Risk Management in Finance - Market risk I

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Outline of Market risk I

- Basic Concepts
- Risk Factor Sensitivities
- Interest Rate Risk

Basic Concepts

Loss Ditribution I

- We denote the *portfolio value* at time t as $V(t)$.
- The *time horizon* is $\triangle t$ (e.g. a day, ten days, a month, a year).
- The *profit* in the time interval from t to $t + \triangle t$ is

$$V(t + \triangle t) - V(t).$$

  The law of this profit is called *profit and loss* $(P \,\&\, L)$ *distribution*.

- The *loss*

$$L_{[t,t+\triangle t]} := -(V(t + \triangle t) - V(t))$$

  is the negative profit. Its law is the *loss distribution*.

Loss Ditribution II

- From now on we consider dates $t_n = n\triangle t$ for $n = 0, 1, 2, \ldots$.
- As a shorthand notation we write $V_n := V(t_n)$ etc. and

$$L_{n+1} := L_{[n\triangle t,(n+1)\triangle t]} = -(V_{n+1} - V_n).$$

- Usually, the present time is denoted by t_n.
- Our goal is to make a statement on the yet unknown loss L_{n+1} given the data up to time t_n.

Modelling

- We suppose that profits are a function of risk factors.
- More specifically,
$$V_n = f(t_n, \mathbf{Z}_n)$$
with some known function f and a random vector
$$\mathbf{Z}_n = (Z_{n,1}, \ldots, Z_{n,d})$$
of *risk factors* as e.g asset prices, interest rates, volatility, etc.
- *Risk factor changes* are denoted by
$$\mathbf{X}_{n+1} := \mathbf{Z}_{n+1} - \mathbf{Z}_n.$$

Modelling II

- So
$$\begin{aligned} L_{n+1} &= -(V_{n+1} - V_n) \\ &= -f(t_{n+1}, \mathbf{Z}_n + \mathbf{X}_{n+1}) + f(t_n, \mathbf{Z}_n) \\ &=: \ell_{[n]}(\mathbf{X}_{n+1}), \end{aligned}$$
with some function $\ell_{[n]}$.
- This randomly changing function $\ell_{[n]}$ is called *loss operator*.
- In general it is nonlinear because the same is true for f.

Example 2.1: A Stock Portfolio

- Consider a stock portfolio with α_i shares of stock i for $i = 1, \dots, d$.
- The price of stock i at time t_n is denoted as $S_{n,i}$, which implies that the portfolio value at time t_n is

$$V_n = \sum_{i=1}^{d} \alpha_i S_{n,i}.$$

- As risk factors we consider the logarithmic stock prices

$$Z_{n,i} := \log S_{n,i}.$$

Example 2.1 continued

- The risk factor changes

$$X_{n+1,i} = \log \frac{S_{n+1,i}}{S_{n,i}}, \; i = 1, \; . \; . \; . \; , d$$

are the logarithmic returns of stocks 1, . . . , d.

- The portfolio value can be expressed in terms of risk factors as

$$V_n = \sum_{i=1}^{d} \alpha_i \exp(Z_{n,i}) = f(t_n, \mathbf{Z}_n)$$

with $f(t, z_1, \ldots, z_d) := \sum_{i=1}^{d} \alpha_i \exp(z_i)$.

Example 2.1 continued

- The loss is

$$
\begin{aligned}
L_{n+1} &= -V_{n+1} + V_n \\
&= -\sum_{i=1}^{d} \alpha_i \left(\exp(Z_{n,i} + X_{n+1,i}) - \exp(Z_{n,i})\right) \\
&= -\sum_{i=1}^{d} \alpha_i S_{n,i} \left(\exp(X_{n+1,i}) - 1\right)
\end{aligned}
$$

- The loss operator is

$$
\ell_{[n]}(x) = -\sum_{i=1}^{d} \alpha_i S_{n,i} \cdot (\exp(x_i) - 1)
$$

Example 2.2: European Call Option

- Consider a portfolio containing a single European call option on a stock.
- According to the Black-Scholes model, the value of the call and hence the portfolio is of the form

$$C(t, S(t), r, \sigma; T, K),$$

  where t denotes the present time, $S(t)$ the present stock price, r the riskless interest rate, σ the volatility of the stock, T the maturity of the option, K the strike price.
- $C(\ldots)$ – the explicitly known function appearing in the Black-Scholes formula.

Example 2.2 continued

- If we consider the stock price, the interest rate, and the volatility as variable, a reasonable vector of risk factors is

$$\mathbf{Z}_n = (\log S_n, r_n, \sigma_n),$$

  where the index n refers to time t_n as usual.

- The corresponding risk factor change is

$$X_{n+1} = \left(\log\left(\frac{S_{n+1}}{S_n}\right), r_{n+1} - r_n, \sigma_{n+1} - \sigma_n\right).$$

Example 2.2 continued

- The loss amounts to

$$\begin{aligned}L_{n+1} &= -C(t_{n+1}, S_{n+1}, r_{n+1}, \sigma_{n+1}; T, K) + C(t_n, S_n, r_n, \sigma_n; T, K)\\ &= -C(t_{n+1}, \exp(Z_{n+1,1}), Z_{n+1,2}, Z_{n+1,3}; T, K)\\ &\quad + C(t_n, \exp(Z_{n,1}), Z_{n,2}, Z_{n,3}; T, K)\end{aligned}$$

- “Theta”: $D_1C(t, S, r, \sigma; T, K) = \dfrac{\partial}{\partial t}C(t, S, r, \sigma; T, K)$

- “Delta”: $D_2C(t, S, r, \sigma; T, K) = \dfrac{\partial}{\partial S}C(t, S, r, \sigma; T, K)$

- $\ldots$

Risk Factor Sensitivity

Risk Factor Sensitivity

- Factor sensitivity gives the change in portfolio value for a predetermined change in one of the underlying risk factors

- If the value of the portfolio is given by

$$V_n = f(t_n, \mathbf{Z}_n),$$

then factor sensitivity measures are given by the partial derivatives

$$f_{z_i}(t_n, \mathbf{Z}_n) = \frac{\partial f}{\partial z_i}(t_n, \mathbf{Z}_n).$$

- The "Greeks" of a derivative portfolio may be considered as factor sensitivity

Risk Factor Sensitivity II

- Advantage: Factor sensitivity provides information about the robustness of the portfolio value with respect to certain events (risk-factor changes).

