FINM2013 Time Series for Finance and Macroeconomics

Chapter 5 Assignment

1. Identify the following as specific ARIMA models. That is, what are p, d, and q and what are the values of the parameters (the ϕ's and θ's)?

   (a) $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$.

   (b) $Y_t = 2Y_{t-1} - Y_{t-2} + e_t$.

   (c) $Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$.

(a) 首先考虑是否平稳，如果平稳 $\Rightarrow$ ARMA(p,q)
如果不平稳 $\Rightarrow$ ARIMA(p,d,q)

ARMA(p,q)与AR(p)特征方程相同，所以类比检验AR(p)的平稳性
检验ARMA(p,q)的平稳性
AR(2)平稳性检验 ① $\phi_2+\phi_1<1$ ② $\phi_2-\phi_1<1$ ③ $|\phi_2|<1$

之后类比检验MA(q)的可逆性检验ARMA(p,q)的可逆性
MA(1)可逆性检验 $|\theta|<1$

it look like ARMA(2,1) model with $\phi_1=1$ $\phi_2=-0.25$ $\theta_1=0.1$

Check the stationarity: ① $\phi_2+\phi_1=0.75<1$
② $\phi_2-\phi_1=-1.25<1$
③ $|\phi_2|=0.25<1$

Thus, the process is stationary

Check the invertibility: $|\theta_1|=0.1<1$

Thus, the process is invertible

$\therefore$ it's stationary and invertible ARMA(2,1) model with $\phi_1=1$ $\phi_2=-0.25$ $\theta_1=0.1$

(b) Firstly, it look like AR(2) model with $\phi_1=2$, $\phi_2=-1$

Check the stationarity: ① $\phi_2+\phi_1=1$

Thus, it's nonstationary process.

$Y_t-Y_{t-1}=Y_{t-1}-Y_{t-2}+e_t \Rightarrow \nabla Y_t=\nabla Y_{t-1}+e_t \Rightarrow \nabla Y_t-\nabla Y_{t-1}=e_t$

Thus, $\nabla^2 Y_t=e_t$, Y_t is an IMA(2,0) model.

(c) it look like ARMA (2,2) model with $\phi_1=0.5$, $\phi_2=-0.5$, $\theta_1=0.5$ $\theta_2=-0.25$

Check the stationarity: ① $\phi_2+\phi_1=0<1$

② $\phi_2-\phi_1=-1<1$

③ $|\phi_2|=0.5<1$

∴ the process is stationary

Check the invertibility: ① $\theta_2+\theta_1=0.25<1$

② $\theta_2-\theta_1=-0.75<1$

③ $|\theta_2|=0.25<1$

∴ The process is invertible

Therefore, it's a stationary and invertible ARMA(2,2) model with $\phi_1=0.5$, $\phi_2=-0.5$ $\theta_1=0.5$ $\theta_2=-0.25$

2. For each of the ARIMA models below, give the values for $E(\nabla Y_t)$ and $Var(\nabla Y_t)$.

(a) $Y_t=\overset{\theta_0}{3}+Y_{t-1}+e_t-0.75e_{t-1}$.

(b) $Y_t=10+1.25Y_{t-1}-0.25Y_{t-2}+e_t-0.1e_{t-1}$.

(c) $Y_t=5+2Y_{t-1}-1.7Y_{t-2}+0.7Y_{t-3}+e_t-0.5e_{t-1}+0.25e_{t-2}$.

模型均值非零时，有 $\mu=\dfrac{\theta_0}{1-\phi_1-\phi_2-\cdots}$

(a)

$$Y_t=3+Y_{t-1}+e_t-0.75e_{t-1}$$

Here $\nabla Y_t=Y_t-Y_{t-1}=3+e_t-0.75e_{t-1}$ so that $E(\nabla Y_t)=3$ and $Var(\nabla Y_t)=(1+0.75^2)\sigma_e^2=\frac{25}{16}\sigma_e^2$

(b) ARMA(1,1)模型 $\begin{cases}\gamma_0=\dfrac{1-2\phi\theta+\theta^2}{1-\phi^2}\sigma_e^2\\ \rho_k=\dfrac{(1-\theta\phi)(\phi-\theta)}{1-2\phi\theta+\theta^2}\phi^{k-1}\ \text{for}\ k\geq1\end{cases}$

$$Y_t=10+1.25Y_{t-1}-0.25Y_{t-2}+e_t-0.1e_{t-1}$$

In this case $\nabla Y_t=Y_t-Y_{t-1}=10+0.25(Y_{t-1}-Y_{t-2})+e_t-0.1e_{t-1}$. So the model is a stationary, invertible ARIMA(1,1,1) model with $\phi=0.25$, $\theta=0.1$. Hence $E(\nabla Y_t)=\frac{\theta_0}{1-\phi}=\frac{40}{3}$. Also, the Variance

