2022-23 First Semester
MATH1053 Linear Algebra I

Assignment 2a

Due Date: 11/Oct/2022 (Tuesday), 11:00 in class.

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- For online students, hand in your homework in **one pdf file** on iSpace.
- **Late submissions or answers without steps won't be graded.**

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1. For each of the following systems, use Gaussian elimination to obtain an equivalent system whose augmented matrix is in *row echelon form*. Solve for all possible solutions.

(a). $$\begin{cases} x_1 & +2x_2 & -x_3 & = 1 \\ 2x_1 & -x_2 & +x_3 & = 3 \\ -x_1 & +2x_2 & +3x_3 & = 7 \end{cases}$$

(b). $$\begin{cases} x_1 & -x_2 & +2x_3 & = 4 \\ 2x_1 & +3x_2 & -x_3 & = 1 \\ 7x_1 & +3x_2 & +4x_3 & = 7 \end{cases}$$

2. For each of the following systems, use Gauss-Jordan elimination to obtain an equivalent system whose augmented matrix is in *reduced row echelon form*. Solve for all possible solutions.

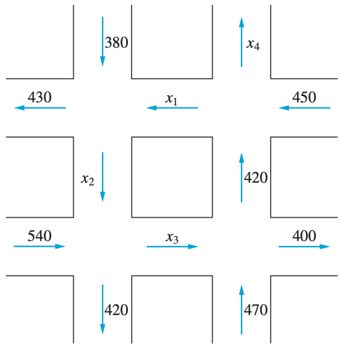
(a). $$\begin{cases} x_1 - 5x_2 = 6 \\ 3x_1 + 2x_2 = 1 \\ 5x_1 + 2x_2 = 1 \end{cases}$$

(b). $$\begin{cases} x_1 + 2x_2 + 3x_3 + 2x_4 + 15x_5 = 1 \\ 2x_1 + 4x_2 - x_3 + 2x_4 + 8x_5 = 6 \\ 3x_1 + 6x_2 - x_3 + 3x_4 + 13x_5 = 8 \end{cases}$$

3. Write out every possible reduced row echelon form for a 3×3 matrix. Make sure you explain carefully why your answer includes *all* possibilities.

For example, the rref of a 3×3 matrix could be the one with 2 pivots: $\begin{pmatrix} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \end{pmatrix}$.

4. In the downtown section of a certain city, two sets of one-way streets intersect as shown in the following figure. The average hourly volume of traffic entering and leaving this section during rush hour is given in the diagram. Determine the values of x_1, x_2, x_3 and x_4 for the following traffic flow diagram, using Gauss-Jordan elimination.



5. Consider a linear system whose augmented matrix is of the form

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right)$$

   (a) For what values of a and b will the system have infinitely many solutions?

   (b) For what values of a and b will the system be inconsistent?

6. Let

$$A = \begin{pmatrix} 2 & 3 & 6 & 4 \\ 4 & 7 & 2 & 5 \\ 0 & 3 & 8 & 7 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 & 6 & 4 \\ 0 & 1 & -10 & -3 \\ 0 & 3 & 8 & 7 \end{pmatrix}, C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{pmatrix} \text{ and } D = \begin{pmatrix} 1 & 20 \\ 3 & 40 \\ 5 & 60 \\ 7 & 80 \end{pmatrix}.$$

   a) Find an elementary matrix E such that $EA = B$.
   b) Find an elementary matrix F such that $CF = D$.
   c) Write down the inverses of E and F.

7. Find the inverse of matrix A and B using elementary row operations on $[A|I]$ and $[B|I]$, respectively.

   (a) $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$; (b) $B = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 2 & 1 \\ -2 & 3 & 5 \end{pmatrix}$.

8. Prove that if A is row equivalent to B and B is row equivalent to C, then A is row equivalent to C.

9. Let A be a 3×3 matrix and suppose that $2\mathbf{a}_1 + \mathbf{a}_2 = 4\mathbf{a}_3$, where $\mathbf{a}_j$ is the j^{th} column of A. How many solutions does the system $A\mathbf{x} = \mathbf{0}$ have? Explain. Is A nonsingular? Explain.

10. True or false. If false, give a counterexample. If true, explain or prove your answer briefly.

(a) If matrix A is in rref, then at least one of the entries in each column must be 1.

(b) Given A, B have same sizes, then $\mathrm{rref}(A) + \mathrm{rref}(B) = \mathrm{rref}(A + B)$.

(c) The product of two $n \times n$ elementary matrices is an elementary matrix.

(d) If A and B are nonsingular $n \times n$ matrices, then $A + B$ is also nonsingular and $(A + B)^{-1} = A^{-1} + B^{-1}$.

11. (*Bonus!*) If $\mathbf{x}$ and $\mathbf{y}$ are nonzero vectors in $\mathbb{R}^n$ and $A = \mathbf{x}\mathbf{y}^T$, show that the row echelon form of A will have exactly one nonzero row.