Risk Management in Finance - Market risk IV – Historical Simulation and Model-Building Approach

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Outline of Market risk IV

- Historical Simulation
- Model-Building Approach

Historical Simulation
Model-Building Approach

Historical Simulation

Historical Simulation

- Collect data on the daily movements in all market variables (known as risk factors)
- The first simulation trial assumes that the percentage changes in all risk factors are as on the first day
- The second simulation trial assumes that the percentage changes in all risk factors are as on the second day
- and so on

Historical Simulation continued

- Suppose we use n days of historical data with today being day n
- Let v_i be the value of a variable on day i
- The ith trial assumes that the value of the market variable tomorrow (i.e., on day n+1) is

$v_n \frac{v_i}{v_{i-1}}$ if percentage changes are matched

$v_n + v_i - v_{i-1}$ if actual changes are matched

Example: Portfolio on Sept 25, 2008 (Table 13.1, page 295)

Index	Amount Invested ($000s)
DJIA	4,000
FTSE 100	3,000
CAC 40	1,000
Nikkei 225	2,000
Total	10,000

U.S. Dollar Equivalent of Stock Indices (Table 13.2, page 295)

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	11,219.38	11,131.84	6,373.89	131.77
1	Aug 8, 2006	11,173.59	11,096.28	6,378.16	134.38
2	Aug 9, 2006	11,076.18	11,185.35	6,474.04	135.94
3	Aug 10, 2006	11,124.37	11,016.71	6,357.49	135.44
….	……	……	…..	……	……
499	Sep 24, 2008	10,825.17	9,438.58	6,033.93	114.26
500	Sep 25, 2008	11,022.06	9,599.90	6,200.40	112.82

Scenarios (Table 13.3, page 296)

$$= 11{,}022.06 \times \frac{11{,}173.59}{11{,}219.38}$$

Scenario Number	DJIA	FTSE	CAC	Nikkei	Portfolio Value	Loss
1	10,977.08	9,569.23	6,204.55	115.05	10,014.334	-14.334
2	10,925.97	9,676.96	6,293.60	114.13	10,027.481	-27,481
3	11,070.01	9,455.16	6,088.77	112.40	9,946.736	53,264
….	……	…..	…..	…..	…..	…..
499	10,831.43	9,383.49	6,051.94	113.85	9,857.465	142.535
500	11,222.53	9,763.97	6,371.45	111.40	10,126.439	-126.439

Losses (Table 13.4, page 297)

Scenario Number	Loss ($000s)
494	477.841
339	345.435
349	282.204
329	277.041
487	253.385
227	217.974
131	205.256

One-day 99% VaR=$253,385

One-day 99% ES is
(477,841+345,435+282,204+277,041)/4=$345,630

We estimate VaR as the fifth worst loss and ES as the average of losses more than VaR. Other estimates are possible (see footnotes 1 and 2)

Stressed VaR and Stressed ES

- Instead of basing calculations on the movements in market variables over the last n days, we can base calculations on movements during a period in the past that would have been particularly bad for the current portfolio
- This produces measures known as "stressed VaR" and "stressed ES"

Accuracy of VaR (page 299-301)

Suppose that x is the qth percentile of the loss distribution when it is estimated from n observations. The standard error of x is

$$\frac{1}{f(x)}\sqrt{\frac{(1-q/100)q/100}{n}}$$

where $f(x)$ is an estimate of the probability density of the loss at the qth percentile calculated by assuming a probability distribution for the loss

Example 13.1 (page 300)

- We estimate the 99-percentile from 500 observations as \$25 million
- We estimate $f(x)$ by approximating the actual empirical distribution with a normal distribution mean zero and standard deviation \$10 million
- The 1-percentile of the approximating distribution is NORMINV(0.99 ,0,10) = 23.26 and the value of $f(x)$ is NORMDIST(23.26,0,10,FALSE)=0.0027
- The estimate of the standard error is therefore

$$\frac{1}{0.0027} \times \sqrt{\frac{0.01 \times 0.99}{500}} = 1.67$$

Weighting of Observations

- Let weights assigned to observations decline exponentially as we go back in time
- Rank observations from worst to best
- Starting at worst observation sum weights until the required percentile is reached