求Var(∇Yt)就是求ARMA(1,1)的γ_0

$$Var(\nabla Y_t)=\frac{1-2\phi\theta+\theta^2}{1-\phi^2}\sigma_e^2=1.024\sigma_e^2$$

(c) 法一:

$$Y_t=5+2Y_{t-1}-1.7Y_{t-2}+0.7Y_{t-3}+e_t-0.5e_{t-1}+0.25e_{t-2}$$

Factoring the AR characteristic polynomial we have $1-2x+1.7x^2-0.7x^3=(1-x)(1-x+0.7x^2)$. This shows that a first difference is needed after which a stationary AR(2) obtains. Thus the model may be rewritten as $\nabla Y_t=5+\nabla Y_{t-1}-0.7\nabla Y_{t-2}+e_t-0.5e_{t-1}+0.25e_{t-2}$ is an ARIMA(2,1,2) with $\phi_1=1$, $\phi_2=-0.75$, $\theta_1=0.5$, $\theta_2=-0.25$, and $\theta_0=5$.

chapter 5里面讲过 ARIMA(p,1,q)并非ARMA(p+1,q)

① ARIMA(p,1,q)模型

$W_t=\nabla Y_t=Y_t-Y_{t-1}$ 服从ARMA(p,q)

∴ $W_t=\nabla Y_t=Y_t-Y_{t-1}=\phi_1W_{t-1}+\phi_2W_{t-2}+\cdots+\phi_pW_{t-p}+e_t-\theta_1e_{t-1}-\theta_2e_{t-2}-\cdots-\theta_qe_{t-q}$

$= \phi_1(Y_{t-1}-Y_{t-2}) + \phi_2(Y_{t-2}-Y_{t-3}) + \cdots + \phi_p(Y_{t-p}-Y_{t-p-1}) + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$

$\Downarrow$ 得到 差分方程形式

$$Y_t = (1+\phi_1)Y_{t-1} + (\phi_2-\phi_1)Y_{t-2} + (\phi_3-\phi_2)Y_{t-3} + \cdots + (\phi_p-\phi_{p-1})Y_{t-p} - \phi_p \boxed{Y_{t-p-1}} + e_t - \theta_1 e_{t-1} - \cdots - \theta_q \boxed{e_{t-q}}$$

又观察可知、以上差分方程形式类似于 ARMA(p+1,q) 但实际上不是！因为特征方程根

不满足：$1-(1+\phi_1)x-(\phi_2-\phi_1)x^2-(\phi_3-\phi_2)x^3-\cdots-(\phi_p-\phi_{p-1})x^p+\phi_p x^{p+1}=0$

$\longrightarrow \boxed{(1-\phi_1 x-\phi_2 x^2-\cdots-\phi_p x^p)(1-x)=0}$ $\longrightarrow$ 特征根$\phi=1$ $\Rightarrow$ first differencing

可知有根为 $\phi=1$. 不满足 ARMA 模型要求的 $|\phi|<1$.

所以 ARIMA(p,1,q) 不是 ARMA(p+1,q)！

W_t的特征方程，根满足绝对值 <1

法二：

ARMA(p,q) 与 AR(p) 验证平稳性 $(p\geq 3)$：① $\phi_1+\phi_2+\cdots+\phi_p<1$

② $|\phi_p|<1$

it looks like ARMA(3, 2) model with $\phi_1=2$, $\phi_2=-1.7$, $\phi_3=0.7$, $\theta_1=0.5$, $\theta_2=-0.25$, $\theta_0=5$

check the stationarity: ① $\phi_1+\phi_2+\phi_3 = 2-1.7+0.7=1$

$\therefore$ it's nonstationary, do the first differencing.

$Y_t - Y_{t-1} = 5 + (Y_{t-1}-Y_{t-2}) - 0.7(Y_{t-2}-Y_{t-3}) + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

$\nabla Y_t = 5 + \nabla Y_{t-1} - 0.7\nabla Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

check the stationarity: ① $\phi_2+\phi_1 = 1-0.7=0.3<1$

② $\phi_2-\phi_1 = -0.7-1 = -1.7<1$

③ $|\phi_2| = 0.7<1$

$\therefore$ stationary.

Actually, it's a ARIMA(2,1,2) model with $\phi_1=1$, $\phi_2=-0.7$, $\theta_1=0.5$, $\theta_2=-0.25$, $\theta_0=5$

$$E(\nabla Y_t) = \frac{\theta_0}{1-\phi_1-\phi_2} = \frac{5}{1-1-(-0.7)} = \frac{50}{7}$$

$W_t = \nabla Y_t$

$W_t = 5 + W_{t-1} - 0.7W_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$

3. Suppose that $Y_t = A + Bt + X_t$,where $\{X_t\}$ is a random walk. First suppose that A and B are constants.

(a) Is $\{Y_t\}$ stationary?

(b) Is $\{\bigtriangledown Y_t\}$ stationary?

