

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

21. October 2021

Deployment of Autonomous Vehicles (AVs)

AVs driving on our roads bring many concerns:

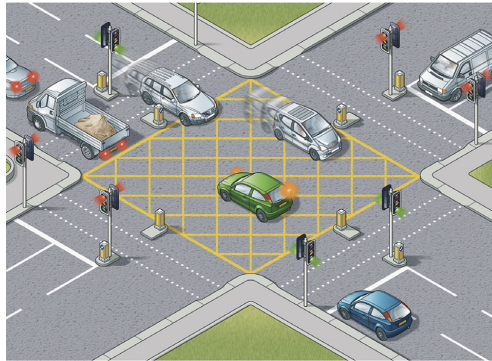


<https://insights.rlist.io/>

Autonomous Vehicles and Traffic Rules

Can AVs behave according to traffic rules?

- How can traffic rules be “**embedded**” into an autonomous system?
- What are the main challenges?



<https://www.gov.uk/guidance/the-highway-code/using-the-road-159-to-203>

Autonomous Vehicles and Traffic Rules

How can traffic rules be “embedded” into an autonomous system?

- Translating rules written in natural language into a machine-readable format
- The rules as stated in a Highway Code are:
 - full of imprecise meanings,
 - ambiguities
 - and sometimes the sequence and dependence of actions described in the rules are not clear.
- **Goal:** Highway Code \Rightarrow Digital Highway Code.

A First attempt: Representing Temporal Aspects of Traffic Rules

Using temporal logic to formalise the UK Road Junction rules

- **Linear Temporal Logic**: brings a proper way to define the sequence and dependence of actions that take place in a road junction scenario.



https://doi.org/10.1007/978-3-030-54994-7_16

[ADF20] Alves, G.V., Dennis, L. & Fisher, M. (2020): *Formalisation and Implementation of Road Junction Rules on an Autonomous Vehicle Modelled as an Agent (FMAS@FMWeek20)*.

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Formalisation of UK Road Junction rules

- **Rule 170:** You should watch out for road users (RU).

Watch out for pedestrians crossing a road junction (JC) into which you are turning.

Do not cross or join a road until there is a safe gap (SG) large enough for you to do so safely.

$$\begin{aligned} & \Box ((\text{watch}(\text{AV}, \text{JC}, \text{RU}) \wedge (\neg \text{cross}(\text{RU}, \text{JC}) \wedge (\text{exists}(\text{SG}, \text{JC})))) \\ & \rightarrow ((\text{exists}(\text{SG}, \text{JC}) \wedge \neg \text{cross}(\text{RU}, \text{JC})) \cup \text{enter}(\text{AV}, \text{JC})))) \end{aligned}$$



[ADF20] Alves, G.V., Dennis, L. & Fisher, M. (2020): *Formalisation and Implementation of Road Junction Rules on an Autonomous Vehicle Modelled as an Agent (FMAS@FMWeek20)*.

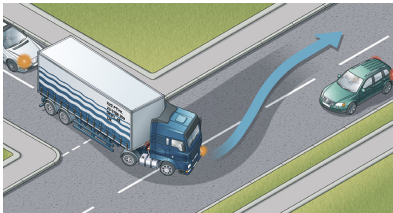
Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Temporal and Spatial Aspects

Are temporal aspects enough?

- For example: how can we represent a **safe gap**?



<https://www.gov.uk/guidance/the-highway-code/using-the-road-159-to-203>

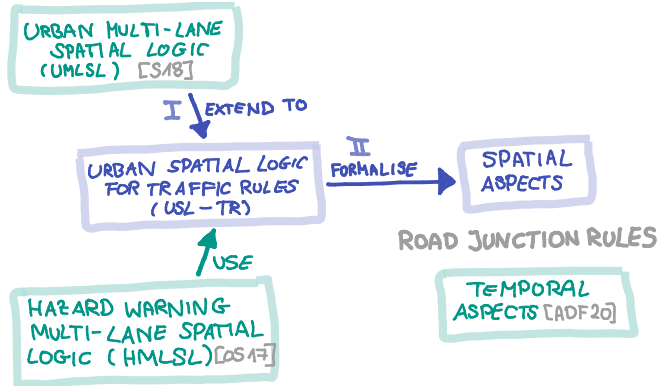
Focus of this talk: Spatial aspects of road junction rules

Outlook/ Destination: Combination of temporal and spatial aspects



Our Contribution

- Part I: Urban Spatial Logic for Traffic Rules (USL-TR)
- Part II: Exemplary formalisation of spatial aspects of UK Road Junction Rules



[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

[ADF20] Alves, G.V., Dennis, L. & Fisher, M. (2020): *Formalisation and Implementation of Road Junction Rules on an Autonomous*

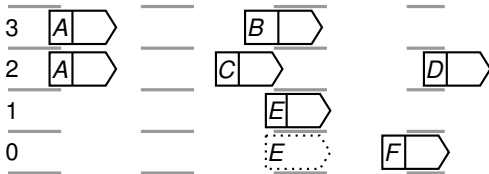
Extending Urban/Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Part I: Urban Spatial Logic for Traffic Rules (USL-TR)

