2023-24 First Semester

MATH2023 Ordinary and Partial Differential Equations (1002)

Assignment 7

Due Date: **23/Nov/2023(Thursday), on or before 16:00, in tutorial class.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps will not be graded.**

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1. Determine the Taylor series about the point x_0 for the given function. Also determine the radius of convergence of the series.

   (a) $e^x, \quad x_0 = 0$

   (b) $x^2, \quad x_0 = -1$

   (c) $\frac{1}{1-x} \quad x_0 = 2$

2. Solve the given differential equation by means of a power series about the given point x_0. Find the first four nonzero terms in each of two linearly independent solutions (unless the series terminates sooner). If possible, find the general term in each solution.

   (a) $y'' - xy' - y = 0, \quad x_0 = 0$

   (b) $y'' - xy' - y = 0, \quad x_0 = 1$

   (c) $(2+4x-2x^2)y'' - 12(x-1)y' - 12y = 0, \quad x_0 = 1$

   (d) $e^x y'' + xy = 0, \quad x_0 = 0$

3. Read Section 5.3 from the **textbook** about determining the radius of convergence for $\frac{Q(x)}{P(x)}$ where P, Q are both polynomials. and practice on the following:
   Determine a lower bound for the radii of convergence of series solutions about the given x_0 for each of the differential equations.

   (a) $xy'' + y = 0; \quad x_0 = 1$

   (b) $(x^2 - 2x - 3)y'' + xy' + 4y = 0; \quad x_0 = 4$ (Example 1 in Sec5.2 of the notes)

   (c) $(x^2 - 2x + 3)y'' + xy' + 4y = 0; \quad x_0 = 4,\ x_0 = -4,\ x_0 = 0$

4. Consider the following differential equation

$$3x^2y'' + 12xy' + 9y = 0, \qquad x > 0$$

   Use the method of change of variables by letting $t = \ln x$ to find out the general solution $y(x)$.