

## PT Assignment 5

1. Let  $X$  be a discrete random variable with probability mass function

$$P_X(k) = \begin{cases} 0.1 & \text{for } k = 0 \\ c & \text{for } k = 1 \\ 0.3 & \text{for } k = 2 \\ 0.2 & \text{for } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find  $E[X]$  and  $Var[X]$ .

(b) Let  $Y = (X - 2)^2$ . Find  $E[Y]$  by

(i) the formula  $\sum (x - 2)^2 P_X(x)$ .

(ii) probability mass function of  $Y$ .

2. Find the mean and variance of the following distributions:

(a) Discrete Uniform: Let  $n \in \mathbb{N}$ .  $P\left(X = \frac{k}{n}\right) = \frac{1}{n}$  for  $k = 1, 2, \dots, n$ .

(b) Poisson: Let  $\lambda > 0$ .  $P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}$  for  $k = 0, 1, 2, \dots$

(c) Binomial: Let  $n \in \mathbb{N}$ ,  $0 \leq p \leq 1$ .  $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$  for  $k = 0, 1, 2, \dots, n$ .

(d) Geometric: Let  $0 < p \leq 1$ .  $P(X = k) = (1 - p)^k p$  for  $k = 0, 1, 2, \dots$

(e) Negative Binomial:  $P(X = n) = \binom{n-1}{r-1} p^r (1 - p)^{n-r}$ ,  $n = r, r + 1, \dots$

3. An automobile insurance company has a block of one-year car insurance policies. The policies are divided into three classes: A, B, and C. A randomly chosen policy has 40% chance of being in class A, 10% in class B, and 50% in class C. The probability that a policy will produce a claim is 20% in class A, 10% in class B and 5% in class C. A class of policies (i.e., either class A, or class B, or class C) is chosen at random, with probability of being chosen proportional to the random chance of a policy being chosen from class (i.e., 40% for class A, 10% for class B, and 50% for class C) and five policies are selected at random from that class. It turns out that exactly one of the five policies produced a claim. What is the probability that these policies are from class A?
4. Let  $X$  have the Poisson distribution such that  $4p(2) = p(1) + p(0)$ . Calculate  $P(X \geq 2 | X \leq 4)$ .
5. A card is drawn at random from an ordinary deck of 52 cards and replaced. This is done a total of 5 independent times. What is the conditional probability of drawing the ace of spades exactly 4 times, given that this ace is drawn at least 4 times.