

# Algoritmul Minmax

---

Miruna-Andreea Zăvelcă  
miruna-andreea.zavelca@unibuc.ro  
Universitatea din București



**Maria**



**Gigel**



**Maria**



**Gigel**

# Joc de sumă zero (0-sum game)

Orice câștig al unui jucător reprezintă o pierdere cu aceeași valoare pentru celălalt jucător.



**Maria**



**Gigel**



0

# Joc cu informație perfectă

Există mai mulți jucători care efectuează mutări alternativ și au informație completă asupra stării curente a jocului.



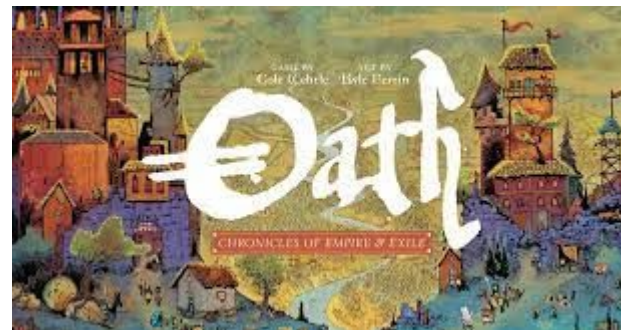
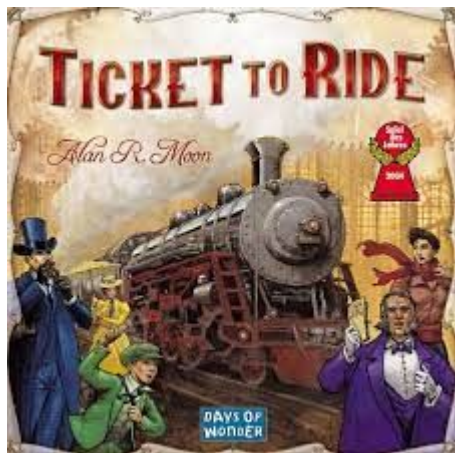
**Maria**



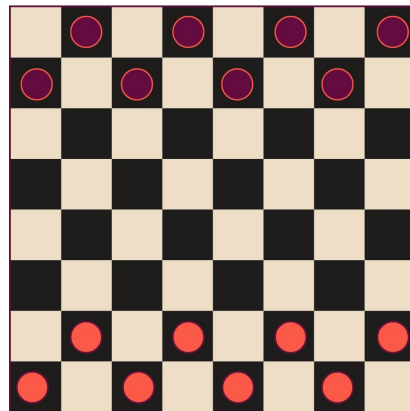
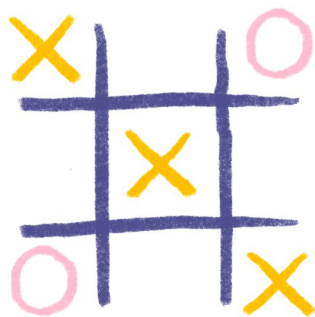
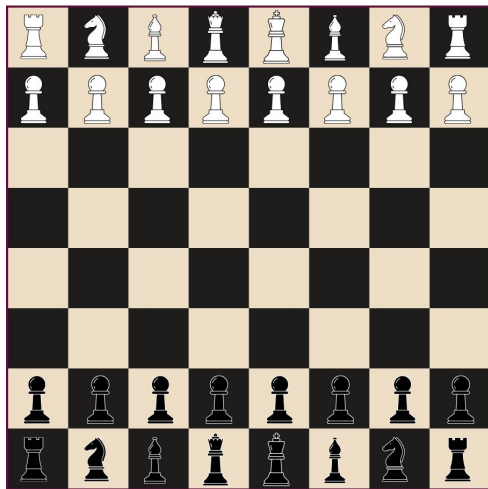
**Gigel**

# Joc cu informație completă

Există mai mulți jucători care efectuează mutări alternativ și cunosc obiectivele celuilalt jucător în fiecare moment.

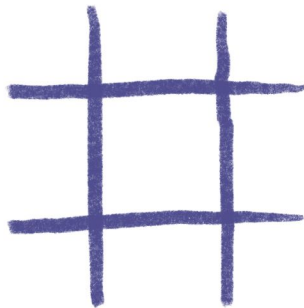


Jocuri cu 2 jucători, informație completă, perfectă și sumă 0



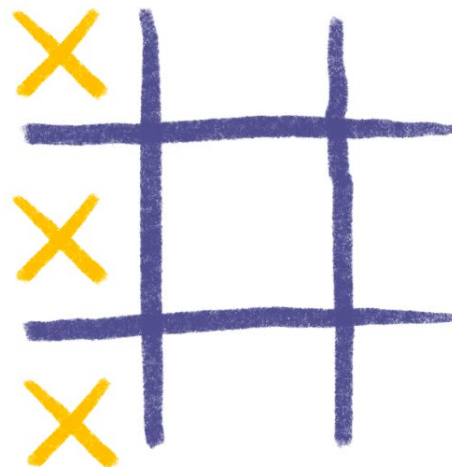
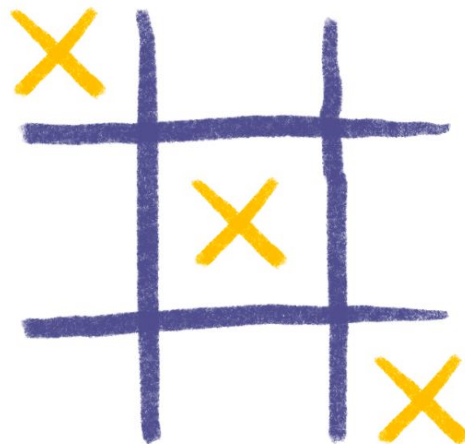
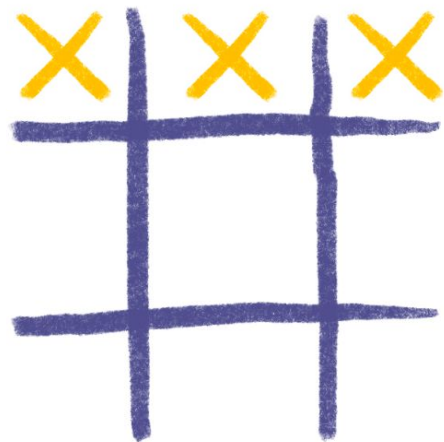


