Risk Management in Finance - Credit Risk

Peng Jin

Altman's Z-score (Manufacturing Companies)

- X_1=Working Capital/Total Assets
- X_2=Retained Earnings/Total Assets
- X_3=EBIT/Total Assets
- X_4=Market Value of Equity/Book Value of Liabilities
- X_5=Sales/Total Assets

$$Z = 1.2X_1+1.4X_2+3.3X_3+0.6X_4+0.99X_5$$

If the $Z > 3.0$ default is unlikely; if $2.7 < Z < 3.0$ we should be on alert. If $1.8 < Z < 2.7$ there is a moderate chance of default; if $Z < 1.8$ there is a high chance of default

Estimating Default Probabilities

- Alternatives:
  - Use historical data
  - Use credit spreads
  - Use Merton’s model

Historical Data

Historical data provided by rating agencies can be used to estimate the probability of default

Cumulative Average Default Rates % (1970-2016, Moody's)

	Time (years)						
	1	2	3	4	5	7	10
Aaa	0.000	0.011	0.011	0.031	0.085	0.195	0.386
Aa	0.021	0.060	0.110	0.192	0.298	0.525	0.778
A	0.055	0.165	0.345	0.536	0.766	1.297	2.224
Baa	0.177	0.461	0.804	1.216	1.628	2.472	3.925
Ba	0.945	2.583	4.492	6.518	8.392	11.667	16.283
B	3.573	8.436	13.377	17.828	21.908	28.857	36.177
Caa-C	10.624	18.670	25.443	30.974	35.543	42.132	50.258

Interpretation

- The table shows the probability of default for companies starting with a particular credit rating
- A company with an initial credit rating of Baa has a probability of 0.177% of defaulting by the end of the first year, 0.461% by the end of the second year, and so on

Do Default Probabilities Increase with Time?

- For a company that starts with a good credit rating default probabilities tend to increase with time
- For a company that starts with a poor credit rating default probabilities tend to decrease with time

Hazard Rate vs. Unconditional Default Probability

- The hazard rate or default intensity is the probability of default over a short period of time conditional on no earlier default
- The unconditional default probability is the probability of default as seen at time zero

Properties of Hazard Rates

- Suppose that $\lambda(t)$ is the hazard rate at time t
- The probability of default between times t and $t+\Delta t$ conditional on no earlier default is $\lambda(t)\Delta t$
- The probability of default by time t is

$$1 - e^{-\bar{\lambda}(t)t}$$

where $\bar{\lambda}(t)$ is the average hazard rate between time zero and time t

Recovery Rate

The recovery rate for a bond is usually defined as the price of the bond 30 days after default as a percent of its face value

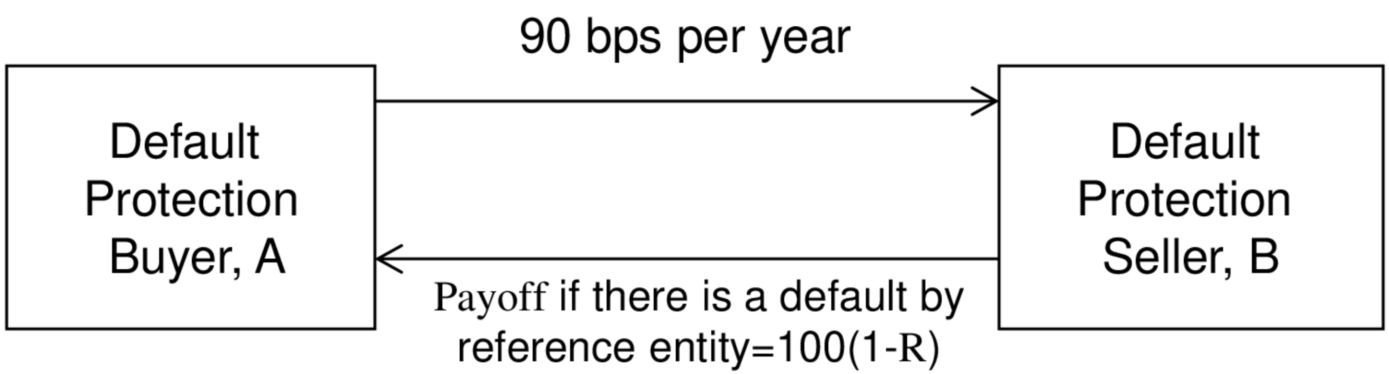
Recovery Rates; Moody's: 1983 to 2016

Class	Ave Rec Rate (%)
First lien bond	52.8
Second lien bond	44.6
Senior unsecured bond	37.2
Senior subordinated bond	31.1
Subordinated bond	31.9
Junior subordinated bond	23.2

