Caculus II Math 1038 (1002&1003)

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Week 4:

1. Quiz One:

   - **Time: 7:00-7:50pm, Thu 16 Mar 2023**
   - Venue: To be updated
   - Content: What you have learnt this semester, Chapter 11 Sequences and Series and Chapter 12 Vector space
   - No calculator, no elctronic devices, no books or notes are allowed. Only bring your pen and your photo ID.

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1. Taylor series:

   (a) Definition of Taylor series

   (b) Taylor Theorem: derivation of the coefficients

   (c) simple examples (need to remember all of them): e^x, $\sin x$, $\cos x$, $\ln x$, $\frac{1}{1-x}$, $\frac{1}{1+x}$, $\sqrt{1+x}$,...

   (d) n-th degree Taylor Polynomials $T_n(x)$

   (e) **Binomial series**: $(1+x)^p$ when p is any real number

      i. Binomial Coefficients

      ii. p can be positive and negative

   (f) Remainder R_n and error estimate.

2. Application of Taylor series (lots of exercise!!)
   The most important application of Taylor series is to use their partial sums, the Taylor Polynomials $T_n(x)$, to **approximate functions**. These partial sums are (finite) polynomials and are easy to compute.

   (a) Approximating functions by polynomials

      i. approximate by Taylor Polynomials: e.g. $e^{0.01}$, $\sqrt{1.02}$.

      ii. maximum error:

         A. Taylor's Inequality:
         $$|R_n| \leq \max \left| \frac{f^{(n+1)}(z)}{(n+1)!} (x-a)^{n+1} \right|$$

         B. Alternating Series Estimating Theorem (for alternating series only!!)
         $$|R_n| \leq |a_{n+1}|$$

   (b) Find the limit of a function without L'Hospital rule.

   (c) [For reading]Taylor series are very easy to work with and are often much preferable to the **messy** function they are approximating. Taylor series have wide reaching applications across mathematics, physics, engineering and other sciences. And the concept of approximating a function, or data, using a series of function is a fundamental tool in modern science and in use in data analysis, cell phones, differential equations, etc.. Taylor series give you a first glimpse into this world of approximation (some other common approximation methods are Fourier series and wavelets, if you are curious you could look those up and read about them).

   (d) In your future courses: Ordinary differential equations, partial differential equations, numerical analysis, machine learning, probability theory, etc.

3. Chapter 12 Vector the Geometry of Space

(a) Definitions
  i. vector $\overrightarrow{v}, \hat{v}, \overline{AB}$: length/magnitude/distance, components of a vector
  ii. scalar c
  iii. coordinate, axes, planes, octant, projection, three dimensional rectangular coordinate system
  iv. surface, solid region,

(b) Formula
  i. Distance formula in 3Ds

(c) Equations
  i. a line: $y = ax + b$
  ii. a surface: $z = f(x, y)$ or $F(x, y, z) = a$
    A. equation of a sphere wth center $C(h, k, l)$:

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

(d) Operations
  i. scalar multiplication

(e) Dot product of two vectors $\overrightarrow{a} \cdot \overrightarrow{b}$ and its properties

$$\overrightarrow{a} \cdot \overrightarrow{b} = |a|\,|b| \cos\theta \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

  i. Orthoganal: $\overrightarrow{a} \cdot \overrightarrow{b} = 0$
  ii. Projection

(f) Cross product

(g) [To be continued...]

4. **About Email writing skills**:
First, you are welcome to communicate with me by **EMAILS** at wenchen@uic.edu.cn if you need help with study Calculus II. But, please learn how to write a proper email.
This is very useful skill which you need to learn before going overseas for study in the future. I hope the content below can be helpful. You can learn how to writing an formal or informal email, by searching or from the following links:

- https://grammar.yourdictionary.com/writing/how-to-write-a-clear-polite-email-to-a-teacher.html

- https://www.wikihow.com/Write-an-Email

- https://www.grammarly.com/blog/email-teacher/

- Even though there are multiple communication modes, email is still considered the **most reliable and formal** mode of communication. Unlike social media platforms or chat applications, email is universal. This makes email one of the most preferred communication tools, **for work and for study!**

- Common problems I have met before:

  – **I don't who you are, and maybe you don't know my name either.** (this does not sound good) Sometimes, the email address does not contain your name, even though it does, I am not sure whether that is your true name or not. I may not reply anonymous email.

  – **Send me multiple emails with one/few sentences, like a chat.** This is not good as it is not efficient and takes up space in my email list. Please think carefully about the content before you send out, gather all your thoughts and questions together, so that I can reply them together in one email.

- Suggestions:

  – Use a **clear** subject line

– Choose the correct greeting and use an appropriate salutation:
Some teachers like to be called by names, like me. You can call me Monica. Some may prefer to be called *Prof./Dr.+ family name.* In most situations, it is NOT a problem calling your teachers directly *their given names.* This does NOT consider impolite. However, if you start your email with your question without calling or start with something general, like "dear teacher", then it sounds bit improper/weird because I would think you don't know who I am, or it might be an email sending to every teacher, or it may be an advertisement or scam. A good way to start an email is like this: *Hi Monica* or *Dear Dr. Chen.* both are proper and polite, and I know this email is for me.

– Structure as a **formal** letter

– Include only necessary information

– Keep your tone respectful