**ECON3133 Introductory Econometrics**
**Solution to Quiz 1**

**I. Multiple choice questions (8 points):**

1. B
2. C
3. B
4. A

**II. Problems (25 points)**

1.

a) Since $\bar{x} = 25.08, \bar{y} = 45.75, \sum_{i=1}^{12}(x_i - \bar{x})(y_i - \bar{y}) = 1097.25, \sum_{i=1}^{12}(x_i - \bar{x})^2 = 980.92$, then we have $\hat{\beta}_1 = 1097.25/980.92 = 1.12$, and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 45.75 - 1.12(25.08) = 17.69$. The fitted linear regression line is $\hat{y} = 17.69 + 1.12x$.

b) $SSE = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 = \sum_{i=1}^{n}(\hat{\beta}_1 x_i - \hat{\beta}_1\bar{x})^2 = \hat{\beta}_1^2\sum_{i=1}^{n}(x_i - \bar{x})^2$.

c) $SST = \sum_{i=1}^{12}(y_i - \bar{y})^2 = 1564.25$. Using result in (b) , we obtain that $SSE = 1.12^2(980.92) = 12.5 = 1227.38$, and $SSR = SST - SSE = 336.87$. So $R^2 = SSE/SST = 1227.38/1564.25 = 0.785$.

d) $\sum_{i=1}^{n}\hat{u}_i^2 = SSR = 336.87, \hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}\hat{u}_i^2 = \frac{1}{10}(336.87) = 33.687$. Thus $\hat{\sigma} = \sqrt{33.687} = 5.804$.

e) $se(\hat{\beta}_1) = \sqrt{\hat{\sigma}^2/SST_x} = 5.804/\sqrt{980.92} = 0.185$.

2. Consider the optimization problem of minimizing the variance of the weighted estimator. If the estimate is to be unbiased, it must be of the form $c_1\hat{\theta}_1 + c_2\hat{\theta}_2$ where $c_1$ and $c_2$ sum to 1. Thus $c_2 = 1 - c_1$. The function to minimize is Min$L$ = $\hat{\theta} = c_1^2 v_1 + (1 - c_1)^2 v_2$. The first order condition is $\partial L/\partial c_1 = 2c_1v_1 - 2(1 - c_1)v_2 = 0$ which implies $c_1 = v_2/(v_1 + v_2)$. An alternative form is obtained by dividing numerator and denominator by $v_1v_2$ to obtain $c_1 = (1/v_1)/(1/v_1 + 1/v_2)$. Thus, the weight is proportional to the inverse of the variance. The estimator with the smaller variance gets the larger weight.

3.

$$\begin{aligned}\hat{\beta}_1^* &= \frac{\sum_{i=1}^{n}(x_i^* - \bar{x}^*)(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i^* - \bar{x}^*)^2} = \frac{\sum_{i=1}^{n}(\mu_1 x_i - \mu_1\bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(\mu_1 x_i - \mu_1\bar{x})^2} \\ &= \frac{\mu_1\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\mu_1^2\sum_{i=1}^{2}(x_i - \bar{x})^2} = \frac{\hat{\beta}_1}{\mu_1}\end{aligned}$$