Chapter Five

Risk, Return, and the Historical Record

Chapter Overview

- Present tools for estimating expected returns and risk from the historical record
  - Interest rates and investments in safe assets
    - History of risk-free investments in the U.S.
  - Scenario analysis of risky investments and the data inputs necessary to conduct it

- Develop statistical tools needed to make inferences from historical time series of portfolio returns

Measuring Returns over Different Holding Periods

- We start with the simplest security, a zero-coupon bond:
  - A bond that pays its owner only one cash flow (*face value, $100 for example*) on the maturity date (*T*).
  - P(T): the price paid today for a zero-coupon bond with maturity date T.
  - Holding period return:

$$r(T) = \frac{100}{P(T)} - 1$$

Measuring Returns over Different Holding Periods

- How should we compare returns on investments with different horizons?
- Re-express each total return as a rate of return over a common period.
- Effective annual rate (EAR), defined as investment return per year

Measuring Returns over Different Holding Periods

Suppose prices of zero-coupon Treasuries with \$100 face value and various maturities are as follows. We find the total return of each security by using Equation 5.6:

Horizon, T	Price, $P(T)$	$[100/P(T)] - 1$	Total Return for Given Horizon
Half-year	\$97.36	$100/97.36 - 1 = 0.0271$	$r_f(0.5) = 2.71\%$
1 year	\$95.52	$100/95.52 - 1 = 0.0469$	$r_f(1) = 4.69\%$
25 years	\$23.30	$100/23.30 - 1 = 3.2918$	$r_f(25) = 329.18\%$

For the 6-month Treasury in Example 5.2, $T = ½$, and $1/T = 2$. Therefore,

$$1 + \text{EAR} = 1.0271^2 = 1.0549 \text{ and EAR} = 5.49\%$$

For the 25-year Treasury in Example 5.2, $T = 25$. Therefore,

$$1 + \text{EAR} = 4.2918^{1/25} = 1.060 \text{ and EAR} = 6.0\%$$

Measuring Returns over Different Holding Periods

- Rates on short-term investments (usually with holding periods less than a year) are in practice often annualized using simple interest that ignores compounding, called annual percentage rates, or APRs.

$$\text{APR} = n \times [(1 + \text{EAR})^{1/n} - 1]$$

Effective Annual Rate (EAR) and Annual Percentage Rate (APR)

- Effective annual rates (EAR) explicitly account for compound interest

$$1 + \text{EAR} = \left(1 + \frac{\text{APR}}{n}\right)^n$$

- Annual percentage rates (APR) are annualized using simple rather than compound interest

$$\text{APR} = n \times [(1 + \text{EAR})^{1/n} - 1]$$

Effective Annual Rate (EAR) and Annual Percentage Rate (APR)

Compounding Period	T	EAR $= [1 + r_f(T)]^{1/T} - 1 = 0.058$		APR $= r_f(T) \times (1/T) = 0.058$	
		$r_f(T)$	APR $= [(1 + \text{EAR})^T - 1]/T$	$r_f(T)$	EAR $= (1 + \text{APR} \times T)^{(1/T)} - 1$
1 year	1.0000	0.0580	0.05800	0.0580	0.05800
6 months	0.5000	0.0286	0.05718	0.0290	0.05884
1 quarter	0.2500	0.0142	0.05678	0.0145	0.05927
1 month	0.0833	0.0047	0.05651	0.0048	0.05957
1 week	0.0192	0.0011	0.05641	0.0011	0.05968
1 day	0.0027	0.0002	0.05638	0.0002	0.05971
Continuous			$r_{cc} = \ln(1 + \text{EAR}) = 0.05638$		EAR $= \exp(r_{cc}) - 1 = 0.05971$

Table 5.1

Annual percentage rates (APR) and effective annual rates (EAR). In the first set of columns, we hold the equivalent annual rate (EAR) fixed at 5.8% and find APR for each holding period. In the second set of columns, we hold APR fixed at 5.8% and solve for EAR.

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Continuous Compounding

- The difference between EAR and APR increases with the frequency of compounding.
- How far will these two rates diverge as the compounding frequency continues to grow?
- Given EAR, as n gets larger, we effectively approach *continuous compounding* (CC).