Credit Default Swaps

- Buyer of the instrument acquires protection from the seller against a default by a particular company or country (the reference entity)
- Example: Buyer pays a premium of 90 bps per year for $100 million of 5-year protection against company X
- Premium is known as the *credit default spread*. It is paid for life of contract or until default
- If there is a default, the buyer has the right to sell bonds with a face value of $100 million issued by company X for $100 million (Several bonds may be deliverable)

CDS Structure



Recovery rate, R, is the ratio of the value of the bond issued by reference entity immediately after default to the face value of the bond

Other Details

- Payments are usually made quarterly in arrears
- In the event of default there is a final accrual payment by the buyer
- Increasingly settlement is in cash and an auction process determines cash amount
- Suppose payments are made quarterly in the example just considered. What are the cash flows if there is a default after 3 years and 1 month and recovery rate is 40%?

Attractions of the CDS Market

- Allows credit risks to be traded in the same way as market risks
- Can be used to transfer credit risks to a third party
- Can be used to diversify credit risks

Credit Indices

- CDX IG: equally weighted portfolio of 125 investment grade North American companies
- iTraxx: equally weighted portfolio of 125 investment grade European companies
- If the five-year CDS index is bid 165 offer 166 it means that a portfolio of 125 CDSs on the CDX companies can be bought for 166bps per company, e.g., $800,000 of 5-year protection on each name could be purchased for $1,660,000 per year. When a company defaults the annual payment is reduced by 1/125.

Use of Fixed Coupons

- Increasingly CDSs and CDS indices trade like bonds
- A coupon and a recovery rate is specified
- There is an initial payments from the buyer to the seller or vice versa reflecting the difference between the currently quoted spread and the coupon

Credit Default Swaps and Bond Yields

- Portfolio consisting of a 5-year par yield corporate bond that provides a yield of 6% and a long position in a 5-year CDS costing 100 basis points per year is (approximately) a long position in a riskless instrument paying 5% per year
- What are arbitrage opportunities in this situation is risk-free rate is 4.5%? What if it is 5.5%?

Risk-free Rate

- The risk-free rate used by bond traders when quoting credit spreads is the Treasury rate
- The risk-free rate traditionally assumed in derivatives markets is the LIBOR/swap rate
- By comparing CDS spreads and bond yields it appears that in normal market conditions traders are assuming a risk-free rate 10 bp less than the LIBOR/swap rates
- In stressed market conditions the gap between the LIBOR/swap rate and the “true” risk-free rate is liable to be much higher

Asset Swaps

- Asset swaps are used by the market as an estimate of the bond yield relative to LIBOR
- The present value of the asset swap spread is an estimate of the present value of the cost of default

Asset Swaps (page 443)

- Suppose asset swap spread for a particular corporate bond is 150 basis points
- One side pays coupons on the bond; the other pays LIBOR+150 basis points. The coupons on the bond are paid regardless of whether there is a default
- In addition there is an initial exchange of cash reflecting the difference between the bond price and $100
- The PV of the asset swap spread is the amount by which the price of the corporate bond is exceeded by the price of a similar risk-free bond when the LIBOR/swap curve is used for discounting

CDS-Bond Basis

- This is the CDS spread minus the Bond Yield Spread
- Bond yield spread is usually calculated as the asset swap spread
- Tended to be positive pre-crisis

Using CDS Prices to Predict Default Probabilities

Average hazard rate between time zero and time t is to a good approximation

$$\bar{\lambda} = \frac{s(t)}{1-R}$$

where $s(t)$ is the credit spread calculated for a maturity of t and R is the recovery rate

More Exact Calculation

- Suppose that a five year corporate bond pays a coupon of 6% per annum (semiannually). The yield is 7% with continuous compounding and the yield on a similar risk-free bond is 5% (with continuous compounding)
- The expected loss from defaults is 8.75. This can be calculated as the difference between the market price of the bond and its risk-free price
- Suppose that the unconditional probability of default is Q per year and that defaults always happen half way through a year (immediately before a coupon payment).

