PT Assignment 13

1. Find the moment generating function of X

(a) Binomial(n, p)

(b) Poisson(λ)

(c) Negative Binomial (r, p)

(d) Uniform (a, b)

(e) Gamma (α, λ)

(f) Normal (μ, σ^2)

2. You are given the joint moment generating function of 2 random variables, X and Y:

$$M_{X,Y}(s,t) = \frac{1}{1-2s-3t+6st}$$

for $s < \frac{1}{2}$ and $t < \frac{1}{3}$. Find $P\left(\min(X,Y) > 0.95\right)$.

3. Let X_1 and X_2 be random variables with joint moment generating function

$$M_{X_1,X_2}(t_1,t_2) = 0.3 + 0.1e^{t_1} + 0.2e^{t_2} + 0.4e^{t_1+t_2}.$$

What is $E[2X_1 - X_2]$?

4. Let X and Y be identically distributed independent random variables such that the moment generating function of $X + Y$ is

$$M(t) = 0.09e^{-2t} + 0.24e^{-t} + 0.34 + 0.24e^{t} + 0.09e^{2t}, -\infty < t < \infty.$$

Calculate P[$X \leq 0$].

5. Let X_1 , $X_2, \cdots, X_n$ be i.i.d. $Exponential(\lambda)$ random variables with $\lambda = 1$. Let

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}.$$

How large n should be such that

$$P\left(0.9 \leq \overline{X} \leq 1.1\right) \geq 0.95?$$

6. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$$\mathrm{E}[X] = 50,\ \mathrm{E}[Y] = 20,\ \mathrm{Var}[X] = 50,\ \mathrm{Var}[Y] = 30,\ \mathrm{Cov}(X, Y) = 10.$$

The totals of hours that different individuals watch movies and sporting events during the three months are mutually independent.

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this three-month period.

Approximate the value of $P[T < 7100]$.

7. Let $X_1, \cdots, X_{100}$ and $Y_1, \cdots, Y_{100}$ be independent random samples from uniform distributions on the intervals $[-10\sqrt{3}, 10\sqrt{3}]$ and $[-30, 30]$, respectively. According to the Central Limit Theorem, what is the approximate value of $P\left(\bar{X} - \bar{Y} < 1\right)$?