2021-22 First Semester
MATH1083 Calculus II (1003)

Assignment 5

Due Date: 11:30am 21/Mar/2021(Tue).

- Write down your **Chinese name** and **student number**. Write neatly on **A4-sized** paper and **show your steps**.
- **Late submissions or answers without details will not be graded.**

1. For two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, with angle θ in between:

   (a) Prove **Cauchy-Schwartz Inequality**

   $$\left|\overrightarrow{a} \cdot \overrightarrow{b}\right| \leq |\overrightarrow{a}|\left|\overrightarrow{b}\right|$$

   (b) Use Cauchy-Schwartz Inequality to prove the **Triangle Inequality**

   $$\left|\overrightarrow{a} + \overrightarrow{b}\right| \leq |\overrightarrow{a}| + \left|\overrightarrow{b}\right|$$

   Hint: *use the fact that*

   $$\left|\overrightarrow{a} + \overrightarrow{b}\right|^2 = \left(\overrightarrow{a} + \overrightarrow{b}\right) \cdot \left(\overrightarrow{a} + \overrightarrow{b}\right)$$

   (c) Prove the **Parallelogram Indentity**:

   $$\left|\overrightarrow{a} + \overrightarrow{b}\right|^2 + \left|\overrightarrow{a} - \overrightarrow{b}\right|^2 = 2|\overrightarrow{a}|^2 + 2\left|\overrightarrow{b}\right|^2$$

   and give a geometric interpretation of the Parallelogram Indentity.

Solution: (a) Using Thoerem that

$$\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}|\left|\overrightarrow{b}\right|\cos\theta$$

The absolute values of the LHS and RHS are equal, and since $|\cos\theta| \leq 1$

$$\left|\overrightarrow{a} \cdot \overrightarrow{b}\right| = |\overrightarrow{a}|\left|\overrightarrow{b}\right||\cos\theta| \leq |\overrightarrow{a}|\left|\overrightarrow{b}\right|$$

(b) Since $|\overrightarrow{a}|^2 = \overrightarrow{a} \cdot \overrightarrow{a}$ and $\overrightarrow{a} \cdot \overrightarrow{b} \leq |\overrightarrow{a}| \cdot \left|\overrightarrow{b}\right|$

$$\begin{aligned}
\left|\overrightarrow{a} + \overrightarrow{b}\right|^2 &= \left(\overrightarrow{a} + \overrightarrow{b}\right) \cdot \left(\overrightarrow{a} + \overrightarrow{b}\right) \\
&= \overrightarrow{a} \cdot \overrightarrow{a} + 2\overrightarrow{a} \cdot \overrightarrow{b} + \overrightarrow{b} \cdot \overrightarrow{b} \\
&\leq |\overrightarrow{a}|^2 + 2|\overrightarrow{a}| \cdot \left|\overrightarrow{b}\right| + \left|\overrightarrow{b}\right|^2 \\
&= \left(|\overrightarrow{a}| + \left|\overrightarrow{b}\right|\right)^2
\end{aligned}$$

Find the square root on both side, we can prove the Triangle Inequality. The geometric interpretation is that in a triangle formed by vectors $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{a} + \overrightarrow{b}$, **the sum of the length of two edges** $|\overrightarrow{a}| + \left|\overrightarrow{b}\right|$ is greater than the **third edge** $\left|\overrightarrow{a} + \overrightarrow{b}\right|$

(c) Since $\left|\overrightarrow{a} + \overrightarrow{b}\right|^2 = \left(\overrightarrow{a} + \overrightarrow{b}\right) \cdot \left(\overrightarrow{a} + \overrightarrow{b}\right)$ and $\left|\overrightarrow{a} - \overrightarrow{b}\right|^2 = \left(\overrightarrow{a} - \overrightarrow{b}\right) \cdot \left(\overrightarrow{a} - \overrightarrow{b}\right)$, then