Basis: Multi-lane Spatial Logic “Universe”

- **Highway Traffic:** Multi-lane Spatial Logic (MLSL) [HLOR11]
 - One-way traffic, lane-change manoeuvres
 - **Hazard Warning Extension HMLSL** [OS17, B18]
 - Proof theory [L15], UPPAAL implementation [S18b]
- **Country Roads:** Extended Multi-lane Spatial Logic (EMLSL) [HLO13]
 - Opposing traffic, overtaking manoeuvres
- **Urban Traffic: Urban Multi-lane Spatial Logic (UMLSL)** [HS16,S17,S18,S21]
 - urban intersections, model and semantics differs greatly from predecessors
 - UPPAAL implementation [S18b, BS19,S21]

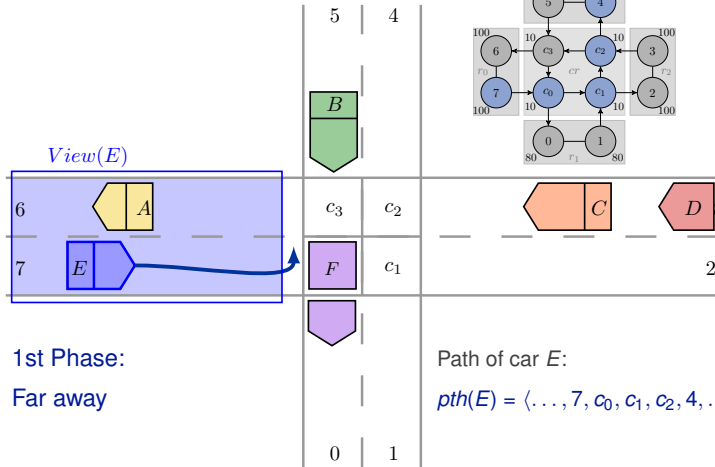
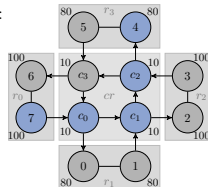


www.wikimedia.org

Basis: Urban Multi-lane Spatial Logic

Urban road network:

\mathcal{N} :

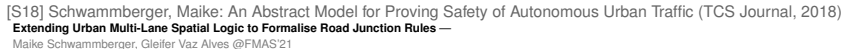


[S18] Schwammberger, Maïke: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

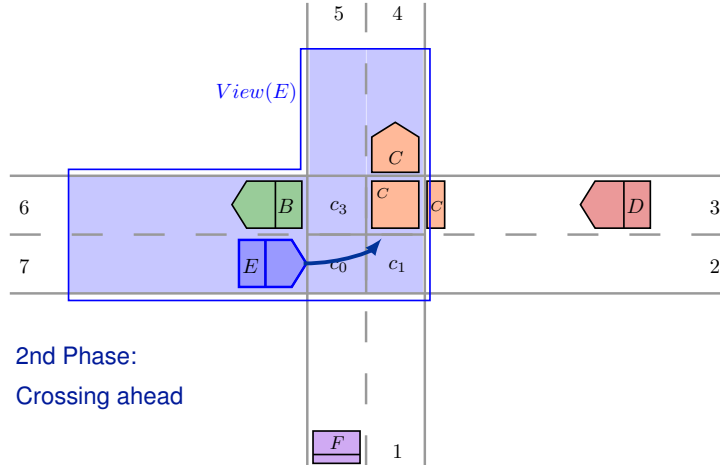
Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maïke Schwammberger, Gleifer Vaz Alves @FMAS'21

Slide 11
21. October 2021



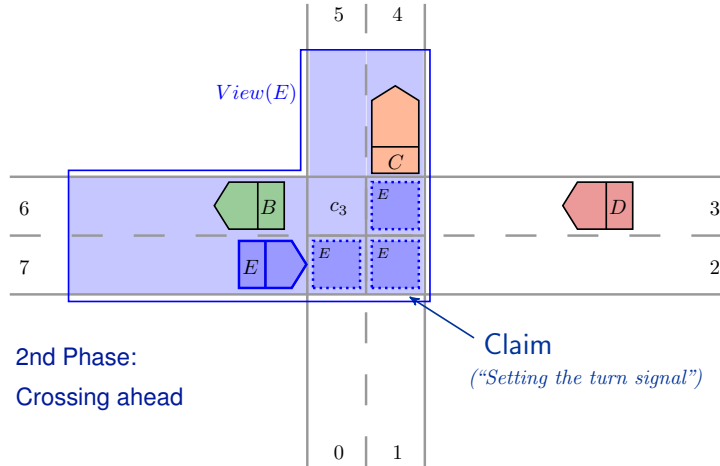
Basis: Urban Multi-lane Spatial Logic



2nd Phase:
Crossing ahead

[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)
Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —
Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Basis: Urban Multi-lane Spatial Logic



