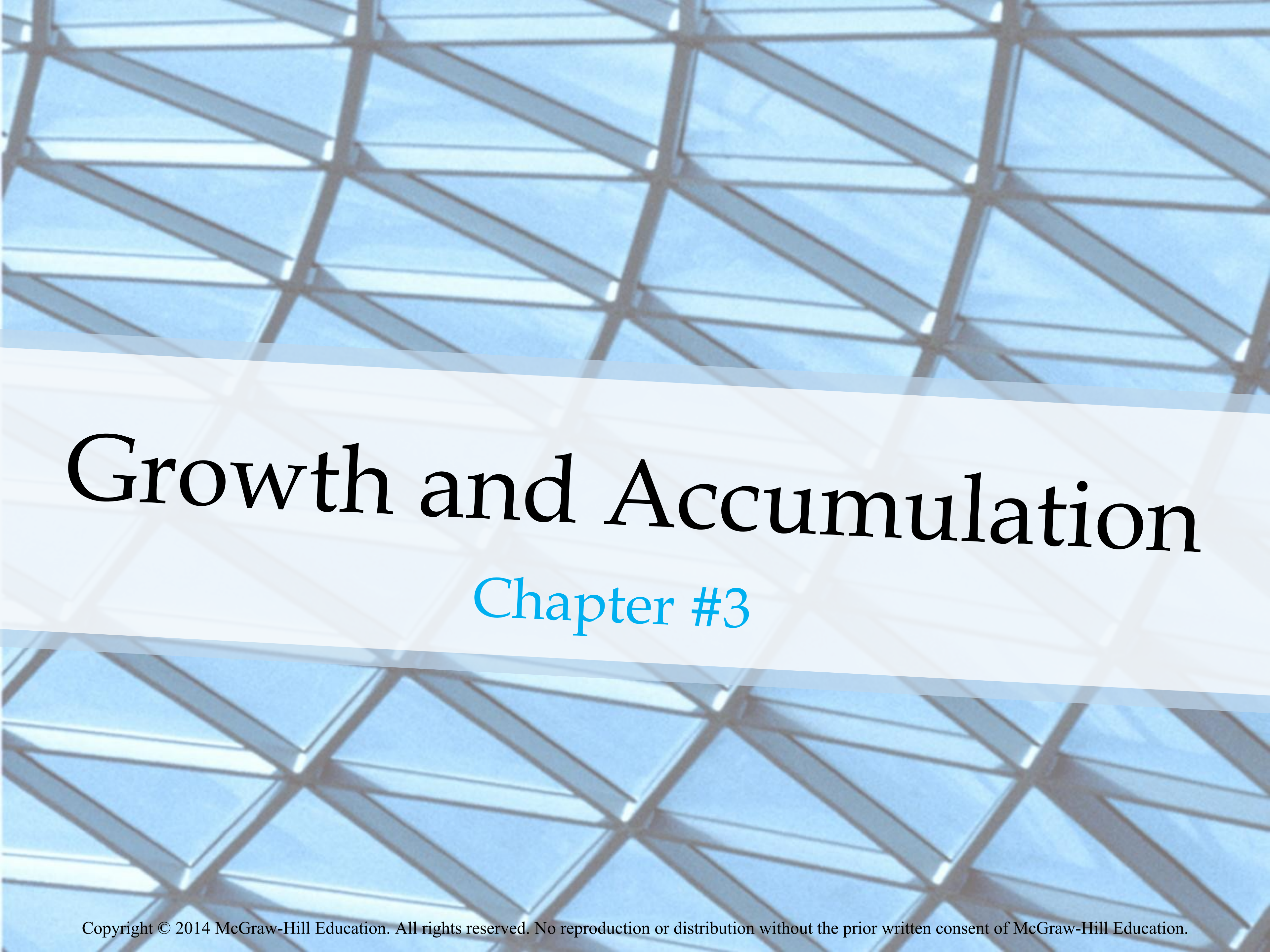

Growth and Accumulation
Chapter #3

Introduction

- Per capita GDP (income per person) increasing over time in industrialized nations, yet stagnant in many developing nations (Ex. U.S. vs. Ghana)
- Growth accounting explains what part of growth in total output is due to growth in different factors of production
- Growth theory explains how economic decisions determine the accumulation of factors of production
  - Ex. How does the rate of saving today affect the stock of capital in the future?

