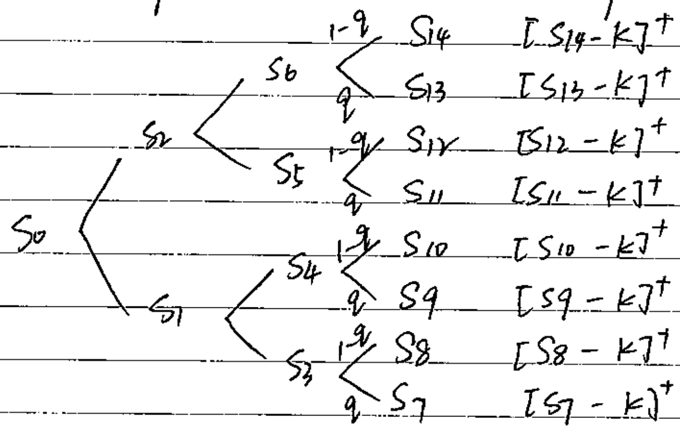
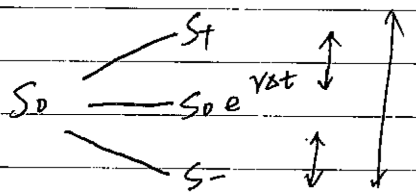
American options

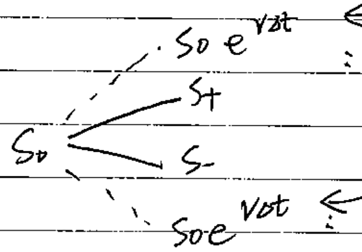
American options can be exercised any time before and on the maturity date.



$\max\{S-K, 0\} = [S-K]^+$



$$q = \frac{S_+ - S_0 e^{r\Delta t}}{S_+ - S_-}$$



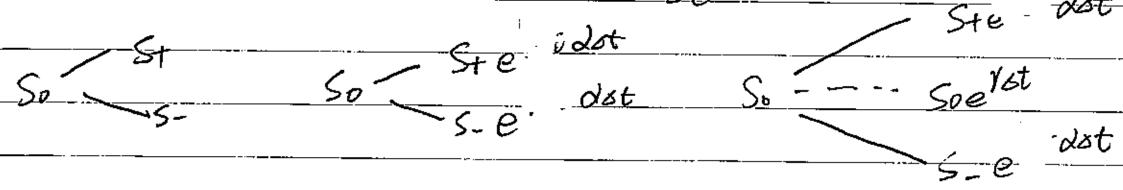
Arbitrage: Short stock & put money in the Bank

Arbitrage: Borrow money from bank & Buy the stock

Dividend:

X_0 $\quad t: Xe^{rt}$

S_0 $\quad Se^{qt}$



$$q = \frac{S_+ e^{\delta\Delta t} - S_0 e^{r\Delta t}}{S_+ e^{\delta\Delta t} - S_- e^{\delta\Delta t}}$$