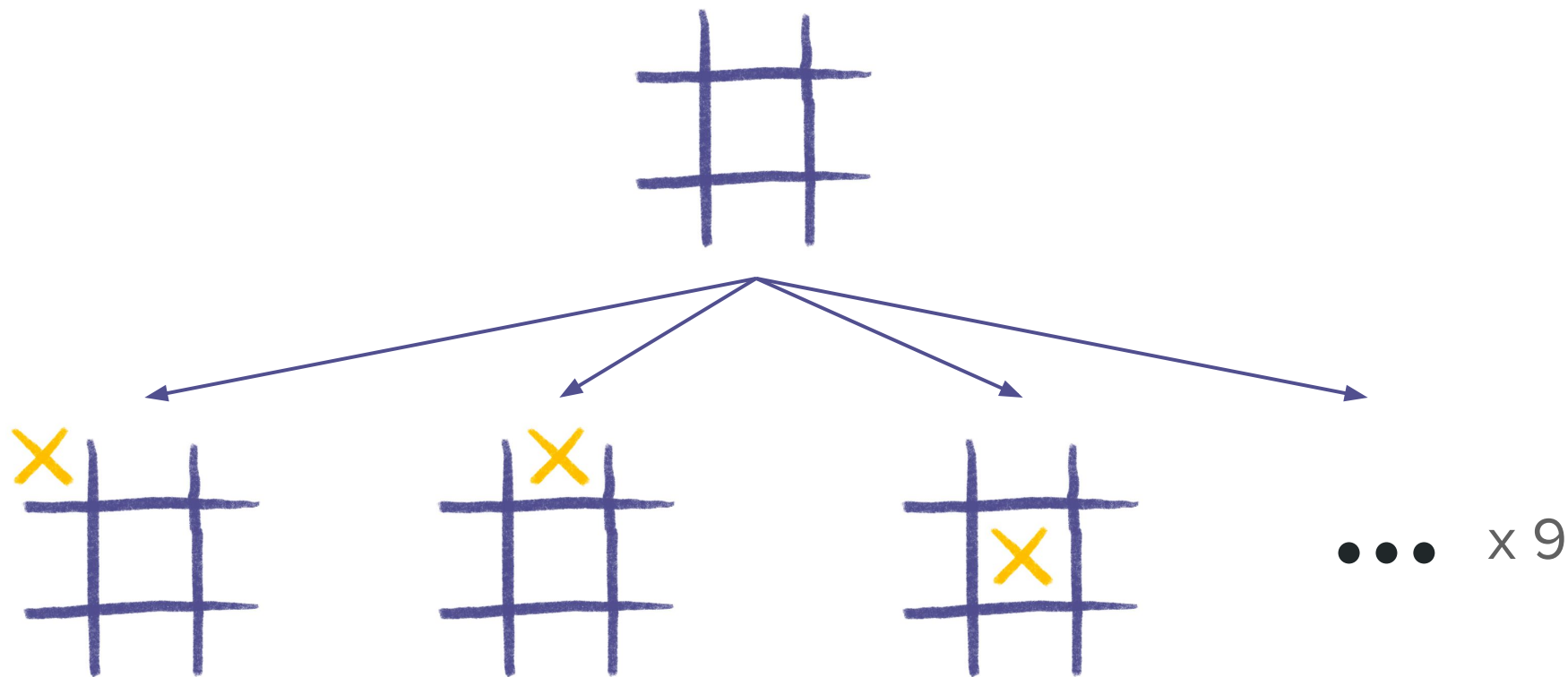
**Maria**



**Gigel**

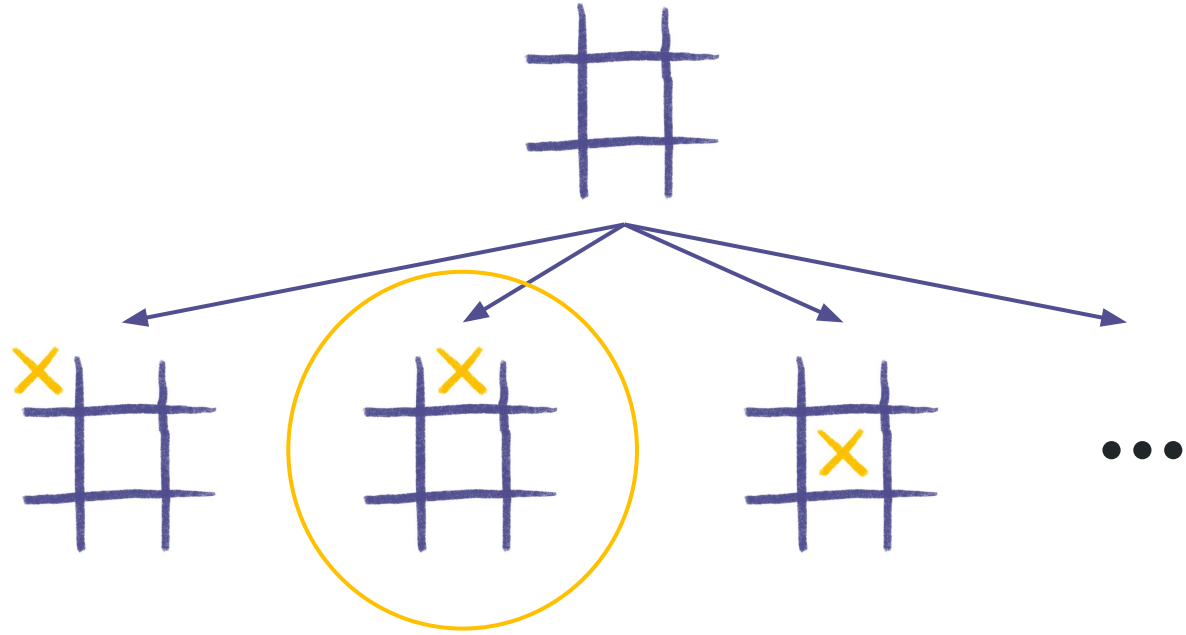






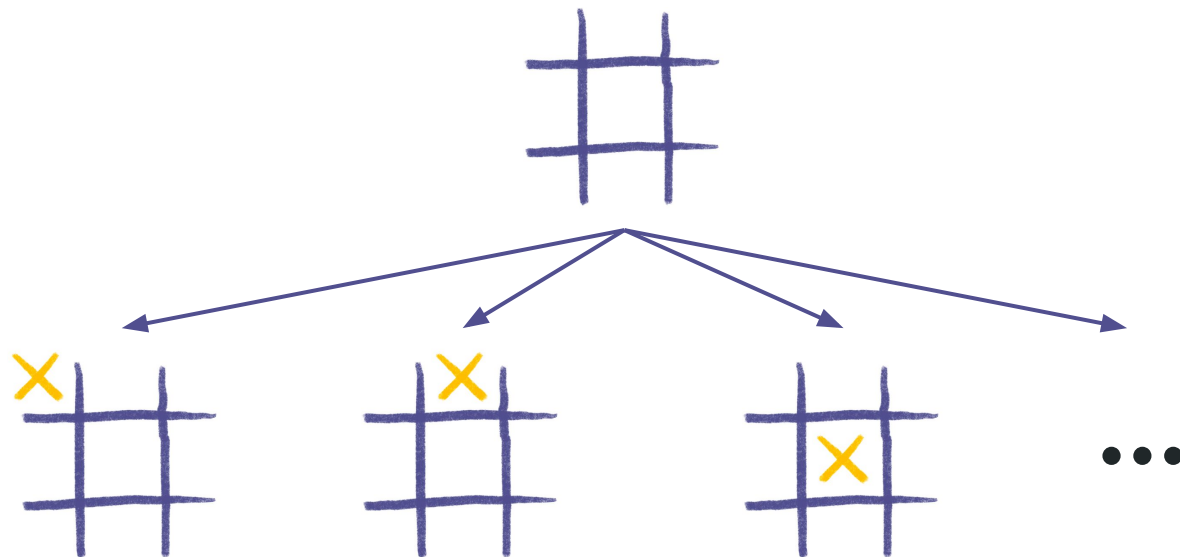


**Gigel**

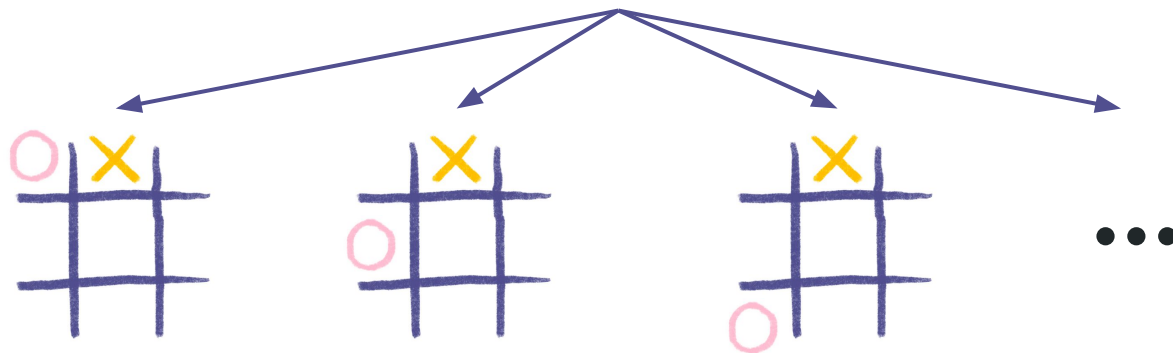




**Gigel**

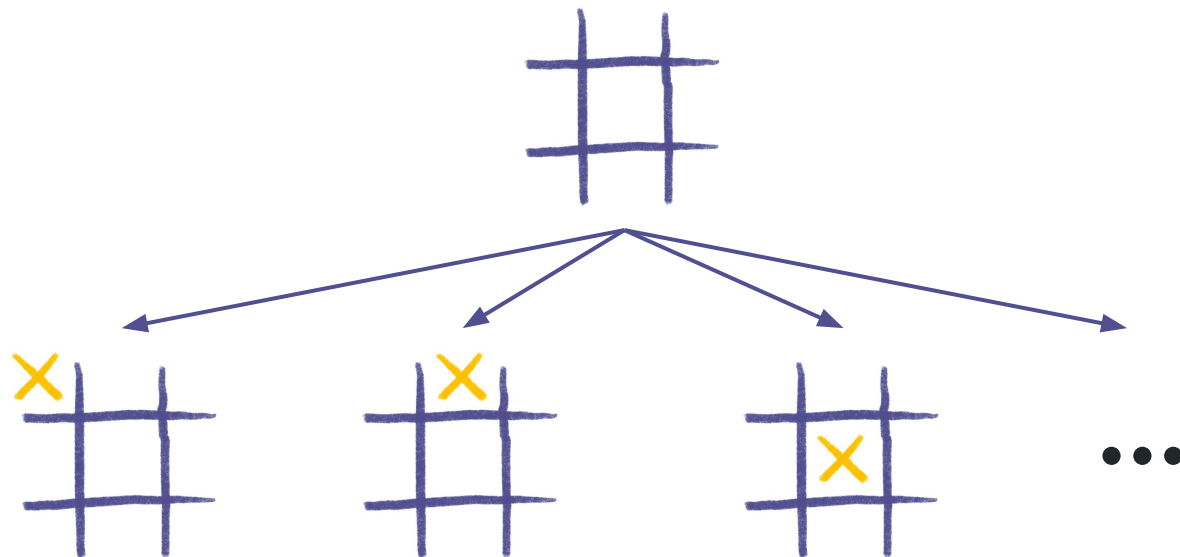


**Maria**

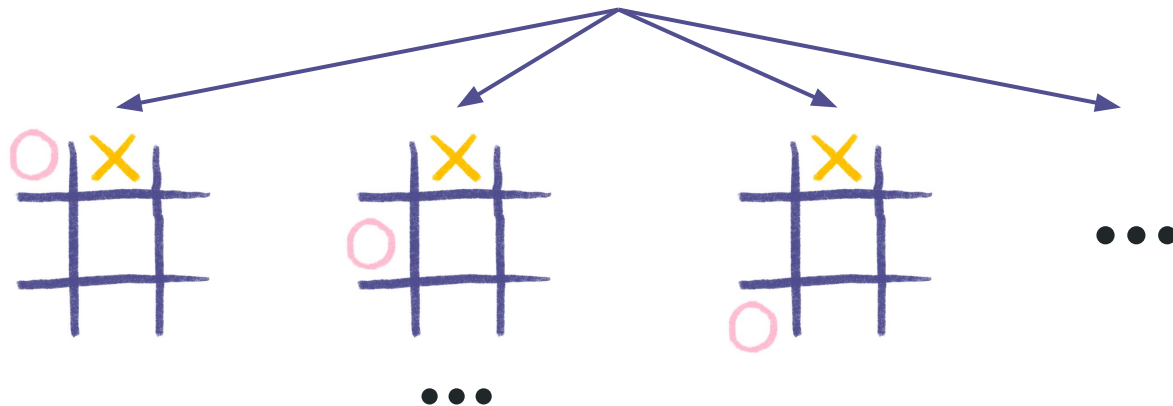




Gigel



Maria



# Cum măsurăm utilitatea?

Utilitatea = valoarea euristică a unui nod

= cât de mult îl apropie de câștig pe jucătorul MAX

# Exemplu pentru X&O:

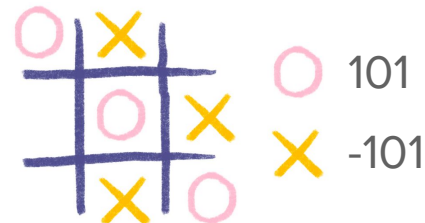
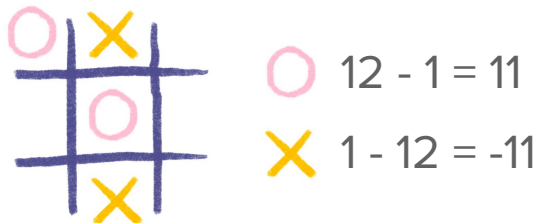
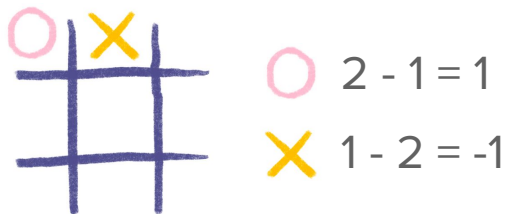
Diferența între scorul jucătorului curent și scorul jucătorului opus, unde scorul unui jucător este:

+ 100 pentru 3 în linie

+10 pentru fiecare 2 în linie (cu o căsuță liberă)

+1 pentru fiecare 1 în linie (cu 2 căsuțe libere)

