Chapter Fifteen

The Term Structure of Interest Rates

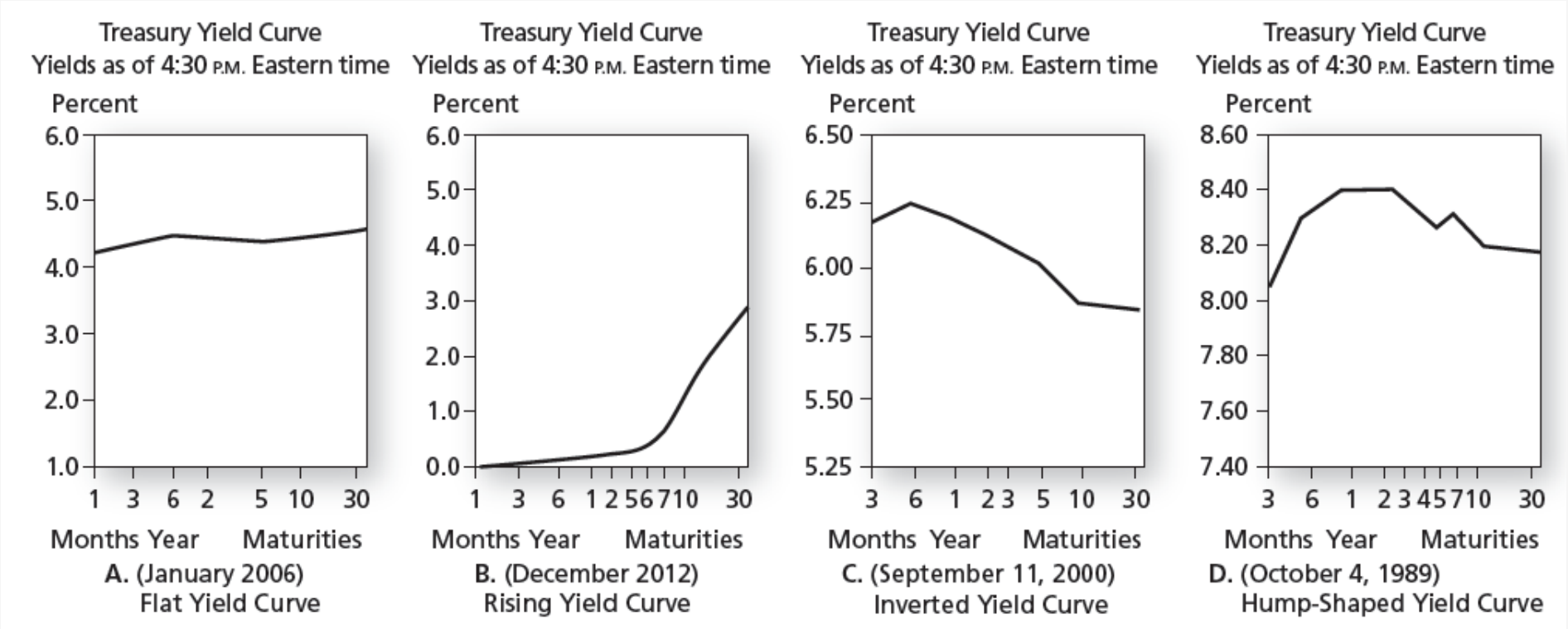
Chapter Overview

- Explore pattern of interest rates for different-term assets
  - Identify factors that account for pattern and determine what information may be gleaned from an analysis of the **term structure of interest rates**
- Demonstrate how prices of Treasury bonds may be derived from prices and yields of stripped zero-coupon Treasury securities
- Examine extent to which term structure reveals market-consensus forecasts of future interest rates

The Yield Curve

- Relationship between yield and maturity may be shown graphically in a **yield curve**
  - Yield curve is a plot of yield to maturity as a function of time to maturity
- Key concern of fixed-income investors
- Central to bond valuation
  - Allows investors to gauge their expectations for future interest rates against those of the market

Treasury Yield Curves



Yield Curve: Bond Pricing

- If yields on different maturity bonds are not equal, how should we value coupon bonds that make payments at many different times?
- The trick is to consider cash flow of each bond as a stand-alone zero-coupon bond
- The value of the bond should be the sum of the values of its parts