$$= \frac{S_+ - S_0 e^{(r-\delta)\Delta t}}{S_+ - S_-}$$

$S_0 \to S_+$, S_-

$S_0 \to S_+ + D(\Delta t, \alpha, S_0, S_+)$

$S_0 \to S_- + D(\Delta t, \alpha, S_0, S_-)$

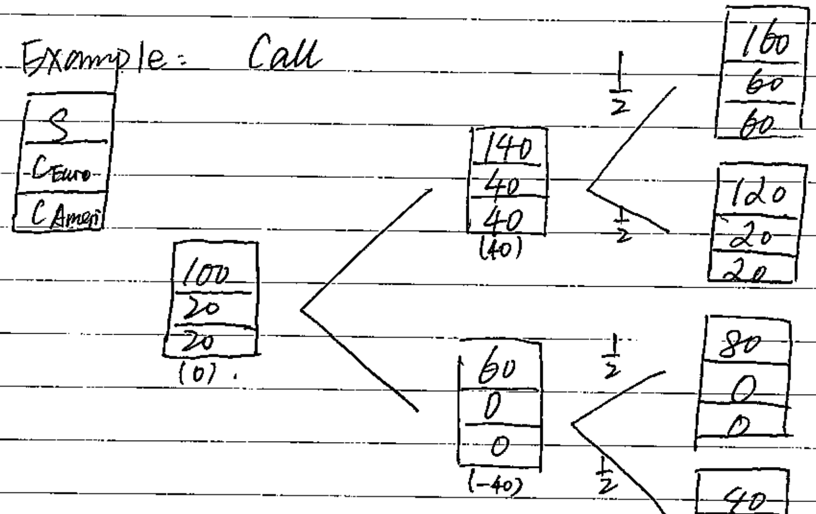
$S_-(e^{\alpha\Delta t} - 1)$

$S_0 \to S_+ + S_0 e^{\alpha\Delta t}$

$S_0 \; \text{---} \; S_0 e^{r\Delta t}$

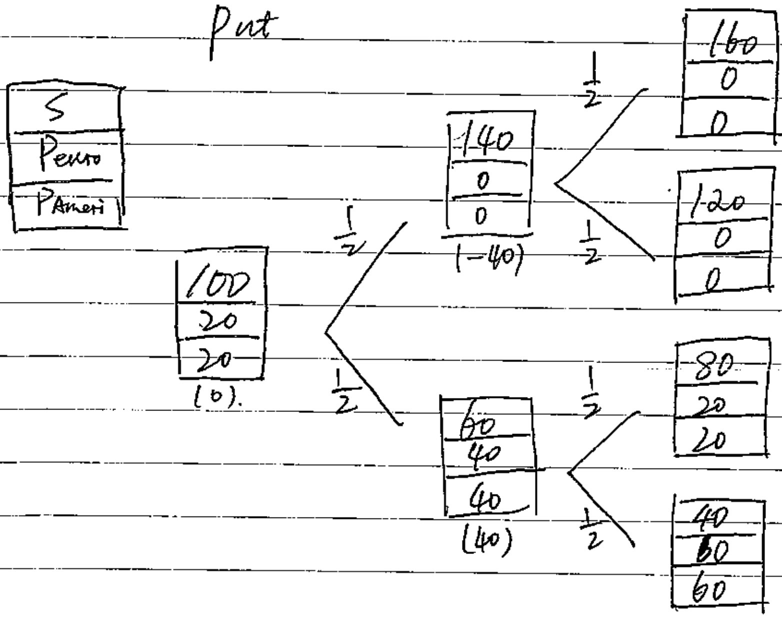
$S_0 \to S_- + S_0 e^{\alpha\Delta t}$

Example: Call



A European call & An American call
with $K=100$, $T=2$, $r=0$, $\alpha=0$

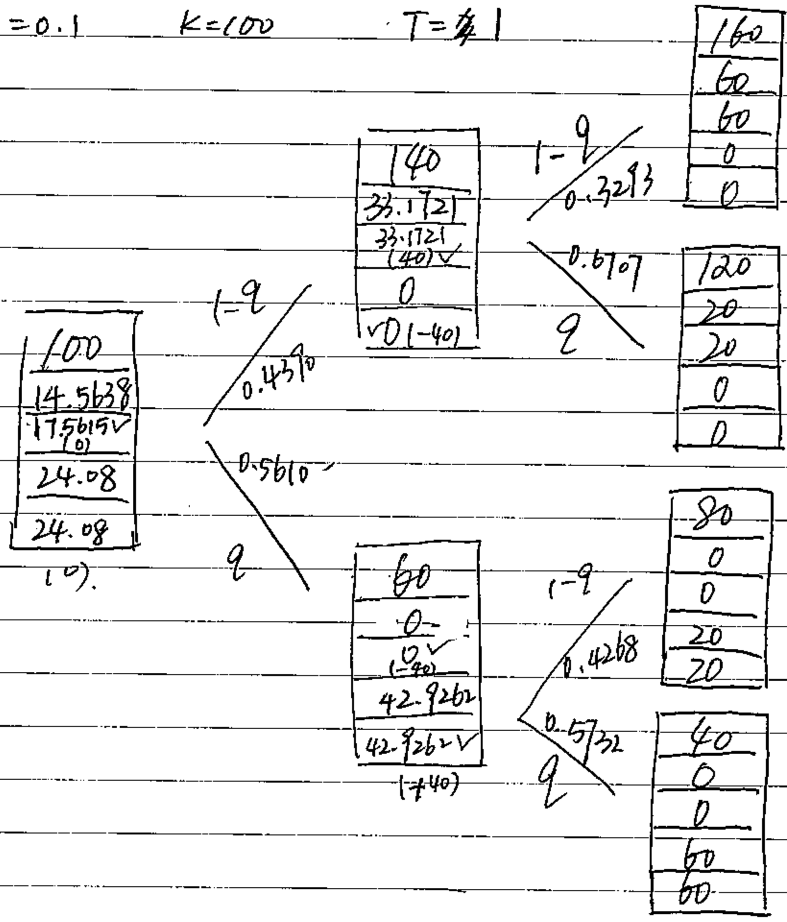
Put



$r=0 \quad \delta=0.1 \quad K=100 \quad T=1$

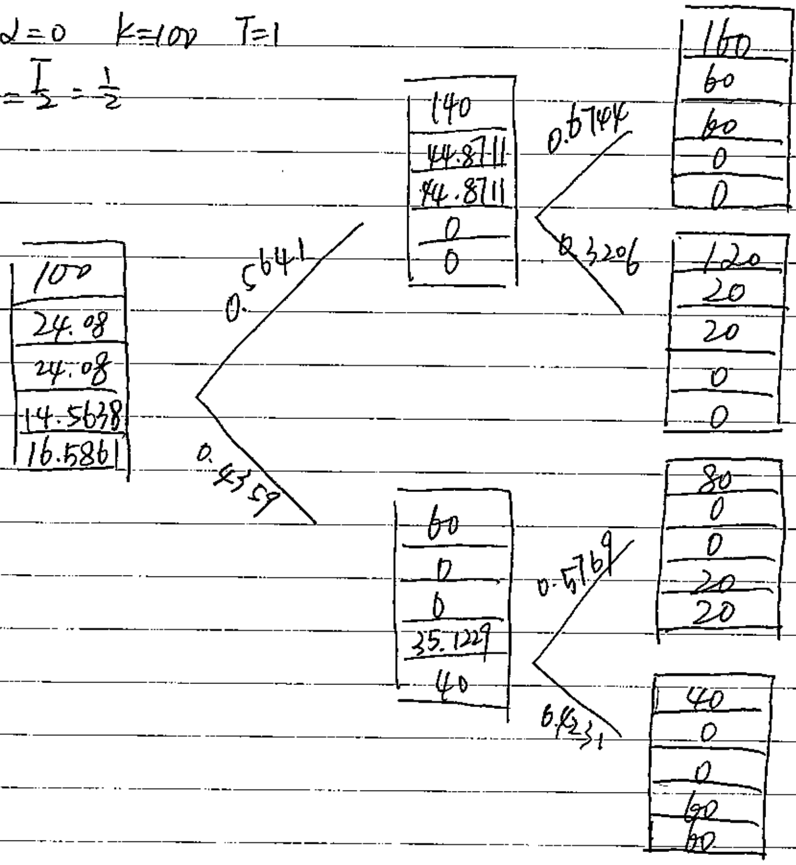