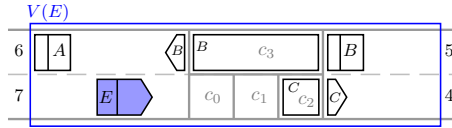
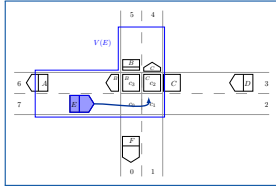
[S18] Schwammbberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)
 Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —
 Maike Schwammbberger, Gleifer Vaz Alves @FMAS'21

3rd Phase:
On crossing

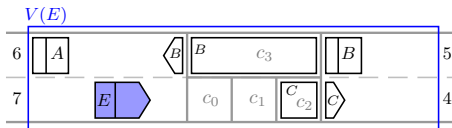
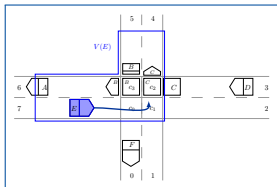


Slide 11
21. October 2021

Basis: Urban Multi-lane Spatial Logic



Basis: Urban Multi-lane Spatial Logic



Example 1: Free space in front of E :

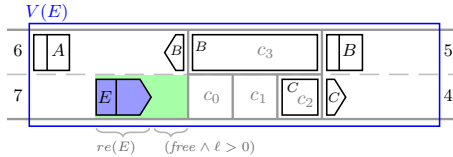
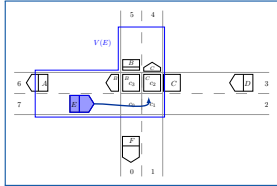
$$\phi_1 \equiv \langle re(E) \wedge (free \wedge \ell > 0) \rangle$$

[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Basis: Urban Multi-lane Spatial Logic



Example 1: Free space in front of E :

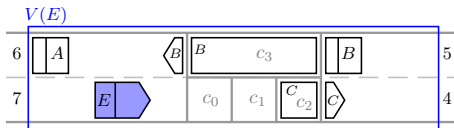
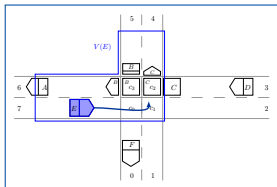
$$\phi_1 \equiv \langle re(E) \wedge (free \wedge \ell > 0) \rangle \quad \checkmark$$

[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Basis: Urban Multi-lane Spatial Logic



Example 1: Free space in front of E :

$$\phi_1 \equiv \langle re(E) \wedge (free \wedge \ell > 0) \rangle \checkmark$$

Example 2: Crossing ahead of E :

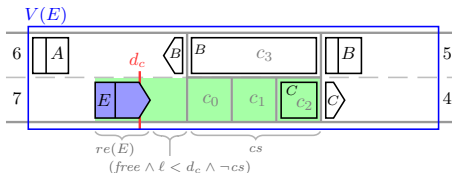
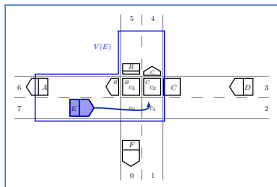
$$ca(E) \equiv \langle re(E) \wedge (free \wedge \ell < d_c \wedge \neg \langle cs \rangle) \wedge cs \rangle$$

[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Basis: Urban Multi-lane Spatial Logic



Example 1: Free space in front of E :

$$\phi_1 \equiv \langle re(E) \wedge (free \wedge \ell > 0) \rangle \quad \checkmark$$

Example 2: Crossing ahead of E :

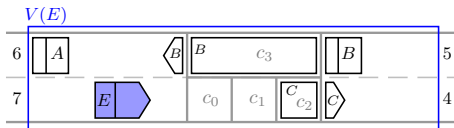
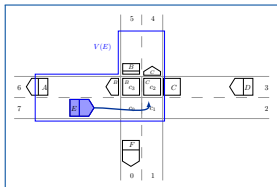
$$ca(E) \equiv \langle re(E) \wedge (free \wedge \ell < d_c \wedge \neg \langle cs \rangle) \wedge cs \rangle \quad \checkmark$$

[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Basis: Urban Multi-lane Spatial Logic



Example 1: Free space in front of E :

$$\phi_1 \equiv \langle re(E) \wedge (free \wedge \ell > 0) \rangle \checkmark$$

Example 2: Crossing ahead of E :

$$ca(E) \equiv \langle re(E) \wedge (free \wedge \ell < d_c \wedge \neg \langle cs \rangle) \wedge cs \rangle \checkmark$$

Example 4: Position of E is on crossing segment:

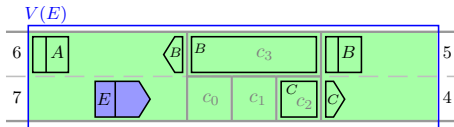
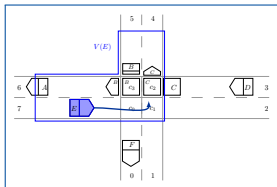
$$oc(E) \equiv \langle re(E) \wedge cs \rangle$$

[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Basis: Urban Multi-lane Spatial Logic



Example 1: Free space in front of E :

$$\phi_1 \equiv \langle re(E) \wedge (free \wedge \ell > 0) \rangle \checkmark$$

Example 2: Crossing ahead of E :

$$ca(E) \equiv \langle re(E) \wedge (free \wedge \ell < d_c \wedge \neg \langle cs \rangle) \wedge cs \rangle \checkmark$$

Example 4: Position of E is on crossing segment:

$$oc(E) \equiv \langle re(E) \wedge cs \rangle \times$$

[S18] Schwammberger, Maike: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

Basis: System Properties \rightsquigarrow Traffic Rules?