Prices and Yields to Maturities on Zero-Coupon Bonds ($1,000 Face Value)

Maturity (years)	Yield to Maturity (%)	Price
1	5%	$952.38 = $1,000/1.05
2	6	$890.00 = $1,000/$1.06^2$
3	7	$816.30 = $1,000/$1.07^3$
4	8	$735.03 = $1,000/$1.08^4$

Valuing Coupon Bonds

- Value a 3-year, 10% coupon bond using discount rates from Table 15.1:

$$\text{Price} = \frac{\$100}{1.05} + \frac{\$100}{1.06^2} + \frac{\$1100}{1.07^3}$$

  - Price = $1,082.17 and YTM = 6.88%
  - 6.88% is less than the 3-year rate of 7%

Bond Pricing: Two Types of Yield Curves

Pure Yield Curve

- Refers to the curve for stripped, or zero-coupon, Treasuries
- May differ significantly from the on-the-run yield curve

On-the-Run Yield Curve

- Refers to the plot of yield as a function of maturity for recently issued coupon bonds selling at or near par value
- Typically published by the financial press

The Yield Curve and Future Interest Rates

- Why is the curve sometimes upward-sloping and other times downward-sloping?
- How do expectations for the evolution of interest rates affect the shape of today's yield curve?
- These questions do not have simple answers, so we will begin with an idealized framework, and then extend to more realistic settings.

