FINM3133 Time Series for Finance and Macroeconomics

Mid-term Test

Date: 25 April 2025
Time allowed: 90 minutes **Full mark: 80**

1. (12 points) Suppose that $Y_t = e_t - e_{t-12}$, where $\{e_t\}$ is white noise process with mean zero and variance σ_e^2.

   (a) (6 points) Is $\{Y_t\}$ stationary? Why?

   (b) (6 points) Find the autocorrelation function for $\{Y_t\}$.

2. (10 points) Let $\{e_t\}$ be a white noise time series with zero mean and variance $\text{Var}(e_t) = \sigma_e^2 > 0$. Calculate the values of ρ_1 and ρ_2, the autocorrelation function ρ_k at lag $k = 1$ and lag $k = 2$ respectively, for the following stationary AR(2) time series

$$X_t = 0.3X_{t-1} - 0.6X_{t-2} + e_t, \quad t \geq 2,$$

   with initials $X_0 = X_1 = 0$.

3. (14 points) Consider the time series model

$$Y_t = 20 + e_t + 0.2e_{t-1}$$

   where $\{e_t\}$ is a white noise process with mean zero and variance σ_e^2.

   (a) (5 points) Is this a stationary time series process? Why?

   (b) (5 points) Is this an invertible time series? Why?

   (c) (4 points) What is the mean of the time series?

4. (16 points) Let $\{e_t\}$ be a zero-mean, unit-variance white noise process. Consider a process that begins at time $t = 0$ and is defined recursively as follows. Let $Y_0 = c_1 e_0$ and $Y_1 = c_2 Y_0 + e_1$, where c_1 and c_2 are constants, and $c_1 > 0$. Then let $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$ for $t > 1$ as in an AR(2) process.

   (a) (6 points) What is the mean of the process $\{Y_t\}$? Why?

   (b) (10 points) For particular values of ϕ_1 and ϕ_2 within the stationarity region for an AR(2) model, what are the values of c_1 and c_2, in terms of ϕ_1 and ϕ_2, such that $Var(Y_0) = Var(Y_1)$ and $Corr(Y_1, Y_0) = Corr(Y_t, Y_{t-1})$ for $t > 1$?

5. (22 points)

   (a) (6 points) Give an expression for the following

$$2X_t - 7X_{t-1} + 9X_{t-2} - 5X_{t-3} + X_{t-4}$$

   in terms of third order difference.

   (b) (16 points) Let $\{e_t\}$ be a white noise time series with zero mean and variance $\text{Var}(e_t) = \sigma_e^2 > 0$. Identify the following as specific ARIMA model. That is, what are p, d, and q and what are the values of the parameters (the ϕ's and θ's)?

      i. $Y_t = 3.2Y_{t-1} - 3.6Y_{t-2} + 1.6Y_{t-3} - 0.2Y_{t-4} + e_t + 0.3e_{t-1}$.

      ii. $Y_t = 2.9Y_{t-1} - 2.8Y_{t-2} + 0.9Y_{t-3} + e_t - e_{t-2}$.

6. (6 points) The following table shows the first 16 values of the sample ACF r_k and sample PACF $\hat{\phi}_{kk}$ for a series of 60 observations of logged quarterly unemployment in the United Kingdom. Identify a model for the series. (*Hint:* Let $n^{-1/2}$ be the approximate standard errors of both the sample autocorrelations and sample partial autocorrelation coefficients.)

k	r_k	$\hat{\phi}_{kk}$	k	r_k	$\hat{\phi}_{kk}$	k	r_k	$\hat{\phi}_{kk}$
1	0.93	0.93	7	0.03	0.05	13	-0.21	0.19
2	0.80	-0.41	8	-0.09	-0.07	14	-0.12	0.20
3	0.65	-0.14	9	-0.16	0.12	15	-0.01	0.03
4	0.49	-0.11	10	-0.22	-0.14	16	0.10	-0.11
5	0.32	-0.07	11	-0.25	0.03			
6	0.16	-0.10	12	-0.25	0.09			