- Disadvantage: It is difficult to aggregate the sensitivity with respect to changes in different risk-factors or aggregate across markets to get an understanding of the overall riskiness of a position.

A trader's Gold Portfolio

Position	Value ($)
Spot Gold	3,180,000
Forward Contracts	– 3,060,000
Futures Contracts	2,000
Swaps	180,000
Options	– 6,110,000
Exotics	125,000
Total	−5,683,000

Delta

- Delta of a portfolio is the partial derivative of a portfolio with respect to the price of the underlying asset (gold in this case)
- Suppose that a \$0.1 increase in the price of gold leads to the gold portfolio decreasing in value by \$100
- The delta of the portfolio is –1000
- The portfolio could be hedged against short-term changes in the price of gold by buying 1000 ounces of gold. This is known as making the portfolio delta neutral

Linear vs Nonlinear Products

- When the price of a product is linearly dependent on the price of an underlying asset a "hedge and forget" strategy can be used

- Non-linear products require the hedge to be rebalanced to preserve delta neutrality

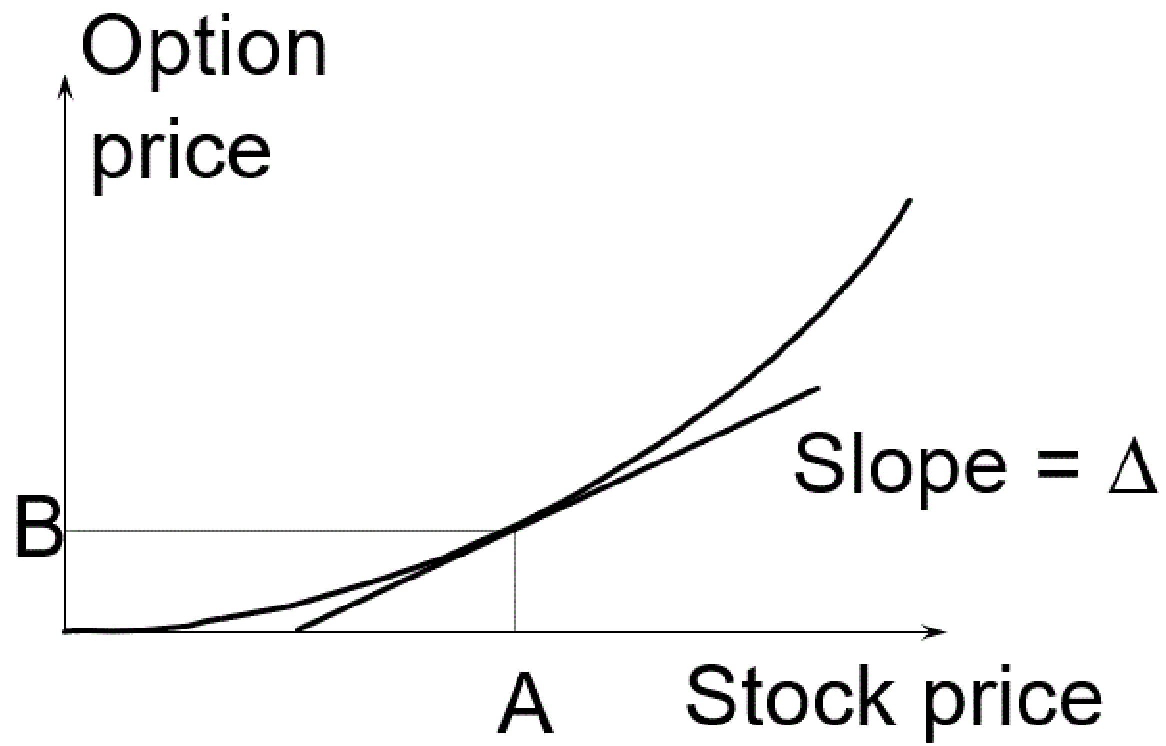
Example of Hedging a Nonlinear Product

- A bank has sold for \$300,000 a European call option on 100,000 shares of a nondividend paying stock
- S_0=49, K=50, r=5%, s = 20%, T = 20 weeks, m = 13%

Example of Hedging a Nonlinear Product

- A bank has sold for \$300,000 a European call option on 100,000 shares of a nondividend paying stock
- S_0=49, K=50, r=5%, s = 20%, T = 20 weeks, m = 13%
- The Black-Scholes-Merton value of the option is \$240,000
- How does the bank hedge its risk to lock in a \$60,000 profit?

Delta of the Option



Delta Hedging

- Initially the delta of the option is 0.522
- The delta of the position is -52,200
- This means that 52,200 shares must purchased to create a delta neutral position
- But, if a week later delta falls to 0.458, 6,400 shares must be sold to maintain delta neutrality
- Tables 8.2 and 8.3 provide examples of how delta hedging might work for the option.

Table 8.2: Option closes in the money

Week	Stock Price	Delta	Shares Purchased
0	49.00	0.522	52,200
1	48.12	0.458	(6,400)
2	47.37	0.400	(5,800)
3	50.25	0.596	19,600
….	…..	….	…..
19	55.87	1.000	1,000
20	57.25	1.000	0

Table 8.3: Option closes out of the money

Week	Stock Price	Delta	Shares Purchased
0	49.00	0.522	52,200
1	49.75	0.568	4,600
2	52.00	0.705	13,700
3	50.00	0.579	(12,600)
….	…..	….	…..
19	46.63	0.007	(17,600)
20	48.12	0.000	(700)

Where the Costs Come From

- Delta hedging a short option position tends to involve selling after a price decline and buying after a price increase

- This is a "sell low, buy high" strategy.

