PT Assignment 3

1. Toss two fair coins, blindfolded. Somebody tells you that you tossed at least one Heads. What is the probability that both tosses are Heads?

2. Let $A \subset B$. Express the following probabilities as simply as possible:

$$P(A \mid B), \quad P\left(A \mid B^c\right), \quad P(B \mid A), \quad P\left(B \mid A^c\right).$$

3. Toss a coin 10 times. What is the probability that your first toss is Head, given

   (a) that exactly 7 Heads are tossed;

   (b) that at least 7 Heads are tossed.

4. An urn initially contains 5 white and 7 black balls. Each time a ball is selected, its color is noted and it is replaced in the urn along with 2 other balls of the same color. Compute the probability that

   (a) the first 2 balls selected are black and the next 2 are white;

   (b) of the first 3 balls selected, exactly 2 are black.

5. Suppose that $E_1, E_2, \ldots, E_n$ are independent events. Show that:

   (a) $E_1^c, E_2^c, \ldots, E_n^c$ are also independent events;

   (b) it holds

$$P\left(E_1 \cup E_2 \cup \cdots \cup E_n\right) = 1 - \prod_{i=1}^{n} \left[1 - P\left(E_i\right)\right].$$

6. Roll a die, then select at random, without replacement, as many cards from the deck as the number shown on the die. What is the probability that you get at least one Ace?

7. In my town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 0.5, and given that it is not rainy, there will be heavy traffic with probability 0.25. If it's rainy and there is heavy traffic, I arrive late for work with probability 0.5. On the other hand, the probability of being late is reduced to 0.125 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. You pick a random day.

   (a) What is the probability that it's not raining and there is heavy traffic and I am not late?

   (b) What is the probability that I am late?

   (c) Given that I arrived late at work, what is the probability that it rained that day?

8. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died during the five-year period. Calculate the probability that the participant was a heavy smoker.