Applied Stochastic Process

Quiz

Date: 23rd October 2024

Time allowed: 55 minutes **Full mark: 100**

1. (32 points) Let $e_1, e_2, \ldots$ be a sequence of independent, identically distributed normal random variables each with zero mean and variance 1, i.e., $e_t \sim N(0,1)$ for all t. Let $Y_1 = e_1$, and then for $t > 1$ define Y_t recursively by $Y_t = Y_{t-1} + e_t$. Here θ_0 is a constant.

   (a) (8 points) Find the mean function for $\{Y_t\}$.

   (b) (12 points) Find the autocovariance function for $\{Y_t\}$.

   (c) (12 points) Determine the joint distribution of (Y_3, Y_4, Y_7).

2. (14 points) Consider a Markov chain with state space $\{0,1\}$ and transition matrix

$$\boldsymbol{P} = \begin{bmatrix} 1/4 & 3/4 \\ 2/3 & 1/3 \end{bmatrix}.$$

   Assuming that the chain starts in state 1 at time $n = 0$, what is the probability that it is in state 0 at time $n = 3$?

3. (36 points) Consider a Markov chain $\{S_0, S_1, S_2, S_3, S_4, S_5\}$ with the following transition matrix

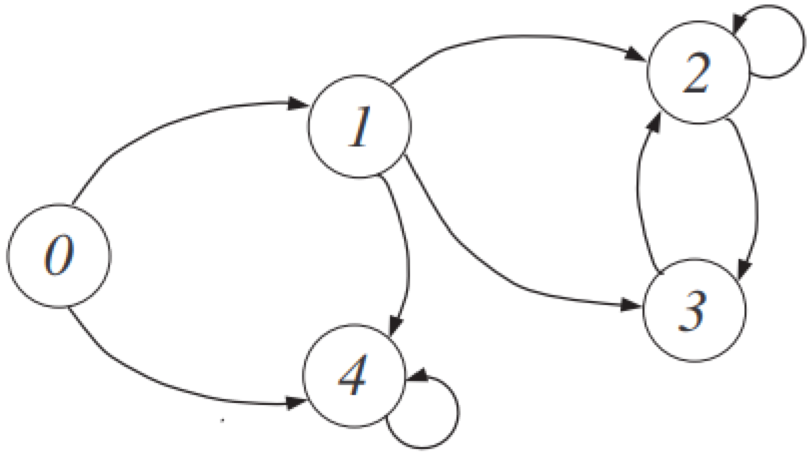
$$\boldsymbol{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1/4 & 1/4 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

   (a) (12 points) Draw the transition graph of this Markov chain;

   (b) (12 points) Sarting from S_2, get $\mathbb{P}$(process enters S_4 and leaves S_4 at the next step);

   (c) (12 points) Sarting from S_2, get $\mathbb{P}$(process enters S_3 for the 1^{st} time at 3^{rd} step)

4. (18 points) Consider the Markov chain in below figure. It is assumed that when there is an arrow from state i to state j, then $p_{i,j} > 0$. There is no arrow from state i to state j, then $p_{i,j} = 0$.



Find the equivalence classes for this Markov chain. Which class is recurrent? Which class is transient?