0 altfel



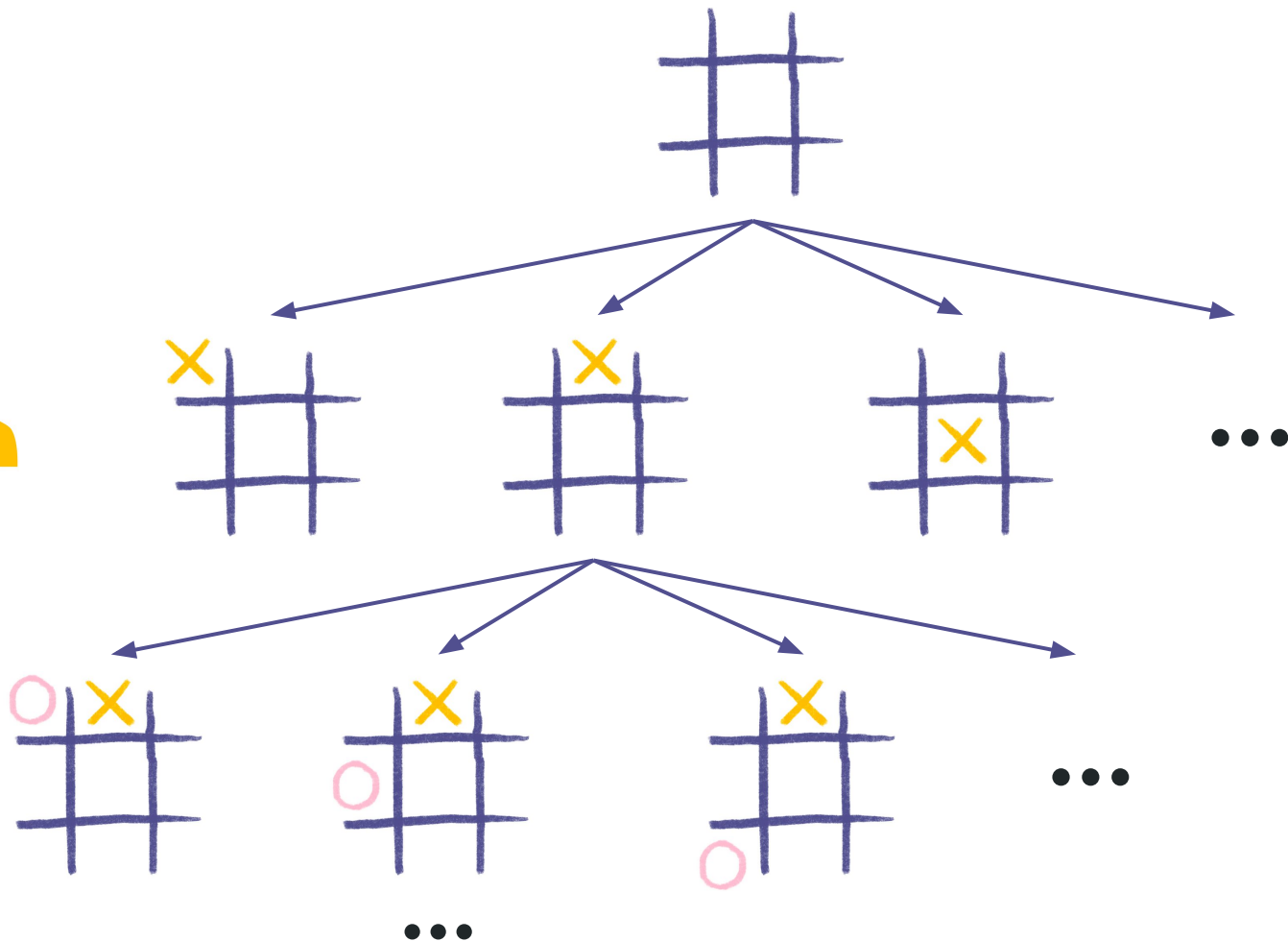
# MINMAX



Max

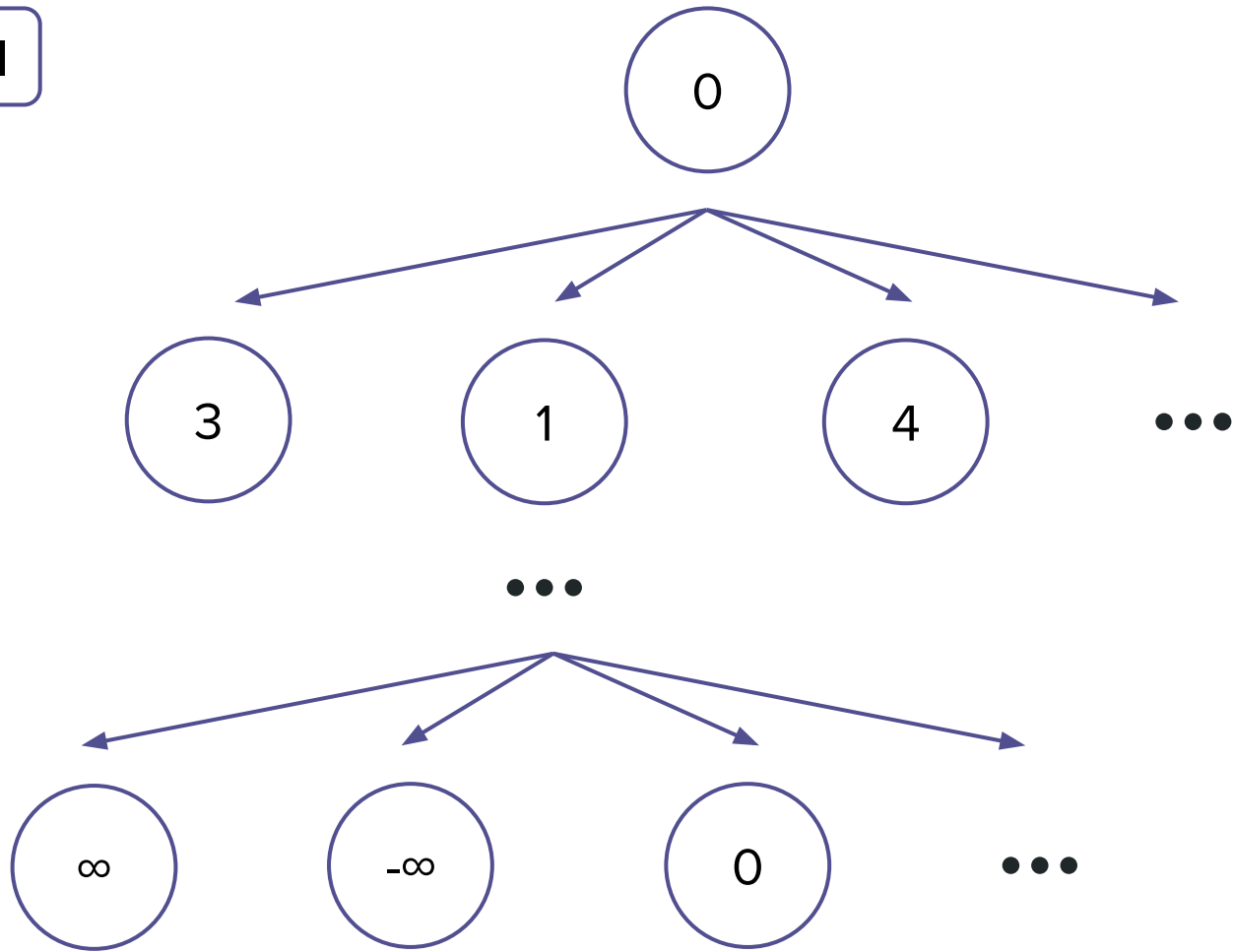


Min





# EURISTICI



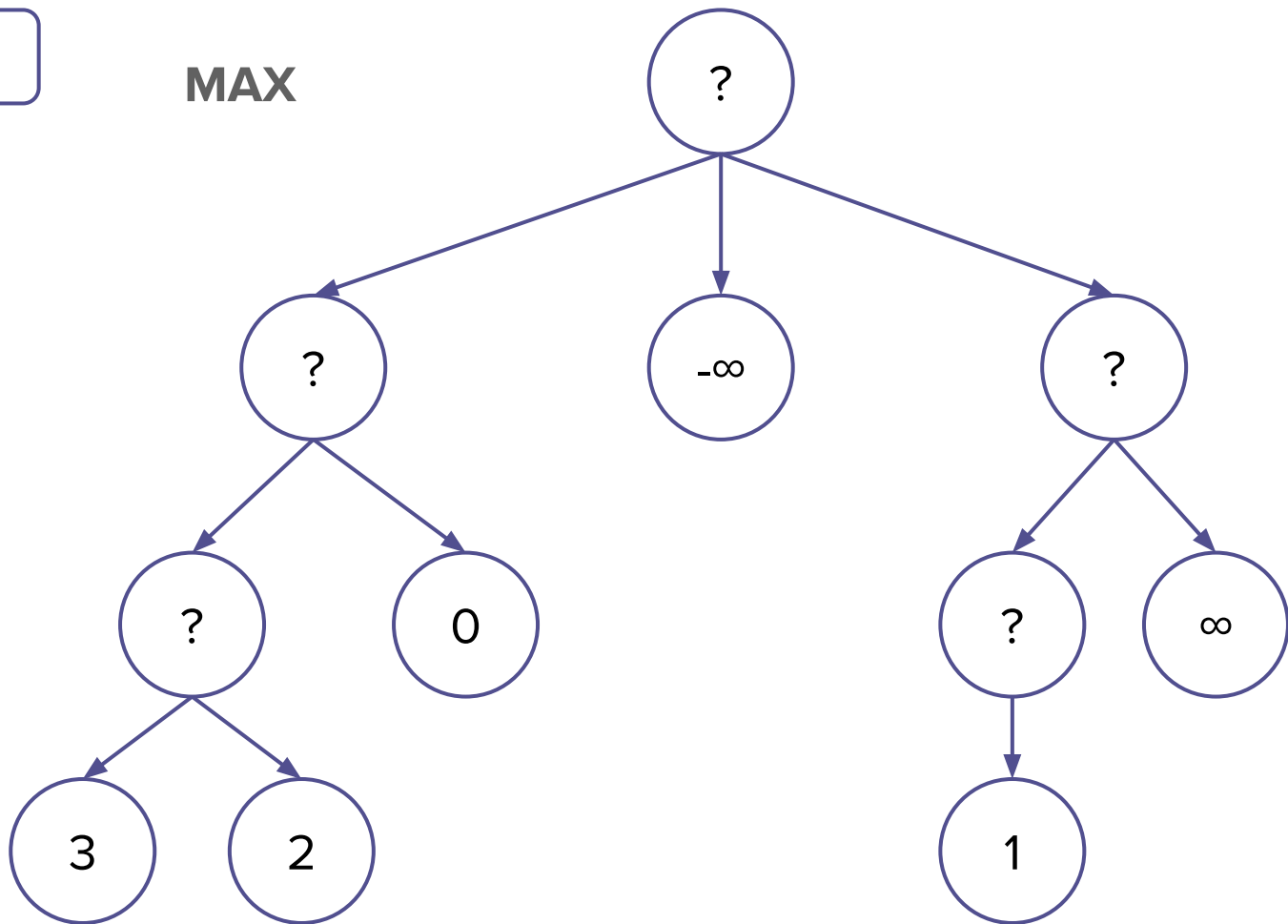
**MINMAX**

**MAX**

**MIN**

**MAX**

**MIN**



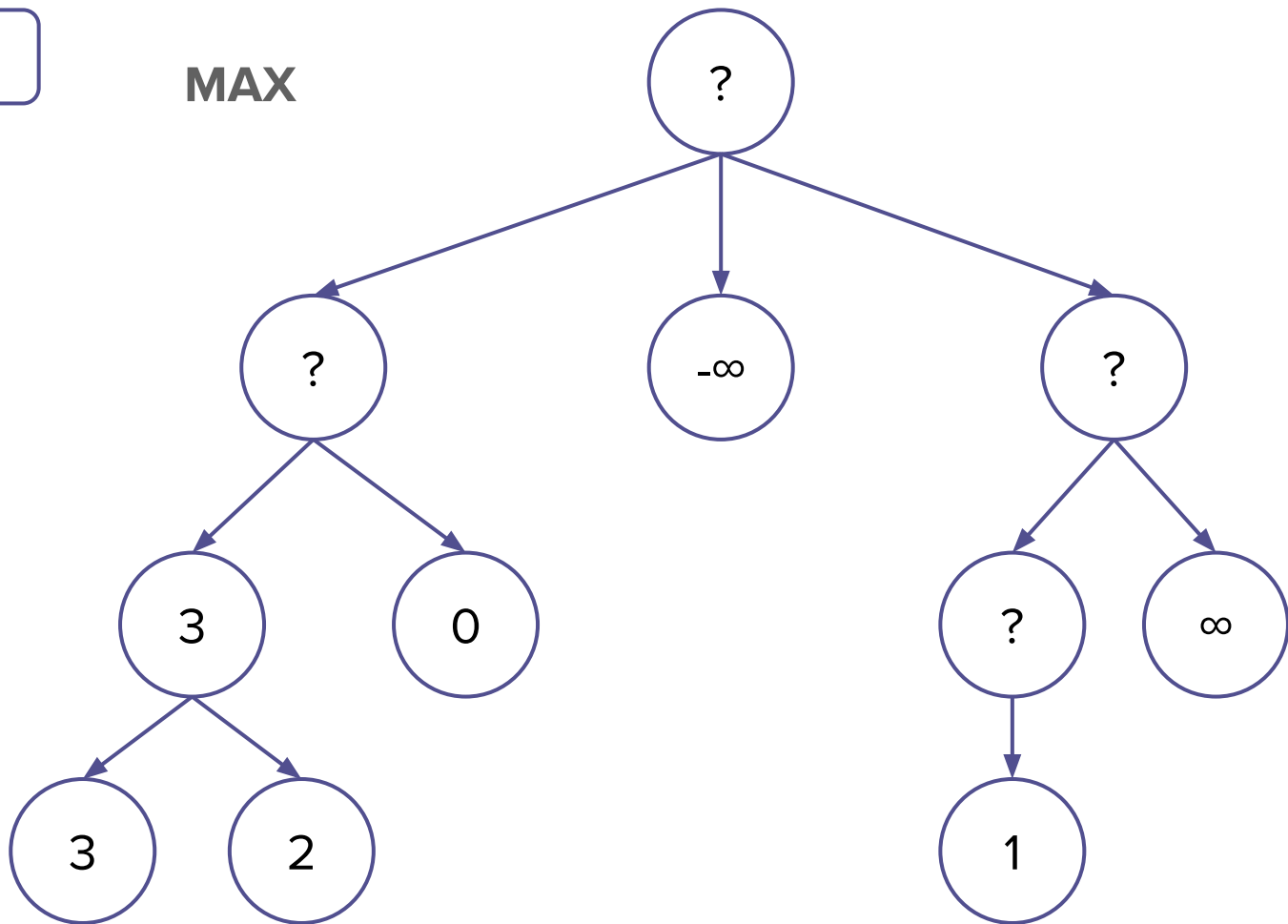
**MINMAX**

**MAX**

**MIN**

**MAX**

**MIN**



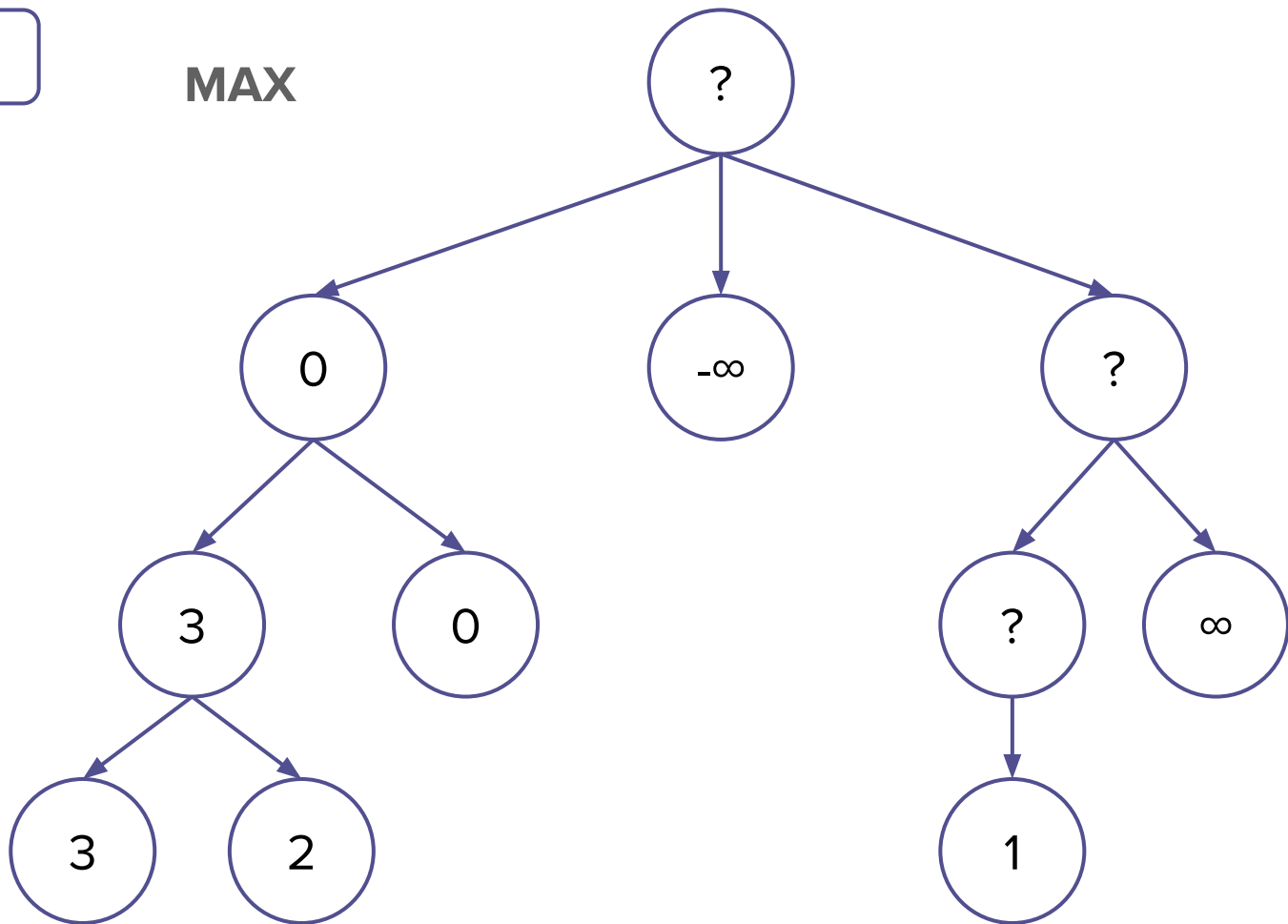
**MINMAX**

**MAX**

**MIN**

**MAX**

**MIN**



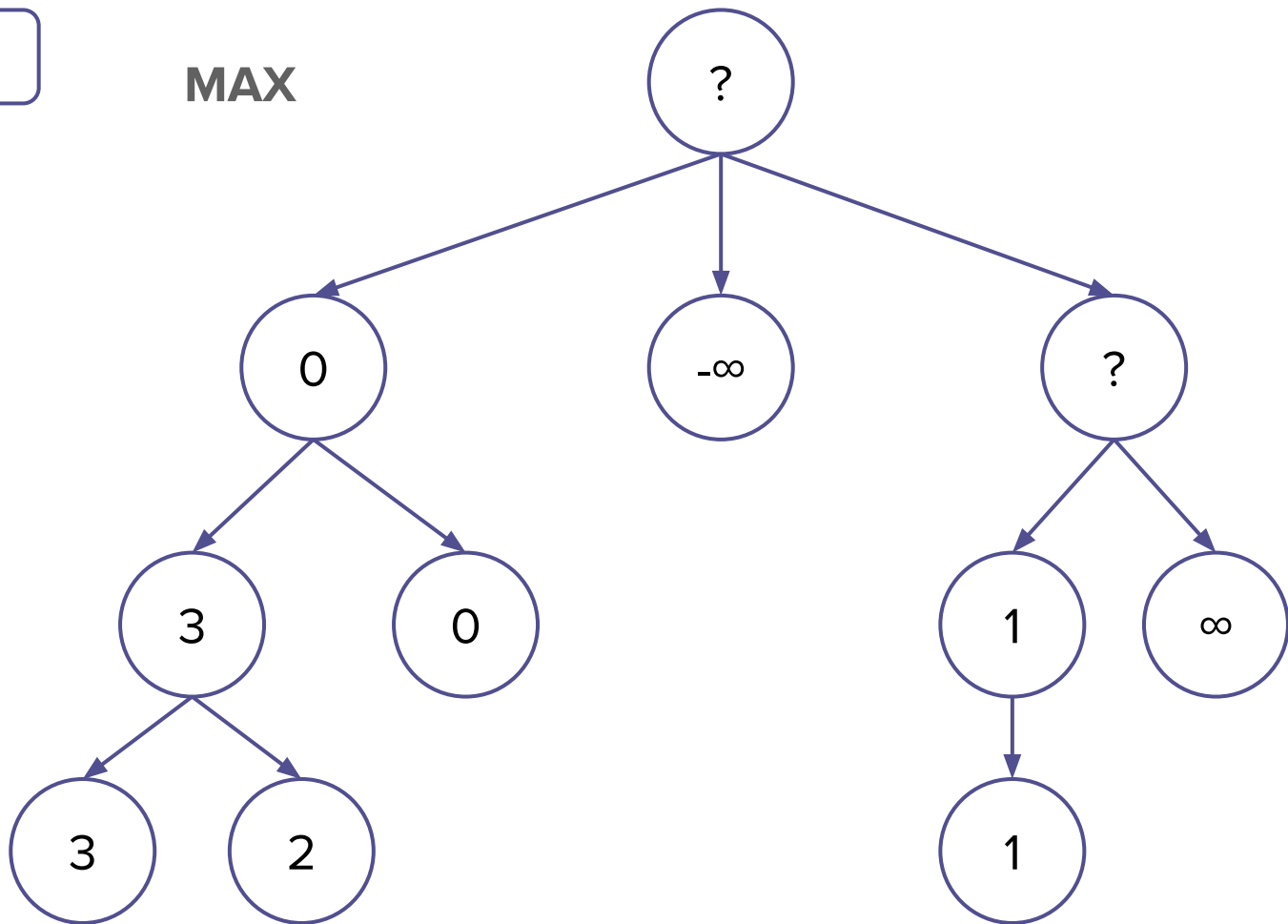
