Chapter 7 Solution

1.

For AR(2) model, we can estimate the parameters using the mothod of moments. The Yule-Walker equation of AR(2):

$$\rho_1 = \phi_1 + \phi_2 \rho_1$$
$$\rho_2 = \phi_1 \rho_1 + \phi_2$$

Replace ρ with r_1 and solve the equation to obtain the estimator for ϕ_1 and ϕ_2 .

$$\hat{\phi}_1 = 1.11 \qquad and \qquad \hat{\phi}_2 = -0.389$$

Then, $\theta_0 = \hat{\mu}(1 - \hat{\phi}_1 - \hat{\phi}_2) = 0.558$ and $\sigma_e^2 = (1 - \hat{\phi}_1 r_1 - \hat{\phi}_2 r_2)s^2 = 1.5325$.

2.

The estimator of ϕ generated by maximum likelihood and least squares is approximately unbiased and normally distributed with $Var(\hat{\phi}) \approx \frac{1-\phi^2}{n}$.

Estimate $\phi = \rho_1$ with 95% confidence that our estimation error is no more than ± 0.1, which means

$$1.96(\sqrt{\frac{1-0.7^2}{n}}) \leq 0.1$$

Thus, n should be larger than 195.9216, ie 196.

3.

(a)

The variances and correlation of the maximum likelihood estimators of ϕ and θ :

$$Var(\hat{\phi}) \approx [\frac{1-\phi^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = [\frac{1-0.5^2}{48}][\frac{1-0.50.45}{0.5-0.45}]^2 = 3.75$$

$$Var(\hat{\theta}) \approx [\frac{1-\theta^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = [\frac{1-0.45^2}{48}][\frac{1-0.50.45}{0.5-0.45}]^2 = 3.99$$

$$Corr(\hat{\phi}, \hat{\theta}) \approx \frac{\sqrt{(1-\phi^2)(1-\theta^2)}}{1-\phi\theta} = \frac{\sqrt{(1-0.5^2)(1-0.45^2)}}{1-0.50.45} = 0.998$$

The standard errors are quite large relative to the quantities being estimated. This is because of the near cancellation of the AR and MA parameters. This is a rather unstable model approaching white noise. The estimators are very highly correlated.

(b)

Repeat part(a)but now with $n = 120$.

$$Var(\hat{\phi}) \approx [\frac{1-\phi^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = [\frac{1-0.5^2}{120}][\frac{1-0.50.45}{0.5-0.45}]^2 = 1.5$$

$$Var(\hat{\theta}) \approx [\frac{1-\theta^2}{n}][\frac{1-\phi\theta}{\phi-\theta}]^2 = [\frac{1-0.45^2}{120}][\frac{1-0.50.45}{0.5-0.45}]^2 = 1.6$$

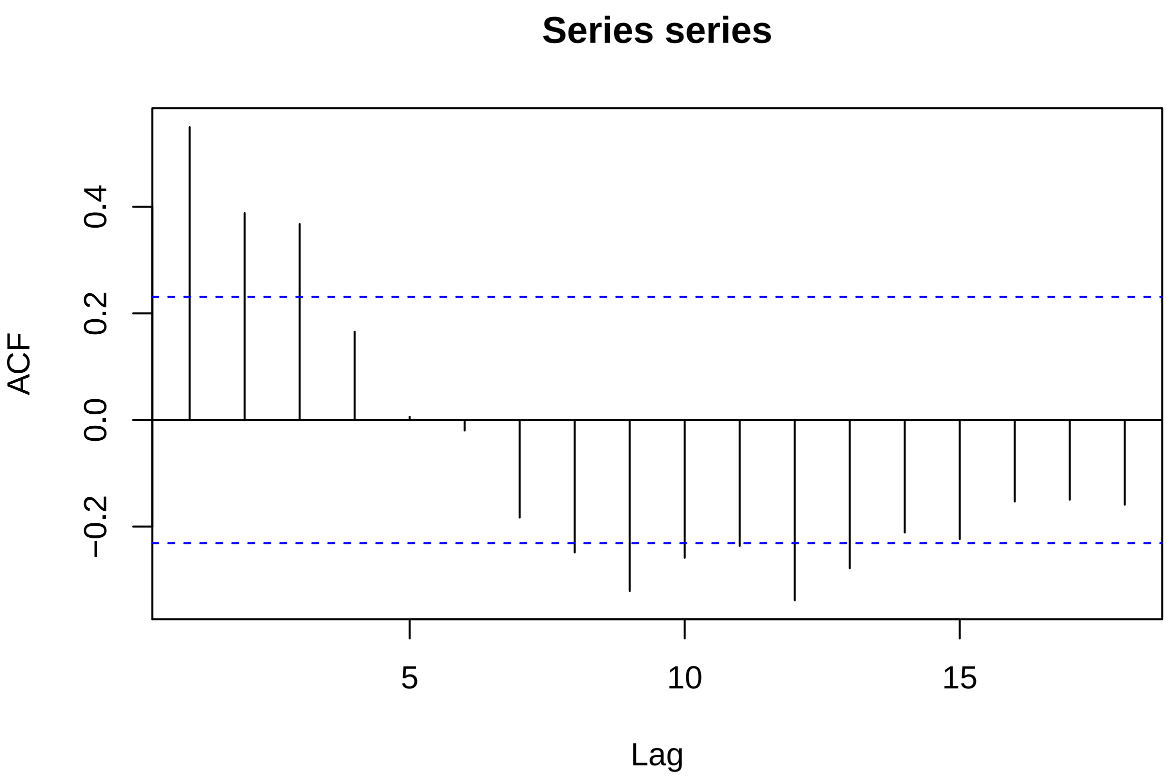
$$Corr(\hat{\phi},\hat{\theta}) \approx \frac{\sqrt{(1-\phi^2)(1-\theta^2)}}{1-\phi\theta} = \frac{\sqrt{(1-0.5^2)(1-0.45^2)}}{1-0.50.45} = 0.998$$

The variance are only smaller and the correlation does not change with n.

