Chapter Sixteen

Managing Bond Portfolios

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Interest Rate Risk

- We have seen already that bond prices and yields are inversely related, and we know that interest rates can fluctuate substantially.
- As interest rates rise and fall, bondholders experience capital losses and gains.
- These gains or losses make fixed-income investments risky, even if the coupon and principal payments are guaranteed.

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Interest Rate Sensitivity (1 of 2)

Observations about the sensitivity of bond prices to changes in market interest rates:

1. Bond prices and yields are inversely related
2. An increase in a bond's yield to maturity results in a smaller price change than a decrease in yield of equal magnitude
3. Prices of long-term bonds tend to be more sensitive to interest rate changes than prices of short-term bonds

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Interest Rate Sensitivity (2 of 2)

4. The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases
   - Although interest rate sensitivity generally increases with maturity, it does so less than proportionally as bond maturity increases.

5. Interest rate risk is inversely related to the bond's coupon rate
   - Prices of low-coupon bonds are more sensitive to changes in interest rates than prices of high-coupon bonds.

6. The sensitivity of a bond's price to a change in its yield is inversely related to the YTM at which the bond is currently selling

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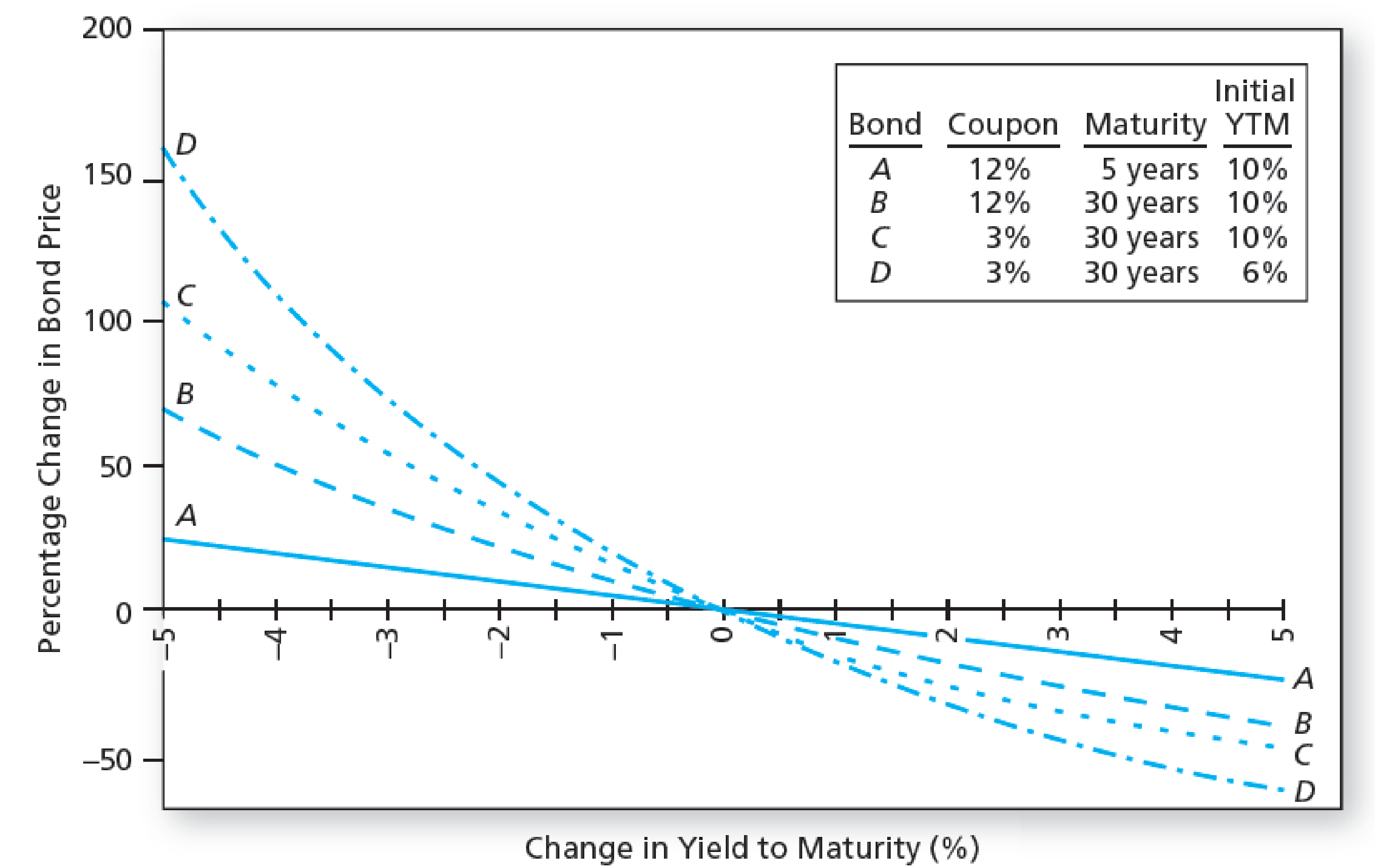
Change in Bond Price as a Function of Change in Yield to Maturity



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Prices of 8% Coupon Bond v.s. Prices of Zero-Coupon Bond

Yield to Maturity (APR)	*T* = 1 Year	*T* = 10 Years	*T* = 20 Years
8%	1,000.00	1,000.00	1,000.00
9%	990.64	934.96	907.99
Fall in price (%)*	0.94%	6.50%	9.20%

*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

Yield to Maturity (APR)	*T* = 1 Year	*T* = 10 Years	*T* = 20 Years
8%	924.56	456.39	208.29
9%	915.73	414.64	171.93
Fall in price (%)*	0.96%	9.15%	17.46%

*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

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Duration

Observations:

– for each maturity, the price of the zero-coupon bond falls by a greater proportional amount than the price of the 8% coupon bond.