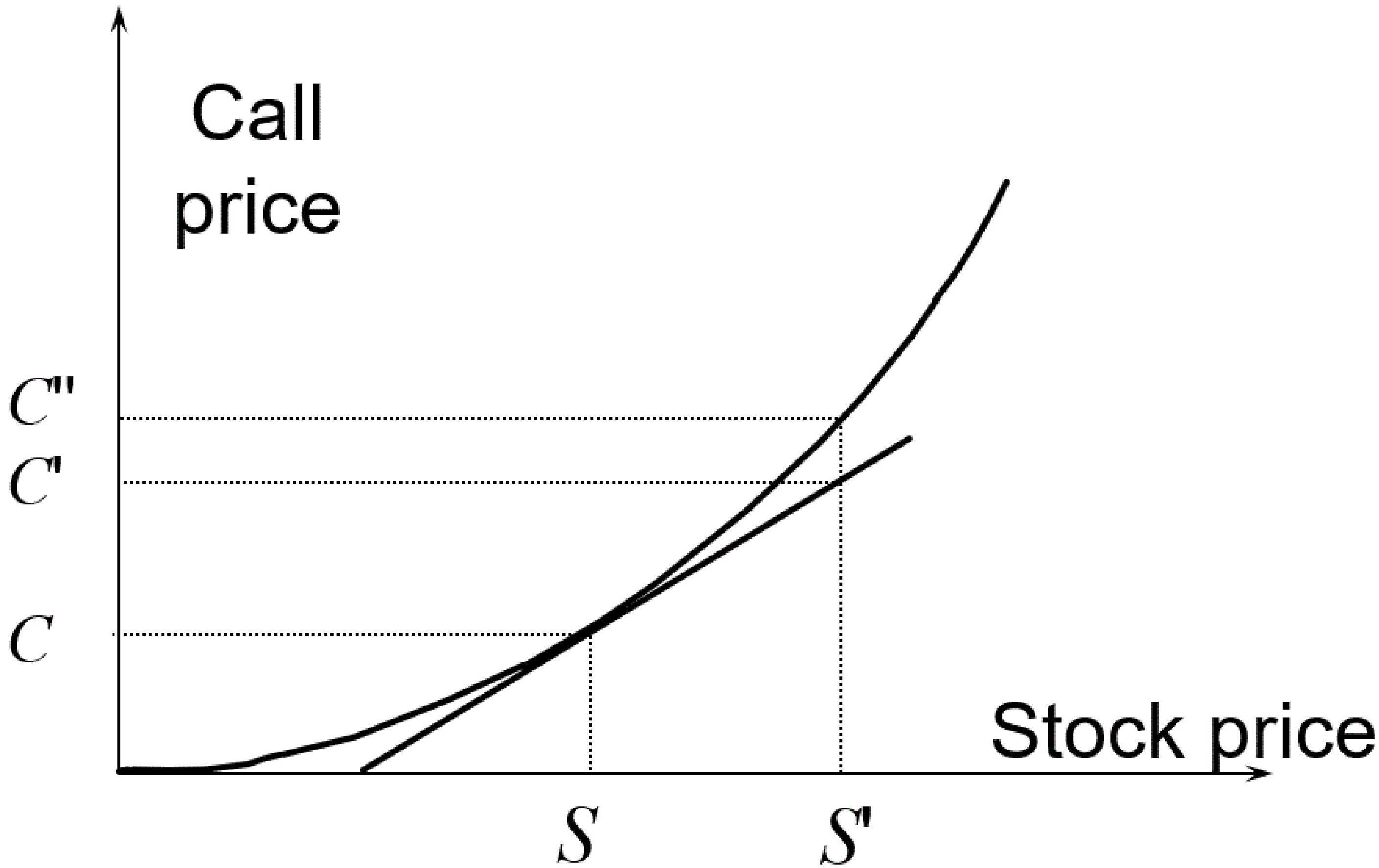
- The total costs incurred are close to the theoretical price of the option

Gamma

- Gamma (Γ) is the rate of change of delta ($\triangle$) with respect to the price of the underlying asset

- Gamma is greatest for options that are close to the money

Gamma Measures the Delta Hedging Errors Caused By Curvature



Vega

- Vega (ν) is the rate of change of the value of a derivatives portfolio with respect to volatility
- Like gamma, vega tends to be greatest for options that are close to the money

Gamma and Vega Limits

- In practice, a traders must keep gamma and vega within limits set by risk management

Theta

- Theta (Θ) of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time
- The theta of a call or put is usually negative. This means that, if time passes with the price of the underlying asset and its volatility remaining the same, the value of the option declines

Rho

- Rho is the partial derivative with respect to a parallel shift in all interest rates in a particular country

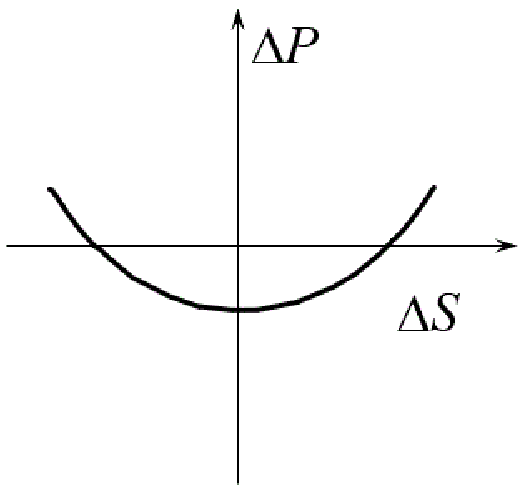
Taylor Series Expansion

$$\Delta P = \frac{\partial P}{\partial S}\Delta S + \frac{\partial P}{\partial t}\Delta t + \frac{1}{2}(\partial^2 P)/(\partial S^2)(\Delta S)^2 + \\ + \frac{1}{2}\frac{\partial^2 P}{\partial t^2}(\Delta t)^2 + \frac{\partial^2 P}{\partial S \partial t}\Delta S \Delta t$$

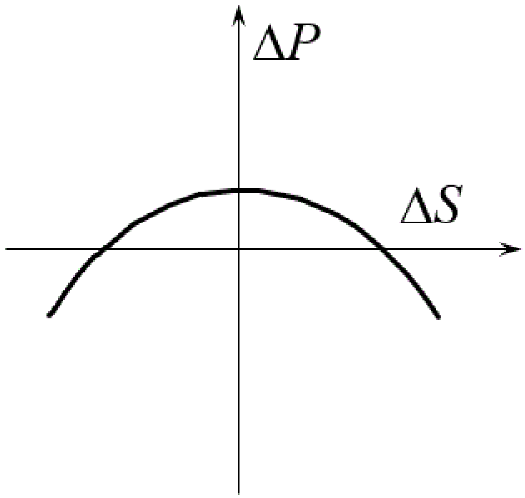
Interpretation of Gamma

- For a delta neutral portfolio,

$$\triangle P \approx \Theta \triangle t + \frac{1}{2}\Gamma \triangle S^2$$



Positive Gamma



Negative Gamma

Taylor Series Expansion when Volatility is Uncertain

$$\Delta P = \frac{\partial P}{\partial S}\Delta S + \frac{\partial P}{\partial \sigma}\Delta\sigma + \frac{\partial P}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 P}{\partial S^2}(\Delta S)^2 + \\ + \frac{1}{2}\frac{\partial^2 P}{\partial \sigma^2}(\Delta\sigma)^2 + \cdots$$