4.

(a)

```
set.seed(54321); series=arima.sim(n=72,list(ar=0.7,ma=-0.4))
acf(series)$acf
```




```
## , , 1
##
##              [,1]
##  [1,]  0.549357167
##  [2,]  0.388096245
##  [3,]  0.367623116
##  [4,]  0.165666847
##  [5,]  0.006112673
```

```
##  [6,] -0.019838269
##  [7,] -0.183102515
##  [8,] -0.248674930
##  [9,] -0.320889586
## [10,] -0.258465587
## [11,] -0.236178041
## [12,] -0.338259006
## [13,] -0.278153979
## [14,] -0.211176145
## [15,] -0.223407235
## [16,] -0.153101240
## [17,] -0.149556740
## [18,] -0.158915347
```

So $\hat{\phi} = \frac{r_2}{r_1} = \frac{0.388096245}{0.549357167} \approx 0.7065$. Recall that $r_1 = \frac{(1-\theta\hat{\phi})(\hat{\phi}-\theta)}{1-2\theta\hat{\phi}+\theta^2}$. The solutions for θ is approximately 0.16572.

(b)

```
arima(series,order=c(1,0,1),method='CSS')
```

```
##
## Call:
## arima(x = series, order = c(1, 0, 1), method = "CSS")
##
## Coefficients:
##          ar1      ma1  intercept
##       0.7655  -0.3605    -0.2444
## s.e.  0.0961   0.1480     0.3075
##
## sigma^2 estimated as 0.868:  part log likelihood = -97.07
```

The estimate of ϕ here is larger than the one obtained by the method-of-moments. However, taking standard errors into account, the two are not significantly different.

(c)

```
arima(series,order=c(1,0,1),method='ML')
```

```
##
## Call:
## arima(x = series, order = c(1, 0, 1), method = "ML")
##
## Coefficients:
##          ar1      ma1  intercept
##       0.7771  -0.3055    -0.0203
## s.e.  0.1190   0.1647     0.3409
##
## sigma^2 estimated as 0.9147:  log likelihood = -99.19,  aic = 204.39
```

The CSS and ML estimates are very close to each other and easily within two standard errors of their true values.

5.

(a)

```r
set.seed(54321)
series=arima.sim(n=48,list(ar=0.7,ma=0.6),innov=rchisq(48,9))
eacf(series)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x o o o o o o o o o  o  o  o
## 1 x o o o o o o o o o o  o  o  o
## 2 x x o o o o o o o o o  o  o  o
## 3 x x x o o o o o o o o  o  o  o
## 4 x o o o o o o o o o o  o  o  o
## 5 o o o o o o o o o o o  o  o  o
## 6 o x o o o o o o o o o  o  o  o
## 7 x o o o o o o o o o o  o  o  o
```

The sample eacf gives a clear indication of the mixed ARMA(1,1) model.

(b)

```r
arima(series,order=c(1,0,1))
```

```
##
## Call:
## arima(x = series, order = c(1, 0, 1))
##
## Coefficients:
##          ar1     ma1  intercept
##       0.6413  0.8026    45.8948
## s.e.  0.1420  0.1164     3.7758
##
## sigma^2 estimated as 29.1:  log likelihood = -150.2,  aic = 306.4
```

Relative to their standard errors, the estimates are not significantly different from their true values.

6.

(a)

```r
data(deere3); arima(deere3,order=c(1,0,0))
```

```
##
## Call:
## arima(x = deere3, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.5255   124.3832
## s.e.  0.1108   394.2067
##
## sigma^2 estimated as 2069355:  log likelihood = -495.51,  aic = 995.02
```

The $\hat{\phi}_1$ coefficient is significantly different from zero.

(b)

```r
arima(deere3, order=c(2,0,0))
```

```
##
## Call:
## arima(x = deere3, order = c(2, 0, 0))
##
## Coefficients:
##          ar1     ar2  intercept
##       0.5211  0.0083   123.2979
## s.e.  0.1310  0.1315   397.6134
##
## sigma^2 estimated as 2069208:  log likelihood = -495.51,  aic = 997.01
```

The $\hat{\phi}_2$ is not statistically significant so the AR(1) model still looks good.

7.

(a)

```r
data(robot); arima(robot,order=c(1,0,0))
```

```
##
## Call:
## arima(x = robot, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.3074     0.0015
## s.e.  0.0528     0.0002
##
## sigma^2 estimated as 6.482e-06:  log likelihood = 1475.54,  aic = -2947.08
```

Notice that both the $\hat{\phi}_1$ and intercept are significantly different from zero statistically.

(b)

```r
arima(robot,order=c(0,1,1))
```

```
##
## Call:
## arima(x = robot, order = c(0, 1, 1))
##
## Coefficients:
##           ma1
##       -0.8713
## s.e.   0.0389
##
## sigma^2 estimated as 6.069e-06:  log likelihood = 1480.95,  aic = -2959.9
```

The $\theta_1 = 0.8713$ coefficient is significantly different from zero statistically.

(c)

The nonstationary IMA(1,1) model has a slightly smaller AIC value but the log likelihoods and AIC values are very close to each other.