2021-22 First Semester
MATH1083 Calculus II (1003)

Assignment 2

Due Date: 11:30am 29/Feb/2021(Wed).

- Write down your **Chinese name** and **student number**. Write neatly on **A4-sized** paper and **show your steps**.
- **Late submissions or answers without details will not be graded.**

1. Using $\epsilon - \delta$ **definition** to prove that the sequence $\{a_n\}$
$$a_n = \frac{1}{e^n}$$
converges.

2. If $\sum a_n$ is convergent and $\sum b_n$ is divergent, show that the series $\sum (b_n - a_n)$ is divergent.
[Hint: proof by **contradiction**]

3. Prove sequence
$$a_n = \frac{2^n n!}{(2n+1)!}$$
is convergent by **squeeze theorem**.

4. Determine whether each imporper integrals is convergent or not, and find the limit if it is convergent.

(a)
$$\int_1^\infty \frac{1}{\sqrt{x}} dx$$

(b)
$$\int_1^\infty \frac{1}{a^x} dx, \qquad a > 1$$

(c)
$$\int_1^\infty \frac{1}{a^x} dx, \qquad a < 1$$

(d)
$$\int_2^\infty \frac{1}{(x-1)(x+2)} dx$$

5. Use **Integral Test** to determine whether the series is convergent or not.
$$\sum_{n=2}^{\infty} \frac{\tan^{-1} n}{1+n^2}$$

6. For the series
$$s = \sum_{n=1}^{\infty} \frac{1}{n^4}$$

(a) Estimate the error if we use s_{10} as an approximation to s.

(b) Find a value of n, so that s_n is within 9×10^{-9} of the sum.

7. For the alternating series

$$s = \sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$$

   (a) Determine whether the series absolutely convergent, conditionally convergent or divergent.

   (b) Is the 100-th partial sum s_{100} an overestimate or underestimate? and explain why.

8. Use the **Ratio Test** to determine whether the series

$$\sum_{n=1}^{\infty} 1 - \frac{2!}{1\cdot 3} + \frac{3!}{1\cdot 3\cdot 5} - \frac{4!}{1\cdot 3\cdot 5\cdot 7} + \cdots + (-1)^n \frac{n!}{1\cdot 3\cdot 5\cdot 7\cdot\cdots\cdot(2n-1)}$$

   is convergent or divergent.

9. Use the **Root Test** to determine whether the series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$$

   is convergent or divergent.