Now suppose that A and B are random variables that are independent of the random walk $\{X_t\}$.

(c) Is $\{Y_t\}$ stationary?

(d) Is $\{\bigtriangledown Y_t\}$ stationary?

Solution:

(a) Since $E(Y_t) = A + Bt$, varies with t, the process Y_t is not stationary.

(b) $Cov(\nabla Y_t, \nabla Y_{t-k}) = Cov(B + \nabla X_t, B + \nabla X_{t-k}) = 0$ for $k > 0$ since ∇X_t is white noise and B is constant.

(c) No, since $E(Y_t) = E(A) + E(B)t$, in general, varies with t, the process Y_t is not stationary.

(d) $Cov(\nabla Y_t, \nabla Y_{t-k}) = Cov(B + \nabla X_t, B + \nabla X_{t-k}) = Var(B)$ for all k. So we do have stationary.

4. Consider two models:

$$\begin{aligned} A : Y_t &= 0.9Y_{t-1} + 0.09Y_{t-2} + e_t \\ B : Y_t &= Y_{t-1} + e_t - 0.1e_{t-1} \end{aligned}$$

(a) Identify each as a specific ARIMA model. That is, what are p, d, and q and what are the values of the parameters, ϕ's and θ's?

(b) In what ways are the two models different?

(c) In what ways are the two models similar? (Compare ψ-weights and π-weights.)

Solution:

(a)

$$A : Y_t = 0.9Y_{t-1} + 0.09Y_{t-2} + e_t$$

Since $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, and $|\phi_2| < 1$, the process is a stationary AR(2) process, with $\phi_1 = 0.9$ and $\phi_2 = 0.09$.

$$B : Y_t = Y_{t-1} + e_t - 0.1e_{t-1}$$

Since $Y_t - Y_{t-1} = e_t - 0.1e_{t-1}$, this is an IMA(1,1) process with $\theta = 0.1$.

(b) One is stationary while the other is nonstationary.

ARMA(p,q) 平稳 ARMA(p,d,q) 不平稳

(c) change AR(2) to general linear process model:

$$\begin{cases} \psi_0 = 1 \\ \psi_1 - \psi_0\phi_1 = 0 \text{ with } \psi_0 = 1 \\ \psi_2 - \psi_1\phi_1 - \psi_0\phi_2 = 0 \\ \vdots \\ \psi_j = \psi_{j-1}\phi_1 + \psi_{j-2}\phi_2 \end{cases} \longrightarrow \begin{cases} \psi_0 = 1 \\ \psi_1 = \phi_1 = 0.9 \\ \psi_2 = \psi_1\phi_1 + \psi_0\phi_2 = 0.9 \times 0.9 + 1 \times 0.09 = 0.9 \\ \psi_3 = \psi_2\phi_1 + \psi_1\phi_2 = 0.9 \times 0.9 + 0.9 \times 0.09 = 0.891 \end{cases}$$

Thus, express AR(2) as MA(∞) $Y_t = e_t + 0.9e_{t-1} + 0.9e_{t-2} + 0.891e_{t-3} + \cdots$

chapter 4 笔记

② 把 AR(2) 表示为 general linear process 形式

$Y_t = e_t + \phi_1 e_{t-1} + (\phi_1^2 + \phi_2)e_{t-2} + (\phi_1^3 + 2\phi_1\phi_2)e_{t-3} + \cdots$ $\begin{cases} \psi_0 = 1 \\ \psi_1 - \phi_1\psi_0 = 0 \\ \vdots \end{cases}$

$$\psi_j - \phi_1\psi_{j-1} - \phi_2\psi_{j-2} = 0$$

代入general linear process

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$$
$$Y_{t-1} = e_{t-1} + \psi_1 e_{t-2} + \psi_2 e_{t-3} + \cdots$$
$$Y_{t-2} = e_{t-2} + \psi_1 e_{t-3} + \psi_2 e_{t-4} + \cdots$$

$$Y_t = \phi_1(e_{t-1} + \psi_1 e_{t-2} + \psi_2 e_{t-3} + \cdots) + \phi_2(e_{t-2} + \psi_1 e_{t-3} + \psi_2 e_{t-4} + \cdots) + e_t$$

代入 Y_{t-1} 与 Y_{t-2} 的general linear process function

$$\therefore Y_t = e_t + \phi_1 e_{t-1} + (\phi_1\psi_1 + \phi_2)e_{t-2} + (\phi_1\psi_2 + \phi_2\psi_1)e_{t-3} + \cdots$$

$$又\ Y_t = e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + \psi_3 e_{t-3} + \cdots$$

所以
$$\begin{cases} \psi_1 = \phi_1 \\ \psi_2 = \phi_1\psi_1 + \phi_2 \\ \psi_3 = \phi_1\psi_2 + \phi_2\psi_1 \\ \vdots \\ \psi_j = \phi_1\psi_{j-1} + \phi_2\psi_{j-2} \end{cases}$$

