FINM3093 Investments

Lecture 4 Exercises

Solutions

1. a. Firm-specific risk is measured by the residual standard deviation. Thus, stock A has more firm-specific risk: 10.3% > 9.1%

   b. Market risk is measured by beta, the slope coefficient of the regression. A has a larger beta coefficient: 1.2 > 0.8

   c. R^2 measures the fraction of total variance of return explained by the market return. A's R^2 is larger than B's: 0.576 > 0.436

   d. Rewriting the SCL equation in terms of total return (r) rather than excess return (R):

$$r_A - r_f = \alpha + \beta \times (r_M - r_f) \Rightarrow$$
$$r_A = \alpha + r_f \times (1-\beta) + \beta \times r_M$$

   The intercept is now equal to:

$$\alpha + r_f \times (1-\beta) = 1\% + r_f \times (1-1.2)$$

   Since r_f = 6%, the intercept would be: $1\% + 6\%(1-1.2) = 1\% - 1.2\% = -0.2\%$

2.

1) The standard deviation of each stock can be derived from the following equation for R^2:

$$R_i^2 = \frac{\beta_i^2 \sigma_M^2}{\sigma_i^2} = \frac{\text{Explained variance}}{\text{Total variance}}$$

Therefore:

$$\sigma_A^2 = \frac{\beta_A^2 \sigma_M^2}{R_A^2} = \frac{0.7^2 \times .20^2}{0.20} = .098$$
$$\sigma_A = .3130 = 31.30\%$$

For stock B:

$$\sigma_B^2 = \frac{\beta_B^2 \sigma_M^2}{R_B^2} = \frac{1.2^2 \times .20^2}{0.12} = .048$$

$$\sigma_B = .6928 = 69.28\%$$

2) The systematic risk for A is:

$$\beta_A^2 \times \sigma_M^2 = 0.70^2 \times 20^2 = 196$$

The firm-specific risk of A (the residual variance) is the difference between A's total risk and its systematic risk:

980 – 196 = 784

The systematic risk for B is:

$$\beta_B^2 \times \sigma_M^2 = 1.20^2 \times 20^2 = 576$$

B's firm-specific risk (residual variance) is:

4,800 – 576 = 4,224

3) The covariance between the returns of A and B is (since the residuals are assumed to be uncorrelated):

$$Cov(r_A, r_B) = \beta_A \beta_B \sigma_M^2 = 0.70 \times 1.20 \times .04 = .0336$$

The correlation coefficient between the returns of A and B is:

$$\rho_{AB} = \frac{Cov(r_A r_B)}{\sigma_A \sigma_B} = \frac{.0336}{.3130 \times .6928} = .1549$$

4) Note that the correlation is the square root of R^2: $\rho = \sqrt{R^2}$

$$Cov(r_{A,} r_M) = \rho \sigma_A \sigma_M = 0.20^{1/2} \times 31.30 \times 20 = 280$$

$$Cov(r_{B,} r_M) = \rho \sigma_B \sigma_M = 0.12^{1/2} \times 69.28 \times 20 = 480$$

5) For portfolio P we can compute:

$\sigma_P = [(0.6^2 \times 980) + (0.4^2 \times 4800) + (2 \times 0.4 \times 0.6 \times 336)]^{1/2} = [1282.08]^{1/2} = 35.81\%$

$\beta_P = (0.6 \times 0.7) + (0.4 \times 1.2) = 0.90$

$\sigma^2(e_P) = \sigma_P^2 - \beta_P^2\sigma_M^2 = 1282.08 - (0.90^2 \times 400) = 958.08$

$\text{Cov}(r_P,r_M) = \beta_P\sigma_M^2 = 0.90 \times 400 = 360$

This same result can also be attained using the covariances of the individual stocks with the market:

$\text{Cov}(r_P,r_M) = \text{Cov}(0.6r_A + 0.4r_B, r_M) = 0.6 \times \text{Cov}(r_A, r_M) + 0.4 \times \text{Cov}(r_B,r_M)$

$= (0.6 \times 280) + (0.4 \times 480) = 360$

6) Note that the variance of T-bills is zero, and the covariance of T-bills with any asset is zero. Therefore, for portfolio Q:

$$\sigma_Q = \left[w_P^2\sigma_P^2 + w_M^2\sigma_M^2 + 2 \times w_P \times w_M \times \text{Cov}(r_P, r_M)\right]^{1/2}$$
$$= \left[(0.5^2 \times 1{,}282.08) + (0.3^2 \times 400) + (2 \times 0.5 \times 0.3 \times 360)\right]^{1/2} = 21.55\%$$

$$\beta_Q = w_P\beta_P + w_M\beta_M = (0.5 \times 0.90) + (0.3 \times 1) + (0.20 \times 0) = 0.75$$

$$\sigma^2(e_Q) = \sigma_Q^2 - \beta_Q^2\sigma_M^2 = 464.52 - (0.75^2 \times 400) = 239.52$$

$$\text{Cov}(r_Q, r_M) = \beta_Q\sigma_M^2 = 0.75 \times 400 = 300$$

3. a. Call the aggressive stock A and the defensive stock D. Beta is the sensitivity of the stock's return to the market return, i.e., the change in the stock return per unit change in the market return. Therefore, we compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in the market return:

