PT Assignment 11

1. A die is rolled twice. Let X equal the sum of the outcomes, and let Y equal the first outcome minus the second. Compute $\mathrm{Cov}(X, Y)$.

2. The joint density function of X and Y is given by

$$f(x, y) = \frac{1}{y} e^{-(y+x/y)}, \quad x > 0, y > 0$$

   Find $E[X], E[Y]$, and show that $\mathrm{Cov}(X, Y) = 1$.

3. Suppose $X \sim$ Uniform $(1, 2)$, and given $X = x, Y$ is exponential with parameter $\lambda = x$. Find $\mathrm{Cov}(X, Y)$.

4. Let $X_1, \ldots$ be independent with common mean μ and common variance σ^2, and set $Y_n = X_n + X_{n+1} + X_{n+2}$. For $j \geq 0$, find $\mathrm{Cov}\,(Y_n, Y_{n+j})$.

5. Show that if $Y = a + bX$, then

$$\rho_{X,Y} = \begin{cases} +1 & \text{if } b > 0 \\ -1 & \text{if } b < 0 \end{cases}$$

6. If X_1, X_2, X_3, and X_4 are (pairwise) uncorrelated random variables, each having mean 0 and variance 1, compute the correlations of
   (a) $X_1 + X_2$ and $X_2 + X_3$;
   (b) $X_1 + X_2$ and $X_3 + X_4$.

7. Let X and Y be 2 random variables with probability mass functions

$$p_X(x) = \begin{cases} 0.9 & x = 1 \\ 0.1 & x = 0 \\ 0 & \text{otherwise} \end{cases} \qquad p_Y(y) = \begin{cases} 0.2 & y = 1 \\ 0.8 & y = 0 \\ 0 & \text{otherwise} \end{cases}$$

   Suppose $\rho_{X,Y} = -0.5$. Find $p_{X,Y}(0, 0), p_{X,Y}(0, 1), p_{X,Y}(1, 0), p_{X,Y}(1, 1)$.

8. You are given that the random variable X is exponential with mean 1, and that the random variable Y is uniformly distributed on the interval $[0, 1]$. Furthermore, it is known that X and Y are independent. Find the density of the joint distribution of $U = XY$ and $V = \frac{X}{Y}$.