– Because we know that long-term bonds are more sensitive to interest rate movements, the greater sensitivity of zeros-coupon bonds suggests that in some sense they must represent a longer-term investment than an equal-time-to-maturity coupon bond.

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Duration

In fact, this insight about the *effective maturity* of a bond can be made mathematically precise.

- The 20-year 8% bond makes many coupon payments, most of which come years before the bond's maturity date.
- Each payment may be considered to have its own 'maturity'.
- The effective maturity of the bond is therefore some sort of average of the maturities of <u>all</u> the cash flows.
- The zero-coupon bond, by contrast, makes only one payment at maturity. Its time to maturity is, therefore, well defind.

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Duration

- To deal with the ambiguity of the ‘maturity’ of a bond making many payments, we need a measure of the average maturity of the bond’s promised cash flows.
- **Macaulay’s duration** equals the weighted average of the times to each coupon or principal payment
  - The weight applied to each payment time is proportion of total value of bond accounted for by that payment (i.e., the PV of the payment divided by the bond price)

- Duration = Maturity for zero coupon bonds
- Duration < Maturity for coupon bonds

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Duration Calculation

- Duration calculation:

$$D = \sum_{t=1}^{T} t \times w_t$$

$$w_t = \frac{CF_t \big/ (1+y)^t}{\text{P}}$$

$CF_t = \text{Cash Flow at Time t}$

$P = \text{Price of Bond}$

$y = \text{Yield to Maturity}$

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Duration Calculation

	A	B	C	D	E	F	G
1			Time until		PV of CF		Column (C)
2			Payment		(Discount rate =		times
3		Period	(Years)	Cash Flow	5% per period)	Weight*	Column (F)
4	**A.** 8% coupon bond	1	0.5	40	38.095	0.0395	0.0197
5		2	1.0	40	36.281	0.0376	0.0376
6		3	1.5	40	34.554	0.0358	0.0537
7		4	2.0	1040	855.611	0.8871	1.7741
8	Sum:				964.540	1.0000	1.8852
9							
10	**B.** Zero-coupon	1	0.5	0	0.000	0.0000	0.0000
11		2	1.0	0	0.000	0.0000	0.0000
12		3	1.5	0	0.000	0.0000	0.0000
13		4	2.0	1000	822.702	1.0000	2.0000
14	Sum:				822.702	1.0000	2.0000
15							
16	Semiannual int rate:	0.05					
17							
18	*Weight = Present value of each payment (column E) divided by the bond price.						

Spreadsheet 16.1

Calculating the duration of two bonds

Column sums subject to rounding error.

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Interest Rate Risk

- Duration helps quantify the relationship that a bond's price sensitivity to interest rate changes increases with maturity.
  - Price change is proportional to duration

$$\frac{\Delta P}{P} = -D \times \left[\frac{\Delta(1+y)}{1+y} \right]$$

  - **Modified duration:** $D^* = D/(1+y)$

$$\frac{\Delta P}{P} = -D^* \Delta y$$

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Interest Rate Risk

Example 16.1 Duration and Interest Rate Risk

Consider the 2-year maturity, 8% coupon bond in Spreadsheet 16.1 making semiannual coupon payments and selling at a price of $964.540, for a yield to maturity of 10%. The duration of this bond is 1.8852 years. For comparison, we will also consider a zero-coupon bond with maturity *and duration* of 1.8852 years. As we found in Spreadsheet 16.1, because the coupon bond makes payments semiannually, it is best to treat one "period" as a half-year. So the duration of each bond is 1.8852 × 2 = 3.7704 (semiannual) periods, with a per-period interest rate of 5%. The modified duration of each bond is therefore 3.7704/1.05 = 3.591 semiannual periods.

Suppose the semiannual interest rate increases from 5% to 5.01%. According to Equation 16.3, the bond prices should fall by

$$\Delta P/P = -D^*\Delta y = -3.591 \times .01\% = -.03591\%$$

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Duration Rules

(1 of 2)

- *Rule 1*
  - The duration of a zero-coupon bond equals its time to maturity
- *Rule 2*
  - Holding maturity constant, a bond's duration is lower when the coupon rate is higher
- *Rule 3*
  - Holding the coupon rate constant, a bond's duration generally increases with its time to maturity

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Duration Rules

(2 of 2)

- *Rule 4*
  - Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower

- *Rule 5*
  - The duration of a level perpetuity is equal to:

$$\frac{1+y}{y}$$

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Duration Rules

(2 of 2)

- *Rule 5*
  - The duration of a level perpetuity is equal to:

$$\frac{1+y}{y}$$

  - For example, at a 10% yield, the duration of a perpetuity that pays $100 once a year forever is 1.1/0.1 = 11 years, but at an 8% yield it is 1.08/0.08 = 13.5 years.

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Exercise

A 9-year bond has a yield of 10% and a duration of 7.194 years. If the market yield changes by 50 basis points, what is the percentage change in the bond's price?

Find the duration of a 6% coupon bond making *annual* coupon payments if it has three years until maturity and has a yield to maturity of 6%.