Chapter Nine

The Capital Asset Pricing Model

Capital Asset Pricing Model (CAPM)

- CAPM is a set of predictions concerning equilibrium expected returns on risky assets
- Based on two sets of assumptions
  - Individual behavior
  - Market structure
- Markowitz established modern portfolio management in 1952
- Sharpe, Lintner and Mossin published CAPM in 1964

Assumptions

1. Individual behavior
   a. Investors are rational, mean-variance optimizers.
   b. Their common planning horizon is a single period.
   c. Investors all use identical input lists, an assumption often termed **homogeneous expectations.** Homogeneous expectations are consistent with the assumption that all relevant information is publicly available.
2. Market structure
   a. All assets are publicly held and trade on public exchanges.
   b. Investors can borrow or lend at a common risk-free rate, and they can take short positions on traded securities.
   c. No taxes.
   d. No transaction costs.

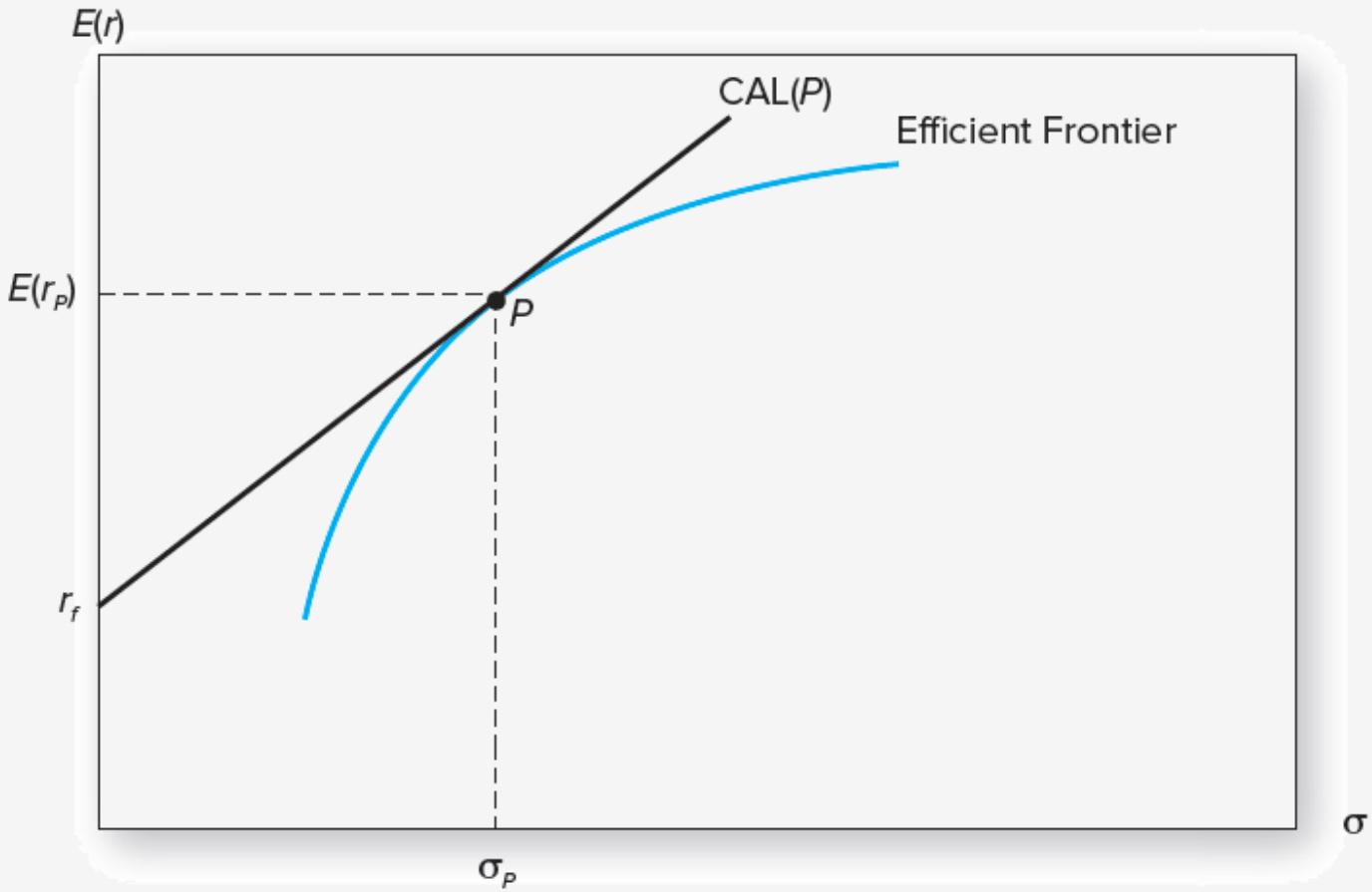
The Market Portfolio

- All investors will hold the same portfolio for risky assets — **market portfolio**

