

Assignment 3 Advanced Financial Mathematics

1. A stochastic process is generated by the equation

$$dX_t = \mu dt + \sigma dW_t.$$

with the initial condition $X_0 = 1$. Which equation governs the process $Y(t, X_t) = (1+t)^2 e^{aX_t}$? Here a is a constant.

2. Consider a stochastic process X_t governed by the equation

$$dX_t = X_t(\mu dt + \sigma dW_t),$$

with unknown constant μ and σ . It is known that the process $Y_t = e^{-3t} X_t^2$ is governed by the equation

$$dY_t = Y_t(dt + dW_t).$$

Determine the value of μ and σ .

3. Find $u(X, t)$ and $v(X, t)$ where

$$dX_t = u dt + v dW_t$$

and

- (a) $X_t = W_t^3$,
- (b) $X_t = -7t^2 + W_t^3$,
- (c) $X_t = g(t)e^{9W_t}$,

where f is a bounded, differentiable function.

4. S_t is generated by the stochastic differential equation

$$dS_t = S_t(\mu dt + \sigma dW_t), \quad \text{with } S_0 = a,$$

where μ , σ and a are constants. Evaluate the probability of $\lambda K \leq S_T \leq 2\lambda K$, where $T > 0$, $\lambda > 0$, and K is a positive constant.

5. A stochastic process is generated by the equation

$$dX_t = \mu dt + \sigma X_t dW_t$$

with the initial condition of $X_0 = 1$. Which equation governs the process $Y(t, X_t) = e^{(1+t)X_t+t^2}$?

6. Consider a stochastic process X_t governed by the equation

$$dX_t = X_t(a(t)dt + b(t)dW_t),$$

where $a(t)$ and $b(t)$ are unknown functions of t . It is known that the process $Y_t = f(t)X_t^2$ is governed by the equation

$$dY_t = Y_t t(dt + 9t^2 dW_t).$$

Determine the functions $a(t)$ and $b(t)$.