2022-23 First Semester
MATH1053 Linear Algebra I

Assignment 1a

Due Date: Week 3, see in HW1b.

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. Let $A = \begin{pmatrix} -2 & 1 & 3 \\ 4 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 1 \\ -4 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & -1 \\ 0 & 6 \\ 2 & 3 \end{pmatrix}$ and $D = \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ -1 & -1 \end{pmatrix}$.
   Find each of the following matrices.
   (a) $2BD - AC$
   (b) DD^T

2. Let $A = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -2 \\ 4 & 1 \end{pmatrix}$, and $C = \begin{pmatrix} 3 & -2 & 0 & 5 \\ 1 & 0 & -3 & 4 \end{pmatrix}$.
   (a) Does the matrix $D = ABC$ exist? If so, what is the $(4,3)$-entry d_{34} of D?
   (b) Does the matrix $H = CAB$ exist? If so, what is the $(2,1)$-entry h_{21} of H?

3. Let a, b, c, d be real numbers.

$$A = \begin{pmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \end{pmatrix}, \quad B = \begin{pmatrix} -a & -b \\ -c & -d \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

   Compute AB and BA.

4. A square matrix A is said to be skew symmetric if $A^T = -A$. Show that if a matrix is skew symmetric, then its diagonal entries must all be 0.

5. An $n \times n$ matrix A is called skew-symmetric if $A^T = -A$.

   (a) Let B be an $n \times n$ matrix. Prove that $B - B^T$ is skew-symmetric.

   (b) Let B be an $n \times n$ matrix. Prove that $B + B^T$ is symmetric.

   (c) Let B be an $n \times n$ matrix. Prove that B can be written as the sum of a symmetric matrix and a skew-symmetric matrix. Also show that such decomposition is unique.

6. Let A and B be symmetric $n \times n$ matrices. Prove that $AB = BA$ **if and only if** AB is also symmetric.

7. Let $A = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Let A^2 represent AA.

   (a) Compute A^2 and A^3.

   (b) What is A^n for any positive integer n? Prove your answer by mathematical induction.