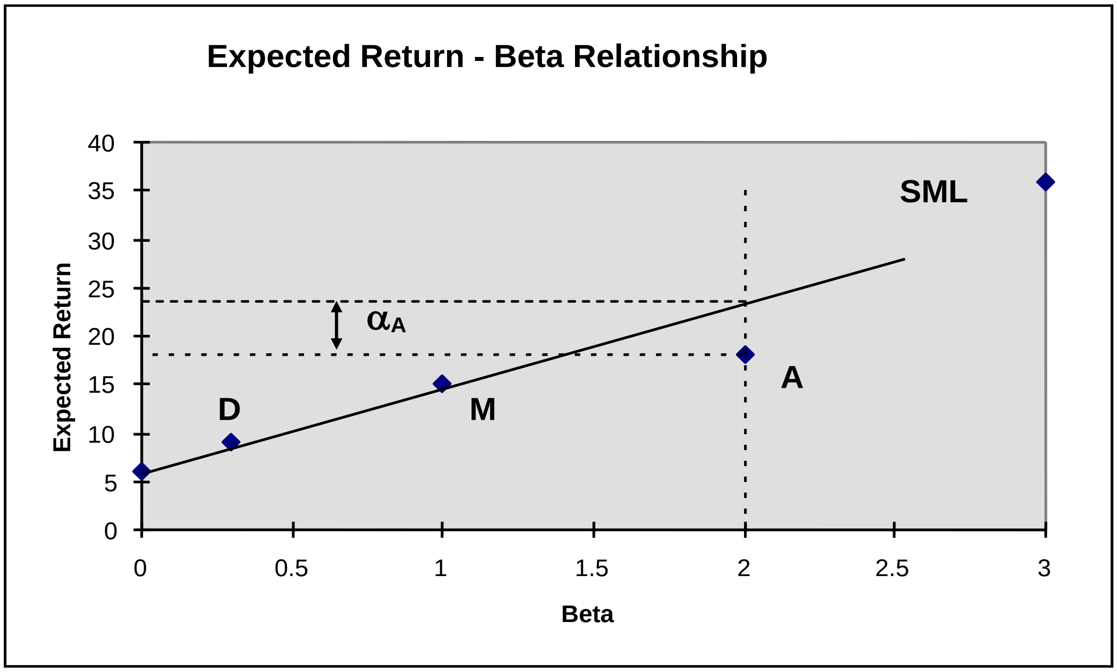
$$\beta_A = \frac{-.02 - .38}{.05 - .25} = 2.00 \qquad \beta_D = \frac{.06 - .12}{.05 - .25} = 0.30$$

b. With the two scenarios equally likely, the expected return is an average of the two possible outcomes:

$E(r_A) = 0.5 \times (-.02 + .38) = .18 = 18\%$

$E(r_D) = 0.5 \times (.06 + .12) = .09 = 9\%$

c. The SML is determined by the market expected return of $[0.5 \times (.25 + .05)] = 15\%$, with $\beta_M = 1$, and $r_f = 6\%$ (which has $\beta_f = 0$). See the following graph:



The equation for the security market line is:

$E(r) = .06 + \beta \times (.15 - .06)$

d. Based on its risk, the aggressive stock has a required expected return of:

$E(r_A) = .06 + 2.0 \times (.15 - .06) = .24 = 24\%$

The analyst's forecast of expected return is only 18%. Thus the stock's alpha is:

α_A = actually expected return – required return (given risk)

$= 18\% - 24\% = -6\%$

Similarly, the required return for the defensive stock is:

$E(r_D) = .06 + 0.3 \times (.15 - .06) = 8.7\%$

The analyst's forecast of expected return for D is 9%, and hence, the stock has a positive alpha:

α_D = Actually expected return – Required return (given risk)

= .09 – .087 = +0.003 = +0.3%

The points for each stock plot on the graph as indicated above.

e. The hurdle rate is determined by the project beta (0.3), not the firm's beta. The correct discount rate is 8.7%, the fair rate of return for stock D.

4.

a. Since the stock's beta is equal to 1.2, its expected rate of return is:

.06 + [1.2 × (.16 – .06)] = 18%

$$E(r) = \frac{D_1 + P_1 - P_0}{P_0} \rightarrow 0.18 = \frac{P_1 - \$50 + \$6}{\$50} \rightarrow P_1 = \$53$$

b. The series of $1,000 payments is a perpetuity. If beta is 0.5, the cash flow should be discounted at the rate:

.06 + [0.5 × (.16 – .06)] = .11 = 11%

PV = $1,000/0.11 = $9,090.91

If, however, beta is equal to 1, then the investment should yield 16%, and the price

paid for the firm should be:

PV = $1,000/0.16 = $6,250

The difference, $2,840.91, is the amount you will overpay if you erroneously assume that beta is 0.5 rather than 1.

c. Using the SML: .04 = .06 + β × (.16 – .06) ⇒ β = –.02/.10 = –0.2

5. r_1 = 19%; r_2 = 16%; β_1 = 1.5; β_2 = 1

a. To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b. If $r_f = 6\%$ and $r_M = 14\%$, then (using the notation alpha for the abnormal return):

$$\alpha_1 = .19 - [.06 + 1.5 \times (.14 - .06)] = .19 - .18 = 1\%$$

$$\alpha_2 = .16 - [.06 + 1 \times (.14 - .06)] = .16 - .14 = 2\%$$

Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If $r_f = 3\%$ and $r_M = 15\%$, then:

$$\alpha_1 = .19 - [.03 + 1.5 \times (.15 - .03)] = .19 - .21 = -2\%$$

$$\alpha_2 = .16 - [.03 + 1 \times (.15 - .03)] = .16 - .15 = 1\%$$

Here, not only does the second investor appear to be the superior stock selector, but the first investor's predictions appear valueless (or worse).