Application to 4-Index Portfolio $\lambda=0.995$ (Table 13.5, page 302)

Scenario Number	Loss ($000s)	Weight	Cumulative Weight
494	477.841	0.00528	0.00528
339	345.435	0.00243	0.00771
349	282.204	0.00255	0.01027
329	277.041	0.00231	0.01258
487	253.385	0.00510	0.01768
227	217.974	0.00139	0.01906

One-day 99% VaR=$282,204

One day 99% ES is
$[0.00528\times477{,}841+0.00243\times345{,}435+(0.01-0.00528-0.00243)\times282{,}204]/0.01 =$
$400,914

Volatility Scaling

- Use a volatility updating scheme to monitor volatilities of all market variables
- If the current volatility for a market variable is β times the volatility on Day i, multiply the percentage change observed on day i by β
- Value of market variable under ith scenario becomes

$$v_n \frac{v_{i-1} + (v_i - v_{i-1})\sigma_{n+1} / \sigma_i}{v_{i-1}}$$

Volatilities (% per Day) Estimated for Next Day in 4-Index Example (Table 13.6, page 304)

Day	Date	DJIA	FTSE	CAC 40	Nikkei
0	Aug 7, 2006	1.11	1.42	1.40	1.38
1	Aug 8, 2006	1.08	1.38	1.36	1.43
2	Aug 9, 2006	1.07	1.35	1.36	1.41
3	Aug 10, 2006	1.04	1.36	1.39	1.37
….	……	……	……	……	……
499	Sep 24, 2008	2.21	3.28	3.11	1.61
500	Sep 25, 2008	2.19	3.21	3.09	1.59

Volatility Adjusted Losses (Table 13.7, page 304)

Scenario Number	Loss ($000s)
131	1,082.969
494	715.512
227	687.720
98	661.221
329	602.968
339	546.540
74	492.764

One-day 99% VaR = $602,968

One-day 99% ES = (1,082,969+715,512+687,720+661,221)/4 = $786,855

Volatility Scaling for the Portfolio

- Monitor variance of simulated losses on the portfolio using EWMA
- If current standard deviation of losses is β_P times the standard deviation of simulated losses on Day i, multiply ith loss given by the standard approach by β_P
- This approach gives VaR and ES as \$627.916 and \$777.545, respectively

The Bootstrap Method to Determine Confidence Intervals

- Suppose there are 500 daily changes
- Calculate a 95% confidence interval for VaR by sampling 500,000 times with replacement from daily changes to obtain 1000 sets of changes over 500 days
- Calcuate VaR for each set and calculate a confidence interval
- This is known as the bootstrap method

Model-Building Approach

The Model-Building Approach

- The main alternative to historical simulation for calculating VaR or ES is to make assumptions about the probability distributions of the returns on the market variables
- This is known as the model building approach (or sometimes the variance-covariance approach)

Microsoft Example (page 318-319)

- We have a position worth $10 million in Microsoft shares
- The volatility of Microsoft is 2% per day (about 32% per year)
- We use N=10 and X=99

Microsoft Example continued

- The standard deviation of the change in the portfolio in 1 day is $200,000
- The standard deviation of the change in 10 days is

$$200{,}000\sqrt{10} = \$632{,}500$$

Microsoft Example continued

- We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)
- We assume that the change in the value of the portfolio is normally distributed
- Since $N(-2.326)=0.01$, the VaR is

$$2.326 \times 632{,}500 = \$1{,}471{,}300$$

AT&T Example

- Consider a position of $5 million in AT&T
- The daily volatility of AT&T is 1% (approx 16% per year)
- The SD per 10 days is

$$50{,}000\sqrt{10} = \$158{,}144$$

- The VaR is

$$158{,}100 \times 2.326 = \$367{,}800$$

Two-Asset Portfolio (page 319-320)