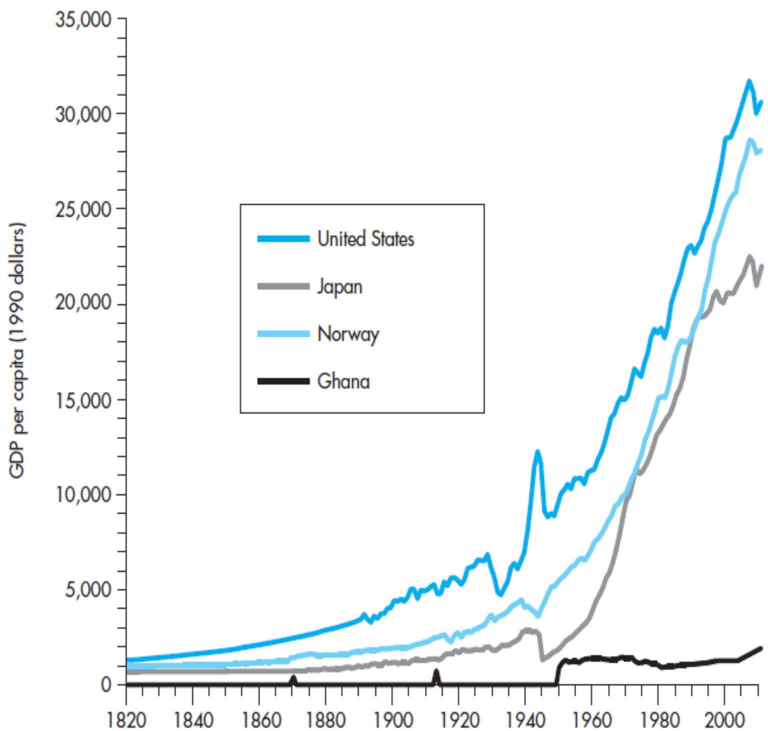


FIGURE 3-1 GDP PER CAPITA FOR FOUR COUNTRIES, 1820–2010.

The Production Function

- The production function defines relationship between inputs and output (Y)
- Use the production function to study two sources of output growth:
  1. Increases in inputs (N, K)
  2. Increases in productivity (technology)

- If N and K are the only inputs, the production function is $Y = AF(K, N)$ (1), Y depends upon inputs and technology (A)
  - An increase in A = increase in productivity $\rightarrow$ output increases for given level of inputs N and K
  - Assume MPN and MPK > 0, so that an increase in inputs $\rightarrow$ increase in output

Growth Accounting Equation

- Equation (1) relates the level of output to the level of inputs and technology
- Transform the production function into growth rate form to show the relationship between input growth and output growth
  - The growth accounting equation is:

$$\frac{\Delta Y}{Y} = \underbrace{(1-\Theta)\times\frac{\Delta N}{N}}_{Nshare \times Ngrowth} + \underbrace{\Theta\times\frac{\Delta K}{K}}_{Kshare \times Kgrowth} + \underbrace{\frac{\Delta A}{A}}_{tech.progress} \quad (2)$$

  - Growth rates of K and N are weighted by their respective income shares, so that each input contributes an amount equal to the product of the input's growth rate and their share of income to output growth

Growth Accounting: Examples

- If $\Theta = 0.25, (1-\Theta) = 0.75$, the growth rates of N and K are 1.2% and 3% respectively, and the rate of technological progress is 1.5%, then output growth is:

$$\frac{\Delta Y}{Y} = (0.75 \times 1.2\%) + (0.25 \times 3\%) + 1.5\% = 3.15\%$$

- Since labor share is greater than capital share, a 1% point increase in labor increases output by more than a 1% point increase in capital
- Suppose the growth rate of capital doubles from 3% to 6%. What is the growth rate of output?

$$\frac{\Delta Y}{Y} = (0.75 \times 1.2\%) \times (0.25 \times 6\%) + 1.5\% = 3.9\%$$

$\rightarrow$ Output increases by less than a percentage point after a 3% point increase in the growth rate of capital

- If the growth rate of labor doubled to 2.4% instead, output growth would increase from 3.15% to 4.05%

Growth In Per Capita Output

- Important to consider per capita output/income since total values might be misleading if population is large (total output can be large even though per capita output/income is low)
  - Income for an average person is estimated by GDP per capita,

    $\rightarrow$ used as an estimate for individual standard of living
- Traditional to use lower case letters for per capita values $\rightarrow$

  $y \equiv \frac{Y}{N}, k \equiv \frac{K}{N}$ where k is the capital-labor ratio
- Additionally, $\frac{\Delta Y}{Y} = \frac{\Delta y}{y} + \frac{\Delta N}{N}, \frac{\Delta K}{K} = \frac{\Delta k}{k} + \frac{\Delta N}{N}$

