FINM3093 Investments

Lecture 3 Exercises

Solutions

1. a. The expected cash flow is: $(0.5 \times \$70{,}000) + (0.5 \times 200{,}000) = \$135{,}000$.

   With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

   $\$135{,}000/1.14 = \$118{,}421$

   b. If the portfolio is purchased for \$118,421 and provides an expected cash inflow of \$135,000, then the expected rate of return [$E(r)$] is as follows:

   $\$118{,}421 \times [1 + E(r)] = \$135{,}000$

   Therefore, $E(r) = 14\%$. The portfolio price is set to equate the expected rate of return with the required rate of return.

   c. If the risk premium over T-bills is now 12%, then the required return is:

   $6\% + 12\% = 18\%$

   The present value of the portfolio is now:

   $\$135{,}000/1.18 = \$114{,}407$

   d. For a given expected cash flow, portfolios that command greater risk premiums must sell at lower prices. The extra discount from expected value is a penalty for risk.

2. When we specify utility by $U = E(r) - 0.5A\sigma^2$, the utility level for T-bills is: 0.07

   The utility level for the risky portfolio is:

   $$U = 0.12 - 0.5 \times A \times (0.18)^2 = 0.12 - 0.0162 \times A$$

   In order for the risky portfolio to be preferred to bills, the following must hold:

   $$0.12 - 0.0162A > 0.07 \Rightarrow A < 0.05/0.0162 = 3.09$$

   A must be less than 3.09 for the risky portfolio to be preferred to bills.

3.

1) Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

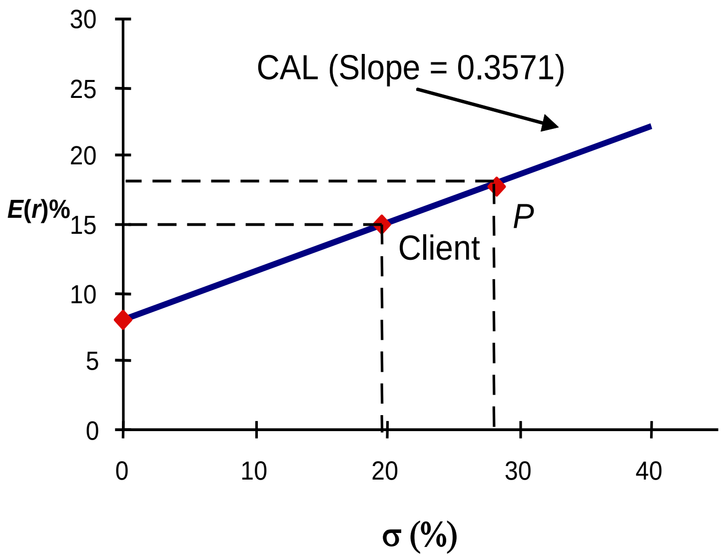
2)

Investment proportions:		30.0% in T-bills
	0.7 × 25% =	17.5% in Stock A
	0.7 × 32% =	22.4% in Stock B
	0.7 × 43% =	30.1% in Stock C

3) Your reward-to-volatility (Sharpe) ratio: $S = \frac{.18 - .08}{.28} = 0.3571$

Client's reward-to-volatility (Sharpe) ratio: $S = \frac{.15 - .08}{.196} = 0.3571$

4)



5) a. $E(r_C) = r_f + y \times [E(r_P) - r_f] = .08 + y \times (.18 - .08)$

If the expected return for the portfolio is 16%, then:

$$16\% = 8\% + 10\% \times y \Rightarrow y = \frac{.16 - .08}{.10} = 0.8$$

Therefore, in order to have a portfolio with expected rate of return equal to 16%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

Client's investment proportions:		20.0% in T-bills
	$0.8 \times 25\% =$	20.0% in Stock A
	$0.8 \times 32\% =$	25.6% in Stock B
	$0.8 \times 43\% =$	34.4% in Stock C

c. $\sigma_C = 0.8 \times \sigma_P = 0.8 \times 28\% = 22.4\%$

6) a. $\sigma_C = y \times 28\%$

If your client prefers a standard deviation of at most 18%, then:

$y = 18/28 = 0.6429 = 64.29\%$ invested in the risky portfolio.

b. $E(r_C) = .08 + .1 \times y = .08 + (0.6429 \times .1) = 14.429\%$

7) a. $y^* = \dfrac{E(r_P) - r_f}{A\sigma_P^2} = \dfrac{0.18 - 0.08}{3.5 \times 0.28^2} = \dfrac{0.10}{0.2744} = 0.3644$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