Change IMA(1,1) to general linear process model.

$$Y_t = \sum_{j=-m}^{t} W_j = e_t + (1-\theta)e_{t-1} + (1-\theta)e_{t-2} + \cdots + (1-\theta)e_{t-m} - \theta e_{t-m-1}$$

$$\begin{cases} \psi_1 = 1-\theta = 0.9 \\ \psi_2 = 1-\theta = 0.9 \\ \vdots \end{cases}$$

Chapter 5 笔记

5. 滑动平均求和过程 IMA(d,q) ⇐ ARIMA(0,d,q)，即不包含自回归项

① IMA(1,1)

W_t 为 MA(q) process, $W_t = \nabla^d Y_t$

$$Y_t = Y_{t-1} + e_t - \theta e_{t-1} \quad = W_t \qquad \nabla Y_t = MA(1) = e_t - \theta e_{t-1}$$

$$Y_t - Y_{t-1} = \nabla Y_t = W_t \Rightarrow W_t = e_t - \theta e_{t-1}$$

所以有
$$\nabla Y_t = e_t - \theta e_{t-1} = W_t$$
$$+\ \nabla Y_{t-1} = e_{t-1} - \theta e_{t-2} = W_{t-1}$$
$$\vdots$$
$$+\ \nabla Y_{-m} = e_{-m} - \theta e_{-m-1} = W_{-m}$$

加和有
$$Y_t = e_t + (1-\theta)e_{t-1} + (1-\theta)e_{t-2} + \cdots + (1-\theta)e_{-m} - \theta e_{-m-1}$$

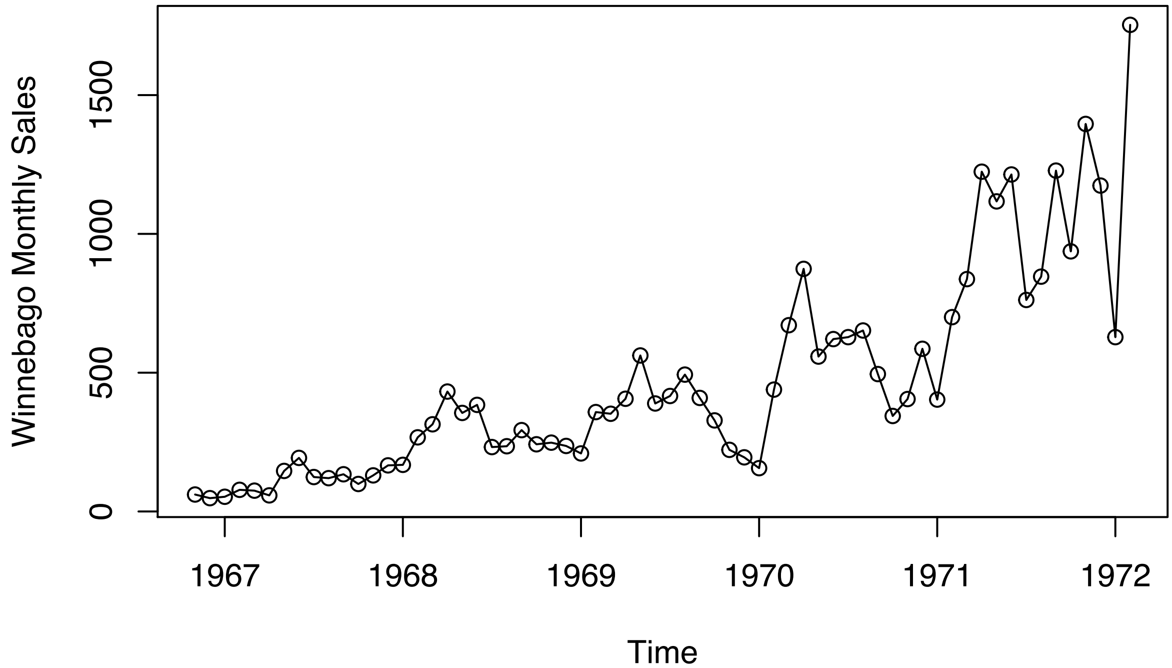
与ARMA模型对比，白噪声权重并非递减，而是都相同

The first two π-weights for the two models are identical and the remaining π-weights are nearly the same. These two models would be essentially impossible to distinguish in practice.

5.