Managing Delta, Gamma, & Vega

- $\triangle$ can be changed by taking a position in the underlying
- To adjust Γ & ν it is necessary to take a position in an option or other derivative

Hedging in Practice

- Traders usually ensure that their portfolios are delta-neutral at least once a day
- Whenever the opportunity arises, they improve gamma and vega
- As portfolio becomes larger hedging becomes less expensive

Static Options Replication

- This involves approximately replicating an exotic option with a portfolio of vanilla options
- Underlying principle: if we match the value of an exotic option on some boundary, we have matched it at all interior points of the boundary
- Static options replication can be contrasted with dynamic options replication where we have to trade continuously to match the option

Scenario Analysis

- A scenario analysis involves testing the effect on the value of a portfolio of different assumptions concerning asset prices and their volatilities

- In this approach we consider a number of possible scenarios, i.e. a number of possible risk-factor changes

  - A scenario may be for instance a 10% rise in a relevant exchange rate and a simultaneous 20% drop in a relevant stock index.

  - The risk is then measured as the maximum loss over all possible (predetermined) scenarios.

Scenario Analysis II

- Fix a number N of possible risk-factor changes, $\mathbf{X} = \{\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_N\}$.
- Each scenario is given a weight, w_i and we write $\mathrm{w} = (w_1, \ldots, w_N)$.
- Consider a portfolio with loss-operator $\ell_{[n]}(\cdot)$.
- The risk of the portfolio is then measured as

$$\psi_{[\mathrm{X},\mathrm{w}]} = \max\{w_1 \ell_{[n]}(\mathrm{x}_1), \cdots, w_N \ell_{[n]}(\mathrm{x}_N)\}.$$

Interest Rate Risk?

Management of Net Interest Income I

- Suppose that the market's best guess is that future short term rates will equal today's rates

- What would happen if a bank posted the following rates?

Maturity (yrs)	Deposit Rate	Mortgage Rate
1	3%	6%
5	3%	6%

- How can the bank manage its risks?

Management of Net Interest Income II

- Most banks have asset-liability management groups to manage interest rate risk
- When long term loans are funded with short term deposits, interest rate swaps can be used to hedge the interest rate risk
- But this does not hedge the liquidity risk

LIBOR Rates and Swap Rates

- LIBOR is short for *London interbank offered rate*
- LIBOR rates are rates with maturities up to one year for interbank transactions where the borrower has a AA-rating

- Swap Rates are the fixed rates exchanged for floating in an interest rate swap agreement

Credit Ratings

Moody's	S&P and Fitch	
Aaa	AAA	Investment grade bonds
Aa	AA	
A	A	
Baa	BBB	
Ba	BB	Non-investment grade bonds
B	B	
Caa	CCC	
Ca	CC	
C	C	

Understanding Swap Rates

- A bank can
  - Lend to a series AA-rated borrowers for ten successive six month periods
  - Swap the LIBOR interest received for the five-year swap rate
- This shows that the swap rate has the credit risk corresponding to a series of short-term loans to AA-rated borrowers

Extending the LIBOR Curve

- Alternative 1: Create a term structure of interest rates showing the rate of interest at which a AA-rated company can borrow now for 1, 2, 3 . . . years

- Alternative 2: Use swap rates so that the term structure represents future short term AA borrowing rates

- Alternative 2 is the usual approach. It creates the LIBOR/swap term structure of interest rates

Risk-Free Rate

- Traders have traditionally assumed that the LIBOR/swap zero curve is the risk-free zero curve
- The Treasury curve is on average about 50 basis points below the LIBOR/swap zero curve
- Treasury rates are considered to be artificially low for a variety of regulatory and tax reasons