$$\begin{aligned}
\left|\overrightarrow{a} + \overrightarrow{b}\right|^2 + \left|\overrightarrow{a} - \overrightarrow{b}\right|^2 &= \left(\overrightarrow{a} + \overrightarrow{b}\right) \cdot \left(\overrightarrow{a} + \overrightarrow{b}\right) + \left(\overrightarrow{a} - \overrightarrow{b}\right) \cdot \left(\overrightarrow{a} - \overrightarrow{b}\right) \\
&= |\overrightarrow{a}|^2 + \left|\overrightarrow{b}\right|^2 + 2\overrightarrow{a}\overrightarrow{b} + |\overrightarrow{a}|^2 + \left|\overrightarrow{b}\right|^2 - 2\overrightarrow{a}\overrightarrow{b} \\
&= 2|\overrightarrow{a}|^2 + 2\left|\overrightarrow{b}\right|^2
\end{aligned}$$

The geometric interpretation is sum of the squares of the lengths of the **two diagonals** of a parallelogram equals the sum of the squares of the lengths of the **four sides**.

2. Find the **area of the parallelogram** with vertices $A(-3,0)$, $B(-1,3)$, $C(5,2)$ and $D(3,-1)$. *[Hint:Parallelogram in 2D space, $A = |\vec{a}|\left|\vec{b}\right|\sin\theta$]*

   Solution: let $\vec{a} = \overrightarrow{AB} = (-1,3)-(-3,0) = (2,3)$ and $\vec{b} = \overrightarrow{AC} = (5,2)-(-3,0) = (8,2)$, then we can find the angle θ between the two vectors first.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\left|\vec{b}\right|} = \frac{16+6}{\sqrt{13}\cdot 2\sqrt{17}} = \frac{11}{\sqrt{221}}$$

   so

$$\sin\theta = \frac{10}{\sqrt{221}}$$

   the area

$$A = |\vec{a}|\left|\vec{b}\right|\sin\theta = \sqrt{13}\cdot 2\sqrt{17}\cdot\frac{10}{\sqrt{221}} = 20$$

3. Find the **area of the parallelogram** with vertices $P(1,0,2)$, $Q(3,3,3)$, $R(7,5,8)$ and $S(5,2,7)$. *[Hint:Parallelogram in 3D space, $A = \left|\left(\vec{a}\times\vec{b}\right)\right|$]*

   Solution: let $\vec{a} = \overrightarrow{PQ} = (3,3,3)-(1,0,2) = (2,3,1)$ and $\vec{b} = \overrightarrow{PR} = (7,5,8)-(1,0,2) = (6,5,6)$, then the area

$$\begin{aligned} A &= \left|\left(\vec{a}\times\vec{b}\right)\right| \\ &= |(2,3,1)\times(6,5,6)| \\ &= |(13,-6,-8))| \\ &= \sqrt{269} \end{aligned}$$

4. Find the **volume of the parallelepiped** determined by the vectors $\vec{a} = (1,2,3)$, $\vec{b} = (-1,1,2)$ and $\vec{c} = (2,1,4)$

   Solution:

$$\begin{aligned} A &= \left|\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right| \\ &= |(1,-5,3)\cdot(2,1,4)| \\ &= 9 \end{aligned}$$

5. If $\vec{a}\times\vec{b} = (1,2,2)$ and $\vec{a}\cdot\vec{b} = \sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.

   Solutino: $\left|\vec{a}\times\vec{b}\right| = 3$, and

$$\begin{aligned} \left|\vec{a}\times\vec{b}\right| &= |\vec{a}|\left|\vec{b}\right|\sin\theta = 3 \\ \vec{a}\cdot\vec{b} &= |\vec{a}|\left|\vec{b}\right|\cos\theta = \sqrt{3} \end{aligned}$$

   therefore

$$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$$

   so

$$\theta = \frac{\pi}{3}$$

6. Show that

$$\left|\vec{a}\times\vec{b}\right|^2 = |\vec{a}|^2\cdot\left|\vec{b}\right|^2 - \left(\vec{a}\cdot\vec{b}\right)^2$$