Continuous Compounding

- The relation of EAR to the annual percentage rate, denoted by r_{cc} for the continuously compounded case, is given by

$$1 + EAR = \exp(r_{cc}) = e^{r_{cc}}$$

$$r_{cc} = \ln(1 + EAR)$$

Continuous Compounding

- With r_{cc}, the total return for any period T, $r_{cc}(T)$ is simply $\exp(T * r_{cc})$. (why?)
- Far simpler than working with the exponents that arise using discrete period compounding.

Interest Rates and Inflation Rates

- Fundamental factors that determine the level of interest rates:
  1. Supply of funds from savers, primarily households
  2. Demand for funds from businesses to be used to finance investments in plant, equipment, and inventories
  3. Government's net demand for funds as modified by actions of the Federal Reserve Bank
  4. Expected rate of inflation

Real and Nominal Rates of Interest

- A guaranteed rate of return specified in dollars does not necessarily lock in the increase in what you can buy with your investment proceeds.

Real and Nominal Rates of Interest

- Suppose one year ago, you deposited $1000 in a 1-year bank deposit
- The (nominal) interest rate: 10%
- You are about to collect $1100 in cash
- What is the real return on your investment?
- Depends on what your money can buy today relative to what you could buy a year ago.

Real and Nominal Rates of Interest

- Suppose the rate of inflation (the percentage change in the CPI, denoted by i) is running at 6%
- Price of a loaf of bread: \$1 v.s. \$1.06
- Loaves you can buy: 1000/1 v.s. 1100/1.06 (1038)
- The rate at which your purchasing power has increased is therefore 3.8%

Real and Nominal Rates of Interest

- With a 10% interest rate, after you net out the 6% reduction in the purchasing power of money, you are left with a net increase in purchasing power of almost 4%.
- Thus we need to distinguish between a **nominal interest rate** and a **real interest rate**

Real and Nominal Rates of Interest

- A **nominal interest rate** is the growth rate of your money
- A **real interest rate** is the growth rate of your purchasing power

$$r_{nom} = \text{ Nominal Interest Rate}$$

$$r_{real} = \text{ Real Interest Rate}$$

$$i = \text{Inflation Rate}$$

$$r_{real} = \frac{r_{nom} - i}{1 + i}$$

$$Note: r_{real} \approx r_{nom} - i$$

Real and Nominal Rates of Interest

- Conventional fixed income investments promise a *nominal* rate of interest
- The real rate of return is risky because the future inflation is uncertain
- You can only infer the expected real rate on these investments with the expectation of the rate of inflation

Interest Rates and Inflation

- Investors care about the real returns
- When inflation is higher, we would expect higher nominal interest rates.

Interest Rates and Inflation

- Irving Fisher (1930) argued that the nominal rate ought to increase one-for-one with expected flation

- If $E(i)$ denotes current expectations of inflation, the Fisher hypothesis is

$$r_{nom} = r_{real} + E\left(i\right)$$

Interest Rates and Inflation

- Takeaways about Fisher hypothesis:
  1. When real rates are stable, changes in nominal rates ought to predict changes in inflation rates.
  2. The empirical validity of the Fisher equation will depend on how well market participants can predict inflation during their investment horizon.

Taxes and the Real Interest Rate

- Tax liabilities are based on *nominal* income and the tax rate determined by the investor's tax bracket

r_{nom} = Nominal Interest Rate

r_{real} = Real Interest Rate

i = Inflation Rate

t = Tax Rate

$$r_{nom} \times (1-t) - i = (r_{real} + i) \times (1-t) - i = r_{real}(1-t) - i \times t$$

