FINM3133 Time Series for Finance and Macroeconomics

Chapter 9 Exercises

1. For an AR(1) model with $Y_t = 12.2, \phi = -0.5$, and $\mu = 10.8$,
   (a) Find $\hat{Y}_t(1)$.
   (b) Calculate $\hat{Y}_t(2)$ in two different ways.
   (c) Calculate $\hat{Y}_t(10)$.

2. Suppose that annual sales (in millions of dollars) of the Acme Corporation follow the AR(2) model $Y_t = 5 + 1.1Y_{t-1} - 0.5Y_{t-2} + e_t$ with $\sigma_e^2 = 2$.
   (a) If sales for 2005, 2006, and 2007 were $9 million, $11 million, and $10 million, respectively, forecast sales for 2008 and 2009.
   (b) Show that $\psi_1 = 1.1$ for this model.
   (c) Calculate 95% prediction limits for your forecast in part (a) for 2008.

3. Simulate an ARMA(1,1) process with $\phi = 0.7, \theta = -0.5$, and $\mu = 100$. Simulate 50 values but set aside the last 10 values to compare forecasts with actual values.
   (a) Using the first 40 values of the series, find the values for the maximum likelihood estimates of ϕ, θ, and μ.
   (b) Using the estimated model, forecast the next ten values of the series. Plot the series together with the ten forecasts. Place a horizontal line at the estimate of the process mean.
   (c) Compare the ten forecasts with the actual values that you set aside.
   (d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?
   (e) Repeat parts (a) through (d) with a new simulated series using the same values of the parameters and same sample size.

4. Simulate an IMA(1,1) process with $\theta = 0.8$ and $\theta_0 = 0$. Simulate 35 values, but set aside the last five values to compare forecasts with actual values.
   (a) Using the first 30 values of the series, find the values for the maximum likelihood estimate of θ.

(b) Using the estimated model, forecast the next five values of the series. Plot the series together with the five forecasts. What is special about the forecasts?

(c) Compare the five forecasts with the actual values that you set aside.

(d) Plot the forecasts together with 95% forecast limits. Do the actual values fall within the forecast limits?

(e) Repeat parts (a) through (d) with a new simulated series using the same values of the parameters and same sample size.

5. Consider the model $Y_t = \beta_0 + \beta_1 t + X_t$, where $X_t = \phi X_{t-1} + e_t$. We assume that β_0, β_1, and ϕ are known. Show that the minimum mean square error forecast l steps ahead can be written as $\hat{Y}_t(l) = \beta_0 + \beta_1(t + l) + \phi^l(Y_t - \beta_0 - \beta_1 t)$.

6. The data file named **deere3** contains 57 consecutive values from a complex machine tool process at Deere & Co. The values given are deviations from a target value in units of ten millionths of an inch. The process employs a control mechanism that resets some of the parameters of the machine tool depending on the magnitude of deviation from target of the last item produced.

   (a) Using an AR(1) model for this series, forecast the next ten values.

   (b) Plot the series, the forecasts, and 95% forecast limits, and interpret the results.

7. The time series in the data file **robot** gives the final position in the "x-direction" after an industrial robot has finished a planned set of exercises. The measurements are expressed as deviations from a target position. The robot is put through this planned set of exercises in the hope that its behavior is repeatable and thus predictable.

   (a) Use an IMA(1,1) model to forecast five values ahead. Obtain 95% forecast limits also.

   (b) Display the forecasts, forecast limits, and actual values in a graph and interpret the results.

   (c) Now use an ARMA(1,1) model to forecast five values ahead and obtain 95% forecast limits. Compare these results with those obtained in part (a).