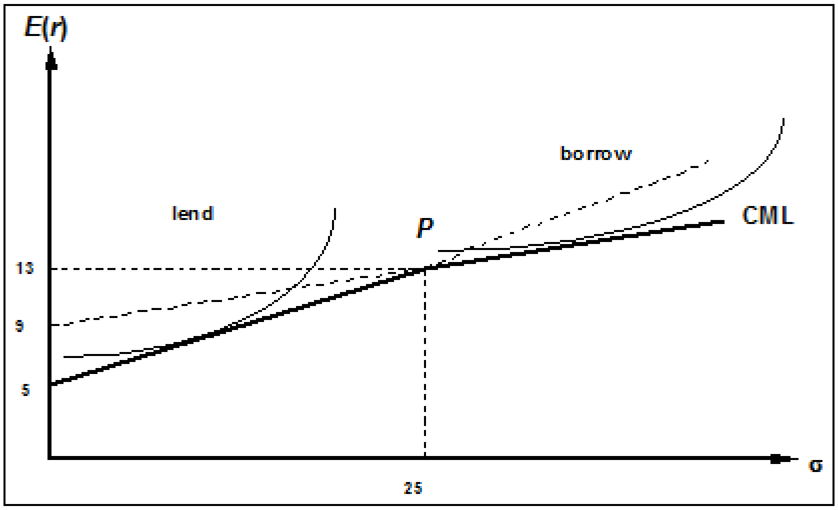
b. $E(r_C) = 0.08 + 0.10 \times y^* = 0.08 + (0.3644 \times 0.1) = 0.1164$ or 11.644%

$\sigma_C = 0.3644 \times 28 = 10.203\%$

4.

1) Data: $r_f = 5\%$, $E(r_M) = 13\%$, $\sigma_M = 25\%$, and $r_f^B = 9\%$

The CML and indifference curves are as follows:



2) For y to be less than 1.0 (that the investor is a lender), risk aversion (A) must be large enough such that:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} < 1 \;\Rightarrow\; A > \frac{0.13 - 0.05}{0.25^2} = 1.28$$

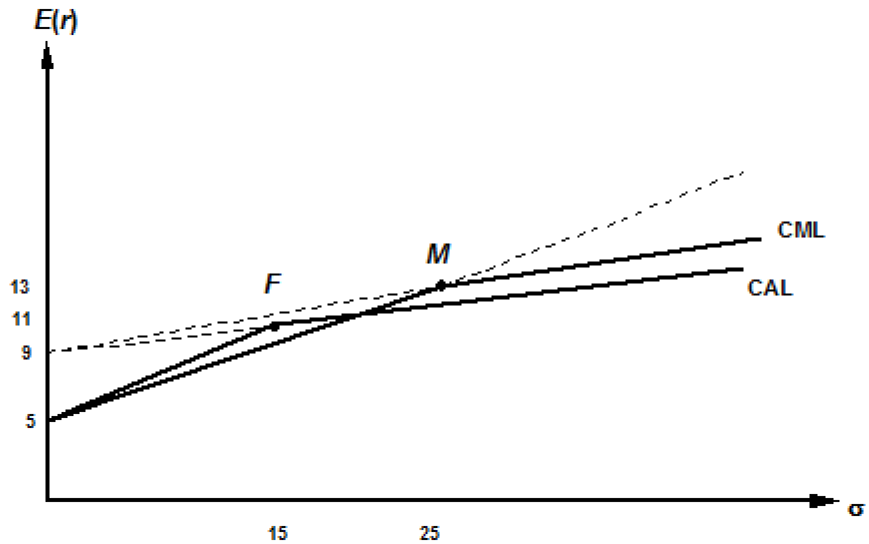
For y to be greater than 1 (the investor is a borrower), A must be small enough:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} > 1 \;\Rightarrow\; A < \frac{0.13 - 0.09}{0.25^2} = 0.64$$

For values of risk aversion within this range, the client will neither borrow nor lend but will hold a portfolio composed only of the optimal risky portfolio:

$y = 1$ for $0.64 \leq A \leq 1.28$

3) The graph for (a) has to be redrawn here, with:
$E(r_P) = 11\%$ and $\sigma_P = 15\%$



