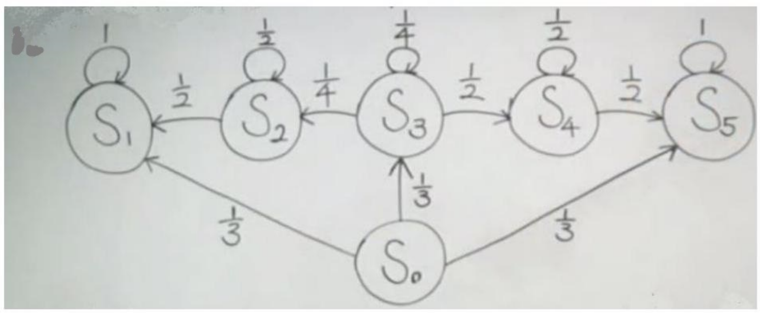
ASP Assignment 4

1. Consider a Markov chain with state space $\{0, 1\}$ and transition matrix

$$\boldsymbol{P} = \begin{bmatrix} 1/3 & 2/3 \\ 3/4 & 1/4 \end{bmatrix}.$$

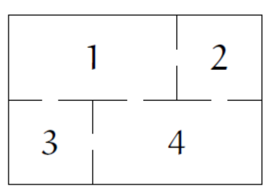
Assuming that the chain starts in state 0 at time $n = 0$, what is the probability that it is in state 1 at time $n = 3$?

2. Consider the following Markov chain.



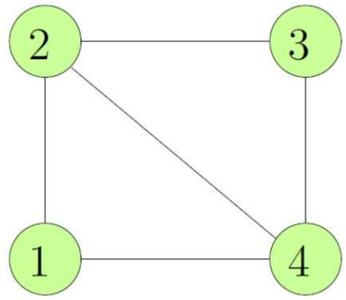
Starting from S_0, find
(a) $\mathbb{P}$(process enters S_2 for the 1^{st} time at k^{th} step)
(b) $\mathbb{P}$(process never enters S_4)
(c) $\mathbb{P}$(process ever enters S_2 and then leaves S_2 at the next step)
(d) $\mathbb{P}$(process enters S_1 for 1^{st} time at 3^{rd} step)
(e) $\mathbb{P}$(process in S_3 at the N^{th} step).

3. (**The rat**) Suppose that a rat wanders aimlessly through the maze pictured below. If the rat always chooses one of the available doors at random, regardless of what's happened in the past, then X_n =the rat's position at time n, defines a Markov chain.



(a) Find the transition matrix for this chain.
(b) Is this chain irreducible?
(c) If the rat starts at 1, what's the probability that it will reach 4 in less than 5 steps?

4. Consider the following Markov chain from Assignment 3.



At every step, a random walker at each state randomly walks, with equal probability, along an edge to another state if such edge exists. Show that the state 1 is recurrent. [**Hint:** use the formula for $\boldsymbol{P}^n$ derived in Assignment 3.]

5. Let $\alpha \geq 0$ be a constant. Assume that a Markov chain X_n has states $0, 1, 2 \ldots$ and transitions from each $i > 0$ to $i + 1$ with probability $1 - \frac{1}{2 \cdot i^\alpha}$ and to 0 with probability $\frac{1}{2 \cdot i^\alpha}$. Moreover, from 0 it transitions to 1 with probability 1.

   (a) Is this chain irreducible?

   (b) Assume that $X_0 = 0$ and let R be the first return time to 0 (i.e., the first time after the initial time the chain is back at the origin). Determine α for which

   $$1 - f_{0,0} = \mathbb{P}(\text{no return to } 0) = \mathbb{P}(R = \infty) = 0.$$

   (Hint: use $\ln(1 + x) \approx x$ for $|x|$ small.)