2022-23 First Semester
MATH1053 Linear Algebra I

Assignment 5b

Due Date: 29/Nov/2022 (Tuesday), 11:00 in class.

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

---

1. Determine the dimension of the following subspaces W by finding a basis
   (a) W is the set of all vectors of the form $(a+b, a-b+2c, b, c)^T$, where $a, b, c \in \mathbb{R}$.
   (b) $W = \{(s+4t, t, s, 2s-t) : s \text{ and } t \text{ are real numbers.}\}$

2. Let $\mathbf{x}_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$.
   (a) Show that $\mathbf{x}_1, \mathbf{x}_2$ and $\mathbf{x}_3$ are linearly dependent.
   (b) Show that $\mathbf{x}_1$ and $\mathbf{x}_2$ are linearly independent.
   (c) Determine the dimension of $\mathrm{Span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ by finding a basis.

3. The vectors
$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \mathbf{x}_4 = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}, \mathbf{x}_5 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},$$
   span $\mathbb{R}^3$. Pare down the set $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5\}$ to form a basis for $\mathbb{R}^3$.

4. Find a basis for the null space of matrices in HW4b-Q4, respectively.

5. In $\mathbb{R}^4$, let U be the subspace of all vectors of the form $(u_1, u_2, 0, 0)^T$, and let V be the subspace of all vectors of the form $(0, v_2, v_3, 0)^T$. What are the dimensions of U, V, $U \cap V$ and $U + V$ (optional)? Find a basis for each of these four subspaces.

6. (*Optional!*) Show that if U and V are subspaces of $\mathbb{R}^n$ and $U \cap V = \{\mathbf{0}\}$, then
$$\dim(U \cup V) = \dim U + \dim V.$$