MATH2033 Mathematical Statistics
Assignment 3

Due Date: **17/Mar/2024(Sunday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.
- **Late submissions**, **answers without details**, or **unrecognizable handwritings** will NOT be graded.

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1. Consider a population consisting of five values $-1, 2, 2, 4$, and 8. Find the population mean and variance. Calculate the sampling distribution of the mean of a sample of size 2 by generating all possible such samples. From them, find the mean and variance of the sampling distribution, and compare the results to Theorems 2.1.4 and 2.2.7 in our slides.

2. Suppose that a sample of size $n = 2$ is drawn from the population of the preceding problem and that the proportion of the sample values that are greater than 3 is recorded. Find the sampling distribution of this statistic by listing all possible such samples. Find the mean and variance of the sampling distribution.

3. Consider a population of size four, the members of which have values x_1, x_2, x_3, x_4.

   (a) If simple random sampling were used, how many samples of size two are there?

   (b) Suppose that rather than simple random sampling, the following sampling scheme is used. The possible samples of size two are

   $$\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_1, x_4\}$$

   and the sampling is done in such a way that each of these four possible samples is equally likely. Is the sample mean unbiased?

4. This is one proof of Lemma 2.2.6 in our slides. Let U_i be a random variable with $U_i = 1$ if the i th population member is in the sample and equal to 0 otherwise.

   (a) Show that the sample mean $\overline{X} = n^{-1} \sum_{i=1}^{N} U_i x_i$.

(b) Show that $P(U_i = 1) = n/N$. Find $E(U_i)$, using the fact that U_i is a Bernoulli random variable.

(c) What is the variance of the Bernoulli random variable U_i?

(d) Noting that U_iU_j is a Bernoulli random variable, find $E(U_iU_j), i \neq j$. (Be careful to take into account that the sample is drawn without replacement.)

(e) Find $\operatorname{Cov}(U_i, U_j), i \neq j$.

(f) Using the representation of $\overline{X}$ above, find $\operatorname{Var}(\overline{X})$.

5. A simple random sample of a population of size 2000 yields the following 25 values:

104	109	111	109	87
86	80	119	88	122
91	103	99	108	96
104	98	98	83	107
79	87	94	92	97

(a) Calculate an unbiased estimate of the population mean.

(b) Calculate unbiased estimates of the population variance and $\operatorname{Var}(\overline{X})$.

(c) Give approximate 95% confidence intervals for the population mean and total.