Growth Accounting Equation In Per Capita Terms

- To translate the growth accounting equation into per capita terms, subtract the population growth rate from both sides of equation (2) and rearrange terms:

$$
\begin{aligned}
\frac{\Delta Y}{Y} - \frac{\Delta N}{N} &= (1-\Theta)\frac{\Delta N}{N} - \frac{\Delta N}{N} + \Theta\frac{\Delta K}{K} + \frac{\Delta A}{A} \\
&= \frac{\Delta N}{N} - \Theta\frac{\Delta N}{N} - \frac{\Delta N}{N} + \Theta\frac{\Delta K}{K} + \frac{\Delta A}{A} \\
&= -\Theta\frac{\Delta N}{N} + \Theta\frac{\Delta K}{K} + \frac{\Delta A}{A} \qquad (3) \\
&= \Theta\left(\frac{\Delta K}{K} - \frac{\Delta N}{N}\right) + \frac{\Delta A}{A}
\end{aligned}
$$

Growth Accounting Equation In Per Capita Terms

- If $\frac{\Delta y}{y} = \frac{\Delta Y}{Y} - \frac{\Delta N}{N}, \frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta N}{N}$, then the growth accounting equation becomes:

$$\frac{\Delta y}{y} = \Theta \frac{\Delta k}{k} + \frac{\Delta A}{A} \quad (4)$$

Per Capita Growth

TABLE 3-1 Postwar Annual Growth Rates
(Percent)

	GDP PER CAPITA			NONRESIDENTIAL CAPITAL STOCK PER CAPITA		
	UNITED STATES	JAPAN	DIFFERENCE	UNITED STATES	JAPAN	DIFFERENCE
1950–1973	2.42	8.01	5.59	1.78	7.95	6.17
1973–1992	1.38	3.03	1.65	2.12	6.05	3.93
1950–1992	1.95	5.73	3.78	1.93	7.09	5.16

Source: Angus Maddison, *Monitoring the World Economy 1820–1992* (Paris: Organization for Economic Cooperation and Development, 1995); and authors' calculations.

Factors Other Than N and K: Human Capital

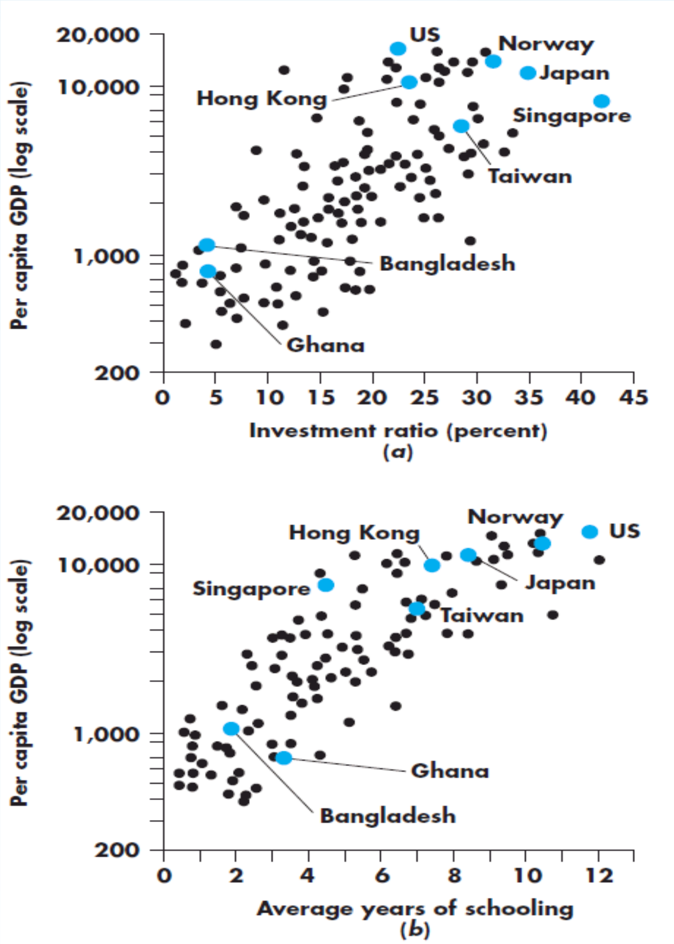
- The production functions shown omit a long list of inputs other than N and K
  - While N and K are the most important factors of production, others matter
- Investment in human capital (H) through schooling and on-the-job training is an important determinant of output in many economies
  - With the addition of H, the production function becomes

$$Y = AF(K, H, N) \quad (5)$$

  - Mankiw, Romer, and Weil (1992) suggest that H contributes equally to Y as K and N $\rightarrow$ factor shares all equal to 1/3

