MATH2033 Mathematical Statistics
Assignment 1

Due Date: **1/Mar/2024(Friday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.
- **Late submissions**, **answers without details**, or **unrecognizable handwritings** will NOT be graded.

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1. (a) If U is uniform on $[0,1]$, find the density function of $\sqrt{U}$.

   (b) If U is uniform on $[-1,1]$, find the density function of U^2.

2. Let $f_{1|2}(x_1 \mid x_2) = c_1 x_1/x_2^2,\ 0 < x_1 < x_2,\ 0 < x_2 < 1$, zero elsewhere, and $f_2(x_2) = c_2 x_2^4,\ 0 < x_2 < 1$, zero elsewhere, denote, respectively, the conditional pdf of X_1, given $X_2 = x_2$, and the marginal pdf of X_2. Determine:

   (a) The constants c_1 and c_2.

   (b) The joint pdf of X_1 and X_2.

   (c) $\mathbb{P}\left(\frac{1}{4} < X_1 < \frac{1}{2} \middle| X_2 = \frac{5}{8}\right)$.

   (d) $\mathbb{P}\left(\frac{1}{4} < X_1 < \frac{1}{2}\right)$.

3. Let $f(x_1, x_2) = 21 x_1^2 x_2^3,\ 0 < x_1 < x_2 < 1$, zero elsewhere, be the joint pdf of X_1 and X_2.

   (a) Find the conditional mean and variance of X_1, given $X_2 = x_2,\ 0 < x_2 < 1$.

   (b) Find the distribution of $Y = \mathbb{E}(X_1 \mid X_2)$.

   (c) Determine $\mathbb{E}(Y)$ and $\operatorname{Var}(Y)$ and compare these to $\mathbb{E}(X_1)$ and $\operatorname{Var}(X_1)$, respectively.

4. Suppose that X_1, X_2, X_3, and X_4 are independent random variables, each with pdf $f_{X_i}(x_i) = 4x_i^3,\ 0 \le x_i \le 1$. Find

   (a) $\mathbb{P}\left(X_1 < \frac{1}{2}\right)$.

   (b) $\mathbb{P}\left(\text{exactly one } X_i < \frac{1}{2}\right)$

   (c) $f_{X_1,X_2,X_3,X_4}(x_1, x_2, x_3, x_4)$

   (d) $F_{X_2,X_3}(x_2, x_3)$.

5. Suppose that $X_1, \ldots, X_{20}$ are independent random variables with density functions

   $$f(x) = 2x, \quad 0 \le x \le 1$$

   Let $S = X_1 + \cdots + X_{20}$. Use the central limit theorem to approximate $\mathbb{P}(S \le 10)$.