- After-tax return falls by the tax rate times the inflation rate

Bills and Inflation, 1926-2018

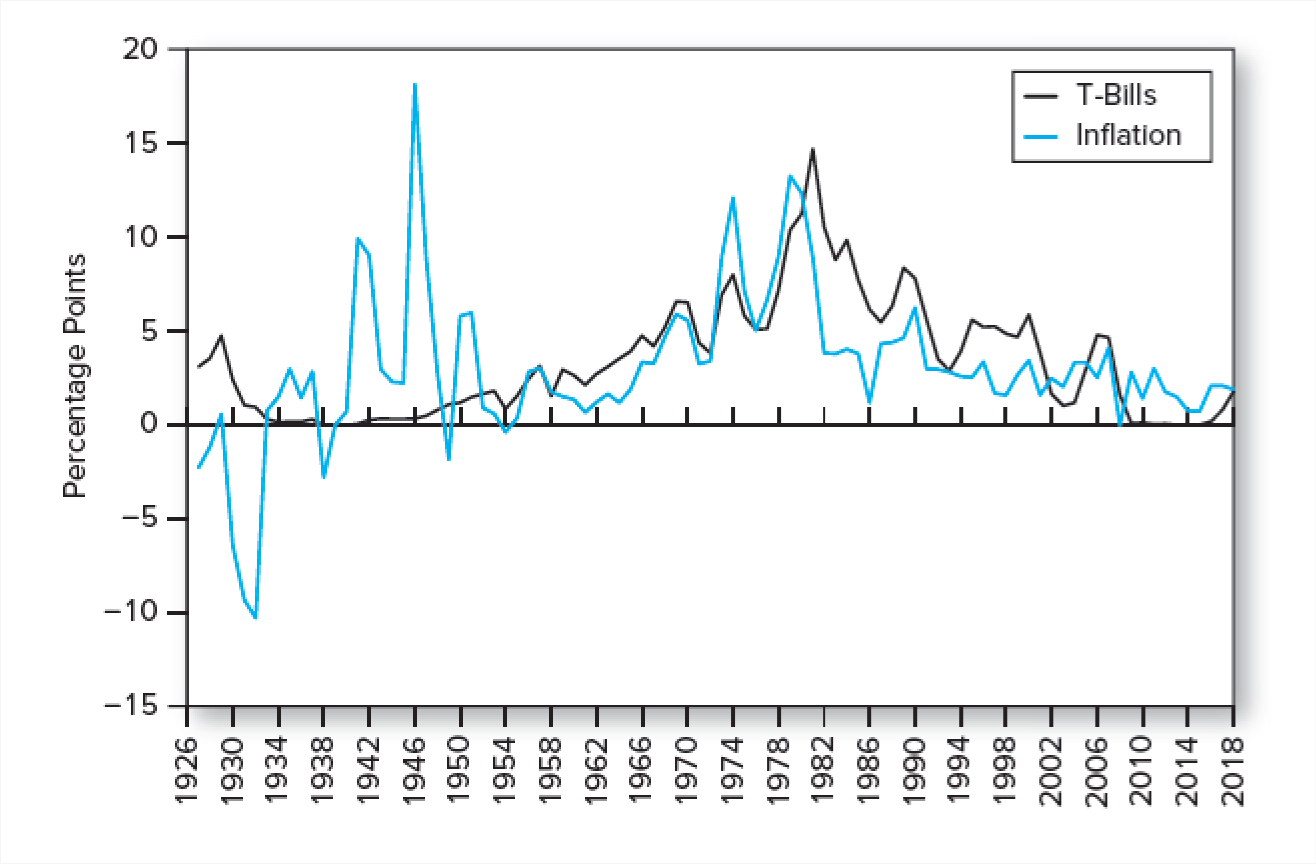
- Fisher equation
  - Predicts the nominal rate of interest should track the inflation rate, leaving the real rate somewhat stable
  - Appears to work far better when inflation is more predictable and investors can more accurately gauge the nominal interest rate they require to provide an acceptable real rate of return

T-Bill Rates, Inflation Rates, and Real Rates, 1926-2018

	Average Annual Rates			Standard Deviation		
	T-Bills	Inflation	Real T-Bill	T-Bills	Inflation	Real T-Bill
Full sample	3.38	3.01	0.46	3.12	4.00	3.76
1927–1951	0.95	1.79	−0.47	1.24	6.04	6.33
1952–2018	4.29	3.46	0.81	3.13	2.84	2.12

Source: Annual rates of return from rolling over 1-month T-bills: Kenneth French; annual inflation rates: Bureau of Labor Statistics.

