

2022-23 First Semester
MATH1053 Linear Algebra I

Assignment 2b

Due Date: 11/Oct/2022 (Tuesday), 11:00 in class.

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
 - For online students, hand in your homework in **one pdf file** on iSpace.
 - **Late submissions or answers without steps won't be graded.**
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1. Let A be an $n \times n$ matrix and let \mathbf{x} and \mathbf{y} be vectors in \mathbb{R}^n . Use the equivalent conditions for nonsingularity to show that if $A\mathbf{x} = A\mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$, then the matrix A must be singular.

2. Let

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}$$

- (a) Express A^{-1} as a product of elementary matrices.
- (b) Express A as a product of elementary matrices.

3. Let A be a real $m \times n$ matrix. Prove that if $A^T A = \mathbf{O}_{n \times n}$, then $A = \mathbf{O}_{m \times n}$.
(Hint: Denote $A = (a_{ij})_{m \times n}$. Write down the expression for the $(i, i)^{\text{th}}$ entry of $A^T A$.)