FINM3133 Time Series for Finance and Macroeconomics

Quiz 2 Solutions

1. (a) The time series is plotted in the attached figure 1. It is not stationary as a clear trend is seen.

   (b) A suitable transformation from Y_t to an acceptable stationary time series X_t is

   $$X_t = \nabla Y_t = Y_t - Y_{t-1}.$$

   The (differenced) time series X_t is plotted in the attached figure 2.

   (c) The estimated autocorrelations at lags 0 to 5 are

k	0	1	2	3	4	5
r_k	1	-0.113	-0.028	-0.087	-0.163	0.155

   (d) The standard error is approximately $n^{-1/2} = 1/\sqrt{20} = 0.224$. Then we get an 95% confidence interval of $(-0.448, 0.448)$. As none of the estimated autocorrelations are outside the limits, we can conclude that X_t is white noise. Thus the model suggested is ARIMA(0,1,0).

2. (a) The process can be written as

   $$(1 - 2B + B^2)Y_t = 10 + (1 - 0.7B)e_t$$

   which is an ARIMA(0,2,1) process. The AR characteristic equation has two unit roots. Thus the process is not stationary. The root of the MA characteristic equation $1 - 0.7x = 0$ is 1.43, outside the unit circle. Thus the process is invertible.

   (b) The process can be written as

   $$(1 - B)(1 - B + 0.5B^2)Y_t = (1 - 0.1B)(1 - 0.3B)e_t$$

   which is an ARIMA(2,1,2) process. The AR characteristic equation has a unit root. Thus the process is not stationary. The roots of the MA characteristic equation are outside the unit circle. Thus the process is invertible.

3. Note that

   $$Y_t = \phi Y_{t-1} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}. \tag{1}$$

   Multiply e_t on both sides of (1),

   $$Y_t e_t = \phi Y_{t-1} e_t + e_t^2 - \theta_1 e_{t-1} e_t - \theta_2 e_{t-2} e_t,$$

take expectation on both sides,

$$E(Y_t e_t) = \sigma_e^2.$$

Multiply e_{t-1} on both sides of (1),

$$Y_t e_{t-1} = \phi Y_{t-1} e_{t-1} + e_t e_{t-1} - \theta_1 e_{t-1} e_{t-1} - \theta_2 e_{t-2} e_{t-1},$$

take expectation on both sides,

$$E(Y_t e_{t-1}) = (\phi - \theta_1)\sigma_e^2.$$

Multiply e_{t-2} on both sides of (1),

$$Y_t e_{t-2} = \phi Y_{t-1} e_{t-2} + e_t e_{t-2} - \theta_1 e_{t-2} e_{t-1} - \theta_2 e_{t-2}^2,$$

take expectation on both sides,

$$E(Y_t e_{t-2}) = \phi(\phi - \theta_1)\sigma_e^2 - \theta_2 \sigma_e^2 = (\phi^2 - \phi\theta_1 - \theta_2)\sigma_e^2.$$

Multiply Y_{t-k} on both sides of (1),

$$Y_t Y_{t-k} = \phi Y_{t-1} Y_{t-k} + e_t Y_{t-k} - \theta_1 e_{t-1} Y_{t-k} - \theta_2 e_{t-2} Y_{t-k}. \tag{2}$$

Choose $k = 0$ in (2),

$$Y_t Y_t = \phi Y_{t-1} Y_t + e_t Y_t - \theta_1 e_{t-1} Y_t - \theta_2 e_{t-2} Y_t,$$

take expectation on both sides,

$$\gamma_0 = \phi\gamma_1 + \sigma_e^2 - \theta_1(\phi - \theta_1)\sigma_e^2 - \theta_2(\phi^2 - \phi\theta_1 - \theta_2)\sigma_e^2. \tag{3}$$

Choose $k = 1$ in (2),

$$Y_t Y_{t-1} = \phi Y_{t-1} Y_{t-1} + e_t Y_{t-1} - \theta_1 e_{t-1} Y_{t-1} - \theta_2 e_{t-2} Y_{t-1},$$

take expectation on both sides,

$$\gamma_1 = \phi\gamma_0 - \theta_1 \sigma_e^2 - \theta_2(\phi - \theta_1)\sigma_e^2. \tag{4}$$

Combining (3) and (4) gives

$$\gamma_0 = \frac{1 - 2\theta_1\phi - 2\theta_2\phi^2 + 2\phi\theta_1\theta_2 + \theta_1^2 + \theta_2^2}{1 - \phi^2}\sigma_e^2,$$

and

$$\gamma_1 = \frac{\phi - \phi^2\theta_1 + \phi\theta_1^2 - \phi^3\theta_2 + \phi^2\theta_1\theta_2 + \phi\theta_2^2 - \theta_1 - \theta_2\phi + \theta_1\theta_2}{1 - \phi^2}\sigma_e^2.$$

So

$$\rho_1 = \frac{\phi - \phi^2\theta_1 + \phi\theta_1^2 - \phi^3\theta_2 + \phi^2\theta_1\theta_2 + \phi\theta_2^2 - \theta_1 - \theta_2\phi + \theta_1\theta_2}{1 - 2\theta_1\phi - 2\theta_2\phi^2 + 2\phi\theta_1\theta_2 + \theta_1^2 + \theta_2^2}.$$