(a)

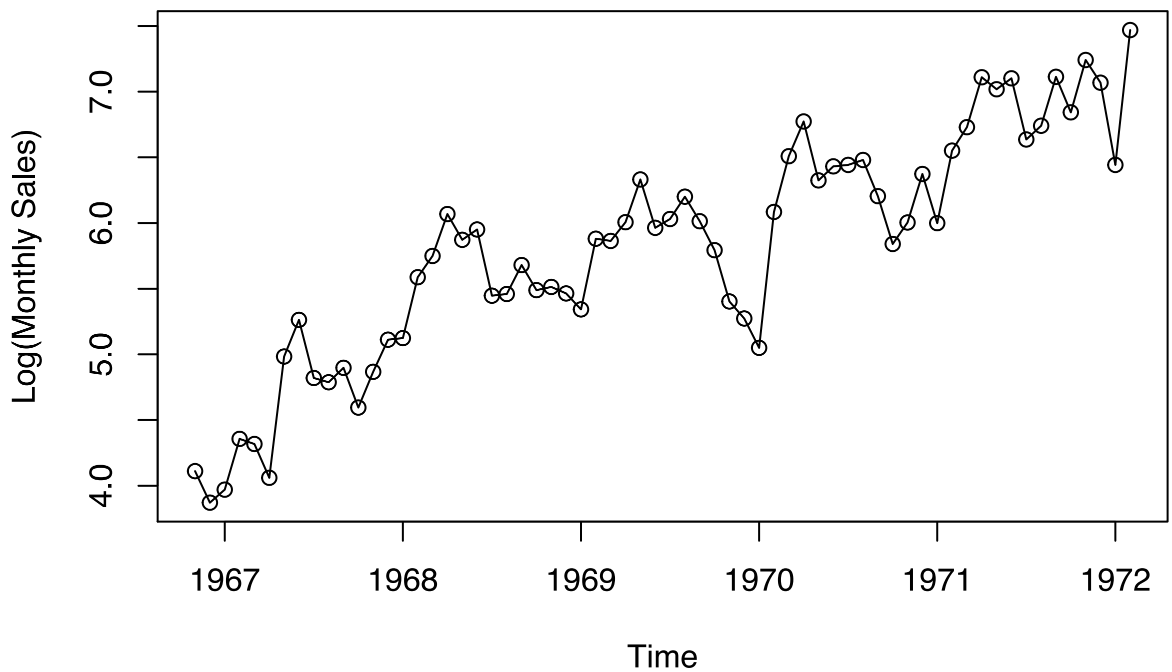
```
data("winnebago")
plot(winnebago,type='o',ylab='Winnebago Monthly Sales')
```



The series increases over time and the variation is larger as the series level gets higher—a series begging us to take logarithms.

(b)

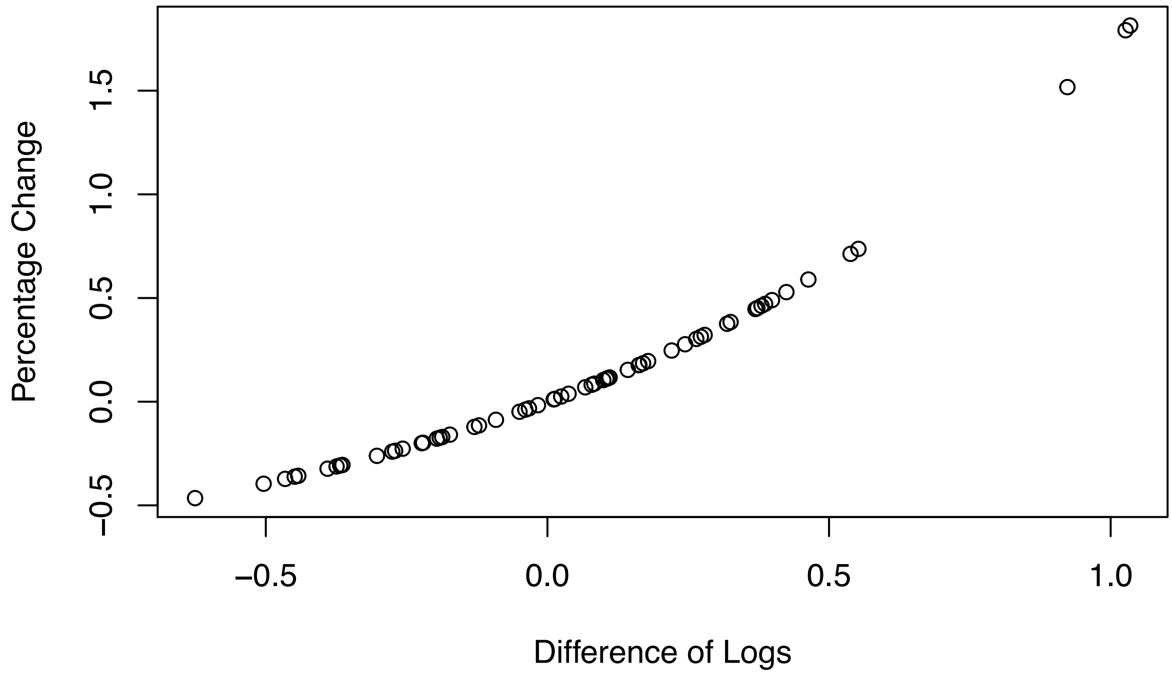
```
plot(log(winnebago),type='o',ylab='Log(Monthly Sales)')
```



The series still increases over time, but now the variation around the general level is quite similar at all levels of the series.