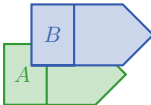
Safety:

Any two cars may never collide.

Safety property as UMLSL formula:

$$Safe \equiv \forall c, d : c \neq d \rightarrow \neg \langle re(c) \wedge re(d) \rangle$$

Example $re(A) \wedge re(B)$ (collision):



[S18] Schwammberger, Maïke: An Abstract Model for Proving Safety of Autonomous Urban Traffic (TCS Journal, 2018)

UMLSL: Fit for Traffic Rules?

UMLSL for the specification of traffic rules...

Useful:

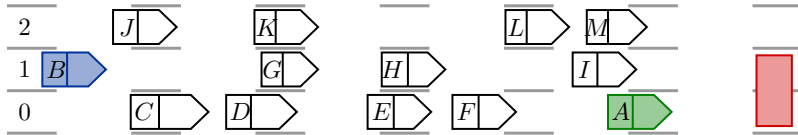
- spatial aspects of manoeuvres at intersections (relative positions of AVs, collisions, collision freedom,...)
- sizes of spatial intervals (distances between cars or to intersection, safe gaps,...)

Missing:

- non-autonomous road users (human-driven cars, cyclists, pedestrians,...)
- static objects (traffic signs, markings on the road, obstacles...)

From Hazards to Road Users and Static Objects

- Hazard Warning Multi-lane Spatial logic (HMLSL) [OS17]
 - Hazard ahead: $hz\text{-}ahead(a) \equiv \langle re(a) \rangle \sim \langle hz \rangle$
 - Only one stationary hazard in the entire world
- Multiple and moving hazards (human-driven cars) for HMLSL [B18]
 - Switching an AV “on” and “off”, place and remove hazards
- [OS17, B18]: Only for highway traffic case



[OS17] Olderog, E.R., Schwammberger, M.: *Formalising a Hazard Communication Protocol with Timed Automata* (Models, Algorithms, Logics and Tools, 2017)

[B18] Bischopink, C.: *Moving Hazards – Reasoning about human drivers in autonomous traffic*. (Master's thesis, University of Oldenburg, 2018)

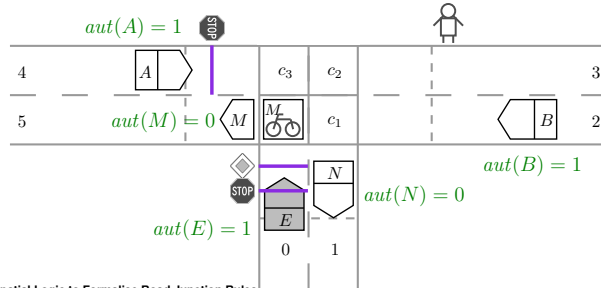
Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammberger, Gleifer Vaz Alves @FMAS'21

From Hazards to Road Users and Objects

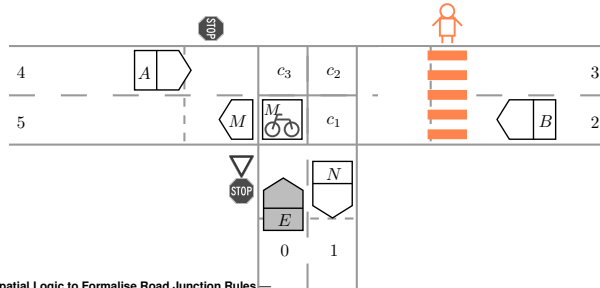
For our extension:

- **Static objects** modelled similar to stationary hazards
 - traffic signs, markings on the road, pedestrians (!),...
 - treated like a **virtual flag** at a specific position (cf. $obj(stop) = \{(0, 98), (4, 198)\}$)
- **Road users** modelled similar to moving hazards (“moving along paths”)
 - AV, human-driven cars, cyclists,...
 - Move along paths in urban road network (i.e. on lane and crossing segments)
 - Switchable **“autonomy flag”** for autonomy status



Pedestrians \approx Objects?

- Pedestrian: Does not move along paths in urban road network
 \Rightarrow Cannot be road user
- No road-side formalised in existing urban road network
 - Even with road side, difficult to assign path to pedestrian
 - Mental models of human needed (cf. ongoing work)
- For now: Pedestrian that crosses blocks all neighbouring lanes at that position
 - $obj(ped) = \{(2, 90), (3, 90)\}$
- Cf. virtual zebra crossing that appears whenever a pedestrian wants to cross a road



Syntax of USL-TR

- Car variables: c, d , object variables: o , real variable r , other variables u, v

USL-TR formulae ϕ :

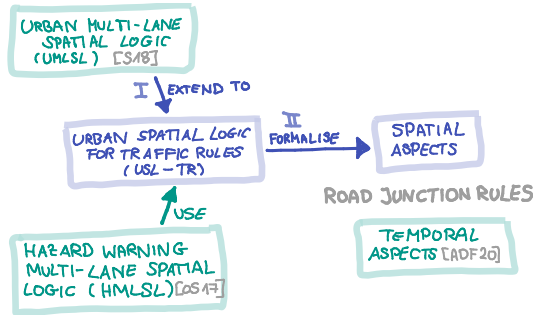
$\phi ::= u = v \mid \ell = r$ (*Atoms, comparison*)

$\mid cs \mid free \mid re(c) \mid cl(c)$ (*Atoms, spatial*)

$\mid ob(o) \mid ru(c)$ (*Atoms, obj. + roadusers*)

$\mid true \mid \neg\phi_1 \mid \phi_1 \wedge \phi_2 \mid \exists c: \phi_1$ (*FOL*)

$\mid \phi_1 \curvearrowright \phi_2 \mid \begin{array}{l} \phi_2 \\ \phi_1 \end{array}$ (*Spatial Chops*)



Part II: Exemplary formalisation of spatial aspects of UK Road Junction Rules

Ambiguity of natural language

“The car hit the boat while it was moving.”



Stanley Forman @twitter.com

Rule 170

1. You should watch out for road users.
2. Watch out for pedestrians crossing a road junction into which you are turning. If they have started to cross they have priority, so give way.
3. Look all around before emerging.
4. Do not cross or join a road until there is a safe gap large enough for you to do so safely.

Traffic rule: “Detection” and “Reaction” part

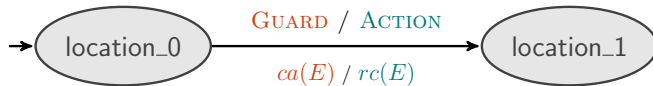
“4. Do not cross or join a road until there is a safe gap large enough [...]”

Detection: “until there is a safe gap large enough [...]”

- Formalise this using USL-TR
- E.g., use this formula as a transition **guard** in a traffic rule controller
 - ⇒ [S18]: Automotive-Controlling Timed Automata (ACTA) to specify traffic manoeuvre controllers
 - ⇒ ACTA: UMLSL formulae as guards and invariants (e.g. “crossing ahead” $ca(E)$)

Reaction: “(Do not) cross or join a road [...]”

- E.g., as an **action** in the traffic rule controller, following the guard
 - ⇒ Controller actions in ACTA
 - ⇒ Cf. $rc(E)$ to reserve crossing segments/ enter crossing



Rule 170 – Part 2

2. “Watch out for pedestrians crossing a road junction into which you are turning. If they have started to cross they have priority, so give way.”

“Pedestrian Ahead” check (Detection):

- (A pedestrian is crossing via a virtual zebra crossing)

$$pa(E) \equiv \langle re(E) \wedge (free \wedge \ell < d_p) \wedge ob(Ped) \rangle. \quad (1)$$

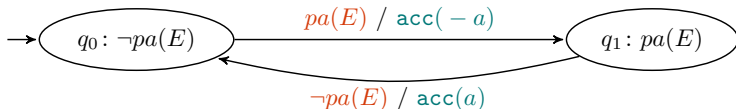
- Adaptation from “crossing ahead check” from [S18]:

$$ca(E) \equiv \langle re(E) \wedge (free \wedge \ell < d_c \wedge \neg \langle cs \rangle) \wedge cs \rangle$$

“Give way” (Reaction):

Traffic rule controller issues braking manoeuvre

(cf. [S21]: Distance Controller on the Dynamic Layer)



Rule 170 – Part 4

4. “Do not cross or join a road until there is a safe gap large enough for you to do so safely.”

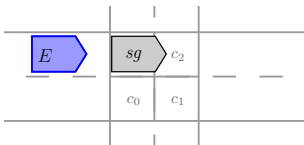
Detection:

“Free safe gap anywhere”:

$$sg(E) \equiv free \wedge \ell \geq size_E. \quad (2)$$

“Free safe gap on an intersection ahead”:

$$sg_I(E) \equiv \langle (re(E) \wedge \neg cs) \wedge (free \wedge \neg cs) \wedge (sg(E) \wedge cs) \rangle, \quad (3)$$



Reaction: “Do not cross or join a road” (until safe gap):

Cf. previous slide, adjust speed to (not) enter/ leave intersection

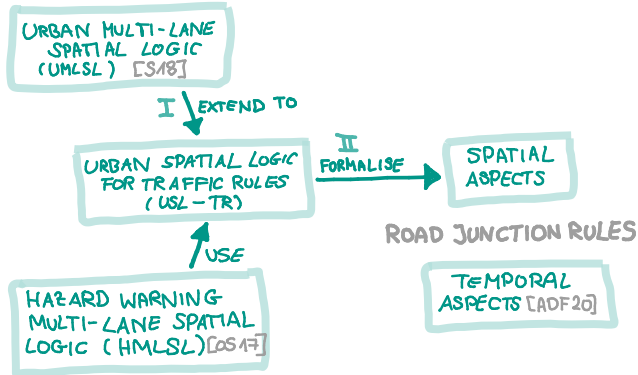
Outlook: More Traffic Rules

USL-TR: Variety of other traffic rules possible:

