2021-22 Second Semester MATH1083 Calculus II

Assignment 1 (1002)

Due Date: 22/Feb/2021(Wed), before 11:30am.

- Write down your **Chinese name** and **student number**. Write neatly on **A4-sized** paper and **show your steps**.
- **Late submissions or answers without details will not be graded.**

1. What is the difference between a sequence and a series?

2. Find the value of c such that
$$\sum_{n=0}^{\infty} e^{nc} = 10$$

3. Determine whether the following sequence converges or diverges. If it converges, find the limit.

   (a)
   $$a_n = (-1)^n \frac{e^n}{n!}$$

   (b)
   $$b_n = n^{-\frac{1}{n}}$$

4. Show the sequence $\{a_n\}$ given by
$$a_1 = \sqrt{2}, \qquad a_{n+1} = \sqrt{2 + a_n}$$
   is monotonic and bounded. Apply Monotonic Sequence Theorem to show
   $$\lim_{n\to\infty} a_n$$
   exists, and find the limit.

5. Determine whether the following sequence converges or diverges.
$$\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$$

6. Show that if $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ is convergent, then
$$\sum_{n=1}^{\infty} \ln(1 + a_n)$$
   is convergent.