MATH2033 Mathematical Statistics
Assignment 2

Due Date: **10/Mar/2024(Sunday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.
- **Late submissions**, **answers without details**, or **unrecognizable handwritings** will NOT be graded.

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1. Let X and Y have a bivariate normal distribution with parameters $\mu_1 = \mu_2 = 0$, $\sigma_1^2 = \sigma_2^2 = 1$, and correlation coefficient ρ. Find the distribution of the random variable $Z = aX + bY$ in which a and b are nonzero constants.

2. Suppose $\mathbf{X}$ is distributed $N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Determine the distribution of the random vector $(X_1 + X_2, X_1 - X_2)$. Show that $X_1 + X_2$ and $X_1 - X_2$ are independent if $\mathrm{Var}(X_1) = \mathrm{Var}(X_2)$.

3. Suppose $\mathbf{X}$ is distributed $N_3(\mathbf{0}, \boldsymbol{\Sigma})$, where
$$\boldsymbol{\Sigma} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$
Find $P((X_1 - 2X_2 + X_3)^2 > 15.36)$.

4. Let X follow a gamma$(r/2, \beta)$ distribution, where r is a positive integer and β is a positive constant. Let $Y = 2\beta X$. What is the distribution of Y?

5. Let F have an F-distribution with parameters r_1 and r_2. Argue that $1/F$ has an F-distribution with parameters r_2 and r_1.

6. If F has an F-distribution with parameters $r_1 = 5$ and $r_2 = 10$, find a and b so that $P(F \leq a) = 0.05$ and $P(F \leq b) = 0.95$, and, accordingly, $P(a < F < b) = 0.90$.

7. Let $T = W/\sqrt{V/r}$, where the independent variables W and V are, respectively, normal with mean zero and variance 1 and chi-square with r degrees of freedom. Show that T^2 has an F-distribution with parameters $r_1 = 1$ and $r_2 = r$.
   Hint: What is the distribution of the numerator of T^2?