2021-22 First Semester
MATH1083 Calculus II (1002)

Assignment 6

Due Date: 11:30am 29/Mar/2023(Wed).

- Write down your **Chinese name** and **student number**. Write neatly on **A4-sized** paper and **show your steps**.
- **Late submissions or answers without details will not be graded.**

1. Match the equation with its graph

21–28 Match the equation with its graph (labeled I–VIII). Give reasons for your choices.

21. $x^2 + 4y^2 + 9z^2 = 1$

22. $9x^2 + 4y^2 + z^2 = 1$

23. $x^2 - y^2 + z^2 = 1$

24. $-x^2 + y^2 - z^2 = 1$

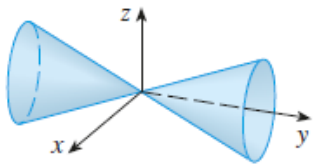
25. $y = 2x^2 + z^2$

26. $y^2 = x^2 + 2z^2$

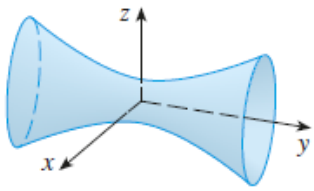
27. $x^2 + 2z^2 = 1$

28. $y = x^2 - z^2$

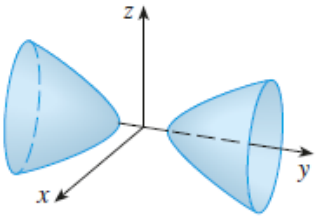
I



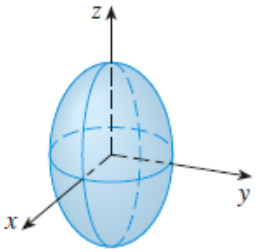
II



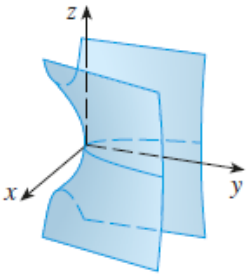
III



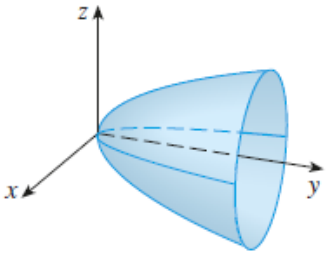
IV



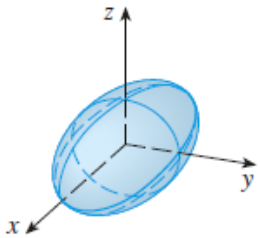
V



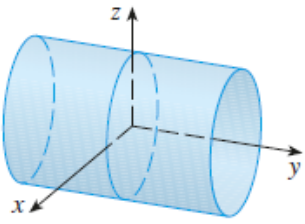
VI



VII



VIII



2. Sketch the region bounded by the surface $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 = 1$ for $1 \leq z \leq 2$

3. Find the limit of the vector function:

$$\lim_{t \to 0} \left(e^{-3t} \vec{i} + \frac{t^2}{\sin^2 t} \vec{j} + \cos 2t \, \vec{k} \right)$$

4. Find the **unit tangent vector** $\vec{T}(t)$ for the given value t: $\vec{r}(t) = \cos t \, \vec{i} + 3t \, \vec{j} + 2 \sin 2t \, \vec{k}$ at $t = 0$

5. Find the parametric equation for the **tanget line** to the curve with the given parametric equations

$$x = t \cos t, \qquad y = t, \qquad z = t \sin t$$

at the point $(-\pi, \pi, 0)$

6. Evaluate the integral

$$\int_0^1 \left(\frac{1}{t+1} \vec{i} + \frac{1}{t^2+1} \vec{j} + \frac{t}{t^2+1} \vec{k} \right) dt$$

7. If $\vec{r}(t) = (t^4, t, t^2)$, find $\vec{r}\,'(t)$, $\vec{T}(1)$, $\vec{r}\,''(t)$ and $\vec{r}\,'(t) \times \vec{r}\,''(t)$

8. If $\vec{u}(t) = (\sin t, \cos t, t)$ and $\vec{v}(t) = (t, \cos t, \sin t)$ use chain rule to find

$$\frac{d}{dt} \left[\vec{u}(t) \cdot \vec{v}(t) \right]$$

9. Find the length of the curve

$$\vec{r}(t) = \cos t \, \vec{i} + \sin t \, \vec{j} + \ln \cos t \, \vec{k} \qquad 0 \leq t \leq \frac{\pi}{4}$$

10. a) Find the arc length function for the curve measured from the point P in the direction of increasing t and then b) reparametrize the curve with respect to arc length starting from P. c) Find the point **4 units** along the curve (in the direction of increasing t) from P.

$$\vec{r}(t) = (5 - t) \, \vec{i} + (4t - 3) \, \vec{j} + 3t \, \vec{k} \qquad P(4, 1, 3)$$

11. 1) Find the unit tangent and unit normal vectors $\vec{T}(t)$ and $\vec{N}(t)$. 2) Find the curvature.

    (a) $\vec{r}(t) = \left\langle t^2, \sin t - t \cos t, \cos t + t \sin t \right\rangle, t > 0$

    (b) $\vec{r}(t) = \left\langle \sqrt{2}t, e^t, e^{-t} \right\rangle, t > 0$

12. Find the curvature

$$\vec{r}(t) = \sqrt{6} t^2 \, \vec{i} + 2t \, \vec{j} + 2t^3 \, \vec{k}$$

Use theorem 10 $\kappa = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$.