Caculus II Math 1038 (1002&1003)

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Week 6: finish Ch12 Vector and geometry of space and start Ch13 Vector-valued function

1. How to evaluate
   - **15%** Assignment (weekly)
   - **10%** Quiz:
     - Quiz One 5%: 16th Mar
     - Quiz Two 5%: Ch14 and part of Ch15
   - **15%** Midterm: Ch11, Ch12 , Ch13, part of Ch14 Partial derivatives
     - **13 Apr** evening. Thu 1.5 hours
   - **60%** Final: Ch11 - Ch15 all
2. Chapter 12 Summary
   (a) Vectors operations: sum, scalar multiplication, dot product and cross product
   (b) Equations
      i. Line: vector, parametric, symmetric
      ii. Plane: vector, parametric
   (c) Distances: point to line, point to plane
   (d) Intersections: line and line (point), line to plane (point), plane to plane (line).
   (e) Surfaces
3. Chapter 13 Vector-valued functions
   (a) vector function: takes **one or more variables** and returns a vector. while real valued function: takes variables and returns a real number.
      i. x: one or more real variables
      ii. y: a vector
      iii. example: vector equation of a line in $\mathbb{R}^3$:

$$\overrightarrow{r}(t) = \langle f(t), g(t), h(t) \rangle$$

      it is a vector function $F : t \mapsto \overrightarrow{r}(t)$
      iv. domain, limits and continuity
   (b) Space curves
      i. Vector function

$$\overrightarrow{r} = \overrightarrow{r}_0 + t\overrightarrow{v}$$

      ii. Parametric function

$$x = f(t), \qquad y = g(t), \qquad z = h(t)$$

      iii. sketch a curve: helix
      iv. Parameterization of a curve:

$$x^2 + y^2 = 1$$

      $x = \cos t$ and $y = \sin t$
   (c) Derivatives:

$$\frac{d\overrightarrow{r}(t)}{dt} = \overrightarrow{r}'(t) = \lim_{h \to 0} \frac{\overrightarrow{r}(t+h) - \overrightarrow{r}(t)}{h}$$

i. tangent vector $\overrightarrow{r}'(t)$

ii. unit tangent vector $\overrightarrow{T}(t)$uk

iii. differentiation rules

- $\overrightarrow{r}'(t)$ is orthogonal to $\overrightarrow{r}(t)$

$$\frac{d}{dt}\left[\overrightarrow{r}(t)\cdot\overrightarrow{r}(t)\right]=\frac{dc}{dt}=0=2\overrightarrow{r}(t)\cdot\overrightarrow{r}'(t)$$

(d) Integrals

i. definition integral

(e) **Arc length of a curve** $(x,y)=(f(t),g(t))$ in $\mathbb{R}^2$

$$\begin{aligned} L &= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt \qquad \text{Pathagoras Theorem} \\ &= \int_a^b \sqrt{[f'(t)]^2+[g'(t)]^2}\,dt \end{aligned}$$

in $\mathbb{R}^3$

$$L=\int_a^b \sqrt{[f'(t)]^2+[g'(t)]^2+[h'(t)]^2}\,dt$$

or

$$L=\int_a^b \left|\overrightarrow{r}'(t)\right| dt$$

Note: $|\overrightarrow{r}'(t)|$ is a scalar, a real function of t, not a vector function. So in order to find the **arc length**, we only need to compute $|\overrightarrow{r}'(t)|$ and then compute an integral on real function.
Let $s(t)$ be part of the arc length, then

$$\frac{ds}{dt}=|\overrightarrow{r}'(t)|$$

It is useful to **parametrize a curve** with respect to arc length $s(t)$ (not t) as

$$\overrightarrow{r}(t(s))$$

because it arises naturally from the shape of the curve and does not depend on a particular coordinate system or a parameterization.

(f) Curvature: how quickly the curve changes directions

i. smooth: $\overrightarrow{r}'(t)$ is continuous and $\overrightarrow{r}'(t)\neq 0$

ii. curvature

$$\kappa=\left|\frac{d\overrightarrow{T}(s)}{ds}\right|=\left|\frac{\overrightarrow{T}'(t)}{\overrightarrow{r}'(t)}\right| \qquad \text{Chain Rule}$$

(g) w