2023-24 First Semester

MATH2043 Ordinary Differential Equations (1002)

Assignment 9

Due Date: **14/Dec/2023(Thursday), on or before 16:00, in tutorial class.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without details will not be graded.**

In each of Problem 1 and 2,

(a) Draw a phase portrait with direction field and a few trajectories.

(b) Find out the general solution in terms of real-valued functions.

(c) Described the behavior of the solutions as $t \to \infty$

1. $\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}$

2. $\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ -4 & 1 \end{pmatrix} \mathbf{x}$

3. Use the result of Q1(b) to solve the following initial value problem

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

4. Consider the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \mathbf{x}$$

Given that $\lambda = 1$ is a triple eigenvalue of the coefficient matrix A.

(a) Show that there is only one linearly independent eigenvector $\vec{\xi}_1$, and then write down the first solution.

(b) Assume a second solution is in the form of

$$\mathbf{x}^{(2)}(t) = [t\vec{\xi} + \vec{\eta}]e^t$$

determine $\vec{\xi}$ and $\vec{\eta}$, then find $\mathbf{x}^{(2)}(t)$.

(c) Assume a third solution is in the form of

$$\mathbf{x}^{(3)}(t) = \left[\frac{t^2}{2}\vec{\xi} + t\vec{\eta} + \vec{\zeta}\right] e^t$$

Determine the relationship among $\vec{\xi}$, $\vec{\eta}$ and $\vec{\zeta}$, then find $\mathbf{x}^{(3)}(t)$.

(d) Write down a fundamental matrix $\Psi(t)$ for the system.

5. (*Optional!*) **Variation of Parameters**
Solve

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^{-2t} \\ -2e^t \end{pmatrix}$$

6. Solve the given boundary value problem or else show that it has no solution.

$$y'' + 2y = x, \quad y(0) = 0, \quad y(\pi) = 0.$$

7. Find the eigenvalues and eigenfunctions of the given boundary value problem. Assume that all eigenvalues are real.

(a)

$$y'' + \lambda y = 0, \quad y'(0) = y'(\pi) = 0.$$

(b)

$$y'' + \lambda y = 0, \quad y'(0) = y(L) = 0.$$