(c)

```
percentage=na.omit((winnebago-zlag(winnebago))/zlag(winnebago))
plot(x=diff(log(winnebago))[-1],y=percentage[-1], ylab='Percentage Change', xlab='Difference of Logs')
```




```
cor(diff(log(winnebago))[-1],percentage[-1])
```

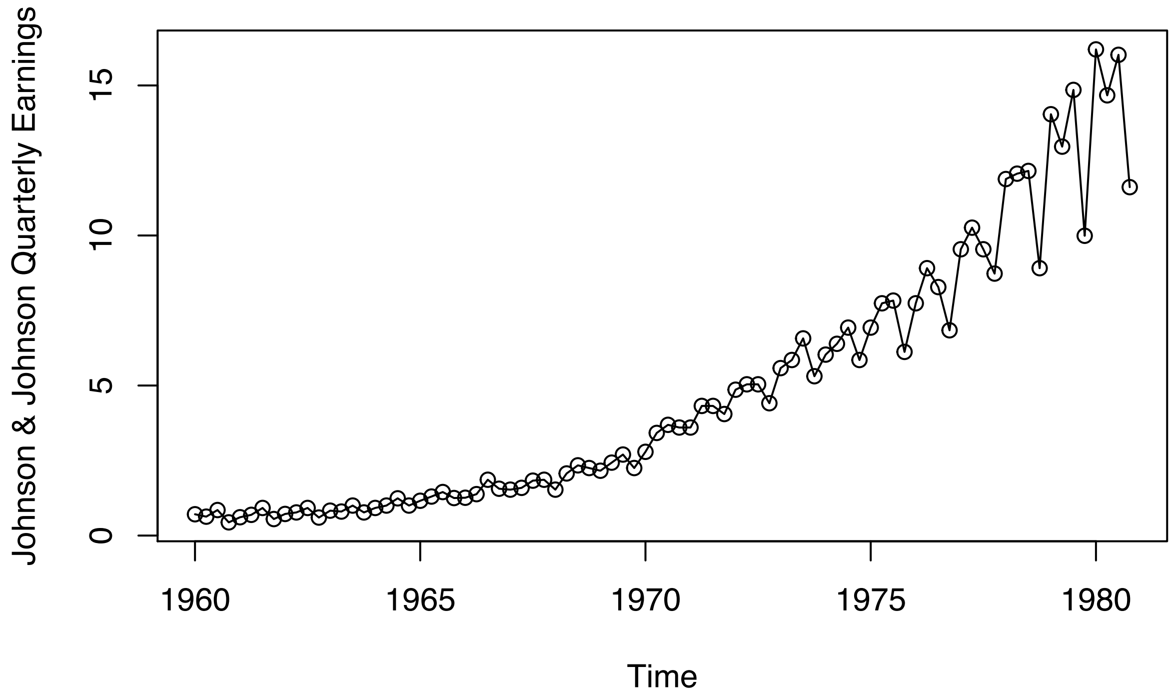
```
## [1] 0.9646886
```

If there were a perfect relationship, the above plot would be a straight line. Clearly, the relationship is good but not perfect. The correlation coefficient in this plot is 0.96 so the agreement is quite good. Of course, there is seasonality in this series that has not been modeled.

6.

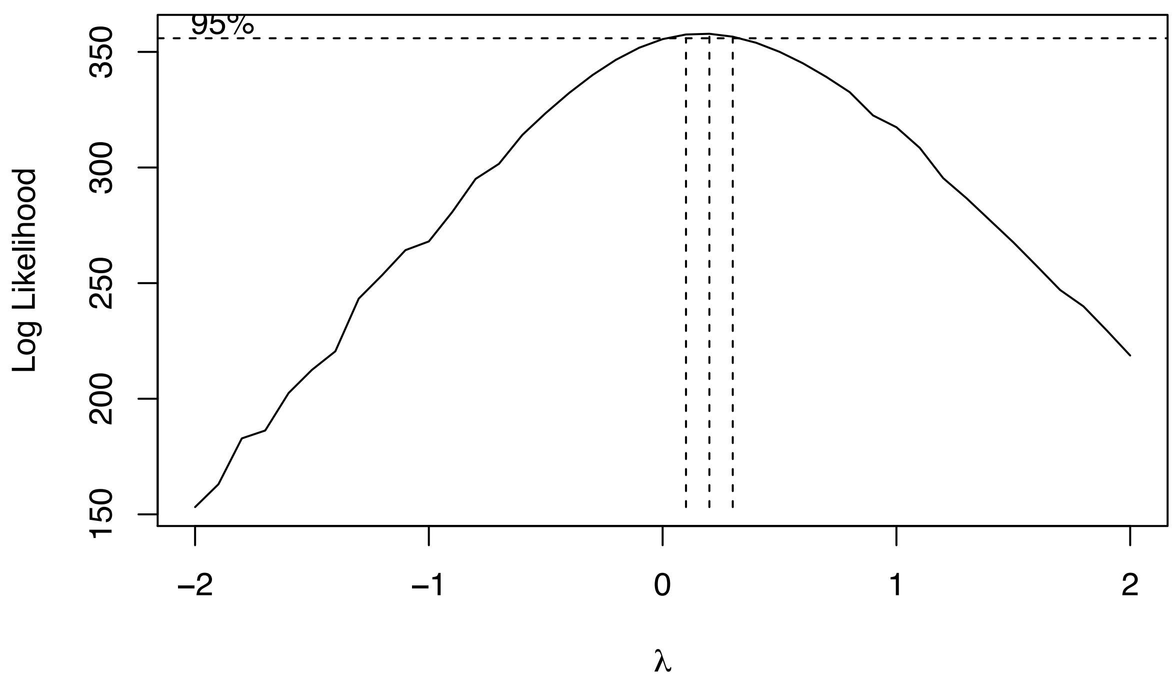
(a)

```
data(JJ); plot(JJ,type='o',ylab='Johnson & Johnson Quarterly Earnings')
```

