

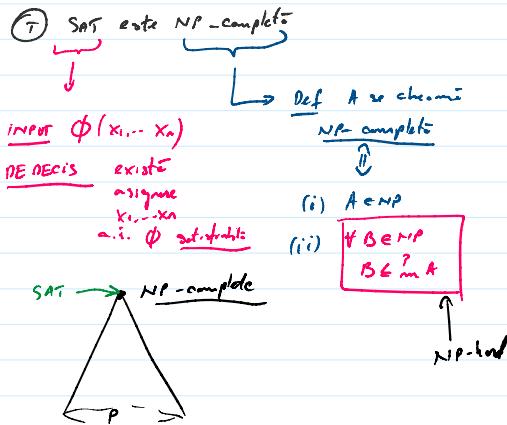
jyc6xvp

 $P \neq NP$

Def $A \leq^P_m B \Leftrightarrow \exists f: Z^* \rightarrow Z^*$
calculable in time polynomial

$$x \mapsto f(x) \text{ or } p(x) \leq^P_m |x|$$

$$x \in A \Leftrightarrow f(x) \in B$$



Ex. 3-SAT

$$\text{INPUT } \Phi(x_1, \dots, x_n) \text{ CNF} \\ |\alpha_i| \leq 3$$

① 3-SAT éste NP-completo

$$\left[\begin{array}{l} \text{Def: } A \text{ éste NPC} \\ B \text{ éste inNPC} \\ A \leq^P_m B \end{array} \right] \Rightarrow B \text{ éste NP-completo}$$

Arrestando dass $\boxed{\text{SAT} \leq^P_m \text{3-SAT}}$

| | |
|-------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ϕ_0 $x \vee y \vee \bar{z} \vee t$ $x \vee \bar{y} \vee \bar{z} \vee \bar{t} \vee u$ | $f(\phi_0)$ $x \vee y \vee \alpha$ $\bar{x} \vee \bar{y} \vee t$ $\bar{z} \vee F \vee u$ <hr/> $x \vee \bar{y} \vee \bar{z} \vee \alpha$ $\bar{t} \vee F \vee u$ <hr/> $x \vee \bar{y} \vee \beta$ $\bar{p} \vee \bar{z} \vee \alpha$ $\bar{z} \vee \bar{t} \vee u$ |
|-------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

◻

Obj 2-SAT é P

① k-SAT $k \geq 3$ NP-completo

$x \vee y \vee z$

$x \vee y \vee z \vee d$
 $x \vee y \vee z \vee \bar{d}$

Obj Dacă $P = NP \Rightarrow \exists A \in NP$
(LAONER) $A \in P$, A nu este NP-complet

Exemplu
Probleme
intermediare
(exemplu)
- Isomorfismul grafurilor
- Factorizing

(7) Variantele SAT sunt fie P în P
(SMTLIB) fie NP-complet.

Exemplu 1-IN-K SAT ($k=3$)

Soluție $\phi = C_1 \wedge C_2 \wedge \dots \wedge C_m$
 $|C_i| = 3$

Vream O alocare A pt variabile x_0, \dots, x_n
care să satisfacă exact un literal
P. fiecare clauză

(7) 1-IN-3 SAT este NP-complet

Dоказ. Arăt $3-SAT \leq_m^P 1-IN-3 SAT$

$P(x_{i,j}, z) \equiv$ "exact
unul din
 $x_{i,j}, z$ "

ϕ

$x \vee y \vee z$

$f(\phi)$
 $R(x, a, d)$
 $R(y, b, d)$
 $R(z, c, e)$
 $R(a, d, f)$
 $R(b, c, \text{FALSE})$



(7) ILP este NP-complet
↑
integer linear programming

Să dăm Un "program linier"
+
restrictii întregitale
,

$$\left\{ \begin{array}{l} \min(2x + 3y + 5z) \\ x - y + z \geq 5 \\ x + y + z \geq 10 \\ x, y, z \geq 0 \end{array} \right.$$

Dacă OPT (intreg) $\leq \alpha$!

Dacă $2-SAT \leq_m^P ILP$

Dann 3-SAT $\in \text{NP}$

$$\phi$$

$$x \vee y \vee z$$

$$f(\phi)$$

$$x + y + (1-z) \geq 1$$

$$x, y, z \in \{0, 1\}$$

[Independent Set]

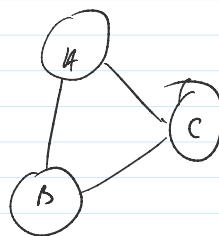
Siede $G = (V, E)$ gegebe
 $1 \leq k \leq n$

Noodeaus $\exists W \subseteq V$
 $|W| = k$

a.i.

$$(a, b) \in E$$

$$a \notin W \text{ dann } b \notin W$$



$$\begin{array}{ll} k=1 & \text{DA} \\ k=2 & \text{NU} \end{array}$$

(T) is este NP-complete

ISENP (G, k) Monter $w = w_1 \dots w_n \in \{0, 1\}^n$

$$(i) \sum w_i = k$$

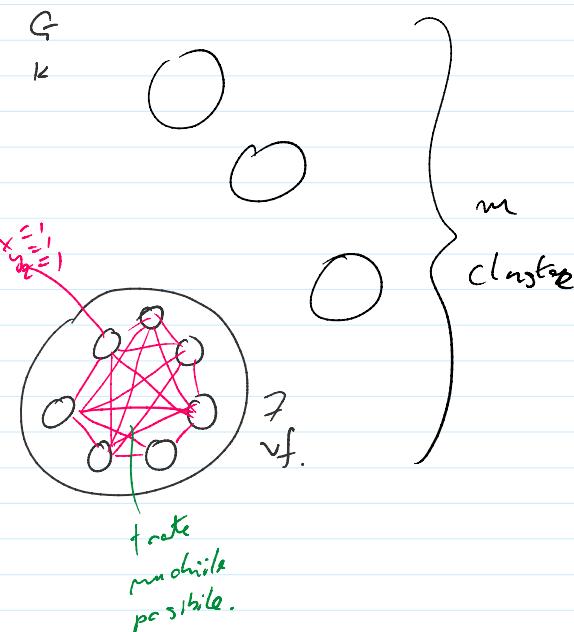
(ii) w codifică un IS.

[3-SAT $\in \text{NP}$ IS]

ϕ m-clause
n-variable

$$C = x \vee y \vee z$$

7 combinații care satisfac C



$$\begin{array}{l}
 \begin{array}{c}
 \overline{x=1} \\
 \overline{y=1} \\
 \overline{z=1}
 \end{array}
 \quad O - ? \rightarrow
 \begin{array}{c}
 \overline{x=0} \\
 \overline{y=1} \\
 \overline{z=0}
 \end{array}
 \\[10pt]
 \text{pura o} \\
 \text{nuchie} \\
 \text{când se!} \\
 \text{se contrapuz}
 \end{array}$$

ϕ satisfacibilitate \Leftrightarrow \exists ună verificare
indep. în G

\Rightarrow Fie A sat. assignment pt ϕ

$x \vee y \vee z \longrightarrow$ vf din cluster

$$\left\{
 \begin{array}{l}
 x = A(x) \\
 y = A(y) \\
 z = A(z)
 \end{array}
 \right.$$

P_1 nu se contrapuz
 P_2 pt că sunt
 consistente cu A

\Rightarrow IS
 de mărire
 în $\vdash G$

\Leftarrow Pr am un IS de mărire în $\vdash G$



1 vîrf pe cluster



A \Leftarrow A($x_1, y_1, 2$) = Formel.

