $E[(aW_{t-s} + bW_s)W_{t-s}]$ for $t > s$.
Solution:

$$\begin{aligned} E(aW_{t-s}^2 + bW_sW_{t-s}) &= aE(W_{t-s}^2) + bE(W_sW_{t-s}) \\ &= a(t-s) + b\min\{s, t-s\} \end{aligned}$$

4. Consider a standard Brownian Motion W_u. Evaluate $E(W_t + W_s^2)^2$ for $t > s$.

Solution:

$$\begin{aligned} E[(W_t + W_s^2)^3] &= E(W_t^3) + 3E(W_t^2W_s^2) + 3E(W_tW_s^4) + E(W_s^6) \\ &= 3E[(W_t - W_s + W_s)^2 \times W_s^2] + 3E[(W_t - W_s + W_s) \times W_s^4] + 15s^3 \\ &= 3(t-s)s + 3 \times 3s^2 + 15s^3 = 3ts + 6s^2 + 15s^3 \end{aligned}$$

5. Consider a standard Brownian Motion W_u. Evaluate $E[[W_t(W_{2t} + W_{5t})]^2]$.

Solution:

$$\begin{aligned} E[[W_t(W_{2t} + W_{5t})]^2] &= E[W_t^2(W_{2t}^2 + 2W_{2t}W_{5t} + W_{5t}^2)] \\ &= E[W_t^2W_{2t}^2 + 2W_t^2W_{2t}W_{5t} + W_t^2W_{5t}^2] \\ &= E[W_t^2W_{2t}^2] + 2E[W_t^2(W_{2t} - W_t + W_t)W_{5t}] + E[W_t^2W_{5t}^2] \\ &= 4t^2 + 2 \times 4t^2 + 7t^2 \\ &= 19t^2 \end{aligned}$$