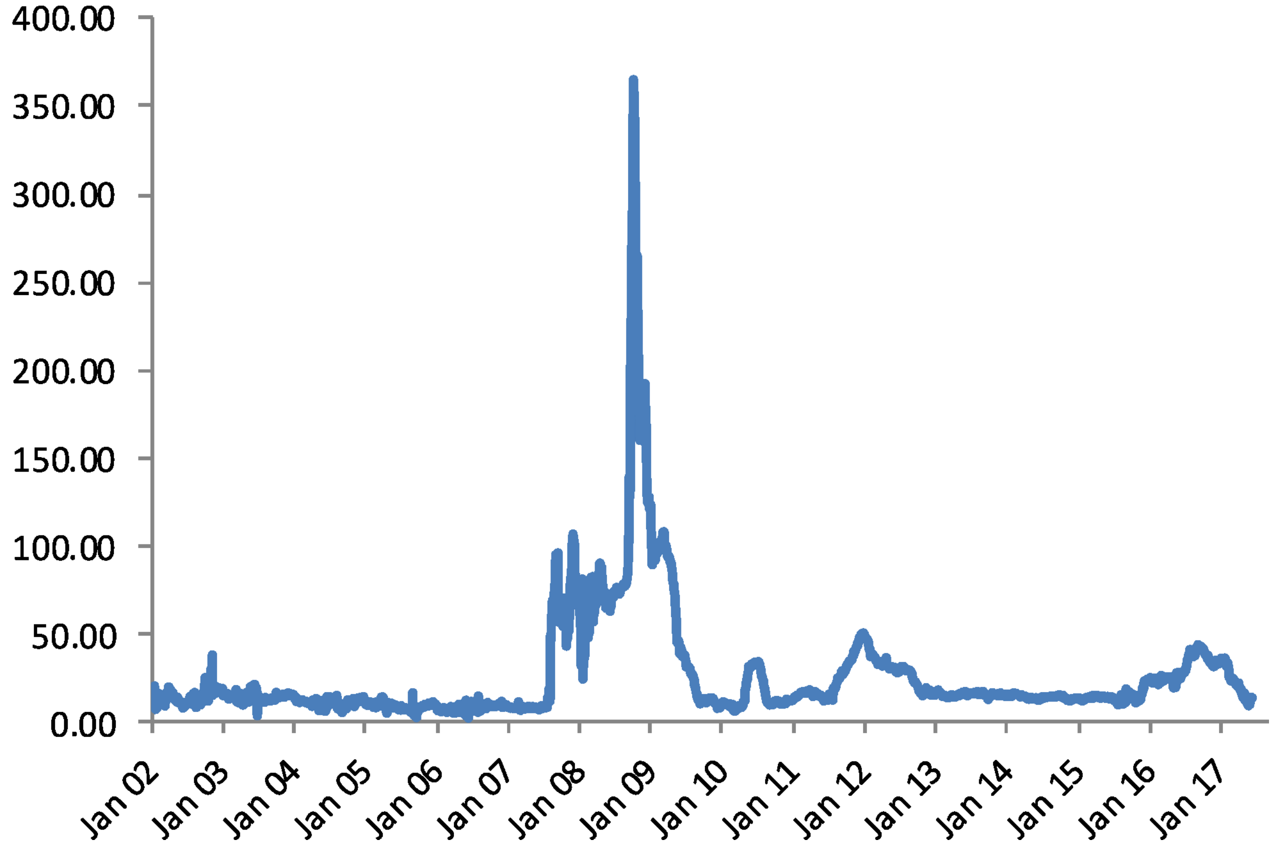
OIS Rate

- LIBOR/swap rates were clearly not “risk-free” during the crisis

- As a result the market has switched to using overnight indexed swap (OIS) rates as proxies for the risk-free rate instead of LIBOR/swap rates

- The OIS rate is the rate swapped for the geometric average of overnight borrowing rates. (In the U.S. the relevant overnight rate is the fed funds rate)

3-month LIBOR-OIS Spread



Repo Rate

- A financial institution owning securities agrees to sell them today for a certain price and buy them back in the future for a slightly higher price
- It is obtaining a secured loan
- The interest on the loan is the difference between the two prices

Duration I

- Duration of a bond that provides cash flow c_i at time t_i is

$$\sum_{i=1}^{n} t_i \left(\frac{c_i e^{-y t_i}}{B} \right)$$

where B is its price and y is its yield (continuously compounded)

- This leads to

$$\frac{\Delta B}{B} = -D \Delta y$$

Duration II

Calculation of Duration for a 3-year bond paying a coupon 10%. Bond yield=12%. :

Time (yrs)	Cash Flow ($)	PV ($)	Weight	Time × Weight
0.5	5	4.709	0.050	0.025
1.0	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2.0	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3.0	105	73.256	0.778	2.333
Total	130	94.213	1.000	2.653

Duration III

- When the yield y is expressed with compounding m times per year

$$\Delta B = -\frac{BD\Delta y}{1 + y/m}$$

- The expression

$$\frac{D}{1 + y/m}$$

is referred to as the "modified duration"

Convexity

The convexity of a bond is defined as

$$C = \frac{1}{B}\frac{d^2B}{dy^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-yt_i}}{B}$$

which leads to

$$\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

Portfolios

- Duration and convexity can be defined similarly for portfolios of bonds and other interest-rate dependent securities

- The duration of a portfolio is the weighted average of the durations of the components of the portfolio. Similarly for convexity.

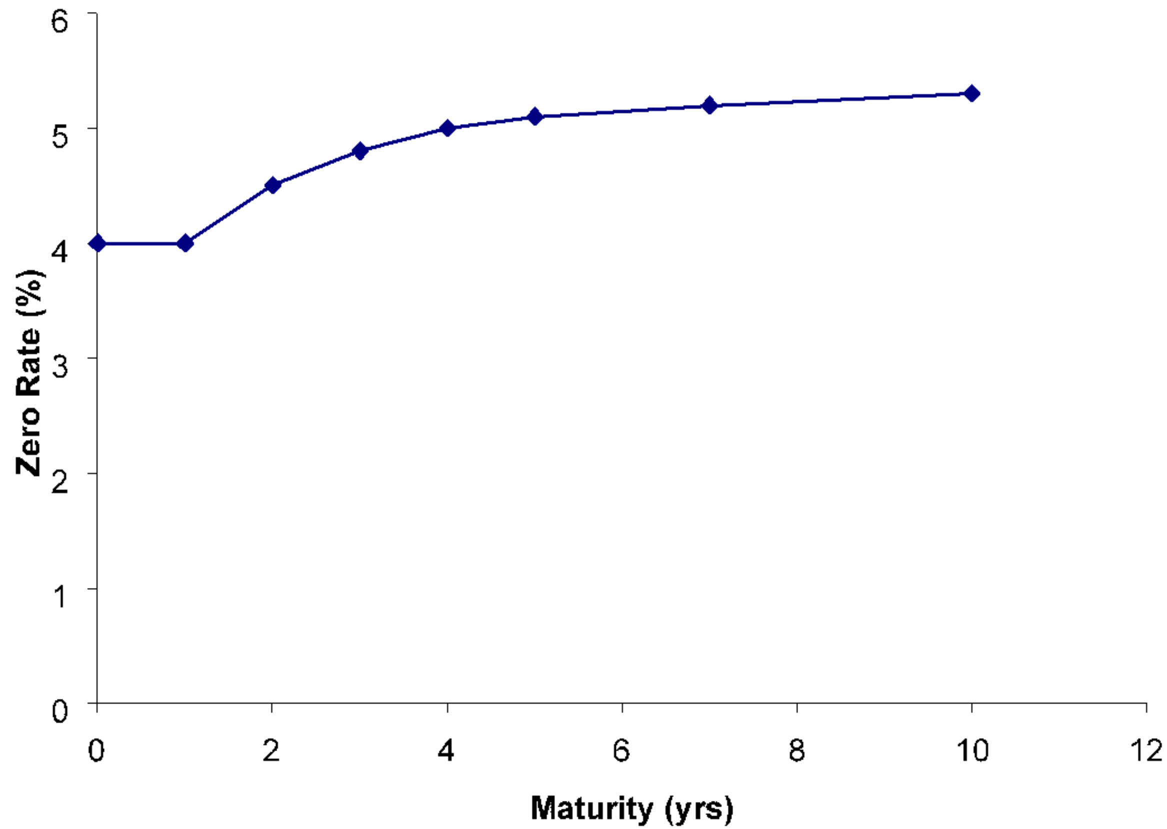
What Duration and Convexity Measure

- Duration measures the effect of a small parallel shift in the yield curve
- Duration plus convexity measure the effect of a larger parallel shift in the yield curve
- However, they do not measure the effect of non-parallel shifts

