**2022-23 First Semester**

**MATH1063 Linear Algebra II (1003)**

Assignment 3

Due Date: **17/Mar/2023 (Friday), 09:00 in tutorial class.**

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**
- For questions marked '*Easy!*', you may skip them if they are really easy for you.

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1. Let $L$ be the linear operator mapping $\mathbb{R}^3$ into $\mathbb{R}^3$ defined by $L(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & 1 \end{bmatrix}$$

and let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

Find the transition matrix $V$ corresponding to a change of basis from $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, and use it to determine the matrix $B$ representing $L$ with respect to $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

2. (a) Consider a vector $\vec{v}$ in $\mathbb{R}^n$, and a scalar $k$. Show that

$$\|k\vec{v}\| = |k|\|\vec{v}\|.$$

(b) Show that if $\vec{v}$ is a nonzero vector in $\mathbb{R}^n$, then $\vec{u} = \dfrac{1}{\|\vec{v}\|}\vec{v}$ is a unit vector.

3. Let $\mathbf{x}$ and $\mathbf{y}$ be vectors in $\mathbb{R}^n$ and define

$$\mathbf{p} = \frac{\mathbf{x}^{\mathbf{T}}\mathbf{y}}{\mathbf{y}^{\mathbf{T}}\mathbf{y}}\mathbf{y} \quad \text{and} \quad \mathbf{z} = \mathbf{x} - \mathbf{p}.$$

Show that $\mathbf{p} \perp \mathbf{z}$.

4. Let $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ be vectors in $\mathbb{R}^3$. If $\mathbf{x}_1 \perp \mathbf{x}_2$ and $\mathbf{x}_2 \perp \mathbf{x}_3$, is it necessarily true that $\mathbf{x}_1 \perp \mathbf{x}_3$? Prove your answer.

5. Find the distance from the point $P(2, 1, -2)$ to the plane $6(x-1)+2(y-3)+3(z+2) = 0$.

6. Find the distance from the point $(1, 2)$ to the line $4x - 3y = 0$.

7. (*Optional!*) Find the point on the line $y = 2x + 2$ that is closest to the point $(5, 2)$.

8. If $Y$ is a subspace of $\mathbb{R}^n$, show that $Y^\perp$ is also a subspace of $\mathbb{R}^n$.

9. (*Easy!*) For each of the following matrices, determine a basis for each of the subspaces $\mathrm{Col}(A^T)$, $N(A)$, $\mathrm{Col}(A)$, and $N(A^T)$:

   (a) $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 0 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 0 \\ 1 & 4 & 2 \end{bmatrix}$

10. Let $V$ be the solution space of the linear system

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 + 2x_2 + 5x_3 + 4x_4 = 0 \end{cases}$$

   Find a basis of $V^\perp$.