2022-23 Second Semester

MATH1063 Linear Algebra II (1003)

Assignment 9

Due Date: **24/May/2023 (Wednesday), 15:00 @T3-602-R25-H2.**

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. Locate all critical points and classify them

   (a) $f(x, y) = e^{-x^2}(y^2 + 1)$

   (b) $f(x, y) = \frac{1}{3}x^3 + xy^2 - 4xy + 1$

   (c) $f(x, y) = x\sin(y)$

2. Show that for every symmetric $n \times n$ matrix A, there exists a symmetric $n \times n$ matrix B such that $B^3 = A$.

3. Let

$$A = \begin{pmatrix} 1 & -1/2 \\ -1/2 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

   (a) Show that A is positive definite and that $\mathbf{x}^T A\mathbf{x} = \mathbf{x}^T B\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^2$.

   (b) The definition of definite matrix can be extended to non-symmetric matrices. But the eigenvalue test and determinant test will no longer work for non-symmetric matrices.
   Use the definition to show that B is positive definite, while B^2 is not.

4. Let A be an $m \times n$ matrix with rank n. Show that the matrix A^TA is symmetric positive definite.

5. Let A be an $n \times n$ symmetric matrix, if A is positive definite, show that A can be written as $A = BB^T$, where B is an $n \times n$ matrix with orthogonal columns.