MATH2033 Mathematical Statistics
Assignment 7 Suggested Solutions

1. (a) the test statistic is
$$Z = \frac{18.2 - 20.0}{\sqrt{16/25}} = -2.25$$
The appropriate 5% critical value is $-z_{0.95} = -1.645$. The observed value of Z is less than -1.645 . Hence, we reject H_0 at the 5% significance level and conclude that the true value of μ in the normal distribution from which the data are sampled satisfies $\mu < 20$.

   (b) the null hypothesis is rejected if
$$\frac{\bar{X} - 20.0}{4/\sqrt{25}} < -1.645$$
or equivalently if
$$\bar{X} < 20.0 - 1.645 \times \frac{4}{\sqrt{25}}$$
The true distribution of $\bar{X}$ is $N(19.0, 16/25)$ and so the probability of rejecting H_0 is
$$\begin{aligned}
&\mathrm{P}\left(\bar{X} < 20.0 - 1.645 \times \frac{4}{\sqrt{25}}\right) \\
&= \mathrm{P}\left(\frac{\bar{X} - 19.0}{4/5} < \frac{20.0 - \left(1.645 \times \frac{4}{5}\right) - 19.0}{4/5}\right) \\
&= \mathrm{P}\left(\frac{\bar{X} - 19.0}{4/5} < -0.395\right) \\
&= \Phi(-0.395) = 0.3464,
\end{aligned}$$
since the true distribution of $\frac{\bar{X}-19.0}{4/5}$ is $N(0,1)$.

2. We want to test $H_0 : \mu = 12.5$ vs $H_1 : \mu > 12.5$ at the 5% significance level. The test statistic is
$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{17.1 - 12.5}{10/\sqrt{21}} = 2.108$$
Under $H_0, T \sim t(20)$. For a one-tailed test at the 5% significance level we will reject H_0 if $T > 1.725$ (from tables). Our observed value of T is greater than 1.725 and so we reject the null hypothesis that $\mu = 12.5$ at the 5% significance level and conclude that $\mu > 12.5$, i.e. the drug 6-mP improves remission times compared to the previous drug treatment.

3. (a) Let X be the number of successful guesses. Then $X \sim \text{bin}(25, p), p \in [0, 1]$. We test $H_0 : p = 0.5$ versus $H_1 : p > 0.5$. The test statistic is X, and the critical region is $\{17, \ldots, 25\}$.

(b) The p-value is

$$P(X \geq 16) = \sum_{k=16}^{25} \binom{25}{k} 0.5^k (1-0.5)^{25-k} = 0.115.$$

(c) No, do not reject H_0 for either value of α_0, since the p-value is greater than 0.1.

4. (a) Given that the technique worked $k = 24$ times during the $n = 52$ occasions it was tried, $z = \frac{24-52(0.40)}{\sqrt{52(0.40)(0.60)}} = 0.91$. The latter is not larger than $z_{.05} = 1.64$, so $H_0 : p = 0.40$ would not be rejected at the $\alpha = 0.05$ level. These data do not provide convincing evidence that transmitting predator sounds helps to reduce the number of whales in fishing waters.

   (b) P-value $= P(Z \geq 0.91) = 0.1814$; H_0 would be rejected for any $\alpha \geq 0.1814$.

5. The null hypothesis would be rejected if $z = \frac{k-200(0.45)}{\sqrt{200(0.45)(0.55)}} \geq 1.08\,(= z_{.14})$. For that to happen, $k \geq 200(0.45) + 1.08 \cdot \sqrt{200(0.45)(0.55)} \doteq 98$.