ECON2113 Macroeconomics

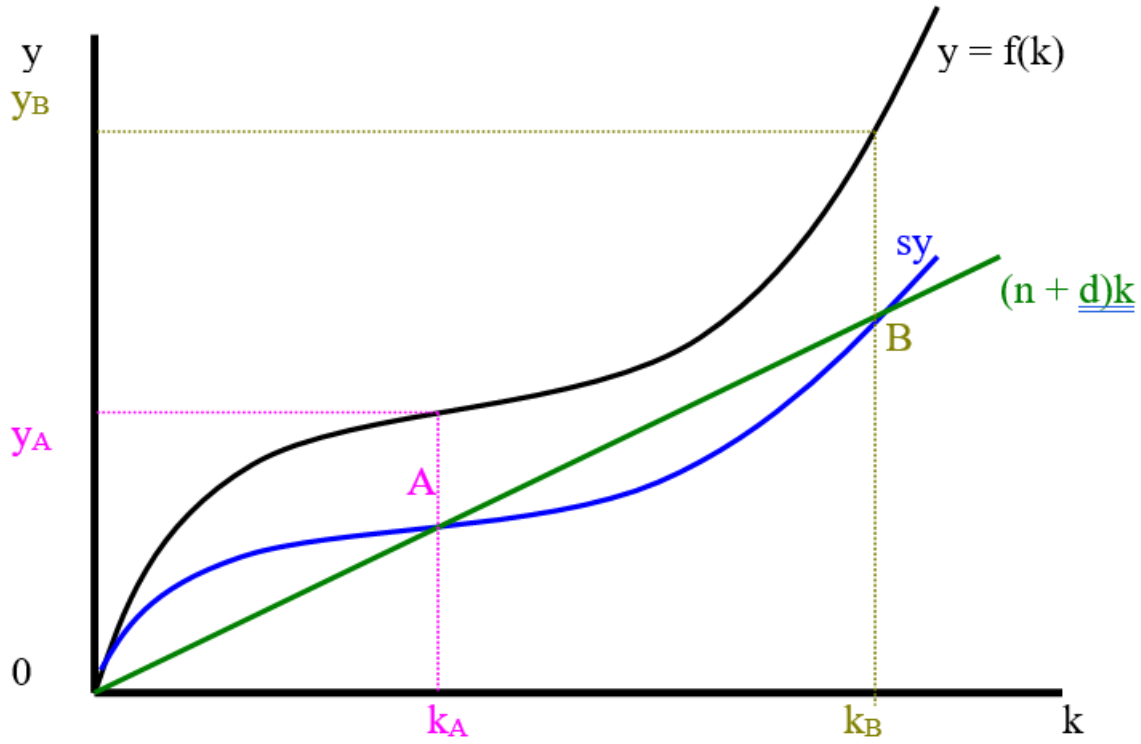
Chapter 4 Exercises

Solutions

1.

a. A production function that displays both a diminishing and a constant marginal product of capital can be displayed by drawing a curved line (as in an exogenous growth model), followed by an upward-sloping line (as in an endogenous growth model). Such a graph is depicted below.

b.



