AFM Assignment 4

1. Solve the PDE $\begin{cases} \frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} \\ U_{(t=0,x)} = x^3 \end{cases}$

2. Solve the PDE $\begin{cases} \frac{\partial U}{\partial t} = t^n \frac{\partial^2 U}{\partial x^2} \\ U_{(t=0,x)} = ax^2 + bx + c \end{cases}$, here a, b and c are constants, n is a positive integer.

3. Solve the PDE $\begin{cases} \frac{\partial U}{\partial t} = e^{-t} \frac{\partial^2 U}{\partial x^2} \\ U_{(t=0,x)} = ax^2 + bx + c \end{cases}$, here a, b and c are constants.

4. Solve the PDE $\begin{cases} \frac{\partial U}{\partial t} = (2 + \sin t) \frac{\partial^2 U}{\partial x^2} \\ U_{(t=0,x)} = e^{\lambda x} \end{cases}$, where λ is a constant.

5. The price of an at-the-money put option with strike price $K = 300$ currently has price \$15. At-the-money means that the current stock price equals the strike price. The option is European style and will mature in 6 months. The interest rate is 3%. What is the price of a call option written on the same stock, with the same strike price and same maturity date?

6. Evaluate $\Delta = \frac{\partial P}{\partial S}$, where P is the Black-Scholes formula of the price of a European put option with no dividend, i.e.,

$$P(t, S) = Ke^{-r(T-t)}\mathrm{N}(-d_2) - S\mathrm{N}(-d_1)$$

where $d_1 = \frac{\ln \frac{S}{K} + (r + \frac{\sigma^2}{2})(T-t)}{\sqrt{\sigma^2(T-t)}}$, and $d_2 = \frac{\ln \frac{S}{K} + (r - \frac{\sigma^2}{2})(T-t)}{\sqrt{\sigma^2(T-t)}}$.

Do not apply the call-put parity. Evaluate the expression $\Delta = \frac{\partial P}{\partial S}$ directly,

7. Based on the call-put parity and the solution of Question **6**, calculate $\frac{\partial C}{\partial S}$, where C is the price of the European call option with the same underlying stock, the same strike price and the same maturity date as the put option in Question **6**.

8. What is the value of an option with the payoff given by Figure 1?

9. What is the value of an option with the payoff given by Figure 2?

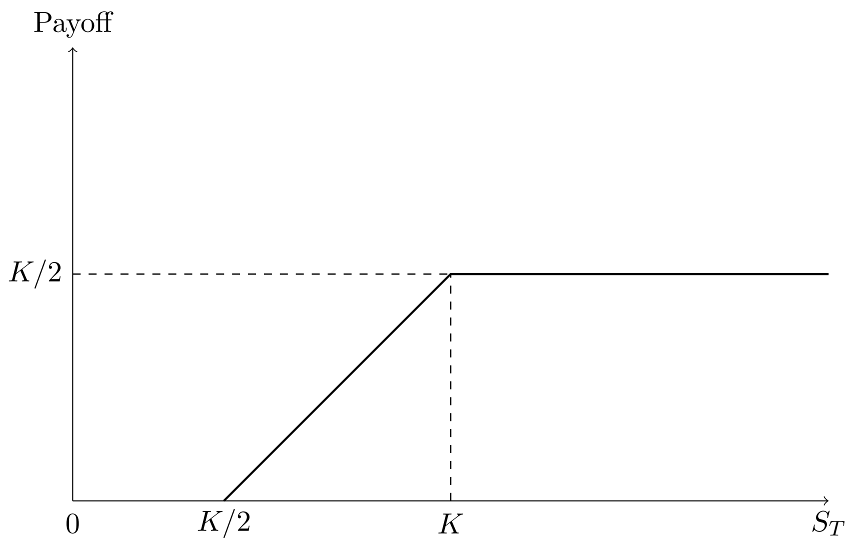

Figure 1:

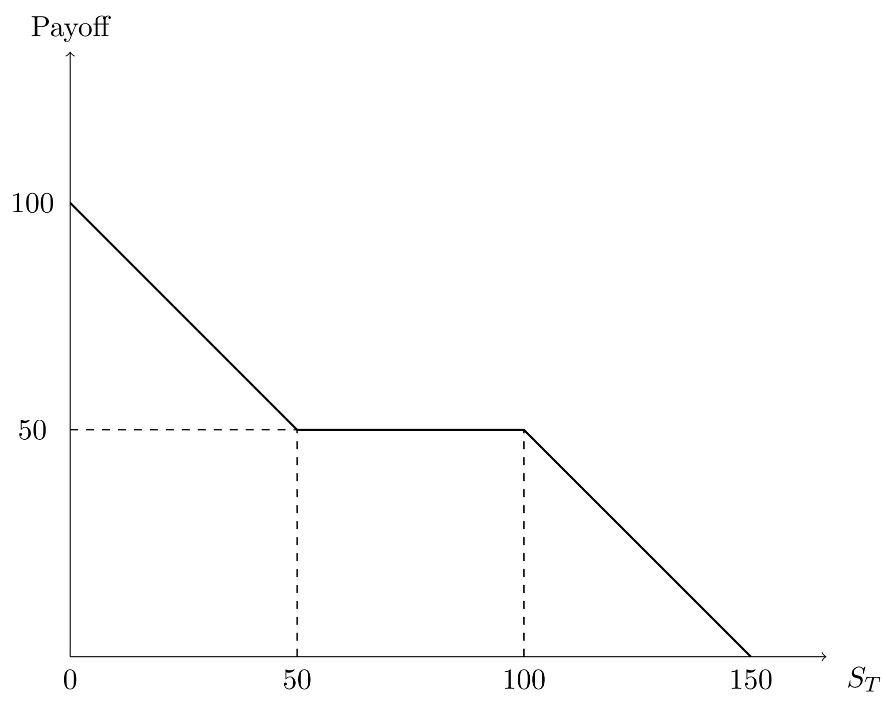

Figure 2: