Week6-chap3.3

 $X_i \sim U(0, \theta)$

1 like [hood function!

$$\pi(x|\theta) = \begin{cases} \frac{1}{2} & x \in [0, \theta] \\ 0 & \text{otherwise} \end{cases}$$

$$\ln z E(x) = \int_{0}^{\theta} x dx = \frac{1}{6} \cdot \frac{1}{2}x^{2} \Big|_{0}^{\theta} = \frac{\theta}{2}$$

$$(X|0) = \begin{cases} \frac{1}{2} & \chi \in [0,0] \\ 0 & \text{otherwise} \end{cases}$$

$$(X) = \int_{0}^{0} x dx = \frac{1}{6} \cdot \frac{1}{2} x^{2} \Big|_{0}^{0} = \frac{0}{2}$$

$$\mathcal{U}_{1} = E(X) = \int_{0}^{\theta} x \frac{1}{\theta} dx = \frac{1}{\theta} \cdot \frac{1}{2}x^{2} \Big|_{0}^{\theta} = \frac{\theta}{2}$$

$$\longrightarrow 0 = 2\mathcal{M}$$

$$\int_{0}^{\infty} x \circ \alpha x = \frac{1}{2} \sum_{i=1}^{\infty} \frac{1}{2}$$

$$= 2M_{i}$$

$$= X_{i} = X \quad \Rightarrow \quad \hat{O}_{mon} = 2\hat{\mathcal{U}}_{i} = 2$$

$$\hat{\mathcal{U}}_{i} = \frac{1}{h} \sum X_{i} = \overline{X} \quad \Rightarrow \quad \hat{0}_{mom} = 2 \hat{\mathcal{U}}_{i} = 2 \overline{X}$$

$$\downarrow, \quad \text{MLE of } 0$$

$$L(0) = \prod_{i=1}^{n} \pi(x_{i}|0) = \left(\frac{1}{0}\right)^{n}$$

$$L(0) = \text{likelihood}$$

$$\text{function}$$

$$L(0) = \ln L(0) z - n \ln 0$$

$$|0\rangle = |n L(0)| = -n |n0|$$

$$|2\rangle = \frac{-n}{2} = 0 \implies \text{there is no statice}$$
because L(0) or

The minimum value of

Maximise
$$L(0)$$

$$\frac{d L(0)}{d0} = \frac{-n}{0} = 0$$

Here is no stationary point!

because $L(0)$ or $L(0)$ is monotonic on $L(0)$

Fisher information

$$\triangle \text{ Maximize } L(0)$$

$$\frac{d L(0)}{d0} = \frac{-n}{0} = 0 \Rightarrow \text{ there is no station}$$

$$\triangle \text{ Fisher information}$$

Maximise
$$L(0)$$

$$\frac{d L(0)}{d0} = \frac{-n}{0} = 0 \Rightarrow \text{there is no station}$$
because $L(0)$ or

Tisher information

Fisher information
$$[(\theta) := E[(\frac{\partial}{\partial \theta} \log \pi(x|\theta))^2] = E[\frac{\partial^2}{\partial \theta^2} \log \pi(x|\theta)]$$

Consider the density function
$$TI(x|0)$$

$$1 = \int_{-\infty}^{\infty} TI(x|0) dx$$

take derivative
$$0 = \frac{\partial}{\partial \theta} | = \frac{\partial}{\partial \theta} \left[\int_{-\infty}^{\infty} \pi(x|\theta) dx \right]$$

$$0 = \int_{-\infty}^{\infty} \frac{\partial}{\partial \theta} \pi(x|\theta) dx$$

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$$\pi(x|\theta) dx$$

$$\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta} \int_{-\infty}^{\infty} \frac{\partial}{\partial \theta} \log \pi(x|\theta) dx$$

$$\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta} \int_{-\infty}^{2} \log \pi(x|\theta) dx$$

$$\frac{\partial}{\partial \theta} = \frac{\partial^{2}}{\partial \theta} \log \pi(x|\theta) dx$$

$$=\int_{\mathbb{R}}\frac{\partial}{\partial \theta^{2}}\log \pi d\theta$$

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$$= \left[\frac{\partial}{\partial \theta^{2}} \left[\frac{\partial}{\partial \theta} \right] \left[\frac{\partial}{\partial \theta} \left[\frac{\partial}$$

By
$$O$$
, we can see
$$E\left(\frac{\partial}{\partial o}\log T(X(o))\right) = 0$$

Here, the function

$$\frac{\partial}{\partial \theta} \log \pi(x_{10})$$

is called the Score function. Recall that it determines the estimating equations for the mle, that is,
$$\hat{O}_{MLE}$$
 Solves $\frac{2}{3} \frac{\partial \log \pi(x_i|0)}{\partial x_i} = 0$

