MATH2033 Mathematical Statistics
Assignment 7

Due Date: **28/Apr/2024(Sunday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.

- **Late submissions**, **answers without details**, or **unrecognizable handwritings** will NOT be graded.

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1. (a) A random sample of $n = 25$ observations is taken from a normal distribution with unknown mean but known variance $\sigma^2 = 16$. The sample mean is found to be $\bar{x} = 18.2$. Test $H_0 : \mu = 20$ vs $H_1 : \mu < 20$ at the 5% significance level.

   (b) Find the probability that we reject H_0 using this testing procedure when the true value of the mean μ is 19.0.

2. The drug 6-mP is used to treat leukaemia. A random sample of 21 patients using 6-mP were found to have an average remission time of $\bar{x} = 17.1$ weeks with a sample standard deviation of $s = 10.00$ weeks. A previously used drug treatment had a known mean remission time of $\mu_0 = 12.5$ weeks. Assuming that the remission times of patients taking 6-mp are normally distributed with both the mean μ and variance σ^2 being unknown, test at the 5% significance level whether the mean remission time of patients taking 6-mp is greater than $\mu_0 = 12.5$ weeks.

3. Someone pretends to have telepathic gifts in the sense that if you randomly draw one card from a set with as many red as black cards, he has probability 0.6 of naming the correct color instead of probability 0.5. To test this, we proceed as follows: we let him guess 25 consecutive times, where the drawn card is put back every time. If he guesses correctly at least 17 times, we believe him; otherwise, we do not.

   (a) Reword this problem in terms of null hypothesis, test statistic, alternative hypothesis, critical region.

   (b) He guesses correctly 16 times. What is the p-value?

   (c) Do we reject H_0 at level $\alpha = 0.05$? And at level $\alpha = 0.10$?

4. Commercial fishermen working certain parts of the Atlantic Ocean sometimes find their efforts hindered by the presence of whales. Ideally, they would like to scare away the whales without frightening the fish. One of the strategies being experimented with is to transmit underwater the sounds of a killer whale. On the fiftytwo occasions that technique has been tried, it worked twenty-four times (that is, the whales immediately left the area). Experience has shown, though, that 40% of all whales sighted near fishing boats leave of their own accord, probably just to get away from the noise of the boat.

   (a) Let $p = P$ (Whale leaves area after hearing sounds of killer whale). Test $H_0 : p = 0.40$ versus $H_1 : p > 0.40$ at the $\alpha = 0.05$ level of significance. Can it be argued on the basis of these data that transmitting underwater predator sounds is an effective technique for clearing fishing waters of unwanted whales?

   (b) Calculate the P-value for these data. For what values of α would H_0 be rejected?

5. This problem is concerned with testing binomial data. Suppose $H_0 : p = 0.45$ is to be tested against $H_1 : p > 0.45$ at the $\alpha = 0.14$ level of significance, where $p = P(i\text{-th trial}$ ends in success). If the sample size is 200, what is the smallest number of successes that will cause H_0 to be rejected?