

# PT Assignment 8

1. Consider the following table:

$x \backslash y$	0	1	2	$p_X(x)$
0	0.1	$a$	$b$	0.45
1	$c$	0.25	$d$	$e$
$p_Y(y)$	0.3	$f$	0.15	

Find

- (a) the values of  $a, b, c, d, e$  and  $f$ .
- (b)  $P(X = Y)$  and  $P(X < Y)$
- (c) Marginal distributions of  $X$  alone and  $Y$  alone.

2. Consider two random variables  $X$  and  $Y$  with joint PMF given in below table.

	$Y = 0$	$Y = 1$	$Y = 2$
$X = 0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X = 1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

- (a) Find  $P(X = 0, Y \leq 1)$
- (b) Find the marginal PMFs of  $X$  and  $Y$ .
- (c) Find  $P(Y = 1 \mid X = 0)$ .
- (d) Are  $X$  and  $Y$  independent?

3. Let

$$f(x, y) = \begin{cases} cx^2y & \text{if } x^2 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine

- (a) the constant  $c$ ,
- (b)  $P(X \geq Y)$ ,
- (c)  $P(X = Y)$ ,
- (d)  $P(X = 2Y)$ .

4. The joint density of  $(X, Y)$  is given by

$$\begin{cases} f(x, y) = 3x & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Compute the conditional density of  $Y$  given  $X = x$ .
- (b) Are  $X$  and  $Y$  independent?

5. Let  $X, Y$  be independent random variables taking values in  $\mathbb{N}$  with

$$P(X = i) = P(Y = i) = \frac{1}{2^i} \quad (i = 1, 2, \dots).$$

Find the following probabilities:

- (a)  $P(\min(X, Y) \leq i)$
- (b)  $P(X = Y)$
- (c)  $P(Y > X)$
- (d)  $P(X \text{ divides } Y)$
- (e)  $P(X \geq kY)$  for a given positive integer  $k$ .

6. Let  $X, Y$  be independent geometric random variables with parameters  $\lambda$  and  $\mu$ . Let  $Z = \min(X, Y)$ . Show  $Z$  is geometric and find its parameter.