2022-23 First Semester
MATH1053 Linear Algebra II (1003)

Assignment 1

Due Date: **3/Mar/2022 (Friday), on or before 09:00 in tutorial class.**

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. Determine whether the following are linear transformations with proofs.

   (a) $L : \mathbb{R}^2 \to \mathbb{R}^2, L(x, y) = (1, y)$;

   (b) $L : P_2 \to P_3$, $L(p(x)) = xp(x)$;

   (c) $L : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}, L(A) = A^T$;

   (d) $L : \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}, L(A) = A + I$;

   (e) $L : \mathbb{R}^{2\times2} \to \mathbb{R}$, $L(A) = \det(A)$.

2. Let T be a linear transformation from P_3 into P_3 such that $T(1) = x$, $T(2x + x^2) = 1 + x$, and $T(x^2 - 1) = 1 + x + x^2$. Find $T(2 - 6x + x^2)$.

3. Find the kernel and range of the following linear transformations, respectively.

   (a) $L : \mathbb{R}^3 \to \mathbb{R}^3$, $L(x, y, z) = (2z, y + 3x, 2x - z)^T$;

   (b) $L : P_3 \to P_3$, $L(p(x)) = p(x) - p'(x)$;

   (c) $L : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$, $L(A) = A - A^T$.

4. For the following linear transformations, find the matrix representation A of L with respect to the standard basis, respectively.

   (a) $L : \mathbb{R}^3 \to \mathbb{R}^2$, $L(\mathbf{x}) = (x_1 + x_2, 0)^T$;

   (b) $L : \mathbb{R}^3 \to \mathbb{R}^3$, $L(\mathbf{x}) = (2x_3, x_2 + 3x_1, 2x_1 - x_3)^T$;

   (c) $L : P_3 \to P_3$, $L(p(x)) = xp'(x)$.

5. Let $E = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $F = \{\mathbf{b}_1, \mathbf{b}_2\}$, where

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{u}_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

and

$$\mathbf{b}_1 = (2, -1)^T, \quad \mathbf{b}_2 = (1, -1)^T$$

For each of the following linear transformations L from $\mathbb{R}^3$ into $\mathbb{R}^2$, find the matrix representing L with respect to the ordered basis E and F:

(a) $L(\mathbf{x}) = (x_2, x_1)^T$

(b) $L(\mathbf{x}) = (2x_2, -x_3 + x_1)^T$

6. Let $L : P_3 \to P_3$ be the differential operator $\dfrac{d}{dx}$. Find the matrix representation of L with respect to the bases $\{1,\ 1 + 2x,\ 4x^2 - 3\}$ and $\{1, x, x^2\}$.

7. Use the "rref" function in MATLAB to find out the reduced row echelon form of each matrix and then determine the kernel and image of the transformations defined by the following matrices $L(\mathbf{x}) = A\mathbf{x}$.

(a). $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$ (b). $A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 1 \end{bmatrix}$ (c). $A = \begin{bmatrix} 4 & 8 & 1 & 1 & 6 \\ 3 & 6 & 1 & 2 & 5 \\ 2 & 4 & 1 & 9 & 10 \\ 1 & 2 & 3 & 2 & 0 \end{bmatrix}$