Week10-2

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$A = PDP^{-1}$

Square matrix
- **Diagonalizable** $\Leftrightarrow$ $A_{n\times n}$ has n linearly independent eigenvectors
  - **Case I**: When A has n distinct eigenvalues
  - **Case II**: When A has $k < n$ distinct eigenvalues, $almu(\lambda_i) = gemu(\lambda_i)$, for each λ_i.
- **Non-diagonalizable** (Defective matrix): When A has less than n linearly independent eigenvectors.

$$A = \underbrace{C}_{\text{nonsingular}}\;\underbrace{J}_{\substack{\text{Jordan form matrix} \\ \text{(upper tri)}}}\;C^{-1}$$

Example: $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ all eigenvalues and li.in. eigenvectors. Diagonalizable?

$$\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & -3 & 1 \\ 1 & -2-\lambda & 1 \\ 1 & -3 & 2-\lambda \end{vmatrix} = (2-\lambda)\left[(-2-\lambda)(2-\lambda)+3\right] - \left[-3(2-\lambda)+3\right] + \left[-3+2+\lambda\right]$$

$$= (2-\lambda)\left[\lambda^2-4+3\right] - \left[3\lambda-3\right] + \left[\lambda-1\right] = -\lambda^3 + \cdots - \lambda$$

$$= (\lambda-1)\left[(2-\lambda)(\lambda+1) - 3 + 1\right]$$

$$= -(\lambda-1)(\lambda-1)\lambda$$

$\lambda_1 = 0$, $\lambda_{2,3} = 1$

For $\lambda_1 = 0$,

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix} \vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\alpha, \quad \alpha \neq 0$$

For $\lambda_{2,3} = 1$,

$$\begin{bmatrix} 1 & -3 & 1 \\ 1 & -3 & 1 \\ 1 & -3 & 1 \end{bmatrix} \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3\alpha-\beta \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\alpha + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\beta, \quad \alpha, \beta \in \mathbb{R}.$$

So $E_0 = span\left\{\underline{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}\right\}$, $E_1 = span\left\{\underline{\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}\right\}$.

$almu(1) = 2$

$gemu(1) = \dim E_1 = 2$

A is diagonalizable.

$D_1 = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}$, $P_1 = \begin{bmatrix} 3 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ | $D_2 = \begin{bmatrix} 1 & & \\ & 0 & \\ & & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$P_1 = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$P_1 = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$