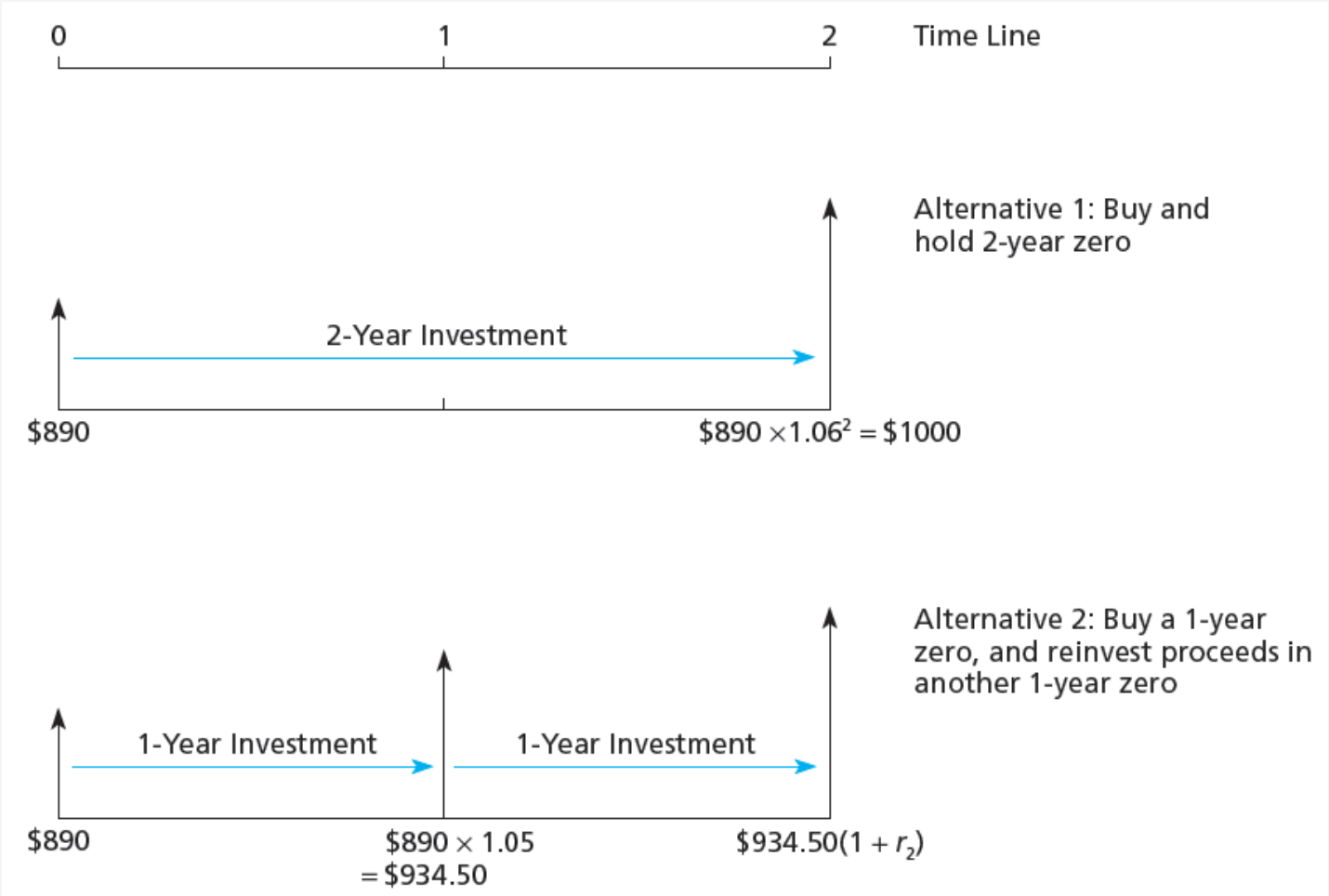
The Yield Curve under Certainty

- If interest rates are certain, what should we make of the fact that the yield on the 2-year zero coupon bond is greater than that on the 1-year zero?
- The upward-slowing yield curve is evidence that short-term rates are going to be higher next year than they are now.

The Yield Curve under Certainty

- Consider two 2-year bond strategies:
  1. Buy the 2-year zero offering a 2-year YTM of 6% and hold it until maturity
     - Face value is $1,000, so it is purchased today for $\$1{,}000/(1.06)^2 = \$890$ and matures in two years to $1,000
     - Total 2-year growth factor is $1,000/$890 = 1.1236
  2. Invest the same $890 in a 1-year zero-coupon bond with a YTM of 5% and upon maturity reinvest the proceeds in another 1-year bond

