FINM3033 Risk Management in Finance

Solution to Assignment 2

Problem 1.

The most recent estimate of the daily volatility of the dollar-sterling exchange rate is 0.6% and the exchange rate at 4:00 p.m. yesterday was 1.5000. The parameter λ in the EWMA model is 0.9. Suppose that the exchange rate at 4:00 p.m. today proves to be 1.4950. How would the estimate of the daily volatility be updated?

Solution:

By the EWMA model, we can update the following day's variance by ${\sigma^2}_{n+1} = \lambda\sigma_n^2 + (1-\lambda)u_n^2$

where $u_n = \frac{S_n - S_{n-1}}{S_{n-1}} = \frac{1.4950-1.5000}{1.5000} = -3.3333 \times 10^{-3}$.

Then ${\sigma^2}_{n+1} = 0.9 * 0.006^2 + (1-0.9) * u_n^2 = 3.3511 \times 10^{-5}$

The daily volatility $\sigma_{n+1} = \sqrt{{\sigma^2}_{n+1}} \approx 0.5789\%$

Problem 2.

Suppose that the parameters in a GARCH(1,1) model are $\alpha = 0.03$, $\beta = 0.95$ and $\omega = 0.000002$.

a) What is the long-run average volatility?

b) If the current volatility is 1.5% per day, what is your estimate of the volatility in 20, 40, and 60 days?

(c) What volatility should be used to price 20-, 40-, and 60-day options?

(d) Suppose that there is an event that increases the volatility from 1.5% per day to 2% per day. Estimate the effect on the volatility in 20, 40, and 60 days.

(e) Estimate by how much the event increases the volatilities used to price 20-, 40-, and 60-day options.

Solution: (a) The long-run average variance, V_L, is

$$\frac{\omega}{1-\alpha-\beta} = \frac{0.000002}{0.02} = 0.0001$$

The long run average volatility is $\sqrt{0.0001} = 0.01$ or 1% per day.

(b) The expected variance in 20 days is

$$0.0001 + 0.98^{20}\left(0.015^2 - 0.0001\right) = 0.000183$$

The expected volatility per day is therefore $\sqrt{0.000183} = 0.0135$ or 1.35%. Similarly the expected volatilities in 40 and 60 days are 1.25% and 1.17%, respectively.

(c) In equation (10.15)$a = \ln(1/0.98) = 0.0202$. The variance used to price 20 -day options is

$$252\left[0.0001 + \frac{1-e^{-0.0202\times 20}}{0.0202\times 20}\left(0.015^2 - 0.0001\right)\right] = 0.051$$

so that the volatility is 22.61%. Similarly, the volatilities that should be used for 40 - and 60 -day options are 21.63% and 20.85% per annum, respectively.

(d) From equation (10.14) the expected variance in 20 days is

$$0.0001 + 0.98^{20}\left(0.02^2 - 0.0001\right) = 0.0003$$

The expected volatility per day is therefore $\sqrt{0.0003} = 0.0173$ or 1.73%. Similarly the expected volatilities in 40 and 60 days are 1.53% and 1.38% per day, respectively.

(e) When today's volatility increases from 1.5% per day (23.81% per year) to 2% per day (31.75% per year) the equation (10.16) gives the 20 -day volatility increase as

$$\frac{1-e^{-0.0202\times 20}}{0.0202\times 20} \times \frac{23.81}{22.61} \times (31.75 - 23.81) = 6.88$$

or 6.88% bringing the volatility up to 29.49%. Similarly the 40 - and 60 -day volatilities increase to 27.63% and 26.11%.

Problem 3.

Suppose that each of two investments has a 4% chance of a loss of \$10 million, a 2% chance of a loss of \$1 million, and a 94% chance of a profit of \$1 million. The investments are independent of each other.

(a) What is the VaR for one of the investments when the confidence level is 95%?

(b) What is the expected shortfall for one of the investments when the confidence level is 95%?

(c) What is the VaR for a portfolio consisting of the two investments when the confidence level is 99%?

(d) What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is 95%?

