ASP Assignment 5

1. Draw the picture of the following Markov chains. Specify the classes of each of the Markov chains, and determine whether they are transient or recurrent. (No need to give any steps.)
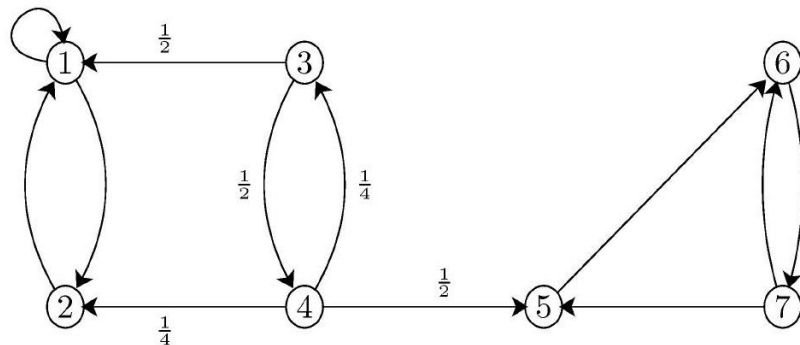
(a) $\boldsymbol{P} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$

(b) $\boldsymbol{P} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

(c) $\boldsymbol{P} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

(d) $\boldsymbol{P} = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$
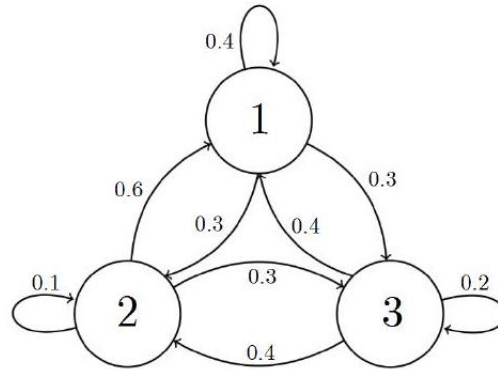
2. Consider the Markov chain in below figure. It is assumed that when there is an arrow from state i to state j, then $p_{i,j} > 0$. There is no arrow from state i to state j, then $p_{i,j} = 0$.

(a) Find the equivalence classes for this Markov chain. Which class is recurrent? Which class is transient?

(b) There are two recurrent classes, R_1, which contains state 1, and R_2, which does not contain state 1. Assuming $X_0 = 3$, find the probability that the chain gets absorbed in R_1.
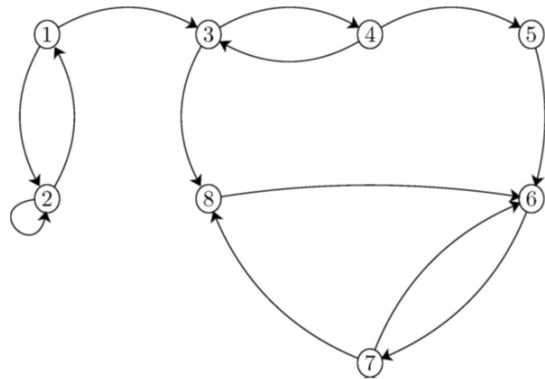
3. Consider the following Markov Chain:

Find the generating functions $f_{1,3}(z)$, $f_{2,3}(z)$, $f_{3,3}(z)$, where

$$f_{i,j}(z) = \sum_{n=1}^{\infty} f_{i,j}^n z^n.$$

4. Consider the Markov chain shown in below figure. It is assumed that when there is an arrow from state i to state j, then $p_{i,j} > 0$.

Find the equivalence classes for this Markov chain. Which class is recurrent? Which class is transient? Calculate the period of each class.