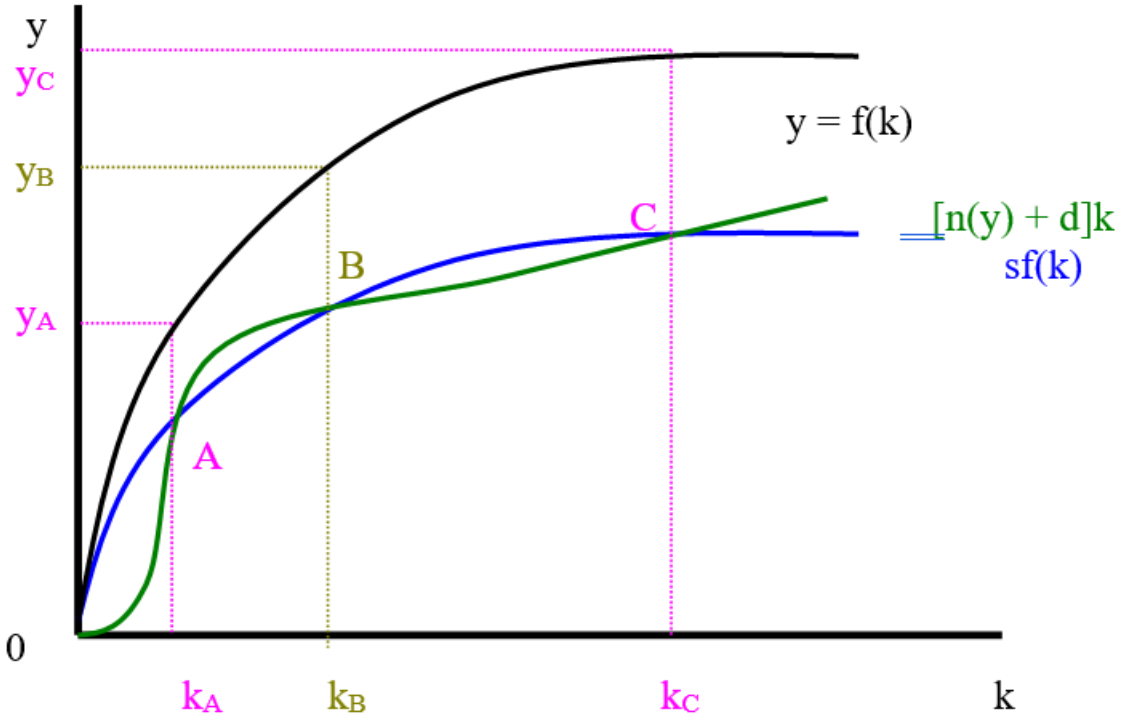
The first equilibrium (Point A) is a stable low-income steady-state equilibrium, and any deviation from that point will cause the economy to adjust again at the same steady-state income level (and capital-output ratio). However, the second equilibrium (Point B) is an unstable high-income steady-state equilibrium. Any deviation from that point will lead to either a lower income steady-state equilibrium back at Point A (if the capital-labor ratio declines) or

ongoing growth (if the capital-labor ratio increases). In the latter case, not only total output but also output per capita will continue to grow.

c. A model like the one in this question can be used to explain how some countries find themselves in situations with no growth and low income while others have ongoing growth and a high level of income. In the first case, a country may have invested mostly in physical capital, leading to some short-term growth at the expense of long-term growth. In the second case, a country may have invested not only in physical capital but also in human capital (education, skills, and training), reaping significant social returns.

2.

a. If population growth is endogenous, that is, if a country can influence the rate of population growth through government policies, then the investment requirement is no longer a straight line. Instead it is curved as depicted below.



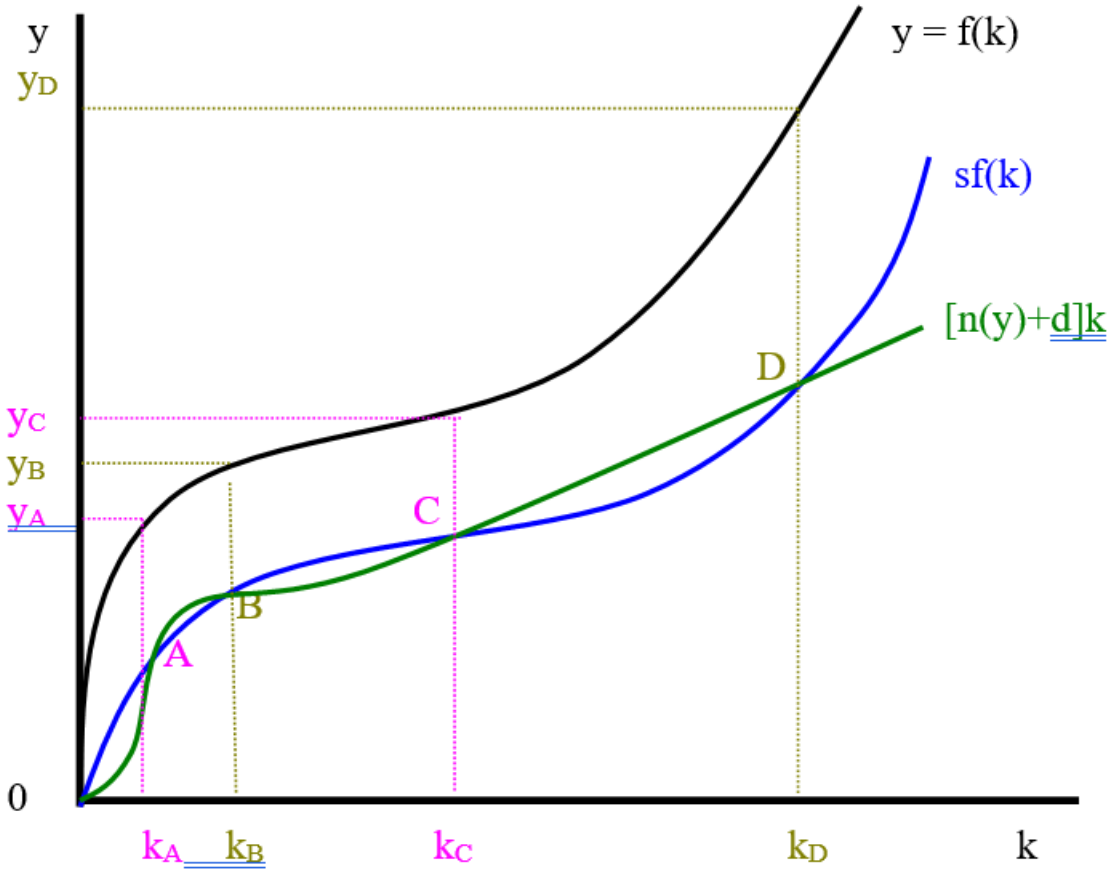
b. The first equilibrium (Point A) is a stable steady-state equilibrium. This is a situation of low income and high population growth, indicating that the country is in a poverty trap. The