S
Ceu
CAm
Peu
PAm

$$q=\frac{S_+ - S_0 e^{(r-d)\Delta t}}{S_+ - S_-}$$

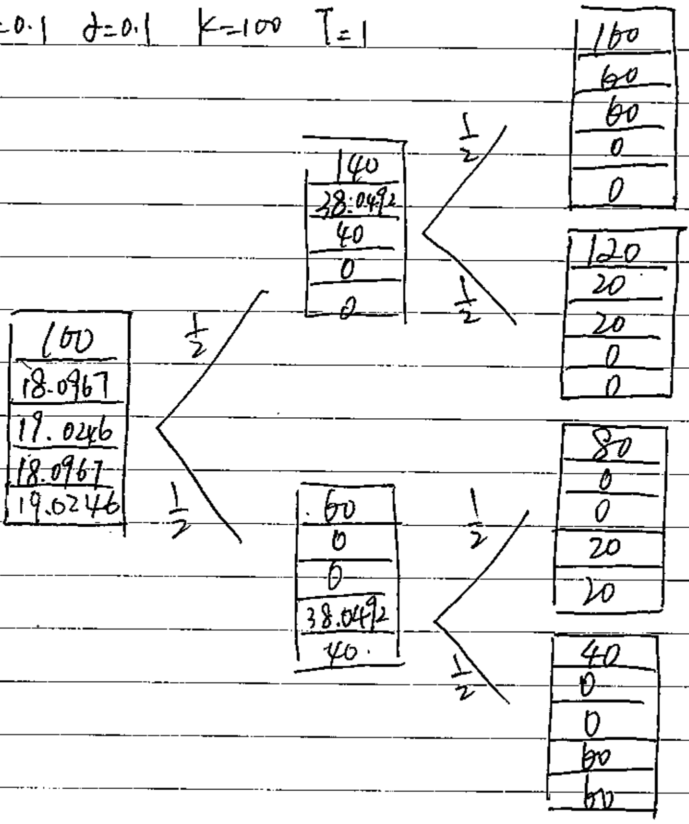


$r=0.1 \quad \delta=0 \quad K=100 \quad T=1$

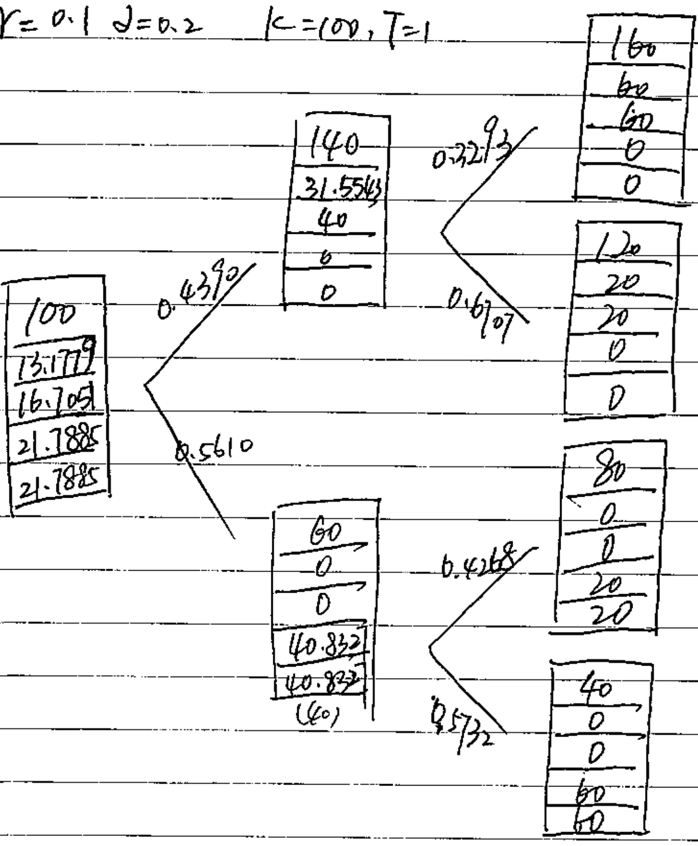
$\Delta t=\frac{T}{2}=\frac{1}{2}$



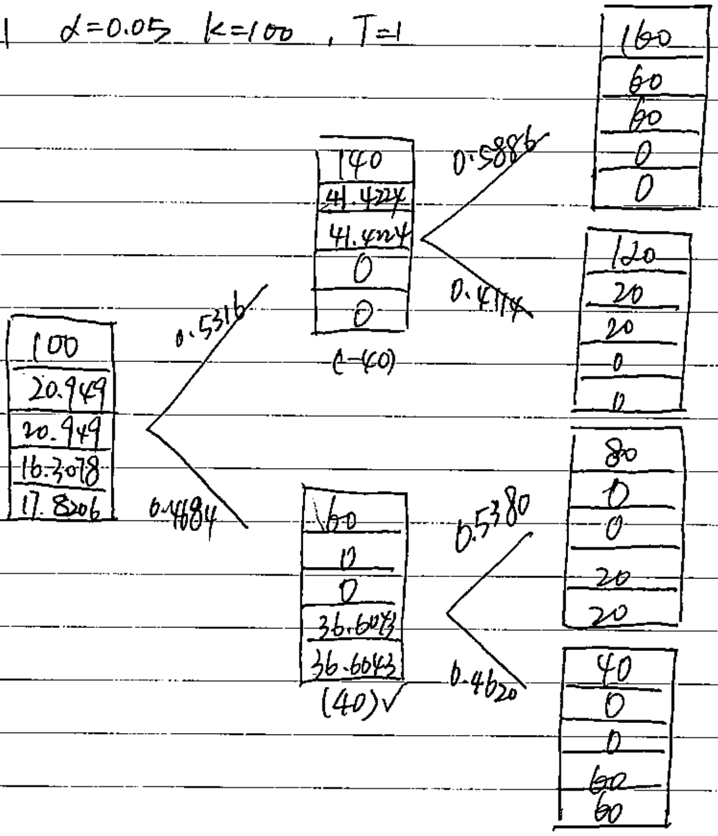
$r=0.1$ $d=0.1$ $K=100$ $T=1$

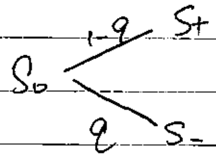


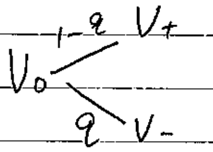
$r=0.1$ $d=0.2$ $K=100$, $T=1$



$r=0.1 \quad \alpha=0.05 \quad k=100, \ T=1$







$$q=\frac{S_+ - S_0 e^{(r-\alpha)\Delta t}}{S_+ - S_-}$$

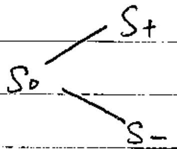
