FINM3093 Investments

Lecture 6 Exercises

Solutions

1.

a.	Zero coupon	8% coupon	10% coupon
Current prices	$463.19	$1,000.00	$1,134.20
b. Price 1 year from now	$500.25	$1,000.00	$1,124.94
c. Price increase	$ 37.06	$ 0.00	− $ 9.26
Coupon income	$ 0.00	$ 80.00	$100.00
Total income	$ 37.06	$ 80.00	$ 90.74
Rate of return	8.00%	8.00%	8.00%
d. Price 1 year from now	$543.93	$1,065.15	$1,195.46
Price increase	$ 80.74	$ 65.15	$ 61.26
Coupon income	$ 0.00	$ 80.00	$100.00
Total income	$ 80.74	$145.15	$161.26
Rate of return	17.43%	14.52%	14.22%

2. a. On a financial calculator, enter the following (you may also use Excel):
n = 40; FV = 1000; PV = –950; PMT = 40

You will find that the yield to maturity on a semiannual basis is 4.26%. This implies a bond equivalent yield to maturity equal to: 4.26% ∗ 2 = 8.52%

Effective annual yield to maturity = $(1.0426)^2$ – 1 = 0.0870 = 8.70%

b. Since the bond is selling at par, the yield to maturity on a semiannual basis is the same as the semiannual coupon rate, i.e., 4%. The bond equivalent yield to maturity is 8%.

Effective annual yield to maturity = $(1.04)^2$ – 1 = 0.0816 = 8.16%

c. Keeping other inputs unchanged but setting PV = –1050, we find a bond equivalent yield to maturity of 7.52%, or 3.76% on a semiannual basis.

Effective annual yield to maturity = $(1.0376)^2 - 1 = 0.0766 = 7.66\%$

3. If the yield to maturity is greater than the current yield, then the bond offers the prospect of price appreciation as it approaches its maturity date. Therefore, the bond must be selling below par value. The coupon rate is less than 9%. If coupon divided by price equals 9%, and price is less than par, then price divided by par is less than 9%.

4. a. The bond sells for $1,276.76 based on the 3% yield to *maturity*.
[n = 60; i = 3; FV = 1000; PMT = 40] (you may also use Excel)

Therefore, yield to *call* is 1.860% semiannually, 3.721% annually.
[n = 10 semiannual periods; PV = –1,276.76; FV = 1100; PMT = 40]

b. If the call price were $1,050, we would set FV = 1,050 and redo part (b) to find that yield to call is 1.471% semiannually, 2.943% annually. With a lower call price, the yield to call is lower.

c. Yield to call is -0.3462% semiannually, -.6924% annually.
[n = 4; PV = −1,276.76; FV = 1100; PMT = 40]

5. a. The yield to maturity on the par bond equals its coupon rate, 8.75%. All else equal, the 4% coupon bond would be more attractive because its coupon rate is far below current market yields, and its price is far below the call price. Therefore, if yields fall, capital gains on the bond will not be limited by the call price. In contrast, the 8.75% coupon bond can increase in value to at most $1,050, offering a maximum possible gain of only 0.5%. The disadvantage of the 8.75% coupon bond, in terms of vulnerability to being called, shows up in its higher *promised* yield to maturity.

b. If an investor expects yields to fall substantially, the 4% bond offers a greater expected return.

c. Implicit call protection is offered in the sense that any likely fall in yields would not be nearly enough to make the firm consider calling the bond. In this sense, the call feature is almost irrelevant.

6.

Maturity	Price	a. YTM	b. Forward Rate
1	$943.40	6.00%	
2	$898.47	5.50%	$(1.055^2/1.06) - 1 = 5.0\%$
3	$847.62	5.67%	$(1.0567^3/1.055^2) - 1 = 6.0\%$
4	$792.16	6.00%	$(1.06^4/1.0567^3) - 1 = 7.0\%$

The expected price path of the 4-year zero coupon bond is shown below. (Note that we discount the face value by the appropriate sequence of forward rates implied by this year's yield curve.) The expected price at the end of the fourth year is $1,000.