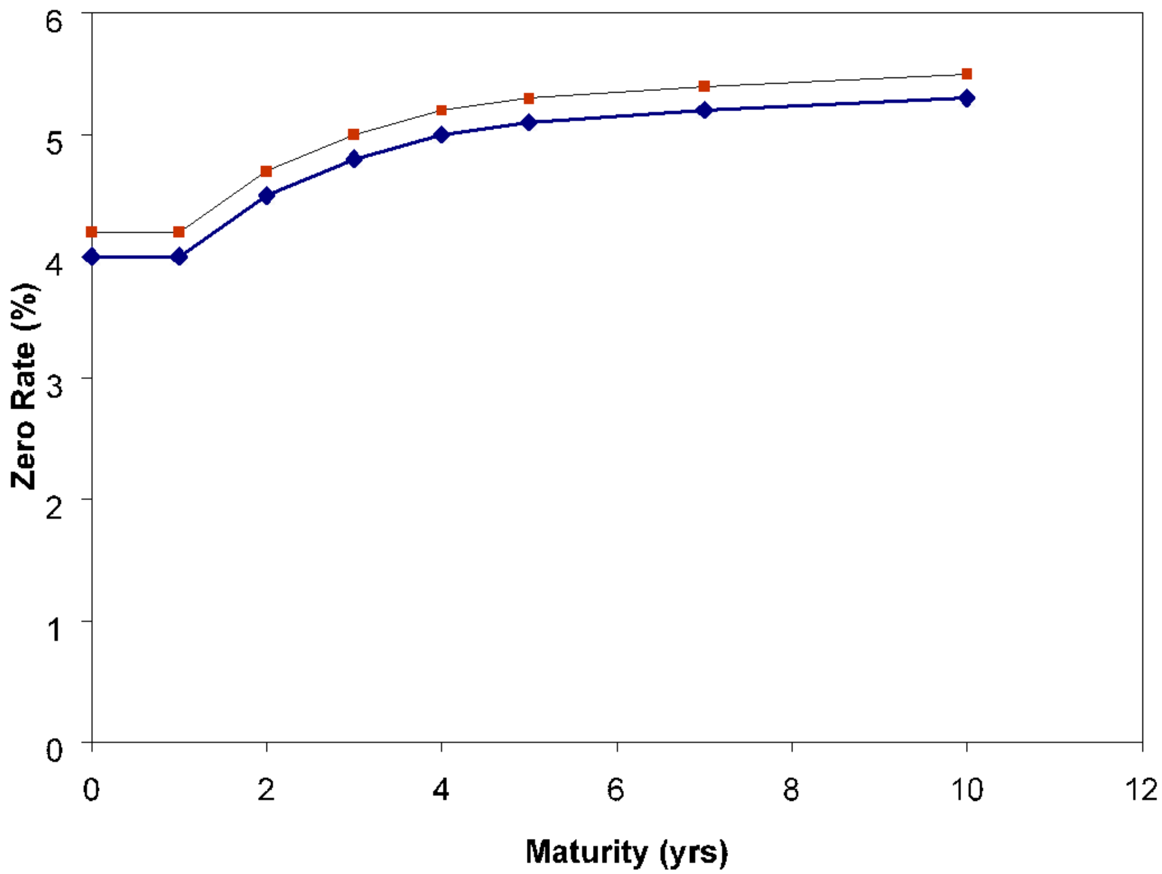
Other Measures

- Dollar Duration: Product of the portfolio value and its duration
- Dollar Convexity: Product of convexity and value of the portfolio
- DV01: Dollar impact of a one-basis-point parallel shift

