Caculus II Math 1038 (1002&1003)

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Week 3:

1. Quiz One:
   - Time: 7:00-7:50pm, Thu 16 Mar 2023
   - Venue: To be updated
   - Content: What you have learnt this semester, Chapter 11 Sequences and Series and Chapter 12 Vector space
   - No calculator, no elctronic devices, no books or notes are allowed. Only bring your pen and your photo ID.

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1. Series function:
   (a) Power series centered at c:

$$\begin{aligned} f(x) &= \sum_{k=0}^{\infty} a_k (x-c)^k \\ &= a_0 + a_1 (x-c) + a_2 (x-c)^2 + a_3 (x-c)^3 + \cdots \end{aligned}$$

   i. a_k: coefficients
   ii. c: center

   (b) domain of a power series: the values of x such that $f(x)$ converges.

2. **Radius of convergence** R, interval of convergence
   (a) Use **ratio or root** test to find R:
   - $$r = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right|, \qquad R = \frac{1}{r}$$

   i. R a real finite number, converges for $x \in (c-R, c+R)$
   ii. $R = 0$, converges only when $x = c$
   iii. $R = \infty$, converges for all $x \in \mathbb{R}$

   (b) check the endpoints $c-R$ and $c+R$ to determine interval of convergence

   - *Radius, Radii pl.*

3. Polynomials $p_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$
   (a) Differentiation
   (b) Integration

4. Representations of functions as power series
   (a) simple example:

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots = \sum_{n=0}^{\infty} x^n \qquad |x| < 1$$

   (b) substitute x by $2x$

$$\frac{1}{1-2x} = 1 + (2x) + (2x)^2 + \cdots = \sum_{n=0}^{\infty} (2x)^n \qquad |x| < \frac{1}{2}$$

(c) Similarly

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 \cdots = \sum_{n=0}^{\infty} (-1)^n x^n \qquad |x| < 1$$

(d) Harder example, using integration

$$\begin{aligned} \tan^{-1} x &= \int \frac{1}{1+x^2} dx \\ &= \int \left(1 - x^2 + x^4 - x^6 + \cdots\right) dx \\ &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{2n+1} \end{aligned}$$

(e) Question: how to find such a power series for any function? Answer: Taylor's expansion.

5. Taylor's series:

(a) Derivation of the coefficients

(b) simple examples (need to remember all of them): e^x, $\sin x$, $\cos x$, $\ln x$, $\frac{1}{1-x}$, $\frac{1}{1+x}$, $\sqrt{1+x}$,...

(c) Binomial series: $(1+x)^p$ when p is any real number

(d) Remainder R_n and error estimate.