FINM3133 Time Series for Finance and Macroeconomics

Chapter 4 Exercises

1. Verify that for an MA(1) process

$$\max_{-\infty<\theta<\infty} \rho_1 = 0.5 \qquad \text{and} \qquad \min_{-\infty<\theta<\infty} \rho_1 = -0.5$$

2. Show that when θ is replaced by $1/\theta$, the autocorrelation function for an MA(1) process does not change.

3. Suppose that $\{Y_t\}$ is an AR(1) process with $-1 < \phi < +1$.

   (a) Find the autocovariance function for $W_t = \bigtriangledown Y_t = Y_t - Y_{t-1}$ in terms of ϕ and σ_e^2.

   (b) In particular, show that $Var(W_t) = 2\sigma_e^2/(1+\phi)$.

4. Describe the important characteristics of the autocorrelation function for the following models: (a) MA(1), (b) MA(2), (c) AR(1), (d) AR(2), and (e) ARMA(1,1).

5. For the ARMA(1,2) model $Y_t = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2}$, show that

   (a) $\rho_k = 0.8\rho_{k-1}$ for $k > 2$.

   (b) $\rho_2 = 0.8\rho_1 + 0.6\sigma_e^2/\gamma_0$.

6. Consider an MA(6) model with $\theta_1 = 0.5, \theta_2 = -0.25, \theta_3 = 0.125, \theta_4 = -0.0625, \theta_5 = 0.03125$, and $\theta_6 = -0.015625$. Find a much simpler model that has nearly the same ψ-weights.

7. Consider an MA(7) model with $\theta_1 = 1, \theta_2 = -0.5, \theta_3 = 0.25, \theta_4 = -0.125, \theta_5 = 0.0625, \theta_6 = -0.03125$, and $\theta_7 = 0.015625$. Find a much simpler model that has nearly the same ψ-weights.