**MINMAX**

**MAX**

**MIN**

**MAX**

**MIN**



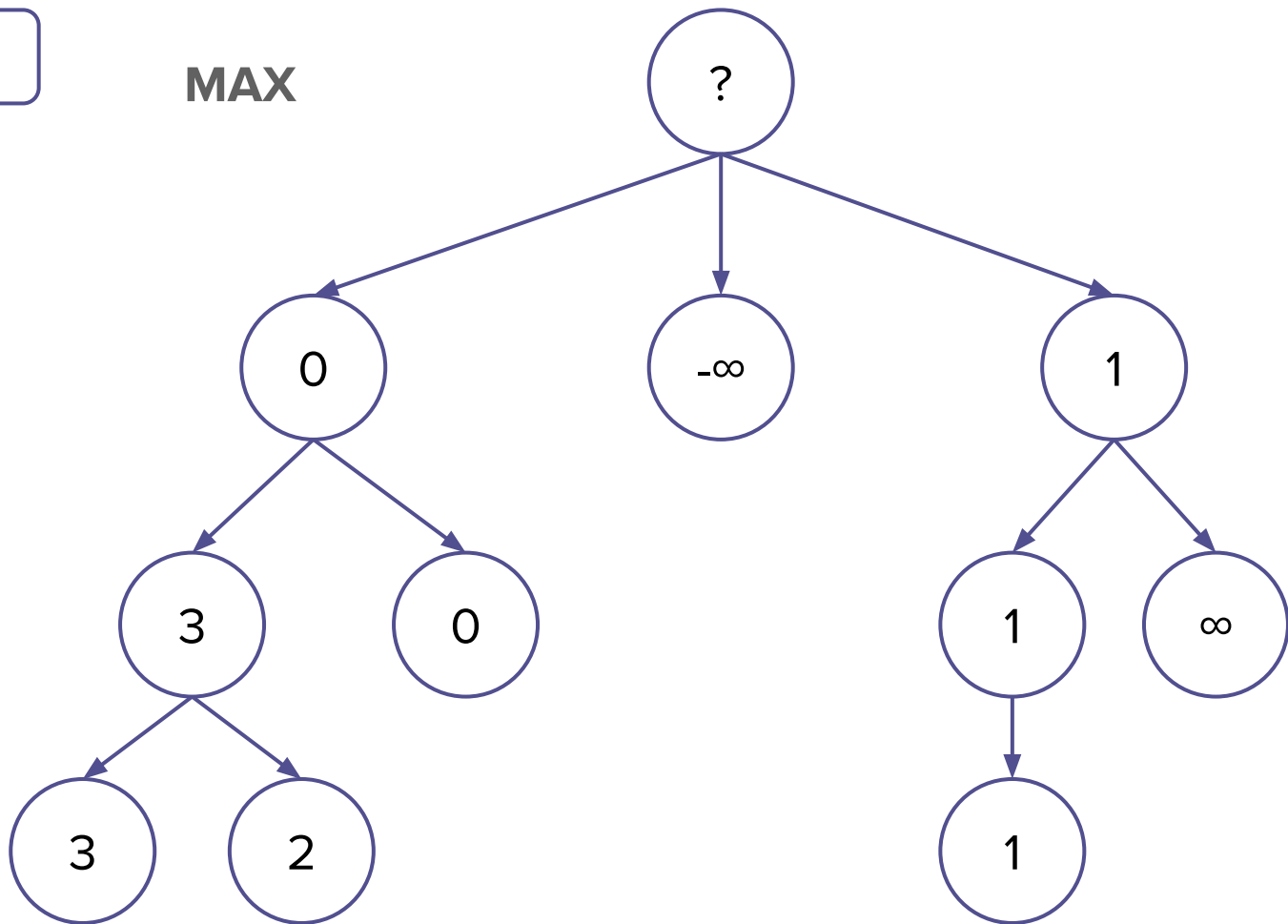
**MINMAX**

**MAX**

**MIN**

**MAX**

**MIN**



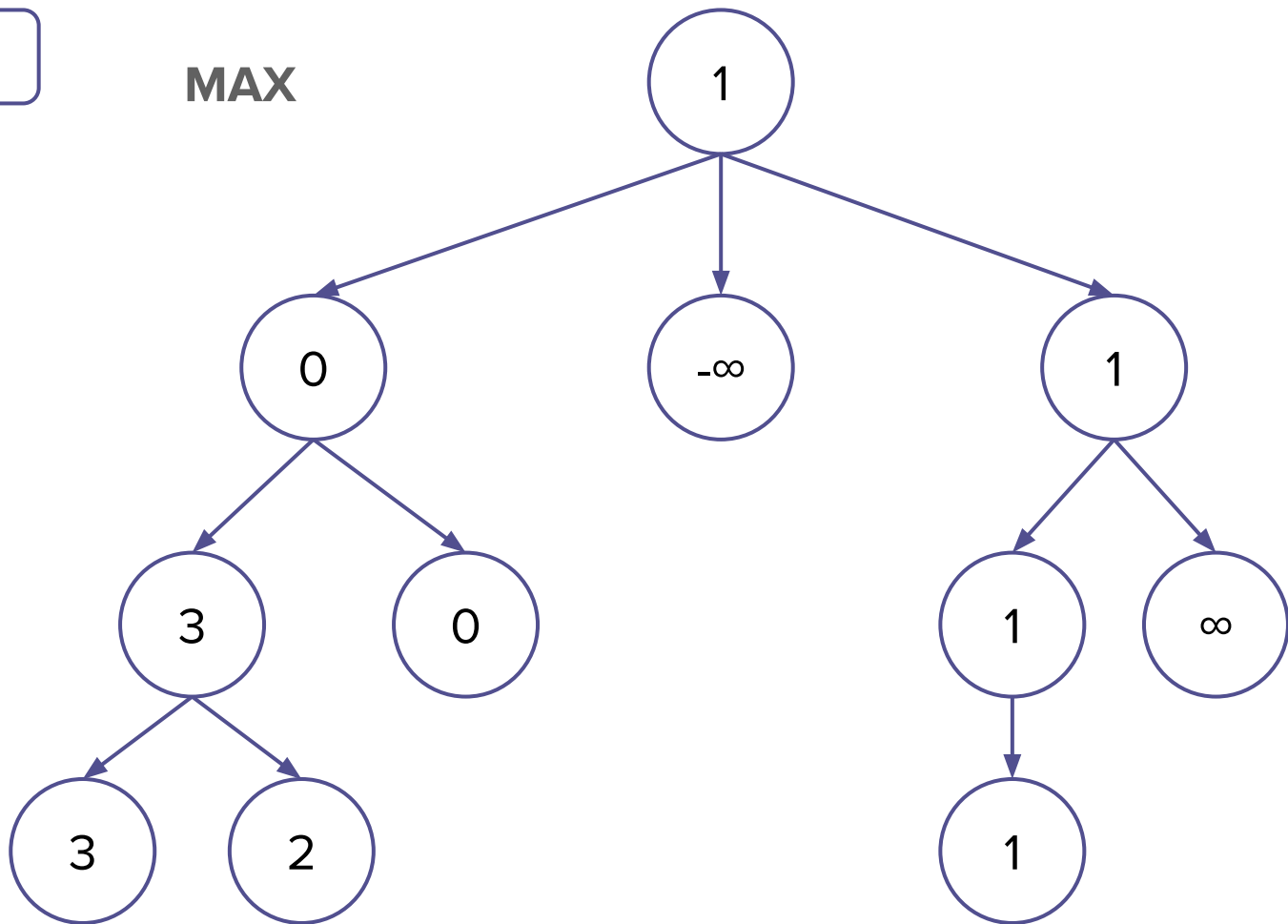
# MINMAX

MAX

MIN

MAX

MIN



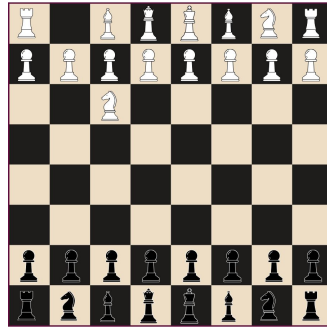
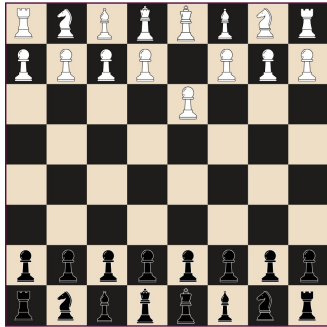
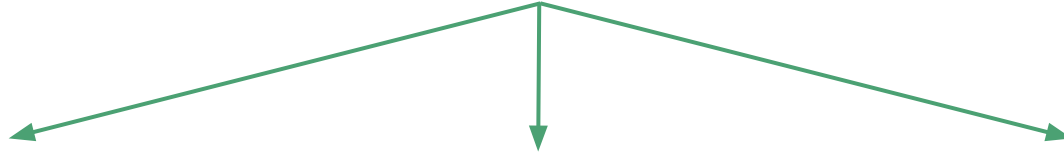
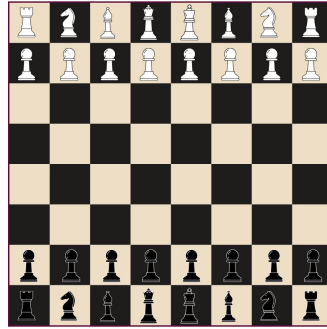
# Pseudocod Minmax

```
function minimax(node, depth, maximizingPlayer) is  
  if depth = 0 or node is a terminal node then  
    return the heuristic value of node  
  if maximizingPlayer then  
    value :=  $-\infty$   
    for each child of node do  
      value := max(value, minimax(child, depth - 1, FALSE))  
    return value  
  else (* minimizing player *)  
    value :=  $+\infty$   
    for each child of node do  
      value := min(value, minimax(child, depth - 1, TRUE))  
  return value
```