6. $\left(|\vec{a}||\vec{b}|\sin\theta\right)^2$
$= |\vec{a}|^2|\vec{b}|^2\sin^2\theta$
$= |\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)$
$= |\vec{a}|^2|\vec{b}|^2 - (|\vec{a}||\vec{b}|\cos\theta)^2$
$= |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

7. Find the vector equation and parametric equations for the line:

(a) The line through the point $(4, 2, -3)$ and parallel to the vector $2\overrightarrow{i} - \overrightarrow{j} + 6\overrightarrow{k}$
Vector equation:

$$\overrightarrow{r} = \left(4\overrightarrow{i} + 2\overrightarrow{j} - 3\overrightarrow{k}\right) + t\left(2\overrightarrow{i} - \overrightarrow{j} + 6\overrightarrow{k}\right), \qquad t \in \mathbb{R}$$

Parametric equation:

$$x = 4 + 2t, \quad y = 2 - t, \quad z = -3 + 6t$$

(b) The line through the point $(8, -1, 3)$ and $(1, 2, 3)$
The direction vector $\overrightarrow{a} = (1, 2, 3) - (8, -1, 3) = (-7, 3, 0)$
Vector equation:

$$\overrightarrow{r} = \left(8\overrightarrow{i} - 1\overrightarrow{j} + 3\overrightarrow{k}\right) + t\left(-7\overrightarrow{i} + 3\overrightarrow{j} + 0\overrightarrow{k}\right), \qquad t \in \mathbb{R}$$

Parametric equation:

$$x = 8 - 7t, \quad y = -1 + 3t, \quad z = 3$$

(c) The line through $(-6, 2, 3)$ and parallel to line $x = y = \frac{z-1}{6}$
The direction vector $\overrightarrow{a} = (1, 1, 7)$
Vector equation:

$$\overrightarrow{r} = \left(-6\overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k}\right) + t\left(\overrightarrow{i} + \overrightarrow{j} + 7\overrightarrow{k}\right), \qquad t \in \mathbb{R}$$

Parametric equation:

$$x = -6 + t, \quad y = 2 + t, \quad z = 3 + 7t$$

(d) The line through $(2, 1, 0)$ and perpendicular to both $\overrightarrow{i} + \overrightarrow{j}$ and $\overrightarrow{j} + \overrightarrow{k}$
The direction vector $\overrightarrow{a} = \left(\overrightarrow{i} + \overrightarrow{j}\right) \times \left(\overrightarrow{j} + \overrightarrow{k}\right) = (1, 1, 0) \times (0, 1, 1) = (1, -1, 1)$
Vector equation:

$$\overrightarrow{r} = \left(2\overrightarrow{i} + 1\overrightarrow{j} + 0\overrightarrow{k}\right) + t\left(\overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}\right), \qquad t \in \mathbb{R}$$

Parametric equation:

$$x = 2 + t, \quad y = 1 - t, \quad z = t$$

8. Find an equation of the plane:

(a) The plane through the point $(3, 2, 1)$ with normal vector $\overrightarrow{i} - \overrightarrow{j} + 2\overrightarrow{k}$

$$(x - 3) - (y - 2) + 2(z - 1) = 0$$

(b) The plane through the point $(5, -2, 4)$and perpendicular to the vector $-\overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k}$

$$-(x - 5) + 2(y + 2) + 3(z - 4) = 0$$

9. The plane that passes through the point $(6, -1, 3)$ and contains the **line** with symmetric equations

$$\frac{x}{3} = y + 4 = \frac{z}{2}$$

Solution: We need look for the normal direction of the plane. Find two points on the line: e.g. $A = (0, -4, 0)$ that is when $x = 0$, $z = 0$ and $y = -4$ on the line. And aother point $B = (6, -2, 4)$. Then let $\overrightarrow{a} = (0, -4, 0) - (6, -1, 3) = (-6, -3, -3)$ and $\overrightarrow{b} = (6, -2, 4) - (6, -1, 3) = (0, -1, 1)$, then

$$\begin{aligned} \overrightarrow{n} &= \overrightarrow{a} \times \overrightarrow{b} \\ &= (-6, -3, -3) \times (0, -1, 1) \\ &= (-6, 6, 6) \end{aligned}$$

plane: $-6(x-6) + 6(y+1) + 6(z-3) = 0$

10. Find the distance

(a) from the point to the given **line**: $(4, 1, -2)$; $x = 1 + t, \quad y = 3 - 2t, \quad z = 4 - 3t$

