**2022-23 Second Semester**
**MATH1063 Linear Algebra II (1003)**

Assignment 8

Due Date: **19/May/2023 (Friday), 09:00 in tutorial class.**

- Write down your **CHN name** and **student number**. Write neatly on **A4-sized** paper (*staple if necessary*) and **show your steps**.
- **Late submissions or answers without steps won't be graded.**

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1. For the following matrix $A$, find an orthogonal matrix $P$ that diagonalizes $A$

   (a) $A = \begin{pmatrix} 0 & -4 \\ -4 & 0 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & 2 \\ 2 & 2 & -1 \end{pmatrix}$

2. Show that $A$ and $A^T$ have the same nonzero singular values. How are their singular value decompositions related?

3. Find a singular value decomposition of $A$ and verify your answer by compute $U\Sigma V^T$

$$A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

4. The matrix

$$A = \begin{pmatrix} 2 & 5 & 4 \\ 6 & 3 & 0 \\ 6 & 3 & 0 \\ 2 & 5 & 4 \end{pmatrix}$$

   has singular value decomposition

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 12 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

   (a) Use the singular value decomposition to find orthonormal bases for $\mathrm{Col}(A^T)$ and $\mathrm{N}(A)$.

   (b) Use the singular value decomposition to find orthonormal bases for $\mathrm{Col}(A)$ and $\mathrm{N}(A^T)$.

5. Prove that if $A$ is a symmetric matrix with eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$, then the singular values of $A$ are $|\lambda_1|, |\lambda_2|, \cdots, |\lambda_n|$.

6. Show that if $\sigma$ is a singular value of $A$ then there exists a nonzero vector $\mathbf{x}$ such that

$$\sigma = \frac{\|A\mathbf{x}\|_2}{\|\mathbf{x}\|_2}$$