For a lending position: $A > \dfrac{0.11 - 0.05}{0.15^2} = 2.67$

For a borrowing position: $A < \dfrac{0.11 - 0.09}{0.15^2} = 0.89$

Therefore, $y = 1$ for $0.89 \le A \le 2.67$

4) The maximum feasible fee, denoted f, depends on the reward-to-variability ratio. For $y < 1$, the lending rate, 5%, is viewed as the relevant risk-free rate, and we solve for f as follows:

$$\frac{.11 - .05 - f}{.15} = \frac{.13 - .05}{.25} \;\Rightarrow\; f = .06 - \frac{.15 \times .08}{.25} = .012, \text{ or } 1.2\%$$

For $y > 1$, the borrowing rate, 9%, is the relevant risk-free rate. Then we notice that, even without a fee, the active fund is inferior to the passive fund because:

$$\frac{.11 - .09 - f}{.15} = 0.13 < \frac{.13 - .09}{.25} = 0.16 \rightarrow f = -.004$$

More risk tolerant investors (who are more inclined to borrow) will not be clients of the fund. We find that f is negative: that is, you would need to *pay* investors to choose your active fund. These investors desire higher risk–higher return complete portfolios and thus are in the borrowing range of the relevant CAL. In this range, the reward-to-variability ratio of the index (the passive fund) is better than that of the managed fund.

5.

1) The parameters of the opportunity set are:

$E(r_S) = 20\%$, $E(r_B) = 12\%$, $\sigma_S = 30\%$, $\sigma_B = 15\%$, $\rho = 0.10$

From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \rho \times \sigma_S \times \sigma_B$]:

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

The minimum-variance portfolio is computed as follows:

$$w_{\text{Min}}(S) = \frac{\sigma_B^2 - \text{Cov}(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2\text{Cov}(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$w_{\text{Min}}(B) = 1 - 0.1739 = 0.8261$

The minimum variance portfolio mean and standard deviation are:

$E(r_{\text{Min}}) = (0.1739 \times .20) + (0.8261 \times .12) = .1339 = 13.39\%$

$$\sigma_{\text{Min}} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B \text{Cov}(r_S, r_B)]^{1/2}$$

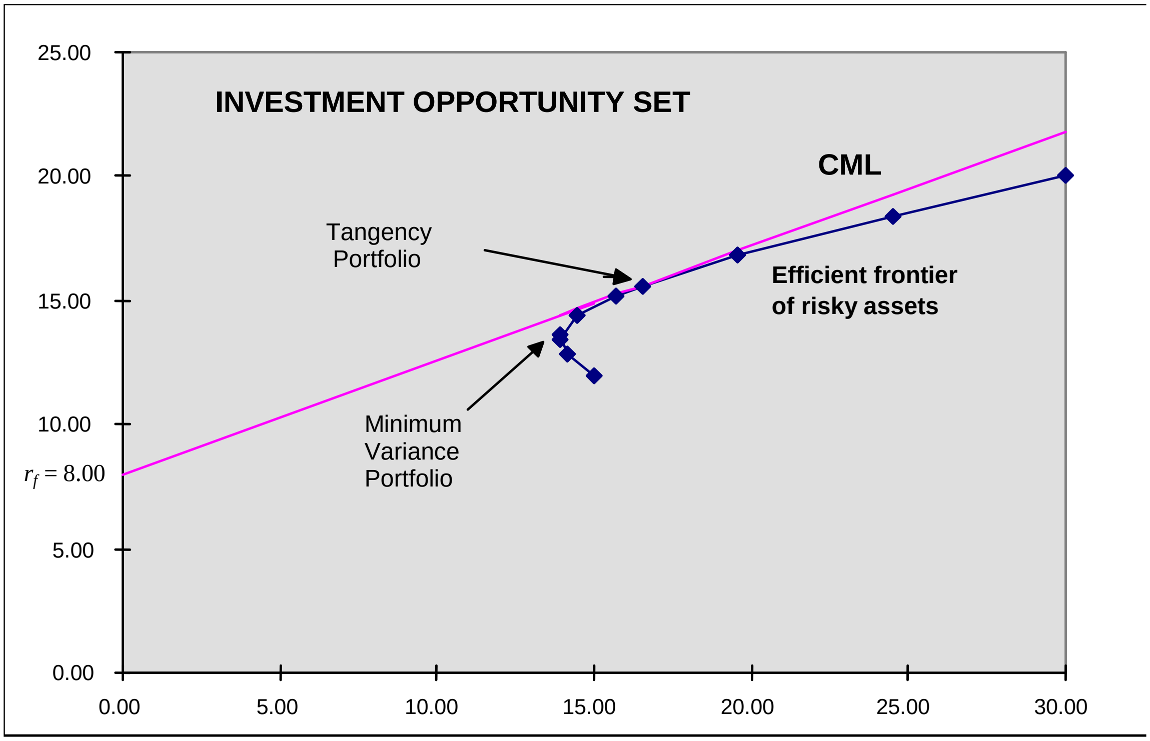
$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$= 13.92\%$