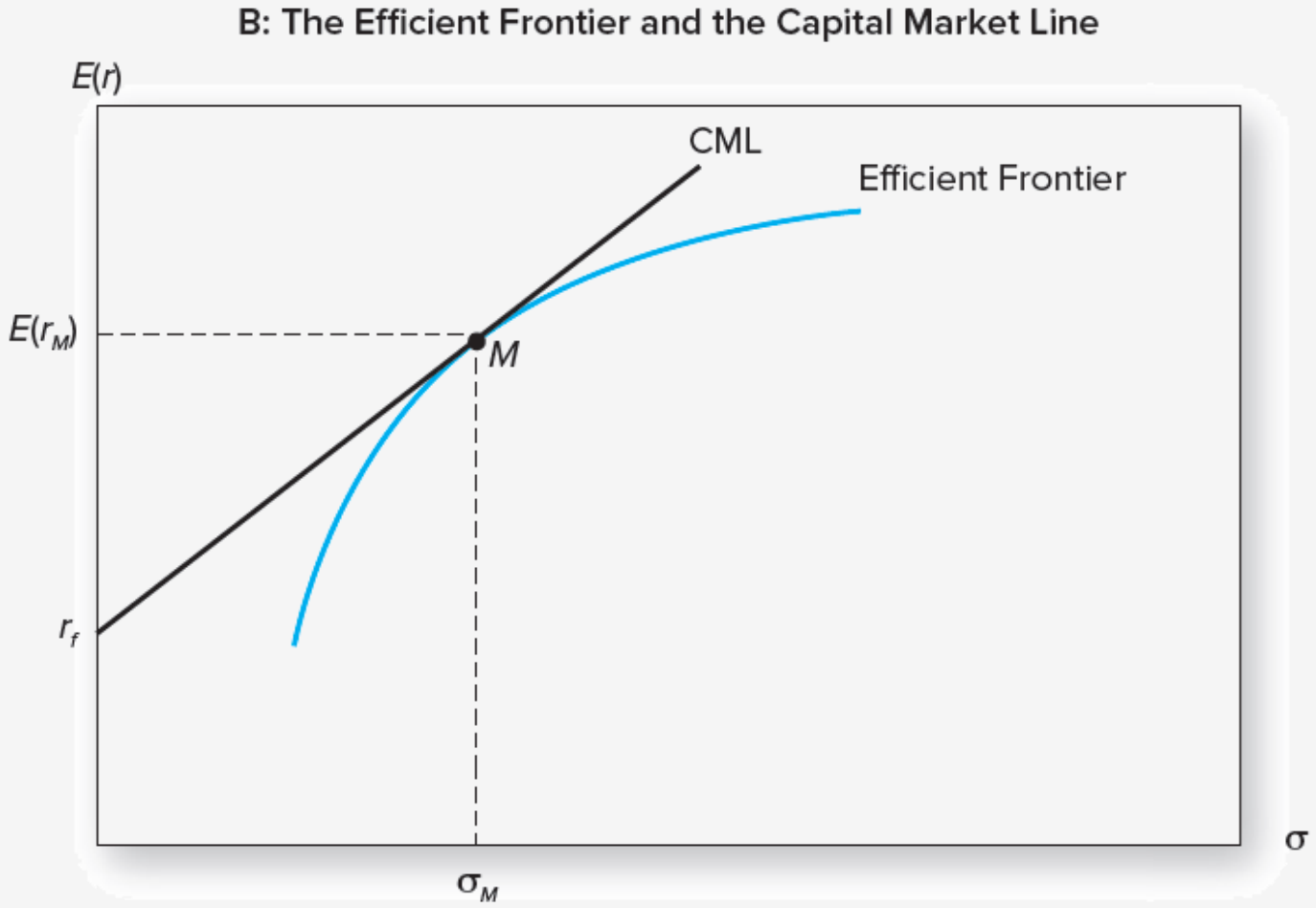
- **Market portfolio** contains all securities
  - Proportion of each stock in this portfolio equals the market value of the stock (price per share times number of shares outstanding) divided by the sum of the market value of all stocks

