PT Assignment 1

1. A box contains three marbles: one red, one green, and one blue. Consider an experiment that consists of taking one marble from the box then replacing it in the box and drawing a second marble from the box.

   (a) What is the sample space? If, at all times, each marble in the box is equally likely to be selected, what is the probability of each point in the sample space?

   (b) Repeat (a) when the second marble is drawn without replacing the first marble.

2. Let E, F, G be three events. Find expressions for the events so that, of E, F, G,

   (a) only F occurs,

   (b) both E and F but not G occur,

   (c) at least one event occurs,

   (d) at least two events occur,

   (e) all three events occur,

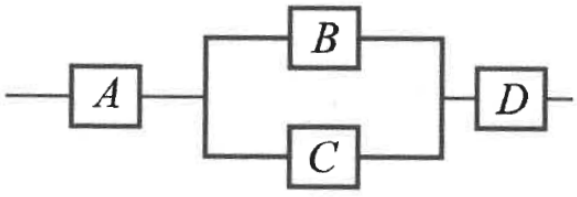
   (f) none occurs,

   (g) at most one occurs,

   (h) at most two occur.

3. It is well known $P(\emptyset) = 0$. Now an event A is such that $P(A) = 0$; ask whether it is true that $A = \emptyset$? If yes, please give a reasonable explanation; otherwise, find a counterexample.

4. Four components A, B, C and D constitute a series-parallel circuit as in the following figure. Let A, B, C and D be the events that the corresponding components work well, respectively.



   (a) Express the event that the circuit works in order in terms of A, B, C and D.

   (b) Express the event that the circuit works out of order in terms of A^c, B^c, C^c and D^c.

5. Suppose that A and B are mutually exclusive events (that is, $A \cap B = \emptyset$) for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that

   (a) either A or B occurs?

   (b) A occurs but B does not?

   (c) both A and B occur?

6. Let $\Omega = \{1, 2, 3\}$ and $\mathcal{A}$ be a σ-algebra over Ω such that $\{1\} \in \mathcal{A}$ and $\{2\} \notin \mathcal{A}$. Write down all elements of $\mathcal{A}$. Explain why $\{1, 2\} \notin \mathcal{A}$.

7. A traditional fair dice is thrown twice. What is the probability that:

   (a) a six turns up exactly once?

   (b) both numbers are odd?

   (c) the sum of the scores is 4?

   (d) the sum of the scores is divisible by 3?