Two 2-Year Investment Programs



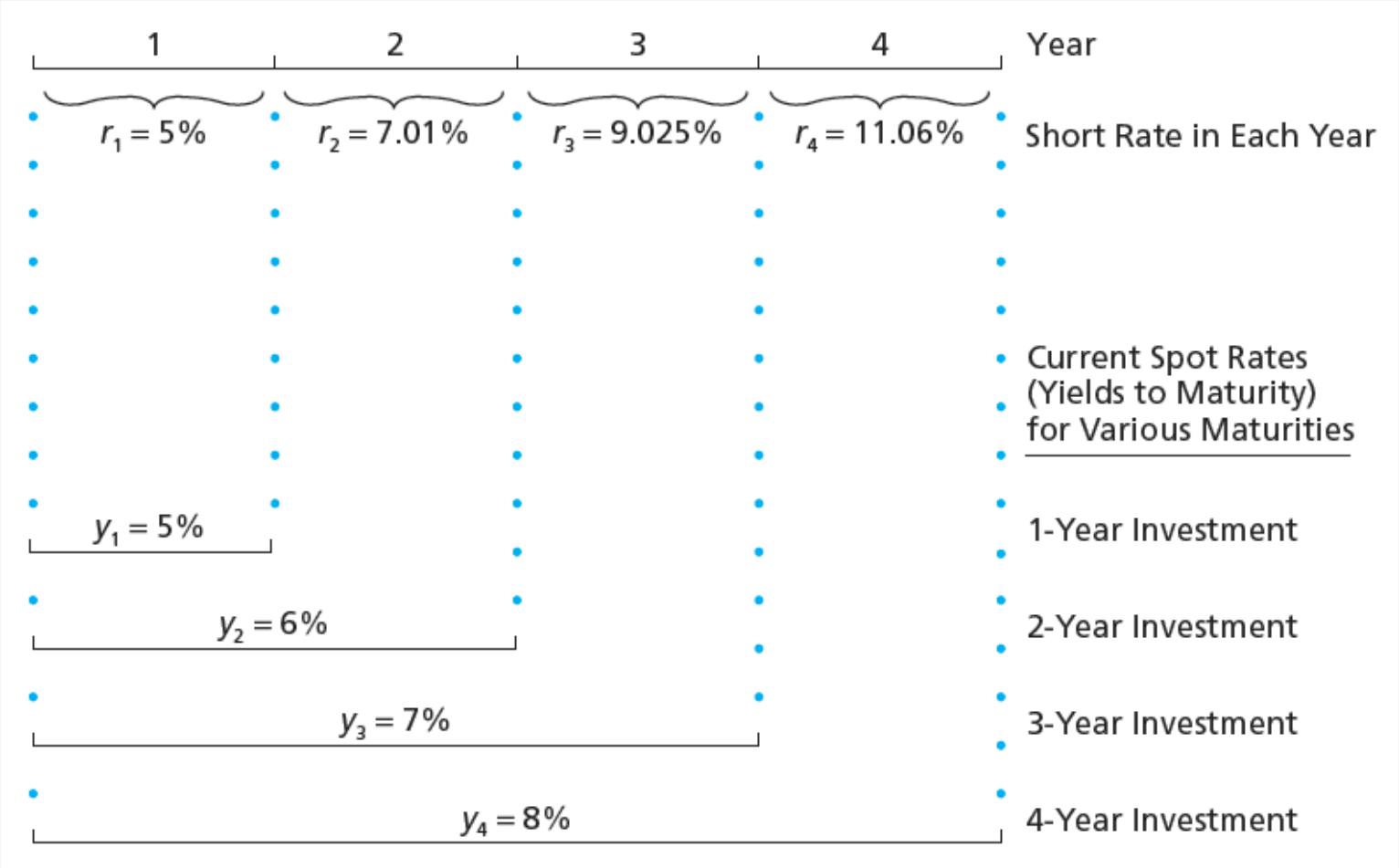
Spot Rates and Short Rates

- To distinguish between yields on long-term bonds versus short-term rates that will be available in the future, practitioners use the following terminology.
- **Spot rate**
  - The rate that prevails *today* for a time period corresponding to the zero's maturity
- **Short rate**
  - Applies for a given time interval (e.g., one year)
  - Refers to the interest rate for that interval available at different points in time
  - In the example, the short rate today is 5%, and the short rate next year will be $r_2 = 7.01\%$.

Spot Rates and Short Rates

- $(1 + y_2)^2 = (1 + r_1)(1 + r_2)$
- When next year's short rate, r_2, is greater than this year's short rate, r_1, $y_2 > r_1$ and the yield curve slopes upward.
- If next year's short rate were less than r_1, the yield curve would slope downward.
- Thus, the yield curve in part reflects the market's assessments of coming interest rates.

Short Rates versus Spot Rates



Forward Rates

- The following equation generalizes our approach to inferring a future short rate from the yield curve of zero-coupon bonds.
- It equates the total return on two n-year investment strategies:
  - Buying and holding an n-year zero coupon bond
  - Buying an (n-1)-year zero and rolling over the proceeds into a 1-year bond

Forward Rates

$$(1 + r_n) = \frac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}}$$

- r_n = short rate in year n
- y_n = YTM of a zero-coupon bond with an n-period maturity

$$(1 + y_n)^n = (1 + y_{n-1})^{n-1} \times (1 + r_n)$$

Forward Rates Continued

- Recognizing that future interest rates are uncertain, we call the interest rate that we infer in this matter the **forward interest rate** rather than the *future short rate*
  - because it need not be the interest rate that actually will prevail at the future date.
  - Rate for 4-year maturity = 8%, rate for 3-year maturity = 7%

$$1+f_4=\frac{(1+y_4)^4}{(1+y_3)^3}=\frac{1.08^4}{1.07^3}=1.1106 \qquad f_4=11.06\%$$

Exercise

The following is a list of prices for zero-coupon bonds of various maturities.

a. Calculate the yield to maturity for a bond with a maturity of (i) one year; (ii) two years; (iii) three years; (iv) four years.

b. Calculate the forward rate for (i) the second year; (ii) the third year; (iii) the fourth year.

Maturity (years)	Price of Bond
1	$943.40
2	898.47
3	847.62
4	792.16