- “Pedestrian ahead check” (cf. formula (1)) \rightsquigarrow “something ahead check”
 - “Stop sign ahead” (UK rule 171), “give-way sign ahead” (UK rule 172), ...
- Safe gaps (of various sizes) needed in several rules
- Any types of objects and road users possible
- Switching from autonomous road user to non-autonomous road user:
Opens Digital Highway Code to semi-autonomous vehicles (SAE 3,4)

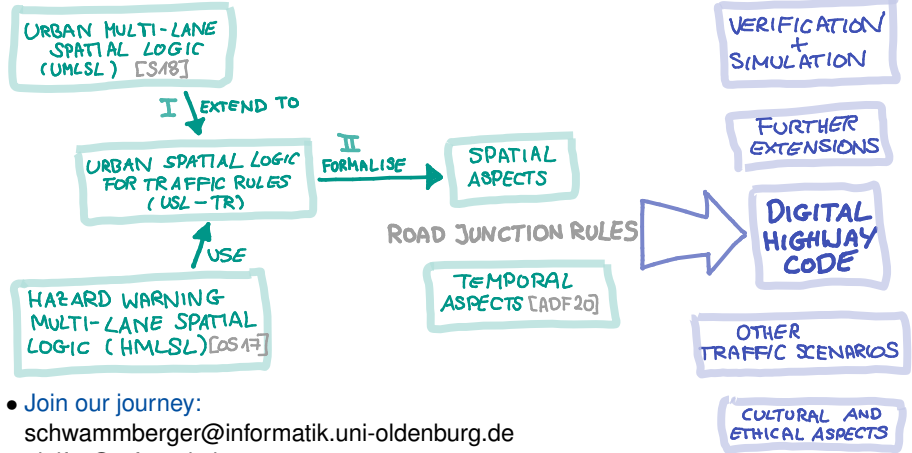
Conclusion and Future Work

- Formalisation of spatial elements of road junction rules



Conclusion and Future Work

- Formalisation of spatial elements of road junction rules
- Pave the way towards a Digital Highway Code



- Join our journey:
schwammberger@informatik.uni-oldenburg.de
gleifer@utfpr.edu.br

Literature



[ADF20] ALVES, G.V., DENNIS, L. & FISHER, M.: *Formalisation and Implementation of Road Junction Rules on an Autonomous Vehicle Modelled as an Agent*. In *Proc. of FMAS@FMWeek20* (2020).



[ADF21] ALVES, G.V., DENNIS, L. & FISHER, M.: *A Double-Level Model Checking Approach for an Agent-Based Autonomous Vehicle and Road Junction Regulations*. In *Journal of Sensor and Actuator Networks*, 10(3) (2021).



[Bi18] BISCHOPINK, C.: *Moving Hazards – Reasoning about human drivers in autonomous traffic*. (Master's thesis, University of Oldenburg, 2018)



[BS19] BISCHOPINK, C. AND SCHWAMMBERGER, M.: *Verification of fair controllers for urban traffic manoeuvres at intersections*. In *Proceedings of FMAS@FMWeek19* (2019).



[OS17] OLDEROG, E.-R. AND SCHWAMMBERGER, M.: *Formalising a Hazard Warning Communication Protocol with Timed Automata*. In: *Models, Algorithms, Logics and Tools*, volume 10460 of *LNCS* (2017).



[S18] SCHWAMMBERGER, M.: *An abstract model for proving safety of autonomous urban traffic*. In *Theoretical Computing Science*, volume 744 (2018).



[S20] SCHWAMMBERGER, M.: *Distributed Controllers for Provably Safe, Live and Fair Autonomous Car Manoeuvres in Urban Traffic*. Doktorarbeit, Universität Oldenburg (2020).

Rule 170 – Part 1

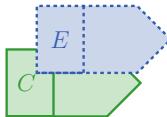
1. “You should watch out for road users.”

- We assume:
 - “always” and “at intersections”
 - “Watch out”: Do not invade spaces others do/ plan to occupy
(Alternative: “Perceive others”, which is a general assumption for AVs)
- From [S18]: Potential Collision Check:

$$pc(c) \equiv c \neq \text{ego} \wedge \langle cl(E) \wedge (re(c) \vee cl(c)) \rangle. \quad (4)$$

⇒ Check for overlaps of own crossing claim $cl(E)$ (“space to occupy in future”) with claims $cl(c)$ / reservations $re(c)$ (“currently occupied space”) of other road user c .

- E.g.: $\neg \exists c : pc(c)$ as invariant in traffic rule controller



Rule 170 – Part 3

3. “Look all around before emerging.”

