Caculus II Math 1038 (1002&1003)

Monica CHEN

Week 1:

1. Self-introduction
   - Lecturer: **Monica** (Dr. Wen CHEN), T3-502-R12 (no key),
     - email: wenchen@uic.edu.cn
     - Lectures: Mon 8-10am 1002; Wed 8-10am 1003; Fri 3-4pm 1002; 5-6pm 1003
     - **Q&A: Fri afternoons**
   - TA: Ms. Mei MING, T3-502-R26-H13
     - Tutorial: to be determined
2. How to evaluate
   - 15% Assignment (weekly)
   - 10% Quiz
   - 15% Midterm
   - 60% Final
3. Rules of this course
   (a) Attendance: check random number of names at the beginning of the class
   (b) Deadline applies strictly for assignments. Submission after deadline will be rejcted.
   (c) Quiz and Exams: **NO** calculator, electronical devices (laptop, pad, phones, smart watches, etc.), books or note.
   (d) No cheating: all cheating behaviours in quiz/exam will be reported to AR.
4. How to study?
   (a) check iSpace regularly
   (b) read (textbook), lecture, read (book & notes), practice (assignment & extra exercise),...**[repeat]**
   (c) definitions, notations, theorems & proofs, examples and more examples.
   (d) time allocation for calculus per week:
      i. preview & review (≥ 3 hours),
      ii. class (3×50 mins lecture, 50 mins tutorial),
      iii. exercise (as much as you can, ≥ 8 hours)
   (e) the merits of attending classes:
      i. English environment for studying mathematics efficiently,
      ii. discover new ways of thinking,
      iii. remove any risk of missing out on something,
      iv. keep in touch with your classmates,
      v. ...
5. What to read?
   - **Textbook**: Smith and Minton, Calculus: *Early Transcendental Functions.* 6th edition. McGraw Hill.
   - Recommended readings (see course syllybus): choose 1 or 2 books
   - Others I will provide later

6. How to find exercises?
    - All the problems on the **Textbook** at least;
    - Find problems with similar difficulty levels as your assignment and quiz.

Course of content:

1. Chapter 11 Sequences, series and power series: 3-4 weeks
2. Chapter 12 Vectors and the geometry of space:1 week
3. Chapter 13 Vector functions: 1 week
4. Chapter 14 Partial derivatives: 3 weeks
5. Chapter 15 Multiple integrals: 4 weeks
6. Chapter 16 Vectors calculus *

Timeline (may be adjusted):

- Week 1-4 Chapter 11
- **Week 5 Quiz One**
- Week 5 Chapter 12
- Week 6 Chapter 13
- **Week 7 Midterm**
- Week 7-10 Chapter 14
- **Week 9 Quiz Two**
- Week 10-13 Chapter 15

Review: Calculus I:

1. **limit** of a function
$$\lim_{x\to a} f(x) = L$$
$$\forall \epsilon > 0, \quad \exists \delta > 0,$$
$$s.t. \qquad |x-a| < \delta, \qquad |f(x) - L| < \epsilon$$
(Notations: $\forall$: for all, $\exists$: exists; Greek letters: ϵ: epsilon, δ: delta.)
2. continuity: A function is **continous** at a number a if
$$\lim_{x\to a} f(x) = f(a)$$
3. differentiation: The **derivative** of a function $f(x)$
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
4. integration: limit of Riemann sum
$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k) \triangle x_k$$
where
$$a = x_1 < x_2 < ... < x_n = b$$

5. Improper Integrals

$$\int_a^{\infty} f(x)dx = \lim_{R\to\infty} \int_a^R f(x)dx$$

If the limit exists, we say that the improper integral converges to a value L. If the limit does not exist, we say that the imprpoer integral diverges: e.g.

$$\int_1^{\infty} \frac{1}{x^2}dx$$

and

$$\int_1^{\infty} \frac{1}{x^p}dx$$

Start of Calculus II: Sequences and series

1. Definitions:
   (a) Sequence: a list of numbers in a definite order
   $$\{a_1, a_2, ...\} \qquad \{a_n\} \qquad \{a_n\}_{n=1}^{\infty}$$
   (b) Limit of a convergent sequence:
   $$\lim_{n\to\infty} a_n = L$$
   (c) Series: the sum of a sequence $\{a_n\}_{n=1}^{\infty}$
   $$s = \sum_{n=1}^{\infty} a_n$$
      i. Partial sum:
      $$s_n = \sum_{k=1}^{n} a_k$$
      ii. Remainder:
      $$R_n = s - s_n = \sum_{k=n+1}^{\infty} a_k$$
   (d) Limit of a convergent series
   $$\lim_{n\to\infty} \sum_{k=1}^{n} a_k = L$$
2. Sequences:
   (a) arithmetic sequence: common difference
   (b) geometric sequence: common ratio
   (c) harmonic sequence: $a_n = 1/n$
   (d) Fibonacci sequence $a_{n+2} = a_n + a_{n+1}$
   (e) alternating sequence: absolute convergence
3. Series:
   (a) geometric series
   (b) **Taylor's series, power series**
   (c) Fourier series, trigometric series
4. Theorems about convergent sequences
   (a) Squeeze Theorem
   (b) Bounded monotonic sequence theorem:

i. a bounded above monotonically increasing sequence converges;
ii. a bounded below monotonically decreasing sequence converges.
iii. a **bounded monotonic** sequence is convergent.

(c) If a sequence converges to L, then every subsequence converges to L.

5. Test for series divergence

(a) If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then

$$\lim_{n\to\infty} a_n = 0$$

Warning! the **converse statement** is not true! counter example: $a_n = 1/n$.
The contrapositive statement is true, which is the **divergence test**: if

$$\lim_{n\to\infty} a_n \neq 0$$

then $\sum_{n=1}^{\infty} a_n$ is divergent.

6. Test for series convergence

(a) Integral Test: f is a continous, positive decreasing function on $[1,\infty)$ and $f(n) = a_n$

$$\int_1^\infty f(x)dx \qquad \text{and} \qquad \sum_{n=1}^{\infty} a_n$$

both converge or both are diverge.

i. **p-series**

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges if $p > 1$ and diverges if $p \leq 1$.

(b) Comparison Test

i. direct comparison test
ii. limit comparison test

(c) Ratio and Root Test

$$r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

i. $0 \leq r < 1$, converges
ii. $r > 1$, diverges
iii. $r = 1$, inconclusive (we cannot make a conclusion, we need another test!)

(d) Root Test

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$$

i. $L < 1$, **absolutely convergent**
ii. $L > 1$, divergent
iii. $L = 1$, inconclusive

(e) Absolute convergence $\sum |a_n|$ implies convergence $\sum a_n$.

7. Growth rates of sequence in order:

$$\ln n, \quad n, \quad n \cdot \ln n, \quad n^2, \quad a^n, \quad n! \quad n^n$$