Capital Allocation Line

A: The Efficient Frontier of Risky Assets with the Optimal CAL



Capital Market Line



The Risk Premium of the Market Portfolio

- Recall that each individual investor chooses a proportion *y*, allocated to the optimal portfolio *M*, such that

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} = \frac{E(R_M)}{A\sigma_M^2}$$

- In the simplified CAPM economy, risk-free investments involve borrowing and lending among investors.
  - Any borrowing position must be offset by the lending position of the creditor.
  - Net borrowing and lending across all investors must be zero.
  - The average position in the risky portfolio is 100%, or $\bar{y} = 1$.

The Risk Premium of the Market Portfolio

- The market risk premium is proportional to its risk and the degree of risk aversion:

$$E(R_M) = \bar{A}\sigma_M^2$$

Where

$\bar{A} =$ representative investor's risk aversion

$\sigma_M^2 =$ variance of the market portfolio

Expected Returns on Individual Securities

- CAPM is built on the insight that the appropriate risk premium on an asset will be determined by its contribution to the risk of investors' overall portfolios
  - All investors use the same input list (i.e., they all end up using the market as their optimal risky portfolio)

Individual Securities: Example

- Covariance of GE return with the market portfolio:

$$\sum_{i=1}^{n} w_i Cov(R_i, R_{GE}) = Cov\left(\sum_{i=1}^{n} w_i R_i, R_{GE}\right)$$

- The reward-to-risk ratio for GE would be:

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{E(R_{GE})}{Cov(R_{GE}, R_M)}$$

GE Example
(1 of 2)

- Reward-to-risk ratio for investment in market portfolio (i.e., **market price of risk**):

$$\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(R_M)}{\sigma^2(R_M)}$$

- Equilibrium dictates all investments should offer the same reward-to-risk ratio

$$\frac{E(R_{GE})}{Cov(R_{Ge}, R_M)} = \frac{E(R_M)}{\sigma^2(R_M)}$$

GE Example
(2 of 2)

- Fair risk premium for GE stock:

$$E(R_{GE}) = \frac{Cov(R_{Ge}, R_M)}{\sigma^2(R_M)} E(R_M)$$

- Restating, we obtain:

$$E(r_{GE}) = r_f + \beta_{GE}\left[E(r_M) - r_f\right]$$

- This **expected return-beta relationship** is the most familiar expression of the CAPM to practitioners.

Expected Return-Beta Relationship

- **Expected return-beta relationship** tells us the total expected rate of return is the sum of the risk-free rate plus a risk premium
  - Risk premium is the product of a "benchmark risk premium" and the relative risk of the particular asset as measured by its beta
- The CAPM predicts that systematic risk should "be priced", meaning that it commands a risk premium, but firm-specific risk should not be priced by the market.

Expected Return-Beta Relationship

- Suppose that some portfolio *P* has weight w_k for stock *k*, *k* = 1,…, n.
  - Expected return on the portfolio is $E(r_P) = \sum_k w_k E(r_k)$
  - Portfolio beta is $\beta_P = \sum_k w_k \beta_k$
- Beta of the market portfolio is 1 because

$$\beta_M = \frac{Cov(R_M, R_M)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1$$

  - Betas greater than 1 are considered aggressive in that investment in high-beta stocks entails above-average sensitivity to market swings.
  - Betas below 1 can be described as defensive.

The Security Market Line

$$E(r_i) = r_f + \beta_i\left[E(r_M) - r_f\right]$$

- The expected return-beta relationship is a reward-risk equation.
  - The beta of a security is the appropriate measure of its risk because beta is proportional to the risk of the security contributes to the optimal risky portfolio.
- The expected return-beta relationship can be portrayed graphically as the **security market line (SML)**.