“Look all around”: Ambiguous meaning, we go further:

$$look(E) \equiv \langle (re(E) \wedge cs) \frown (\neg cs \wedge sg(E)) \rangle \wedge \neg \exists c: pc(c) \wedge \neg pa(E). \quad (5)$$

- $\langle (re(E) \wedge cs) \frown (\neg cs \wedge sg(E)) \rangle$: it is checked that car E is on an intersection ($re(E) \wedge cs$) and that after the intersection, there is a free safe gap for car E available for E to emerge into ($\neg cs \wedge sg(E)$, cf. formula (2)),
- $\neg \exists c: pc(c)$: it is checked that no potential collisions with other road users exist ($\neg \exists c: pc(c)$), and
- $\neg pa(E)$: we ensure that no pedestrian is ahead (cf. formula (1)).

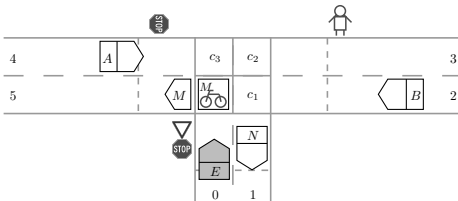
“Emerging” (from an intersection):

Adjust speed to leave intersection (cf. part 2)

USL-TR: Traffic Snapshot Extension

We extend the Definition of a traffic snapshot from [S18]. Given an arbitrary road user identifier $C \in \mathbb{I}$ and a static object $O \in \mathbb{O}$ the new elements in $TS = (\star, obj, aut)$, are defined as follows:

- $obj: \mathbb{O} \rightarrow \mathcal{P}((\mathbb{L} \cup \mathbb{CS}) \times \mathbb{R})$ such that $obj(O)$ yields a set of 2-tuples of each a lane resp. crossing segment $s \in \mathbb{L} \cup \mathbb{CS}$ together with a real position of O on the respective segment s and
- $aut: \mathbb{I} \rightarrow \mathbb{B}$ indicates whether an element $C \in \mathbb{I}$ is an AV or a non-autonomous road user.



Example: $obj(stop) = \{(0, 98), (4, 198)\}$

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

Maike Schwammburger, Gleifer Vaz Alves @FMAS'21

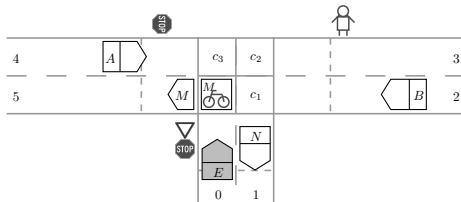
USL-TR: Abstract Model

Placing and removing static objects:

Consider a current traffic snapshot $TS = (\star, obj, aut)$, where \star again marks those traffic snapshot elements that were introduced in **Sch18-TCS** and that are not of concern for this definition. For all $O \in \mathbb{O}$, $s \in \mathbb{L} \cup \mathbb{CS}$ and $p \in \mathbb{R}$ the following transitions hold:

$$TS \xrightarrow{\text{place}(O, s, p)} TS' \Leftrightarrow TS' = (\star, \text{obj}', \text{aut}) \wedge \text{obj}' = \text{obj} \oplus \{O \mapsto \text{obj}(O) \cup (s, p)\}$$

$$TS \xrightarrow{rm(O,s,p)} TS' \Leftrightarrow TS' = (\star, obj', aut) \wedge obj' = obj \oplus \{O \mapsto obj(O) \setminus \{(s,p)\}\}$$



Example: With `rm(stop, 0, 98)`, the instance of the stop sign at lane 0 at position 98 is removed

Extending Urban Multi-Lane Spatial Logic to Formalise Road Junction Rules —

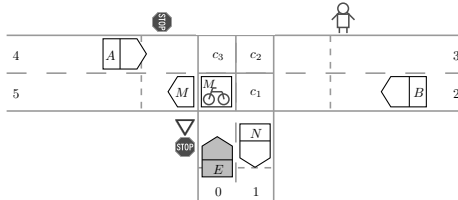
Maíke Schwammberger, Gleifer Vaz Alves @FMAS'21

USL-TR: Abstract Model

Switching “AV” on and off:

Consider a current traffic snapshot $TS = (\star, obj, aut)$. For all $C \in \mathbb{I}$ the following transition holds.

$$TS \xrightarrow{\text{switch}(C)} TS' \quad \Leftrightarrow \quad TS' = (\star, obj', aut) \wedge aut' = aut \oplus \{C \mapsto \neg aut(C)\}$$



Example: On calling $\text{switch}(A)$, the status $aut(A) = 1$ of the AV A is changed to $aut(A) = 0$. With this, A is considered a non-autonomous road user.