# Alpha-Beta Pruning

---



... x 20



**Min**



**Max**

# Deep Blue vs. Kasparov (1996)



# ALPHA-BETA

MAX

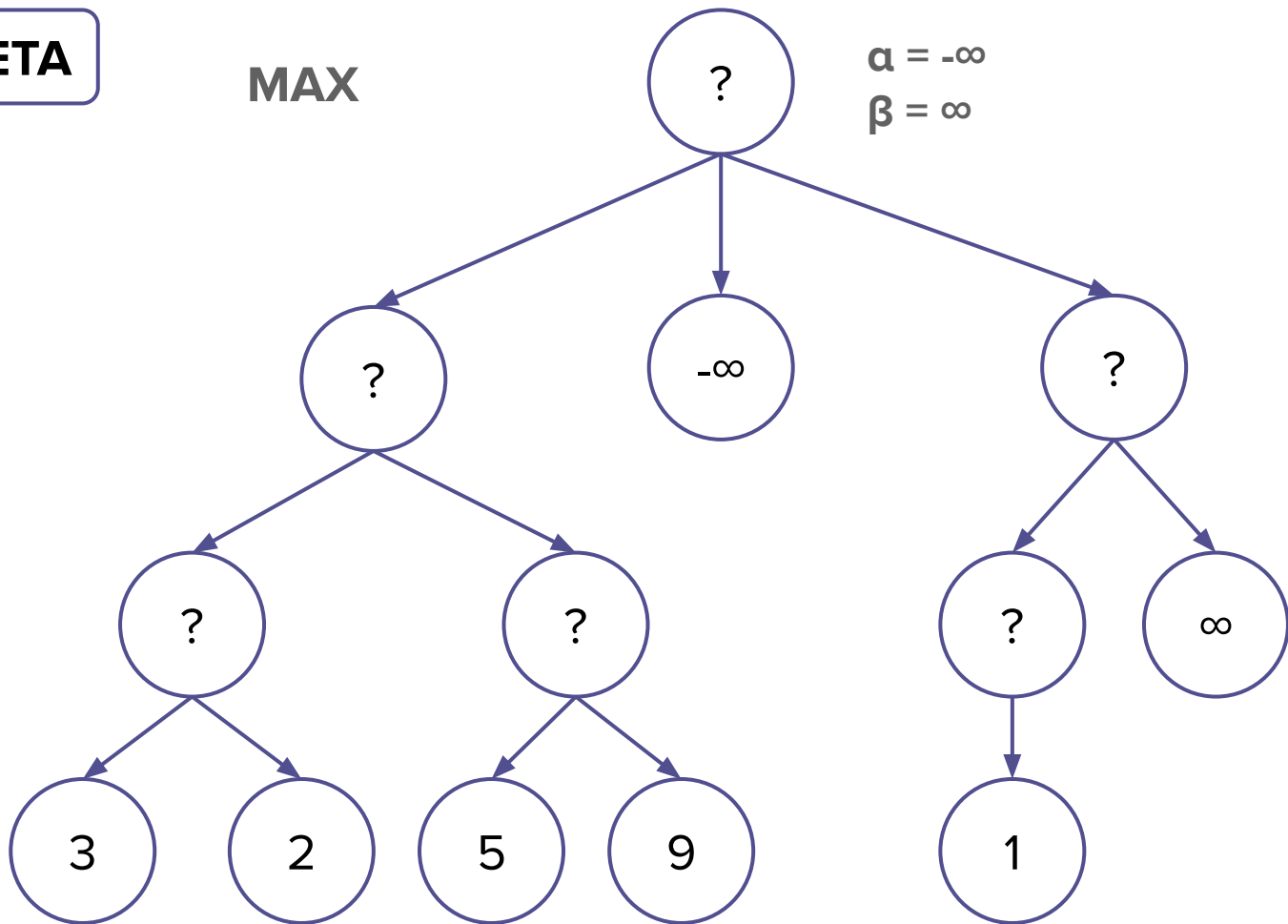
$\alpha = -\infty$

$\beta = \infty$

MIN

MAX

MIN



# ALPHA-BETA

MAX

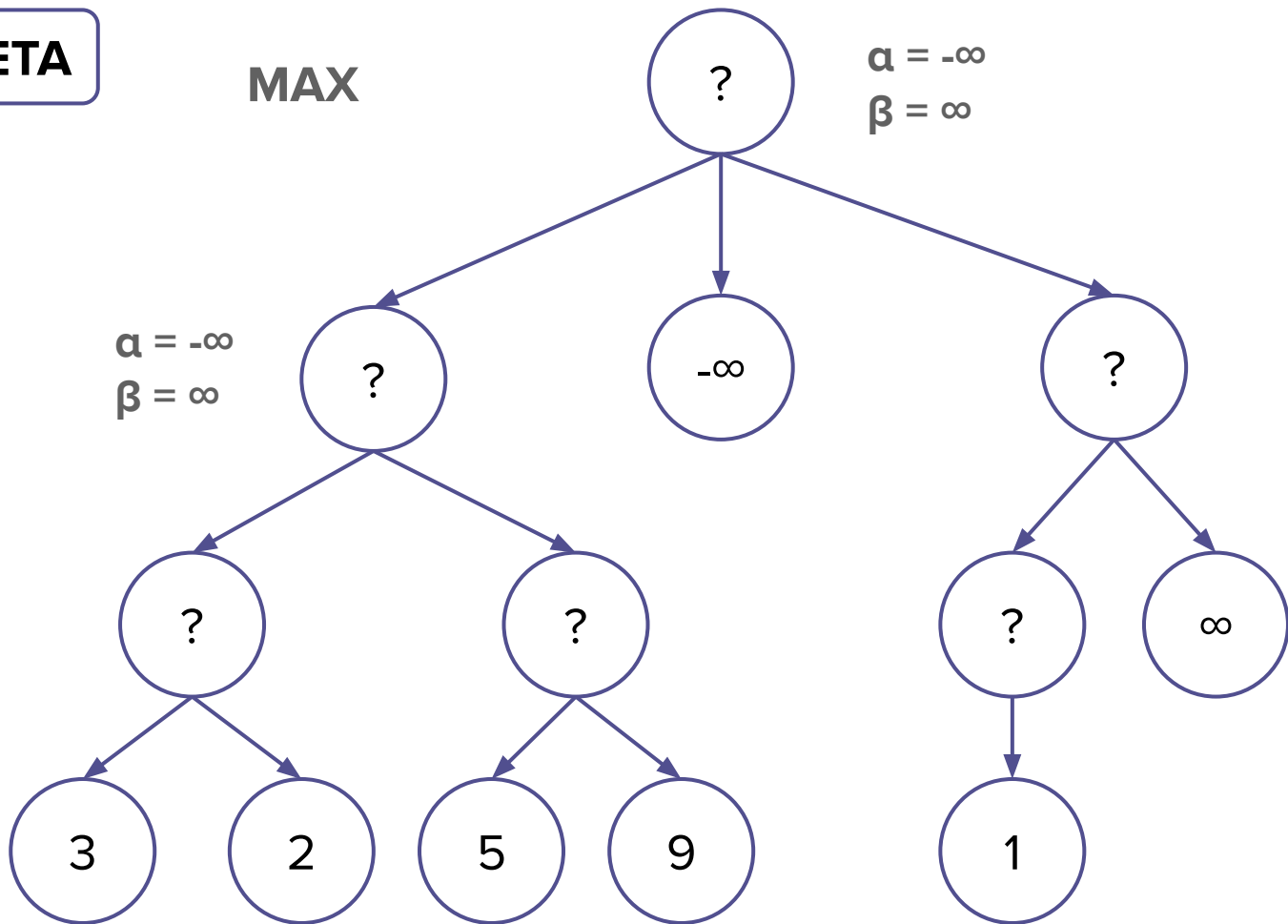
$\alpha = -\infty$   
 $\beta = \infty$

MIN

$\alpha = -\infty$   
 $\beta = \infty$

MAX

MIN



# ALPHA-BETA

MAX

$\alpha = -\infty$   
 $\beta = \infty$

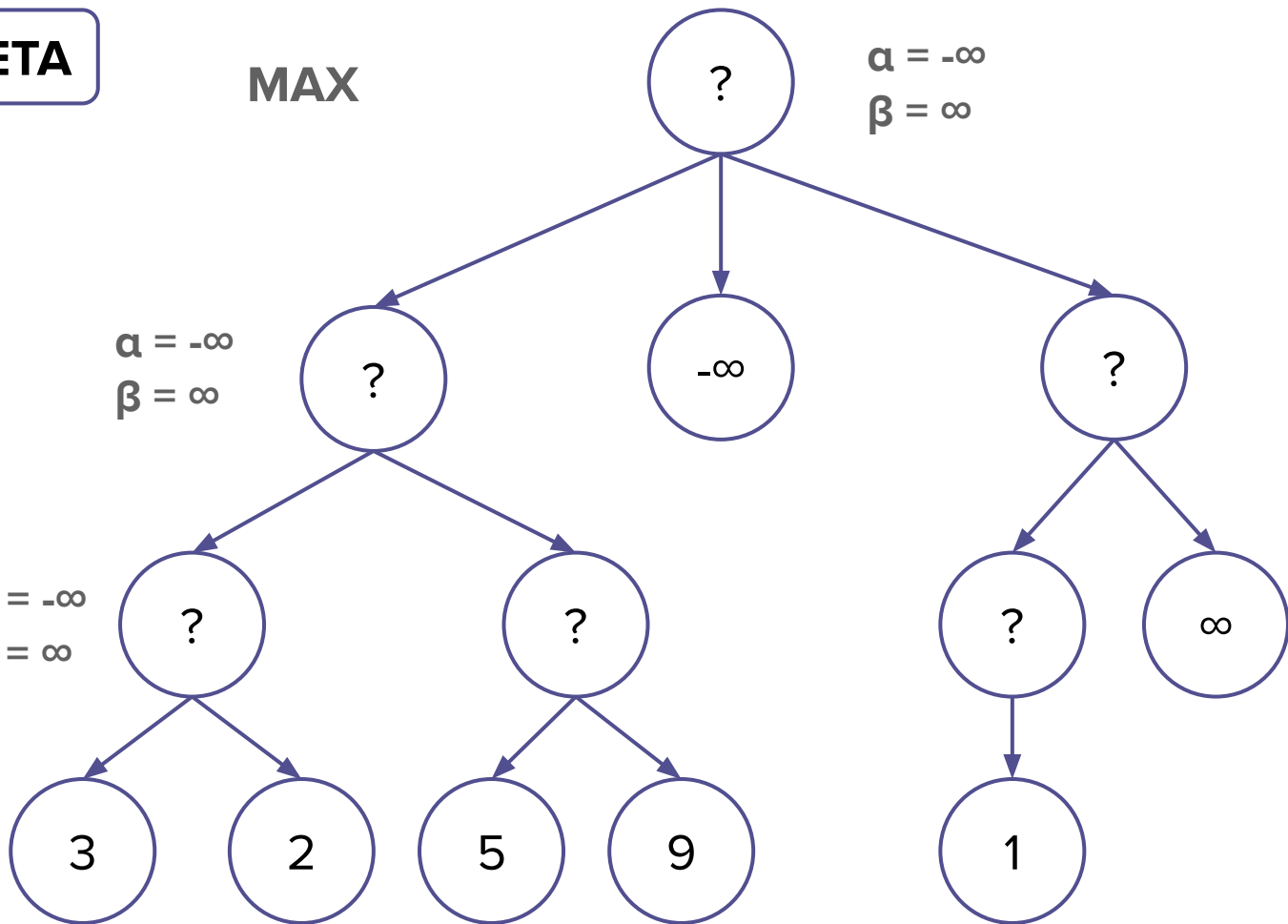
MIN

$\alpha = -\infty$   
 $\beta = \infty$

MAX

$\alpha = -\infty$   
 $\beta = \infty$

MIN



# ALPHA-BETA

MAX

$\alpha = -\infty$   
 $\beta = \infty$

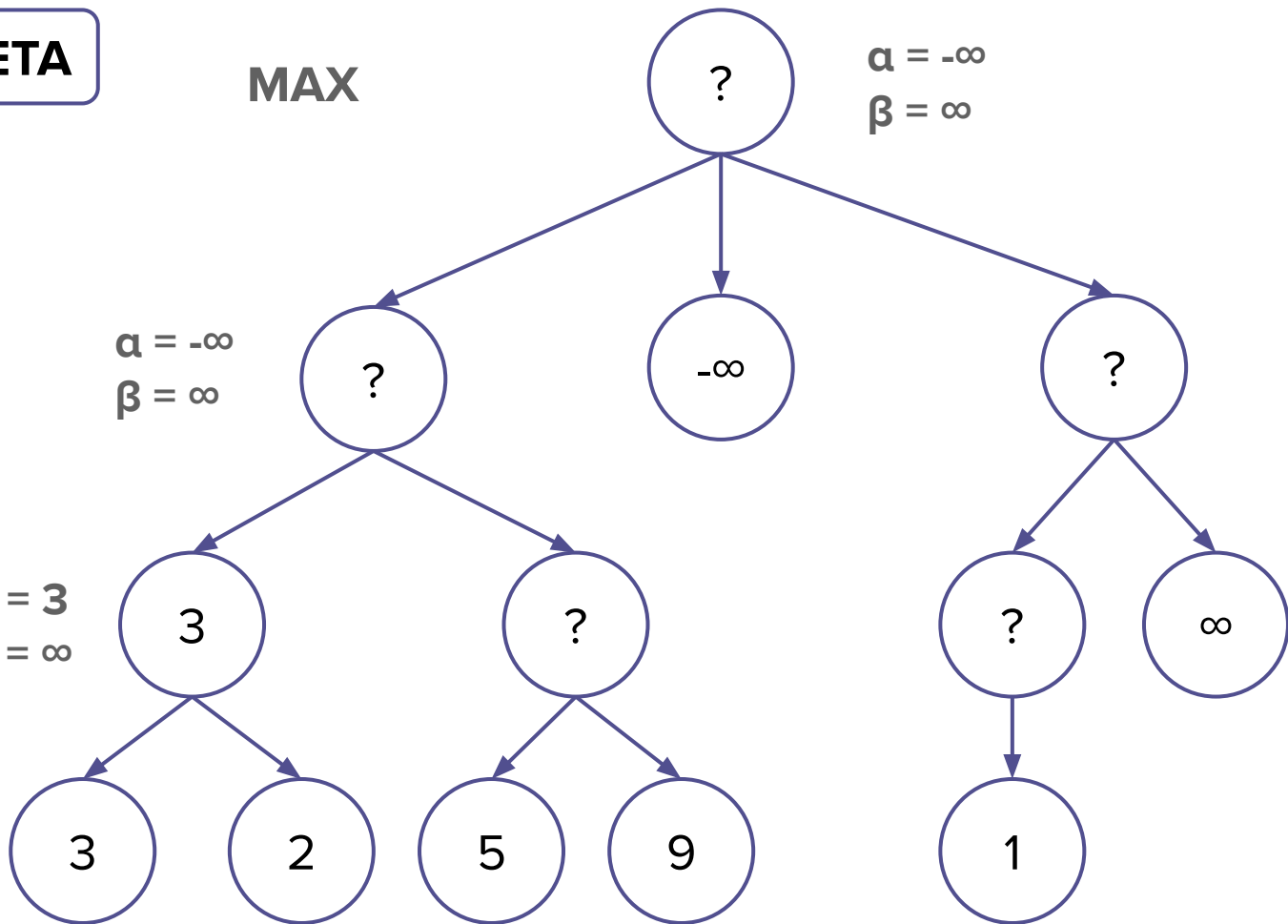
MIN

$\alpha = -\infty$   
 $\beta = \infty$

MAX

$\alpha = 3$   
 $\beta = \infty$

MIN





# ALPHA-BETA

MAX

$\alpha = -\infty$   
 $\beta = \infty$

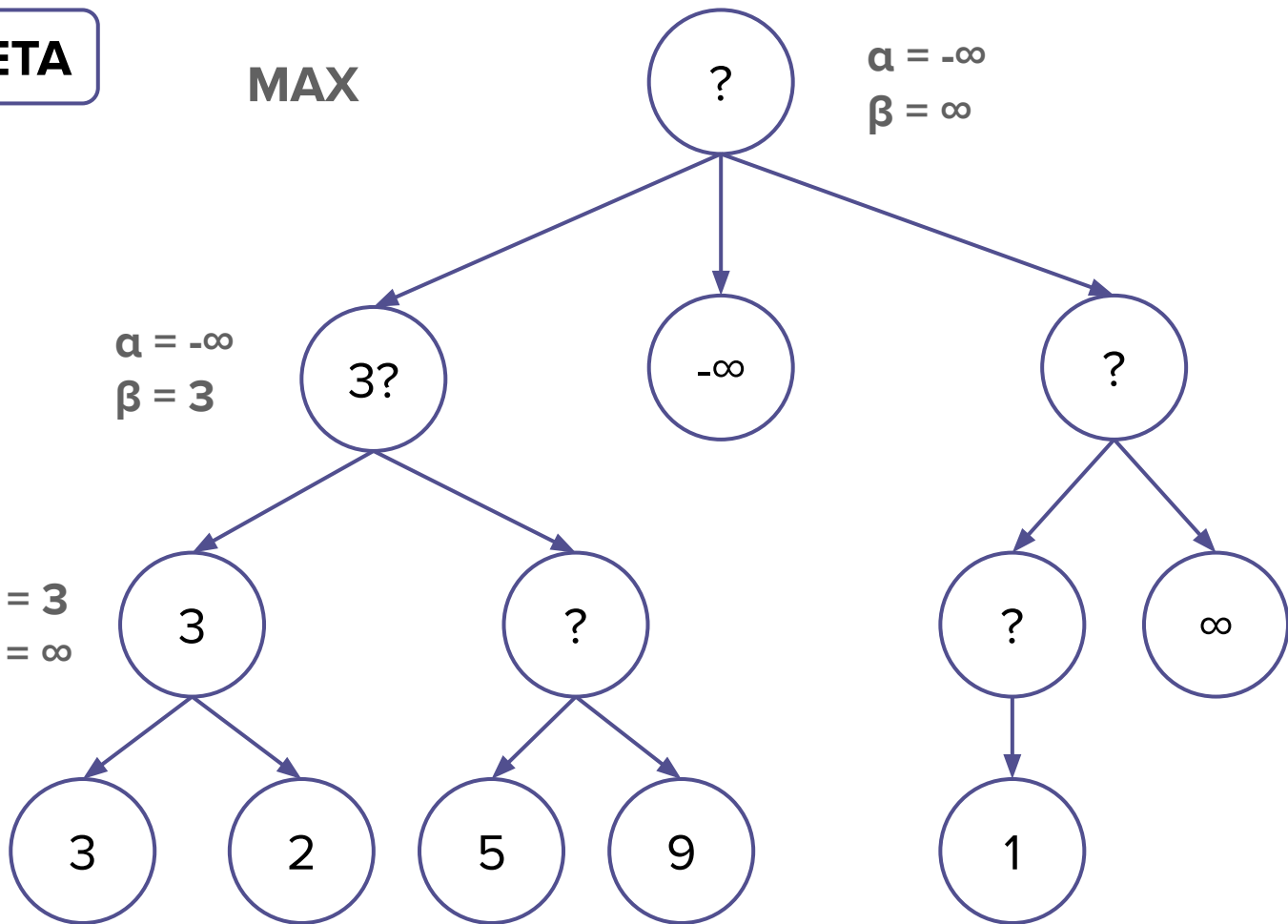
MIN

$\alpha = -\infty$   
 $\beta = 3$

MAX

$\alpha = 3$   
 $\beta = \infty$

MIN



# ALPHA-BETA

MAX

$\alpha = -\infty$   
 $\beta = \infty$

MIN

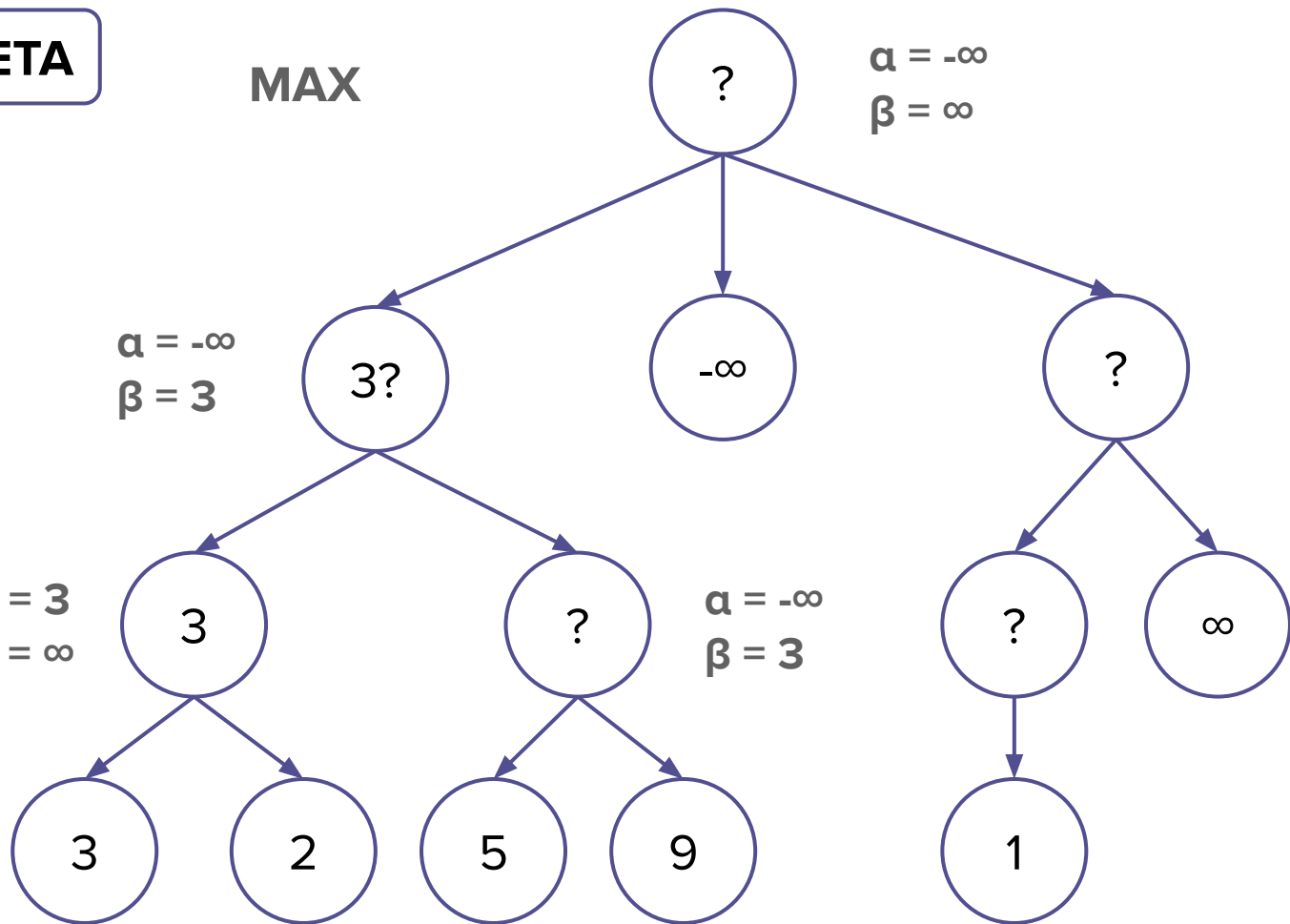
