FINM3033 Risk Management in Finance

Solution to Assignment 1

Problem 1.
A bank estimates that its profit next year is normally distributed with a mean of 0.8% of assets and the standard deviation of 2% of assets. How much equity (as a percentage of assets) does the company need to be (a) 99% sure that it will have a positive equity at the end of the year and (b) 99.9% sure that it will have positive equity at the end of the year? Ignore taxes.

Solution: (a) The bank can be 99% certain that profit will better than 0.8–2.33×2 or –3.85% of assets. It therefore needs equity equal to 3.85% of assets to be 99% certain that it will have a positive equity at the year end. (b) The bank can be 99.9% certain that profit will be greater than 0.8 – 3.09 × 2 or –5.38% of assets. It therefore needs equity equal to 5.38% of assets to be 99.9% certain that it will have a positive equity at the year end.

Problem 2.
A financial institution has the following portfolio of over-the-counter options on pounds:

Type	Position	Delta of Option	Gamma of Option	Vega of Option
Call	–1,000	0.5	2.2	1.8
Call	-500	0.8	0.6	0.2
Put	-2000	–0.40	1.3	0.7
Call	-500	0.70	1.8	1.4

A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8. Suppose that a second traded option, with a delta of 0.1, a gamma of 0.5, and a vega of 0.6, is also available. How could the portfolio be made delta, gamma, and vega neutral?

Solution: Let w_1 be the position in the first traded option and w_2 be the position in the second traded option. We require:

$$6,000 = 1.5w_1 + 0.5w_2$$
$$4,000 = 0.8w_1 + 0.6w_2.$$

The solution to these equations can easily be seen to be w_1 = 3,200, w_2 = 2,400. The whole portfolio then has a delta of –450 + 3,200 × 0.6 + 2,400 × 0.1 = 1,710. Therefore the portfolio can be made delta, gamma and vega neutral by taking a long position in 3,200 of the first traded option, a long position in 2,400 of the second traded option and a short position in £1,710.

Problem 3.
A company's investments earn LIBOR minus 0.5%. Explain how it can use the quotes in Table 5.5 from our textbook to convert them to (a) three-, (b) five-, and (c) ten-year fixed-rate investments.

Solution: (a) By entering into a three-year swap where it receives 2.97% and pays LIBOR the company earns 2.47% for three years. (b) By entering into a five-year swap where it receives 3.26% and pays LIBOR the company earns 2.76% for five years. (c) By entering into a swap where it receives 3.48% and pays LIBOR for ten years the company earns 2.98% for ten years.

Problem 4.
Prove (a) that the definitions of duration in equations (9.1) and (9.3) from our textbook are the same when y is continuously compounded and (b) that when y is compounded m times per year they are the same if the right-hand side of equation (9.3) is divided by $1 + y/m$.

Solution: (a) When y is continuously compounded, we get from (9.1) (or (9.2)) that

$$\begin{aligned} D &= -\frac{\partial B}{\partial y} \cdot \frac{1}{B} = -\frac{\sum_{i=1}^{n} c_i(-t_i)e^{-yt_i}}{B} \\ &= \sum_{i=1}^{n} t_i \left(\frac{c_i e^{-yt_i}}{B} \right) = \sum_{i=1}^{n} t_i \left(\frac{v_i}{B} \right), \end{aligned} \tag{0.1}$$

where $v_i := c_i e^{-yt_i}$ is the present value of the ith payment of the bond. The equation (0.1) shows that the definitions of duration in equations (9.1) and (9.3) are the same. (b) When y is compounded m times per year, we get from (9.1) (or (9.2)) that

$$\begin{aligned} D &= -\frac{\partial B}{\partial y} \cdot \frac{1}{B} = -\frac{\partial \left(\sum_{i=1}^{n} c_i (1 + y/m)^{-mt_i} \right)}{\partial y} \cdot \frac{1}{B} \\ &= \sum_{i=1}^{n} \frac{c_i m t_i (1 + y/m)^{-mt_i - 1} \cdot 1/m}{B} \\ &= \sum_{i=1}^{n} t_i \frac{c_i (1 + y/m)^{-mt_i}}{B(1 + y/m)} \\ &= \sum_{i=1}^{n} t_i \frac{v_i}{B(1 + y/m)}, \end{aligned} \tag{0.2}$$

where $v_i := c_i (1 + y/m)^{-mt_i}$ is the present value of the ith payment of the bond. The equation (0.2) shows that the definition of duration in equation (9.1) is equal to the one given in (9.3) divided by $1 + y/m$.

Problem 5.
A five-year bond with a face value of \$100 and a yield of 10% (continuously compounded) pays an 8% coupon at the end of each year. (a) What is the bond's price? (b) What is the bond's duration? (c) Use the duration to calculate the effect on the bond's price of a 0.2% decrease in its yield. (d) Recalculate the bond's price on the basis of a 9.8% per annum yield and compare the result with your answer to (c).

Solution: (a) The bond's price is

$$8e^{-1\times 0.1} + 8e^{-2\times 0.1} + 8e^{-3\times 0.1} + 8e^{-4\times 0.1} + 108e^{-5\times 0.1} = 90.58,$$

(b) the bond's duration is

$$1\times\frac{8e^{-1\times 0.1}}{90.58} + 2\times\frac{8e^{-2\times 0.1}}{90.58} + 3\times\frac{8e^{-3\times 0.1}}{90.58} + 4\times\frac{8e^{-4\times 0.1}}{90.58} + 5\times\frac{108e^{-5\times 0.1}}{90.58} = 4.274,$$

years, (c) the duration formula shows that when the yield decreases by 0.2% the bond's price increases by $0.002 \times 4.274 \times 90.58 = 0.77$, and (d) recomputing the bond's price with a yield of 9.8% gives a price of

$$8e^{-1\times 0.098} + 8e^{-2\times 0.098} + 8e^{-3\times 0.098} + 8e^{-4\times 0.098} + 108e^{-5\times 0.098} = 91.36,$$

which is approximately consistent with (a) and (c).