I(0) gets larger as $0 \rightarrow 1$ or $0 \rightarrow 0$.

$$= E \left[\frac{\partial^{2}}{\partial \theta^{2}} \log \pi(X|\theta) \right] + \int_{\mathbb{R}} \left(\frac{\partial}{\partial \theta} \log \pi(X|\theta) \right)^{2} \cdot \pi(X|\theta) dx$$

$$= E \left[\frac{\partial^{2}}{\partial \theta^{2}} \log \pi(X|\theta) \right] + F \left[\left(\frac{\partial}{\partial \theta} \log \pi(X|\theta) \right)^{2} \right]$$

$$0 = \int_{\mathbb{R}} \frac{\partial^{2}}{\partial \theta^{2}} \log \pi(x|\theta) \cdot \pi(x|\theta) dx + \int_{\mathbb{R}} \frac{\partial}{\partial \theta} \log \pi(x|\theta) \frac{\partial}{\partial \theta} \pi(x|\theta) dx$$

$$= E \left[\frac{\partial^{2}}{\partial \theta^{2}} \log \pi(x|\theta) \right] + \int_{\mathbb{R}} \frac{\partial}{\partial \theta} \log \pi(x|\theta) \frac{\partial}{\partial \theta} \pi(x|\theta) dx$$

$$\pi(x|\theta) dx$$

$$\int_{\mathbb{R}} \frac{\partial}{\partial x} \int_{\mathbb{R}} \frac{$$

$$\int_{\mathbb{R}} \frac{\partial}{\partial \theta} \left[\frac{\partial}{\partial \theta} \right] \frac{\partial}{\partial$$

Uniform on [0,0]

instead, consider a set of new random variables

X(i): the ith smallest value in the sample

 $\hat{\mathcal{O}}_{MLE} = X_{(n)}$

Minimize the value of 0, so that L10) to be as large as possible.

The minimum value of Q it can attain based on a given sample is

i.e. $0 \leq X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)} \leq 0$

T(X(0))

order statistics

$$\pi(x_{10})$$

$$\int_{0}^{\infty} (x | \theta)$$

$$\frac{\partial}{\partial \theta} \pi(x|\theta)$$

$$\pi(x|\theta)$$

$$\frac{\partial}{\partial \theta} \pi(x|\theta)$$

$$\pi(x|\theta)$$

$$\left(\frac{1}{2} \right)^{2}$$

$$\frac{\mathrm{d} g \cdot a_{S}}{\mathrm{I}(0)}$$

$$\mathrm{I}(8) := \mathrm{E}\left[\left(\frac{\partial}{\partial \theta} \log \pi(X|\theta)\right)^{2}\right] = -\mathrm{E}\left[\frac{\partial^{2}}{\partial \theta^{2}} \log \pi(X|\theta)\right]$$

$$Var\left(\frac{\partial}{\partial Q} \log \pi(\chi(Q))\right) = I(Q) - Q^2 = I(Q)$$
then function

$$\frac{1}{2} \frac{\partial \log \pi(x_i(0))}{\partial x_{i=1}} = 0$$
single

Single Single Single Single Single Sormation for a Bernoulli R.V.) Let
$$X \sim Be$$

$$\log \pi(x|0) = x \ln \theta + (1-x) \ln (1-\theta)$$

$$\frac{\partial \log \pi(x|0)}{\partial \theta} = \frac{x}{\theta} - \frac{1-x}{1-\theta}$$

$$\frac{\partial^2}{\partial \theta^2} \log \pi(x|\theta) = -\frac{x}{\theta^2} - \frac{1-x}{(1-\theta)^2}$$

ormation for a Bernoulli R.V.) Let
$$X \sim \log \pi(x|\theta) = x \ln \theta + (1-x) \ln (1-\theta)$$

$$\frac{\partial \log \pi(x|\theta)}{\partial \theta} = \frac{x}{\theta} - \frac{1-x}{1-\theta}$$

E.g. (Information for a Bernoulli R.V.) Let
$$X \sim Bernoulli (B)$$
, Thus
$$\log \pi(x|B) = x \ln B + (1-x) \ln (1-B)$$

$$\frac{\partial \log \pi(x|B)}{\partial x} = \frac{x}{B} - \frac{1-x}{1-B}$$

 $1(0) = -E\left[\frac{\partial^2}{\partial \theta^2}\log \pi(x|\theta)\right] = E\left[\frac{X}{\theta^2} + \frac{1-X}{(1-\theta)^2}\right]$ $= \frac{1}{Q^2} \cdot Q + \frac{1}{(1-Q)^2} (1-Q) = \frac{1}{Q(1-Q)}$