Solution: Let the point $P = (4, 1, -2)$ and find two points **on the line**: $A = (1, 3, 4)$, $B = (2, 1, 1)$, let $\overrightarrow{a} = \overrightarrow{AB} = (2, 1, 1) - (1, 3, 4) - = (1, -2, -3)$, and $\overrightarrow{b} = \overrightarrow{AP} = (4, 1, -2) - (1, 3, 4) = (3, -2, -6)$

$$\overrightarrow{a} \times \overrightarrow{b} = (-6, 3, -4)$$

the distance

$$d = \frac{\left|\overrightarrow{a} \times \overrightarrow{b}\right|}{|\overrightarrow{a}|} = \frac{\sqrt{61}}{\sqrt{14}}$$

[Remark: $\overrightarrow{a}$ is like the base edge of a parallelogram, which is on the line, $\overrightarrow{b}$ is the other edge, with point P outside the line. The height of the parallelogram is the distance from P to the line.]

(b) from the point to the given **plane**: $(1, 2, 4)$, $3x + 2y + 6z = 5$

Solution: Let the point $P = (1, 2, 4)$. Find three points on the line: $A = (1, 1, 0)$, $B = (-1, 1, 1)$, $C = (-1, -2, 2)$,

let $\overrightarrow{a} = \overrightarrow{AB} = (-1, 1, 1) - (1, 1, 0) = (-2, 0, 1)$ $\quad \overrightarrow{b} = \overrightarrow{AC} = (-1, -2, 2) - (1, 1, 0) = (-2, -3, 2)$, and $\overrightarrow{c} = \overrightarrow{AP} = (1, 2, 4) - (1, 1, 0) = (0, 1, 4)$

$$\overrightarrow{a} \times \overrightarrow{b} = (-2, 0, 1) \times (-2, -3, 2) = (3, 2, 6)$$

the distance

$$d = \frac{\left|\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \overrightarrow{c}\right|}{\left|\overrightarrow{a} \times \overrightarrow{b}\right|} = \frac{\sqrt{26}}{7}$$

Solution 2 [easier!]: We can see the distance from P to this plane as the absolute value of the scalar projection of $\overrightarrow{AP}$ onto the normal vector $\overrightarrow{n} = (3, 2, 6)$

$$d = \frac{\left|\overrightarrow{n} \cdot \overrightarrow{AP}\right|}{\overrightarrow{n}} = \frac{\sqrt{26}}{7}$$

Solution 3, directly use the formula where $(x_1, y_1, z_1) = (1, 2, 4)$ and $(a, b, c) = (3, 2, 6)$ and $d = -5$.

$$d = \frac{\left|\overrightarrow{n} \cdot \overrightarrow{AP}\right|}{|\overrightarrow{n}|} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{\sqrt{26}}{7}$$

(c) between two parallel planes: $2x - 3y + z = 4, \quad 4x - 6y + 2z = 3$

Solution: Find one point on the plane $4x - 6y + 2z = 3$, $P = (0, 0, 3/2)$ and three points on the plane $2x - 3y + z = 4$, $A = (0, 0, 4)$, $B = (2, 0, 0)$ and $C = (5, 2, 0)$let $\overrightarrow{a} = \overrightarrow{AB} = (2, 0, 0) - (0, 0, 4) = (2, 0, -4)$ $\quad \overrightarrow{b} = \overrightarrow{AC} = (5, 2, 0) - (0, 0, 4) = (5, 2, -4)$, and $\overrightarrow{c} = \overrightarrow{AP} = (0, 0, 3/2) - (0, 0, 4) = (0, 0, -5/2)$