Interest Rates and Inflation, 1926-2018



Risk and Risk Premiums:
Holding Period Returns

- Sources of investment risk
  - Macroeconomic fluctuations
  - Changing fortunes of various industries
  - Firm-specific unexpected developments
- *Holding period return (HPR),* or realized rate of return, is based on the price per share at year's end and any cash dividends collected

$$\text{HPR} = \frac{\text{Ending price of a share} - \text{Beginning price} + \text{Cash dividend}}{\text{Beginning price}}$$

Risk and Risk Premiums:
Expected Return and Standard Deviation (1 of 2)

- The list of possible HPRs along with the associated probabilities is called the probability distribution of the HPR
- Expected returns

$$E(r) = \sum_s p(s) \times r(s)$$

  - $p(s)$ = probability of each scenario
  - $r(s)$ = HPR in each scenario
  - s = scenario

Risk and Risk Premiums:
Expected Return and Standard Deviation (2 of 2)

- Variance (VAR):

$$\sigma^2 = \sum_s p(s) \times \left[r(s) - E(r) \right]^2$$

- Standard Deviation (STD):

$$\text{STD} = \sqrt{\sigma^2}$$

Risk and Risk Premiums

	A	B	C	D	E	F	G	H	I
1									
2									
3	Purchase Price =		$100			T-Bill Rate =	0.04		
4									
5							Squared		Squared
6	State of the		Year-End	Cash		Deviations	Deviations	Excess	Deviations
7	Market	Probability	Price	Dividends	HPR	from Mean	from Mean	Returns	from Mean
8	Excellent	0.25	126.50	4.50	0.3100	0.2124	0.0451	0.2700	0.0451
9	Good	0.45	110.00	4.00	0.1400	0.0424	0.0018	0.1000	0.0018
10	Poor	0.25	89.75	3.50	–0.0675	–0.1651	0.0273	–0.1075	0.0273
11	Crash	0.05	46.00	2.00	–0.5200	–0.6176	0.3815	–0.5600	0.3815
12	Expected Value (mean)			SUMPRODUCT(B8:B11, E8:E11) =	0.0976				
13	Variance of HPR					SUMPRODUCT(B8:B11, G8:G11) =	0.0380		
14	Standard Deviation of HPR					SQRT(G13) =	0.1949		
15	Risk Premium						SUMPRODUCT(B8:B11, H8:H11) =	0.0576	
16	Standard Deviation of Excess Return							SQRT(SUMPRODUCT(B8:B11, I8:I11)) =	0.1949

Spreadsheet 5.1

Scenario analysis of holding-period return of the stock-index fund

Risk and Risk Premiums:
Excess Returns and Risk Premiums

- **Risk premium** is the difference between the *expected* HPR and the **risk-free rate**
  - Provides compensation for the risk of an investment
- **Risk-free rate** is the rate of interest that can be earned with certainty
  - Commonly taken to be the rate on short-term T-bills
- Difference between actual rate of return and risk-free rate is called **excess return**
- **Risk aversion** dictates the degree to which investors are willing to commit funds to stocks

Learning from Historical Returns

Expected Returns and the Arithmetic Average

- When using historical data, each observation is treated as an equally likely "scenario"
- Expected return, *E(r)*, is estimated by arithmetic average of sample rates of return

$$E(r) = \sum_{s=1}^{n} p(s)r(s) = \frac{1}{n}\sum_{s=1}^{n} r(s)$$

$$= \text{Arithmetic average of historic rates of return}$$

Geometric (Time-Weighted) Average Return

- Geometric rate of return
  - Intuitive measure of performance over the sample period is the (fixed) annual HPR that would compound over the period to the same terminal value obtained from the sequence of actual returns in the time series

$$(1 + g)^n = \text{Terminal value}$$

$$g = \text{Terminal value}^{1/n} - 1$$

Learning from Historical Returns:
Variance and Standard Deviation

- Estimated variance
  - Expected value of squared deviations

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{s=1}^{n}\left[r(s)-\bar{r}\right]^2$$