Factors Other Than N and K: Human Capital

- Figure 3-2 (a) illustrates a positive relationship between the rate of investment and per capita output and income across many selected nations
- Figure 3-2 (b) illustrates a similar relationship between human capital, using years of schooling as a proxy for H, and per capita output and income across many selected nations



Growth Theory: The Neoclassical Model

- Neoclassical growth theory focuses on K accumulation and its link to savings decisions (Robert Solow)
- Begin with a simplifying assumption: no technological progress $\rightarrow$ economy reaches a long run level of output and capital = steady state equilibrium
  - *The steady state equilibrium for the economy is the combination of per capita GDP and per capita capital where the economy will remain at rest, or where per capita economic variables are no longer changing*

    OR

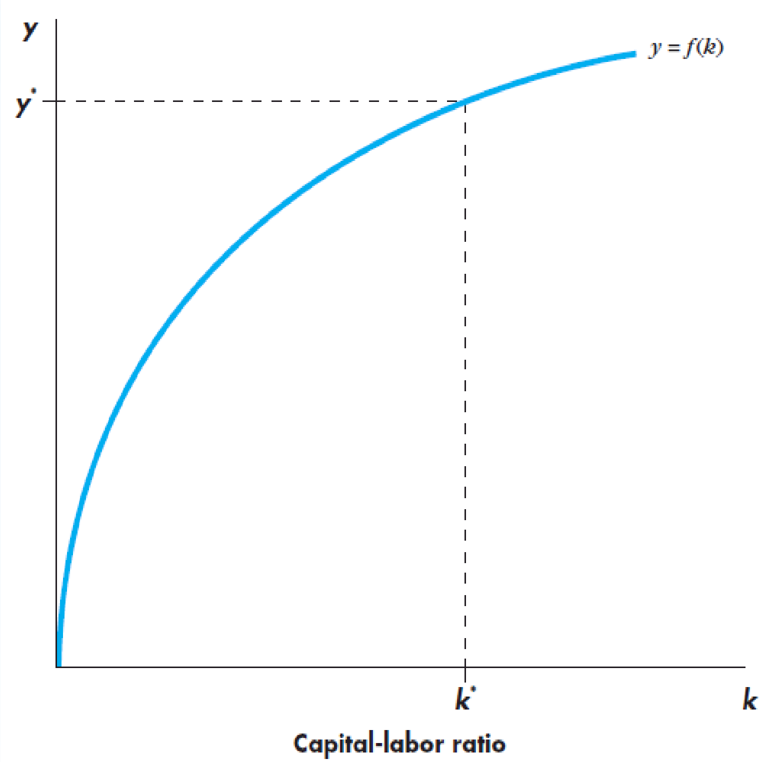
    $$\Delta y = 0, \Delta k = 0$$

Growth Theory: The Neoclassical Model

- Present growth theory in three broad steps:
  1. Examine the economic variables that determine the economy's steady state
  2. Study the transition from the economy's current position to the steady state
  3. Add technological progress to the model

Determinants of the Economy's Steady State

- The production function in per capita form is y = f(k) (6) and is depicted in Figure 3-3
  - As capital increases, output increases, but at a decreasing rate → diminishing MPK
- An economy is in a steady state when per capita income and capital are constant
  - Arrive at steady state when investment required to provide new capital for new workers and to replace worn out machines = savings generated by the economy



Savings and Investment

- The investment required to maintain a given level of k depends on the population growth rate and the depreciation rate (n and d respectively)
  - Assume population grows at a constant rate, $n = \frac{\Delta N}{N}$, so the economy needs nk of investment for new workers
  - Assume depreciation is a constant, d, of the capital stock, adding dk of needed investment
  - $\rightarrow$ The total required investment to maintain a constant level of k is (n+d)k
- If savings is a constant function of income, s, then per capital savings is sy
  - If income equals production, then sy = sf(k)

