ASP Assignment 3

1. The Smiths receive the paper every morning and place it on a pile after reading it. Each afternoon, with probability 1/3, someone takes all the papers in the pile and puts them in the recycling bin. Also, if ever there are at least five papers in the pile, Mr. Smith (with probability 1) takes the papers to the bin. Consider the number of papers in the pile in the evening. Is it reasonable to model this by a Markov chain? If so, what are the state space and transition matrix?

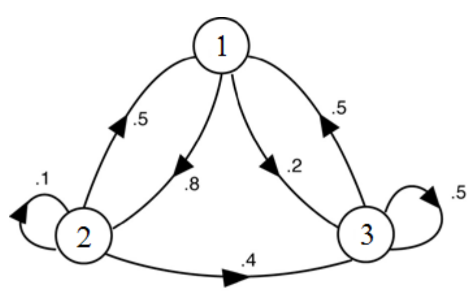
2. Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2.

   (a) Suppose we let the state at time n depend only on whether or not it is raining at time n. Explain why it is not a Markov chain.

   (b) We define that the process is in state 0 if it rained both today and yesterday, state 1 if it rained today but not yesterday, state 2 if it rained yesterday but not today, state 3 if it did not rain either yesterday or today. Find the transition probability matrix of this Markov chain.

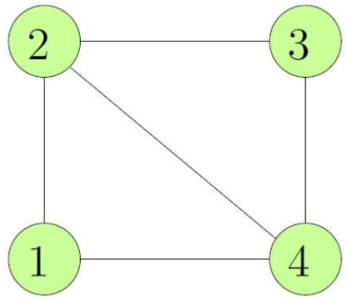
   (c) Suppose that it has rained neither yesterday nor the day before yesterday. What is the probability that it will rain tomorrow?

3. Consider the following Markov chain:



   Find $\boldsymbol{P}^n$.

4. Consider the following Markov chain.



At every step, a random walker at each state randomly walks, with equal probability, along an edge to another state if such edge exists. Find $\boldsymbol{P}^n$.