The Security Market Line

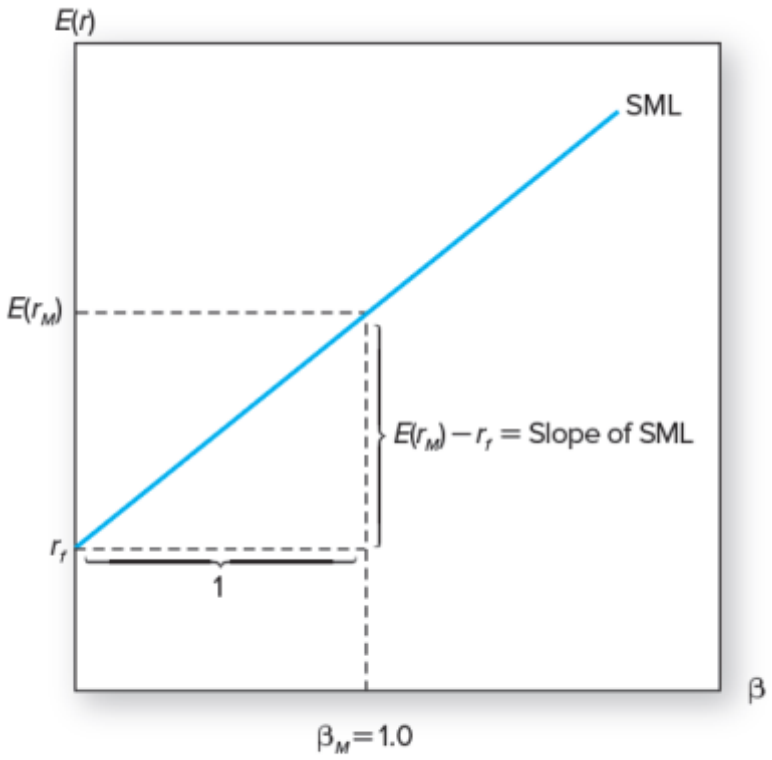


Figure 9.2 The security market line

Security Market Line v.s. Capital Market Line

- CML: the risk premium of *efficient portfolios* as a function of portfolio standard deviation.
  - Takes the standard deviation as a valid measure of risk for efficiently diversified portfolios that are candidates for an investor's overall portfolio.

- SML: *individual asset* risk premiums as a function of asset risk.
  - The relevant measure of risk for individual assets held as parts of well-diversified portfolios is the contribution of the asset to the portfolio variance (measured by β_i), rather than the asset's standard deviation or variance.
  - Valid for both efficient portfolios and individual assets.

The Security Market Line

- The security market line provides a benchmark for the evaluation of investment performance.
- Given the risk of an investment, as measured by its beta, the SML provides the required rate of return necessary to compensate investors for risk.
- “Fairly priced” assets lie exactly on the SML (namely, their expected returns are commensurate with their risk).

The Security Market Line

- Suppose that SML relation is used as a benchmark to assess the fair expected return on a risky asset.
- Then security analysis is performed to calculate the return a money manager actually expects.
  - (Notice that we depart here from the simple CAPM world in that some investors now apply their own unique analysis to derive an “input list” that may differ from that of their competitors).

The Security Market Line

- If a stock is perceived to be a good buy, or underpriced, it will provide an expected return in excess of the fair return stipulated by the SML.
  - Underpriced stocks plot above the SML: given their betas, their expected returns are greater than dictated by the CAPM.
- Overpriced stocks plot below the SML.

The SML and a Positive-Alpha Stock

- The difference between the fair and actually expected rates of return on a stock is called the stock's **alpha,** denoted by α.
- The portfolio manager will increase the weights of securities with positive alphas and decrease the weights of securities with negative alphas.

The SML and a Positive-Alpha Stock

Example:

- The market return is expected to be 14%.
- A stock has a beta of 1.2.
- The T-bill rate is 6%.
- The SML would predict an expected return on the stock: 6%+1.2*8% = 15.6%.
- If one believed the stock would provide an expected return of 17%, the implied alpha would be 1.4%.