Solution for the Steady State

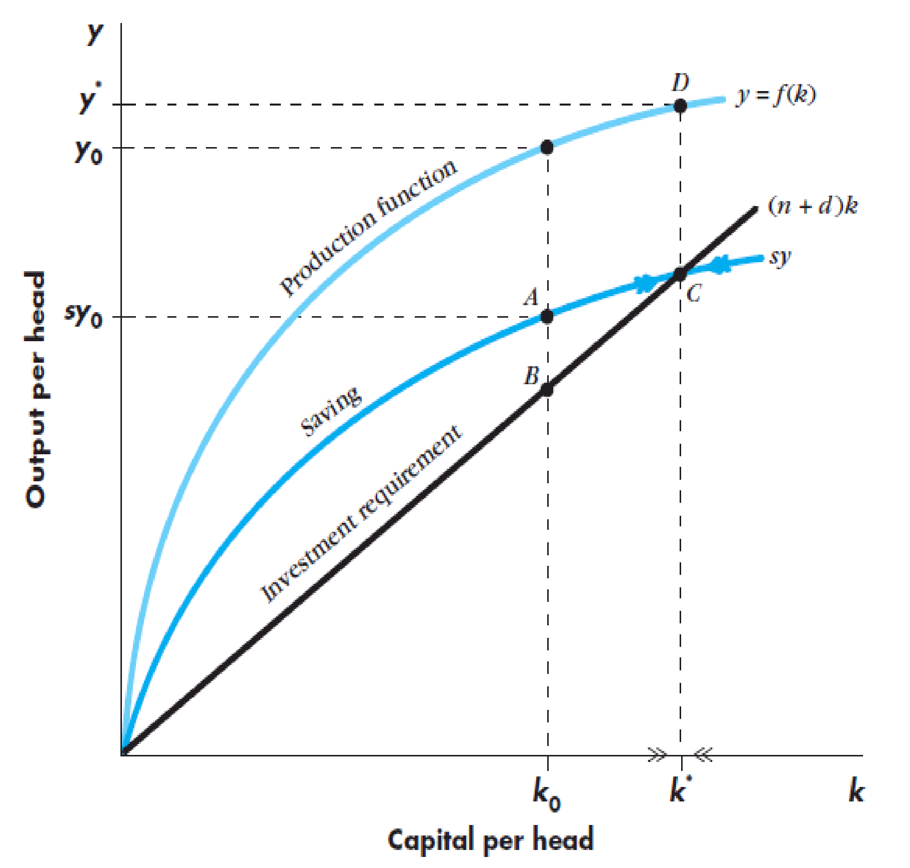
- Δk is the excess of saving over required I:

$$\Delta k = sy - (n+d)k \quad (7)$$

- Δk = 0 in the steady state and occurs at values of y* and k*, satisfying

$$sy^* = sf(k^*) = (n+d)k^* \quad (8)$$

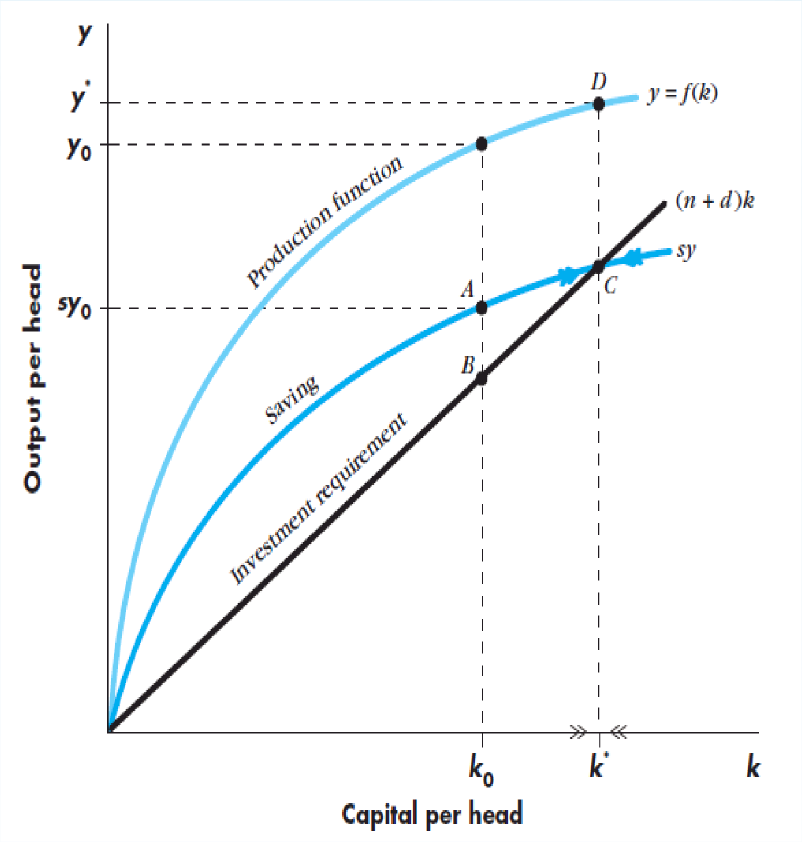
- In Figure 3-4, savings and required investment are equal at point C with a steady state level of capital k*, and steady state level of income y* at point D