$\alpha = -\infty$   
 $\beta = 3$

MAX

$\alpha = 3$   
 $\beta = \infty$

$\alpha = -\infty$   
 $\beta = 3$

MIN



# ALPHA-BETA

MAX

$\alpha = -\infty$   
 $\beta = \infty$

MIN

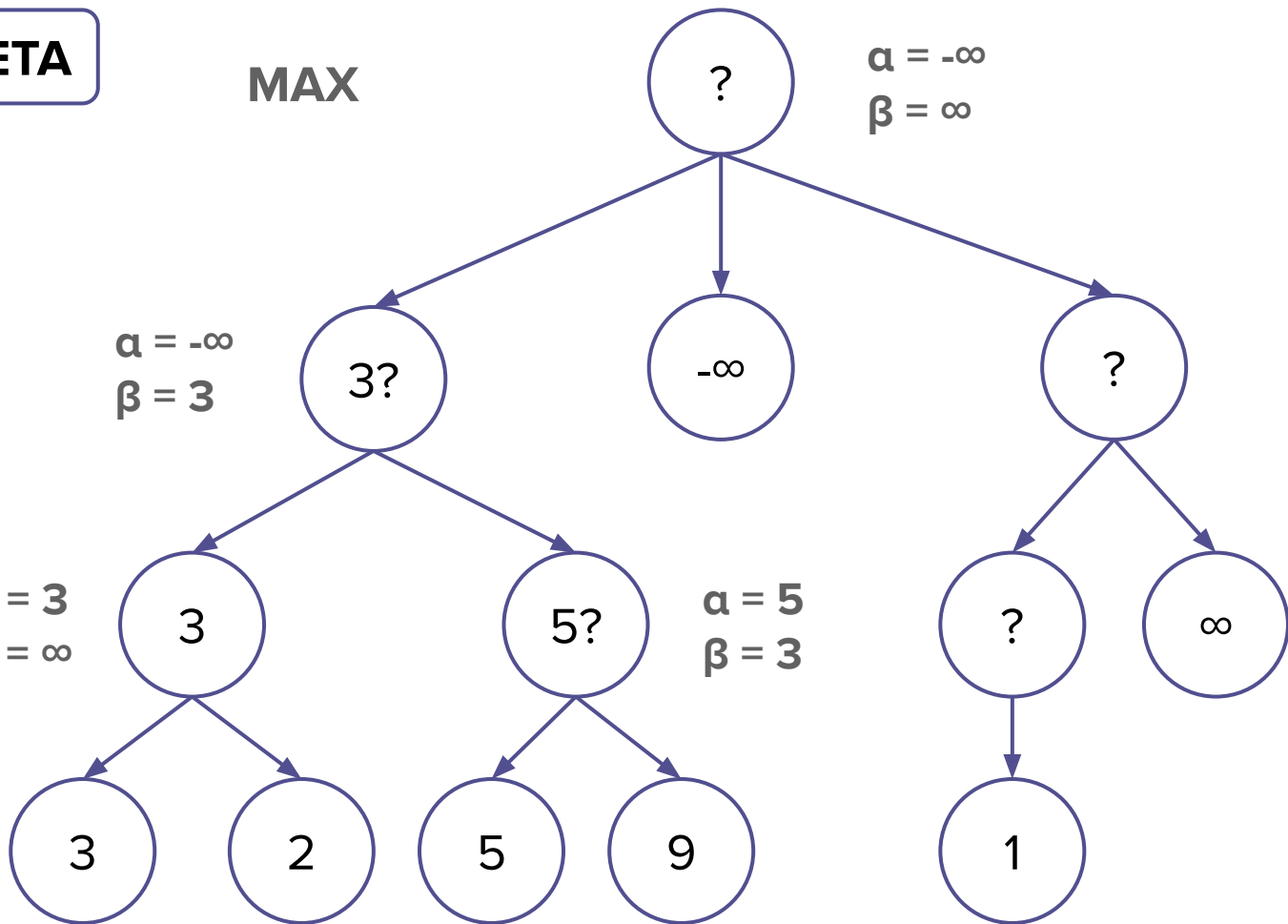
$\alpha = -\infty$   
 $\beta = 3$

MAX

$\alpha = 3$   
 $\beta = \infty$

$\alpha = 5$   
 $\beta = 3$

MIN



# ALPHA-BETA

MAX

$\alpha = -\infty$   
 $\beta = \infty$

MIN

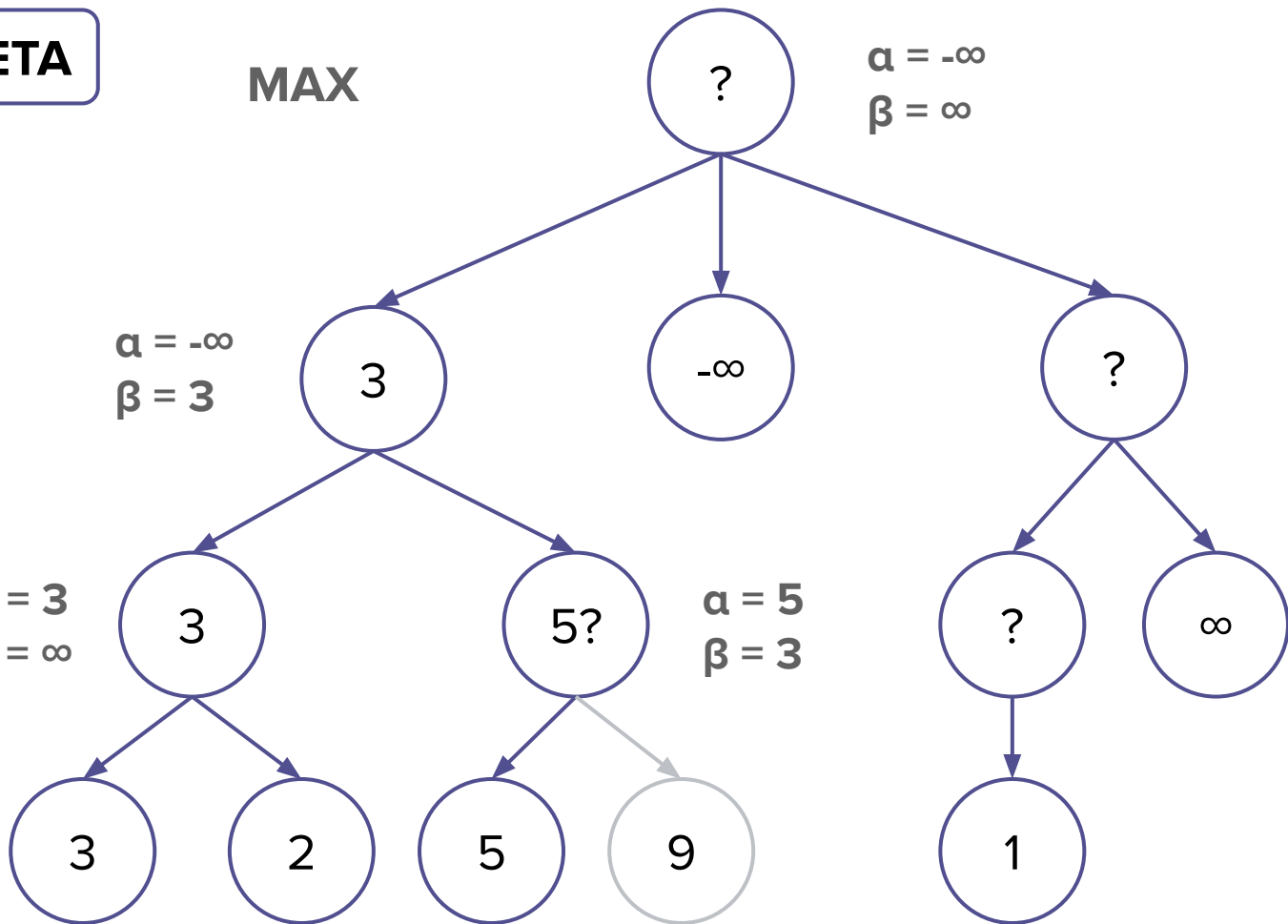
$\alpha = -\infty$   
 $\beta = 3$

MAX

$\alpha = 3$   
 $\beta = \infty$

$\alpha = 5$   
 $\beta = 3$

MIN



# ALPHA-BETA

MAX

$\alpha = 3$   
 $\beta = \infty$

MIN

$\alpha = -\infty$   
 $\beta = 3$

$\alpha = 3$   
 $\beta = 1$

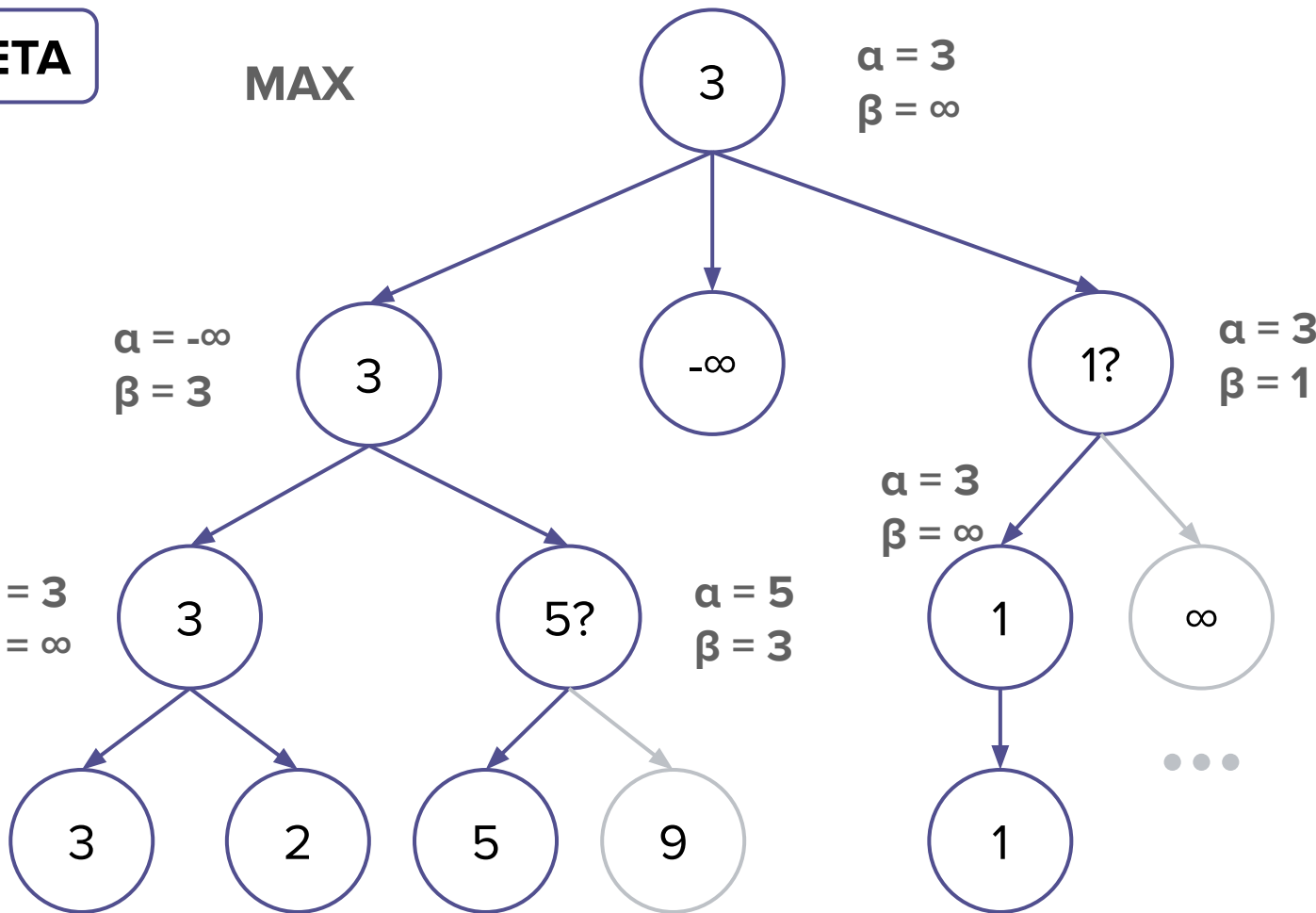
MAX

$\alpha = 3$   
 $\beta = \infty$

$\alpha = 5$   
 $\beta = 3$

$\alpha = 3$   
 $\beta = \infty$

MIN



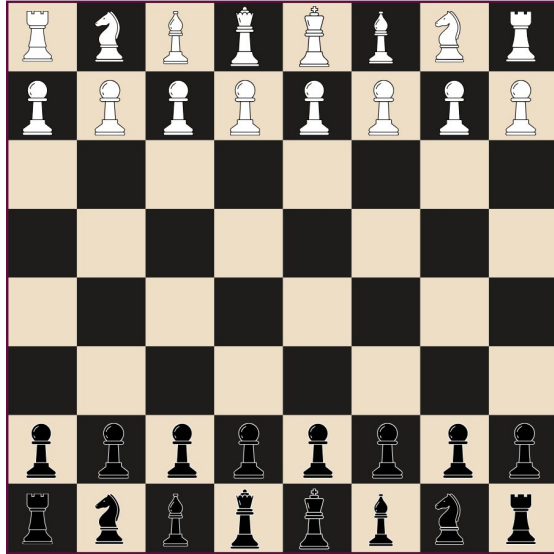
# Pseudocod Alpha-Beta Prunning

```
function alphabeta(node, depth,  $\alpha$ ,  $\beta$ , maximizingPlayer) is
  if depth == 0 or node is terminal then
    return the heuristic value of node
