2022-23 First Semester
MATH1053 Linear Algebra I

Assignment 1b

Due Date: Submit HW1a+1b together on or before 20/Sep/2022 (Tuesday), 11:00 in class.

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. Let A be an $n \times n$ matrix and let $\mathbf{x}$ and $\mathbf{y}$ be vectors in $\mathbb{R}^n$. Show that if $A\mathbf{x} = A\mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$, then the matrix A must be singular.

2. Prove that if A is nonsingular, then A^T is nonsingular and $(A^T)^{-1} = (A^{-1})^T$.

3. Show that if A is a symmetric invertible matrix, then A^{-1} is also symmetric.

4. Suppose A is a square matrix of order n such that $A^2 = O$.
   (a) Let α, β be scalars. Show that $(I_n + \alpha A)(I_n + \beta A) = I_n + (\alpha + \beta)A$.
   (b) By taking $\beta = -\alpha$ in part (a), show that $I_n + \alpha A$ is nonsingular and find its inverse.

5. True or false. If false, give a counterexample. If true, explain or prove your answer briefly.
   (a) If $A = A^{-1}$, then A must be equal to either I or $-I$.
   (b) If A and B are singular $n \times n$ matrices, then $A + B$ is also singular.
   (c) If A and B are nonsingular matrices, then $(AB)^T$ is nonsingular and
   $$((AB)^T)^{-1} = (A^{-1})^T(B^{-1})^T$$

6. Write an augmented matrix for each of the following systems

   (a). $\begin{cases} 2x_1 - x_2 = 3 \\ -4x_1 + 2x_2 = -6 \end{cases}$ (b). $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ x_1 + 3x_2 + 2x_3 = 1 \\ -x_2 + x_3 = 0 \end{cases}$

7. (a) Perform elementary row operations on the augmented matrices you obtained in Q6, in order to find an equivalent triangular system, respectively.
   (b) Based on the results of part (a), determine if the systems are consistent or not.