The Growth Process

- The critical elements in the transition from the initial k to k* are the rate of savings and investment compared to the rate of population and depreciation growth
- Suppose start at k: $sy > (n+d)k$
  Savings exceeds the investment required to maintain a constant level of k

  $\rightarrow$ k increases until reach k* where savings equals required investment

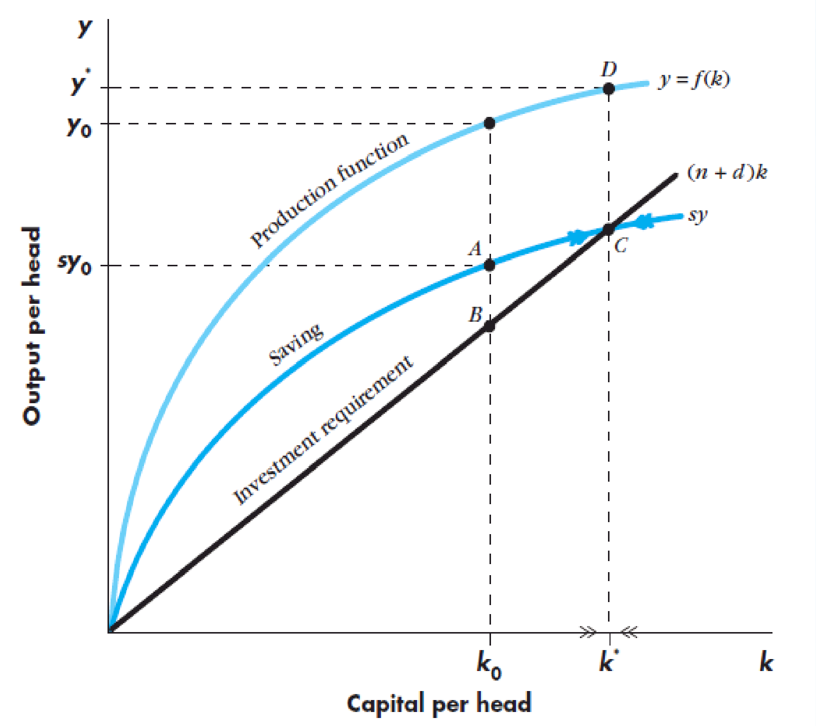


The Growth Process

Conclusions:

1. Countries with equal savings rates, rates of population growth, and technology should converge to equal incomes, although the convergence process may be slow
2. At the steady state, k and y are constant, so aggregate income grows at the same rate as the rate of population growth, n

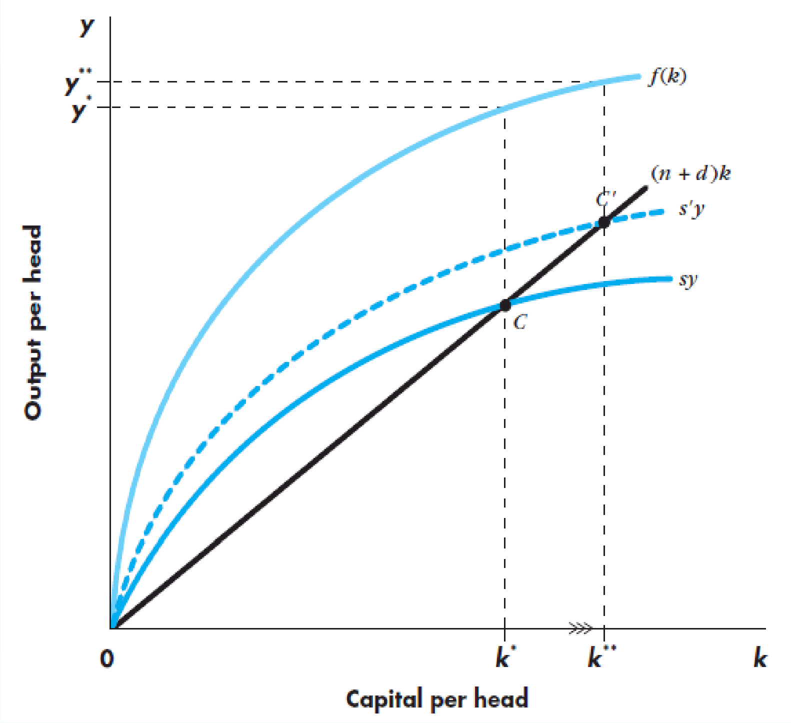
→ Steady state growth rate is not affected by s