(e) Show that in this example VaR does not satisfy the subadditivity condition, whereas expected shortfall does.

Solution:

Let L_1 and L_2 be the loss of these two investments respectively. Then $L = L_1 + L_2$ is the total loss.

(a) A loss of \$1 million extends from the 94 percentile point of the loss distribution to the 96 percentile point. The 95%VaR is therefore \$1 million.

(b) Let F_1 be the distribution function of L_1. Then $F_1(1) = P(L_1 \leq 1) = 0.96$. Then

$$\lambda = \frac{F_1(1) - 0.95}{1 - 0.95} = \frac{0.96 - 0.95}{0.05} = 0.2.$$

The expected shortfall for one of the investments is

$$\begin{aligned}\mathrm{ES}_{0.95}(L_1) &= \lambda \cdot \mathrm{VaR}_{0.95}(L_1) + (1-\lambda) \cdot E[L_1 | L_1 > \mathrm{VaR}_{0.95}(L_1)] \\ &= 0.2 + 0.8 \times 10 = 8.2.\end{aligned}$$

(c) For a portfolio consisting of the two investments there is a $0.04 \times 0.04 = 0.0016$ chance that the loss is \$20 million; there is a $2 \times 0.04 \times 0.02 = 0.0016$ chance that the loss is \$11 million; there is a $2 \times 0.04 \times 0.94 = 0.0752$ chance that the loss is \$9 million; there is a $0.02 \times 0.02 = 0.0004$ chance that the loss is \$2 million; there is a $2 \times 0.2 \times 0.94 = 0.0376$ chance that the loss is zero; there is a $0.94 \times 0.94 = 0.8836$ chance that the profit is \$2 million. It follows that the 95% VaR is \$9 million.

(d) Let F be the distribution function of L. Then $F(9) = P(L \leq 9) = 1 - P(L > 9) = 1 - 0.0016 - 0.0016 = 0.9968$. Then

$$\lambda = \frac{F(9) - 0.95}{1 - 0.95} = \frac{0.9968 - 0.95}{0.05} = 0.936.$$

The expected shortfall for one of the investments is

$$\begin{aligned}\mathrm{ES}_{0.95}(L) &= \lambda \cdot \mathrm{VaR}_{0.95}(L) + (1-\lambda) \cdot E[L | L > \mathrm{VaR}_{0.95}(L)] \\ &= 0.936 \times 9 + 0.064 \times \frac{0.0016 \times 20 + 0.0016 \times 11}{0.0016 + 0.0016} \\ &= 0.936 \times 9 + 0.064 \times 15.5 = 9.416.\end{aligned}$$

(e) VaR does not satisfy the subadditivity condition because $9 > 1 + 1$. However, expected shortfall does because $9.416 < 8.2 + 8.2$.

Problem 4.

Suppose that the change in the value of a portfolio over a one-day time period is normal with a mean of zero and a standard deviation of \$2 million; what is

(a) the one-day 97.5% VaR,

(b) the five-day 97.5% VaR, and

(c) the five-day 99% VaR?

Solution: The loss distribution of the portfolio follows a normal distribution, which can be expressed as Loss $\sim$ N(0,4).

a) Set one day 97.5% VaR as X. We have:

$Prob(Loss \leq \text{X}) = 97.5\%.$
$Prob(\frac{Loss-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}) = 97.5\%.$
$Prob(\phi \leq \frac{X-0}{2}) = 97.5\%.$
$\frac{X}{2} = N^{-1}(97.5\%) = 1.96.$
$X = 3.92.$
The one-day 97.5%VaR is 3.92.
b) Since the portfolio follows a zero-mean normal distribution,
T-day VaR = 1-day VaR $\times\sqrt{T}$
The 5-day 97.5% VaR = $3.92\times\sqrt{5} = 8.77$
c) Set the 1-day 99% VaR as X.

$Prob(\phi \leq \frac{X-0}{2}) = 99\%.$
$\frac{X}{2} = 2.33.$
$X = 4.66$
The 5-day 99% VaR = $4.66\times\sqrt{5} = 10.42$