FACULTY OF NATURAL, MATHEMATICAL & ENGINEERING SCIENCES

DEPARTMENT OF INFORMATICS

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Formalising Mathematics to Obtain Verified AI and Optimisation Software

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A Motivation: Software and Hardware Failures

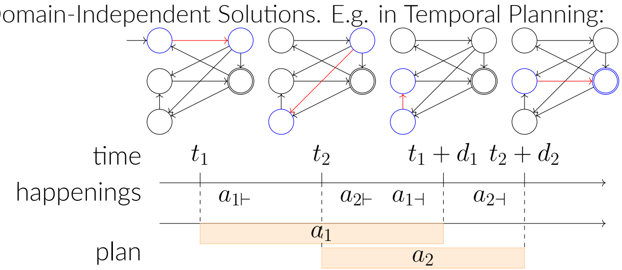
- Post Office Scandal(SF, > 900 postmasters wrongly convicted)
- Pentium FDIV (HF, flawed floating-point unit in processor)
- Therac-25 (SF, excessively high doses in radiation therapy)
- Boeing 737 (SF, screen blackouts when approaching specific runways, flight control)

Interactive Theorem Proving (ITP) and Isabelle

- Interactive Theorem Proving = Writing (partial) proofs s.t. they can be completed + checked by software
- Interactive Theorem Prover/Proof Assistant = Software to complete and check proofs
- We use Isabelle/HOL (others Coq/Rocq, Lean, etc.)
- Trustworthy Software
- In-depth understanding/explanation of behaviour

AI Planning

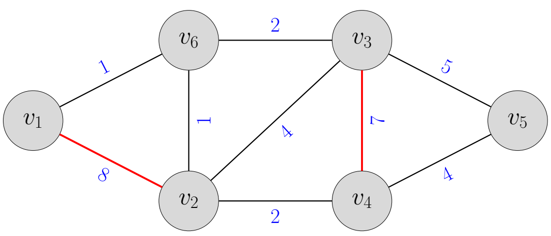
- See: https://formplan.github.io/
- Abstract Problem Representation: (1) Logic: E.g. s ⊨ ∀r::robot. ∀o::object. weight(o) ≤ capacity(r) ∧ ¬broken(r) ∧ ... ⟶ can_carry(r,o)
  (2) Probabilities: E.g. Markov Decision Processes
- Domain-Independent Solutions. E.g. in Temporal Planning:



  Find a schedule of concurrent actions that transform the inital state until a goal is satisfied. E.g. s ⊨ ∀x::task. completed(x)
- Isabelle formalisation of a part of the Planning Domain Definition Language (PDDL) and its semantics based on abstract timed transition systems.

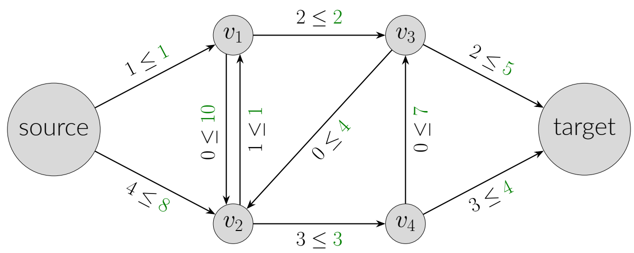
Examples for Combinatorial Optimisation

- Matching (certain one-to-one pairings allowed, selection from these, e.g. by Edmonds' Blossom Algorithm):



Combinatorial Optimisation (continued)

- Flows (send liquid from souce to target while respecting connection capacities, e.g. by Ford-Fulkerson method):



Verifying Algorithms

- Planning and CO problems can be solved by algorithms.
- Isabelle/HOL formalisation of mathematical theory to verify these algorithms.
- Library for graph algorithms (together with collaborators): flows, matchings, matroids, spanning trees etc.
- Running Time: Time waiting for solution in relation to size of problem description
- Correctness guarantee for result if using verified code

Applications of Planning and CO Problems

Matching:

- Kidney Exchange Programs (KEP): E.g. Blossom Algorithm in UK and Scandinavian KEP [2, 8]
- Student-College Matching with preferences and fairness [1]
- Auctions and Market Design: E.g. Google's Adwords (display adds suitable to queries avrriving over time) [16].

Flows:

- Electricity Grid Congestion, Pipe Network Analysis, and other transportation ubject to capacities and costs
- Circuit and other Hardware Design. Esp. Very Large Scale Integration (VLSI), e.g. decrease of chip area by 23% [9], reduction of wirelength by 10% [11], minimise cell movement [4], almost optimum solution of placement in VLSI legalisation [5], and others [7, 12]).

Planning:

- Transportation, Logistics [19], Robotics [6], Space [13, 10, 14, 17] (e.g. optmising data up- and downlink times) + Terrestrial Exploration [15, 3], Enterprise [18].

Further Remarks

Given the advantages of verification, also consider:

- Verification Effort vs. Increased Trustworthiness
- Unverified Assumptions (e.g. Compiler, Hardware)
- Good Running Time (thus high complexity) vs. Verification Effort