$$V_0^{no} = e^{-r\Delta t}\left[(1-q)\cdot V_+ + q\cdot V_-\right].$$

$$V_0^{exc} = F(S_0)$$

Exercise occurs when $V_0^{exer} > V_0^{no.}$

$$F(S_0) > e^{-r\Delta T}\left[(1-q)\cdot V_+ + qV_-\right].$$

$$\text{Call}= \ S_0 - k > e^{-r\Delta T}\left[V_+ - q(V_+ - V_-)\right]$$



$$S_0 - k > e^{-r\Delta T}\left[S_+ - k - \frac{S_+ - S_0 e^{(r-\alpha)\Delta t}}{S_+ - S_-}\left((S_+ - k) - (S_- - k)\right)\right]$$

$$S_0 - k > e^{-r\Delta t}\left[-k + S_0 e^{(r-\alpha)\Delta t}\right]$$

$$> -k e^{-r\Delta t} + S_0 e^{-\alpha\Delta t}$$

$$S_0(1-e^{-\alpha\Delta t}) > k(1-e^{-r\Delta t})$$

$$S_0 > \frac{k(1-e^{-r\Delta t})}{(1-e^{-\alpha\Delta t})} = \frac{k(1-(1-r\Delta t))}{1-(1-\alpha\Delta t)} = k\frac{r}{\alpha}.$$

put= $k - S_0 > e^{-r\Delta t}[(1-q)V_+ - qV_-]$

$$k - S_0 > -[-ke^{-r\Delta t} + S_0 e^{-d\Delta t}]$$

$$-S_0(1-e^{-d\Delta t}) > -k(1-e^{-r\Delta t})$$

$$S_0 < \frac{k(1-e^{-r\Delta t})}{(1-e^{-d\Delta t})}.$$

If $V_-=0$, Call: $S_0 - k > e^{-r\Delta t}[V_+ - qV_+]$

$$S_0 - k > e^{-r\Delta t}\left[S_+ - k - \frac{S_+ - S_0 e^{(r-d)\Delta t}}{S_+ - S_-}(S_+ - k)\right]$$

$$S_0 - k > e^{-r\Delta t}(1-q)(S_+ - k)$$

$$S_0 - k > e^{-r\Delta t}\left(\frac{S_0 e^{(r-d)\Delta t} - S_-}{S_+ - S_-}\right)(S_+ - k).$$