2022-23 First Semester

MATH1053 Linear Algebra I

Assignment 3a

Due Date: 18/Oct/2022 (Tuesday), 11:00 in class.

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. Find the LU decomposition of the following matrix and show your steps.

$$A = \begin{pmatrix} 1 & 4 & 2 \\ -2 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix}.$$

2. Let A and B be $n \times n$ matrices and define the $2n \times 2n$ matrices S and M by

$$S = \begin{pmatrix} I & A \\ O & I \end{pmatrix}, \quad M = \begin{pmatrix} AB & O \\ B & O \end{pmatrix}.$$

   (a) Let the inverse matrix of S be $S^{-1} = \begin{pmatrix} D_1 & D_3 \\ O & D_2 \end{pmatrix}$ for some $n \times n$ matrices D_1, D_2, D_3. Determine D_1, D_2, D_3.

   (b) Use part (a) to compute the blocks form of the product $S^{-1}MS$.

3. Let $A = \begin{pmatrix} A_{11} & A_{12} \\ O & A_{22} \end{pmatrix}$ where all four blocks are $n \times n$ matrices. If A_{11} and A_{22} are nonsingular,

   (a) show that A must also be nonsingular and that A^{-1} must be of the form $\begin{pmatrix} A_{11}^{-1} & C \\ O & A_{22}^{-1} \end{pmatrix}$.

   (b) Determine the matrix C in part (a).

4. Compute the determinant of the following matrices using cofactor expansion,

$$A = \begin{pmatrix} -1 & -1 & 2 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & -3 \\ 4 & 0 & 1 \\ -2 & -1 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 3 & 2 & 4 \\ 1 & 6 & 4 & 8 \\ 1 & 3 & 0 & 0 \\ 2 & 6 & 4 & 8 \end{pmatrix}.$$

5. Find all the values λ such that $\det \begin{pmatrix} 5-\lambda & 6 \\ 3 & -2-\lambda \end{pmatrix} = 0.$

6. Evaluate the following determinant by cofactor expansion. Write your answer as a polynomial in x.

$$\begin{vmatrix} a-x & b & c \\ 1 & -x & 0 \\ 0 & 1 & -x \end{vmatrix}$$

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