  if maximizingPlayer then
    value :=  $-\infty$ 
    for each child of node do
      value := max(value, alphabeta(child, depth - 1,  $\alpha$ ,  $\beta$ , FALSE))
      if value >  $\beta$  then
        break (*  $\beta$  cutoff *)
     $\alpha$  := max( $\alpha$ , value)
  return value
else
  value :=  $+\infty$ 
  for each child of node do
    value := min(value, alphabeta(child, depth - 1,  $\alpha$ ,  $\beta$ , TRUE))
    if value <  $\alpha$  then
      break (*  $\alpha$  cutoff *)
   $\beta$  := min( $\beta$ , value)
return value
```

# Q&A

---

Mulțumesc pentru atenție!







**Maria**



**Gigel**



**Maria**



**Gigel**

# Mai multe variații

- 3-player Minmax
- $\text{Max}^n$
- Multiplayer Alpha-Beta Prunning
- Monte Carlo Tree Search
- 0-sum 3-person game
- Expectiminimax:
  - Pentru jocuri cu informatie incompleta (ex: dat cu zarul, move by nature)
  - Environmentul nu are niciun scop / beneficiu în jocul nostru, așadar nu poate juca optim
  - Pentru mutarea lui algoritmul presupune media probabilităților peste toți fiii posibili