- Unbiased estimated standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left[r(s)-\bar{r}\right]^2}$$

Learning from Historical Returns:
The Reward-to-Volatility (Sharpe) Ratio

- Investors price risky assets so that the risk premium will be commensurate with the risk of expected excess returns
  - Best to measure risk by the standard deviation of excess, not total, returns
- Sharpe ratio
  - Evaluates performance of investment managers

$$\text{Sharpe ratio} = \frac{\text{Risk premium}}{\text{SD of excess return}}$$

The Normal Distribution
(1 of 2)

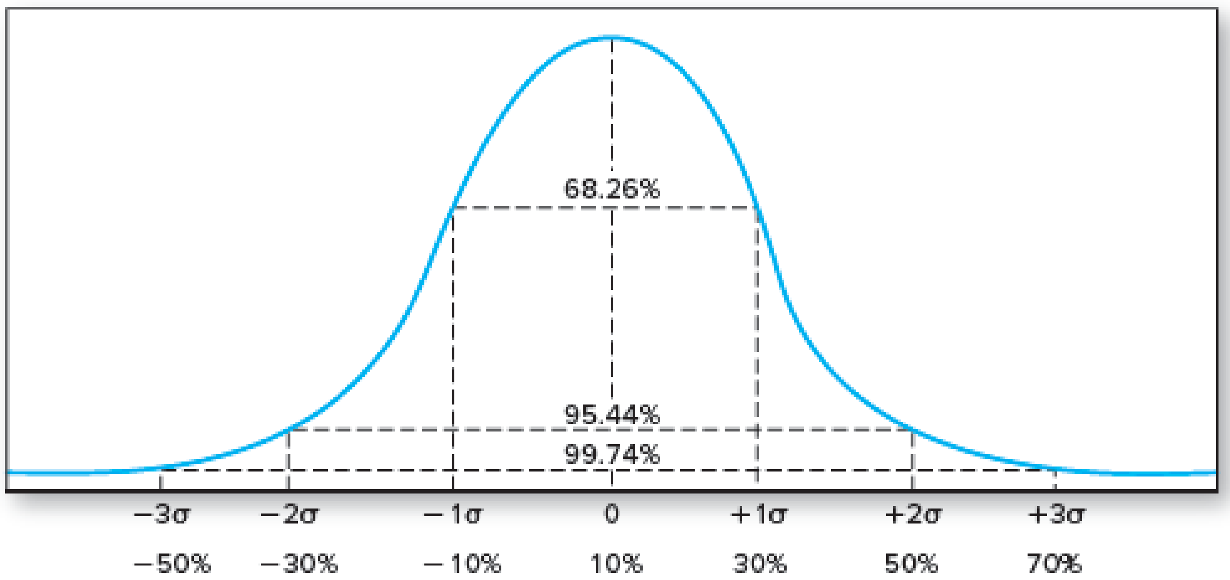


Figure 5.3 The normal distribution with mean 10% and standard deviation 20%.

The Normal Distribution
(2 of 2)

- Normal distribution is a bell-shaped probability distribution that characterizes many natural phenomena
  - E.g., heights and weights of newborns, test scores, etc.
- Investment management is more manageable when returns can be well approximated by the normal distribution
  - Symmetric
  - Stable
  - Only mean and standard deviation are needed to estimate future scenarios
  - Statistical relation between returns can be summarized with a single correlation coefficient

Deviations from Normality and Tail Risk

- Normality of excess returns hugely simplifies portfolio selection:
  1. Normality assures us that standard deviation is a complete measure of risk
  2. Hence, the Sharpe ratio is a complete measure of portfolio performance
- However, deviations from normality of asset returns are potentially significant and dangerous to ignore!