$$\overrightarrow{a} \times \overrightarrow{b} = (2, -1, 0) \times (5, 2, -4) = (8, -12, 4)$$

and

$$\left|\overrightarrow{a} \times \overrightarrow{b}\right| = 4\sqrt{14} \qquad \left|\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \overrightarrow{c}\right| = 10$$

the distance

$$d = \frac{\left|\left(\overrightarrow{a} \times \overrightarrow{b}\right) \cdot \overrightarrow{c}\right|}{\left|\overrightarrow{a} \times \overrightarrow{b}\right|} = \frac{5}{2\sqrt{14}}$$

Solution 2: $\overrightarrow{n} = (2, -3, 1)$

$$d = \frac{\left|\overrightarrow{n} \cdot \overrightarrow{AP}\right|}{|\overrightarrow{n}|} = \frac{(3/2 \cdot 1 - 4)}{\sqrt{14}} = \frac{5}{2\sqrt{14}}$$

$|\vec{v}|$ ① R^2: $\vec{v}=(v_1, v_2)$ $|\vec{v}|=\sqrt{v_1^2+v_2^2}$

② R^3: $\vec{v}=(v_1, v_2, v_3)$ $|\vec{v}|=\sqrt{v_1^2+v_2^2+v_3^2}$

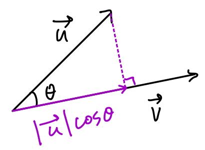
unit vector $\dfrac{\vec{v}}{|\vec{v}|}$

dot plot $\vec{u}\cdot\vec{v}$ ① R^2 $\vec{u}=(u_1, u_2)$ $\vec{v}=(v_1, v_2)$ $\vec{u}\cdot\vec{v}=u_1v_1+u_2v_2$

R^3 $\vec{u}=(u_1, u_2, u_3)$ $\vec{v}=(v_1, v_2, v_3)$ $\vec{u}\cdot\vec{v}=u_1v_1+u_2v_2+u_3v_3$

② $\vec{v}\cdot\vec{v}=|\vec{v}|^2$ $\theta=0$ $\cos\theta=1$

③ $\vec{u}\cdot\vec{v}=|\vec{u}||\vec{v}|\cos\theta$



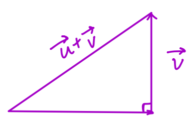
④ $|\vec{u}\cdot\vec{v}|\leq|\vec{u}||\vec{v}|$

1) $|\vec{u}\cdot\vec{v}|=|\vec{u}||\vec{v}| \longleftrightarrow$ $\vec{u}$ and $\vec{v}$ are parallel.

2) $\vec{u}\cdot\vec{v}=0 \xleftrightarrow{(|\vec{u}|\neq 0,\ |\vec{v}|\neq 0)}$ $\vec{u}$ and $\vec{v}$ are perpendicular / orthogonal

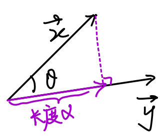
$\downarrow$

$|\vec{u}+\vec{v}|^2=|\vec{u}|^2+|\vec{v}|^2$



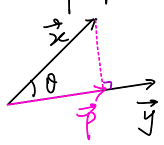
$|\vec{u}|^2+|\vec{v}|^2=|\vec{u}+\vec{v}|^2$ if and only if $\vec{u}\cdot\vec{v}=0$

projection ① scalar projection onto y $\alpha=|\vec{x}|\cos\theta=\dfrac{\vec{x}\cdot\vec{y}}{|\vec{y}|}$



② vector projection onto y $p=\alpha\dfrac{\vec{y}}{|\vec{y}|}=\dfrac{\vec{x}\cdot\vec{y}}{|\vec{y}|^2}\vec{y}$

在方向上的单位向量.



cross product ① $\vec{u}=(u_1, u_2, u_3)$ or $u_1i+u_2j+u_3k$ $\vec{v}=(v_1, v_2, v_3)$ or $v_1i+v_2j+v_3k$