- Now consider a portfolio consisting of both Microsoft and AT&T
- Suppose that the correlation between the returns is 0.3

S.D. of Portfolio

- A standard result in statistics states that

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

- In this case $\sigma_X = 200{,}000$ and $\sigma_Y = 50{,}000$ and $\rho = 0.3$. The standard deviation of the change in the portfolio value in one day is therefore 220,200

VaR for Portfolio

- The 10-day 99% VaR for the portfolio is

$$220{,}200 \times \sqrt{10} \times 2.326 = \$1{,}620{,}100$$

- The benefits of diversification are

(1,471,300+367,800)–1,620,100=$219,000

- What is the incremental effect of the AT&T holding on VaR?

ES Results (10-day 99%)

- Microsoft shares: $1,687,000
- AT&T shares: $421,700
- Portfolio: $1,857,600

The Linear Model

We need to assume

- The daily change in the value of a portfolio is linearly related to the daily returns from market variables
- The returns from the market variables are normally distributed
- The market variable are referred to as risk factors

Markowitz Result for Variance of Return on Portfolio

$$\text{Variance of Portfolio Return} = \sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij} w_i w_j \sigma_i \sigma_j$$

w_i is weight of ith asset in portfolio

σ_i^2 is variance of return on ith asset in portfolio

ρ_{ij} is correlation between returns of ith and jth assets

Similar Result for Portfolio Linear Dependent on *n* Risk Factors

$$\Delta P = \sum_{i=1}^{n} \delta_i \Delta x_i$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \delta_i \delta_j \sigma_i \sigma_j$$

$$\sigma_P^2 = \sum_{i=1}^{n} \delta_i^2 \sigma_i^2 + 2 \sum_{i<j} \rho_{ij} \delta_i \delta_j \sigma_i \sigma_j$$

where:

Δx_i is percentage change in risk factor i in one day

σ_P is the SD of the dollar change in the portfolio value per day

σ_i is the SD of Δx_i (i.e., daily volatility of the ith risk factor)

δ_i is the value of the investment in the *i*-th asset (risk factor *i*)

Alternative Expressions for σ_P^2 in terms of covariances

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{cov}_{ij}\, \delta_i \delta_j$$

$$\sigma_P^2 = \delta^{\mathbf{T}} C \delta$$

where δ is the column vector whose *i*th element is δ_i, $\delta^{\mathbf{T}}$ is its transpose, and C is the covariance matrix whose elements are cov_{ij}

Four Index Example Using Last 500 Days of Data to Estimate Covariances

	Equal Weight	EWMA : λ=0.94
One-day 99% VaR	$217,757	$471,025
One-day 99% ES	$249,476	$539,637

Volatilities and Correlations Increased in Sept 2008 (pages 324-325)

Volatilities (% per day)

	DJIA	FTSE	CAC	Nikkei
Equal Weights	1.11	1.42	1.40	1.38
EWMA	2.19	3.21	3.09	1.59

Correlations

$$\begin{pmatrix} 1 & 0.489 & 0.496 & -0.062 \\ 0.489 & 1 & 0.918 & 0.201 \\ 0.496 & 0.918 & 1 & 0.211 \\ -0.062 & 0.201 & 0.211 & 1 \end{pmatrix}$$

Equal weights

$$\begin{pmatrix} 1 & 0.611 & 0.629 & -0.113 \\ 0.611 & 1 & 0.971 & 0.409 \\ 0.629 & 0.971 & 1 & 0.342 \\ -0.113 & 0.409 & 0.342 & 1 \end{pmatrix}$$

EWMA

Basic Model Building Approach: Summary

- Fast
- Can easily be used with volatility updating
- Requires linearity (no options)
- Assumes multivariate normal distribution
- Used for investment portfolios