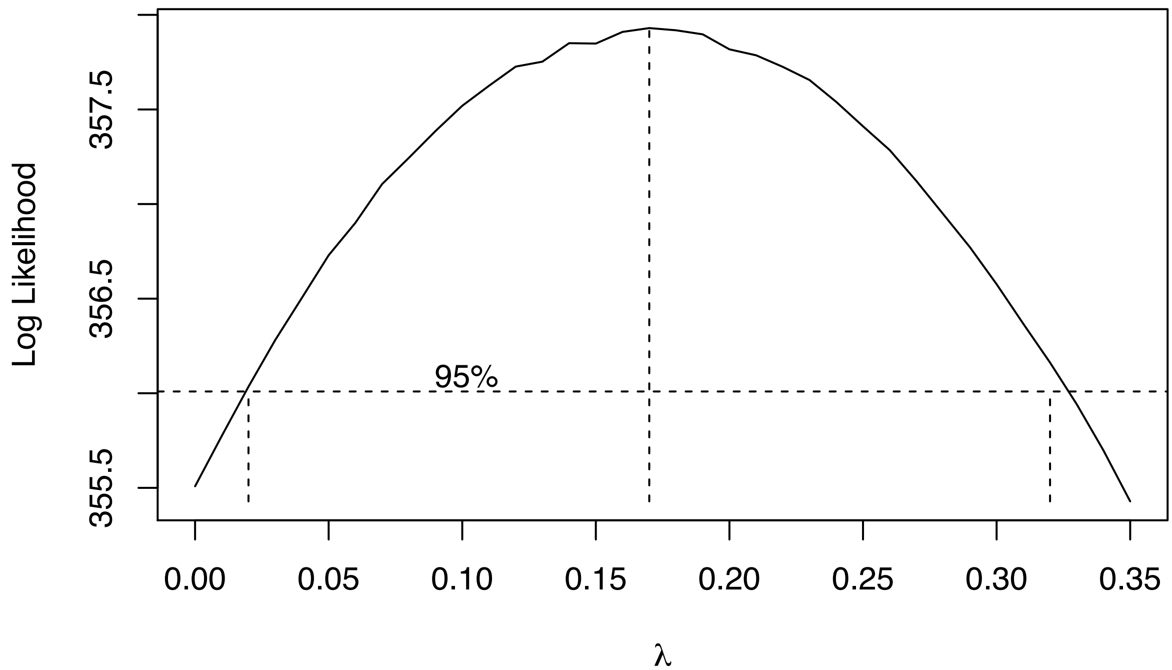


(b)

```
BC=BoxCox.ar(JJ)
```



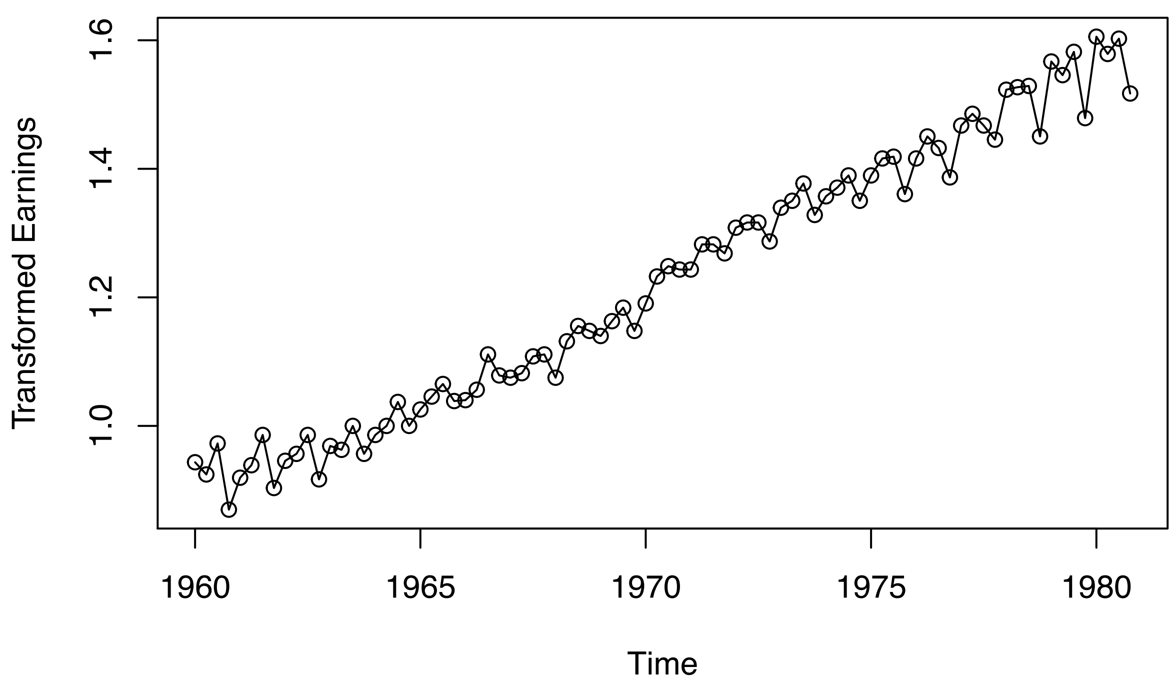

```
BC=BoxCox.ar(JJ,lambda=seq(0.0,0.35,0.01))
```



The plot on the left shows the initial default Box-Cox analysis. The plot on the right shows more detail as the range for the lambda parameter has been restricted to 0.0 to 0.35. The maximum likelihood estimate of lambda is 0.17 and the 95

(c)

