2022-23 First Semester
MATH1053 Linear Algebra I

Assignment 5a

Due Date: 29/Nov/2022 (Tuesday), 11:00 in class.

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

---

1. Which of the following sets are spanning sets for $\mathbb{R}^3$? Justify your answers.
   (a) $\{(2,1,-2)^T, (3,2,-2)^T, (2,2,0)^T\}$.
   (b) $\{(1,1,3)^T, (0,2,1)^T\}$.
2. Determine if the following vectors are linearly independent, and justify your answers:
   (a) $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}$ in $\mathbb{R}^3$
   (b) $\begin{pmatrix}2\\1\\-2\end{pmatrix}, \begin{pmatrix}3\\2\\-2\end{pmatrix}, \begin{pmatrix}2\\2\\0\end{pmatrix}$ in $\mathbb{R}^3$
   (c) 2, x^2, x, $2x+3$ in P_3
3. Let $A \in \mathbb{R}^{m\times n}$. Show that if A has linearly independent column vectors, then $N(A) = \{\mathbf{0}\}$.
4. Let $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k$ be linearly independent vectors in $\mathbb{R}^n$, and let A be a nonsingular $n\times n$ matrix. Define $\mathbf{y}_i = A\mathbf{x}_i$ for $i = 1, \cdots, k$. Show that $\mathbf{y}_1, \cdots, \mathbf{y}_k$ are linearly independent.
5. Show that any finite set of vectors that contains the zero vector must be linearly dependent.
6. Let $\mathbf{v}_1$, and $\mathbf{v}_2$ be two vectors in a vector space V. Show that $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly dependent if and only if one vector is a scalar multiple of the other.
7. Let $V = M_{3\times 3}(\mathbb{R})$. Let W be the set of all symmetric matrices in V.
   (a) Show that W is a subspace of V.
   (b) Compute $\dim W$.