An Increase in the Savings Rate

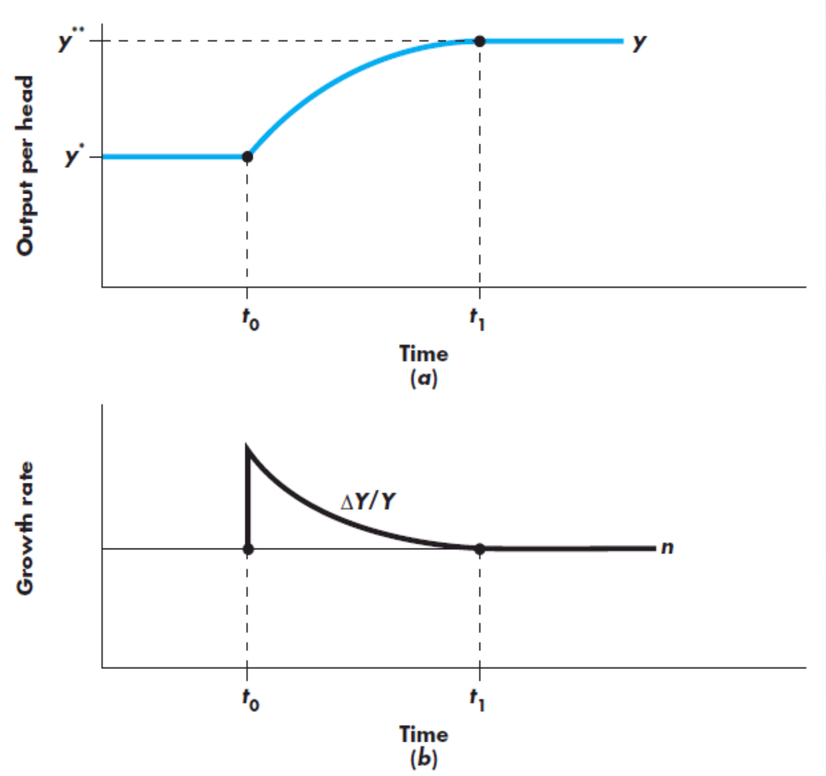
- According to neoclassical growth theory, savings does not affect the growth rate in the long run → WHY?
- Suppose savings rate increases from s to s′: $sy > (n+d)k$
  - When s increases, at k*, thus k increases to k** (and y to y**) at point C′
  - At point C′, the economy returns to a steady state with a growth rate of n

  → Increase in s will increase levels of y* and k*, but not the growth rate of y



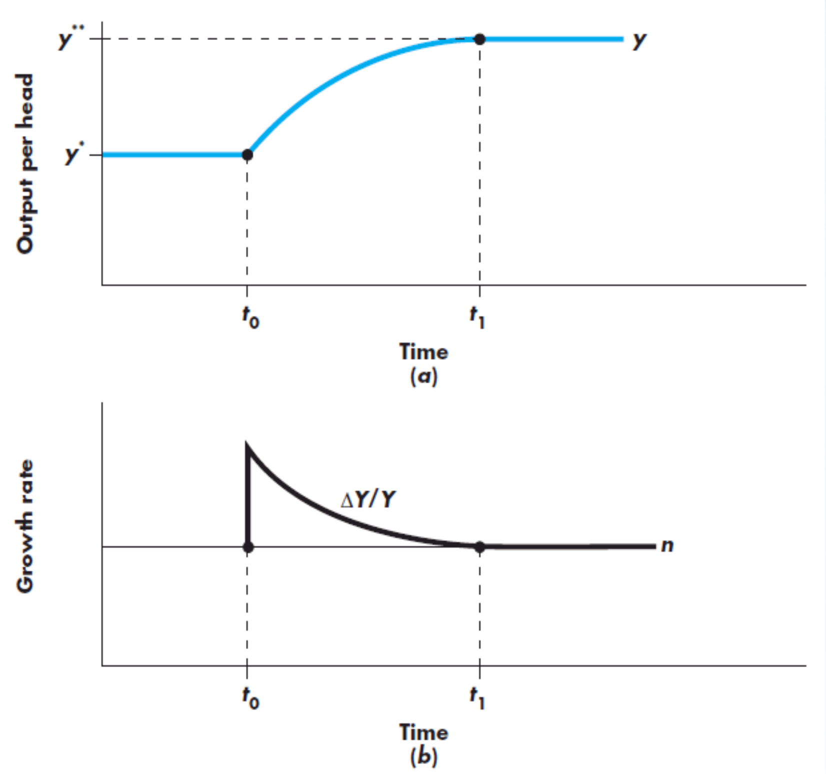
The Transition Process: s to s′

- In the transition process, the higher savings rate increases growth rate of output and growth rate of per capita output
  - k increases from k* to k** → k grows faster than the labor force and depreciation
- Figure 3-6 (a) shows the transition from y* to y** between t_0 and t_1
  - After the savings rate increases, so does savings and investment, resulting in an increase in k and y
  - Y continues to increase at a decreasing rate until reach new steady state at y**