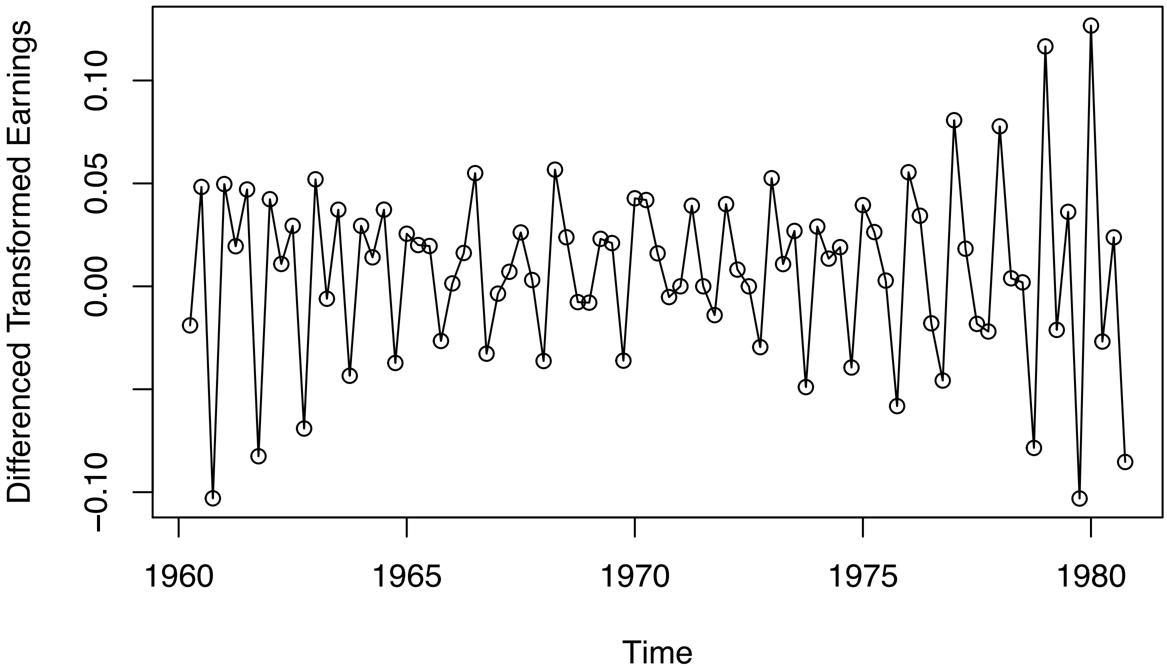
```
plot((JJ)^0.17,type='o',ylab='Transformed Earnings')
```



The variance has been stabilized but the strong trend must be accounted for before we can entertain a stationary model.

(d)

```
plot(diff((JJ)^0.17),type='o',ylab='Differenced Transformed Earnings')
```



The trend is now gone but the variation does not appear to be constant across time and there may be quarterly sea- sonality to deal with