Beginning of Year	c. Expected Price	d. Expected Rate of Return
1	$\$792.16$	$(\$839.69/\$792.16) - 1 = 6.00\%$
2	$\dfrac{\$1{,}000}{1.05 \times 1.06 \times 1.07} = \$839.69$	$(\$881.68/\$839.69) - 1 = 5.00\%$
3	$\dfrac{\$1{,}000}{1.06 \times 1.07} = \$881.68$	$(\$934.58/\$881.68) - 1 = 6.00\%$
4	$\dfrac{\$1{,}000}{1.07} = \$934.58$	$(\$1{,}000.00/\$934.58) - 1 = 7.00\%$

7.

a. $$P = \frac{\$9}{1.07} + \frac{\$109}{1.08^2} = \$101.86$$

b. The yield to maturity is the solution for y in the following equation:

$$\frac{\$9}{1+y} + \frac{\$109}{(1+y)^2} = \$101.86$$

[Using a financial calculator, enter n = 2; FV = 100; PMT = 9; PV = –101.86; Compute i]
YTM = 7.958%

c. The forward rate for next year, derived from the zero-coupon yield curve, is the solution for f_2 in the following equation:

$$1+f_2=\frac{(1.08)^2}{1.07}=1.0901 \Rightarrow f_2 = 0.0901 = 9.01\%.$$

Therefore, using an expected rate for next year of r_2 = 9.01%, we find that the forecast bond price is:

$$P=\frac{\$109}{1.0901}=\$99.99$$

d. If the liquidity premium is 1% then the forecast interest rate is:

$E(r_2) = f_2$ – liquidity premium = 9.01% – 1.00% = 8.01%

The forecast of the bond price is:

$$\frac{\$109}{1.0801}=\$100.92$$

8. The price of the coupon bond, based on its yield to maturity, is:

[$120 × Annuity factor (5.8%, 2)] + [$1,000 × PV factor (5.8%, 2)] = $1,113.99

If the coupons were stripped and sold separately as zeros, then, based on the yield to maturity of zeros with maturities of one and two years, respectively, the coupon payments could be sold separately for:

$$\frac{\$120}{1.05}+\frac{\$1{,}120}{1.06^2}=\$1{,}111.08$$

The arbitrage strategy is to buy zeros with face values of $120 and $1,120, and respective maturities of one year and two years, and simultaneously sell the coupon bond. The profit equals $2.91 on each bond.

9. a. The one-year zero-coupon bond has a yield to maturity of 6%, as shown below:

$$\$94.34=\frac{\$100}{1+y_1} \Rightarrow y_1 = 0.06000 = 6.000\%$$

The yield on the two-year zero is 8.472%, as shown below:

$$\$84.99=\frac{\$100}{(1+y_2)^2} \Rightarrow y_2 = 0.08472 = 8.472\%$$

b. The price of the coupon bond is: $\frac{\$12}{1.06}+\frac{\$112}{(1.08472)^2}=\$106.51$

Therefore: yield to maturity for the coupon bond = 8.333%

[On a financial calculator, enter: n = 2; PV = –106.51; FV = 100; PMT = 12]

c. $f_2 = \dfrac{(1+y_2)^2}{1+y_1} - 1 = \dfrac{(1.08472)^2}{1.06} - 1 = 0.1100 = 11.00\%$

d. $\text{Expected price} = \dfrac{\$112}{1.11} = \$100.90$

(Note that next year, the coupon bond will have one payment left.)