$\vec{u}\times\vec{v}$

$$\vec{u}\times\vec{v}=\begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}=i\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}-j\begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}+k\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$=(u_2v_3-u_3v_2)i-(u_1v_3-u_3v_1)j+(u_1v_2-u_2v_1)k$$

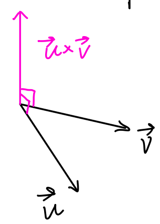
② $i\ j\ k$ 1) $i\times j=k$ $j\times k=i$ $k\times i=j$

2) $i\times i=j\times j=k\times k=0$

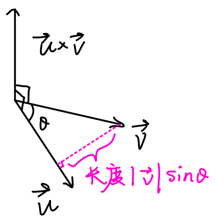
③ $\vec{u}\times\vec{v}=-(\vec{v}\times\vec{u})$

④ $(\vec{u}\times\vec{v})\cdot\vec{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = (\vec{v}\times\vec{w})\cdot\vec{u} = \begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$

⑤ $\vec{u}\times\vec{v}$ is perpendicular to $\vec{u}$ and $\vec{v}$.



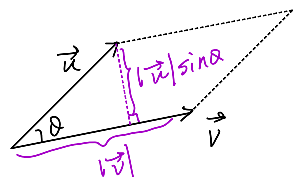
⑥ $|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$



1) $\vec{u}\times\vec{v}=0 \longleftrightarrow \vec{u}$ and $\vec{v}$ are parallel

☆ 2) **area of parallelgram**

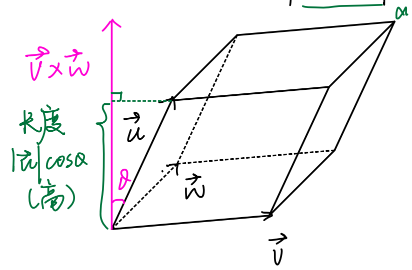
R^2: Area $= |\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$



R^3: Area $= |\vec{u}\times\vec{v}|$

☆ 3) **volume of parallelepiped**

$\vec{u}\cdot(\vec{v}\times\vec{w}) = |\vec{u}|\underbrace{|\vec{v}\times\vec{w}|}_{\text{area of parallelgram.}}\cos\theta$



☆ **equation** ① Vector equation $P = \underset{\text{through point } P_0}{P_0} + s\underset{\text{direction vector}}{u}$

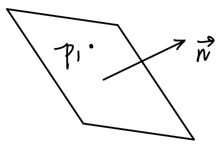
② parameter equation $P_0=(x_0, y_0, z_0)$ $u=(a, b, c)$

$(x, y, z) = (x_0+sa,\ y_0+sb,\ z_0+sc)$

③ equation of the plane

法一: normal vector $\vec{n}=(a,b,c)$ $P_1=(x_1,y_1,z_1)$

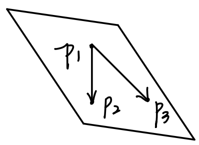
△ 随便取个平面上的点



$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

or $ax+by+cz+d=0$ $d=-ax_1-by_1-cz_1$

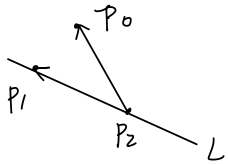
法二: 找平面上三点 P_1, P_2, P_3



$(x,y,z)=P_1+s\overrightarrow{P_1P_2}+t\overrightarrow{P_1P_3}$

两平面夹角为法向量(normal vector)夹角

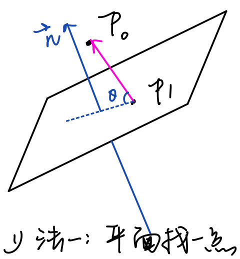
distance ① 点到线问题 $P_0(a_1,b_1,c_1)$ Line $ax+by+cz+d=0$



在线上找到两点 P_1, P_2

$$d=\frac{|\overrightarrow{P_2P_0}\times\overrightarrow{P_2P_1}|}{|\overrightarrow{P_2P_1}|}$$