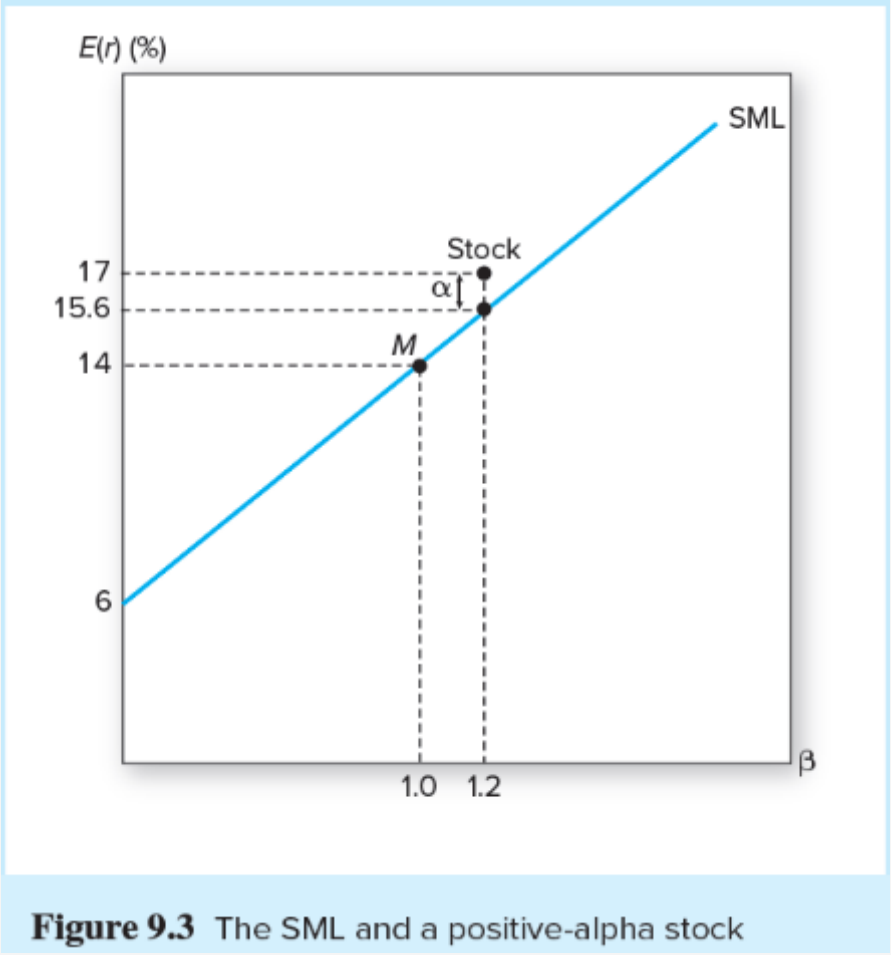


Figure 9.3 The SML and a positive-alpha stock

The CAPM and the Single-Index Market

- Index model states the realized excess return on any stock is the sum of the following:
  - Realized excess return due to marketwide factors
  - A nonmarket premium
  - Firm-specific outcomes

$$R_i = \alpha_i + \beta_i R_M + e_i$$

- $E(R_i) = \alpha_i + \beta_i E(R_M)$

The CAPM and the Single-Index Market

- The expected return-beta relationship of the CAPM:

$$E(R_i) = \beta_i E(R_M)$$

- Please refer to the textbook for more explanations.

Summary

- Assumptions of CAPM
  - Individual behavior and Market structure
  - All investors will hold the same portfolio for risky assets – market portfolio
- Market risk premium $E(R_M) = \bar{A}\sigma_M^2$

Summary

- Which statement is **not** true regarding the market portfolio?

  A) It includes all publicly-traded financial assets.

  B) It lies on the efficient frontier.

  C) All securities in the market portfolio are held in proportion to their market values.

  D) It is the tangency point between the capital market line and the indifference curve.

  E) All of the options are true.

Summary

- CAPM model
  - The CAPM predicts that systematic risk should be priced.
  - Security market line $E(r_i) = r_f + \beta_i[E(r_M) - r_f]$
  - Beta $\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)}$
    - β = 1: Market portfolio;
    - β > 1: Aggressive stocks; β < 1: Defensive stocks
  - Alpha

    α = actually expected return – required return (obtained by CAPM)
    - α = 0: fairly price stocks;
    - α > 0: underpriced stocks; α < 0: overpriced stocks

Summary

- Are the following true or false?
  a. Stocks with a beta of zero offer an expected rate of return of zero.
  b. The CAPM implies that investors require a higher return to hold highly volatile securities.
  c. You can construct a portfolio with beta of .75 by investing .75 of the investment budget in T-bills and the remainder in the market portfolio

Summary

- If the simple CAPM is valid, is the following situation possible?

Portfolio	Expected Return	Beta
Risk-free	10%	0
Market	18%	1.0
A	16%	0.9