2022-23 Second Semester

MATH1063 Linear Algebra II (1003)

Assignment 7

Due Date: **28/Apr/2023 (Friday), 09:00 in tutorial class.**

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. Let
$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
   (a) Diagonalize A.

   (b) Is A orthogonally diagonalizable? Why?

2. Let $A = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{pmatrix}$. Orthogonally diagonalize A.

3. Let A be an $n \times n$ Hermitian matrix and let $\mathbf{x} \in \mathbb{C}^n$. Show that if $c = \mathbf{x}^H A \mathbf{x}$, then c is real.

4. Let A be an $n \times n$ matrix with an eigenvalue λ and let $\mathbf{x}$ be an eigenvector of A with respect to λ. Let S be a nonsingular $n \times n$ matrix and let α be a scalar. Show that if
$$B = \alpha I - SAS^{-1}, \quad \mathbf{y} = S\mathbf{x}$$
   then $\mathbf{y}$ is an eigenvector of B. Then determine the eigenvalue of B corresponding to $\mathbf{y}$.

5. Let A be a real-valued square matrix and λ be an eigenvalue of A. Show that for any positive integer k,
$$\mathrm{N}(A - \lambda I)^k \subseteq \mathrm{N}(A - \lambda I)^{k+1}.$$

6. For a real-valued matrix A, show that if A is orthogonally diagonalizable, then A must be symmetric.
   Remarks: This exercise completes the spectral theorem for real symmetric matrices. For a real-valued matrix A, it is orthogonally diagonalizable **if and only if** A is symmetric.

7. A matrix is *skew-Hermitian* if $A^H = -A$. Show that if A is skew-Hermitian and λ is an eigenvalue of A, then λ is purely imaginary.

8. Let A be a Hermitian matrix. The eigenvalues are $\lambda_1, \cdots, \lambda_n$ with the corresponding orthonormal eigenvectors $\mathbf{u}_1, \cdots, \mathbf{u}_n$. Show that
$$A = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^H + \lambda_2 \mathbf{u}_2 \mathbf{u}_2^H + \cdots + \lambda_n \mathbf{u}_n \mathbf{u}_n^H.$$