Starting Zero Curve



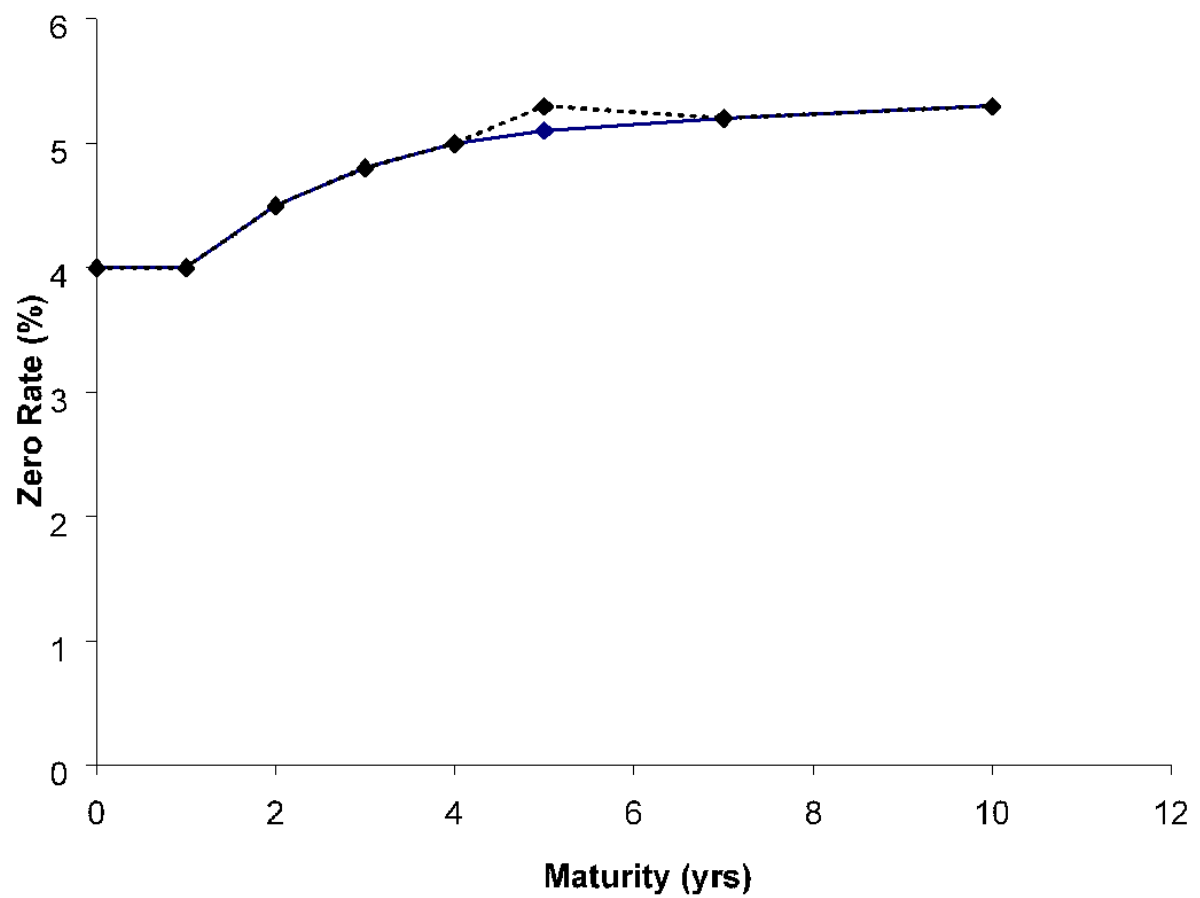
Parallel Shift



Partial Duration I

A partial duration calculates the effect on a portfolio of a change to just one point on the zero curve

Partial Duration II



Partial Duration III: Example

Maturity yrs	1	2	3	4	5	7	10	Total
Partial duration	0.2	0.6	0.9	1.6	2.0	–2.1	–3.0	0.2

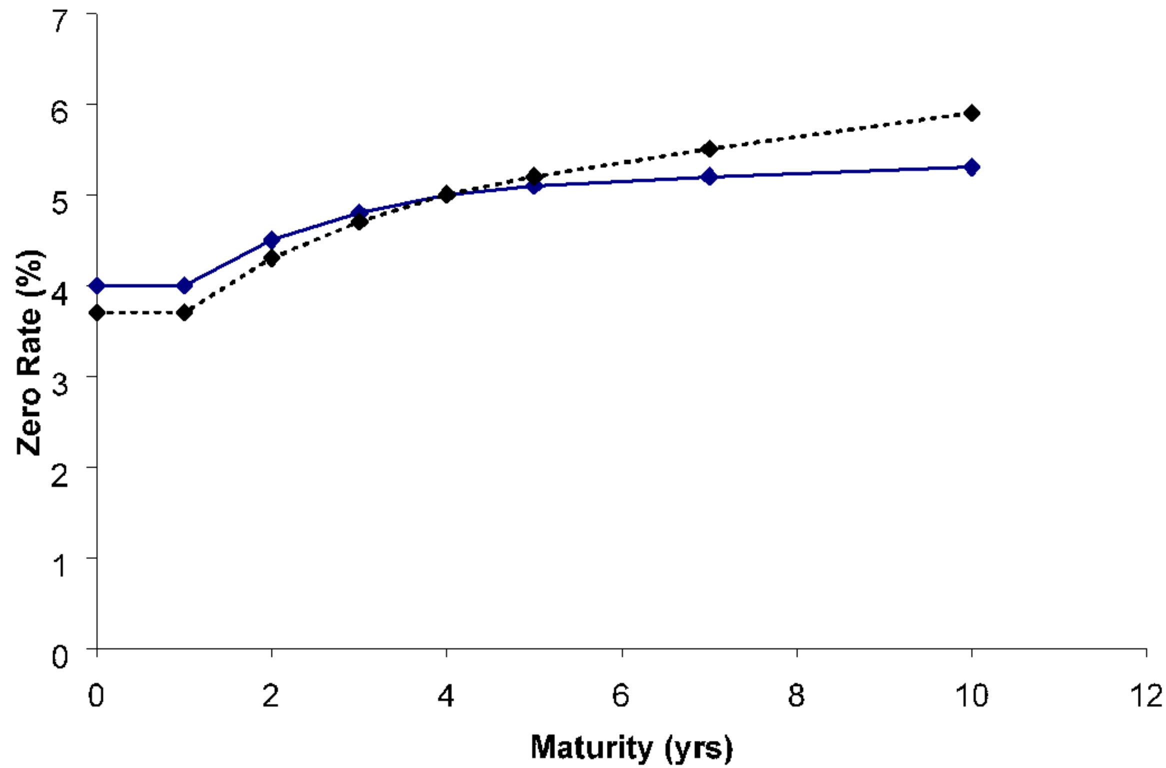
Partial Duration IV

Partial Durations Can Be Used to Investigate the Impact of Any Yield Curve Change

- Any yield curve change can be defined in terms of changes to individual points on the yield curve
- For example, to define a rotation we could change the 1-, 2-, 3-, 4-, 5-, 7, and 10-year maturities by -3ϵ, -2ϵ, $-\epsilon$, 0, ϵ, 3ϵ, 6ϵ