Deviations from Normality and Tail Risk

Skewness

- Standard measure of asymmetry in the probability distribution of returns is called the *skew of the distribution*
  - SD (Sharpe ratio) no longer a complete measure of risk (performance)

$$Skew = Average\left[\frac{(R-\bar{R})^3}{\hat{\sigma}^3}\right]$$

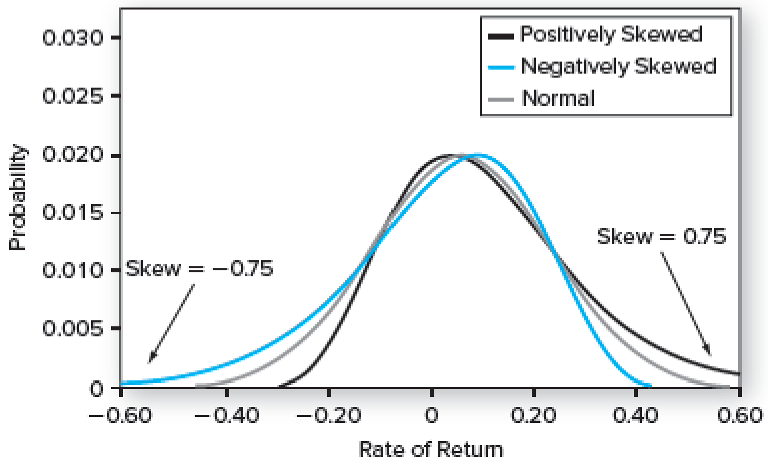
Kurtosis

- **Kurtosis** concerns the likelihood of extreme values on *either* side of the mean at the expense of a smaller likelihood of moderate deviations
  - Measures the degree of fat tails

$$Kurtosis = Average\left[\frac{(R-\bar{R})^4}{\hat{\sigma}^4}\right]-3$$

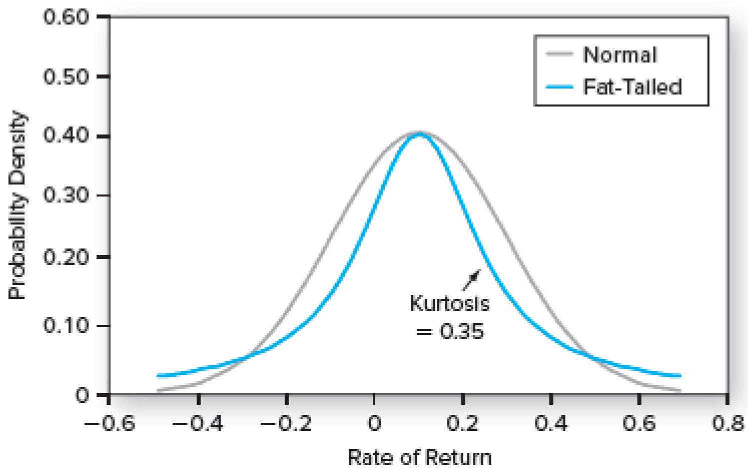
Skewness and Kurtosis

Normal and Skewed Distributions



(mean = 6%, SD = 17%)

Normal and Fat-Tailed Distributions



(mean = .1, SD = .2)

Normality and Risk Measures:
Downside Risk

- Measures of downside risk
  - **Value at risk (VaR)**
    - Loss that will be incurred in the event of an extreme adverse price change with some given, usually low, probability
  - **Expected shortfall (ES)**
    - Expected loss on a security conditional on returns being in the left tail of the probability distribution
  - **Lower partial standard deviation (LPSD)**
    - SD computed using only the portion of the return distribution below a threshold such as the risk-free rate of the sample average
  - Relative frequency of large, negative 3-sigma returns