2)

Proportion in Stock Fund	Proportion in Bond Fund	Expected Return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39	82.61	13.39	13.92	minimum variance
20.00	80.00	13.60	13.94	
40.00	60.00	15.20	15.70	
45.16	54.84	15.61	16.54	tangency portfolio
60.00	40.00	16.80	19.53	
80.00	20.00	18.40	24.48	
100.00	0.00	20.00	30.00	

Graph shown below.



3) The above graph indicates that the optimal portfolio is the tangency portfolio with expected return approximately 15.6% and standard deviation approximately 16.5%.

4) The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{[E(r_S)-r_f]\times\sigma_B^2 - [E(r_B)-r_f]\times Cov(r_S,r_B)}{[E(r_S)-r_f]\times\sigma_B^2 + [E(r_B)-r_f]\times\sigma_S^2 - [E(r_S)-r_f+E(r_B)-r_f]\times Cov(r_S,r_B)}$$

$$= \frac{[(.20-.08)\times 225]-[(.12-.08)\times 45]}{[(.20-.08)\times 225]+[(.12-.08)\times 900]-[(.20-.08+.12-.08)\times 45]} = 0.4516$$

$$w_B = 1-0.4516 = 0.5484$$

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P) = (0.4516 \times .20) + (0.5484 \times .12) = .1561$

$= 15.61\%$

$\sigma_p = [(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2}$

$= 16.54\%$

5) The reward-to-volatility ratio of the optimal CAL is:

$$\frac{E(r_p)-r_f}{\sigma_p} = \frac{.1561-.08}{.1654} = 0.4601$$

6) The investor has a risk aversion of A=3, Therefore, she will invest

$$y = \frac{.1561-.08}{3\times .1654^2} = 0.9272 = 92.72\%$$

of her wealth in this risky portfolio. The resulting investment composition will be stock fund: 0.9272 $\times$ .4516 = 41.87% and bond fund: 0.9272$\times$ .5484 = 50.85%. The remaining 7.28% will be invested in the risk-free asset (money market fund).

7) a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

$$E(r_C) = r_f + \frac{E(r_p)-r_f}{\sigma_P}\sigma_C = .08 + 0.4601\sigma_C$$

If $E(r_C)$ is equal to 14%, then the standard deviation of the portfolio is 13.04%.

b. To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1 - y) \times r_f + y \times E(r_P) = r_f + y \times [E(r_P) - r_f] = .08 + y \times (.1561 - .08)$$

Setting $E(r_C)$ = 14% we find: y = 0.7884 and $(1 - y)$ = 0.2119 (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7884 × 0.4516 = 0.3560

Proportion of bonds in complete portfolio = 0.7884 × 0.5484 = 0.4323

8) Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (w_S) and the appropriate proportion in the bond fund ($w_B = 1 - w_S$) as follows:

$0.14 = 0.20 \times w_S + 0.12 \times (1 - w_S) = 0.12 + 0.08 \times w_S \Rightarrow w_S = 0.25$

So the proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

$\sigma_P = [(0.25^2 \times 900) + (0.75^2 \times 225) + (2 \times 0.25 \times 0.75 \times 45)]^{1/2} = 14.13\%$

This is considerably greater than the standard deviation of 13.04% achieved using T-bills and the optimal portfolio.

6. Since Stock A and Stock B are perfectly negatively correlated, a risk-free portfolio can be created and the rate of return for this portfolio, in equilibrium, will be the risk-free rate. To find the proportions of this portfolio [with the proportion w_A invested in Stock A and $w_B = (1 - w_A)$ invested in Stock B], set the standard deviation equal to zero. With perfect negative correlation, the portfolio standard deviation is:

σ_P = Absolute value $[w_A\sigma_A - w_B\sigma_B]$

$0 = 5 \times w_A - [10 \times (1 - w_A)] \Rightarrow w_A = 0.6667$

The expected rate of return for this risk-free portfolio is:

$E(r) = (0.6667 \times 10) + (0.3333 \times 15) = 11.667\%$

Therefore, the risk-free rate is: 11.667%