2022-23 First Semester
MATH1053 Linear Algebra I

Assignment 4a

Due Date: 15/Nov/2022 (Tuesday), 11:00 in class.

- Write down your **CHN name** and **student ID**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. (a) Use Cramer's rule to solve the following systems.

   (b) Find A^{-1} using the adjoint of A. Then solve the system by computing $\mathbf{x} = A^{-1}\mathbf{b}$.

$$\begin{cases} x_1 & +2x_2 & = 3 \\ 3x_1 & -x_2 & = 1 \end{cases}, \qquad \begin{cases} x_1 & +2x_2 & +x_3 = 1 \\ & -x_2 & +x_3 = 2 \\ 2x_1 & +3x_2 & -2x_3 = 3 \end{cases}.$$

2. Let A and B be $n \times n$ matrices. Prove that if $AB = I$, then $BA = I$.
   What is the significance of the result in terms of the definition of a nonsingular matrix?

3. If A is nonsingular, find out the product $A\mathrm{adj}A$. What if A is singular?

4. Let Z denote the set of all integers with addition defined in the usual way and define scalar multiplication, denoted $\circ$, by

$$\alpha \circ k = [[\alpha]] \cdot k \quad \text{for all} \quad k \in Z$$

   where $[[\alpha]]$ denotes the greatest integer less than or equal to α. For example,

$$2.25 \circ 4 = [[2.25]] \cdot 4 = 2 \cdot 4 = 8$$

   Show that Z, together with these operations, is not a vector space. Which axioms fail to hold?

5. Let V denote the set

$$\{p(x) |\ p(0) = 1,\ p(x) \in P\}$$

   where P denotes the set of all polynomials of any finite degree with real number coefficients. Under the standard addition and scalar multiplication for functions, determine whether V over $\mathbb{R}$ is a vector space.

6. Let $\mathbb{R}^+$ be the set of all positive real numbers. Show that $\mathbb{R}^+$ is a vector space over $\mathbb{R}$ under the addition

$$x \boxplus y = xy, \quad x, y \in \mathbb{R}^+$$

and the scalar multiplication

$$\alpha \odot x = x^\alpha, \quad x \in \mathbb{R}^+, \alpha \in \mathbb{R}.$$

7. Let V be the set of all ordered pairs of real numbers with addition defined by

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$

and scalar multiplication defined by

$$\alpha \circ (x_1, x_2) = (\alpha x_1, x_2).$$

Is V a vector space over $\mathbb{R}$ with these operations? Justify your answer.