USL-TR: Semantics

With respect to a traffic snapshot TS , a virtual view $V = (L, X, E)$ and a valuation of variables ν , with $c \in CVar$ and $o \in OVar$, the *satisfaction* of the spatial USL-TR atoms $re(c)$, $ru(c)$ and $ob(o)$ is defined as follows:

$$TS, V, \nu \models re(c) \Leftrightarrow \#L = 1 \text{ and } |X| > 0 \text{ and } aut(c) = true \text{ and } \forall s_i: L(1); \exists X_i \subseteq X \bullet$$

$$s_i \in cres(\nu(c)) \cup res(\nu(c)) \text{ and } (s_i, X_i) \in seg_V(\nu(c)) \text{ and } X \subseteq \bigcup_{i=1}^{\#L(1)} X_i \quad (6)$$

$$TS, V, \nu \models ru(c) \Leftrightarrow \#L = 1 \text{ and } |X| > 0 \text{ and } aut(c) = false \text{ and } \forall s_i: L(1); \exists X_i \subseteq X \bullet$$

$$s_i \in cres(\nu(c)) \cup res(\nu(c)) \text{ and } (s_i, X_i) \in seg_V(\nu(c)) \text{ and } X \subseteq \bigcup_{i=1}^{\#L(1)} X_i \quad (7)$$

$$TS, V, \nu \models ob(o) \Leftrightarrow \#L = 1 \text{ and } \#L(1) = 1 \text{ and } |X| = 0 \text{ and } \exists s: L(1); \exists p: \mathbb{R} \bullet$$

$$X = [p, p] \text{ and } (s, p) \in obj(o) \quad (8)$$

A Second attempt on Temporal Aspects of Traffic Rules

An Agent-based Approach with Model Checking techniques



Journal of
*Sensor and
Actuator Networks*

an Open Access Journal by MDPI



A Double-Level Model Checking Approach for an Agent-Based Autonomous Vehicle and Road Junction Regulations

Gleifer Vaz Alves; Louise Dennis; Michael Fisher

J. Sens. Actuator Netw. 2021, Volume 10, Issue 3, 41

An Agent-based Approach with Model Checking techniques

AV modelled as an agent

- **Agent**: suitable abstraction to represent the high-level decisions of an AV.
- We apply model checking both at design and development levels.

[ADF21] Alves, G.V., Dennis, L. & Fisher, M. (2021): *A Double-Level Model Checking Approach for an Agent-Based Autonomous Vehicle and Road Junction Regulations* (*Journal of Sensor and Actuator Networks* 10(3)).

Literature: MLSL Universe



[HLOR11] HILSCHER, M., LINKER, S., OLDEROG, E.-R. AND RAVN, A.P.: *An Abstract Model for Proving Safety of Multi-Lane Traffic Manoeuvres*. In: *Proceedings of the 13th ICFEM*, volume 6991 of LNCS (2011).



[HLO13] HILSCHER, M., LINKER, S. AND OLDEROG, E.-R.: *Proving Safety of Traffic Manoeuvres on Country Roads*. In: *Theories of Programming and Formal Methods – Essays Dedicated to Jifeng He on the Occasion of His 70th Birthday*, volume 8051 of LNCS (2013).



[FHO15] FRÄNZLE, M. AND HANSEN, M.R. AND ODY, H.: *No Need Knowing Numerous Neighbours*. In: *Correct System Design Symposium*, volume 9360 of LNCS (2015).



[L15] LINKER, S.: *Proofs for Traffic Safety – Combining Diagrams and Logic*. PhD thesis (2015).



[O15] ODY, H.: *Undecidability Results for Multi-Lane Spatial Logic*. In: *ICTAC 2017*, volume 9399 of LNCS (2015).



[HS16] HILSCHER, M. AND SCHWAMMBERGER, M.: *An Abstract Model for Proving Safety of Autonomous Urban Traffic*. In: *Proceedings of International Colloquium on Theoretical Aspects of Computing (ICTAC)*, volume 9965 of LNCS (2016).



[O17] ODY, H.: *Monitoring of Traffic Manoeuvres with Imprecise Information*. In: *Proceedings of First Workshop on Formal Verification of Autonomous Vehicles*, volume 257 of EPTCS (2017).



[L17a] LINKER, S.: *Spatial Reasoning About Motorway Traffic Safety with Isabelle/HOL*. In: *IFM*, volume 10510 of LNCS (2017).



[L17b] LINKER, S.: *Hybrid Multi-Lane Spatial Logic*. In: *Archive of Formal Proofs* (2017).



[S17] SCHWAMMBERGER, M.: *Imperfect Knowledge in Autonomous Urban Traffic Manoeuvres*. In: *Proceedings of FVAV@IFM17*, volume 257 of EPTCS (2017).



[O18] OLDEROG, E.-R.: *Space for Traffic Manoeuvres: An Overview*. In: *Symposium on Real-Time and Hybrid Systems*, Springer (2018).



[S18b] SCHWAMMBERGER, M.: *Introducing Liveness into Multi-lane Spatial Logic lane change controllers using UPPAAL*. In: *Proceedings of SCAV@CPSWeek18*, volume 269 of EPTCS (2018).



[O19] ODY, H.: *Monitoring of Traffic Manoeuvres with Imprecise Information*. PhD thesis, submitted Nov. 2019.

[S21] SCHWAMMBERGER, M.: *Proving Properties of Autonomous Car Manoeuvres in Urban Traffic*. In *Journal for Information Technology* (2021).