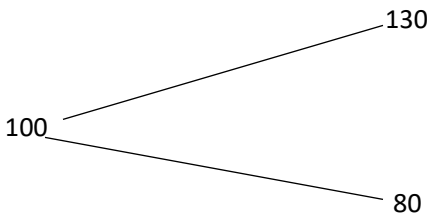
Fin. Math. Assignment 5

1. Stock prices are modeled with the following 1- period binomial tree, the period being 6 months.



For a European call option on the stock expiring in 6 months, the strike price is 104. The continuously compounded risk-free interest rate is 6%. Determine the change in the premium for the call option if the continuous dividend rate for the stock increases from 0 to 2%.

2. Prices of a stock are modeled with a 1-period binomial tree. You are given:
   (i) The stock's initial price is 80.
   (ii) The continuously compounded risk-free interest rate is 5%.
   (iii) The stock pays continuous dividends at a rate of 2%.
   (iv) The risk-neutral probability of an increase in stock price is 0.55.
   (v) In the binomial tree, u and d are selected so that their arithmetic average is 1.

   A European put option on the stock expiring in 6 months has strike price 90. Determine the option premium.

3. Future stock prices are modeled with a 1-period binomial tree, the period being 6 months.
   (i) The stock's current price is 20.
   (ii) The continuously compounded risk-free interest rate is 3%.
   (iii) The stock pays continuous dividends at a rate of 1%.

   The number of shares in the replicating portfolio for a European call option on the stock expiring in 6 months with strike price 20 is 0.4. Determine the number of shares in the replicating portfolio for a European put option on the stock expiring in 6 months with strike price 20.

4. Future stock prices are modeled with a 1-period binomial tree, with each period one year. Consider a European call option on a futures contract on the stock expiring in 1

year. You are given:

(i) $u = e^{0.2}, d = e^{-0.2}$

(ii) The price of the underlying stock is 200.

(iii) The underlying stock pays no dividends.

(iv) The underlying futures contract is a 1-year contract.

(v) The strike price is 210.

(vi) The continuously compounded risk-free interest rate is 5%.

Determine the option premium.

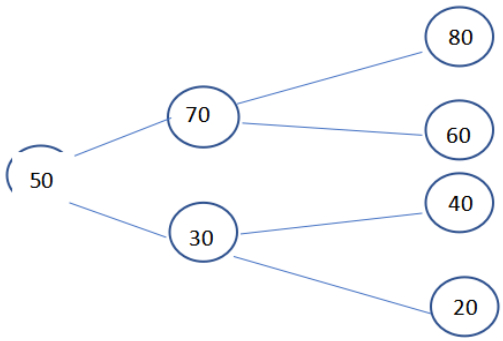


Figure 1: stock price

5. The price of a stock follows the binomial tree shown in Figure 1 with interest rate $\mathrm{r} = 0.1$ and dividend rate $\mathrm{q} = 0.05$. Let X be an American put option with maturity at tick-time 2 and strike price 50. What is the fair price for X?

6. The price of a stock follows the binomial tree shown in Figure 1 with interest rate $\mathrm{r} = 0.1$ and dividend rate $\mathrm{q} = 0.05$. Let X be an American call option with maturity at tick-time 2 and strike price 50. What is the fair price for X?

7. The price of a stock follows the binomial tree shown in Figure 1 with interest rate $\mathrm{r} = 0.05$. The stock pays continuous dividend at the rate $\delta = 0.05$. What is the price of an Asian (arithmetic) average-strike put option?