2023-24 First Semester

MATH2023 Ordinary and Partial Differential Equations (1002)

Assignment 3

Due Date: **12/Oct/2023(Thursday), on or before 16:00, in tutorial class.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without details will not be graded.**

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1. Find the solution of the given initial value problem.

   (a) $y'' + 3y' = 0, \quad y(0) = -2, \quad y'(0) = 3$

   (b) $y'' + 2y' - 3y = 0, \quad y(0) = 0, \quad y'(0) = 1$

   (c) $y'' - 10y' + 24y = 0, \quad y(0) = \alpha, \quad y'(0) = \beta.$

2. (a) Verify that $y_1(t) = 1$ and $y_2(t) = t^{1/2}$ are two solutions of the differential equation

$$yy'' + (y')^2 = 0, \quad \text{for } t > 0.$$

   (b) Then show that $y = c_1 + c_2 t^{1/2}$ is not, in general, a solution of this equation.

   (c) Explain why this result does not contradict the principle of superposition?

3. (Equations with the Dependent Variable Missing.) Solve the following ODE.

   (a) $y'' + t(y')^2 = 0.$

   (b) $y'' + y' = e^{-t}$

   Hint: For a second order DE in the form $y'' = f(t, y')$

   1. Let $v = y'$, then $\dfrac{dv}{dt} = y''$.
   2. Then the original DE becomes a first order equation of the form $v' = f(t, v)$.
   3. Solve for v first, then solve $y' = v$ for y.

4. **Exact Equations.** The equation

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

   is said to be exact if it can be written in the form

$$[P(x)y']' + [f(x)y]' = 0,$$

where $f(x)$ is to be determined in terms of $P(x)$, $Q(x)$, and $R(x)$. The latter equation can be integrated once immediately, resulting in a first order linear equation for y that can be solved as in Section 2.1. By equating the coefficients of the preceding equations and then eliminating $f(x)$, show that a necessary condition for exactness is

$$P''(x) - Q'(x) + R(x) = 0.$$

It can be shown that this is also a sufficient condition.

Determine whether the given equation is exact. If it is, then solve the equation.

(a) $x^2y'' + xy' - y = 0, \quad x > 0.$

(b) $y'' + 2x^2y' + xy = 0.$

5. If the following Bessel's equation has y_1 and y_2 as a fundamental set of solutions

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0, \quad \nu \in \mathbb{R},$$

and if $W(y_1, y_2)(1) = 1$. Find the Wronskian $W(y_1, y_2)(x)$.