second equilibrium (Point B) is an unstable steady-state equilibrium. This is a situation of medium income and low population growth. The third equilibrium (Point C) is again a stable steady-state equilibrium. This is a situation of high income and low population growth. In none of these three cases do we have ongoing growth. For any capital stock $k < k_B$, the economy will adjust to k_A, but for any capital stock $k > k_B$ or $k > k_c$, the economy will adjust to k_C. During the adjustment process to any of these two steady-state equilibria, the change in output per capita only will be transitory.

c. To escape the poverty trap (Point A), a country has several possibilities: First, it can somehow find the means to increase the capital-labor ratio above a level consistent with Point B (perhaps by borrowing funds or seeking direct foreign investment). Second, it can increase the savings rate such that the savings function no longer intersects the investment requirement curve at either Point A or Point B. Third, it can decrease the rate of population growth through specifically designed policies, such that the investment requirement shifts down and no longer intersects with the savings function at Points A or B.

3.

a. If we incorporate endogenous population growth into a two-sector model of growth, we get a curved investment requirement line and a production function with first a diminishing and then a constant marginal product of capital as depicted below. (Note that the savings and production functions have similar shapes.)



b. Here we should have four intersections of the savings function sf(k) and the investment requirement [n(y)+d]k. The first equilibrium (at Point A) is a stable low-income steady-state equilibrium. Any deviation from that point will cause the economy to eventually adjust again at the same steady-state income level (and capital-output ratio). The second equilibrium (at Point B) is an unstable low-income equilibrium. Any deviation from that point will lead to either a lower income steady-state equilibrium at Point A (if the capital-labor ratio declines) or a higher income steady-state equilibrium at Point C (if the capital-labor ratio increases). Since Point C is a stable equilibrium, the economy will settle back at that point whether the capital-labor ratio is below or above k_C. Point D is again an unstable equilibrium but at a high level of income. Any deviation from that point will lead to either a lower income steady-state equilibrium at Point C (if the capital-labor ratio declines) or ongoing growth (if the capital-labor ratio increases).

c. This model is more inclusive than either of the two models discussed previously and thus has

greater explanatory power. While the graphical analysis is far more complicated, we can now see much more clearly that a poor country cannot escape the poverty trap at Point A unless it somehow succeeds in increasing the capital-labor ratio (and thus per-capita income) beyond the level at Point B. The model can also explain why a high-income country can experience ongoing growth.

4.

a. The production function is of the form

$$Y = K^{1/2}(AN)^{1/2} = K^{1/2}(4[K/N]N)^{1/2} \quad = K^{1/2}(4K)^{1/2} = 2K.$$

From this we can see that a = 2 since

$$y = Y/N = 2(K/N) \Longrightarrow y = 2k.$$

b. Since a = y/k = 2, it follows that the growth rate of output and capital is

$$\Delta y/y = \Delta k/k = g = sa - (n + d) = (0.1)2 - (0.02 + 0.03) = 0.15 = 15\%.$$

c. The term "a" in the equation above stands for the marginal product of capital. By assuming that the level of labor-augmenting technology (A) is proportional to the capital-labor ratio (k), it is implied that the level of technology depends on the amount of capital per worker that we have, which may not be realistic.

d. In this model, we have a constant marginal product of capital and therefore we have an endogenous growth model.