CalebECON2103 Microeconomics

Chapter 5 Exercises

Solutions

1.

a. The expected return, *ER*, of Sam's investment is

$$ER = (0.999)(-1{,}000{,}000) + (0.001)(1{,}000{,}000{,}000) = \$1000.$$

The variance is

$$\sigma^2 = (0.999)(-1{,}000{,}000 - 1000)^2 + (0.001)(1{,}000{,}000{,}000 - 1000)^2\text{, or}$$

$$\sigma^2 = 1{,}000{,}998{,}999{,}000{,}000.$$

b. Suppose the insurance guarantees that Sam will receive the expected return of \$1000 with certainty regardless of the outcome of his SCAM project. Because Sam is risk neutral and because his expected return is the same as the guaranteed return with insurance, the insurance has no value to Sam. He is just as happy with the uncertain SCAM profits as with the certain outcome guaranteed by the insurance policy. So Sam will not pay anything for the insurance.

c. The entry of the Japanese lowers Sam's probability of a high payoff. For example, assume that the probability of the billion-dollar payoff cut in half. Then the expected outcome is:

$$ER = (0.9995)(-\$1{,}000{,}000) + (0.0005)((\$1{,}000{,}000{,}000) = -\$499{,}500.$$

Therefore you should raise the policy premium substantially. But Sam, not knowing about the Japanese entry, will continue to refuse your offers to insure his losses.

2.

a. Natasha is risk averse. To show this, assume that she has \$10,000 and is offered a gamble of a \$1000 gain with 50% probability and a \$1000 loss with 50% probability. The utility of her current income of \$10,000 is $u(10) = \sqrt{10(10)} = 10$. Her expected utility with the gamble is:

$$EU = (0.5)(\sqrt{10(11)}) + (0.5)(\sqrt{10(9)}) = 9.987 < 10.$$

She would avoid the gamble. If she were risk neutral, she would be indifferent between the \$10,000 and the gamble, and if she were risk loving, she would prefer the gamble.

You can also see that she is risk averse by noting that the square root function increases at a decreasing rate (the second derivative is negative), implying diminishing marginal utility.

b. The utility of her current salary is $\sqrt{10(40)} = 20$. The expected utility of the new job is

$$EU = (0.6)(\sqrt{10(44)}) + (0.4)(\sqrt{10(33)}) = 19.85,$$

which is less than 20. Therefore, she should not take the job. You can also determine that Natasha should reject the job by noting that the expected value of the new job is only \$39,600, which is less than her current salary. Since she is risk averse, she should never accept a risky salary with a lower expected value than her current certain salary.

c. This question assumes that Natasha takes the new job (for some unexplained reason). Her expected salary is 0.6(44,000) + 0.4(33,000) = \$39,600. The risk premium is the amount Natasha would be willing to pay so that she receives the expected salary for certain rather than the risky salary in her new job. In (b) we determined that her new job has an expected utility of 19.85. We need to find the certain salary that gives Natasha the same utility of 19.85, so we want to find I such that $u(I) = 19.85$. Using her utility function, we want to solve the following equation: $\sqrt{10I} = 19.85$. Squaring both sides, $10I = 394.0225$, and $I = 39.402$. So Natasha would be equally happy with a certain salary of \$39,402 or the uncertain salary with an expected value of \$39,600. Her risk premium is \$39,600 – 39,402 = \$198. Natasha would be willing to pay \$198 to guarantee her income would be \$39,600 for certain and eliminate the risk associated with her new job.

3.

a. The expected value of the return on investment A is

$$EV = (0.1)(300) + (0.8)(250) + (0.1)(200) = \$250.$$

The variance on investment A is

$$\sigma^2 = (0.1)(300 - 250)^2 + (0.8)(250 - 250)^2 + (0.1)(200 - 250)^2 = 500,$$

and the standard deviation on investment A is $\sigma = \sqrt{500} = \$22.36$.

The expected value of the return on investment B is

$$EV = (0.3)(300) + (0.4)(250) + (0.3)(200) = \$250.$$

The variance on investment B is

$$\sigma^2 = (0.3)(300 - 250)^2 + (0.4)(250 - 250)^2 + (0.3)(200 - 250)^2 = 1500,$$

and the standard deviation on investment B is $\sigma = \sqrt{1500} = \$38.73$.

b. Jill's expected utility from investment *A* is

$$EU = (0.1)(5 \times 300) + (0.8)(5 \times 250) + (0.1)(5 \times 200) = 1250.$$

Jill's expected utility from investment *B* is

$$EU = (0.3)(5 \times 300) + (0.4)(5 \times 250) + (0.3)(5 \times 200) = 1250.$$

Since both investments give Jill the same expected utility she will be indifferent between the two. Note that Jill is risk neutral, so she cares only about expected values. Since investments *A* and *B* have the same expected values, she is indifferent between them.

c. Ken's expected utility from investment *A* is

$$EU = (0.1)(5)\sqrt{300} + (0.8)(5)\sqrt{250} + (0.1)(5)\sqrt{200} = 78.98.$$

Ken's expected utility from investment *B* is

$$EU = (0.3)(5)\sqrt{300} + (0.4)(5)\sqrt{250} + (0.3)(5)\sqrt{200} = 78.82.$$

Ken will choose investment *A* because it has a slightly higher expected utility. Notice that Ken is risk averse, and since the two investments have the same expected return, he prefers the investment with less variability.

d. Laura's expected utility from investment A is

$$EU = (0.1)(5 \times 300^2) + (0.8)(5 \times 250^2) + (0.1)(5 \times 200^2) = 315{,}000.$$

Laura's expected utility from investment *B* is

$$EU = (0.3)(5 \times 300^2) + (0.4)(5 \times 250^2) + (0.3)(5 \times 200^2) = 320{,}000.$$

Laura will choose investment *B* since it has a higher expected utility. Notice that Laura is a risk lover, and since the two investments have the same expected return, she prefers the investment with greater variability.