Historic Returns on Risky Portfolios

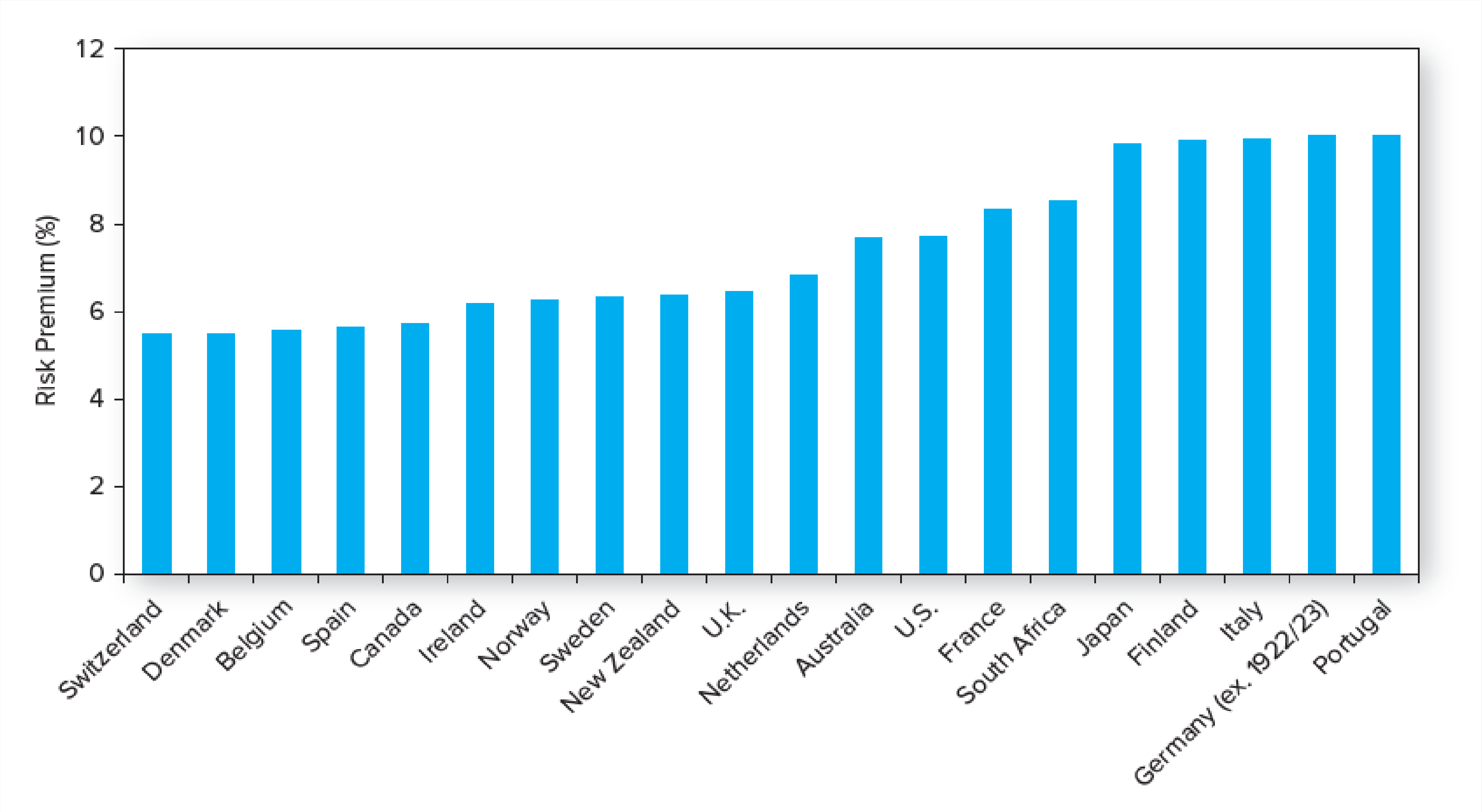
	T-Bills	T-Bonds	Stocks
Average	3.38%	5.58%	11.72%
Risk premium	N/A	2.45	8.34
Standard deviation	3.12	11.59	20.05
Max	14.71	41.68	57.35
Min	−0.02	−25.96	−44.04

Historic Returns on Risky Portfolios:
A Global View of the Historical Record

- Century-plus-long history (1900 – 2017) of average excess returns in 20 stock markets
  - Mean annual excess return across these countries was 7.4% and the median was 6.6%
  - U.S. performance consistent with international experience

- Tremendous variability in year-by-year returns

Average Excess Returns in 20 Countries, 1900 - 2017



Normality and Long-Term Investments

- **Lognormal distribution**
  - Probability distribution that characterizes a variable whose log has a normal (bell-shaped) distribution
  - Use of continuously compounded returns instead of effective annual returns
- Short-run versus long-run risk