MATH2033 Mathematical Statistics
Assignment 6

Due Date: **21/Apr/2024(Sunday), on or before 16:00, on iSpace.**

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper and **show your steps**. Hand in your homework in **one pdf file** on iSpace.
- **Late submissions**, **answers without details**, or **unrecognizable handwritings** will NOT be graded.

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1. In HW5-Problem 3, find the asymptotic variance of the mle.

2. Suppose that an i.i.d. sample of size 15 from a normal distribution gives $\overline{x} = 10$ and $s^2 = 25$. Find 90% confidence intervals for μ and σ^2.

3. Suppose that $X \sim \text{Bin}(n, p)$.

   (a) Show that the mle of p is $\hat{p} = X/n$.

   (b) Show that mle of part (a) attains the Cramér-Rao lower bound.

   (c) Construct an approximate 90% confidence interval for p.

4. Let $X \sim N(0, \theta)$, $0 < \theta < \infty$.

   (a) Find the Fisher information $I(\theta)$.

   (b) Find the asymptotic variance of the mle $\hat{\theta}_{MLE}$.

   (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta}_{MLE} - \theta)$?

5. This problem is concerned with the estimation of the variance of a normal distribution with unknown mean from a sample $X_1, \cdots, X_n$ of i.i.d. normal random variables. In answering the following questions, use the fact that (from Theorem 1.7.35 of Chap 1)

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

   and that the mean and variance of a chi-square random variable with r df are r and $2r$, respectively.

(a) Which of the following estimates is unbiased?

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2, \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

(b) Which of the estimate in part (a) gives the smaller MSE?

(c) For what value of k does $k\sum_{i=1}^{n}(X_i - \overline{X})^2$ have the minimal MSE?

6. A coin is thrown independently 10 times to test the hypothesis that the probability of heads is $\frac{1}{2}$ versus the alternative that the probability is not $\frac{1}{2}$. The test rejects the null hypothesis that $p = 1/2$ if either 0 or 10 heads are observed. What is the significance level of the test?

7. Suppose that the distributions that we consider in this problem are all **normal with known variance**. State the decision rule that would be used to test the following hypotheses. Evaluate the appropriate test statistic and state your conclusion.

(a) $H_0 : \mu = 120$ versus $H_1 : \mu < 120$; $\quad \bar{y} = 114.2,\ n = 25,\ \sigma = 18,\ \alpha = 0.08.$

(b) $H_0 : \mu = 42.9$ versus $H_1 : \mu \neq 42.9$; $\quad \bar{y} = 45.1,\ n = 16,\ \sigma = 3.2,\ \alpha = 0.01.$

(c) $H_0 : \mu = 14.2$ versus $H_1 : \mu > 14.2$; $\quad \bar{y} = 15.8,\ n = 9,\ \sigma = 4.1,\ \alpha = 0.13.$