Calculations

Time (yrs)	Def Prob	Recovery Amount	Risk-free Value	Loss	Discount Factor	PV of Exp Loss
0.5	Q	40	106.73	66.73	0.9753	65.08Q
1.5	Q	40	105.97	65.97	0.9277	61.20Q
2.5	Q	40	105.17	65.17	0.8825	57.52Q
3.5	Q	40	104.34	64.34	0.8395	54.01Q
4.5	Q	40	103.46	63.46	0.7985	50.67Q
Total						288.48Q

Calculations continued

- We set $288.48Q = 8.75$ to get $Q = 3.03\%$
- This analysis can be extended to allow defaults to take place more frequently
- With several bonds we can use more parameters to describe the default probability distribution

Real World vs Risk-Neutral Default Probabilities

- The default probabilities backed out of bond prices or credit default swap spreads are risk-neutral default probabilities
- The default probabilities backed out of historical data are real-world default probabilities

A Comparison

- Calculate 7-year hazard rates from the Moody's data (1970-2013). These are real world default probabilities)
- Use Merrill Lynch data (1996-2007) to estimate average 7-year hazard rates from bond prices (these are risk-neutral default intensities)
- Assume a risk-free rate equal to the 7-year swap rate minus 10 basis points

The Data

Rating	Cumulative 7-year default probability(%): 1970-2016	Average 7- year credit spread (bp): 1996-2007
Aaa	0.195	35.74
Aa	0.525	43.67
A	1.297	68.68
Baa	2.472	127.53
Ba	11.667	280.28
B	28.857	481.04
Caa	42.132	1,103.70

Real World vs Risk Neutral Default Probabilities , 7 year averages (Table 19.5, page 448)

Rating	Historical Hazard Rate (% per annum)	Hazard Rate from bonds (% per annum)	Ratio	Difference
Aaa	0.028	0.596	21.4	0.568
Aa	0.075	0.728	9.7	0.653
A	0.186	1.145	6.1	0.959
Baa	0.358	2.126	5.9	1.768
Ba	1.772	4.671	2.6	2.889
B	4.864	8.017	1.6	3.153
Caa	7.814	18.395	2.4	10.581

Risk Premiums Earned By Bond Traders

Rating	Bond Yield Spread over Treasuries (bps)	Spread of risk-free rate used by market over Treasuries (bps)	Spread to compensate for default rate in the real world (bps)	Extra Risk Premium (bps)
Aaa	78	42	2	34
Aa	86	42	5	39
A	111	42	11	58
Baa	169	42	21	106
Ba	322	42	106	174
B	523	42	292	189
Caa	1146	42	469	635

Possible Reasons for These Results

(The third reason is the most important)

- Corporate bonds are relatively illiquid
- The subjective default probabilities of bond traders may be much higher than the estimates from Moody's historical data
- **Bonds do not default independently of each other. This leads to systematic risk that cannot be diversified away.**
- Bond returns are highly skewed with limited upside. The non-systematic risk is difficult to diversify away and may be priced by the market

Which World Should We Use?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis

Merton's Model

- Merton's model regards the equity as an option on the assets of the firm
- In a simple situation the equity value is

$$\max(V_T - D, 0)$$

where V_T is the value of the firm and D is the debt repayment required

Equity vs. Assets

An option pricing model enables the value of the firm's equity today, E_0, to be related to the value of its assets today, V_0, and the volatility of its assets, σ_V

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}}\ ; \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

Volatilities

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0$$

This equation together with the option pricing relationship enables V_0 and σ_V to be determined from E_0 and σ_E

Example

- A company's equity is \$3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the debt is \$10 million and time to debt maturity is 1 year
- Solving the two equations yields V_0=12.40 and σ_v=21.23%

Example continued

- The probability of default is $N(-d_2)$ or 12.7%
- The market value of the debt is 9.40
- The present value of the promised payment is 9.51
- The expected loss is about 1.2%
- The recovery rate is 91%

The Implementation of Merton's Model to estimate real-world default probability (e.g. Moody's KMV)

- Choose time horizon
- Calculate cumulative obligations to time horizon. We denote it by D
- Use Merton's model to calculate a theoretical probability of default
- Use historical data to develop a one-to-one mapping of theoretical probability into real-world probability of default.
- Assumption is that the rank ordering of probability of default given by the model is the same as that for real world probability of default
- A distance to default measure is $\dfrac{\ln(V_0) - \ln(D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}$

Risk-neutral vs. Real World

- The average growth rate of the value of the assets, V, is greater in the real world than in the risk-neutral world
- This means that V has more chance of dropping below the default point in a risk-neutral world than in the real world
- This explains why risk-neutral default probabilities are higher than real-world default probabilities

The Implementation of Merton's Model continued. Estimation of risk-neutral default probability (e.g. by CreditGrades)

- Same approach can be used
- In this case the assumption is that the rank ordering of probability of default given by the model is the same as that for real world probability of default