Partial Duration V

Combining Partial Durations to Create Rotation in the Yield Curve



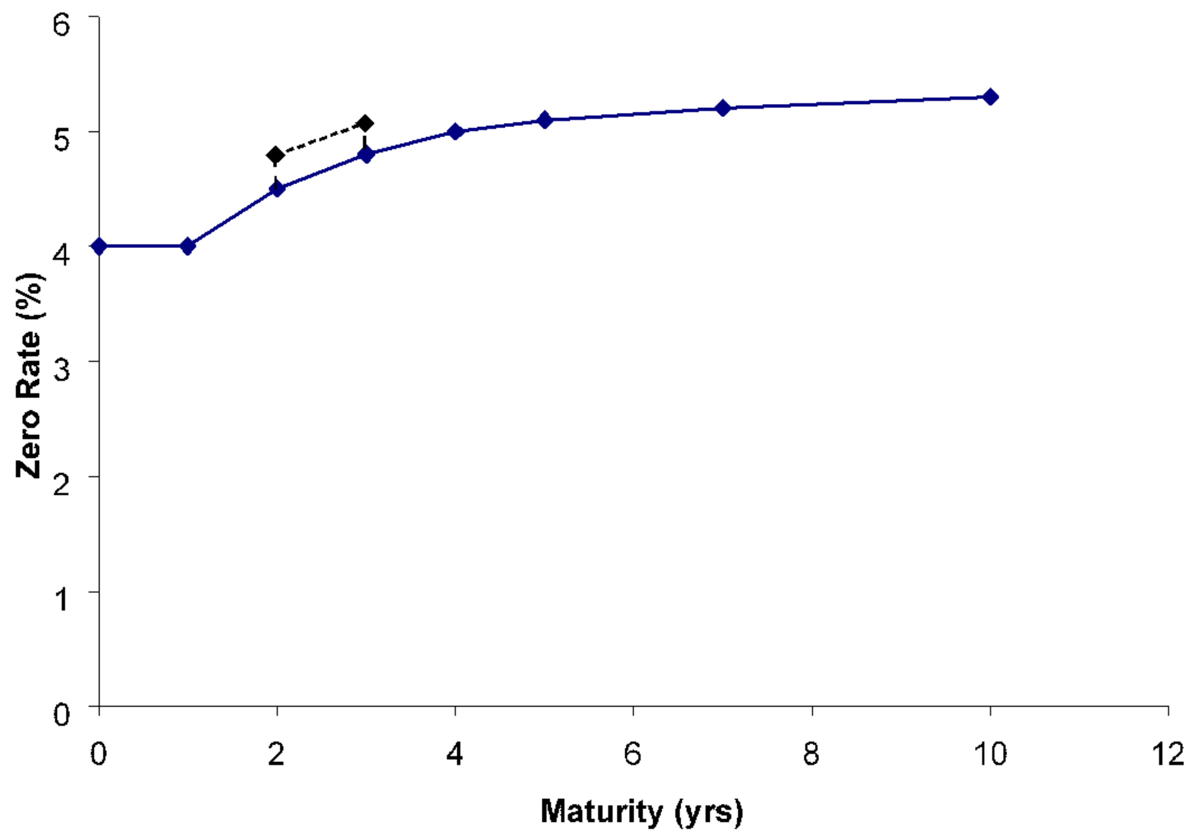
Impact of Rotation

- The impact of the rotation on the proportional change in the value of the portfolio in the example is

$$-[0.2\times(-3\epsilon) + 0.6\times(-2\epsilon) \ldots + (-3.0)\times(+6\epsilon)] = 25.0\epsilon$$

Alternative approach

Bucket the yield curve and investigate the effect of a small change to each bucket



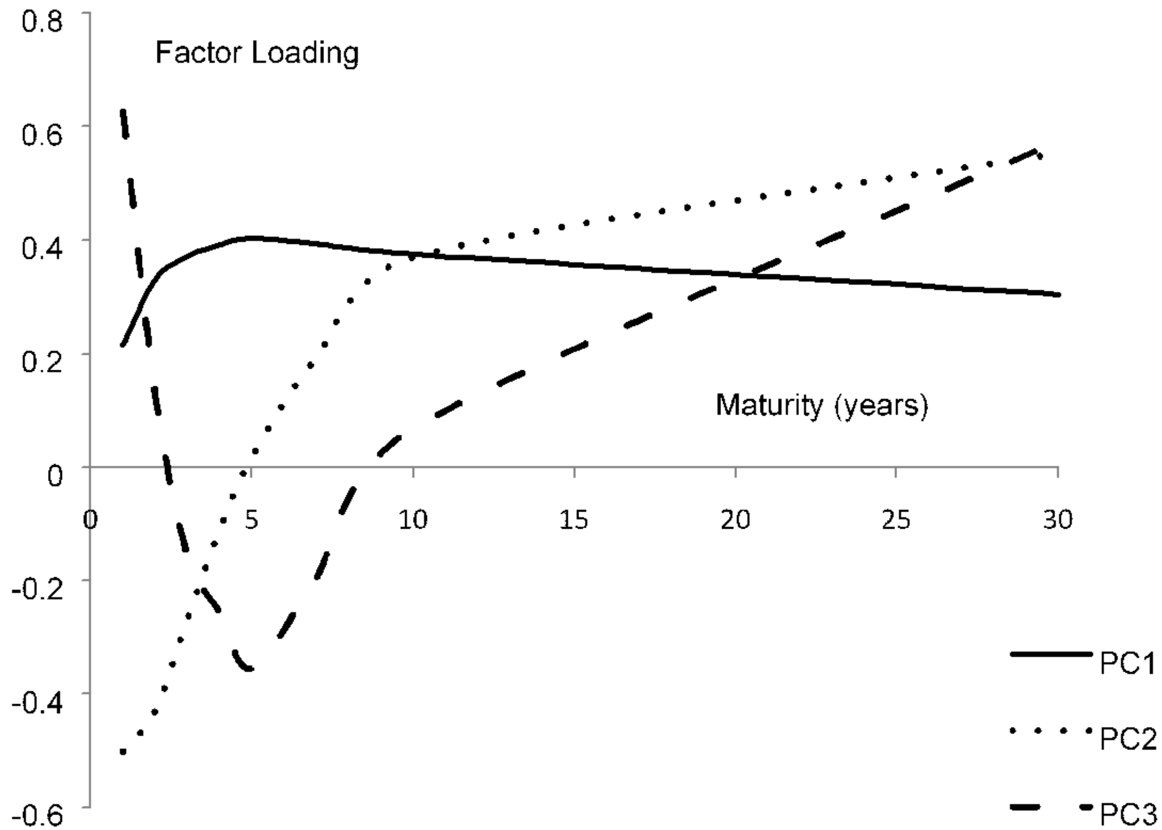
Principal Components Analysis

- Attempts to identify standard shifts (or factors) for the yield curve so that most of the movements that are observed in practice are combinations of the standard shifts

Results

- The first factor is a roughly parallel shift (90.9% of variance explained)
- The second factor is a twist 6.8% of variance explained)
- The third factor is a bowing (1.3% of variance explained)

The Three Factors



Calculating Multiple Deltas

Alternatives for Calculating Multiple Deltas to Reflect Non-Parallel Shifts in Yield Curve

- Shift individual points on the yield curve by one basis point (the partial duration approach)
- Shift segments of the yield curve by one basis point (the bucketing approach)
- Shift quotes on instruments used to calculate the yield curve
- Calculate deltas with respect to the shifts given by a principal components analysis.

Gamma for Interest Rates

- Gamma has the form

$$\frac{\partial^2 P}{\partial x_i \partial x_j}$$

where x_i and x_j are yield curve shifts considered for delta

- To avoid information overload one possibility is consider only $i = j$
- Another is to consider only parallel shifts in the yield curve and calculate convexity
- Another is to consider the first two or three types of shift given by a principal components analysis

Vega for Interest Rates

- One possibility is to make the same change to all interest rate implied volatilities. (However implied volatilities for long-dated options change by less than those for short-dated options.)
- Another is to do a principal components analysis on implied volatility changes for the instruments that are traded