② 点到面距离 $P_0=(x_1,y_1,z_1)$ plane $ax+by+cz+d=0$



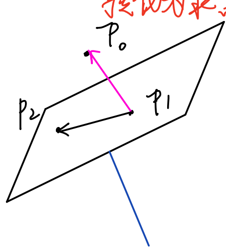
1) 法一: 平面找一点 P_1

$$D=|\overrightarrow{P_0P_1}|\sin\theta=\frac{|\overrightarrow{P_0P_1}\times\vec{n}|}{|\vec{n}|}$$

2) 法二: $D=\frac{|ax_1+by_1+cz_1+d|}{|\vec{n}|}$

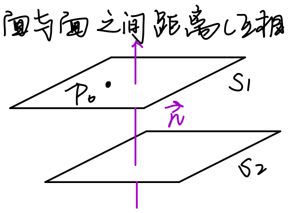
3) 法三: 在平面上找两点 P_1, P_2

转化为求点到线距离



③ 面与面之间距离(互相平行)   plane $D_1: a_1x+b_1y+c_1z+d_1=0$ $(a_1,b_1,c_1)\parallel(a_2,b_2,c_2)$

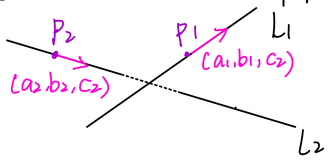
$D_2: a_2x+b_2y+c_2z+d_2=0$



平面S_1找一点P_0, 计算P_0到平面S_2距离

转化为求点到面距离

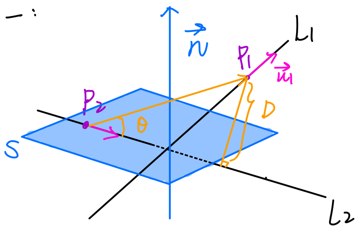
④ 线与线之间距离（非平行，不相交 ⇒ skew lines）



line1: $\overset{P_1}{(x_1,y_1,z_1)}+\overset{\vec{u}_1}{(a_1,b_1,c_1)}t$

line2: $\overset{P_2}{(x_2,y_2,z_2)}+\overset{\vec{u}_2}{(a_2,b_2,c_2)}t$

1) 法一：



根据 $\vec{u}_1$ 与 $\vec{u}_2$ 找到法向量 $\vec{n}=(a,b,c)$

创立经过点 P_2 且法向量为 $\vec{n}$ 的平面S

$$a(x-x_2)+b(y-y_2)+c(z-z_2)=0$$

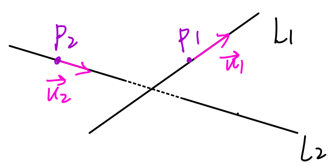
之后求点 P_1 到平面S的距离.

⇑

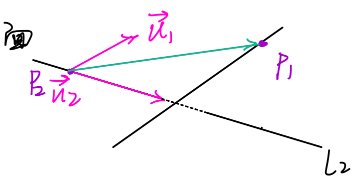
转化为求点到面距离

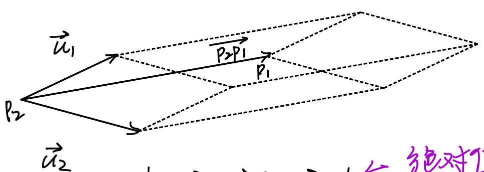
$$D=|\overrightarrow{P_1P_2}|\sin\theta=\frac{|a(x_1-x_2)+b(y_1-y_2)+c(z_1-z_2)|}{|\vec{n}|}$$

2) 法二：



平移 $\vec{u}_1$, $\vec{u}_2$ 到一个平面





扩展成平行六面体（parallelpiped）

求距离转化为求平行六面体的高.

$$D=\frac{|(\vec{u}_2\times\vec{u}_1)\cdot\overrightarrow{P_2P_1}|}{|\vec{u}_2\times\vec{u}_1|}$$

← 绝对值

→ 体积

→ 底面积