1. (i) Annually compounded interest rate. 一年复利一次 $i=6\%$.

$$1+i = e^{r} = 1+6\% \Rightarrow r=\ln(1.06)\approx 5.827\%.$$

连续复利

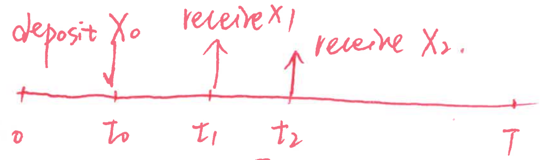
(ii) annual interest rate compounded daily 一年每天复利

$$1+i = \left(1+\frac{i^{(365)}}{365}\right)^{365} \qquad i^{(365)} \approx 5.827\%$$

2. (i) $e^{3\%+4\%+5\%\times 2} = e^{r_{ef}\cdot 4} \Rightarrow \cancel{r_{ef}} \approx \; r_{ef} \approx 4.25\%$

(ii). $e^{3\%+4\%+5\%\times 2} = (1+r_{ef})^4 \Rightarrow r_{ef} \approx 4.343\%$

3.



$$e^{\int_t^T \delta(t)\,dt} = e^{\int_t^T \frac{a}{b+t'}\,dt} = e^{a\ln(b+t')\big|_t^T} = e^{a[\ln(b+T)-\ln(b+t)]}$$

$$= e^{a\ln\frac{b+T}{b+t}} = \left(\frac{b+T}{b+t}\right)^a$$

$$V(T) = X_0 e^{\int_{t_0}^T \delta(t)\,dt} + (-X_1)e^{\int_{t_1}^T \delta(t)\,dt} + (-X_2)e^{\int_{t_2}^T \delta(t)\,dt}$$

$$= X_0\left(\frac{b+T}{b+t_0}\right)^a - X_1\left(\frac{b+T}{b+t_1}\right)^a - X_2\left(\frac{b+T}{b+t_2}\right)^a$$

4. $A(0)=\$100$.

$$\begin{cases} c_0=100 \\ c_0+c_1+c_2=120 \\ c_0+2c_1+4c_2=135 \end{cases} \Rightarrow \begin{cases} c_0=100 \\ c_1=22.5 \\ c_2=-2.5 \end{cases} \Rightarrow A(t)=-2.5t^2+22.5t+100.$$

$$\delta(t) = \frac{A'(t)}{A(t)} = \frac{9-2t_2}{-t_2^2+9t_1+40}. \left(\text{or } \frac{22.5-5t_2}{-2.5t_2^2+22.5t_1+100} \right)$$

$$\begin{cases} A(0)e^{90\delta} = 3A(0) \\ A'(0)\left(1+\frac{\delta}{4}\right)^{\frac{1}{4}t} = 4A'(0) \end{cases}$$ convertible once every 4 yrs ⇒ 每4年复利一次.

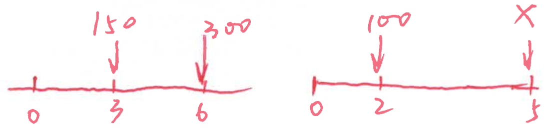
$$\Rightarrow \begin{cases} e^{90\delta}=3 \\ (1+4\delta)^{\frac{1}{4}t}=4 \end{cases} \Rightarrow \begin{cases} \delta=\frac{\ln 3}{90} \\ \left(1+\frac{4\ln 3}{90}\right)^{\frac{1}{4}t}=4 \end{cases} \Rightarrow t=\frac{4\ln 4}{\ln\left(1+\frac{4\ln 3}{90}\right)} \approx 116.327$$

6. $A(t)=A(0)e^{\int \delta_A(t)\,dt}=A(0)e^{\int 3a\,dt}=A(0)e^{3at}$

$B(t)=B(0)e^{\int \delta_B(t)\,dt}=B(0)e^{\int at\,dt}=B(0)e^{\frac{1}{2}at^2}=A(0)e^{\frac{1}{2}at^2}$

$e^{3at}=e^{\frac{1}{2}at^2}\Rightarrow t=6$

7.



由 $e^{\int_{t_1}^{t_2}\delta(t)\,dt}=e^{\int_{t_1}^{t_2}\frac{4}{2+t}dt}=e^{4\ln(2+t)|_{t_1}^{t_2}}=e^{4[\ln(2+t_2)-\ln(2+t_1)]}$

$=e^{\ln\left(\frac{2+t_2}{2+t_1}\right)^4}=\left(\frac{2+t_2}{2+t_1}\right)^4$

$P_1=150e^{-\int_0^3\delta(t)\,dt}+300e^{-\int_0^6\delta(t)\,dt}$

$=150e^{\int_3^0\delta(t)\,dt}+300e^{\int_6^0\delta(t)\,dt}$

$=150\left(\frac{0+2}{3+2}\right)^4+300\left(\frac{0+2}{6+2}\right)^4\approx 5.0119$

$P_2=100e^{-\int_0^2\delta(t)\,dt}+Xe^{-\int_0^5\delta(t)\,dt}$

$=100e^{\int_2^0\delta(t)\,dt}+Xe^{\int_5^0\delta(t)\,dt}$

$=100\left(\frac{0+2}{2+2}\right)^4+X\left(\frac{0+2}{5+2}\right)^4=5.0119$

$X\approx -185.796.$

8. Fund X: $A_X(t)=A_X(0)(1+r_1)^t$ $\quad\delta_X(t)=\frac{A_X'(t)}{A_X(t)}=\ln(1+r_1)$

Fund Y: $A_Y(t)=A_Y(0)(1+r_2t)$ $\quad\delta_Y(t)=\frac{A_Y'(t)}{A_Y(t)}=\frac{r_2}{1+r_2t}$

$\delta_X(t)=\delta_Y(t)\Rightarrow \ln(1+r_1)=\frac{r_2}{1+r_2t}\Rightarrow t=\frac{1}{\ln(1+r_1)}-\frac{1}{r_2}.$

$A_X(t)-A_Y(t)=\left[(1+r_1)^{\frac{1}{\ln(1+r_1)}-\frac{1}{r_2}}-\frac{r_2}{\ln(1+r_1)}\right]A(0).$

9. Fund X: $P(1+\frac{5\%}{12})^{12t}$

Fund Y: $Pe^{\int_t^T\delta(t)\,dt}=Pe^{\int_t^T\frac{1}{t+10}dt}=P\left(\frac{T+10}{t+10}\right).$

$P(1+\frac{5\%}{12})^{12\times 10}=2000\Rightarrow P\approx 1214.3221.$

$Z=1214.3221\times\left(\frac{10+10}{0+10}\right)=2\times 1214.3221=2428.6442.$