The Transition Process: s to s′

- Higher savings rate increases the growth rate of output and the growth rate of per capita output
  - Follows from fact that k increases from k* to k** → only way to achieve an increase in k is for k to grow faster than the labor force and depreciation
- Figure 3-6 (b) illustrates the growth rate of Y between t_0 and t_1
  - The increase in s increases the growth rate of Y due to the faster growth in capital, $\frac{\Delta Y}{Y} > n$
  - As capital accumulates, the growth rate returns to n



Population Growth

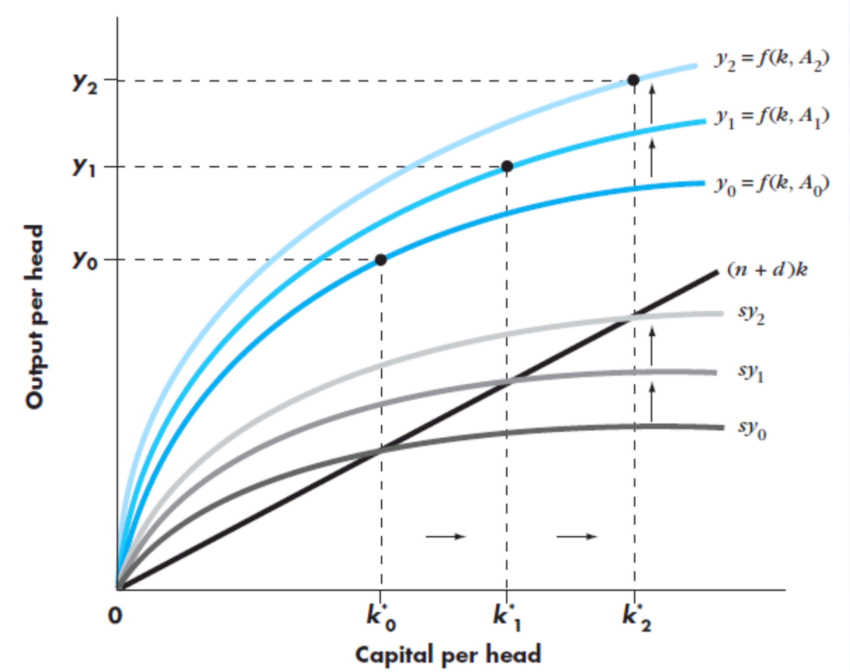
- An increase in the population growth rate is illustrated by an increase in $(n+d)k \rightarrow$ rotate line up and to the left
  - An increase in n reduces the steady state level of k and y
  - An increase in n increases the steady state rate of growth of aggregate output

$\rightarrow$ The decline in per capita output as a consequence of increased population growth is a phenomenon observed in many developing countries (discussed in Chapter 4)

- Conversely, a decrease in the population growth rate is illustrated by a decrease in $(n+d)k \rightarrow$ rotate line down and to the right
  - A decrease in n increases the steady state level of k and y
  - A decrease in n decreases the steady state rate of growth of aggregate output

Growth w/ Exogenous Technological Change

- Thus far have assumed technology is constant, $\frac{\Delta A}{A} = 0$ for simplicity, but need to incorporate to explain long term growth theory



- If rate of growth is defined as $g = \frac{\Delta A}{A}$, the production function, y = Af(k), increases at g percent per year (Fig. 3-7)
- Savings function grows in a parallel fashion, and y* and k* increase over time

How Is A Incorporated?

- The technology parameter can enter the production function in several ways:
  1. Technology can be labor augmenting, or new technology increases the productivity of labor → $Y = F(K, AN)$
     - Equation (4) becomes $\frac{\Delta y}{y} = \Theta \frac{\Delta k}{k} + (1-\Theta)\frac{\Delta A}{A}$ and y* and k* both increase at the rate of technological progress, g
  2. Technology can augment all factors, or represent total factor productivity → Y = AF(K,N)
     - Equation (4) is $\frac{\Delta y}{y} = \Theta \frac{\Delta k}{k}$ and $g = \frac{\Delta y}{y} - \Theta \frac{\Delta k}{k}$