FINM3133 Time Series for Finance and Macroeconomics

Chapter 2 Exercises

1. Suppose $Y_t = 5 + 2t + X_t$, where $\{X_t\}$ is a zero-mean series with autocovariance function γ_k.
   (a) Find the mean function for $\{Y_t\}$.
   (b) Find the autocovariance function for $\{Y_t\}$.
   (c) Is $\{Y_t\}$ stationary? Why or why not?

2. Let $\{X_t\}$ be a stationary time series, and define $Y_t = \begin{cases} X_t & \text{for } t \text{ odd} \\ X_t + 3 & \text{for } t \text{ even.} \end{cases}$
   (a) Show that $Cov(Y_t, Y_{t-k})$ is free of t for all lags k.
   (b) Is $\{Y_t\}$ stationary?

3. Suppose that $\{Y_t\}$ is stationary with autocovariance function γ_k.
   (a) Show that $W_t = \bigtriangledown Y_t = Y_t - Y_{t-1}$ is stationary by finding the mean and autocovariance function for $\{W_t\}$.
   (b) Show that $U_t = \bigtriangledown^2 Y_t = \bigtriangledown[Y_t - Y_{t-1}] = Y_t - 2Y_{t-1} + Y_{t-2}$ is stationary. (You need not find the mean and autocovariance function for $\{U_t\}$.)

4. Let $Y_t = e_t - \theta(e_{t-1})^2$. For this exercise, assume that the white noise series is normally distributed.
   (a) Find the autocorrelation function for $\{Y_t\}$.
   (b) Is $\{Y_t\}$ stationary?

5. Let $\{Y_t\}$ be stationary with autovariance function γ_k. Let $\overline{Y} = \frac{1}{n}\sum_{t=1}^{n} Y_t$. Show that

$$
\begin{aligned}
Var(\overline{Y}) &= \frac{\gamma_0}{n} + \frac{2}{n}\sum_{k=1}^{n-1}\left(1 - \frac{k}{n}\right)\gamma_k \\
&= \frac{1}{n}\sum_{k=-n+1}^{n-1}\left(1 - \frac{|k|}{n}\right)\gamma_k
\end{aligned}
$$

6. Let $Y_1 = \theta_0 + e_1$, and then for $t > 1$ define Y_t recursively by $Y_t = \theta_0 + Y_{t-1} + e_t$. Here θ_0 is a constant. The process $\{Y_t\}$ is called a **random walk with drift**.

(a) Show that Y_t may be rewritten as $Y_t = t\theta_0 + e_t + e_{t-1} + \cdots + e_1$.

(b) Find the mean function for Y_t.

(c) Find the autocovariance function for Y_t.

7. Consider the standard random walk model where $Y_t = Y_{t-1} + e_t$ with $Y_1 = e_1$.

   (a) Use the representation of Y_t above to show that $\mu_t = \mu_{t-1}$ for $t > 1$ with initial condition $\mu_1 = E(e_1) = 0$. Hence show that $\mu_t = 0$ for all t.

   (b) Similarly, show that $Var(Y_t) = Var(Y_{t-1}) + \sigma_e^2$ for $t > 1$ with $Var(Y_1) = \sigma_e^2$ and hence $Var(Y_t) = t\sigma_e^2$.

   (c) For $0 \leq t \leq s$, use $Y_s = Y_t + e_{t+1} + e_{t+2} + \cdots + e_s$ to show that $Cov(Y_t, Y_s) = Var(Y_t)$ and, hence, that $Cov(Y_t, Y_s) = \min(t, s)\sigma_e^2$.