Expected holding period return =

$$\frac{\$12 + (\$100.90 - \$106.51)}{\$106.51} = 0.0600 = 6.00\%$$

This holding period return is the same as the return on the one-year zero.

e. If there is a liquidity premium, then: $E(r_2) < f_2$

$E(\text{Price}) = \dfrac{\$112}{1+E(r_2)} > \$100.90$

$E(\text{HPR}) > 6\%$

10. a. **YTM = 6%**

(1) Time until Payment (Years)	(2) Cash Flow	(3) PV of CF (Discount Rate = 6%)	(4) Weight	(5) Column (1) × Column (4)
1	$	$	0.0566	0.0566
2	60.00	53.40	0.0534	0.1068
3	1,060.00	890.00	0.8900	2.6700
	Column sums	$1,000.00	1.0000	2.8334

Duration = 2.833 years

b. **YTM = 10%**

(1) Time until Payment (Years)	(2) Cash Flow	(3) PV of CF (Discount Rate = 10%)	(4) Weight	(5) Column (1) × Column (4)
1	$	$ 54.55	0.0606	0.0606
2	60.00	49.59	0.0551	0.1102
3	1,060.00	796.39	0.8844	2.6532
	Column sums	$900.53	1.0000	2.8240

Duration = 2.824 years, which is less than the duration at the YTM of 6%.

c. For a semiannual 6% coupon bond selling at par, we use the following parameters: coupon = 3% per half-year period, y = 3%, T = 6 semiannual periods.

(1) Time until Payment (Years)	(2) Cash Flow	(3) PV of CF (Discount Rate = 3%)	(4) Weight	(5) Column (1) × Column (4)
1	$ 30.00	$ 29.13	0.02913	0.02913
2	30.00	28.28	0.02828	0.05656
3	30.00	27.45	0.02745	0.08236
4	30.00	26.65	0.02665	0.10662
5	30.00	25.88	0.02588	0.12939
6	1030.00	862.61	0.86261	5.17565
	Column sums	$1000.00	1.00000	5.57971

D = 5.5797 half-year periods = 2.7899 years

If the bond's yield is 10%, use a semiannual yield of 5% and semiannual coupon of 3%:

(1) Time until Payment (Years)	(2) Cash Flow	(3) PV of CF (Discount Rate = 5%)	(4) Weight	(5) Column (1) × Column (4)
1	$ 30.00	$ 28.57	0.03180	0.03180
2	30.00	27.21	0.03029	0.06057
3	30.00	25.92	0.02884	0.08653
4	30.00	24.68	0.02747	0.10988

5	30.00	23.51	0.02616	0.13081
6	1030.00	768.60	0.85544	5.13265
	Column sums	$898.49	1.00000	5.55223

D = 5.5522 half-year periods = 2.7761 years

11. a. Bond price decreases by $80.00, calculated as follows:

$$10 \times 0.01 \times 800 = 80.00$$

b. $\frac{1}{2} \times 120 \times (0.015)^2 = 0.0135 = 1.35\%$

c. 9/1.10 = 8.18

d. (i)

e. (iii)

12. a. $\text{Modified duration} = \dfrac{\text{Macaulay duration}}{1 + \text{YTM}} = \dfrac{10}{1.08} = 9.26 \text{ years}$

b. For option-free coupon bonds, modified duration is a better measure of the bond's sensitivity to changes in interest rates. Maturity considers only the final cash flow, while modified duration includes other factors, such as the size and timing of coupon payments, and the level of interest rates (yield to maturity). Modified duration indicates the approximate percentage change in the bond price for a given change in yield to maturity.

c. i. Modified duration increases as the coupon decreases.
ii. Modified duration decreases as maturity decreases.

d. Convexity measures the curvature of the bond's price-yield curve. Such curvature means that the duration rule for bond price change (which is based only on the slope of the curve at the original yield) is only an approximation. Adding a term to account for the convexity of the bond increases the accuracy of the approximation. That convexity adjustment is the last term in the following equation:

$$\frac{\Delta P}{P} = (-D^* \times \Delta y) + \left[\frac{1}{2} \times \text{Convexity} \times (\Delta y)^2\right]$$